ASSIGNMENT-2

Fuzzy Sets And Fuzzy Logic

Q1. Let $\tilde{A}$, $=\{(a_1, 0.2), (a_2, 0.4), (a_3, 0.6)$, $\tilde{B}$, $=\{(b_1, 0.3), (b_2, 0.4), (b_3, 0.5), (b_4, 0.2)$ are two fuzzy sets write the matrix representation of fuzzy relation $\tilde{R}$, on $\tilde{A}$, and $\tilde{B}$.

Q2. Prove the following:

(i) if the fuzzy relation R is symmetric then so R^{-1}

(ii) R is symmetric if R= R^{-1}

(iii) If R is transitive relation, then so R^{-1}

(∵Here R is the fuzzy relation)

Q3. Suppose $\tilde{R}_1$ is a relation check it is equivalence relation or not.

$$\tilde{R}_1 = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{array} \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 \\ 1 & 0.8 & 0 & 0.1 & 0.2 \\ 0.8 & 1 & 0.4 & 0 & 0.9 \\ 0 & 0.4 & 1 & 0 & 0 \\ 0.1 & 0 & 0 & 1 & 0.5 \\ 0.2 & 0.9 & 0 & 0.5 & 1 \end{bmatrix}$$

Q4. Cut $\tilde{R}$ and $\tilde{S}$ are two fuzzy relation on x×y. Then membership function of two fuzzy relation.

$$\mu_{\tilde{R}} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{bmatrix} y_1 & y_2 & y_3 & y_4 \\ 0.8 & 1 & 0.1 & 0.7 \\ 0 & 0.8 & 0 & 8 \\ 0.9 & 1 & 0.1 & 0.8 \end{bmatrix} \qquad \mu_{\tilde{S}} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{bmatrix} y_1 & y_2 & y_3 & y_4 \\ 0.4 & 0 & 0.9 & 0.6 \\ 0.9 & 0.4 & 0.5 & 0.7 \\ 0.3 & 0 & 0.8 & 0.5 \end{bmatrix}$$

Then find $u_{\tilde{R}\cap\tilde{S}}$, $u_{\tilde{R}\cup\tilde{S}}$, $u_{\tilde{R}}C$, $u_{\tilde{S}}C$

Q5. $\tilde{R}_1$ and $\tilde{R}_2$ are two fuzzy relations on x× y and y × z respectively.

x = $\{x_1, x_2, x_3\}$. y= $\{y_1, y_2, y_3\}$ and z= $\{z_1, z_2, z_3\}$

$\tilde{R}$={ $((x_1,y_2)0.1)$, $((x_1,y_2),0.2)$, $((x_1,y_3),0)$, $((x_1,y_4),1)$, $((x_1,y_5),0.7)$,

$((x_2,y_1),0.3)$, $((x_2,y_2),0.5)$, $((x_2,y_3),0)$, $((x_2,y_4),0.2)$, $((x_2,y_5),1)$,

$((x_3,y_1),0.8)$, $((x_3,y_2),0)$, $((x_3,y_3),1)$, $((x_3,y_4),0.4)$, $((x_3,y_5),0.3)$}

and $\tilde{R}_2$ ={ $((y_1,z_2)0.9)$, $((y_1,z_2),0)$, $((y_1,z_3),0.3)$, $((y_1,z_4),0.6)$,

$((y_2,z_1),0.2)$, $((y_2,z_2),1)$, $((y_2,z_3),0.8)$, $((y_2,z_4),0)$,

$((y_3, z_1), 0.8), ((y_3, z_2), 0). ((y_3, z_3), 0.7), ((y_3, z_4), 1),$

$((y_4, z_1), 0.4), ((y_4, z_2), 0.2), ((y_4, z_3), 0.3) ((y_4, z_4), 0),$

$((y_5, z_1), 0), ((y_5, z_2), 1), ((y_5, z_3), 0), ((y_5, z_4), 0.8)\}$

Find $\tilde{R}_1 o \ \tilde{R}_2$ where 'o' composition as max-min composition.

Q6. Determine transitive max-min closure $\tilde{R}_T$(x,x) for the fuzzy relation $\tilde{R}$ (x,x) given as:

$$\tilde{R} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_4 & x_5 \end{array} \\ \begin{bmatrix} 0.7 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0.4 & 0 & 0 \\ 0 & 0 & 0.8 & 0 \end{bmatrix} \end{array}$$

Q7. Verify the following relation as a similarity relation equivalence relation

$$\tilde{R} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array} \begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\ \begin{bmatrix} 1 & 0.2 & 1 & 0.6 & 0.2 & 0.6 \\ 0.2 & 1 & 0.2 & 0.2 & 0.8 & 0.2 \\ 1 & 0.2 & 1 & 0.6 & 0.2 & 0.6 \\ 0.6 & 0.2 & 0.6 & 1 & 0.2 & 0.8 \\ 0.2 & 0.8 & 0.2 & 0.2 & 1 & 0.2 \\ 0.6 & 0.2 & 0.6 & 0.8 & 0.2 & 1 \end{bmatrix} \end{array}$$

Find the similarity tree of the similarity relation.

Q8. Consider a fuzzy relation $\tilde{R}$ (x,x) defined on x=$\{x_1, x_2, x_3, x_4, x_5\}$ with membership matrix as:

$$\tilde{R} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{array} \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \begin{bmatrix} 1 & 0.3 & 0.7 & 0.2 & 0 \\ 0.3 & 1 & 0.5 & 0.6 & 0.9 \\ 0.7 & 0.5 & 1 & 0.2 & 0.2 \\ 0.2 & 0.6 & 0.2 & 1 & 0.8 \\ 0 & 0.9 & 0.2 & 0.8 & 1 \end{bmatrix} \end{array}$$

Is this relation as compatibility relation? show it by graphically.

Q9. Compose the following two fuzzy relation $\tilde{R}_1$ and $\tilde{R}_2$using (i)max product (ii) max-average composition.

$$\tilde{R}_1: \begin{array}{c} \\ x_1 \\ x_2 \end{array} \begin{array}{c} \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \begin{bmatrix} 0.3 & 0 & 0.7 & 0.3 \\ 0 & 1 & 0.2 & 0 \end{bmatrix} \end{array} \quad \tilde{R}_2: \begin{array}{c} y_1 \\ y_2 \\ y_3 \\ y_4 \end{array} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0.5 & 0.4 \\ 0.7 & 0.8 & 0.6 \\ 0 & 0 & 0 \end{bmatrix}$$

Q10. The following membership matrix defines fuzzy Partial ordering $\tilde{R}$ on the set x={a, b, c, d, e}

$$\tilde{R} = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} 1 & 0.7 & 0 & 1 & 0.7 \\ 0 & 1 & 0 & 0.9 & 0 \\ 0.5 & 0.7 & 1 & 1 & 0.8 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0.1 & 0 & 0.9 & 1 \end{bmatrix} \end{array}$$

Under this ordering determine the "undominated" and "undominating" elements if any.

Q11. Let the fuzzy relations $\tilde{R}$(x,x) and $\tilde{Q}$(y,y), x∈X, y∈Y be defined on the sets:

x={a, b, c, d}, y={α, β, γ} as

$$\tilde{R} = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.8 & 0 \\ 1 & 0 & 0 & 0.4 \\ 0 & 0.4 & 0 & 0 \end{bmatrix} \end{array} \qquad \tilde{Q} = \begin{array}{c} \\ \alpha \\ \beta \\ \gamma \end{array} \begin{array}{c} \begin{array}{ccc} \alpha & \beta & \gamma \end{array} \\ \begin{bmatrix} 0.6 & 0.8 & 0 \\ 1 & 0.8 & 0 \\ 1 & 0 & 0.8 \end{bmatrix} \end{array}$$

(i) Let h map a_1b of x to α of γ and c of x to β of γ and d of x of γ to Y.

(ii) Let h map a of x to α of Y, b,c of X to β of γ and d of X to γ to Y.

(iii) Let h map a, b of X to α of Y and c, d of X to γ to Y

Which of the above three mapping from γ to $\tilde{R}$ (x) to $\tilde{h}$(x)

ASSIGNMENT-3

Q1. Let $\tilde{f}$(x)= (f(x), s(x), t(x)) be fuzzy function with mean function f(x)=x^2 and the spread fucntions(x)=$\frac{x}{4}$ and t(x)= $\frac{x}{2}$

Also Let L(x) =$\frac{1}{1+x^2}$ and R(x)=$\frac{1}{1+2|x|}$

Evaluate the integral $\int_1^4 \tilde{f}(x)dx.$

Q2. Let $\tilde{a}$={(4,0.8), (5,1), (6,0.4)} and $\tilde{b}$={(6,0.7), (7,1), (8,0.2) and f(x)=2 x∈[2,4] then evaluate:

$$\int_{\tilde{b}}^{\tilde{a}} \tilde{f}(x)dx$$

Q3. Let f(x)=2x-3 and g(x)=2x+5

$\tilde{a}$={(1,0.8), (2,1), (3,0.4)}, $\tilde{b}$={(3,0.7), (4,1), (5,0.3) then evaluate

(i) $\int_{\tilde{b}}^{\tilde{a}} f(x)dx$

(ii) (ii) $\int_{\tilde{b}}^{\tilde{a}} g(x)dx$

(iii) $\int_{\tilde{b}}^{\tilde{a}} \text{fog}(x)dx$

(iv) $\int_{\tilde{b}}^{\tilde{a}} gof(x)dx$

Q4. Suppose f(x) =x^3 and $\tilde{x}${(-1,0.4), (0,1), (1,0.6) then evaluate derivative of f(x) at $\tilde{x}$.

Q5. Let f(x)= x^2+1, g(x)=2-x and $\tilde{a}$ and $\tilde{b}$ be triangular fuzzy numbers $\tilde{a} = (1,2,3)$

$\int_{\tilde{a}}^{\tilde{b}} f(x)dx$ and $\int_{\tilde{a}}^{\tilde{b}} g(x)dx$

is $\int_{\tilde{a}}^{\tilde{b}} f(x)dx$+$\int_{\tilde{a}}^{\tilde{b}} g(x)dx$=$\int_{\tilde{a}}^{\tilde{b}} f(x) + g(x)dx$?

Q6. Let $\tilde{x}_0$={(-1, 4), (0,1), (1,6), f(x)= x^3+3, g(x)=2x+3, Compute f'($\tilde{x}_0$) and g'($\tilde{x}_0$). Is f' f'($\tilde{x}_0$) ⊗ g'($\tilde{x}_0$)=(f'+g') $\tilde{x}_0$ in this case.

Q7. Let f(x)=2x^3+ (x-1), g(x)= 2x^3- (x+1) $\tilde{x}_0$={(-1,5), (0,8), (1,1) (2,8) (3,4). Compute f'($\tilde{x}_0$) and g' ($\tilde{x}_0$) and verify whether f'($\tilde{x}_0$) ⊗ g'($\tilde{x}_0$)=(f'+g') $\tilde{x}_0$.

Q8. Determine the maximizing set of

$$f(x)=\begin{cases} 2x^2-3, & -2 \le x \le 2 \\ 5, & elsewhere \end{cases}$$

Q9. Let f(x)= $\sin x, x \in R$. Then f is real valued function in X=R. Determine maximizing set of f(x).

Q10. Answer Yes or No

(i) A fuzzy function is a generalization of a classical function.

(ii) Every fuzzy function can be regarded as a fuzzy relation.

(iii) It is possible to defined a fuzzy function is helpful in determining extrema of fuzzy functions.

(iv) Concept of maximizing set of fuzzy function is helpful in determining extrema of fuzzy functions.

(v) Maximizing set of crisp function can be defined.

(vi) The definition of integral of crisp function over a fuzzy domain as well as of a fuzzy function over a crisp domain are in conformity with extension principle.

(vii) The relation $\int_{\tilde{a}}^{\tilde{b}}((f \otimes g)x)dx = \int_{a}^{b} f(x) \otimes \int_{\tilde{a}}^{\tilde{b}} g(x)\,dx$, holds always.

(viii) The relation $\int_{\tilde{a}}^{\tilde{b}}((f \dot{+} g))dx = \int_{a}^{b} f(x) + \int_{\tilde{a}}^{\tilde{b}} g\,dx$, is always true.

(ix) The relation $\int_{\tilde{a}}^{\tilde{b}} f(x)dx = \int_{a}^{c} f(x) + \int_{\tilde{c}}^{\tilde{b}} f(x)\,dx$, is always true.

(x) $f'(\tilde{x}_0) + g'(\tilde{x}_0) = (f'+g')\,\tilde{x}_0$ may not be always true.

Q11. Use extension principle to find to find the image of $\tilde{A}$ for the function x^4+x^2-1, where $\tilde{A}$ is fuzzy number around 3 characterized by [(1,2),(2,6),(3,1),(4,6),(5,2)}.

Q12. Let x={0, 0.1, 0.2,.......,1} be the universe of discourse and Let $\tilde{A} \subseteq X$

$P(\widetilde{A})_{True}$={(0.5, 0.6), (0.6, 0.7), (0.7, 0.8), (0.8, 0.9)(0.9,1)(1,1)}

$P(\widetilde{B})_{True}$={(0.2, 0.3), (0.3, 0.8), (0.5, 0.1), (0.7, 0.4)(0.8,0.9)(0.9,0.7)}

a. $P(\widetilde{A})$ Negation
b. $P(\widetilde{A})$ very true
c. $P(\widetilde{A})$ Fairly true
d. $P(\widetilde{A})$very very true
e. $P(\widetilde{A}) \wedge P(\widetilde{B})$
f. $P(\widetilde{A}) \vee P(\widetilde{B})$
g. $P(\widetilde{A}) \rightarrow P(\widetilde{B})$

Q13. Let x=[x_1,x_2,x_3] and y=[y_1,y_2] assume the proposition ($\widetilde{A}$)={(x_1,0.5), (x_2,1), (x_3,0.6) and ($\widetilde{B}$)={(y_1,1), (y_2,0.4).

Then given that x is $(\widetilde{A})^1$={(x_1,0.6), (x_2,0.9), (x_3,0.7) and $(\widetilde{B})^1$ such that Y is $(\widetilde{B})^1$.

Hint: R={ ($\tilde{A}$x$\widetilde{B}$) ∪ $\tilde{A}$xY}

$B^1 = A^1 \circ R$

Q14. In the above question assume that the proposition y is $(\widetilde{B})^1$ is given where

$(\widetilde{B})^1$={(y_1,0.9), (y_2,0.7) Find $(\widetilde{A})^1$ s.t x is $(\widetilde{A})^1$

these membership functions and truth values defined in some person x to determine truth value of various propositions such as.

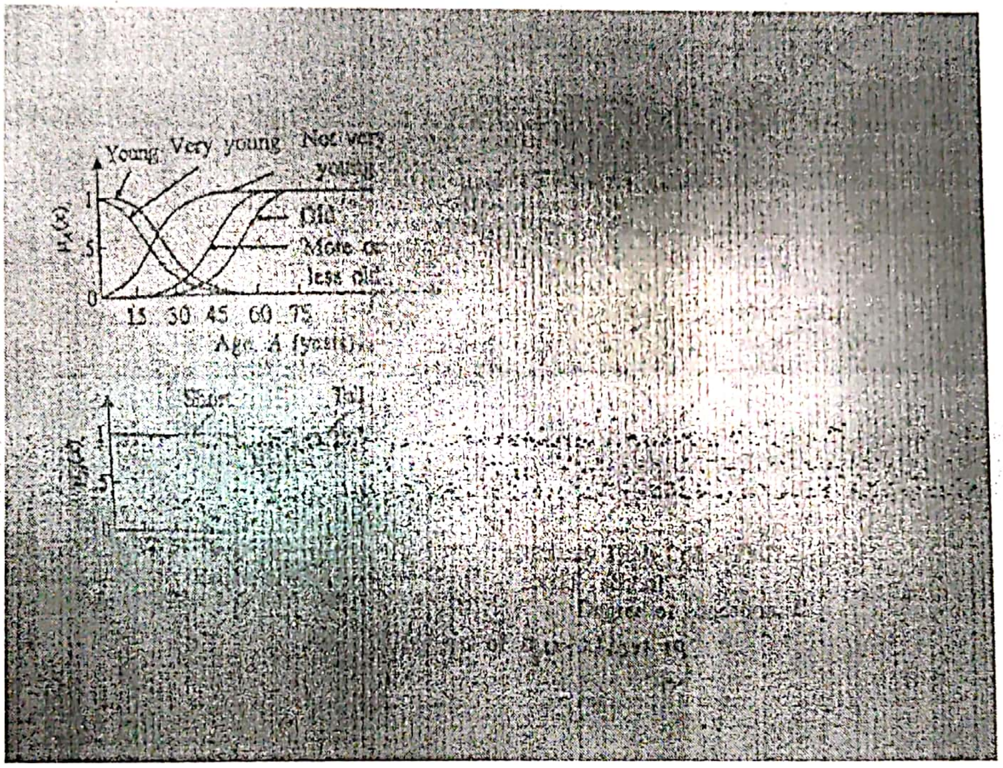
(i) x is highly educated and not very young is very true.

(ii) x is very young, tall, not heavy and somewhat educated is true.

(iii) x is more or less old and highly educated is fairly true.

(iv) x is very heavy or old and not highly educated is fairly true.

(v) x is short, not very young, and highly educated is very true.



Q20. Suppose we have a conditional and qualified proposition *p*: if x is $\tilde{A}$ they *y* is $\tilde{B}$ is very true, where $\tilde{A} = \{(x_1,.8),(x_2,.6),(x_3,.5),(x_4,.4)\}$ and $\tilde{B} = \{(y_1,.2),(y_2,.5),(y_3,.6)\}$. Let *s(a)* = a^2, given $\tilde{A}' = \{(x_1,.9),(x_2,.8),(x_3,.5),(x_4,.5)\}$. Find $\tilde{B}'$ so that *y* is $\tilde{B}'$ is very true.

Q21. Let $X = \{1,2,3,4\}$, and let $\tilde{A} = \{(1,0),(2,.2),(3,.6),(4,1)\}$ denote fuzzy set large *X*. Also let

$$\tilde{R} = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & .5 & 0 & 0 \\ .5 & 1 & .5 & 0 \\ 0 & .5 & 1 & .5 \\ 0 & 0 & .5 & 1 \end{bmatrix} \end{array}$$

Specify approximately equal relation. Then using max-min composition find $\tilde{B}$ .which is approximately equal to $\tilde{A}$.

Q22. Let the fuzzy set $\tilde{A}$ of small positive integers be defined as $\tilde{A}$ = $\{(1,1),(2,.5),(3,.4),(4,.2)\}$ on the universe $X = \{1,2,3,4,5\}$. Let the fuzzy relation

$$\tilde{R} = \begin{array}{c|ccccc} & a & b & c & d & e \\ \hline a & 1 & .8 & 0 & 0 & 0 \\ b & .8 & 1 & .8 & 0 & 0 \\ c & 0 & .8 & 1 & .8 & 0 \\ d & 0 & 0 & .8 & 1 & .8 \\ e & 0 & 0 & 0 & .8 & 1 \end{array}$$

define the fuzzy set 'almost equal integers'. What will be the membership function of the fuzzy set 'small equal integers' if it is interpreted as max-min composition $\tilde{A}$ o $\tilde{R}$?

Q23. Suppose a person's idea of the attractiveness of a car is based on its top speed (km/hr) and the mileage (km/litre) which it gives under normal road conditions and these are given by fuzzy sets $\tilde{A}$ and $\tilde{B}$ defined as

$\tilde{A} = \{(100,.5),(120,.7),(140,.8),(160,1)\}$

$\tilde{B} = \{(10,.6),(12,.8),(15,1)\}$

Use max-min composition to determine his index of an attractive car for different combinations of mileage and top speed. Under this criteria, what is the top speed and mileage of the car which will be most attractive to him?

Q24. Let fuzzy sets $\tilde{A}$ and $\tilde{B}$ defined as under respectively denote the redness and ripeness of six observed tomatoes.

Redness: $\tilde{A} = \{(1,.7),(2,.4),(3,.6),(4,.5),(5,.8),(6,.2)\}$

Ripeness: $\tilde{B} = \{(1,.8),(2,.5),(3,.6),(4,.6),(5,.9),(6,.1)\}$

Use: (i) Zadeh's maximum, (ii) Standard sequence and (iii) Gogueh's fuzzy implications to check the inference 'redness of a tomato implies its ripeness'. Is the conclusion same in all the three cases?