

Discretionary Note

Anish Krishna Lakkapragada

IF YOU USE THIS FILE TO CHEAT, YOU ARE NOT ONLY STUPID BUT YOU ARE CHEATING YOURSELF OUT OF THE ABILITY TO FALL IN LOVE WITH MATH. Furthermore, I am not smarter than you and my solutions did not always get a perfect score.

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Question 1

$$(a) \quad T_1 = \frac{(40 + 30 + T_3 + T_2)}{4} \rightarrow 4T_1 - T_3 - T_2 = 70$$

$$T_2 = \frac{(10 + 30 + T_4 + T_1)}{4} \rightarrow 4T_2 - T_4 - T_1 = 40$$

$$T_3 = \frac{(20 + 40 + T_4 + T_1)}{4} \rightarrow 4T_3 - T_4 - T_1 = 60$$

$$T_4 = \frac{(20 + 10 + T_2 + T_3)}{4} \rightarrow 4T_4 - T_3 - T_2 = 30$$

$$(b) \quad A = \begin{bmatrix} 4 & -1 & -1 & 0 \\ -1 & 4 & 0 & -1 \\ -1 & 0 & 4 & -1 \\ 0 & -1 & -1 & 4 \end{bmatrix}$$

Applying G.E.:

$$\left[\begin{array}{cccc|cccc} 4 & -1 & -1 & 0 & 1 & 0 & 0 & 0 \\ -1 & 4 & 0 & -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 4 & -1 & 0 & 0 & 1 & 0 \\ 0 & -1 & -1 & 4 & 0 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_1 \leftrightarrow R_4} \left[\begin{array}{cccc|cccc} 0 & -1 & -1 & 4 & 0 & 0 & 0 & 1 \\ -1 & 4 & 0 & -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 4 & -1 & 0 & 0 & 1 & 0 \\ 4 & -1 & -1 & 0 & 1 & 0 & 0 & 0 \end{array} \right] \xrightarrow{R_2 = R_2 + R_1, R_3 = R_3 + R_1} \left[\begin{array}{cccc|cccc} 0 & -1 & -1 & 4 & 0 & 0 & 0 & 1 \\ 0 & 3 & -1 & 3 & 0 & 1 & 0 & 1 \\ 0 & -1 & 3 & 3 & 0 & 0 & 1 & 1 \\ 4 & -1 & -1 & 0 & 1 & 0 & 0 & 0 \end{array} \right]$$

$$\left[\begin{array}{cccc|cccc} 0 & -1 & -1 & 4 & 0 & 0 & 0 & 1 \\ 0 & 3 & -1 & 3 & 0 & 1 & 0 & 1 \\ 0 & -1 & 3 & 3 & 0 & 0 & 1 & 1 \\ 4 & -1 & -1 & 0 & 1 & 0 & 0 & 0 \end{array} \right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{cccc|cccc} 0 & 3 & -1 & 3 & 0 & 1 & 0 & 1 \\ 0 & -1 & -1 & 4 & 0 & 0 & 0 & 1 \\ 0 & -1 & 3 & 3 & 0 & 0 & 1 & 1 \\ 4 & -1 & -1 & 0 & 1 & 0 & 0 & 0 \end{array} \right] \xrightarrow{R_3 = R_3 + R_2, R_4 = R_4 + 4R_2} \left[\begin{array}{cccc|cccc} 0 & 3 & -1 & 3 & 0 & 1 & 0 & 1 \\ 0 & -1 & -1 & 4 & 0 & 0 & 0 & 1 \\ 0 & -2 & 2 & 7 & 0 & 0 & 1 & 2 \\ 4 & -5 & -5 & -4 & 1 & 0 & 0 & 4 \end{array} \right]$$

$$\left[\begin{array}{cccc|cccc} 0 & 3 & -1 & 3 & 0 & 1 & 0 & 1 \\ 0 & -1 & -1 & 4 & 0 & 0 & 0 & 1 \\ 0 & -2 & 2 & 7 & 0 & 0 & 1 & 2 \\ 4 & -5 & -5 & -4 & 1 & 0 & 0 & 4 \end{array} \right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{cccc|cccc} 0 & -1 & -1 & 4 & 0 & 0 & 0 & 1 \\ 0 & 3 & -1 & 3 & 0 & 1 & 0 & 1 \\ 0 & -2 & 2 & 7 & 0 & 0 & 1 & 2 \\ 4 & -5 & -5 & -4 & 1 & 0 & 0 & 4 \end{array} \right] \xrightarrow{R_2 = R_2 + 3R_1, R_3 = R_3 + 2R_1, R_4 = R_4 + 4R_1} \left[\begin{array}{cccc|cccc} 0 & -1 & -1 & 4 & 0 & 0 & 0 & 1 \\ 0 & 0 & -4 & 15 & 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & 15 & 0 & 2 & 1 & 4 \\ 4 & -1 & -1 & 0 & 1 & 0 & 0 & 0 \end{array} \right]$$

$$\left[\begin{array}{cccc|cccc} 0 & -1 & -1 & 4 & 0 & 0 & 0 & 1 \\ 0 & 0 & -4 & 15 & 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & 15 & 0 & 2 & 1 & 4 \\ 4 & -1 & -1 & 0 & 1 & 0 & 0 & 0 \end{array} \right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{cccc|cccc} 0 & 0 & -4 & 15 & 0 & 1 & 3 & 4 \\ 0 & -1 & -1 & 4 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 15 & 0 & 2 & 1 & 4 \\ 4 & -1 & -1 & 0 & 1 & 0 & 0 & 0 \end{array} \right] \xrightarrow{R_2 = R_2 + R_1, R_3 = R_3 + 4R_2, R_4 = R_4 + 4R_2} \left[\begin{array}{cccc|cccc} 0 & 0 & -4 & 15 & 0 & 1 & 3 & 4 \\ 0 & -1 & -1 & 4 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 15 & 0 & 2 & 1 & 4 \\ 4 & -5 & -5 & -4 & 1 & 0 & 0 & 4 \end{array} \right]$$

✓ (A-1)

$$R_3 + 2R_2 \rightarrow R_3$$

Question 1 continued

(c) $AT = X$

$$T = A^{-1}X$$

$$T = \begin{bmatrix} \frac{7}{24} & \frac{1}{12} & \frac{1}{12} & \frac{1}{24} \\ \frac{1}{12} & \frac{7}{24} & \frac{1}{24} & \frac{1}{12} \\ \frac{1}{12} & \frac{1}{24} & \frac{7}{24} & \frac{1}{12} \\ \frac{1}{24} & \frac{1}{12} & \frac{1}{12} & \frac{7}{24} \end{bmatrix} \begin{bmatrix} 30 \\ 40 \\ 60 \\ 30 \end{bmatrix} = \begin{bmatrix} 30 \\ 22.5 \\ 27.5 \\ 20 \end{bmatrix}$$

Question 2

To find the basis of $N(L_A)$, we solve $AX=0$

$$\begin{pmatrix} 3 & 5 & -4 & 2 & 0 \\ 2 & 4 & -6 & 3 & 0 \\ 11 & 17 & -8 & 9 & 0 \end{pmatrix} \xrightarrow{\substack{3R_2 - 2R_1 \rightarrow R_2 \\ 3R_3 - 11R_1 \rightarrow R_3}} \begin{pmatrix} 3 & 5 & -4 & 2 & 0 \\ 0 & 2 & -10 & 5 & 0 \\ 0 & -4 & 20 & -10 & 0 \end{pmatrix} \xrightarrow{\substack{2R_2 + R_3 \rightarrow R_3 \\ 2R_1 - 5R_2 \rightarrow R_1}} \begin{pmatrix} 6 & 0 & 42 & -21 & 0 \\ 0 & 2 & -10 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$x_1 + 7x_3 - 3.5x_4 = 0$$

$$2x_2 - 10x_3 + 5x_4 = 0$$

$$x_1 = 3.5x_4 - 7x_3$$

$$x_2 = 5x_3 - 2.5x_4$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 3.5t_4 - 7t_3 \\ 5t_3 - 2.5t_4 \\ t_3 \\ t_4 \end{pmatrix} = t_3 \begin{pmatrix} -7 \\ 5 \\ 1 \\ 0 \end{pmatrix} + t_4 \begin{pmatrix} 3.5 \\ -2.5 \\ 0 \\ 1 \end{pmatrix}$$

Thus, the basis for $N(L_A)$ can be given by $\left\{ \begin{pmatrix} -7 \\ 5 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 3.5 \\ -2.5 \\ 0 \\ 1 \end{pmatrix} \right\}$.

The set of solutions to the given system is given by the span of this basis.

Question 3

$$AX=b$$

$$A^T X = b$$

$$[A|b] = \left(\begin{array}{cccc|c} 1 & -3 & -3 & -14 & 8 \\ 2 & -6 & -3 & -1 & -5 \\ 3 & -9 & -5 & -6 & -4 \end{array} \right) \xrightarrow{\substack{R_2 - 2R_1 \rightarrow R_2 \\ R_3 - 3R_1 \rightarrow R_3}} \left(\begin{array}{cccc|c} 1 & -3 & -3 & -14 & 8 \\ 0 & 0 & 3 & 27 & -21 \\ 0 & 0 & 4 & 36 & -28 \end{array} \right)$$

$$\left(\begin{array}{cccc|c} 1 & -3 & 0 & 13 & -13 \\ 0 & 0 & 1 & 9 & -7 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \xleftarrow{\substack{R_2 \rightarrow R_2 \\ \frac{R_2}{3} \rightarrow R_2}} \left(\begin{array}{cccc|c} 1 & -3 & 0 & 13 & -13 \\ 0 & 0 & 3 & 27 & -21 \\ 0 & 0 & 4 & 36 & -28 \end{array} \right) \xleftarrow{R_1 + R_2 \rightarrow R_1}$$

$$3R_3 - 4R_2 \rightarrow R_3$$

$$\begin{cases} x_1 - 3x_2 + 13x_4 = -13 \Rightarrow x_1 = -13 + 3x_2 - 13x_4 = -13 + 3t_2 - 13t_4 \\ x_3 + 9x_4 = -7 \Rightarrow x_3 = -7 - 9t_4 \end{cases}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -13 + 3t_2 - 13t_4 \\ t_2 \\ -7 - 9t_4 \\ t_4 \end{pmatrix} = \begin{pmatrix} -13 \\ 0 \\ -7 \\ 0 \end{pmatrix} + t_2 \begin{pmatrix} 3 \\ 1 \\ 0 \\ 0 \end{pmatrix} + t_4 \begin{pmatrix} -13 \\ 0 \\ -9 \\ 1 \end{pmatrix}$$

Vectors in \mathbb{R}^4 whose image is equal to $b \in \mathbb{R}^3$ under linear map given in A.

The basis of $N(A)$ can be given by $\left\{ \begin{pmatrix} 3 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -13 \\ 0 \\ 9 \\ 1 \end{pmatrix} \right\}$.

Question 4

(i)

$$[T]^\beta = \begin{bmatrix} 1 & -2 & 4 & -8 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 \end{bmatrix}$$

$$T(1) = (1, 1, 1, 1)$$

$$T(x) = (-2, -1, 1, 2)$$

$$T(x^2) = (4, 1, 1, 4)$$

$$T(x^3) = (-8, -1, 1, 8)$$

(ii)

$$Tx = \begin{bmatrix} 1 \\ 3 \\ 3 \\ 33 \end{bmatrix}$$

$$\left[T \mid \begin{bmatrix} 1 \\ 3 \\ 3 \\ 33 \end{bmatrix} \right] \rightarrow \left[\begin{array}{cccc|c} 1 & -2 & 4 & -8 & 1 \\ 1 & -1 & 1 & -1 & 3 \\ 1 & 1 & 1 & 1 & 3 \\ 1 & 2 & 4 & 8 & 33 \end{array} \right] \xrightarrow{\substack{R_2 - R_1 \rightarrow R_2 \\ R_3 - R_1 \rightarrow R_3 \\ R_4 - R_1 \rightarrow R_4}} \left[\begin{array}{cccc|c} 1 & -2 & 4 & -8 & 1 \\ 0 & 1 & -3 & 7 & 2 \\ 0 & 3 & -3 & 9 & 12 \\ 0 & 4 & 0 & 16 & 32 \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 3 & 0 & 0 & 6 & 21 \\ 0 & 2 & 0 & 2 & 10 \\ 0 & 0 & 6 & -12 & 6 \\ 0 & 0 & 0 & 12 & 12 \end{array} \right]$$

$$\begin{aligned} R_4 - 2R_3 &\rightarrow R_4 \\ 2R_2 + R_3 &\rightarrow R_2 \\ 3R_1 + R_3 &\rightarrow R_1 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 6 & 5 \\ 0 & 1 & -3 & 7 & 2 \\ 0 & 0 & 6 & -12 & 18 \\ 0 & 0 & 12 & -12 & 24 \end{array} \right]$$

$$\begin{aligned} R_3 - 3R_2 &\rightarrow R_3 \\ R_4 - 4R_2 &\rightarrow R_4 \\ 2R_2 + R_1 &\rightarrow R_1 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 6 & 0 & 0 & 0 & 30 \\ 0 & 12 & 0 & 0 & 48 \\ 0 & 0 & 6 & 6 & 18 \\ 0 & 0 & 0 & 12 & 12 \end{array} \right] \xrightarrow{\substack{R_1 \rightarrow R_1 \\ R_2 \rightarrow R_2 \\ R_3 \rightarrow R_3 \\ R_4 \rightarrow R_4}} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 & 4 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{array} \right]$$

Thus, we get this polynomial as:

$$f(x) = 5 + 4x + 3x^2 + x^3$$

Question 5

$$V = \{ (x_1, x_2, x_3, x_4, x_5) : x_1 - 2x_2 + 3x_3 - x_4 + 2x_5 = 0 \}$$

is finally a subspace of \mathbb{R}^5 .

We first find the basis of V :

$$x_1 = 2x_2 - 3x_3 + x_4 - 2x_5$$

$$\begin{aligned} (x_1, x_2, x_3, x_4, x_5) &= (2t_2 - 3t_3 + t_4 - 2t_5, t_2, t_3, t_4, t_5) \\ &= t_2(2, 1, 0, 0, 0) + t_3(-3, 0, 1, 0, 0) + t_4(1, 0, 0, 1, 0) + t_5(-2, 0, 0, 0, 1) \end{aligned}$$

Thus, the basis for V , B_V is given by

$$B_V = \{ (2, 1, 0, 0, 0), (-3, 0, 1, 0, 0), (1, 0, 0, 1, 0), (-2, 0, 0, 0, 1) \}$$

Given set $S = \{v\} = \{ (1, 2, 1, 0, 0) \}$:

$$\begin{pmatrix} 1 & 2 & -3 & 1 & -2 \\ 2 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{\substack{R_3 - R_1 \rightarrow R_3 \\ 2R_1 - R_2 \rightarrow R_2}} \begin{pmatrix} 1 & 2 & -3 & 1 & -2 \\ 0 & 3 & -6 & 2 & -4 \\ 0 & -2 & 4 & -1 & 2 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{aligned} &\xrightarrow{\substack{R_3 + 2R_5 \rightarrow R_3 \\ 2R_5 + R_4 \rightarrow R_5}} \begin{pmatrix} 3 & 0 & 3 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{\substack{R_3 - R_4 \rightarrow R_3 \\ R_3 + R_1 \rightarrow R_1 \\ R_2 - 2R_3 \rightarrow R_2}} \begin{pmatrix} 3 & 0 & 3 & -1 & 2 \\ 0 & 3 & -6 & 2 & -4 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{\substack{3R_1 - 2R_2 \rightarrow R_1 \\ 3R_3 + 2R_2 \rightarrow R_3}} \\ &\xrightarrow{\substack{R_1 \rightarrow R_1 \\ R_2 \rightarrow R_2 \\ R_4 \rightarrow R_4}} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{aligned}$$

Question 5 continued

Thus,

$$\{(1, 2, 1, 0, 0), (2, 1, 0, 0, 0), (1, 0, 0, 1, 0), (-2, 0, 0, 0, 1)\}$$

is a basis for V that includes v .