

Discretionary Note

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IF YOU USE THIS FILE TO CHEAT, YOU ARE NOT ONLY STUPID BUT YOU ARE CHEATING YOURSELF OUT OF THE ABILITY TO FALL IN LOVE WITH MATH. Furthermore, I am not smarter than you and my solutions did not always get a perfect score.

CONTENT STARTS ON NEXT PAGE.

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Problem set 3
Due Thursday February 6 at 11pm

Exercise 3.1 (5 points). Prove that \mathbb{N} is not bounded above in \mathbb{R} .

Exercise 3.2 (10 points; Rudin 2.2+2.3, modified). A real number x is called *algebraic* if there exist $n \in \mathbb{N}$ and $a_0, \dots, a_n \in \mathbb{Z}$, with $a_0 \neq 0$, such that

$$a_0 x^n + a_1 x^{n-1} + \dots + a_{n-1} x + a_n = 0.$$

- (1) Prove that $\sqrt{5}$ and $\sqrt{2 + \sqrt{3}}$ are algebraic.
- (2) Prove that the set of all algebraic real numbers is countable. (You may use without proof the fact that a polynomial of degree n has at most n roots.)
- (3) Prove that there exist real numbers which are not algebraic.

Exercise 3.3 (5 points). Suppose $a, b \in \mathbb{R}$ with $a < b$. Prove that there are uncountably many irrational numbers in the interval (a, b) .

Exercise 3.4 (10 points). Are the following sets finite, countable or uncountable? Prove your answers.

- (1) The set of all finite subsets of \mathbb{N} .
- (2) The set of all subsets of \mathbb{N} .
- (3) The set of all functions $f : \mathbb{Q} \rightarrow \mathbb{Q}$.

Exercise 3.5 (10 points). For each of the following, determine whether X with the distance function d is a metric space, and prove your answer.

- (1) $X = \mathbb{R}, d(x, y) = |x^2 - y^2|$
 - (2) $X = \mathbb{R}, d(x, y) = |x - 2y|$
 - (3) $X = \mathbb{R}, d(x, y) = \frac{|x-y|}{1+|x-y|}$
 - (4) $X = \mathbb{R}^2, d(x, y) = |x_1 - y_1| + |x_2 - y_2|$ [where $x = (x_1, x_2)$ and $y = (y_1, y_2)$]
 - (5) $X = \mathbb{R}^2, d(x, y) = |x_1 - y_2| + |x_2 - y_1|$ [where $x = (x_1, x_2)$ and $y = (y_1, y_2)$]
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