

## Discretionary Note

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**IF YOU USE THIS FILE TO CHEAT, YOU ARE NOT ONLY STUPID BUT YOU ARE CHEATING YOURSELF OUT OF THE ABILITY TO FALL IN LOVE WITH MATH.** Furthermore, I am not smarter than you and my solutions did not always get a perfect score.

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# Math 244 - Problem Set 4

due Friday, February 21, 2025, at 11:59pm

## Section 3.2

1. How many permutations of  $\{1, 2, \dots, n\}$  have a single cycle?
2. For a permutation  $p : X \rightarrow X$ , let  $p^k$  denote the permutation arising by a  $k$ -fold composition of  $p$ , i.e.,  $p^1 = p$  and  $p^k = p \circ p^{k-1}$ . Define a relation  $\approx$  on the set  $X$  as follows:  $i \approx j$  if and only if there exists a  $k \geq 1$  such that  $p^k(i) = j$ . Prove that  $\approx$  is an equivalence relation on  $X$ , and that its classes are the cycles of  $p$ .

## Section 3.3

7. How many functions  $f : \{1, 2, \dots, n\} \rightarrow \{1, 2, \dots, n\}$  are there that are *monotonic*; that is, for  $i < j$  we have  $f(i) \leq f(j)$ ? *The textbook has a hint to this problem in the back.*
21. (optional bonus problem) Draw a triangle  $ABC$ . Draw  $n$  points lying on the side  $AB$  (but different from  $A$  and  $B$ ) and connect them all by segments to the vertex  $C$ . Similarly, draw  $n$  points on the side  $AC$  and connect them to  $B$ .
  - (a) How many intersections of the drawn segments are there? Into how many regions is the triangle  $ABC$  partitioned by the drawn segments?
  - (b) Draw  $n$  points on the side  $BC$  and connect them to  $A$ . Assume that no 3 of the drawn segments intersect at a single point. How many intersections are there now?
  - (c) How many regions are there in the situation of (b)?

Section 3.7

3. (Sieve of Eratosthenes) How many numbers are left in the set  $\{1, 2, \dots, 1000\}$  after all multiples of 2, 3, 5, and 7 are crossed out?

Section 3.8

4. Prove the equation

$$D(n) = n! - nD(n-1) - \binom{n}{2}D(n-2) - \dots - \binom{n}{n-1}D(1) - 1.$$