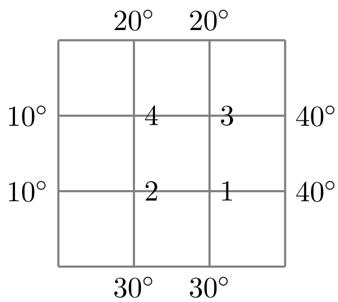
Math 226- HW 7 Due: Oc 29 by Midnight

1. An important concern in the study of heat transfer is to determine the steady-state temperature distribution of a thin plate when the temperature around the boundary is known. Assume the plate shown in the figure represents a cross section of a metal beam, with negligible heat flow in the direction perpendicular to the plate. Let T_1, T_2, T_3, T_4 denote the temperatures at the four interior nodes of the mesh in the figure. The temperature at a node is approximately equal to the average of the four nearest nodes- to the left, above, to the right, and below. For instance $T_1 = (40 + 30 + T_3 + T_2)/4$



   a) (4 points) Write a system of four equations whose solution gives estimates for the temperatures T_1, T_2, T_3, T_4
   b) (6 points) Write down the coefficient matrix and find its inverse.
   c) (2 points) Use part a) to give the estimated values of T_1, T_2, T_3, T_4.

2. (6 points) Find a basis of the kernel (that is $N(L_A)$) of the linear mappings given by

$$A = \begin{pmatrix} 3 & 5 & -4 & 2 \\ 2 & 4 & -6 & 3 \\ 11 & 17 & -8 & 4 \end{pmatrix}$$

   Use it to describe the solutions to

$$\begin{aligned} 3x_1 + 5x_2 - 4x_3 + 2x_4 &= 0 \\ 2x_1 + 4x_2 - 6x_3 + 3x_4 &= 0 \\ 11x_1 + 17x_2 - 8x_3 + 4x_4 &= 0 \end{aligned}$$

3. (12 points) Find all vectors in a space $\mathbb{R}^4$, whose image is equal to the vector $b \in \mathbb{R}^3$ under the linear map $\mathbb{R}^4 \to \mathbb{R}^3$ given by the matrix A. (That is find the solutions to $A\vec{x} = \vec{b}$). Use these vectors to give a basis to $N(L_A)$.

$$A = \begin{pmatrix} 1 & -3 & -3 & -14 \\ 2 & -6 & -3 & -1 \\ 3 & -9 & -5 & -6 \end{pmatrix}, \quad b = \begin{pmatrix} 8 \\ -5 \\ -4 \end{pmatrix}$$

4. (10 points) Consider the map $T : P_3(\mathbb{R}) \to \mathbb{R}^4$ such that $T(f) = (f(-2), f(-1), f(1), f(2))$.

   i) (5 points) Let $\gamma = \{1, x, x^2, x^3\}$, and β be the standard basis for $\mathbb{R}^4$. Compute $[T]_\gamma^\beta$.
   ii) (5 points) Use $[T]_\gamma^\beta$ to find a third degree polynomial such that $f(-2) = 1$, $f(-1) = 3$, $f(1) = 13$, $f(2) = 33$.

5. (10 points)Read “An Interpretation of the Reduced Row Echelon Form” from the book (pg 189-194). Then answer the following question

$$V = \{(x_1, x_2, x_3, x_4, x_5) : x_1 - 2x_2 + 3x_3 - x_4 + 2x_5 = 0\}$$

   Note that $v = (1, 2, 1, 0, 0)$ is a vector in V. Use column correspondance to find a basis for V that includes v.

Practice Problems

1. Let $T, U : V \to W$ be linear transformations.
   a) Prove that $R(U+T) \subseteq R(T) + R(U)$.
   b) Prove that if W is finite dimensional, then $rank(T+U) \leq rank(T) + rank(U)$
   c) Deduce that $rank(A+B) \leq rank(A) + rank(B)$ for any $m \times n$ matrix.
2. Let $A = [a_{ji}]_{n\times n}$, $B = [b_{ji}]_{n\times n}$ be matrices. The trace of A is defined by

$$tr(A) = \sum_{i=1}^{n} a_{ii}$$

   Prove that
   a) $tr(A+B) = tr(A) + tr(B)$
   b) $tr(AB) = tr(BA)$
   c) Let Q be invertible matrix then $tr(QAQ^{-1}) = tr(A)$
3. Section 2.3: 1-13, 16-19
4. Section 2.4: 1-19
5. Section 2.5: 1-7
6. Section 3.2: 1-10