

Discretionary Note

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IF YOU USE THIS FILE TO CHEAT, YOU ARE NOT ONLY STUPID BUT YOU ARE CHEATING YOURSELF OUT OF THE ABILITY TO FALL IN LOVE WITH MATH. Furthermore, I am not smarter than you and my solutions did not always get a perfect score.

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Math 244 - Problem Set 4

due Friday, February 21, 2025, at 11:59pm

Section 3.2

1. How many permutations of $\{1, 2, \dots, n\}$ have a single cycle?
2. For a permutation $p : X \rightarrow X$, let p^k denote the permutation arising by a k -fold composition of p , i.e., $p^1 = p$ and $p^k = p \circ p^{k-1}$. Define a relation \approx on the set X as follows: $i \approx j$ if and only if there exists a $k \geq 1$ such that $p^k(i) = j$. Prove that \approx is an equivalence relation on X , and that its classes are the cycles of p .

Section 3.3

7. How many functions $f : \{1, 2, \dots, n\} \rightarrow \{1, 2, \dots, n\}$ are there that are *monotonic*; that is, for $i < j$ we have $f(i) \leq f(j)$? *The textbook has a hint to this problem in the back.*
21. (optional bonus problem) Draw a triangle ABC . Draw n points lying on the side AB (but different from A and B) and connect them all by segments to the vertex C . Similarly, draw n points on the side AC and connect them to B .
 - (a) How many intersections of the drawn segments are there? Into how many regions is the triangle ABC partitioned by the drawn segments?
 - (b) Draw n points on the side BC and connect them to A . Assume that no 3 of the drawn segments intersect at a single point. How many intersections are there now?
 - (c) How many regions are there in the situation of (b)?

Section 3.7

3. (Sieve of Eratosthenes) How many numbers are left in the set $\{1, 2, \dots, 1000\}$ after all multiples of 2, 3, 5, and 7 are crossed out?

Section 3.8

4. Prove the equation

$$D(n) = n! - nD(n-1) - \binom{n}{2}D(n-2) - \dots - \binom{n}{n-1}D(1) - 1.$$