PSETs Landing Page*

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This is the documentation for using my PSET PDFs responsibly. I post these LaTEX'd PSETs (1) as an education resource for friends at other universities, fellow Yalies, and all those interested and (2) for quick reference. These PSETs are not to be used irresponsibly; only look at the solution after giving each problem an honest attempt. **If YOU USE THESE PSETS TO CHEAT, YOU ARE NOT ONLY STUPID BUT YOU ARE CHEATING YOURSELF OUT OF THE ABILITY TO FALL IN LOVE WITH MATH.** Furthermore, I am not smarter than you and my solutions did not always get a perfect score.

The general format for accessing the (one-indexed) `N`th assigned PSET PDF of a Yale course with course number `CODE` is:

`https://anish.lakkapragada.com/notes/TYPE-CODE/psets/N.pdf`

where `TYPE` is `stats` or `math`. Similarily, to access my solution for this PSET you can go to:

`https://anish.lakkapragada.com/notes/TYPE-CODE/sols/N.pdf`

These PSETs and associated solution PDFs are synchronized daily at 4:20AM with my computer files through a Cronjob Shell Script. If you want to contribute any corrections, please email `anish.lakkapragada@yale.edu`.

*Note that PDF here is referring to Portable Document Format, not to be confused with the veritable Probability Density Function.

Problem set 3
Due Thursday February 6 at 11pm

Exercise 3.1 (5 points). Prove that $\mathbb{N}$ is not bounded above in $\mathbb{R}$.

Exercise 3.2 (10 points; Rudin 2.2+2.3, modified). A real number x is called *algebraic* if there exist $n \in \mathbb{N}$ and $a_0, \ldots, a_n \in \mathbb{Z}$, with $a_0 \neq 0$, such that

$$a_0 x^n + a_1 x^{n-1} + \cdots + a_{n-1} x + a_n = 0.$$

(1) Prove that $\sqrt{5}$ and $\sqrt{2+\sqrt{3}}$ are algebraic.

(2) Prove that the set of all algebraic real numbers is countable. (You may use without proof the fact that a polynomial of degree n has at most n roots.)

(3) Prove that there exist real numbers which are not algebraic.

Exercise 3.3 (5 points). Suppose $a, b \in \mathbb{R}$ with $a < b$. Prove that there are uncountably many irrational numbers in the interval (a, b).

Exercise 3.4 (10 points). Are the following sets finite, countable or uncountable? Prove your answers.

(1) The set of all finite subsets of $\mathbb{N}$.

(2) The set of all subsets of $\mathbb{N}$.

(3) The set of all functions $f : \mathbb{Q} \to \mathbb{Q}$.

Exercise 3.5 (10 points). For each of the following, determine whether X with the distance function d is a metric space, and prove your answer.

(1) $X = \mathbb{R}$, $d(x, y) = |x^2 - y^2|$

(2) $X = \mathbb{R}$, $d(x, y) = |x - 2y|$

(3) $X = \mathbb{R}$, $d(x, y) = \frac{|x-y|}{1+|x-y|}$

(4) $X = \mathbb{R}^2$, $d(x, y) = |x_1 - y_1| + |x_2 - y_2|$ [where $x = (x_1, x_2)$ and $y = (y_1, y_2)$]

(5) $X = \mathbb{R}^2$, $d(x, y) = |x_1 - y_2| + |x_2 - y_1|$ [where $x = (x_1, x_2)$ and $y = (y_1, y_2)$]