MC-405 GRAPH THEORY

Assignment-I

1. Model the following situation as (possibly weighted, possibly directed) graph. Draw the graph, and give the corresponding adjacency matrix.
   It is well-known that in the Netherlands, there is a 2-lane highway from Amsterdam to Breda, another 2-lane highway from Amsterdam to Cappele aan den IJssel, a 3-lane highway from Breda to Dordrecht, a 1-lane road from Breda to Ede and another one from Dordrecht to Ede, and a 5-lane superhighway from Cappele aan den IJssel to Ede.

2. Are the sequences given below graphical?

   (a) $(6, 6, 5, 4, 3, 3, 2)$

   (b) $(6, 6, 5, 4, 3, 3, 1)$

3. Give an example of a degree sequence that is realizable as the degree sequence by only a disconnected graph.

4. A graph G has adjacency matrix

$$A = \begin{pmatrix} 0 & 1 & 1 & 2 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

   (a) Is G a simple graph?

   (b) What is the degree sequence of G?

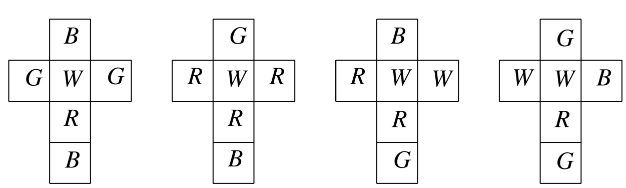
   (c) How many edges does G have?

5. If δ and Δ are respectively the minimum and maximum of the degrees of a graph G, show that $\delta \leq \frac{2m}{n} \leq \Delta$, where G is $(n; m)$ graph.

6. Using techniques from graph theory, show that

$$1 + 2 + \ldots + n = \frac{n(n+1)}{2}.$$

7. The figure below shows four unwrapped cubes that form the Instant Insanity Puzzle. The letters R,W,B and G stand for the colors red, white, blue and green. The object of the puzzle is to stack the blocks in a pile of 4 in such a way that each of the colors appears exactly once on each of the four sides of the stack.



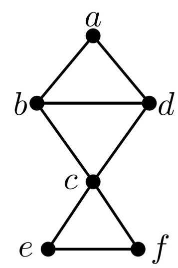
8. Prove that a simple graph with n vertices and k components can have at most $(n-k)(n-k+1)/2$ edges.

9. Show that for a simple bipartite graph $m \leq \frac{n^2}{4}$,where $G = (n; m)$.

10. Show that every simple graph of order n is isomorphic to a subgraph of the complete graph with n vertices.

11. Show that two graphs need not be isomorphic even when they both have the same order and same size.

12. For each of the following sequences of vertices,state whether or not it represents a walk, path, closed walk, circuit or cycle in the graph illustrated.



   (a) abcefcbd
   (b) abcefcd
   (c) abcefcdba
   (d) bcefcdb
   (e) bcdb
   (f) abefcd

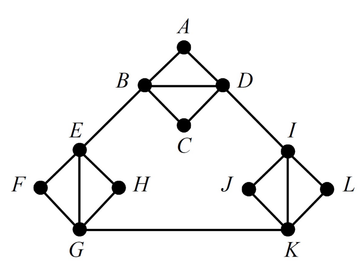
13. Show that it is not possible to have a group of 7 persons such that each knows exactly 3 persons in the group.

14. Show that the complement of a bipartite graph need not be a bipartite graph.

15. A simple graph that is isomorphic to its complement is called self-complementary graph. Find a self-complementary graph of order 4.

16. State and prove the characterization of Eulerian graph. Also give an example of an Eulerian graph with 5 vertices and 8 edges.

17. Apply Fleurys algorithm, beginning with vertex A, to find an Eulerian trail in the following graph.In applying the algorithm, at each stage chose the edge (from those available) which visits the vertex which comes first in alphabetical order.



18. Draw a graph which is:

(a) Hamilton and Eulerian

(b) Hamilton and non-Eulerian

(c) Non-Hamilton and Eulerian

(d) Non-Hamilton and non-Eulerian

19. Define and draw Petersen graph. Is Petersen graph Hamiltonian? If yes, then write down the path.

20. Prove that a simple connected graph with n vertices and m edges is Hamiltonian if $m \geq (n^2 - 3n + 6)/2$.

21. Show that a simple connected graph G with n vertices ,$n \geq 3$ is Hamiltonian if $deg(v) \geq (n/2) \, \forall \, v \in V(G)$.

22. Let G be a connected graph with n vertices,$n > 2$ and no loops or multiple edges.G has Hamiltonian circuit if $deg(u) + deg(v) \geq n$, where u, v are non-adjacent to each other.

23. Consider the following weighted graph G. Apply Dijkstra's algorithm to the vertex v_0.