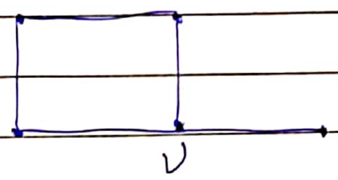
Q6) If a vertex ~~of~~ v of a graph G lies on cycle of G, then v is not a cut vertex.

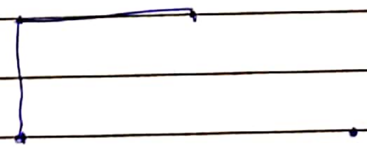
This statement is false as we can construct a graph G where a vertex v lies on a cycle but removing it will clearly make the graph dis-connected.

Let G be :-



G

Here the vertex v lies on C_4 and after removing it we get :-

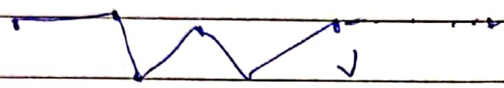


G − v

This is clearly disconnected, hence v was a cut vertex and ~~so~~ the statement above is false.

b) If a vertex of a graph G does not lie on any cycle of graph G, then v is a cut vertex.

We prove this by contradiction. Let us assume that a vertex v in G is not a cut vertex and lies on no cycle.



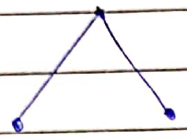
We remove v from G to obtain $G - v$ and as v is not a cut vertex, the graph $G - v$ will be connected. This would imply that 2 vertices u, w that were initially connected in G u & w ~~as~~ ~~are~~ through v are now still connected, which ~~are~~ means some other path exists as well between u & w.

This implies that the path u & w ~~go~~ through v and u & w without ~~to~~ v forms a cycle, which is a contradiction. $\Rightarrow\Leftarrow$

Hence proved ∎

c) A tree of order 3 or more has more cut-vertices than end vertices.

This is false.



The above tree of order 3 has one cut vertex, but only 2 end vertices.

Hence cut vertices < End vertices.

The statement is false.