Discretionary Note

Anish Krishna Lakkapragada

IF YOU USE THIS FILE TO CHEAT, YOU ARE NOT ONLY STUPID BUT YOU ARE CHEATING YOURSELF OUT OF THE ABILITY TO FALL IN LOVE WITH MATH. Furthermore, I am not smarter than you and my solutions did not always get a perfect score.

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Math 244 - Problem Set 1

due Friday, January 24, 2025, at 11:59pm

Section 1.2

5. Is a "cancellation" possible for the Cartesian product? That is, if $X \times Y = X \times Z$ holds for some sets X, Y, Z, does it necessarily follow that $Y = Z$?

6. Prove that for any two sets A, B we have

$$(A \setminus B) \cup (B \setminus A) = (A \cup B) \setminus (A \cap B).$$

   Note from Prof. Hall: $A \setminus B$ is the "difference" of A and B, i.e., the set of all elements belonging to A but not to B.

Section 1.3

2. The numbers $F_0, F_1, F_2, \ldots$ are defined as follows (this is a definition by mathematical induction, by the way): $F_0 = 0, F_1 = 1, F_{n+2} = F_{n+1} + F_n$ for $n = 0, 1, 2, \ldots$. Prove that for any $n \geq 0$ we have $F_n \leq ((1 + \sqrt{5})/2)^{n-1}$ (see also Section 12.3).

5. (optional bonus problem) In ancient Egypt, fractions were written as sums of fractions with numerator 1. For instance, $\frac{3}{5} = \frac{1}{2} + \frac{1}{10}$. Consider the following algorithm for writing a fraction $\frac{m}{n}$ in this form ($1 \leq m < n$): write the fraction $\frac{1}{\lceil n/m \rceil}$, calculate the fraction $\frac{m}{n} - \frac{1}{\lceil n/m \rceil}$, and if it is nonzero repeat the same step. Prove that this algorithm always finishes in a finite number of steps. *Note from Prof. Hall: $\lceil x \rceil$ is the "ceiling" of x, i.e., the least integer greater than or equal to x. The textbook has a hint to this problem in the back.*

Section 1.4

2. Find an example of:

    (a) A one-to-one function $f : \mathbf{N} \to \mathbf{N}$ which is not onto

    (b) A function $f : \mathbf{N} \to \mathbf{N}$ which is onto but not one-to-one.

6. Prove that the following two statements about a function $f : X \to Y$ are equivalent (X and Y are some arbitrary sets):

    (i) f is one-to-one.

    (ii) For any set Z and any two distinct functions $g_1 : Z \to X$ and $g_2 : Z \to X$ the composed functions $f \circ g_1$ and $f \circ g_2$ are also distinct.

    (First, make sure you understand what it means that two functions are equal and what it means that they are distinct.)