

## Discretionary Note

Anish Krishna Lakkapragada

**IF YOU USE THIS FILE TO CHEAT, YOU ARE NOT ONLY STUPID BUT YOU ARE CHEATING YOURSELF OUT OF THE ABILITY TO FALL IN LOVE WITH MATH.** Furthermore, I am not smarter than you and my solutions did not always get a perfect score.

**CONTENT STARTS ON NEXT PAGE.**

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**Math 226- HW 6 Due: October 14 by Midnight**

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1. (24 points) Let  $A$  be  $m \times n$ ,  $B$  be  $n \times k$ ,  $P, Q$  be  $n \times n$  matrices
  - a) (10 points) Show that  $(AB)^t = B^t A^t$ . (Use the definition of matrix multiplication)
  - b) (6 points) Show that  $P$  is invertible then  $P^t$  is invertible, and  $(P^t)^{-1} = (P^{-1})^t$
  - c) (8 points) Show that if  $PQ$  is invertible then  $(PQ)^t$  is invertible and  $((PQ)^t)^{-1} = (P^{-1})^t (Q^{-1})^t$ .
2. (11 points) Let  $A = [\alpha_{ij}]_{n \times n}$ . Trace of  $A$  is defined as  $Tr(A) = \sum_{i=1}^n \alpha_{ii}$ .
  - a) (7 points) Let  $B = [\beta_{ij}]_{n \times n}$ , prove that  $Tr(AB) = Tr(BA)$ .
  - b) (4 points) Use part a) to show if  $A$  and  $B$  are similar matrices then  $Tr(A) = Tr(B)$
3. (8 points) Show that if  $A$  is  $n \times 1$  matrix, and  $B$  is  $1 \times n$  matrix then the  $n \times n$  matrix  $AB$  has rank at most 1. Conversely if  $C$  is any  $n \times n$  matrix having rank 1, then there exist  $n \times 1$  matrix  $A$ , and  $1 \times n$  matrix  $B$  such that  $C = AB$ .
4. (14 points) In  $\mathbb{R}^2$ , let  $L$  be the line  $y = mx$ , where  $m \neq 0$ , and  $T$  be reflection of  $\mathbb{R}^2$  about  $L$ . (We did something similar to the following logic in the class for projection operator)
  - a) (2 points) Find a vector  $v_1 \in \mathbb{R}^2$  such that  $T(v_1) = v_1$ , and a vector  $v_2 \in \mathbb{R}^2$  such that  $T(v_2) = -v_2$
  - b) (2 points) Let  $\beta' = \{v_1, v_2\}$  where  $v_1, v_2$  are the vectors that you found in part a. Find the matrix representation for  $[T]_{\beta'}$ .
  - c) (6 points) Let  $\beta = \{e_1, e_2\}$ . Find  $Q = [I_{\mathbb{R}^2}]_{\beta}^{\beta'}$ , and  $Q^{-1} = [I_{\mathbb{R}^2}]_{\beta'}^{\beta}$ .
  - d) (4 points) Find an expression for  $T$  using  $[T]_{\beta} = Q^{-1}[T]_{\beta'}Q$ .