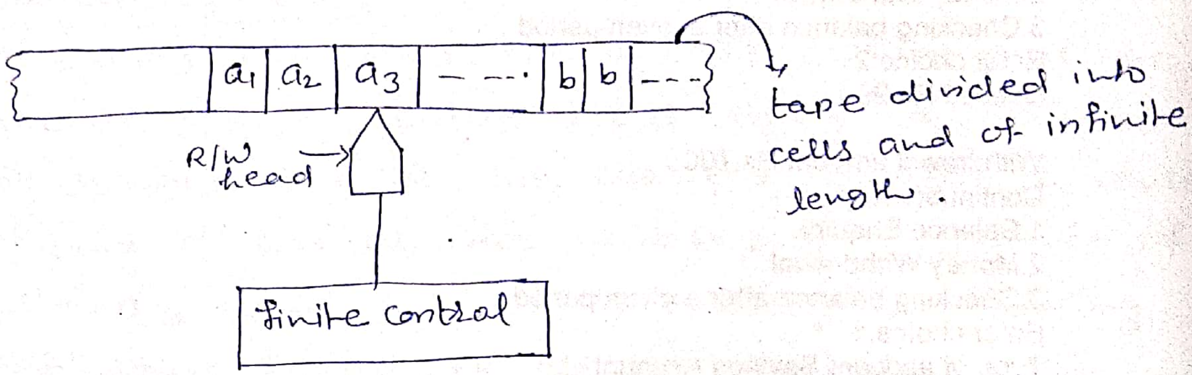
Turing Machines

The turing machine can be thought of as finite control connected to a R/W (read / write) head. It has one tape which is divided into a no. of cells.



Each cell can store only one symbol. In one move, the machine examines the present symbol under the R/W head on the tape and the present state to determine

(i) a new symbol to be written on the tape

(ii) a motion of the R/W head along the tape; either the head moves one cell left (L), or one cell right (R).

(iii) the next state of the machine, and

(iv) whether to halt or not.

Defn. A Turing machine M is a 7-tuple $(Q, \Sigma, \Gamma, \delta, q_0, b, F)$ where

(i) Q is a finite non empty set of states

(ii) Γ is a finite nonempty set of tape symbols

(iii) $b \in \Gamma$ is the blank.

(iv) Σ is a nonempty set of input symbols and $\Sigma \subseteq \Gamma$ & $b \notin \Sigma$

(v) δ is the transition fn. mapping (q, x) onto (q', y, D) where D denotes the direction of movement of R/W head; $D = L$ or R, $y, x \in \Gamma$, $q, q' \in Q$.

(vi) $q_0 \in Q$ is the initial state

(vii) $F \subseteq Q$ is the set of final states.

(δ may not defined for some elts. of $Q \times \Gamma$.)

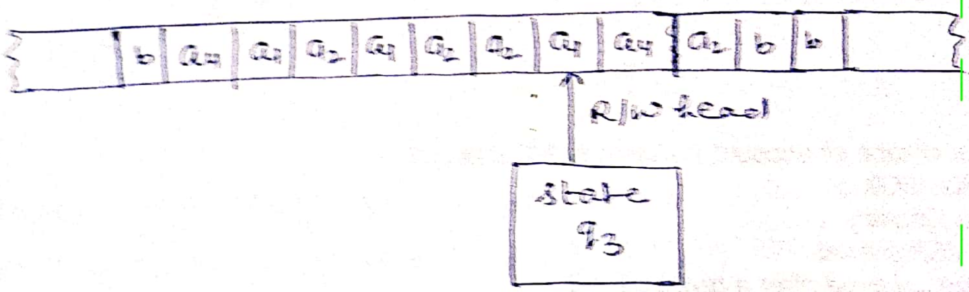
Representation of Turing Machines

We can describe a TM by (i) instantaneous descriptions using move-relation, (ii) transition table, (iii) transition diagram.

Representation By ID:

An ID of a TM 'M' is a string $\alpha \beta \gamma$, where β is the present state of M, the entire input string is split as $\alpha\gamma$, the 1st symbol of γ is the current symbol a under R/W head and γ has all the subsequent symbols of the input string, and the string α is the substring of the input string formed by all the symbols to the left of a.

Ex:



The present symbol under the R/W head is a_1. The present state is q_3. So a_1 is written to the right of q_3.

ID is given by $a_4 a_1 a_2 a_4 a_2 a_2 q_3 a_1 a_4 a_2$

Moves in a TM

Suppose $\delta(q, x_i) = (p, y, L)$. The input string to be processed is $x_1 x_2 \cdots x_n$, and the present symbol under R/W head is x_i. So the ID before processing x_i is

$$x_1 x_2 \cdots x_{i-1}\, q\, x_i \cdots x_n$$

After processing x_i, the resulting ID is

$$x_1 x_2 \cdots x_{i-2}\, p\, x_{i-1}\, y\, x_{i+1} \cdots x_n$$

This change is represented by

$$x_1 x_2 \cdots x_{i-1}\, q\, x_i \cdots x_n \vdash x_1, \cdots x_{i-2}\, p\, x_{i-1}\, y\, x_{i+1} \cdots x_n$$

If $i = 1$, the resulting ID is $p\, y\, x_2 x_3 \cdots x_n$.

If $\delta(q, x_i) = (p, y, R)$ then the change of ID is represented by

$$x_1 x_2 \cdots x_{i-1}\, q\, x_i \cdots x_n \vdash x_1 x_2 \cdots x_{i-1}\, y\, p\, x_{i+1} \cdots x_n$$

If $i = n$, the resulting ID is $x_1 x_2 \cdots x_{n-1}\, y\, p\, b$.

Representation By Transition Table

If $\delta(q, a) = (\gamma, \alpha, \beta)$, we write $\alpha\beta\gamma$ under the a-col. and in the q-row. So, if we get $\alpha\beta\gamma$ in the table, it means that α is written in the current cell, β gives for the movement of the head and γ denotes the new state into which the TM enters.

Ex:

Present state	Tape Symbols b	0	1
$\rightarrow q_1$	$1Lq_2$	$0Rq_1$	–
q_2	bRq_3	$0Lq_2$	$1Lq_2$
q_3	–	bRq_4	bRq_5
q_4	$0Rq_5$	$0Rq_4$	$1Rq_4$
q_5	$0Lq_2$	–	–

Ex 1. Consider the TM given in the above table. Draw the computation of the input string 00b.

Soln. For the input string 00b, we get the following sequence:

$q_1 00b \vdash 0q_1 0b \vdash 00q_1 b \vdash 0q_2 01 \vdash q_2 001$

$\vdash q_2 b001 \vdash bq_3 001 \vdash bbq_4 01 \vdash bb0q_4 1 \vdash bb01q_4 b$

$\vdash bb010q_5 b \vdash bb01q_2 00 \vdash bb0q_2 100 \vdash bbq_2 0100$

$\vdash bq_2 b0100 \vdash bbq_3 0100 \vdash bbbq_4 100 \vdash bbb1q_4 00$

$\vdash bbb10q_4 0 \vdash bbb100q_4 b \vdash bbb1000q_5 b$

$\vdash bbb100q_2 00 \vdash bbb10q_2 000 \vdash bbb1q_2 0000$

$\vdash bbbq_2 10000 \vdash bbq_2 b10000 \vdash bbbq_3 10000 \vdash bbbbq_5 0000$

Ex 2. Draw the computation of the input string 00110.

Representation By Transition Diagram

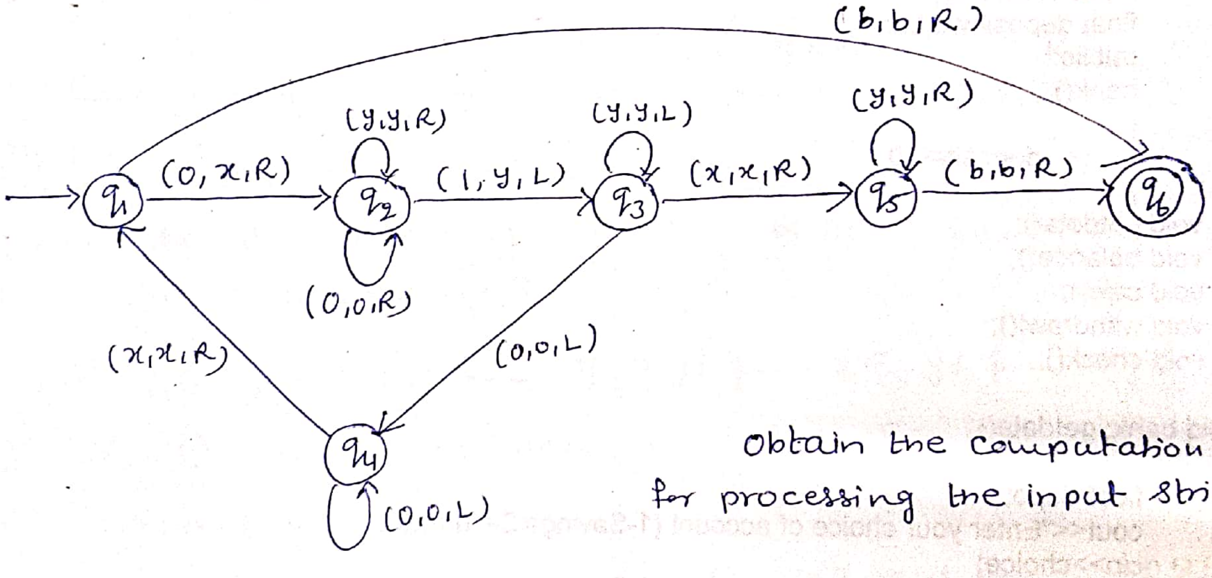
The states are represented by vertices, Directed edges are used to represent transition of states. The labels are triples (α, β, γ), where $\alpha, \beta \in \Gamma$ and $\gamma \in \{L, R\}$.
When there is a directed edge from q_i to q_j with label (α, β, γ) it means that

$$\delta(q_i, \alpha) = (q_j, \beta, \gamma).$$

During the processing of an input string, suppose the TM enters q_i and R/W head scans the present symbol α. As a result, the symbol β is written in the cell under the R/W head. The R/W head moves to the left or right, depending upon γ, and the new state is q_j.

Every edge can be represented by 5-tuple $(q_i, \alpha, \beta, \gamma, q_j)$.

Ex.



Obtain the computation seq. of M for processing the input string 0011.

Soln. The initial tape input is b0011b.

$$\underset{q_1}{b\overset{\downarrow}{0}011b} \xrightarrow{(0,x,R)} \underset{q_2}{bx\overset{\downarrow}{0}11b} \xrightarrow{(0,0,R)} \underset{q_2}{bx0\overset{\downarrow}{1}1b} \xrightarrow{(1,y,L)} \underset{q_3}{bx\overset{\downarrow}{0}y1b} \xrightarrow{(0,0,L)} \underset{q_4}{b\overset{\downarrow}{x}0y1b}$$

$$\xrightarrow{(x,x,R)} \underset{q_1}{bx\overset{\downarrow}{0}y1b} \xrightarrow{(0,x,R)} \underset{q_2}{bxx\overset{\downarrow}{y}1b} \xrightarrow{(y,y,R)} \underset{q_2}{bxxy\overset{\downarrow}{1}b} \xrightarrow{(1,y,L)} \underset{q_3}{bxx\overset{\downarrow}{y}yb}$$

$$\xrightarrow{(y,y,L)} \underset{q_3}{bx\overset{\downarrow}{x}yyb} \xrightarrow{(x,x,R)} \underset{q_5}{bxx\overset{\downarrow}{y}yb} \xrightarrow{(y,y,R)} \underset{q_5}{bxxy\overset{\downarrow}{y}b}$$

$$\xrightarrow{(y,y,R)} \underset{q_5}{bxxyy\overset{\downarrow}{b}} \xrightarrow{(b,b,R)} \underset{q_6}{bxxyyb\overset{\downarrow}{b}}$$

Language acceptability by TM.

Let $M = (Q, \Sigma, \Gamma, \delta, q_0, b, F)$ be a TM. $w \in \Sigma^*$ is said to be accepted by M if $q_0 w \vdash^* \alpha_1 p \alpha_2$ for some $p \in F$ and $\alpha_1, \alpha_2 \in \Gamma^*$.

M does not accept w if the machine M either halts in a nonaccepting (non final) state or does not halt.

Ex consider the TM M described by the table below:

State	0	1	x	y	b
$\rightarrow q_1$	xRq_2	–	–	–	bRq_5
q_2	$0Rq_2$	yLq_3	–	yRq_2	–
q_3	$0Lq_4$	–	xRq_5	yLq_3	–
q_4	$0Lq_4$	–	xRq_1	–	–
q_5	–	–	–	yRq_5	bRq_6
q_6	–	–	–	–	–

Describe the processing of (a) 011, (b) 0011, (c) 001. which strings are accepted by M?

Soln. (a) $q_1011 \vdash xq_211 \vdash q_3xy1 \vdash xq_5y1 \vdash xyq_51$

As $\delta(q_5, 1)$ is not defined, M halts, so the string 011 is not accepted by M.

(b) $q_1 0011 \vdash xq_2011 \vdash x0q_211 \vdash xq_30y1 \vdash q_4x0y1 \vdash xq_10y1$

$\vdash xxq_2y1 \vdash xxyq_21 \vdash xxq_3yy \vdash xq_3xyy \vdash xxq_5yy$

$\vdash xxyq_5y \vdash xxyyq_5b \vdash xxyybq_6$

M halts, As q_6 is a final state, so the string 0011 is accepted by M.

(c) $q_1001 \vdash xq_201 \vdash x0q_21 \vdash xq_30y \vdash q_4x0y$

$\vdash xq_10y \vdash xxq_2y \vdash xxyq_2$

M halts and q_2 is non final, so the string 001 is not accepted by M.

Designing of T.M.

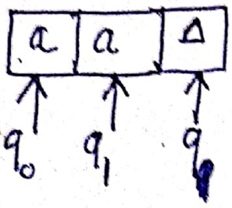
Design the turing machines for following languages:

① $L_1 = \{a^n \mid n \geq 1\}$ ② $L_2 = \{a^{2n} \mid n \geq 1\}$ ③ $L_3 = (a+b)^*$

⑤ $L_5 = \{a^n b^n \mid n \geq 1\}$ ④ $L_4 = \{$ string over $\{0,1\}^*$ starting with 00 $\}$

Sol. ① $L_1 = \{a^n \mid n \geq 1\}$

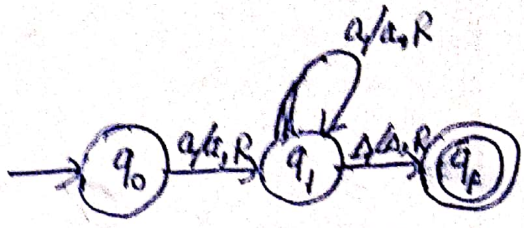
$= a\,a^*$



$\delta(q_0, a) = (q_1, a, R)$

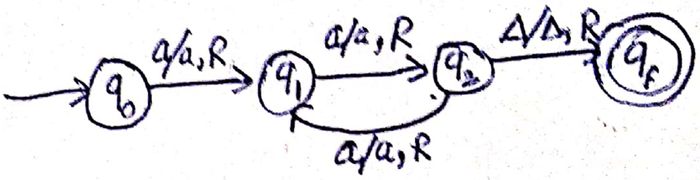
$\delta(q_1, \Delta) = (q_f, \Delta, R)$

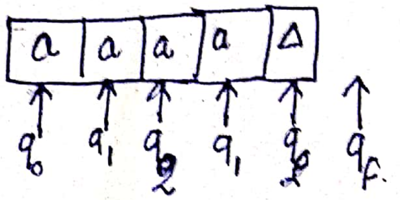
$\delta(q_1, a) = (q_1, a, R)$



② $L_2 = \{a^{2n} \mid n \geq 1\}$

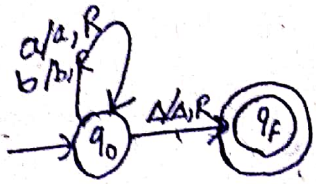
$= aa\,(aa)^*$





③ $L_3 = (a+b)^*$

$S \mapsto aS \mid bS \mid \Lambda$



④ $L_4 = \{$ string over $\{0,1\}^*$ starting with 00 $\}$

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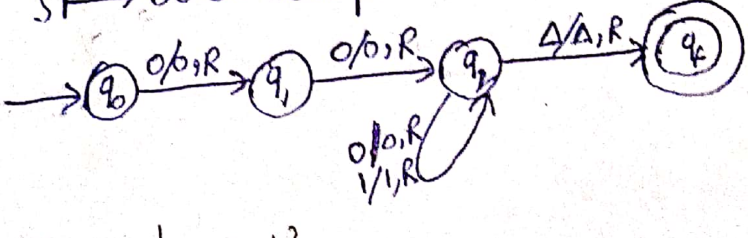
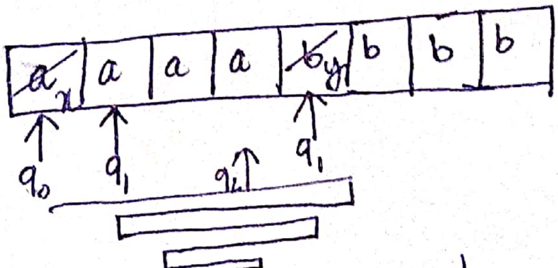
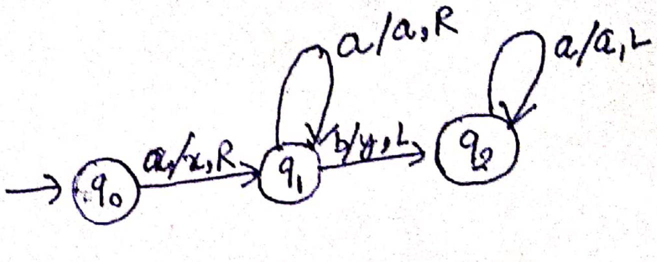
$S \mapsto 000S \mid 001S \mid \Lambda$



⑤ $\{a^n b^n \mid n \geq 1\}$





replace first a by x and leave next a's as it is