PSETs Landing Page*

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This is the documentation for using my PSET PDFs responsibly. I post these LaTEX'd PSETs (1) as an education resource for friends at other universities, fellow Yalies, and all those interested and (2) for quick reference. These PSETs are not to be used irresponsibly; only look at the solution after giving each problem an honest attempt. **If YOU USE THESE PSETS TO CHEAT, YOU ARE NOT ONLY STUPID BUT YOU ARE CHEATING YOURSELF OUT OF THE ABILITY TO FALL IN LOVE WITH MATH.** Furthermore, I am not smarter than you and my solutions did not always get a perfect score.

The general format for accessing the (one-indexed) `N`th assigned PSET PDF of a Yale course with course number `CODE` is:

`https://anish.lakkapragada.com/notes/TYPE-CODE/psets/N.pdf`

where `TYPE` is `stats` or `math`. Similarily, to access my solution for this PSET you can go to:

`https://anish.lakkapragada.com/notes/TYPE-CODE/sols/N.pdf`

These PSETs and associated solution PDFs are synchronized daily at 4:20AM with my computer files through a Cronjob Shell Script. If you want to contribute any corrections, please email `anish.lakkapragada@yale.edu`.

*Note that PDF here is referring to Portable Document Format, not to be confused with the veritable Probability Density Function.

Problem set 5
Due Friday February 20 at 11pm

Exercise 5.1 (5 points). Show that the subset of $\mathbb{R}^2$ given by

$$E = \{(x, y) \in \mathbb{R}^2 \mid x < y\}$$

is open.

Exercise 5.2 (5 points). Show that the union of a finite number of compact sets is compact.

Exercise 5.3 (5 points; Rudin 2.14). Prove directly that the interval $(0, 1) \subset \mathbb{R}$ is not compact, by giving an example of an open cover of $(0, 1)$ which has no finite subcover. Include a proof that your cover has no finite subcover.

Exercise 5.4 (20 points; Rudin 2.19). If A and B are subsets of a metric space X, we say A and B are *separated* if $A \cap \overline{B} = \emptyset$ and $\overline{A} \cap B = \emptyset$. (We used this notion when we defined connectedness.)

(1) If A and B are disjoint closed sets in some metric space X, prove that A and B are separated.

(2) Prove the same for disjoint open sets.

(3) Fix $p \in X$ and $\delta > 0$. Define $A = \{q \in X \mid d(p, q) < \delta\}$. Define $B = \{q \in X \mid d(p, q) > \delta\}$. Prove that A and B are separated.

(4) Prove that every connected metric space with at least two points is uncountable. (Hint: use the previous part.)

Exercise 5.5 (5 points; Rudin 2.22, modified). Given a metric space X and a set $E \subset X$, we say E is *dense* in X if $\overline{E} = X$. Prove that $\mathbb{Q}$ is dense in $\mathbb{R}$.

Exercise 5.6 (not for credit; Rudin 2.8). Is every point of every open set $E \subset \mathbb{R}^2$ a limit point of E? Answer the same question for closed sets in $\mathbb{R}^2$.