Functional Dependencies

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Functional Dependencies

- Motivation is – create 'good' tables
- For example:

  Table1(roll_no, course_id, grade, name, address)

  Is this table good or bad?

Roll_no	Course_id	Grade	Name	Address
2017/MC/24	MC302	A	Aman	Prime
2017/MC/24	MC304	A+	Aman	Prime
2017/MC/24	MC306	A	Aman	Prime

- Table1 is bad. Why?
- Answer – Redundancy-
  - Space
  - Inconsistency
  - Updation anomalies
- What caused the problem?
- Answer – name depends on roll_no

Functional Dependencies

- Definition – $a \rightarrow b$
- a functionally determines b
- If you know ‘a’, there is only one ‘b’ to match

Formally:

X → Y implies (t1[x1] = t2[x1] then t1[y1] = t2[y1])

if two tuples agree on the “X” attribute, the *must* agree on the “Y” attribute, too

- ‘X’ and ‘Y’ can be set of attributes
- Other examples of functional dependencies:

Roll_no	Course_id	Grade	Name	Address
2017/MC/24	MC302	A	Aman	Prime
2017/MC/24	MC304	A+	Aman	Prime
2017/MC/24	MC306	A	Aman	Prime

Roll_no → *name, address*

Roll_no, course_id → grade

Closure

- **Closure** of a set of FD: all implied FDs
- For example –

  Roll_no → *name, address*

  Roll_no, course_id → grade

Imply

  Roll_no, course_id → *grade, name, address*

  Roll_no, course_id → roll_no

- How to find all the implied ones, systematically?

Armstrong's Axioms

- "Armstrong's axioms" guarantee soundness and completeness:
- ***Reflexivity:***

$$Y \subseteq X \Rightarrow X \rightarrow Y$$

eg., roll_no, name -> roll_no

- ***Augmentation***

$$X \rightarrow Y \Rightarrow XW \rightarrow YW$$

- eg., roll_no → name then roll_no, grade-> name, grade

Armstrong’s Axioms

- ***Transitivity***

$$X \to Y \text{ and } Y \to Z \Rightarrow X \to Z$$

For example, roll_no → address, and

address → HRA_rate

THEN:

roll_no -> HRA_rate

Armstrong's Axioms

Reflexivity:

$$Y \subseteq X \Rightarrow X \rightarrow Y$$

Augmentation

$$X \rightarrow Y \Rightarrow XW \rightarrow YW$$

Transitivity

$$X \rightarrow Y \text{ and } Y \rightarrow Z \Rightarrow X \rightarrow Z$$

'sound' and 'complete'

Armstrong's axioms

- Additional rules:
- Union

$$X \rightarrow Y \text{ and } X \rightarrow Z \Rightarrow X \rightarrow YZ$$

- Decomposition

$$X \rightarrow YZ \Rightarrow X \rightarrow Y \text{ and } X \rightarrow Z$$

- Pseudo-transitivity

$$X \rightarrow Y \text{ and } YW \rightarrow Z \Rightarrow XW \rightarrow Z:$$

Prove 'Union', 'Decomposition' and 'pseudo-transitivity' from Armstrong's axioms.

Prove 'Union' from three axioms:

$$\left.\begin{aligned} X \rightarrow Y \quad (1) \\ X \rightarrow Z \quad (2) \end{aligned}\right\}$$

$$(1) + augm.\ w/\ Z \Rightarrow XZ \rightarrow YZ\ (3)$$

$$(2) + augm.\ w/\ X \Rightarrow XX \rightarrow XZ\ (4)$$

$$but\ XX\ is\ X;\ thus$$

$$(3) + (4)\ and\ transitivity \Rightarrow X \rightarrow YZ$$

FDs – Closure F+

- Given a set F of FD (on a schema)
- F+ is the set of all implied FD. Eg.,

table1(roll_no, course_id, grade, name, address)

$$\left.\begin{array}{l}\text{roll\_no, course\_id} \rightarrow \text{grade} \\ \text{roll\_no} \rightarrow \text{name, address}\end{array}\right\} F$$

Closure F+

$$\left.\begin{array}{l}\text{Roll\_no, course\_id} \rightarrow \text{grade} \\ \text{Roll\_no} \rightarrow \text{name, address} \\ \text{Roll\_no} \rightarrow \text{roll\_no} \\ \text{Roll\_no, course\_id} \rightarrow \text{address} \\ \text{Course\_id, address} \rightarrow \text{course\_id}\end{array}\right\} F+$$

FDs – Closure A+

- Given a set F of FD (on a schema)
- A+ is the set of all attributes determined by A:

table1(roll_no, course_id, grade, name, address)

roll_no, course_id → grade

Roll_no → name, address

F

{roll_no}+ =??

{roll_no}+ = {roll_no, name, address}

{course_id}+ = ??

{course_id, roll_no}+ = ??

FDs – A+ closure

if A+ = {all attributes of table}

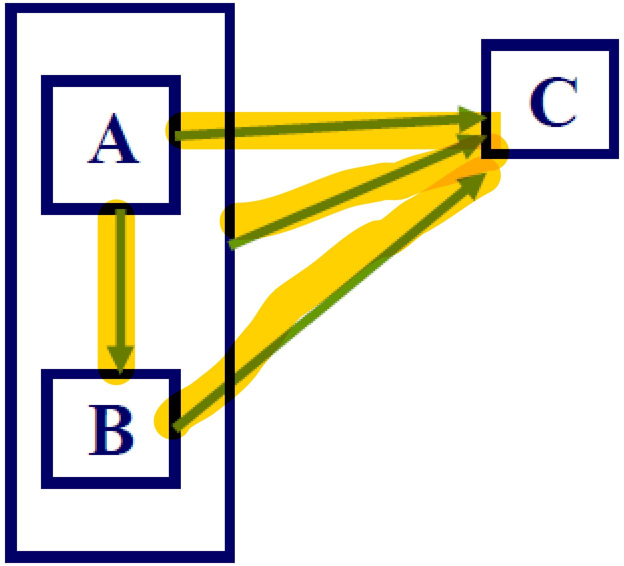
then ‘A’ is a **superkey**

AB → C (1)

A → BC (2)

B → C (3)

A → B (4)



Canonical Cover F_c

Given a set F of FD (on a schema)

F_c is a minimal set of equivalent FD. Eg.,

table1(roll_no, course_id, grade, name, address)

$$\left.\begin{array}{l} \text{roll\_no, course\_id} \rightarrow \text{grade} \\ \text{Roll\_no} \rightarrow \text{name, address} \\ \text{Roll\_no,name} \rightarrow \text{name, address} \\ \text{roll\_no, course\_id} \rightarrow \text{grade, name} \end{array}\right\} F$$

F_c

$$\boxed{\begin{array}{l} roll\_no, course\_id \rightarrow grade \\ Roll\_no \rightarrow name, address \end{array}}$$

$$\left.\begin{array}{l} roll\_no, course\_id \rightarrow grade \\ Roll\_no \rightarrow name, address \\ Roll\_no,name \rightarrow name, address \\ roll\_no, course\_id \rightarrow grade, name \end{array}\right\} F$$

FDs – Canonical cover F_c

- Why do we need it?
- define it properly
- compute it efficiently

FDs – Canonical cover F_c

- Why do we need it?
  - easier to compute candidate keys
- Define it properly – three properties
  - 1) the RHS of every FD is a single attribute
  - 2) the closure of F_c is identical to the closure of F
    - i.e., F_c and F are equivalent
  - 3) F_c is minimal
    - i.e., if we eliminate any attribute from the LHS or RHS of a FD, property #2 is violated
    - we need to eliminate ‘extraneous’ attributes.
- An attribute is ‘extraneous’ if
    - the closure is the same, before and after its elimination
    - or if F-before implies F-after and vice-versa

Canonical cover F_c

$$\left.\begin{array}{l}
\text{roll_no, course_id} \rightarrow \text{grade} \\
\text{Roll_no} \rightarrow \text{name, address} \\
\text{Roll_no,}\cancel{\text{name}} \rightarrow \text{name, address} \\
\text{roll_no, course_id} \rightarrow \text{grade, }\cancel{\text{name}}
\end{array}\right\} F$$

Algorithm for Canonical cover F_c

- examine each FD; drop extraneous LHS or RHS attributes; or redundant FDs
- make sure that FDs have a single attribute in their RHS
- repeat until no change

For example: Trace algorithm for

$AB \rightarrow C$ (1)

$A \rightarrow BC$ (2)

$B \rightarrow C$ (3)

$A \rightarrow B$ (4)

Canonical cover F_C

- Step 1: Split (2)

$AB \rightarrow C$	(1)
$A \rightarrow B$	(2’)
$A \rightarrow C$	(2’’)
$B \rightarrow C$	(3)
$A \rightarrow B$	(4)

Step 2: Delete redundant FDs

$AB \rightarrow C$	(1)
$\cancel{A \rightarrow B}$	(2’)
$A \rightarrow C$	(2’’)
$B \rightarrow C$	(3)
$A \rightarrow B$	(4)

$AB \rightarrow C$	(1)
$A \rightarrow C$	(2’’)
$B \rightarrow C$	(3)
$A \rightarrow B$	(4)

- Step 3: (2'') is redundant (implied by (4), (3) and transitivity)

$AB \to C$	(1)	$AB \to C$	(1)
$\cancel{A \to C}$	(2'')		
$B \to C$	(3)	$B \to C$	(3)
$A \to B$	(4)	$A \to B$	(4)

- Step 4: in (1), 'A' is extraneous:

$AB \to C$	(1)	$B \to C$	(1')
$B \to C$	(3)	$B \to C$	(3)
$A \to B$	(4)	$A \to B$	(4)

Canonical Cover F_c

- Step 5: (1') is redundant

$\cancel{B \rightarrow C}$ (1')

$B \rightarrow C$ (3)

$A \rightarrow B$ (4)

$B \rightarrow C$ (3)
$A \rightarrow B$ (4)

- **nothing is extraneous**
- **all RHS are single attributes**
- **final and original set of FDs are equivalent (same closure)**

Before

$AB \rightarrow C$ (1)

$A \rightarrow BC$ (2)

$B \rightarrow C$ (3)

$A \rightarrow B$ (4)

After

$B \rightarrow C$ (3)

$A \rightarrow B$ (4)

Equivalence of set of FDs

- Given - 2 sets of FDs, E and F
- If every dependency of E can be inferred from F, then E is covered by F, and vice versa.
- If $E^+ = F^+$, E covers F and F covers E
- Check – F covers E
  - For all $X \rightarrow Y$ in E, calculate X^+
  - Check if X^+ includes the attributes in Y.
- For example: $E = \{A \rightarrow C, AC \rightarrow D, E \rightarrow AD, E \rightarrow H\}$

  $F = \{A \rightarrow CD, E \rightarrow AH\}$

Check – F covers E?

E ={A → *C, AC → D, E → AD, E → H*}

F = {A → *CD, E → AH*}

In E:

A → *C*
Compute A^+ using FDs in F
A^+ = {A, C, D}

AC → *D*
Compute $\{AC\}^+$ using FDs in F
$\{AC\}^+$ = {A, C, D}

E → *AD*
Compute $\{E\}^+$ using FDs in F
$\{E\}^+$ = {E, A, H, C, D}

E → *H*
Compute $\{E\}^+$ using FDs in F
$\{E\}^+$ = {E, A, H, C, D}

- Thus, F covers E.

Check – E covers F?

E =$\{A \rightarrow C, AC \rightarrow D, E \rightarrow AD, E \rightarrow H\}$

F = $\{A \rightarrow CD, E \rightarrow AH\}$

In F:

$A \rightarrow CD$
Compute A^+ using FDs in E
$A^+ = \{A, C, D\}$

$E \rightarrow AH$
Compute $\{E\}^+$ using FDs in E
$\{E\}^+ = \{E, A, D, H, C\}$

Thus, E covers F.

Is E equivalent to F?

- E is covered by F, and
- F is covered by E

- Thus, E is equivalent to F.

Question 1:

- Find canonical cover for the following sets of dependencies:
  - E = {B → *A, D → A, AB → D*}
  - F = {A → *BCDE, CD → E*}