Discretionary Note

Anish Krishna Lakkapragada

IF YOU USE THIS FILE TO CHEAT, YOU ARE NOT ONLY STUPID BUT YOU ARE CHEATING YOURSELF OUT OF THE ABILITY TO FALL IN LOVE WITH MATH. Furthermore, I am not smarter than you and my solutions did not always get a perfect score.

CONTENT STARTS ON NEXT PAGE.

To access the general instructions for this repository head here.

Math 226- HW 7 Due: Oc 29 by Midnight

1. An important concern in the study of heat transfer is to determine the steady-state temperature distribution of a thin plate when the temperature around the boundary is known. Assume the plate shown in the figure represents a cross section of a metal beam, with negligible heat flow in the direction perpendicular to the plate. Let T_1, T_2, T_3, T_4 denote the temperatures at the four interior nodes of the mesh in the figure. The temperature at a node is approximately equal to the average of the four nearest nodes- to the left, above, to the right, and below. For instance $T_1 = (40 + 30 + T_3 + T_2)/4$
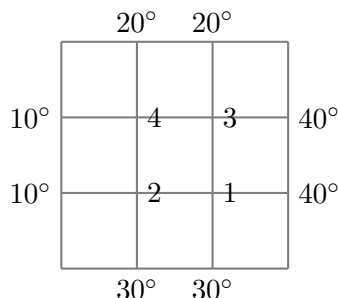
   a) (4 points) Write a system of four equations whose solution gives estimates for the temperatures T_1, T_2, T_3, T_4

   b) (6 points) Write down the coefficient matrix and find its inverse.

   c) (2 points) Use part a) to give the estimated values of T_1, T_2, T_3, T_4.

2. (6 points) Find a basis of the kernel (that is $N(L_A)$) of the linear mappings given by

$$A = \begin{pmatrix} 3 & 5 & -4 & 2 \\ 2 & 4 & -6 & 3 \\ 11 & 17 & -8 & 4 \end{pmatrix}$$

   Use it to describe the solutions to

$$3x_1 + 5x_2 - 4x_3 + 2x_4 = 0$$
$$2x_1 + 4x_2 - 6x_3 + 3x_4 = 0$$
$$11x_1 + 17x_2 - 8x_3 + 4x_4 = 0$$

3. (12 points) Find all vectors in a space $\mathbb{R}^4$, whose image is equal to the vector $b \in \mathbb{R}^3$ under the linear map $\mathbb{R}^4 \to \mathbb{R}^3$ given by the matrix A. (That is find the solutions to $A\vec{x} = \vec{b}$). Use these vectors to give a basis to $N(L_A)$.

$$A = \begin{pmatrix} 1 & -3 & -3 & -14 \\ 2 & -6 & -3 & -1 \\ 3 & -9 & -5 & -6 \end{pmatrix}, \quad b = \begin{pmatrix} 8 \\ -5 \\ -4 \end{pmatrix}$$

4. (10 points) Consider the map $T : P_3(\mathbb{R}) \to \mathbb{R}^4$ such that $T(f) = (f(-2), f(-1), f(1), f(2))$.

   i) (5 points) Let $\gamma = \{1, x, x^2, x^3\}$, and β be the standard basis for $\mathbb{R}^4$. Compute $[T]_\gamma^\beta$.

   ii) (5 points) Use $[T]_\gamma^\beta$ to find a third degree polynomial such that $f(-2) = 1$, $f(-1) = 3$, $f(1) = 13$, $f(2) = 33$.

5. (10 points)Read "An Interpretation of the Reduced Row Echelon Form" from the book (pg 189-194). Then answer the following question

$$V = \{(x_1, x_2, x_3, x_4, x_5) : x_1 - 2x_2 + 3x_3 - x_4 + 2x_5 = 0\}$$

   Note that $v = (1, 2, 1, 0, 0)$ is a vector in V. Use column correspondance to find a basis for V that includes v.

Practice Problems

1. Let $T, U : V \to W$ be linear transformations.

   a) Prove that $R(U + T) \subseteq R(T) + R(U)$.

   b) Prove that if W is finite dimensional, then $rank(T + U) \leq rank(T) + rank(U)$

   c) Deduce that $rank(A + B) \leq rank(A) + rank(B)$ for any $m \times n$ matrix.

2. Let $A = [a_{ji}]_{n \times n}$, $B = [b_{ji}]_{n \times n}$ be matrices. The trace of A is defined by

$$tr(A) = \sum_{i=1}^{n} a_{ii}$$

   Prove that

   a) $tr(A + B) = tr(A) + tr(B)$

   b) $tr(AB) = tr(BA)$

   c) Let Q be invertible matrix then $tr(QAQ^{-1}) = tr(A)$

3. Section 2.3: 1-13, 16-19

4. Section 2.4: 1-19

5. Section 2.5: 1-7

6. Section 3.2: 1-10