# PSETs Landing Page*

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This is the documentation for using my PSET PDFs responsibly. I post these LaTEX'd PSETs (1) as an education resource for friends at other universities, fellow Yalies, and all those interested and (2) for quick reference. These PSETs are not to be used irresponsibly; only look at the solution after giving each problem an honest attempt. **If YOU USE THESE PSETS TO CHEAT, YOU ARE NOT ONLY STUPID BUT YOU ARE CHEATING YOURSELF OUT OF THE ABILITY TO FALL IN LOVE WITH MATH.** Furthermore, I am not smarter than you and my solutions did not always get a perfect score.

The general format for accessing the (one-indexed) `N`th assigned PSET PDF of a Yale course with course number `CODE` is:

`https://anish.lakkapragada.com/notes/TYPE-CODE/psets/N.pdf`

where `TYPE` is `stats` or `math`. Similarily, to access my solution for this PSET you can go to:

`https://anish.lakkapragada.com/notes/TYPE-CODE/sols/N.pdf`

These PSETs and associated solution PDFs are synchronized daily at 4:20AM with my computer files through a Cronjob Shell Script. If you want to contribute any corrections, please email `anish.lakkapragada@yale.edu`.

*Note that PDF here is referring to Portable Document Format, not to be confused with the veritable Probability Density Function.

**Problem set 6**
**Due Friday February 27 at 11pm**

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**Exercise 6.1 (10 points; Rudin 2.25, modified).** Suppose that $K$ is a compact metric space. Prove that $K$ has a dense subset which is at most countable. (Hint: first show that for every $n \in \mathbb{N}$, there are finitely many neighborhoods of radius $1/n$ whose union covers $K$.)

**Exercise 6.2 (10 points).** Suppose $(x_n)$ is a sequence in a metric space $X$ that converges to some $x \in X$. Show that the set $\{x_n \mid n \in \mathbb{N}\} \cup \{x\}$ is compact.

**Exercise 6.3 (10 points).** Use the definition of convergence to show that $\lim_{n\to\infty} \dfrac{2n+1}{3n-1} = \dfrac{2}{3}$.

**Exercise 6.4 (10 points).** Let $\{x_n\}$ be a sequence of real numbers converging to $x \in \mathbb{R}$. Show that $|x_n| \to |x|$. Is the converse true?

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**Exercise 6.5 (not for credit).** Show that the set $\{p \in \mathbb{Q} \mid 2 < p^2 < 3\}$ is closed and bounded in $\mathbb{Q}$ (with the usual metric), but that it is not compact. (Note: This implies that the Heine-Borel theorem does not hold for $\mathbb{Q}$!)

**Exercise 6.6 (not for credit).** Let $\{x_n\}$ and $\{y_n\}$ be sequences of real numbers. Prove that if $x_n \to x$ and $y_n \to y$, then $\lim_{n\to\infty}(x_n + y_n) = x + y$.