Math 244 - Problem Set 3

due Monday, February 10, 2025, at 11:59pm

Section 2.3

1. How many linear extensions of $\mathcal{B}_2$ are there, and what about $\mathcal{B}_3$?

5. (optional bonus problem) ~~Prove that not every finite poset admits an embedding into the ordered set of triples of real numbers as in Example 2.1.1.~~ *Note from Prof. Hall: This problem as stated seems to be unnecessarily difficult, so we are replacing it with a simpler problem:* Prove that not every finite poset admits an embedding into the poset $(\mathbb{N}^2, \preceq)$, where $(x_1, y_1) \preceq (x_2, y_2)$ if and only if $x_1 \leq x_2$ and $y_1 \leq y_2$.

Section 2.4

3. Find a sequence of real numbers of length 16 that contains no monotone subsequence of length 5. *Note from Prof. Hall: The version in the textbook has "17" instead of "16", but this is a typo.*

4. Prove the following strengthening of Theorem 2.4.6: Let k, ℓ be natural numbers. Then every sequence of real numbers of length $k\ell+1$ contains a nondecreasing subsequence of length $k+1$ or a decreasing subsequence of length $\ell + 1$.

Section 3.1

2. Determine the number of ordered pairs (A, B), where $A \subseteq B \subseteq \{1, 2, \ldots, n\}$.

6. Show that a natural number $n \geq 1$ has an odd number of divisors (including 1 and itself) if and only if $\sqrt{n}$ is an integer. *The textbook has a hint to this problem in the back.*