Neuronal Signal Modulation by Dendritic Geometry

by

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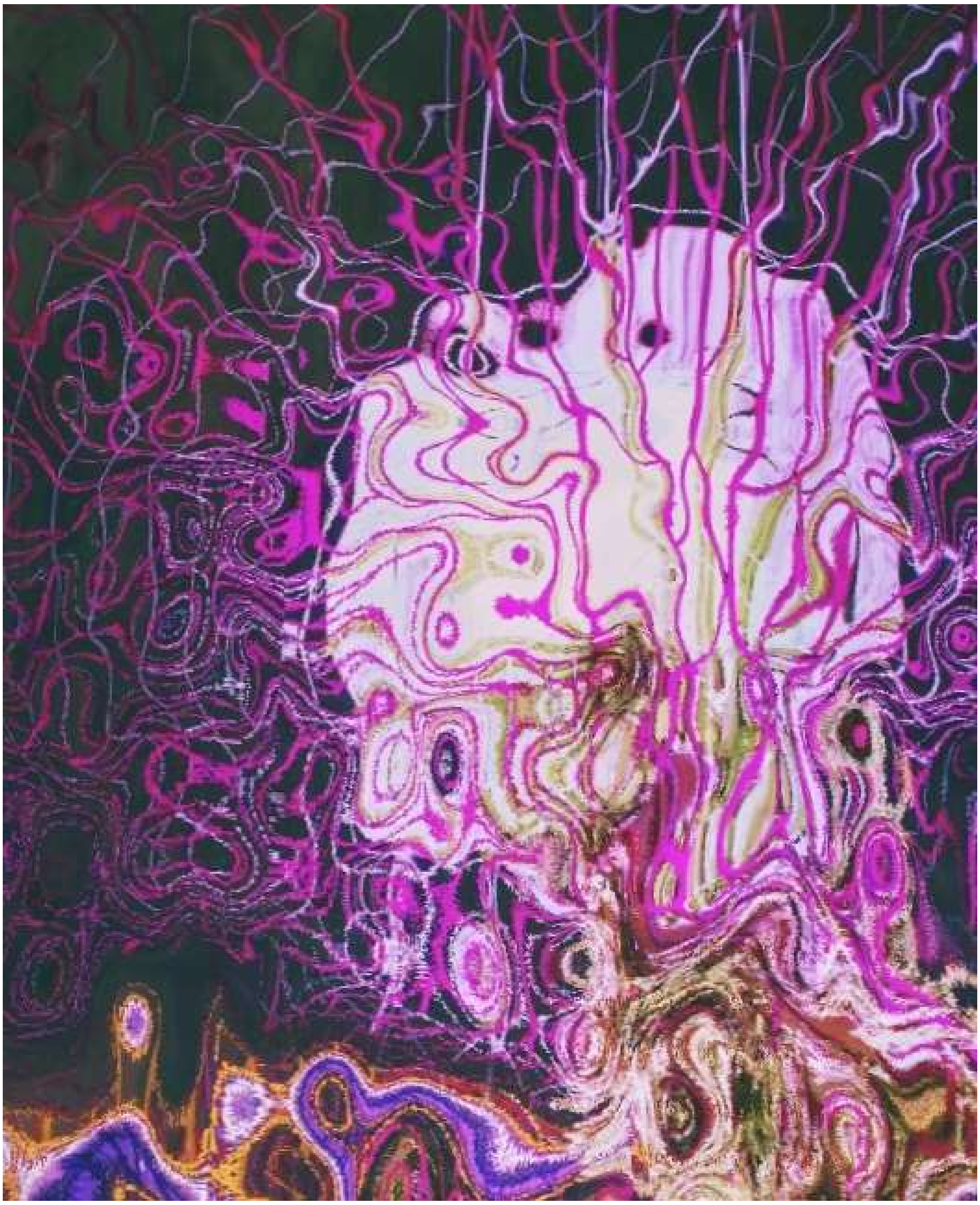
Cogito, ergo sum.

– René Descartes



Si Xu (*Thinking*). Xiaohe Lu, 2018.

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Abstract

Neurons are the basic units in nervous systems. They transmit electrical signals along neurites and at synapses. Neuronal morphology, mainly dendritic geometry, is famous for anatomical diversity, and many neuronal types are named after their morphologies. As well as distributions of ion channels on cell membranes, dendritic geometries contribute significantly to distinct behaviours of signal filtration and integration in different types of neurons (even under the circumstances of receiving same inputs *in vitro*).

In this thesis, I mainly address the importance of dendritic geometry, by investigating its modulation on neuronal signals via mathematical and computational approaches. By "geometry", I consider both branching structures of entire dendritic trees and tapered structures of individual dendritic branches.

The mathematical model of dendritic electro-physiology is established by generalising the classical cable theory. It forms the theoretical benchmark for this thesis to study neuronal signal modulation on branching dendrites with tapered branches. A novel method to solve for analytical response functions on such structures is developed. It permits theoretical analysis of a neuron as an electrical circuit, and enables accurate and efficient simulations on neuronal networks.

By invetigating simplified but representative dendritic geometries, it is found that the tapered dendrite amplifies signals in comparison to the non-tapered dendrite. This modulation is almost a local effect, which is merely influenced by the global geometry. Nonetheless, the global geometry has a stronger impact on signal amplitudes, and even more on phases.

In addition, the methodology employed in this thesis is perfectly compatible with other existing methods, which makes it straightforward to recruit stochasticity and non-linearity into the framework. Future works of large-scale neural networks can easily adapt this work to improve computational efficiency, while preserving a large amount of biophysical details.