# Discretionary Note

Anish Krishna Lakkapragada

# IF YOU USE THIS FILE TO CHEAT, YOU ARE NOT ONLY STUPID BUT YOU ARE CHEATING YOURSELF OUT OF THE ABILITY TO FALL IN LOVE WITH MATH. Furthermore, I am not smarter than you and my solutions did not always get a perfect score.

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# Math 244 - Problem Set 2

due Monday, February 3, 2025, at 11:59pm

### Section 1.5

5. Prove the associativity of composing relations: if R, S, T are relations such that  $(R \circ S) \circ T$  is well-defined, then  $R \circ (S \circ T)$  is also well-defined and equals  $(R \circ S) \circ T$ .

## Section 1.6

- 3. Prove that a relation R is transitive if and only if  $R \circ R \subseteq R$ .
- 6. Describe all relations on a set X that are equivalences and orderings at the same time.

### RESP Section 2.1

- 4. Let  $(X, \leq), (Y, \preceq)$  be ordered sets. We say that they are isomorphic (meaning that they "look the same" from the point of view of ordering) if there exists a bijection  $f: X \to Y$  such that for every  $x, y \in X$ , we have  $x \leq y$  if and only if  $f(x) \leq f(y)$ . Note from Prof. Hall: In the book this problem has four parts, but you are only being asked to do parts (a) and (b).
  - (a) Draw Hasse diagrams for all non-isomorphic 3-element posets.
- (b) Prove that any two *n*-element linearly ordered sets are isomorphic.

  The textbook has a hint to this problem in the back.

# Section 2.2

- 2. (a) Consider the set  $\{1, 2, ... n\}$  ordered by the divisibility relation | (see Example 2.1.2). What is the maximum possible number of elements of a set  $X \subseteq \{1, 2, ... n\}$  that is ordered linearly by the relation | (such a set X is called a chain)?
- RESPONSITED Solve the same question for the set  $2^{\{1,2,\dots n\}}$  ordered by the relation RESPONSIBLY  $\subseteq$  (see Example 2.1.3).
  - 3. (optional bonus problem) Let  $le(X, \preceq)$  denote the number of linear extensions of a partially ordered set  $(X, \preceq)$ . Prove:
    - (a)  $le(X, \preceq) = 1$  if and only if  $(X, \preceq)$  is a linear ordering;
    - (b)  $le(X, \preceq) \leq n!$ , where n = |X| (you may want to read Chapter 3 first).