

## Discretionary Note

Anish Krishna Lakkapragada

**IF YOU USE THIS FILE TO CHEAT, YOU ARE NOT ONLY STUPID BUT YOU ARE CHEATING YOURSELF OUT OF THE ABILITY TO FALL IN LOVE WITH MATH.** Furthermore, I am not smarter than you and my solutions did not always get a perfect score.

**CONTENT STARTS ON NEXT PAGE.**

To access the general instructions for this repository head [here](#).

# Math 244 - Problem Set 1

due Friday, January 24, 2025, at 11:59pm

## Section 1.2

5. Is a “cancellation” possible for the Cartesian product? That is, if  $X \times Y = X \times Z$  holds for some sets  $X, Y, Z$ , does it necessarily follow that  $Y = Z$ ?
6. Prove that for any two sets  $A, B$  we have

$$(A \setminus B) \cup (B \setminus A) = (A \cup B) \setminus (A \cap B).$$

*Note from Prof. Hall:  $A \setminus B$  is the “difference” of  $A$  and  $B$ , i.e., the set of all elements belonging to  $A$  but not to  $B$ .*

## Section 1.3

2. The numbers  $F_0, F_1, F_2, \dots$  are defined as follows (this is a definition by mathematical induction, by the way):  $F_0 = 0, F_1 = 1, F_{n+2} = F_{n+1} + F_n$  for  $n = 0, 1, 2, \dots$ . Prove that for any  $n \geq 0$  we have  $F_n \leq ((1 + \sqrt{5})/2)^{n-1}$  (see also Section 12.3).
5. (optional bonus problem) In ancient Egypt, fractions were written as sums of fractions with numerator 1. For instance,  $\frac{3}{5} = \frac{1}{2} + \frac{1}{10}$ . Consider the following algorithm for writing a fraction  $\frac{m}{n}$  in this form ( $1 \leq m < n$ ): write the fraction  $\frac{1}{\lceil n/m \rceil}$ , calculate the fraction  $\frac{m}{n} - \frac{1}{\lceil n/m \rceil}$ , and if it is nonzero repeat the same step. Prove that this algorithm always finishes in a finite number of steps. *Note from Prof. Hall:  $\lceil x \rceil$  is the “ceiling” of  $x$ , i.e., the least integer greater than or equal to  $x$ . The textbook has a hint to this problem in the back.*

Section 1.4

2. Find an example of:
  - (a) A one-to-one function  $f : \mathbf{N} \rightarrow \mathbf{N}$  which is not onto
  - (b) A function  $f : \mathbf{N} \rightarrow \mathbf{N}$  which is onto but not one-to-one.
6. Prove that the following two statements about a function  $f : X \rightarrow Y$  are equivalent ( $X$  and  $Y$  are some arbitrary sets):
  - (i)  $f$  is one-to-one.
  - (ii) For any set  $Z$  and any two distinct functions  $g_1 : Z \rightarrow X$  and  $g_2 : Z \rightarrow X$  the composed functions  $f \circ g_1$  and  $f \circ g_2$  are also distinct.

(First, make sure you understand what it means that two functions are equal and what it means that they are distinct.)