

## Discretionary Note

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**IF YOU USE THIS FILE TO CHEAT, YOU ARE NOT ONLY STUPID BUT YOU ARE CHEATING YOURSELF OUT OF THE ABILITY TO FALL IN LOVE WITH MATH.** Furthermore, I am not smarter than you and my solutions did not always get a perfect score.

**CONTENT STARTS ON NEXT PAGE.**

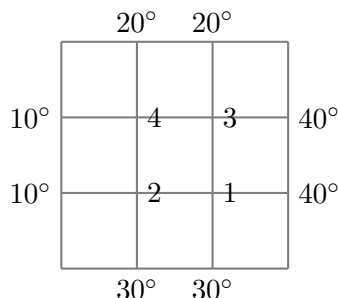
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## Math 226- HW 7 Due: Oc 29 by Midnight

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1. An important concern in the study of heat transfer is to determine the steady-state temperature distribution of a thin plate when the temperature around the boundary is known. Assume the plate shown in the figure represents a cross section of a metal beam, with negligible heat flow in the direction perpendicular to the plate. Let  $T_1, T_2, T_3, T_4$  denote the temperatures at the four interior nodes of the mesh in the figure. The temperature at a node is approximately equal to the average of the four nearest nodes- to the left, above, to the right, and below. For instance  $T_1 = (40 + 30 + T_3 + T_2)/4$



- (4 points) Write a system of four equations whose solution gives estimates for the temperatures  $T_1, T_2, T_3, T_4$
  - (6 points) Write down the coefficient matrix and find its inverse.
  - (2 points) Use part a) to give the estimated values of  $T_1, T_2, T_3, T_4$ .
2. (6 points) Find a basis of the kernel (that is  $N(L_A)$ ) of the linear mappings given by

$$A = \begin{pmatrix} 3 & 5 & -4 & 2 \\ 2 & 4 & -6 & 3 \\ 11 & 17 & -8 & 4 \end{pmatrix}$$

Use it to describe the solutions to

$$\begin{aligned} 3x_1 + 5x_2 - 4x_3 + 2x_4 &= 0 \\ 2x_1 + 4x_2 - 6x_3 + 3x_4 &= 0 \\ 11x_1 + 17x_2 - 8x_3 + 4x_4 &= 0 \end{aligned}$$

3. (12 points) Find all vectors in a space  $\mathbb{R}^4$ , whose image is equal to the vector  $b \in \mathbb{R}^3$  under the linear map  $\mathbb{R}^4 \rightarrow \mathbb{R}^3$  given by the matrix  $A$ . (That is find the solutions to  $A\vec{x} = \vec{b}$ ). Use these vectors to give a basis to  $N(L_A)$ .

$$A = \begin{pmatrix} 1 & -3 & -3 & -14 \\ 2 & -6 & -3 & -1 \\ 3 & -9 & -5 & -6 \end{pmatrix}, \quad b = \begin{pmatrix} 8 \\ -5 \\ -4 \end{pmatrix}$$

4. (10 points) Consider the map  $T : P_3(\mathbb{R}) \rightarrow \mathbb{R}^4$  such that  $T(f) = (f(-2), f(-1), f(1), f(2))$ .
- (5 points) Let  $\gamma = \{1, x, x^2, x^3\}$ , and  $\beta$  be the standard basis for  $\mathbb{R}^4$ . Compute  $[T]_{\gamma}^{\beta}$ .
  - (5 points) Use  $[T]_{\gamma}^{\beta}$  to find a third degree polynomial such that  $f(-2) = 1, f(-1) = 3, f(1) = 13, f(2) = 33$ .
5. (10 points) Read “An Interpretation of the Reduced Row Echelon Form” from the book (pg 189-194). Then answer the following question

$$V = \{(x_1, x_2, x_3, x_4, x_5) : x_1 - 2x_2 + 3x_3 - x_4 + 2x_5 = 0\}$$

Note that  $v = (1, 2, 1, 0, 0)$  is a vector in  $V$ . Use column correspondance to find a basis for  $V$  that includes  $v$ .

## Practice Problems

1. Let  $T, U : V \rightarrow W$  be linear transformations.
  - a) Prove that  $R(U + T) \subseteq R(T) + R(U)$ .
  - b) Prove that if  $W$  is finite dimensional, then  $\text{rank}(T + U) \leq \text{rank}(T) + \text{rank}(U)$
  - c) Deduce that  $\text{rank}(A + B) \leq \text{rank}(A) + \text{rank}(B)$  for any  $m \times n$  matrix.
2. Let  $A = [a_{ji}]_{n \times n}$ ,  $B = [b_{ji}]_{n \times n}$  be matrices. The trace of  $A$  is defined by

$$\text{tr}(A) = \sum_{i=1}^n a_{ii}$$

Prove that

- a)  $\text{tr}(A + B) = \text{tr}(A) + \text{tr}(B)$
  - b)  $\text{tr}(AB) = \text{tr}(BA)$
  - c) Let  $Q$  be invertible matrix then  $\text{tr}(QAQ^{-1}) = \text{tr}(A)$
3. Section 2.3: 1-13, 16-19
4. Section 2.4: 1-19
5. Section 2.5: 1-7
6. Section 3.2: 1-10