

## Discretionary Note

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**IF YOU USE THIS FILE TO CHEAT, YOU ARE NOT ONLY STUPID BUT YOU ARE CHEATING YOURSELF OUT OF THE ABILITY TO FALL IN LOVE WITH MATH.** Furthermore, I am not smarter than you and my solutions did not always get a perfect score.

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**Problem set 4**  
**Due Sunday February 16 at 11pm**

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**Exercise 4.1 (10 points).** Let  $X$  be any set, let  $d : X \times X \rightarrow \mathbb{R}$  be the discrete metric, defined by

$$d(x, y) = \begin{cases} 0 & x = y \\ 1 & x \neq y \end{cases}$$

for all  $x, y \in X$ .

- (1) Prove that, with this distance function,  $X$  is a metric space.
- (2) For any  $x \in X$ , what is  $N_\epsilon(x)$  when  $\epsilon = \frac{1}{2}$ , 1, and 2?
- (3) Which subsets of  $X$  are open? Which are closed?

**Exercise 4.2 (6 points; Rudin 2.5).** Construct a bounded set of real numbers with exactly three limit points (using the standard metric on  $\mathbb{R}$ ). (You need not prove carefully what the limit points are; it is sufficient to give the set, give bounds for the set, and state what are the limit points.)

**Exercise 4.3 (24 points; Rudin 2.9).** Let  $E$  be a subset of a metric space. Define the *interior* of  $E$ , denoted  $E^\circ$ , to be the set of all interior points of  $E$ .

- (1) Prove that  $E^\circ$  is always open.
- (2) Prove that  $E$  is open if and only if  $E^\circ = E$ .
- (3) Prove that, if  $G$  is an open subset of  $E$ , then  $G \subset E^\circ$ .
- (4) Prove that the complement of  $E^\circ$  is the closure of the complement of  $E$ .
- (5) Do  $E$  and  $\bar{E}$  always have the same interiors?
- (6) Do  $E$  and  $E^\circ$  always have the same closures?