Math 226- HW 6 Due: October 14 by Midnight

1. (24 points) Let A be $m \times n$, B be $n \times k$, P, Q be $n \times n$ matrices
   a) (10 points) Show that $(AB)^t = B^t A^t$. (Use the definition of matrix multiplication)
   b) (6 points) Show that P is invertible then P^t is invertible, and $(P^t)^{-1} = (P^{-1})^t$
   c) (8 points) Show that if PQ is invertible then $(PQ)^t$ is invertible and $((PQ)^t)^{-1} = (P^{-1})^t (Q^{-1})^t$.

2. (11 points) Let A =$[\alpha_{ij}]_{n\times n}$. Trace of A is defined as $Tr(A) = \sum_{i=1}^{n} \alpha_{ii}$.
   a) (7 points) Let $B = [\beta_{ij}]_{n\times n}$, prove that $Tr(AB) = Tr(BA)$.
   b) (4 points) Use part a) to show if A and B are similar matrices then $Tr(A) = Tr(B)$

3. (8 points) Show that if A is $n \times 1$ matrix, and B is $1 \times n$ matrix then the $n \times n$ matrix AB has rank at most 1. Conversely if C is any $n \times n$ matrix having rank 1, then there exist $n \times 1$ matrix A , and $1 \times n$ matrix B such that $C = AB$.

4. (14 points) In $\mathbb{R}^2$, let L be the line $y = mx$, where $m \neq 0$, and T be reflection of $\mathbb{R}^2$ about L. (We did something similar to the following logic in the class for projection operator)
   a) (2 points) Find a vector $v_1 \in \mathbb{R}^2$ such that $T(v_1) = v_1$, and a vector $v_2 \in \mathbb{R}^2$ $T(v_2) = -v_2$
   b) (2 points) Let $\beta' = \{v_1, v_2\}$ where v_1, v_2 are the vectors that you found in part a. Find the matrix representation for $[T]_{\beta'}$.
   c) (6 points) Let $\beta = \{e_1, e_2\}$. Find $Q = [I_{\mathbb{R}^2}]_{\beta}^{\beta'}$, and $Q^{-1} = [I_{\mathbb{R}^2}]_{\beta'}^{\beta}$.
   d) (4 points) Find an expression for T using $[T]_\beta = Q^{-1}[T]_{\beta'} Q$.