Discretionary Note

Anish Krishna Lakkapragada

IF YOU USE THIS FILE TO CHEAT, YOU ARE NOT ONLY STUPID BUT YOU ARE CHEATING YOURSELF OUT OF THE ABILITY TO FALL IN LOVE WITH MATH. Furthermore, I am not smarter than you and my solutions did not always get a perfect score.

CONTENT STARTS ON NEXT PAGE.

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MATH 241 PSET 7

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1.

Because F is a continuous function and is strictly increasing (i.e. $\frac{dF}{dx} > 0$) for $x > 0$, this means that $F^{-1}(x)$ exists. Thus, by Universality of the Uniform, $X = F^{-1}(U)$ gives us an r.v. with a CDF of F. We compute $F^{-1}(u)$ below:

$$\begin{aligned} F(x) &= 1 - e^{-x^3} \\ u &= 1 - e^{-x^3} \\ e^{-x^3} &= 1 - u \\ x^3 &= -ln(1-u) \\ x &= [-ln(1-u)]^{\frac{1}{3}} \end{aligned}$$

Thus, $F^{-1}(u) = [-ln(1-u)]^{\frac{1}{3}}$ and so the r.v. $X = F^{-1}(U) = [-ln(1-U)]^{\frac{1}{3}}$ has the CDF F.

2.

The general form of a transformation of the standard normal distribution is $Y = \mu + \sigma Z$, where $Z \sim \mathcal{N}(0,1)$ and $Y \sim \mathcal{N}(\mu, \sigma^2)$. Thus, for a distribution Y with $\sigma = \sqrt{4} = 2$ and $\mu = 1$, $Y = 1 + 2Z$.

3.

Let us define ϵ as an r.v. where $\epsilon \sim \mathcal{N}(0, 0.04)$. The probability that the observed distance is within 0.4 meters of the true distance is given by $P(-0.4 \le \epsilon \le 0.4) = \Phi(\frac{0.4-0}{\sqrt{0.04}}) - \Phi(\frac{-0.4-0}{\sqrt{0.04}}) = \Phi(2) - \Phi(-2) = \Phi(2) - (1 - \Phi(2)) = 2\Phi(2) - 1$.

In numerical terms, we can calculate $P(-0.4 \le \epsilon \le 0.4)$ by first computing $\Phi(2)$:

$$\Phi(2) = \int_{-\infty}^{2} \frac{1}{\sqrt{2\pi(0.04)}} e^{-\frac{x^2}{2(0.04)}}\, dx \approx 0.977$$

and so, we get that $P(-0.4 \le \epsilon \le 0.4) = 2\Phi(2) - 1 = 2(0.977) - 1 = 0.954$.

4.

The distribution $X - Y$ is given by the addition of distributions $X \sim \mathcal{N}(a, b)$ and $-Y \sim \mathcal{N}(-c, d)$. Thus, $X - Y = X + (-Y)$ is distributed by $\mathcal{N}(a - c, b + d)$. The standardized version of $X - Y$ is given by $\frac{X-Y-\mu}{\sigma}$, where $\mu = 0$ and $\sigma = \sqrt{b+d}$. Re-expressed, the standardized version of $X - Y$ is given by the following: $\frac{X-Y}{\sqrt{b+d}}$. Given F as the CDF of $X - Y$, $P(X - Y < 0) = F(0) = \Phi(\frac{0}{\sqrt{b+d}}) = \Phi(0) = 0.5$ as normal distributions (i.e. $X - Y$) are symmetric.

5.

Let us define the r.v. $N \sim Pois(20 * 0.1)$ as the number of emails arrived in the first 0.1 hours. Note that the rate parameter of N is $20 * 0.1 = 2$ emails as we are expecting 20 emails/hour over a duration of 0.1 hours. Using count-time duality:

$$
\begin{aligned}
P(T > 0.1) = P(N < 3) &= P(N = 0) + P(N = 1) + P(N = 3) \\
&= \frac{e^{-2}2^0}{0!} + \frac{e^{-2}2^1}{1!} + \frac{e^{-2}2^2}{2!} = e^{-2}[1 + 2 + 2] = 5e^{-2}
\end{aligned}
$$

6. Anish Lakkapragada. I worked independently.