PSETs Landing Page*

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This is the documentation for using my PSET PDFs responsibly. I post these LaTEX'd PSETs (1) as an education resource for friends at other universities, fellow Yalies, and all those interested and (2) for quick reference. These PSETs are not to be used irresponsibly; only look at the solution after giving each problem an honest attempt. **If YOU USE THESE PSETS TO CHEAT, YOU ARE NOT ONLY STUPID BUT YOU ARE CHEATING YOURSELF OUT OF THE ABILITY TO FALL IN LOVE WITH MATH.** Furthermore, I am not smarter than you and my solutions did not always get a perfect score.

The general format for accessing the (one-indexed) `N`th assigned PSET PDF of a Yale course with course number `CODE` is:

`https://anish.lakkapragada.com/notes/TYPE-CODE/psets/N.pdf`

where `TYPE` is `stats` or `math`. Similarily, to access my solution for this PSET you can go to:

`https://anish.lakkapragada.com/notes/TYPE-CODE/sols/N.pdf`

These PSETs and associated solution PDFs are synchronized daily at 4:20AM with my computer files through a Cronjob Shell Script. If you want to contribute any corrections, please email `anish.lakkapragada@yale.edu`.

*Note that PDF here is referring to Portable Document Format, not to be confused with the veritable Probability Density Function.

Math 226- HW 6 Due: October 14 by Midnight

1. (24 points) Let A be $m \times n$, B be $n \times k$, P, Q be $n \times n$ matrices
   a) (10 points) Show that $(AB)^t = B^t A^t$. (Use the definition of matrix multiplication)
   b) (6 points) Show that P is invertible then P^t is invertible, and $(P^t)^{-1} = (P^{-1})^t$
   c) (8 points) Show that if PQ is invertible then $(PQ)^t$ is invertible and $((PQ)^t)^{-1} = (P^{-1})^t (Q^{-1})^t$.

2. (11 points) Let A =$[\alpha_{ij}]_{n\times n}$. Trace of A is defined as $Tr(A) = \sum_{i=1}^{n} \alpha_{ii}$.
   a) (7 points) Let $B = [\beta_{ij}]_{n\times n}$, prove that $Tr(AB) = Tr(BA)$.
   b) (4 points) Use part a) to show if A and B are similar matrices then $Tr(A) = Tr(B)$

3. (8 points) Show that if A is $n \times 1$ matrix, and B is $1 \times n$ matrix then the $n \times n$ matrix AB has rank at most 1. Conversely if C is any $n \times n$ matrix having rank 1, then there exist $n \times 1$ matrix A , and $1 \times n$ matrix B such that $C = AB$.

4. (14 points) In $\mathbb{R}^2$, let L be the line $y = mx$, where $m \neq 0$, and T be reflection of $\mathbb{R}^2$ about L. (We did something similar to the following logic in the class for projection operator)
   a) (2 points) Find a vector $v_1 \in \mathbb{R}^2$ such that $T(v_1) = v_1$, and a vector $v_2 \in \mathbb{R}^2$ $T(v_2) = -v_2$
   b) (2 points) Let $\beta' = \{v_1, v_2\}$ where v_1, v_2 are the vectors that you found in part a. Find the matrix representation for $[T]_{\beta'}$.
   c) (6 points) Let $\beta = \{e_1, e_2\}$. Find $Q = [I_{\mathbb{R}^2}]_{\beta}^{\beta'}$, and $Q^{-1} = [I_{\mathbb{R}^2}]_{\beta'}^{\beta}$.
   d) (4 points) Find an expression for T using $[T]_\beta = Q^{-1}[T]_{\beta'}Q$.