PSETs Landing Page*

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This is the documentation for using my PSET PDFs responsibly. I post these LaTEX'd PSETs (1) as an education resource for friends at other universities, fellow Yalies, and all those interested and (2) for quick reference. These PSETs are not to be used irresponsibly; only look at the solution after giving each problem an honest attempt. **If YOU USE THESE PSETS TO CHEAT, YOU ARE NOT ONLY STUPID BUT YOU ARE CHEATING YOURSELF OUT OF THE ABILITY TO FALL IN LOVE WITH MATH.** Furthermore, I am not smarter than you and my solutions did not always get a perfect score.

The general format for accessing the (one-indexed) `N`th assigned PSET PDF of a Yale course with course number `CODE` is:

`https://anish.lakkapragada.com/notes/TYPE-CODE/psets/N.pdf`

where `TYPE` is `stats` or `math`. Similarily, to access my solution for this PSET you can go to:

`https://anish.lakkapragada.com/notes/TYPE-CODE/sols/N.pdf`

These PSETs and associated solution PDFs are synchronized daily at 4:20AM with my computer files through a Cronjob Shell Script. If you want to contribute any corrections, please email `anish.lakkapragada@yale.edu`.

*Note that PDF here is referring to Portable Document Format, not to be confused with the veritable Probability Density Function.

MATH 241 PSET 7

October 31, 2024

1.

Because F is a continuous function and is strictly increasing (i.e. $\frac{dF}{dx} > 0$) for $x > 0$, this means that $F^{-1}(x)$ exists. Thus, by Universality of the Uniform, $X = F^{-1}(U)$ gives us an r.v. with a CDF of F. We compute $F^{-1}(u)$ below:

$$F(x) = 1 - e^{-x^3}$$
$$u = 1 - e^{-x^3}$$
$$e^{-x^3} = 1 - u$$
$$x^3 = -ln(1-u)$$
$$x = [-ln(1-u)]^{\frac{1}{3}}$$

Thus, $F^{-1}(u) = [-ln(1-u)]^{\frac{1}{3}}$ and so the r.v. $X = F^{-1}(U) = [-ln(1-U)]^{\frac{1}{3}}$ has the CDF F.

2.

The general form of a transformation of the standard normal distribution is $Y = \mu + \sigma Z$, where $Z \sim \mathcal{N}(0,1)$ and $Y \sim \mathcal{N}(\mu, \sigma^2)$. Thus, for a distribution Y with $\sigma = \sqrt{4} = 2$ and $\mu = 1$, $Y = 1 + 2Z$.

3.

Let us define ϵ as an r.v. where $\epsilon \sim \mathcal{N}(0, 0.04)$. The probability that the observed distance is within 0.4 meters of the true distance is given by $P(-0.4 \leq \epsilon \leq 0.4) = \Phi(\frac{0.4-0}{\sqrt{0.04}}) - \Phi(\frac{-0.4-0}{\sqrt{0.04}}) = \Phi(2) - \Phi(-2) = \Phi(2) - (1 - \Phi(2)) = 2\Phi(2) - 1$.

In numerical terms, we can calculate $P(-0.4 \leq \epsilon \leq 0.4)$ by first computing $\Phi(2)$:

$$\Phi(2) = \int_{-\infty}^{2} \frac{1}{\sqrt{2\pi(0.04)}} e^{-\frac{x^2}{2(0.04)}}\, dx \approx 0.977$$

and so, we get that $P(-0.4 \leq \epsilon \leq 0.4) = 2\Phi(2) - 1 = 2(0.977) - 1 = 0.954$.

4.

The distribution $X - Y$ is given by the addition of distributions $X \sim \mathcal{N}(a, b)$ and $-Y \sim \mathcal{N}(-c, d)$. Thus, $X - Y = X + (-Y)$ is distributed by $\mathcal{N}(a - c, b + d)$. The standardized version of $X - Y$ is given by $\frac{X-Y-\mu}{\sigma}$, where $\mu = 0$ and $\sigma = \sqrt{b + d}$. Re-expressed, the standardized version of $X - Y$ is given by the following: $\frac{X-Y}{\sqrt{b+d}}$. Given F as the CDF of $X - Y$, $P(X - Y < 0) = F(0) = \Phi(\frac{0}{\sqrt{b+d}}) = \Phi(0) = 0.5$ as normal distributions (i.e. $X - Y$) are symmetric.

5.

Let us define the r.v. $N \sim Pois(20 * 0.1)$ as the number of emails arrived in the first 0.1 hours. Note that the rate parameter of N is $20 * 0.1 = 2$ emails as we are expecting 20 emails/hour over a duration of 0.1 hours. Using count-time duality:

$$\begin{aligned} P(T > 0.1) = P(N < 3) &= P(N = 0) + P(N = 1) + P(N = 3) \\ &= \frac{e^{-2}2^0}{0!} + \frac{e^{-2}2^1}{1!} + \frac{e^{-2}2^2}{2!} = e^{-2}[1 + 2 + 2] = 5e^{-2} \end{aligned}$$

6. Anish Lakkapragada. I worked independently.