PSETs Landing Page*

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This is the documentation for using my PSET PDFs responsibly. I post these LaTEX'd PSETs (1) as an education resource for friends at other universities, fellow Yalies, and all those interested and (2) for quick reference. These PSETs are not to be used irresponsibly; only look at the solution after giving each problem an honest attempt. **If YOU USE THESE PSETS TO CHEAT, YOU ARE NOT ONLY STUPID BUT YOU ARE CHEATING YOURSELF OUT OF THE ABILITY TO FALL IN LOVE WITH MATH.** Furthermore, I am not smarter than you and my solutions did not always get a perfect score.

The general format for accessing the (one-indexed) `N`th assigned PSET PDF of a Yale course with course number `CODE` is:

`https://anish.lakkapragada.com/notes/TYPE-CODE/psets/N.pdf`

where `TYPE` is `stats` or `math`. Similarily, to access my solution for this PSET you can go to:

`https://anish.lakkapragada.com/notes/TYPE-CODE/sols/N.pdf`

These PSETs and associated solution PDFs are synchronized daily at 4:20AM with my computer files through a Cronjob Shell Script. If you want to contribute any corrections, please email `anish.lakkapragada@yale.edu`.

*Note that PDF here is referring to Portable Document Format, not to be confused with the veritable Probability Density Function.

MATH 241 PSET 1

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1.

(a)

The number of games played in total will just be the number of unique pairs which can be drawn from n unique players. This can be computed as the number $g = \binom{n}{2} = \frac{n!}{2!(n-2)!} = \frac{n(n-1)}{2}$.

Because each single game has either two outcomes, win or lose, the number of outcomes can be given by 2^g or $2^{\frac{n(n-1)}{2}}$. This is essentially sampling with replacement for g games across 2 possible outcomes (win or loss.)

(b)

As stated in (a), the number of games played in total will just be the number of unique pairs which can be drawn from n unique players. This is sampling without replacement where order does not matter. Thus the number of games played in total is given by $\binom{n}{2} = \frac{n!}{2!(n-2)!} = \frac{n(n-1)}{2}$.

2.

(a)

The total number of unique combinations for choosing 7 courses across the 20 possible courses is given by $\binom{20}{7}$. The number of these combinations which are invalid, where no statistics courses are chosen, is given by $\binom{15}{7}$ which represents choosing all 7 courses from the 15 non-statistics courses. Thus the number of valid combinations where at least one statistics course is chosen is given by $\binom{20}{7} - \binom{15}{7}$.

(b)

The expression $\binom{5}{1} * \binom{19}{6}$ double counts certain schedules which will have the same classes. This is because even if $\binom{5}{1}$ will choose a unique course, this unique course can be part of the six chosen courses in $\binom{19}{6}$. Thus this expression is wrong as it overestimates the number of valid schedules possible.

3.

(a)

The number of functions from A to B is the problem of sampling with replacement, in a case where order does matter. Each of the n elements in A can be mapped to one of the m elements in B, giving the number of possible functions as m^n.

(b)

If $m < n$, the number of one-to-one functions from A to B is zero as not every element in A can be mapped to a unique element in B. If $m \geq n$, the number of unique one-to-one functions which can map all n elements in A to a unique element in B, which has m elements, is the problem of sampling n elements from m choices without replacement. Thus, the number of one-to-one functions from A to B is given by ${}^mP_n = \frac{m!}{(m-n)!}$.

4.

We solve this problem by first computing the number of permutations where three consecutive numbers were pressed. Given that there are seven combinations of numbers where three consecutive numbers are pressed, and that each of these sets of three numbers can be ordered in 3! unique ways, the number of permutations where three consecutive numbers are pressed is given by $|A| = 7 * 3! = 42$.

We now compute the size of the sample space, denoted by $|S|$. The sample space S gives the total number of unique permutations of buttons pressed. Note that each of the three people in the elevator have 9 choices (to choose a number from $2 \dots 10$, and if their number of choice is already chosen, do nothing). This is the problem of sampling with replacement in a case where order matters ($n = 9, k = 3$), and so $|S| = 9^3$.

Thus, $P(A) = \frac{|A|}{|S|} = \frac{42}{9^3}$.

5.

The probability that at least one location has more than one number stored there can be given by $1 - P(A^c)$, where A^c is the set of possible outcomes where each of the k phone numbers are stored in a different location. To compute $|A^c|$, we observe that the problem of uniquely allocating k phone numbers to n possible locations is essentially sampling k locations from n choices without replacement. This gives us $|A^c| = \frac{n!}{(n-k)!}$. The sample space of outcomes is the number of different ways to allocate k phone numbers to n locations (this time allowing ≥ 1 phone numbers to be stored in the same location). This is sampling k locations from n choices with replacement. This leaves us with the sample space size $|S| = n^k$. Thus we can compute $P(A^c) = \frac{|A^c|}{|S|} = \frac{n!}{(n-k)!n^k}$. This gives us the answer to this question as $1 - \frac{n!}{(n-k)!n^k}$.

6.

Anish Lakkapragada. I worked independently.