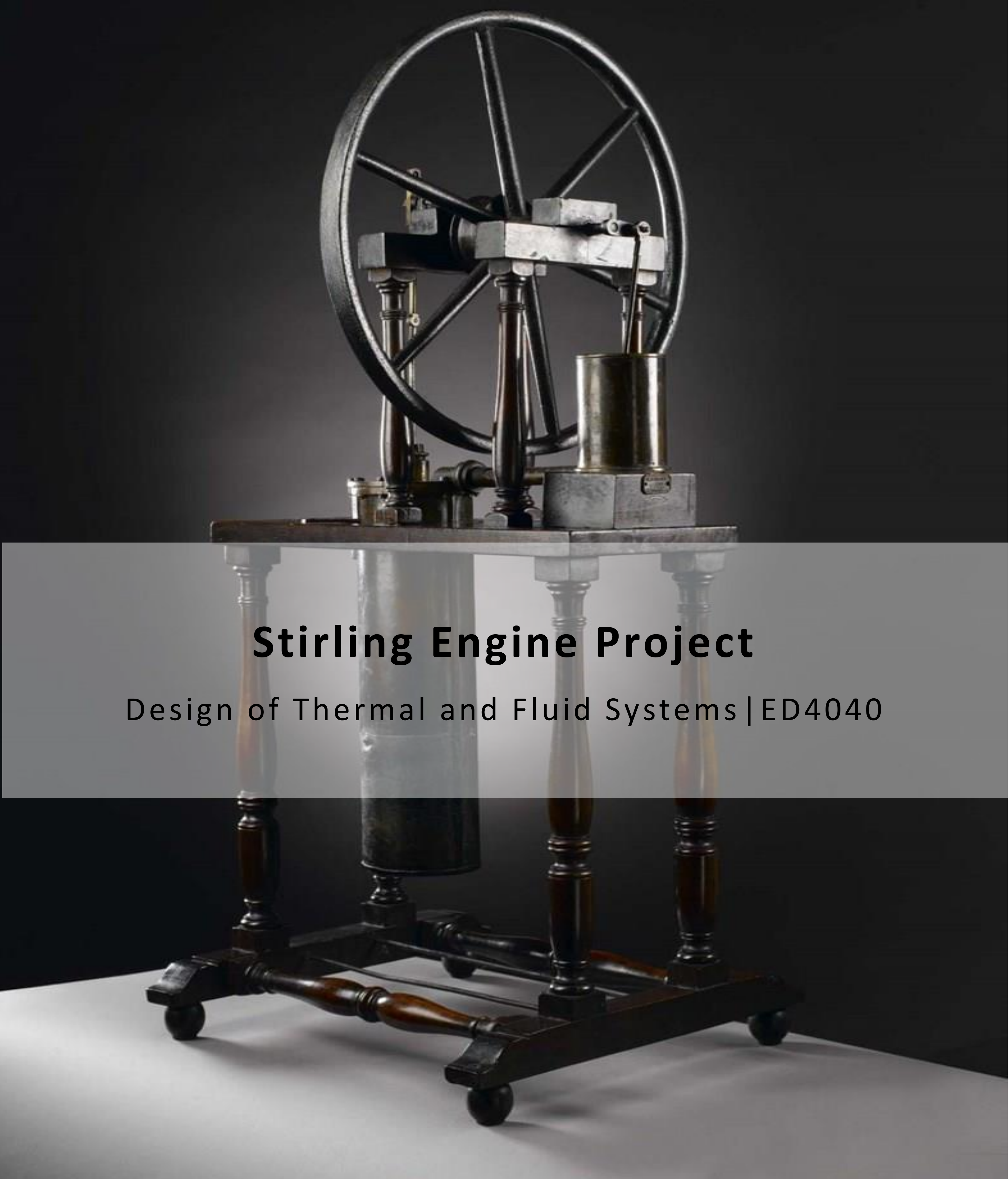

Stirling Engine Project

Design of Thermal and Fluid Systems|ED4040

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ED4040 | TEAM 8

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Introduction

This report contains a detailed study and design for a Stirling engine used to pump water using solar energy as energy source. We used python for calculations and observations and the same is being attached to this report.

Application

The Stirling Engine Generator is a sealed high efficiency heat engine that is driven by the radiant energy supplied from the sun or any other source of external heat. **A solar powered Stirling engine** is a reciprocating piston engine that uses solar radiation to produce heat in place of traditional fossil fuels. These offer several advantages over traditional engines that run on fossil fuels since solar energy is abundantly available and inexhaustible.

Design of Stirling Engine

The aim of this study is to find a feasible solution which may lead to a preliminary conceptual design of a workable **solar-powered LTD (Low-temperature differential) Stirling engine**. Since this Engine is designed for use in rural areas; the engine design should be as simple as possible. The most appropriate type of solar-powered Stirling engine would be the LTD Stirling engine. **The engine design should be that of a beta-configuration, LTD Stirling engine.** **Reasons for using beta – Stirling engine -**

- The Gamma Stirling Engine is not commonly used in residential or small-scale applications. This is because it is typically larger and more expensive than other types of Stirling engines, making it more suitable for industrial settings.

- **Higher Efficiency at Lower Temperatures**: Beta Stirling engines are more suitable for solar applications because they can operate efficiently at lower temperature differentials. Solar thermal collectors often produce lower temperature heat, and beta Stirling engines can effectively utilize this heat to generate power. In contrast, alpha and gamma Stirling engines typically require higher temperature differentials to achieve optimal efficiency.

- **Simplified Design:** Beta Stirling engines have a simpler design compared to alpha and gamma engines. They have only one power piston and one displacer piston, resulting in fewer moving parts and reduced complexity. This simplicity leads to easier maintenance, lower manufacturing costs, and improved reliability.

- **Compact Size and Portability:** Beta Stirling engines can be designed to be compact and portable, making them suitable for smaller-scale solar-powered pump systems. Their smaller size allows for easier integration into the overall pump setup, especially in remote or off-grid locations where space may be limited.

Heat Source

Solar energy is considered as the main renewable energy source we can use because it is available in abundance and also, in the form that can be harnessed easily.

A **parabolic reflector is a reflective device** used to accumulate or projection of energy such as light, sound, or radio waves. Its shape is part of a circular paraboloid, that is, the surface caused by a parabola gyrating around its axis.

Parabolic reflectors can be used to gather and accumulate energy entering the reflector at a specific angle. Similarly, energy emitted from the focus to the dish can be transferred outwards in a beam that is parallel to the axis of the dish.

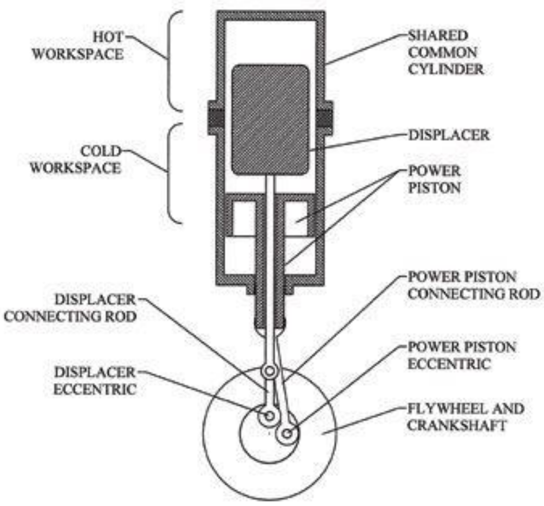


Fig – 1 Beta-Stirling engine

Working and Operation

Basic Components: A beta Stirling engine consists of several key components, including a heat source, a working fluid (typically a gas such as helium or hydrogen), a hot side, a cold side, a displacer piston, and a power piston.

Cyclic Process: The beta Stirling engine operates on a cyclic process involving the compression and expansion of the working fluid. This cycle consists of four phases: heating, expansion, cooling, and compression.

Heating Phase: The heat source, such as a burner or solar concentrator, provides thermal energy to the hot side of the engine. The heat causes the working fluid in the hot side to expand and increase in pressure.

Expansion Phase: The increased pressure from the heating phase forces the power piston to move outward, performing mechanical work. This movement of the power piston drives the connected machinery or generator.

Cooling Phase: As the power piston moves outward, it pushes the displacer piston. The displacer piston transfers the working fluid from the hot side to the cold side of the engine. This transfer of gas displaces the hot working fluid, allowing it to cool down.

Compression Phase: In the compression phase, the displacer piston moves back, pushing the working fluid from the cold side to the hot side. This compresses the gas and increases its pressure, preparing it for the next heating phase.

Repeat Cycle: The cyclic process repeats continuously, with the displacer piston and power piston moving back and forth, driven by the temperature differentials between the hot and cold sides of the engine. The heat input and cooling output are regulated to maintain the cyclic operation.

Regenerator: Some beta Stirling engines incorporate a regenerator, which is a heat exchanger that helps improve the engine's efficiency. The regenerator stores and releases heat as the working fluid moves back and forth, allowing for better heat transfer and reducing energy losses.

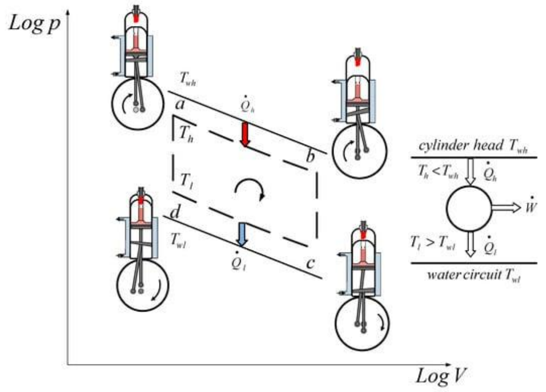


Fig -2 Stirling cycle

Calculations

The second order methods take into account the kinematics of the pistons and the internal and external irreversibility of the machine. In addition, the non-uniformity distribution regarding the space and time of the working fluid in the engine it is taken into consideration by dividing it into three, five, or more volumes, with which a characteristic temperature is associated. These methods use the hypothesis of independency of the energy losses.

The isothermal analysis considers the expansion and the heat exchange with the source in the same hot, isothermal volume, and the compression and the heat exchange with the sink in the same cold, isothermal volume.

A classical way to model this engine with some realism is to use the **Schmidt model**. This analysis relies on the division of the engine into three spaces: **expansion, compression, and regeneration spaces as shown in the below figure:**

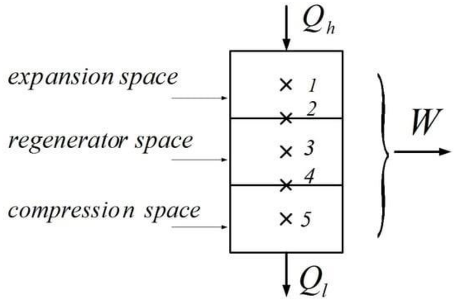


Fig 3- Representation of the machine spaces with their boundaries.

Assumptions

The performance of the engine can be calculated a P-V diagram. The volume in the engine is easily calculated by using the internal geometry. When the volume, mass of the working gas and the temperature is decided, the pressure is calculated using an ideal gas method as shown in equation.

$$PV = mRT$$

The engine pressure can be calculated under following assumptions:

- There is no pressure loss in the heat exchangers and there are no internal pressure differences.
- The expansion process and the compression process change isothermal.
- Conditions of the working gas are changed as an ideal gas.
- The expansion dead space maintains the expansion gas temperature – T_e, the compression dead space maintains the compression gas temperature – T_c during the cycle.
- The regenerator gas temperature is an average of the expansion gas temperature – T_e and the compression gas temperature – T_c.
- The expansion space - VE and the compression space - VC changes according sine curves.

Calculations

Assuming 300kg of water to be lifted per minute from a depth of 20m and ejecting it with a speed of 50m/s.

$$Total\ energy = mgh + 0.5 * m * v^2$$

$$T.E = 300 * 9.8 * 20 + 0.5 * 300 * 50^2$$

$$T.E = 433.8\ kJ$$

$$Total\ Power\ required = \frac{T.E}{60\ secs}$$

$$Total\ power\ required = 7.23\ kW$$

The Stirling engine which is present on the focal length of the parabolic concentrator, receives energy after reflection from the parabolic concentrator.

The fluid present inside is helium gas. Assuming the temperature to which gas gets heated to after light rays fall on it is **650 degrees Celsius** and temperature of the gas after it releases the heat is **45 degrees Celsius.**

$$efficiency = 1 - \frac{45 + 273}{650 + 273}$$

$$efficiency = 65.54\ \%$$

$$Power\ in = efficiency * Power\ out$$

$$\boldsymbol{Power\ in = 4.78\ kW}$$

Hence, the radius of parabolic disk:

Assuming there will be constant amount of heat received by the parabolic

$$V_B = \frac{V_{SE} + V_{SC}}{2} - \sqrt{\frac{V_{SE}^2 + V_{SC}^2}{4} - \frac{V_{SE}V_{SC}}{2}\cos dx}$$

Total volume – V:

$$V = V_E + V_R + V_C$$

reflector. Amount of energy received per sec is given by

$$Power\ in = solar\ constant(W/m2) * Area(m2)$$

$Q_{h,rev} =$

radius(r) of parabolic dish = 2.12m

Schmidt Analysis

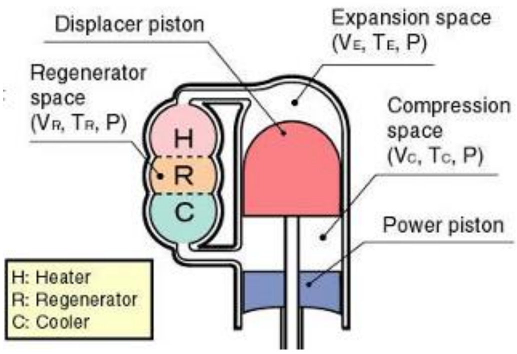


Fig – 4 Beta-Stirling engine with Schmidt model

Referring to above diagram –

- *V_E - expansion momental volume*
- *V_C - compression momental volume*
  V_{SE} - swept volume of expansion piston
- *V_{SC} – swept volume of power piston*
- *dx - phase angle between displacer and power piston*
- *V_B – overlap volume*
- *No. of moles of gas = 0.5*

$$V_E = \frac{V_{SE}}{2}(1 - \cos x) + V_{DE} \qquad (24)$$

$$V_C = \frac{V_{SE}}{2}(1 + \cos x) + \frac{V_{SC}}{2}\{1 - \cos(x - dx)\} + V_{DC}$$

The engine pressure - P based on the mean- pressure - P_{mean}

$$P = \frac{P_{mean}\sqrt{1-c^2}}{1 - c \cdot \cos(x-a)}$$

Several ratios and coefficients are defined as follows –

$$t = \frac{T_C}{T_E} \quad v = \frac{V_{SC}}{V_{SE}} \quad X_B = \frac{V_B}{V_{SE}} \quad X_{DE} = \frac{V_{DE}}{V_{SE}} \quad X_{DC} = \frac{V_{DC}}{V_{SE}} \quad X_R = \frac{V_R}{V_{SE}}$$

$$a = \tan^{-1}\frac{v\sin dx}{t + \cos dx + 1}$$

$$S = t + 2tX_{DE} + \frac{4tX_R}{1+t} + v + 2X_{DC} + 1 - 2X_B$$

$$B = \sqrt{t^2 + 2(t-1)v\cos dx + v^2 - 2t + 1}$$

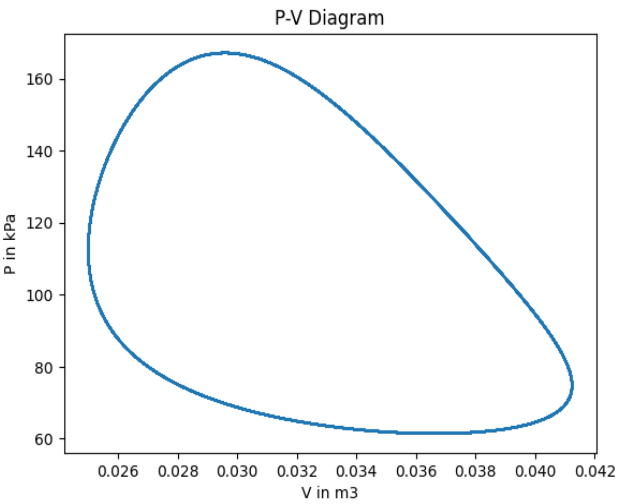
$$c = \frac{B}{S}$$

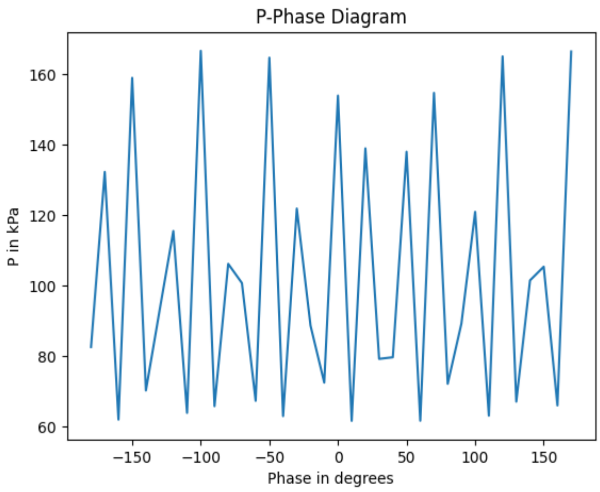
Python Code

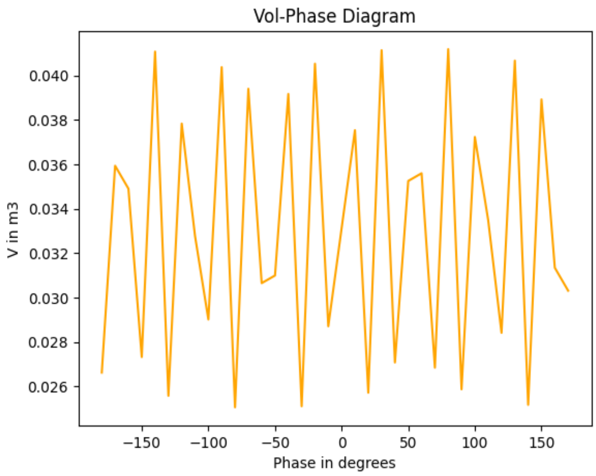
[Dofts - Beta-Striling engine.ipynb]

Output Obtained

Swept volume by expansion piston: **0.025 m3**
Max Pressure: **167.14 kPa**
Work done per cycle: **556.319 J**
Power output: **7417.588 Watts**
Indicated efficiency: **0.655471289274**







Calculating angular velocity
In the case of the ideal Stirling cycle, the quantities of heat transferred in the isothermal process in the heat exchangers is as follows:

$$Qin_f low = p_{\max} V_{\max} \frac{\ln \varepsilon}{\varepsilon} = E_\varepsilon$$

$$e = Vmax/Vmin$$

If the regeneration is perfect, the heat stored on the regenerator during the d-a transformation and released during the reversible process b-c is as follows:

$$Q_{reg\_T} = mc_v(T_h - T_l) = \frac{mRT_h}{\gamma - 1}\left(1 - \frac{T_l}{T_h}\right) = \frac{p_{\max}V_{\max}}{\varepsilon(\gamma - 1)}\left(1 - \frac{T_l}{T_h}\right)$$

For imperfect regeneration, stop loss 'k' is defined as:

$$k = \frac{1 - \eta_{reg}}{\ln \varepsilon (\gamma - 1)}$$

$$\eta_{reg} = \frac{Q_{reg\_T} - Q_{p,reg}}{Q_{reg\_T}}$$

Taking n_{reg} = *70 %*

Hence, the total heat, Qh, delivered to the working gas is the sum of the isothermally delivered heat, Qh.rev, and the added heat, Qp.reg, as a result of the imperfect regeneration. The total heat, Ql, released from the gas is the sum of the isothermally released heat, Ql.rev, and the added heat, Qp.reg.

$$Q_h = Q_{h.rev} + Q_{p.reg} = E_\varepsilon\left[1 + k\left(1 - \frac{T_l}{T_h}\right)\right]$$

$$|Q_l| = |Q_{l.rev}| + Q_{p.reg} = E_\varepsilon\left[\frac{T_l}{T_h} + k\left(1 - \frac{T_l}{T_h}\right)\right]$$

The heat flow transferred on the hot sink and cold source can also be obtained when taking into account the engine rotational speed, *n*, as follows:

$$\dot{Q}_{out} = n|Q_{out}| = nE_\varepsilon\left[1 + k\left(1 - \frac{T_l}{T_h}\right)\right] = K_h(T_{wh} - T_h)$$

K_h (conductance of aluminium) = 3.5×10^7

Th (max temperature obtained from graph shown at the end): 502 K
Twh (wall temperature): 502.0013

$$n = \frac{3.5*10^7*(Twh-502)}{556.31\ J}$$

$$n = \frac{3.5*10^7*(502.0013-502)}{556.31\ J}$$

$$n = 83.77\ rad/s$$

And we get Qh = 912.04 J
Ql = 355.65 J

Take the rpm rate of the Stirling engine flywheel to be about 800 rpm to produce 7.23 kW of power.

$$angular\ vel = 2*pi*800/60$$

$$angular\ vel = 83.77\ rad/s$$

$Thus,$

$$Torque = Power/\ angular\ vel$$

$$\boldsymbol{Torque = 86.3\ N.m}$$

Design of Stirling Engine

The characteristics required for this type of design are as given below:
- The diameters of the displacer and displacer cylinder must be large.
- The length of the displacer must be small.
- The heat transfer must be effective on the surface of both plate ends of the displacer.

As can be seen in the pages of *Model Engineer*, the problem faced in making hot air engines is not so much on of evolving an efficient design but making one that actually works. The following designs have been evolved by experiment and give a fairly compact engine layout, which though thermodynamically not very efficient is capable of giving a reasonable power output.

Stirling's design of 1815 with the following design parameter:

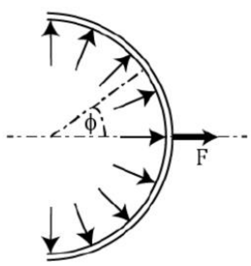
- length of heater chamber = 2/3**L**
- length of cooler = 1/3**L**
- swept volume of displacer = 1.5 times swept volume of piston cylinder.
- length of displacer = 2/3**L** and stroke = 1/3**L**.

With the availability of dimeter of 6.4 cm of displacer cylinder. The layout of the Stirling engine is done with the following design parameter:

- length of heater chamber = 2/3L=2/3*19.2 = **12.8cm**
- length of cooler = 1/3L=1/3*19.2 = **6.4cm**
- length of displacer = 2/3L=2/3*19.2
- = **12.8cm**
- Diameter of displacer(d) = 0.93* diameter of displacer cylinder = **17.67cm**

Calculation of hoop stress and cylinder wall thickness

Let us first consider the force that exerted on each half of a thin-walled cylindrical section as a consequence of the internal pressure. In order to determine this force, we need to integrate the x-component (i.e. in an arbitrarily chosen radial direction) of the pressure force (i.e $P\delta A$) acting over the side of the cylindrical vessel as shown below:



However, we can write $\delta A = L\,\pi D \frac{\delta\phi}{2\pi}$ where $\delta\phi$ is a small increment of angle, hence

$$F = \frac{LD}{2}\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} P\cos\phi\, d\phi$$

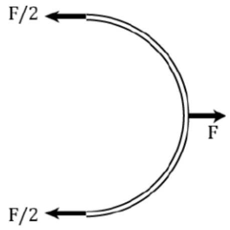
$$F = \frac{PLD}{2}\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \cos\phi\, d\phi$$

$$F = \frac{PLD}{2}\,[\sin\phi]_{-\pi/2}^{\pi/2}$$

$$F = PLD$$

i.e., the projected area of the surface in the plane perpendicular to the x axis multiplied by the pressure.

Now, if the vessel is to maintain its shape this force must be balances by an equal and opposite force in the wall of the vessel as shown below



But this force will be distributed over an area equal to the product of the cylindrical section length L and the wall thickness t so we have

$$\sigma_{Hoop} = \frac{PLD}{2Lt} = \frac{PD}{2t}$$

Considering 0.6 as safety factor, we get:

$$\boldsymbol{t = \frac{PD}{2 * 0.6 * Max\ tensile\ strength}}$$

$$\boldsymbol{t = \frac{167.14\ kPa\ *\ 0.19m}{2 * 0.6 * 90\ MPa}}$$

$$\boldsymbol{t = 4\ mm}$$

Here:
- P = Max pressure obtained from the P-V graph (shown at the end) = 167.14 kPa
- D = Diameter of the displacer cylinder obtained from the max swept volume (the length is 12.8cm, obtained from the PV graph)
- Max tensile strength = 90 MPa for Al

- **We are considering aluminium as our material for our design because:** Aluminium has excellent thermal conductivity, allowing it to efficiently transfer heat between different components of the Stirling engine.
- The lightweight nature of aluminium helps reduce the overall weight of the engine, making it easier to handle and potentially improving its portability.

Heat Loss due to Conduction

Energy lost by internal conduction, through the solid matrix of the regenerator between hot and cold parts are calculated by the expression:

$$\delta\dot{W}_{Pcdr} = k_{cdr}\frac{A_r}{L_r}(T_{r-h} - T_{f-r})$$

Losses of energy by external conduction through the regenerator which is not adiabatic:

$$\delta\dot{W}_{Pext} = (1 - \varepsilon)\ (\delta\dot{Q}_{r1} + \delta\dot{Q}_{r2})$$

The pressure losses by friction in the regenerator which leads to power losses:

$$\delta\dot{W}_{Pch} = -\frac{\Delta p\ \dot{m}}{\rho}$$

The losses of energy by Shuttle effect which comes from the Shuttle movement of the displacer piston between the heater and the cooler:

$$\delta\dot{W}_{Pshtl} = \frac{0.4\,Z^2\,k_{pis}\,D_d}{J\,L_d}\left(T_d - T_c\right)$$

Losses by irreversibility effect of the compression and relaxation:

$$\delta\dot{W}_{Pirrc} \cong \sqrt{\frac{1}{32}\omega\gamma^3(\gamma-1)T_{pac}P_{cmoy}k_{pac}}\left(\frac{\Delta V_c}{V_{cmoy}}\right)^2 A_{pac}$$

The dynamic model variables are given on the basis of the energy and mass conversation balance:

$$\delta\dot{W} + C_p T_E \dot{m}_E - C_p T_S \dot{m}_S = P\frac{dV}{dt} + C_v \frac{d(mT)}{dt} + \sum Diss$$

$\sum Diss$: is the energy losses sum.

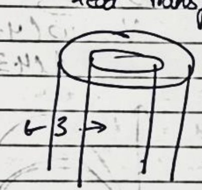
The values of Qh and Ql have been calculated in during the calculation of angular velocity in the previous section.

The heat transfer occurring through the walls of the Stirling engine has been determined to have a thickness of approximately 3mm using stress calculations. By considering the conductance of aluminium as 170W/mK and the heat transfer coefficient of air as 25W/m^2, we can apply the principle of Biots law. This law can be expressed as B = hL/k, where B represents the Biots number, h denotes the heat transfer coefficient, L signifies the characteristic length, and k represents the thermal conductivity. In this case, the calculated Biots number is 0.002, which is considerably smaller than 0.1, allowing us to make a reasonable assumption.

Heat flow through regenerator
h for helium in forced convection = 250 W/mK

Assume:
Length l = 1m
Thickness t = 0.001m
Radius r = 0.01m

Heat transfer through walls



involves convection due to atmosphere and conduction through walls

$$R_{conduction} = \frac{L}{KA}$$

L is only 3mm
so R_{cond} is very less
implying infinite conductance.

So using $Bi = \frac{hL}{K} = 0.002 << 0.1$

$$\therefore \rho c \frac{dT}{dt} = -hA(T - T_a)$$

$$\boxed{\frac{T_f - T_a}{T_i - T_a} = e^{-\frac{hAt}{\rho c}}}$$

For hot piston

$$\boxed{T_h = T_a + (T_i - T_a) e^{-\frac{hAt}{\rho c}}}$$

For cold piston

$$T_L = T_a + (T_i - T_a) e^{-\frac{h_{air} A}{\rho c} t}.$$

Heat transfer through regenerator

→ conduction

→ convection

· conduction $= k(2\pi r t) \times \left(\frac{T_H - T_L}{l}\right)$ [t is thickness and l is length]

convection $= h \times \pi r^2 \times (T_H - T_L)$

comparing the two

Using Bi.

which is 0.01 again conduction is negligible

Heat transfered is $h(\pi r^2)(T_H - T_L)$

$= 250 \times 3.14 \times 10^{-4} \times (923 - 313)$

$= 46.1\ J/s$

Heat lost due to convection by air

If there is linear temperature drop

Heat loss due to convection will be same as if the entire pipe was at average temperature.

$Q_{loss} = h_{air} \times 2\pi r l \times \left(\frac{T_H + T_L}{2} - T_{air}\right)$

$= 230\ J/s$

References

- *Schmidt-model*
- *chrome-extension://efaidnbmnnnibpcajpcglclefindmkaj/https://iopscience.iop.org/article/10.1088/1757-899X/376/1/012022/pdf*
- *https://www.mdpi.com/1099-4300/22/11/1278*
- *http://www.bekkoame.ne.jp/~khirata/academic/schmidt/schmidt.ht*
- *https://www.mdpi.com/1099-4300/22/11/1278*
- *https://www.studocu.com/en-gb/document/swansea-university/process-equipment-design-selection-control/derivation-of-the-expression-for-hoop-and-longitudinal-stresses-in-cylindrical-vessels/8628809*