Discretionary Note

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IF YOU USE THIS FILE TO CHEAT, YOU ARE NOT ONLY STUPID BUT YOU ARE CHEATING YOURSELF OUT OF THE ABILITY TO FALL IN LOVE WITH MATH. Furthermore, I am not smarter than you and my solutions did not always get a perfect score.

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Math 244 - Problem Set 1

due Friday, January 24, 2025, at 11:59pm

Section 1.2

5. Is a "cancellation" possible for the Cartesian product? That is, if $X \times Y = X \times Z$ holds for some sets X, Y, Z, does it necessarily follow that $Y = Z$?

6. Prove that for any two sets A, B we have

$$(A \setminus B) \cup (B \setminus A) = (A \cup B) \setminus (A \cap B).$$

Note from Prof. Hall: $A \setminus B$ is the "difference" of A and B, i.e., the set of all elements belonging to A but not to B.

Section 1.3

2. The numbers $F_0, F_1, F_2, \ldots$ are defined as follows (this is a definition by mathematical induction, by the way): $F_0 = 0, F_1 = 1, F_{n+2} = F_{n+1} + F_n$ for $n = 0, 1, 2, \ldots$. Prove that for any $n \geq 0$ we have $F_n \leq ((1 + \sqrt{5})/2)^{n-1}$ (see also Section 12.3).

5. (optional bonus problem) In ancient Egypt, fractions were written as sums of fractions with numerator 1. For instance, $\frac{3}{5} = \frac{1}{2} + \frac{1}{10}$. Consider the following algorithm for writing a fraction $\frac{m}{n}$ in this form ($1 \leq m < n$): write the fraction $\frac{1}{\lceil n/m \rceil}$, calculate the fraction $\frac{m}{n} - \frac{1}{\lceil n/m \rceil}$, and if it is nonzero repeat the same step. Prove that this algorithm always finishes in a finite number of steps. *Note from Prof. Hall: $\lceil x \rceil$ is the "ceiling" of x, i.e., the least integer greater than or equal to x. The textbook has a hint to this problem in the back.*

Section 1.4

2. Find an example of:

   (a) A one-to-one function $f : \mathbf{N} \to \mathbf{N}$ which is not onto

   (b) A function $f : \mathbf{N} \to \mathbf{N}$ which is onto but not one-to-one.

6. Prove that the following two statements about a function $f : X \to Y$ are equivalent (X and Y are some arbitrary sets):

   (i) f is one-to-one.

   (ii) For any set Z and any two distinct functions $g_1 : Z \to X$ and $g_2 : Z \to X$ the composed functions $f \circ g_1$ and $f \circ g_2$ are also distinct.

   (First, make sure you understand what it means that two functions are equal and what it means that they are distinct.)