

Discretionary Note

Anish Krishna Lakkapragada

IF YOU USE THIS FILE TO CHEAT, YOU ARE NOT ONLY STUPID BUT YOU ARE CHEATING YOURSELF OUT OF THE ABILITY TO FALL IN LOVE WITH MATH. Furthermore, I am not smarter than you and my solutions did not always get a perfect score.

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Problem set 6
Due Friday February 27 at 11pm

Exercise 6.1 (10 points; Rudin 2.25, modified). Suppose that K is a compact metric space. Prove that K has a dense subset which is at most countable. (Hint: first show that for every $n \in \mathbb{N}$, there are finitely many neighborhoods of radius $1/n$ whose union covers K .)

Exercise 6.2 (10 points). Suppose (x_n) is a sequence in a metric space X that converges to some $x \in X$. Show that the set $\{x_n \mid n \in \mathbb{N}\} \cup \{x\}$ is compact.

Exercise 6.3 (10 points). Use the definition of convergence to show that $\lim_{n \rightarrow \infty} \frac{2n+1}{3n-1} = \frac{2}{3}$.

Exercise 6.4 (10 points). Let $\{x_n\}$ be a sequence of real numbers converging to $x \in \mathbb{R}$. Show that $|x_n| \rightarrow |x|$. Is the converse true?

Exercise 6.5 (not for credit). Show that the set $\{p \in \mathbb{Q} \mid 2 < p^2 < 3\}$ is closed and bounded in \mathbb{Q} (with the usual metric), but that it is not compact. (Note: This implies that the Heine-Borel theorem does not hold for \mathbb{Q} !)

Exercise 6.6 (not for credit). Let $\{x_n\}$ and $\{y_n\}$ be sequences of real numbers. Prove that if $x_n \rightarrow x$ and $y_n \rightarrow y$, then $\lim_{n \rightarrow \infty} (x_n + y_n) = x + y$.