PSETs Landing Page*

Anish Krishna Lakkapragada

This is the documentation for using my PSET PDFs responsibly. I post these LaTEX'd PSETs (1) as an education resource for friends at other universities, fellow Yalies, and all those interested and (2) for quick reference. These PSETs are not to be used irresponsibly; only look at the solution after giving each problem an honest attempt. **If YOU USE THESE PSETS TO CHEAT, YOU ARE NOT ONLY STUPID BUT YOU ARE CHEATING YOURSELF OUT OF THE ABILITY TO FALL IN LOVE WITH MATH.** Furthermore, I am not smarter than you and my solutions did not always get a perfect score.

The general format for accessing the (one-indexed) `N`th assigned PSET PDF of a Yale course with course number `CODE` is:

`https://anish.lakkapragada.com/notes/TYPE-CODE/psets/N.pdf`

where `TYPE` is `stats` or `math`. Similarily, to access my solution for this PSET you can go to:

`https://anish.lakkapragada.com/notes/TYPE-CODE/sols/N.pdf`

These PSETs and associated solution PDFs are synchronized daily at 4:20AM with my computer files through a Cronjob Shell Script. If you want to contribute any corrections, please email `anish.lakkapragada@yale.edu`.

*Note that PDF here is referring to Portable Document Format, not to be confused with the veritable Probability Density Function.

Math 244 - Problem Set 3

due Monday, February 10, 2025, at 11:59pm

Section 2.3

1. How many linear extensions of $\mathcal{B}_2$ are there, and what about $\mathcal{B}_3$?

5. (optional bonus problem) ~~Prove that not every finite poset admits an embedding into the ordered set of triples of real numbers as in Example 2.1.1.~~ *Note from Prof. Hall: This problem as stated seems to be unnecessarily difficult, so we are replacing it with a simpler problem:* Prove that not every finite poset admits an embedding into the poset $(\mathbb{N}^2, \preceq)$, where $(x_1, y_1) \preceq (x_2, y_2)$ if and only if $x_1 \leq x_2$ and $y_1 \leq y_2$.

Section 2.4

3. Find a sequence of real numbers of length 16 that contains no monotone subsequence of length 5. *Note from Prof. Hall: The version in the textbook has "17" instead of "16", but this is a typo.*

4. Prove the following strengthening of Theorem 2.4.6: Let k, ℓ be natural numbers. Then every sequence of real numbers of length $k\ell+1$ contains a nondecreasing subsequence of length $k+1$ or a decreasing subsequence of length $\ell + 1$.

Section 3.1

2. Determine the number of ordered pairs (A, B), where $A \subseteq B \subseteq \{1, 2, \ldots, n\}$.

6. Show that a natural number $n \geq 1$ has an odd number of divisors (including 1 and itself) if and only if $\sqrt{n}$ is an integer. *The textbook has a hint to this problem in the back.*