PSETs Landing Page*

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This is the documentation for using my PSET PDFs responsibly. I post these LaTEX'd PSETs (1) as an education resource for friends at other universities, fellow Yalies, and all those interested and (2) for quick reference. These PSETs are not to be used irresponsibly; only look at the solution after giving each problem an honest attempt. **If YOU USE THESE PSETS TO CHEAT, YOU ARE NOT ONLY STUPID BUT YOU ARE CHEATING YOURSELF OUT OF THE ABILITY TO FALL IN LOVE WITH MATH.** Furthermore, I am not smarter than you and my solutions did not always get a perfect score.

The general format for accessing the (one-indexed) Nth assigned PSET PDF of a Yale course with course number CODE is:

https://anish.lakkapragada.com/notes/TYPE-CODE/psets/N.pdf

where TYPE is stats or math. Similarily, to access my solution for this PSET you can go to:

https://anish.lakkapragada.com/notes/TYPE-CODE/sols/N.pdf

These PSETs and associated solution PDFs are synchronized daily at 4:20AM with my computer files through a Cronjob Shell Script. If you want to contribute any corrections, please email anish.lakkapragada@yale.edu.

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*Note that PDF here is referring to Portable Document Format, not to be confused with the veritable Probability Density Function.

Math 244 - Problem Set 4

due Friday, February 21, 2025, at 11:59pm

Section 3.2

1. How many permutations of $\{1, 2, \ldots, n\}$ have a single cycle?

2. For a permutation $p : X \to X$, let p^k denote the permutation arising by a k-fold composition of p, i.e., $p^1 = p$ and $p^k = p \circ p^{k-1}$. Define a relation $\approx$ on the set X as follows: $i \approx j$ if and only if there exists a $k \geq 1$ such that $p^k(i) = j$. Prove that $\approx$ is an equivalence relation on X, and that its classes are the cycles of p.

Section 3.3

7. How many functions $f : \{1, 2, \ldots, n\} \to \{1, 2, \ldots, n\}$ are there that are *monotonic*; that is, for $i < j$ we have $f(i) \leq f(j)$? *The textbook has a hint to this problem in the back.*

21. (optional bonus problem) Draw a triangle ABC. Draw n points lying on the side AB (but different from A and B) and connect them all by segments to the vertex C. Similarly, draw n points on the side AC and connect them to B.

   (a) How many intersections of the drawn segments are there? Into how many regions is the triangle ABC partitioned by the drawn segments?

   (b) Draw n points on the side BC and connect them to A. Assume that no 3 of the drawn segments intersect at a single point. How many intersections are there now?

   (c) How many regions are there in the situation of (b)?

Section 3.7

3. (Sieve of Eratosthenes) How many numbers are left in the set $\{1, 2, \ldots 1000\}$ after all multiples of $2, 3, 5$, and 7 are crossed out?

Section 3.8

4. Prove the equation

$$D(n) = n! - nD(n-1) - \binom{n}{2}D(n-2) - \ldots - \binom{n}{n-1}D(1) - 1.$$