Q4) Consider 2 i.i.d random variables X and Y each having uniform distribution between the intervals 0 and 1. Define $Z = X+Y$. Prove that

$$E(X|Z) = \frac{Z}{2}$$

Ans.) We are given that X and Y are i.i.d and also have same probability distribution, hence

$$Z = X+Y$$

$$E(X|Z) = E(Y|Z)$$

$$E(X|Z) = E(Z-Y|Z)$$
$$= E(Z|Z) - E(Y|Z)$$

$$E(X|Z) + E(Y|Z) = E(Z|Z) \quad \text{— (1)}$$

We also know that $E(Z|Z) = Z$ — (2)

Putting (2) in (1), we get

$$E(X|Z) + E(Y|Z) = Z$$

Also, $E(X|Z) = E(Y|Z)$, so,

$$E(X|Z) + E(X|Z) = Z$$
$$2E[X|Z] = Z$$
$$\boxed{E(X|Z) = \frac{Z}{2}}$$

Hence proved

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