(1) Let $S(T)$ be the price of stock at time T. All of the following options have exercise time T and strike price K. Give the payoff at time T that is earned by an investor who =)

(i) one call and one put option

- if $S(T) > K$ call option will be exercised
  put option will not be exercised

payoff for call option = $S(T) - K$
for put option = 0
=) total = $S(T) - K$

- if $S(T) < K$ call option will not be exercised
  put option will be exercised

payoff for call option = 0
for put option = $K - S(T)$
total = $K - S(T)$

- if $S(T) = K$ both options will not be exercised

payoff for both = 0
=) total payoff = 0

(ii) two calls and one sold short share of stock

- if $S(T) > K$ call option will be exercised

→ since due to the call option exercised we will have two stock, out of which we will return the stock which we sold short

payoff = $2(S(T) - K) - S(T)$
= $S(T) - 2K$

- if $S(T) \leq K$ call option will not be exercised

payoff = $-S(T)$

(iii) one share of stock, one sold call

- if $S(T) \leq K$ call option is not exercised

payoff = $S(T)$

- if $S(T) > K$ call option is exercised

payoff = $(K - S(T) + S(T))$ (he will not have the stock now)
= K

② A certain stock is selling for Rs. 50. The feeling is that for each month, for the next two months, the stock price will rise by 10% or fall by 10%. Assuming that the risk free rate is 1%, calculate the price of the European call with the strike price of Rs. 48.

Solution =>

$S(0) = ₹\,50$

$u = 1.1$ (grows by 10%)

$d = 0.9$ (falls by 10%)

$r = 1\% = 0.01$ $\qquad R = 1 + r = 1.01$

$X = ₹\,48$

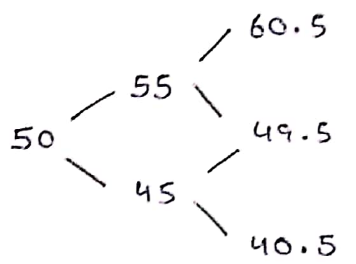
$S^u = S(0) \times u = ₹\,50 \times 1.1 = ₹\,55$

$S^d = S(0) \times d = ₹\,50 \times 0.9 = ₹\,45$

$S^{uu} = S(0) \times u \times u = ₹\,50 \times 1.1 \times 1.1 = ₹\,60.5$

$S^{ud} = S(0) \times u \times d = ₹\,50 \times 1.1 \times 0.9 = ₹\,49.5$

$S^{dd} = S(0) \times 0.9 \times 0.9 = ₹\,40.5$



$$p^* = \frac{R-d}{u-d} = \frac{1.01 - 0.9}{1.1 - 0.9} = \frac{0.11}{0.2} = \frac{11}{20}$$

$$1 - p^* = \frac{9}{20}$$

$C^{uu} = [S^{uu} - K]^+ = \max(60.5 - 48; 0) = ₹\,12.5$

$C^{ud} = [S^{ud} - K]^+ = ₹\,1.5$

$C^{dd} = [S^{dd} - K]^+ = ₹\,0$

$$C^u = \frac{1}{R}\left[p^* C^{uu} + (1-p^*) C^{ud}\right]$$

$$= \frac{1}{1.01}\left[\frac{11}{20} \times 12.5 + \frac{9}{20} \times 1.5\right]$$

$c^{u} = ₹\, 7.47525$

$c^{d} = \frac{1}{R}\left[p^{*} c^{ud} + (1-p^{*})\, c^{dd} \right]$

$= \frac{1}{1.01}\left[\frac{11}{20} \times 1.5 + \frac{9}{20} \times 0 \right]$

$= ₹\, 0.81683$

$c(0) = \frac{1}{R}\left[p^{*} c^{u} + (1-p^{*})\, c^{d} \right]$

$= \frac{1}{1.01}\left[\frac{11}{20} \times 7.47525 + \frac{9}{20} \times 0.81683 \right]$

$= \underline{\underline{₹\, 4.4346}}$

(3) Consider the data $S(0) = 60$, $K = 62$, $u = 1.1$, $d = 0.95$, $r = 0.03$ and $T = 3$. Find $C^E(0)$ and $P^E(0)$.

Solution

$S(0) = 60$ $\quad K = 62$ $\quad u = 1.1$ $\quad d = 0.95$

$r = 0.03$ $\quad \Rightarrow R = 1 + r = 1.03$ $\quad T = 3$

$S^u = S(0) \times u = 60 \times 1.1 = 66$

$S^d = S(0) \times d = 60 \times 0.95 = 57$

$S^{uu} = S(0) \times u^2 = 60 \times 1.1 \times 1.1 = 72.6$

$S^{ud} = S(0) \times u \times d = 60 \times 1.1 \times 0.95 = 62.7$

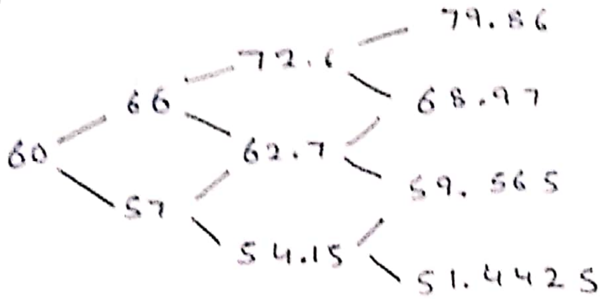
$S^{dd} = S(0) \times d^2 = 60 \times 0.95 \times 0.95 = \cancel{62}.54.15$

$S^{uuu} = S(0) \times u^3 = 60 \times 1.1 \times 1.1 \times 1.1 = 79.86$

$S^{uud} = S(0) \times u^2 \times d = 60 \times 1.1 \times 1.1 \times 0.95 = 68.97$

$S^{udd} = S(0) \times u \times d^2 = 60 \times 1.1 \times 0.95 \times 0.95 = 59.565$

$S^{ddd} = S(0) \times d^3 = 60 \times 0.95 \times 0.95 \times 0.95 = 51.4425$



$$p^* = \frac{R-d}{u-d} = \frac{1.03 - 0.95}{1.1 - 0.95} = \frac{8}{15}$$

$$1 - p^* = \frac{7}{15}$$

$C^{uuu} = [S^{uuu} - K]^+ = 17.86$

$C^{uud} = [S^{uud} - K]^+ = 6.97$

$C^{udd} = [S^{udd} - K]^+ = 0$

$C^{ddd} = [S^{ddd} - K]^+ = 0$

$$C^{uu} = \frac{1}{1.03}\left[\frac{8}{15} \times 17.86 + \frac{7}{15} \times 6.97\right]$$

$$= 12.4058$$

$$c^{ud} = \frac{1}{1.03}\left[\frac{8}{15} \times 6.97 + \frac{7}{15} \times 0\right]$$

$$= 3.6091$$

$$c^{dd} = \frac{1}{1.03}\left[\frac{8}{15} \times 0 + \frac{7}{15} \times 0\right]$$

$$= 0$$

$$c^{u} = \frac{1}{1.03}\left[\frac{8}{15} \times 12.4058 + \frac{7}{15} \times 3.6091\right]$$

$$= 8.0589$$

$$c^{d} = \frac{1}{1.03}\left[\frac{8}{15} \times 3.6091 + \frac{7}{15} \times 0\right]$$

$$= 1.8688$$

$$c(0) = \frac{1}{1.03}\left[8.0589 \times \frac{8}{15} + 1.8688 \times \frac{7}{15}\right]$$

$$= ₹\ \underline{\underline{5.0196}}$$

$$P(0) = C(0) - S(0) + Ke^{-rT}$$

$$= 5.0196 - 60 + 62 \times e^{-0.03 \times 3}$$

$$= ₹\ \underline{\underline{1.6833}}$$

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(4) Let $S(0) = 120$, $u = 1.2$, $d = 0.9$ and $r = 1\%$. Consider a call option with strike price $K = 120$ and $T = 2$. Find the option pricing and the replicating strategy.

<u>Solution</u>

$S(0) = 120 \quad u = 1.2 \quad d = 0.9 \quad r = 0.01$

$R = 1 + r = 1.01 \quad K = 120 \quad T = 2$

$S^u = S(0) \times u = 120 \times 1.2 = 144$

$S^d = S(0) \times d = 120 \times 0.9 = 108$

$S^{uu} = S(0) \times u^2 = 120 \times 1.2 \times 1.2 = 172.8$

$S^{ud} = S(0) \times u \times d = 120 \times 1.2 \times 0.9 = 129.6$

$S^{dd} = S(0) \times d^2 = 120 \times 0.9 \times 0.9 = 97.2$

$$\begin{tikzcd}[column sep=small, row sep=small]
 & & 172.8 \\
 & 144 \arrow[ur] \arrow[dr] & \\
120 \arrow[ur] \arrow[dr] & & 129.6 \\
 & 108 \arrow[ur] \arrow[dr] & \\
 & & 97.2
\end{tikzcd}$$

$c^{uu} = [S^{uu} - K]^+ = 52.8$

$c^{ud} = [S^{ud} - K]^+ = 9.8$

$c^{dd} = [S^{dd} - K]^+ = 0$

$$p^* = \frac{R-d}{u-d} = \frac{1.01 - 0.9}{1.2 - 0.9}$$

$$= \frac{0.11}{0.3} = \frac{11}{30}$$

$$1 - p^* = \frac{19}{30}$$

$$c^d = \frac{1}{1.01}\left[\frac{11}{30} \times 9.8 + \frac{19}{30} \times 0\right]$$

$$= \frac{1078}{303}$$

$$c^u = \frac{1}{1.01}\left[\frac{11}{30} \times 52.8 + \frac{19}{30} \times 9.8\right]$$

$$= \frac{7670}{303}$$

$$c(0) = \frac{1}{1.01}\left[\frac{11}{30} \times \frac{7670}{303} + \frac{19}{30} \times \frac{1078}{303}\right]$$

$$= ₹\ 11.4206$$

(5) A non-dividend paying stock is currently selling at Rs 100 with annual volatility 18%. Assume that the continuously compounded risk free interest rate is 4%. Using a two period CRR binomial model, find the price of one European call option on this stock with strike price of ₹ 80, and time to expiration 3 years.

Solution =>

$S(0) = ₹100$ $\sigma = 18\% = 0.18$

$r = 4\% = 0.04$ $\Delta t = 2$ years $T = 3$ years

$X = ₹ 80$

$u = e^{\sigma\sqrt{\Delta t}} = e^{0.18 \times \sqrt{2}}$

$= 1.2899$

$d = \frac{1}{u} = 0.7753$

$S^{u} = S(0) \times u = ₹100 \times 1.2899 = ₹ 128.99$

$S^{d} = S(0) \times d = ₹ 100 \times 0.7753 = ₹ 77.53$

$S^{uu} = S(0) \times u^2 = ₹ 100 \times 1.2899 \times 0.7753$

$= ₹ 166.3842$

$S^{ud} = S(0) \times u \times d = ₹ 100$

$S^{dd} = S(0) \times d^2 = ₹ 60.1091$

166.3842

128.99

100 100

77.53

60.1091

$C^{uu} = [S^{uu} - K]^{+} = \max(166.384 - 80, 0)$

$= 86.3842$

$C^{ud} = [S^{ud} - K]^{+} = 20$

$C^{dd} = [S^{dd} - K]^{+} = 0$

$R = e^{r\Delta t} = e^{0.04 \times 2}$

$= 1.0833$

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$$p^* = \frac{R-d}{u-d} = \frac{1.0833 - 0.7753}{1.2899 - 0.7533}$$

$$= 0.5985$$

$$C(0) = e^{-r\Delta t}\left[p^{*2} C_{uu} + 2p^*(1-p^*) C_{ud} + (1-p^*)^2 C_{dd}\right]$$

$$= e^{-0.04\times 2}\Big[(0.5985)\ 86.3842$$

$$+ 2\times(0.5985)\times(1-0.5985)\times 20 + 0\Big]$$

$$= ₹\ \underline{\underline{37.5882}}$$

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(6) Consider the following data: $S(0) = ₹51$, $K = ₹50$, $\sigma = 30\%$, $r = 8\%$. Assuming the Black Scholes framework and that the stock pays no dividend, compute 3 months European call price and 3 months European put price using the Black Scholes formula. Also compute the put price using the put-call parity. Are the two values same?

Solution.

$S(0) = ₹51$ $\quad K = ₹50$ $\quad \sigma = 0.3$ $\quad r = 0.08$

$T = 3\text{months} = 3/12 = 1/4$

$$d_1 = \frac{\ln\left(\frac{S(0)}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$= \frac{\ln\left(\frac{51}{50}\right) + \left(0.08 + \frac{0.3^2}{2}\right)\frac{1}{4}}{0.3 \times \sqrt{\frac{1}{4}}}$$

$$= 0.3404$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$= 0.3404 - 0.3\sqrt{\frac{1}{4}}$$

$$= 0.1904$$

$\phi(d_1) = 0.6332$ $\qquad \phi(d_2) = 0.5755$

$$C(0) = S(0)\,\phi(d_1) - Ke^{-rT}\phi(d_2)$$

$$= 51 \times 0.6332 - 50 \times e^{-0.08 \times \frac{1}{4}} \times 0.5755$$

$$= ₹\,4.0879$$

$\phi(-d_1) = 0.3668$ $\qquad \phi(-d_2) = 0.4245$

$$P(0) = Ke^{-rT}\phi(-d_2) - S(0)\,\phi(-d_1)$$

$$= 50 \times e^{-0.08 \times \frac{1}{4}} \times 0.4245 - 51 \times 0.3668$$

$$= ₹\,2.0979$$

using put call parity

$$P(0) = C(0) - S(0) + Ke^{-rT}$$

$$= 4.0879 - 51 + 50 \times e^{-0.08 \times \frac{1}{4}}$$

$$= ₹\ \underline{\underline{2.0978}}$$

both the values are same.

(7) The price of a stock is Rs. 260. A 6 month European call option on the stock with strike price Rs. 256 is priced using Black Scholes formula. It is given that continuously compounded risk free rate is 4%, the stock pays no dividend, the volatility of the stock is 25%. Determine the price of the call option.

Solution

$S(0) = ₹260$ $\quad T = 6 \text{ months} = 1/2$

$r = 4\% = 0.04$ $\quad \sigma = 25\% = 0.25$ $\quad K = ₹256$

$$d_1 = \frac{\ln\left(\frac{S(0)}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$= \frac{\ln\left(\frac{260}{256}\right) + \left(0.04 + \frac{0.25^2}{2}\right) \times \frac{1}{2}}{0.25 \times \sqrt{\frac{1}{2}}}$$

$$= 0.2892$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$= 0.2892 - 0.25\sqrt{\frac{1}{2}}$$

$$= 0.1124$$

$\phi(d_1) = 0.6138$ $\quad \phi(d_2) = 0.5448$

$$C(0) = S(0)\,\phi(d_1) - Ke^{-rT}\phi(d_2)$$

$$= 260 \times 0.6138 - 256 \times e^{-0.04 \times 1/2} \times 0.5448$$

$$= ₹\,22.8809$$

(8) You own 100 shares of a stock whose current price is Rs. 42. You would like to hedge your downside exposure by buying 6 months European put option with a strike price of Rs. 40. It is given that the continuously compounded risk-free rate is 5%, the stock pays no dividend, the stock volatility is 22%. Assuming the Black Scholes framework determine the cost of the put option.

Solution

$S(0) = ₹ 42$ $\qquad T = 6 \text{ months} = 6/12 = 1/2$

$K = ₹ 40$ $\qquad r = 5\% = 0.05$ $\qquad \sigma = 22\% = 0.22$

$$d_1 = \frac{\ln\left[\frac{S(0)}{K}\right] + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$= \frac{\ln\left[\frac{42}{40}\right] + \left[0.05 + \frac{0.22^2}{2}\right] \times \frac{1}{2}}{0.22 \times \sqrt{\frac{1}{2}}}$$

$= 0.5521$

$d_2 = d_1 - \sigma\sqrt{T}$

$= 0.5521 - 0.22 \times \sqrt{\frac{1}{2}}$

$= 0.3965$

$\phi(-d_1) = 0.2904$ $\qquad \phi(-d_2) = 0.3459$

$P(0) = Ke^{-rT}\phi(-d_2) - S(0)\phi(-d_1)$

$= 40 \times e^{-0.05 \times \frac{1}{2}} \times 0.3459 - 42 \times 0.2904$

$= ₹ 1.2976$ for one stock

(9) Consider purchase of 100 units of 3 month Rs. 25 strike European call option. It is given that the stock is currently selling for Rs. 20, the continuous compounding risk-free interest is 5%, the stock volatility is 24% per annum. If the stock pays dividend continuously at the rate of 3% per annum, determine the price of block of 100 call options, assuming the Black Scholes framework.

Solution

$T = 3 \text{ months} = 3/12 = 1/4 \text{ year}$

$K = ₹25$ $\quad$ $S(0) = ₹20$

$r = 0.05$ $\quad$ $\sigma = 24\% = 0.24$

$r_{div} = 3\% = 0.03$ (continuously)

$$S'(0) = S(0)\, e^{-r_{div} T}$$

$$= 20 \times e^{-0.03 \times \frac{1}{4}}$$

$$= 19.8505$$

$$d_1 = \frac{\ln\left[\frac{S'(0)}{K}\right] + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$= \frac{\ln\left(\frac{19.8505}{25}\right) + \left(0.05 + \frac{0.24^2}{2}\right) \times \frac{1}{4}}{0.24 \times \sqrt{\frac{1}{4}}}$$

$$= -1.7579$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$= -1.7579 - 0.24 \times \sqrt{\frac{1}{4}}$$

$$= -1.8779$$

$\phi(d_1) = 0.0394$ $\quad$ $\phi(d_2) = 0.0302$

$$C(0) = S'(0)\,\phi(d_1) - Ke^{-rT}\phi(d_2)$$

$$= 19.8505 \times 0.0394 - 25 \times e^{-0.05 \times \frac{1}{4}} \times 0.0302$$

$= ₹0.0365$ (for 1 share)

for 100 shares $\Rightarrow$ ₹ 3.65

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