PSETs Landing Page*

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This is the documentation for using my PSET PDFs responsibly. I post these LaTEX'd PSETs (1) as an education resource for friends at other universities, fellow Yalies, and all those interested and (2) for quick reference. These PSETs are not to be used irresponsibly; only look at the solution after giving each problem an honest attempt. **If YOU USE THESE PSETS TO CHEAT, YOU ARE NOT ONLY STUPID BUT YOU ARE CHEATING YOURSELF OUT OF THE ABILITY TO FALL IN LOVE WITH MATH.** Furthermore, I am not smarter than you and my solutions did not always get a perfect score.

The general format for accessing the (one-indexed) `N`th assigned PSET PDF of a Yale course with course number `CODE` is:

`https://anish.lakkapragada.com/notes/TYPE-CODE/psets/N.pdf`

where `TYPE` is `stats` or `math`. Similarily, to access my solution for this PSET you can go to:

`https://anish.lakkapragada.com/notes/TYPE-CODE/sols/N.pdf`

These PSETs and associated solution PDFs are synchronized daily at 4:20AM with my computer files through a Cronjob Shell Script. If you want to contribute any corrections, please email `anish.lakkapragada@yale.edu`.

*Note that PDF here is referring to Portable Document Format, not to be confused with the veritable Probability Density Function.

Problem set 4
Due Sunday February 16 at 11pm

Exercise 4.1 (10 points). Let X be any set, let and $d : X \times X \to \mathbb{R}$ be the discrete metric, defined by

$$d(x,y) = \begin{cases} 0 & x = y \\ 1 & x \neq y \end{cases}$$

for all $x, y \in X$.

(1) Prove that, with this distance function, X is a metric space.

(2) For any $x \in X$, what is $N_\epsilon(x)$ when $\epsilon = \frac{1}{2}$, 1, and 2?

(3) Which subsets of X are open? Which are closed?

Exercise 4.2 (6 points; Rudin 2.5). Construct a bounded set of real numbers with exactly three limit points (using the standard metric on $\mathbb{R}$). (You need not prove carefully what the limit points are; it is sufficient to give the set, give bounds for the set, and state what are the limit points.)

Exercise 4.3 (24 points; Rudin 2.9). Let E be a subset of a metric space. Define the *interior* of E, denoted E°, to be the set of all interior points of E.

(1) Prove that E° is always open.

(2) Prove that E is open if and only if $E^\circ = E$.

(3) Prove that, if G is an open subset of E, then $G \subset E^\circ$.

(4) Prove that the complement of E° is the closure of the complement of E.

(5) Do E and $\overline{E}$ always have the same interiors?

(6) Do E and E° always have the same closures?