

Que 1:-

Ans 1:-

$B(0) = ₹100$ → current bond price

$B(1) = ₹110$ → bond price after 1 period.

$S(0) = ₹80$ → current stock price.

$V(t) = xS(t) + yB(t)$ → total wealth of an investor holding x stock shares and y bonds at time t

$$V(0) = ₹10000$$

Let the investor hold x stock shares and y bond shares at time instant $t=0$,

$$x = 10000 \times \left(\frac{3}{8}\right) \times \left(\frac{1}{80}\right) = 75$$

$$y = 10000 \times \left(\frac{2}{3}\right) \times \left(\frac{1}{100}\right) = 40$$

$(75, 40)$ → portfolio.

$(₹10000)$ → value of portfolio at $t=0$

$$S(1) = \begin{cases} ₹100 & \text{with } p=0.8 \\ ₹60 & \text{with } p=0.2 \end{cases}$$

$$V(1) = \begin{cases} ₹11900 & p=0.8 \\ ₹8900 & p=0.2 \end{cases}$$

$$R_U = \frac{U(1) - U(0)}{U(0)} = \text{return on the portfolio}$$

$$R_U = \begin{cases} 0.19 & p=0.8 \\ -0.11 & p=0.2 \end{cases}$$

$$\begin{aligned} E(R_U) &= 0.19 \times 0.8 - 0.11 \times 0.2 \\ &= 0.152 - 0.022 \\ &= 0.16 \rightarrow \text{expected return} \end{aligned}$$

$$\begin{aligned} \text{Risk } (\sigma_U) &= \sqrt{(0.19 - 0.16)^2 \times 0.8 + (-0.11 - 0.16)^2 \times 0.2} \\ &= 0.123 \end{aligned}$$

Ans 2:- $B(0) = ₹ 90$

$S(0) = ₹ 25$

$B(1) = ₹ 100$

$S(1) = \begin{cases} ₹ 30 & \text{with probability } p \\ ₹ 20 & \text{with probability } (1-p) \end{cases}$

portfolio : (10, 15) value of portfolio at $t=0$

$$\begin{aligned} U(0) &= ₹ 10 \times ₹ 90 + ₹ 15 \times ₹ 25 \\ &= ₹ (250 + 1350) \\ &= ₹ 1600 \end{aligned}$$

$$U(1) = \begin{cases} ₹ (10 \times 30 + 15 \times 100) = ₹ 1800 \\ ₹ (10 \times 20 + 15 \times 100) = ₹ 1700 \end{cases}$$

$$k_u = \begin{cases} 0.125 \\ 0.0625 \end{cases}$$

Expected return = $0.125p + 0.0625q$

Ans 3 :-

$$B(0) = ₹100$$

$$B(1) = ₹110$$

$$S(0) = ₹80$$

$$S(1) = \begin{cases} ₹100 & p = 0.8 \\ ₹60 & p = 0.2 \end{cases}$$

$$U(0) = ₹10000$$

$$\lambda = \frac{5000}{80} = 62.5$$

$$\gamma = \frac{5000}{100} = 50$$

$$U(1) = \begin{cases} 62.5 \times 100 + 50 \times 110 = ₹11750 \\ 62.5 \times 60 + 50 \times 110 = ₹9250 \end{cases}$$

$$K(U) = \begin{cases} 0.175 \\ -0.075 \end{cases}$$

$$\begin{aligned} E(KU) &= 0.175 \times 0.8 - 0.075 \times 0.2 \\ &= 0.14 - 0.015 \\ &= 0.125 \quad (\text{Expected return}) \end{aligned}$$

$$\begin{aligned} \text{Risk } (\sigma_U) &= \sqrt{(0.175 - 0.125)^2 \times 0.8 + (-0.075 - 0.125)^2 \times 0.2} \\ &= 0.084 \end{aligned}$$

Ans

Ans 4:-

$$B(0) = ₹ 90$$

$$S(0) = ₹ 25$$

$$B(1) = ₹ 100$$

$$S(1) = \begin{cases} ₹ 30 & \text{with probability } p \\ ₹ 20 & \text{" " " } 1-p \end{cases}$$

$$V(1) = \begin{cases} ₹ 1160 & \text{" " " } p \\ ₹ 1040 & \text{" " " } 1-p \end{cases}$$

Portfolio = (x, y)
no. of shares \swarrow \searrow no. of bonds

$$x \times 30 + y \times 100 = 1160$$

$$x \times 20 + y \times 100 = 1040$$

$$10x = 120$$

$$x = 12$$

— (1)

$$y = \frac{1160 - 360}{100}$$

= 8 — (2)

from (1) & (2)

$$V(0) = 2(12 \times 25 + 8 \times 90)$$

$$= ₹ 1020$$

Ans 7:-

Now Arbitrage principle :-

There is no admissible portfolio with initial value $V(0) = 0$ such that $V(1) > 0$ with non-zero profit.

Let's suppose $V(0) = 0$;

£10000 is borrowed from bank.

① we will buy pounds from dealer B we get,
$$\frac{10000}{80} = 125 \text{ pound.}$$

② Investing it in bank for 1 year we get;
$$(125 + 125 \times 0.06) \text{ pound} = 132.5 \text{ pound.}$$

③ we sell the pound for rupee to dealer A we get :-

$$₹ (132.5 \times 79) = ₹ 10467.5$$

④ we returned the borrowed amount with interest to the bank i.e. $₹ 10000 + ₹ 400$ interest
$$= ₹ 10400$$

⑤ profit :- $₹ 10467.5 - ₹ 10400$
$$= ₹ 67.5 > 0 \text{ (Arbitrage exist!!)}$$

Ans 8:-

$$B(0) = ₹100$$

$$B(1) = ₹110$$

$$S(0) = ₹50$$

Let the forward price be F :-

// Case 1:- Short forward contract

If we sell at a fixed price F

- Borrow \$50 (we start from $V(0)=0$)
- Buy asset for $S(0) = \$50$

Portfolio :- $(1, -1/2, -1)$

Now, we will sell the asset at F and return the amount \$55 to the borrower
the profit = $\$(F - 55)$

Now for no arbitrage condition.

$$F - 55 \leq 0$$

$$\Rightarrow F \leq 55 \quad - (1)$$

// Case 2:- Long forward contract

If we buy at F ~~asset~~. at $t=1$ then,

• Sell short the asset at \$ 50

• Investing risk free

we get \$ 55 from investment, we get the asset at F , we will return the asset to the owner

$$\text{profit} = \$55 - F$$

for no arbitrage :-

$$55 - F \leq 0$$

$$\Rightarrow 55 \leq F \quad - (2)$$

from (1) & (2)

$$F = \$55$$

Ans 9:- strike price = ₹ 30
price of option = ₹ 4

Investor is able to make a gain if the price of the commodity (P) becomes less than ₹ 34 in future.

as if $P < 34$

then he/she can ~~sell~~ sell the commodity at ₹ 30 and buying it again at cheaper price making a profit of $(30+4) - P$

$$= \underline{\underline{₹ (34 - P)}}$$