PSETs Landing Page*

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This is the documentation for using my PSET PDFs responsibly. I post these LaTEX'd PSETs (1) as an education resource for friends at other universities, fellow Yalies, and all those interested and (2) for quick reference. These PSETs are not to be used irresponsibly; only look at the solution after giving each problem an honest attempt. **If YOU USE THESE PSETS TO CHEAT, YOU ARE NOT ONLY STUPID BUT YOU ARE CHEATING YOURSELF OUT OF THE ABILITY TO FALL IN LOVE WITH MATH.** Furthermore, I am not smarter than you and my solutions did not always get a perfect score.

The general format for accessing the (one-indexed) `N`th assigned PSET PDF of a Yale course with course number `CODE` is:

`https://anish.lakkapragada.com/notes/TYPE-CODE/psets/N.pdf`

where `TYPE` is `stats` or `math`. Similarily, to access my solution for this PSET you can go to:

`https://anish.lakkapragada.com/notes/TYPE-CODE/sols/N.pdf`

These PSETs and associated solution PDFs are synchronized daily at 4:20AM with my computer files through a Cronjob Shell Script. If you want to contribute any corrections, please email `anish.lakkapragada@yale.edu`.

*Note that PDF here is referring to Portable Document Format, not to be confused with the veritable Probability Density Function.

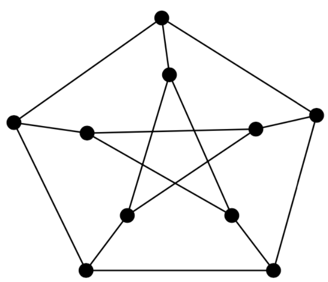
Math 244 - Problem Set 5

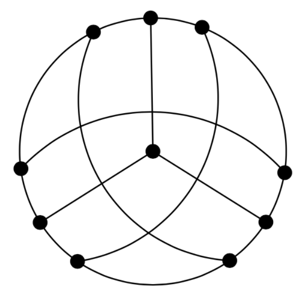
due Friday, March 7, 2025, at 11:59pm

Section 4.1

1. (optional bonus problem)

   (a) Find an isomorphism of the following graphs:

   (b) Show that both the graphs above are isomorphic to the following graph: the vertex set is $\binom{\{1,2,\ldots,5\}}{2}$ (unordered pairs of numbers), and two vertices $\{i,j\}$ and $\{k,\ell\}$ ($i,j,k,\ell \in \{1,2,\ldots,5\}$) form an edge if and only if $\{i,j\} \cap \{k,\ell\} = \emptyset$.

3. An *automorphism* of a graph $G = (V,E)$ is any isomorphism of G and G, i.e., any bijection $f : V \to V$ such that $\{u,v\} \in E$ if and only if $\{f(u), f(v)\} \in E$. A graph is called *asymmetric* if its only automorphism is the identity mapping (each vertex is mapped to itself).

   (a) Find an example of an asymmetric graph with at least two vertices.

   (b) Show that no asymmetric graph G exists with $1 < |V(G)| \leq 5$.

Section 4.2

1. Prove that the complement of a disconnected graph G is connected. (The *complement* of a graph $G = (V, E)$ is the graph $(V, \binom{V}{2} \setminus E)$.)

10. Show that a graph G contains a triangle (i.e., a K_3) if and only if there exist indices i and j such that both the matrices A_G and A_G^2 have the entry (i, j) nonzero, where A_G is the adjacency matrix of G.

Section 4.3

5. Draw all nonisomorphic graphs with score $(6, 3, 3, 3, 3, 3, 3)$. Prove that none was left out!

12. A graph G is called k-regular if all its vertices have degree exactly k. Determine all (k, n) such that there exists a k-regular graph on n vertices. *The textbook has a hint to this problem in the back.*