

Q10) Consider a stock whose value $S(t)$ follows SDE

$dS = r \cdot S dt + \sigma S dw$ and has a current price $S(0)$.
What is the probability the call option is in the money based on a strike price $K = 1.25 S(0)$ at time of expiration T ? Given that $T = 0.5$, $r = 0.04$ and $\sigma = 0.10$

Ans) We can see that above stochastic differential equation (SDE) follows a geometric Brownian motion.

$$\mu = \frac{\ln\left(\frac{S(0)}{K}\right) + \left(\frac{\mu - \sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$= \frac{\ln\left(\frac{S(0)}{1.25 S(0)}\right) + \left(\frac{0.04 - 0.01}{2}\right)0.5}{0.1\sqrt{0.5}}$$

$$= -2.908$$

$$\text{Probability of call option} = 1 - N_c(-2.908)$$

$$= 0.998$$

$$\text{Hence } \boxed{\text{solution (probability)} = 0.998}$$