

Neuronal Signal Modulation by Dendritic Geometry

by

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Thesis

Submitted to the University of Warwick

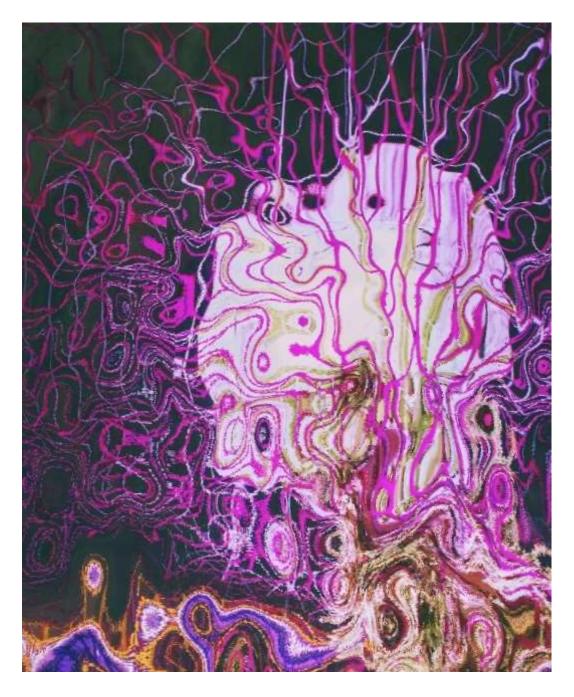
for the degree of

Doctor of Philosophy

Centre for Complexity Science

July 2018





 ${\bf Si}~{\bf Xu}$ (Thinking). Xiaohe Lu, 2018.

Contents

Acknowledgments				
Declar	ations		v	
Abstra	act		vi	
Abbre	viation	as	vii	
Chapte	er 1 I	ntroduction	1	
1.1	Overv	iew of neuroscience	2	
1.2	Summ	nary of methodology	4	
1.3	Outlin	ne of thesis	7	
$\mathbf{Chapt}_{\mathbf{c}}$	er 2 I	Dendritic Morphology and Electro-Physiology	9	
2.1	Branc	hing structures of dendritic trees	10	
	2.1.1	Digital reconstructions	11	
	2.1.2	Metric graphs	14	
	2.1.3	Point neurons	15	
2.2	Mathematical models of membrane potentials			
	2.2.1	Electrical circuits	17	
	2.2.2	Spiking neurons	21	
	2.2.3	Synaptic activities	24	
2.3	Cable theory on dendritic trees			
	2.3.1	Cable equations	27	
	2.3.2	Boundary conditions	32	
	2.3.3	Green's functions	34	
Chapte	er 3 1	Method of Local Point Matching	39	
3.1	Frame	ework of sum-over-trips	40	
	3.1.1	Random walks on passive dendrites	40	

	3.1.2	Green's functions on resonant dendrites	44		
	3.1.3	Summary of sum-over-trips	48		
3.2	Method of local point matching				
	3.2.1	Convergence of sum-over-trips	50		
	3.2.2	Deriviation of local point matching	53		
	3.2.3	Summary of local point matching	56		
3.3	Results on arbitrary dendritic trees				
	3.3.1	Properties of Green's functions	57		
	3.3.2	Features of local morphology	58		
	3.3.3	Responses at steady states	61		
Chapt	er 4 S	Sum-Over-Trips with Taper	64		
4.1	Mathe	ematical models of single tapered cables	65		
	4.1.1	Cable equations with reducible taper	66		
	4.1.2	Cable equations with realistic taper	72		
	4.1.3	Parabolic taper and Exponential type	78		
4.2	Sum-c	over-trips with reducible taper	81		
	4.2.1	Green's functions with reducible taper	81		
	4.2.2	Deriviation of generalised node factors	83		
	4.2.3	Generalised sum-over-trips framework	88		
4.3	Sum-over-trips with general taper				
	4.3.1	General green's functions on two realistic tapers	93		
	4.3.2	Sum-over-trips by finite element method	96		
	4.3.3	Sum-over-trips with heterogeneous electrical properties	99		
Chapt	er 5 I	Response Functions and Numerical Results	103		
5.1	Single	e neuron with single dendritic branch	104		
5.2	Single	e neuron with "Y"-shaped dendritic tree	114		
5.3	Two r	neurons coupled by gap junctions	118		
Chapt	er 6 (Conclusion	127		
6.1	Thesis	s summary	128		
6.2	Further works				
6.3	Misce	llaneous notes	131		

Abstract

Neurons are the basic units in nervous systems. They transmit electrical signals along neurites and at synapses. Neuronal morphology, mainly dendritic geometry, is famous for anatomical diversity, and many neuronal types are named after their morphologies. Dendritic geometries, as well as distributions of ion channels on cell membranes, contribute significantly to distinct behaviours of signal filtration and integration in different types of neurons (even under the circumstances of receiving same inputs in vitro).

In this thesis, I mainly address the importance of dendritic geometry, by investigating its modulation on neuronal signals via mathematical and computational approaches. By "geometry", I consider both branching structures of entire dendritic trees and tapered structures of individual dendritic branches.

The mathematical model of dendritic electro-physiology is established by generalising the classical cable theory. It forms the theoretical benchmark for this thesis to study neuronal signal modulation on branching dendrites with tapered branches. A novel method to solve for analytical response functions on such structures is developed. It permits theoretical analysis of a neuron as an electrical circuit, and enables accurate and efficient simulations on neuronal networks.

By investigating simplified but representative dendritic geometries, it is found that the tapered dendrite amplifies signals in comparison to the non-tapered dendrite. This modulation is almost a local effect, which is merely influenced by the global geometry. Nonetheless, the global geometry has a stronger impact on signal amplitudes, and even more on phases.

In addition, the methodology employed in this thesis is perfectly compatible with other existing methods, which makes it straightforward to recruit stochasticity and non-linearity into the framework. Future works of large-scale neural networks can easily adapt this work to improve computational efficiency, while preserving a large amount of biophysical details.