

EC-353 Computer Vision

Topic: Bayes Law.

 $x \rightarrow$ original data $y \rightarrow$ noisy Data

$$p(x) \cdot p\left(\frac{y}{x}\right) = p(y) p\left(\frac{x}{y}\right)$$

 $p(x)$ - a-priori $p(y)$ - a-priori $p\left(\frac{x}{y}\right)$ - a-posteriori $p\left(\frac{y}{x}\right)$ - a-prioriMA \rightarrow Filters (Moving Average) \Downarrow ZerosDenominator - Poles

$$H(z) = \frac{N(z)}{D(z)} - \text{Autoregressive Systems}$$

ARMA Models - Auto Regressive Moving
average Models

A A B A A T

$$FT(f(t)) = f(w)$$

$$FT(f(w)) = f(t)$$

$$E[e^{jw t}] = \int_{-\infty}^{\infty} e^{jw t} f(t) dt$$

$$E[X] = \sum x \cdot f(x)$$

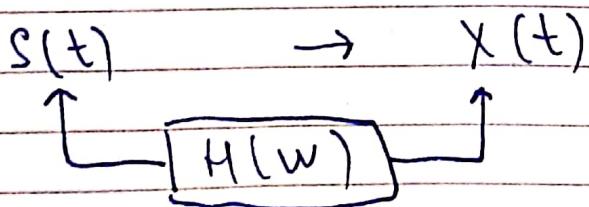
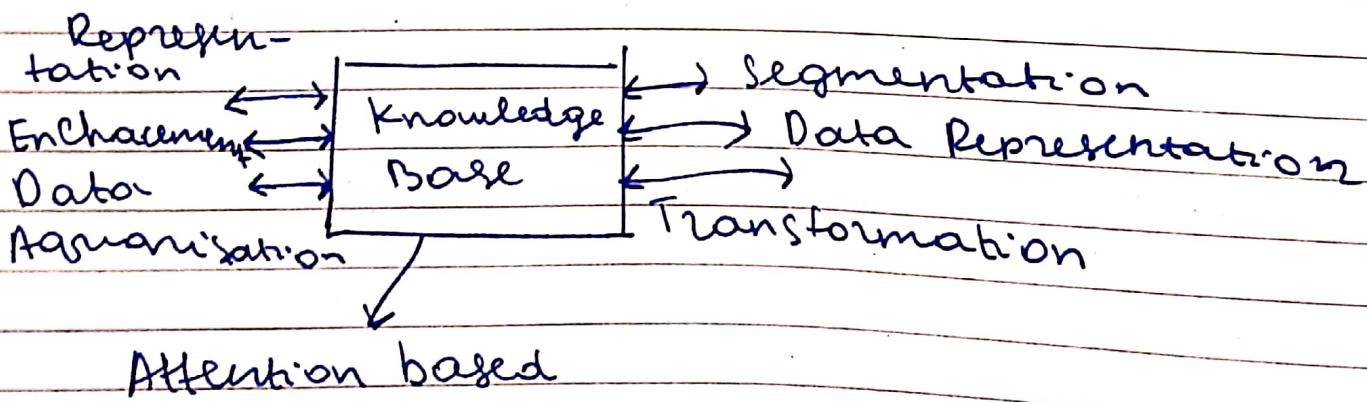
$$= \int_{-\infty}^{\infty} x \cdot f(x) dx$$

Hadamard Transform

$$H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$H_4 = \begin{bmatrix} H_2 & H_2 \\ H_2 & -H_2 \end{bmatrix}, \quad H_8 = \begin{bmatrix} H_4 & H_4 \\ H_4 & -H_4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \quad \begin{array}{l} 1 - 1 + 1 - 1 = 0 \\ 1 + 1 + 1 - 1 = 0 \\ 1 - 1 - 1 + 1 = 0 \end{array}$$



DT. 20 08 2019

Topic : Computer vision. Concepts

$$OTF = \frac{F(w)}{|F(w)|_{\max}}$$

$$R(w) = \int_{-\infty}^{\infty} f(t) e^{-jw t} dt$$

\downarrow \downarrow
 F A

$$B = AFA^T - \text{Diagonal matrix}$$

A - Orthogonal matrix

$B = AFA^T \Rightarrow$ Diagonal matrix
+ve definite.
 \Downarrow
eigen vectors

Every transform is a comprehension technique in itself

$$\text{Avg} = \frac{a+b+c}{3} = \frac{1}{3}a + \frac{1}{3}b + \frac{1}{3}c$$

Statistical avg = $ap(a) + bp(b) + cp(c)$

$$\begin{aligned} p(a) &= p(b) = p(c) = 1 \\ p(a) + p(b) + p(c) &= 1 \end{aligned}$$

Ensemble avg = $ap(a) + bp(b) + cp(c)$
 $p(a) \neq p(b) \neq p(c)$
 $p(a) + p(b) + p(c) = 1$

Spiral

Probabilities are unequal

$$MA = ap(a) + bp(b) + cp(c)$$

$$; p(a) \neq p(b) \neq p(c)$$

$$\& p(a) + p(b) + p(c) \neq 1$$

Probability density function

Uniform density function

Gaussian density function

RDF

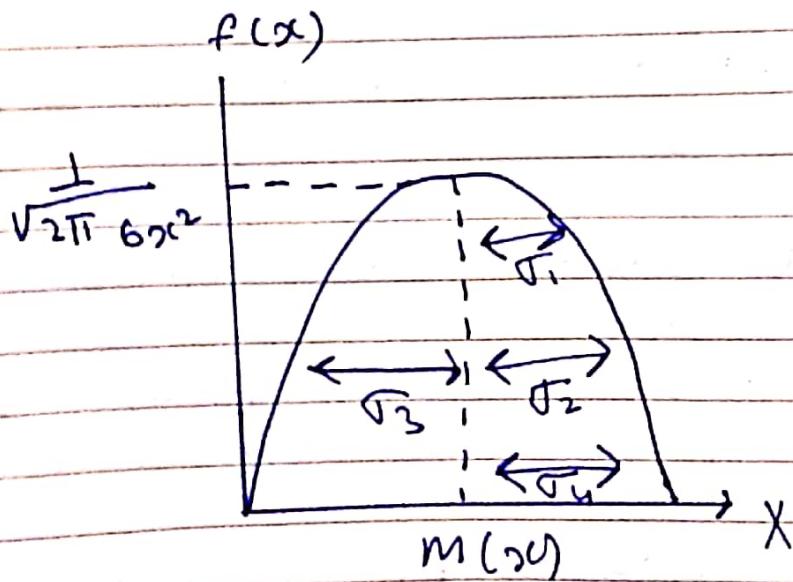
Naragami density function

Binomial density function

$$GMM \ gdf = p_1 f_1 + p_2 f_2 + \dots + p_n f_n$$

Gaussian density function

$$f_{\mu}(x) = \frac{1}{\sqrt{2\pi} \sigma_x^2} e^{-(x-\mu)^2 / 2\sigma_x^2}$$



White noise / colored noise.

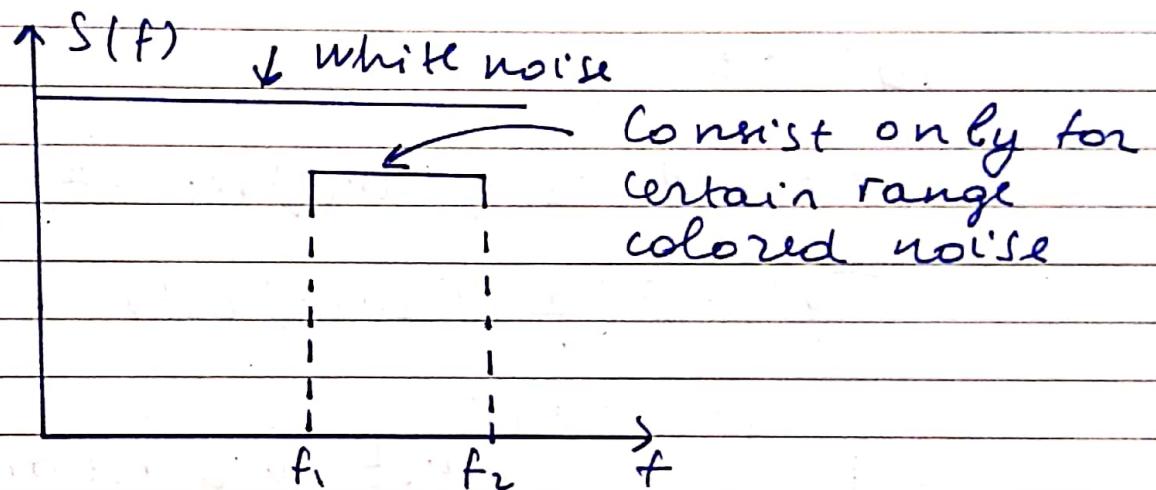
$$\sum_{t=0}^{\infty} x(t) \times x(t-T) = R(t) \forall t$$

Weiner - Khintchin theorem

$$S(f) = FT(R(T))$$

$$R(T) = F(T)^{-1}(S(f))$$

Power spec density gives behaviour of signal



• Entropy is a measure of info content whenever and whatever function manner you can make that entropy to the max for the given noise that condition when evaluated comes out to be gaussian espⁿ (noise is gaussian)

$$x(t) = s(t) + w(t)$$

$$s(t) = [A \quad -A] NR 2$$

$$y(t) = \int_0^T (x(t) - 1) dt$$

$$= \int_0^T (-A) dt + \int_0^T w(t) dt$$

$$y(t) = -AT + \int_0^T w(t) dt$$

$$y(t) = y_{B/T} = T = -A + \frac{1}{T} \int_0^T w(t) dt$$

$$m_x = -A$$

$$\sigma_y^2 = E(y - m_x)^2$$

$$= E(-A + \frac{1}{T} \int_0^T w(t) dt + A)$$

$$\sigma_y^2 = E(\frac{1}{T} \int_0^T w(t) dt)$$

$$= E \left[\frac{1}{T^2} \int_0^T \int_0^T w(t) w(u) dt du \right]$$

$$= \frac{1}{T^2} \int_0^T \int_0^T E(w(t) w(u) dt du)$$

$$= \frac{1}{T^2} \int_0^T \int_0^T R(t-u) dt du$$

$$= \frac{1}{T^2} \int_0^T \int_0^T \frac{N_2}{2} f(t-u) dt du$$

$$= \frac{N_2}{2T^2} \int_0^T 1 \cdot dt \quad \{t=u\}$$

DT.

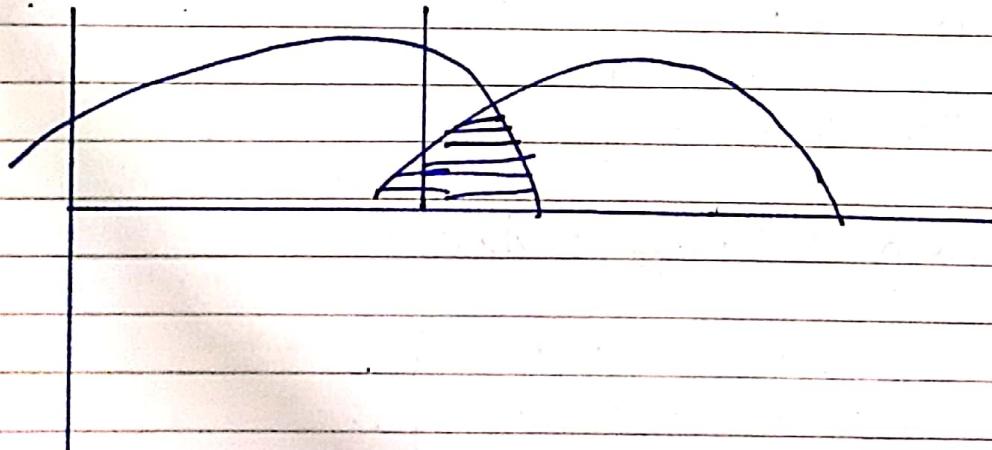
$$= \frac{N_0}{2T^2} \times T = \frac{N_0}{2T}$$

$$f(y|0) = \frac{1}{\sqrt{2\pi} \frac{N_0}{2T}} e^{-(y+A)^2/2} \times \frac{N_0}{2T}$$

Let $T=1$

$$f(y|0) = \frac{1}{\sqrt{\pi} N_0} e^{-(y+A)^2/2} \cdot \frac{N_0}{2T}$$

$$f(y|1) = \frac{1}{\sqrt{\pi} N_0} e^{-(y-A)^2/2} \cdot \frac{N_0}{2T}$$



$$P_{01} = \int_0^\infty f(y|0) dy$$

Spiral

Topic: Geometrical Transforms

Translation:

$$x^* = x + \Delta x$$

$$y^* = y + \Delta y$$

$$z^* = z + \Delta z$$

$$\begin{bmatrix} x^* \\ y^* \\ z^* \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \Delta x \\ 0 & 1 & 0 & \Delta y \\ 0 & 0 & 1 & \Delta z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Scaling

$$T = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation (Along z -Axis)

$$T = \begin{bmatrix} \cos \theta & \sin \theta & 0 & 0 \\ -\sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation (Along x -axis)

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & \sin \theta & 0 \\ 0 & -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Unitary Transforms

Walsh Transform

$$N = 2^n ; \quad s = 2^s \quad n = 3$$

$$W(x, u) = \frac{1}{N} \prod_{i=1}^{n-1} (-1)^{b_i(x) b_{n-i-1}(u)}$$

Kernel transform

$$b_k(z) = b_k(110)$$

$z = 6 - (110)$ k_{th} bit of binary representation of 2

$$b_0 = 0 \quad b_1 = 1 \quad b_2 = 1$$

$$b=3 \quad N=8 \quad W(0,0) = \frac{1}{8} \left[\prod_{i=0}^2 (-1)^{b_i(0)} b_{2-i}(0) \right]$$

y \ x	0	1	2	3	4	5	6	7
0	+	+	+	+	+	+	+	+
1	+	+	+	-	-	-	-	-
2								
3								
4								
5								
6								

HAAR Transform

Shant Transform

Sine Transform

Cosine Transform

$$f(u) \sum_{x=0}^{n-1} f(x) \cos \left[\frac{(2x+1)u\pi}{2N} \right]$$

$u = 0 \dots (n-1)$

$$\mathcal{L}(u) = \frac{1}{\sqrt{N}} ; u=0$$

$$= \sqrt{\frac{2}{N}} ; \text{ else}$$

x	0 1 2 3 4 5 6 7 8
0	1 1 1 1 1 1 1 1 1
1	1 1
2	
3	

Hotteling Transform

$$[X] = \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_{n-1} \end{bmatrix} \quad m_x = E[X]$$

$$C_x = [(x - m_x) \cdot (x - m_x)^T]$$

1) Real
2) Symmetrical | Toeplitz

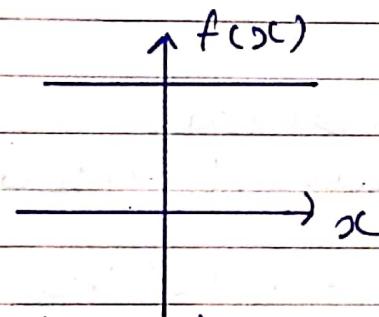
$$A(i) = A(i-j)$$

x_i and x_j are uncorrelated

$$c_{ij} = c_{ji}$$

$$m_x = \frac{1}{m} \sum_{i=0}^{m-1} x_i \cdot f(x) \rightarrow \text{UDF}$$

If $f(x)$ is or UDF (uniformly distributed function) then statistical average will be $= E.A$ (Ensemble average)



$$y = A(x - m_x)$$

Transform - ortho-normal Matrix

$$A \cdot J_x \cdot A^T = \text{Diagonal Matrix}$$

↙
Eigenvalues

$$\frac{MSE}{X \bar{X}}$$

$$\sum_{j=1}^n l_j - \sum_{j=1}^k l_j; k < n = \sum_{j=k+1}^n l_j$$

K.L. Transform

Let $C_x = E[(x - m_x)(x - m_x)^T]$

$A \rightarrow$ Transformation Matrix

$A C_V A^T \Rightarrow$ Eigen Vectors

Considering few top eigenvectors

Principal Components

Edge Detection

Template Matching Gradient Based

The template matching uses local edge gradient magnitude by the approximately maximum of the responses of edge mask

$$g = \max [g_i], i=1 \dots N$$

(gradient mask)

8-12

In differential gradient the local edge magnitude is computed vectorially as a vector using a non-linear transform

$$g_{\infty} = \sqrt{g_x^2 + g_y^2}$$

$$g_x = \max [g_x], i=1 \dots N$$

$$g_y = \max [g_y], i=1 \dots N$$

$$\theta = \arctan(g_y / g_x)$$

Sobel operator

$$S_x = \begin{bmatrix} -1 & 0 & -1 \\ -2 & 0 & 2 \\ -1 & 0 & -1 \end{bmatrix}$$

$$S_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

Prewitt Operators

$$P_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \quad 0^\circ \quad \begin{bmatrix} -1 & 1 & 1 \\ -1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$P_y = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix} \quad 45^\circ \quad \begin{bmatrix} 1 & 1 & 1 \\ -1 & 2 & 1 \\ -1 & -1 & 1 \end{bmatrix}$$

$$Ku\text{sh} = \begin{bmatrix} -3 & -3 & 5 \\ -3 & 0 & 5 \\ -3 & -3 & 5 \end{bmatrix} \quad 45^\circ \quad \begin{bmatrix} -3 & 5 & 5 \\ -3 & 0 & 5 \\ -3 & -3 & -3 \end{bmatrix}$$

Types of edges:

- 1) Step edge
- 2) Slant edge
- 3) Signal / Planar edge
- 4) Triangular edge
- 5) Impulse edge

DT.

$$0^\circ \quad \begin{bmatrix} -A & 0 & A \\ -B & 0 & B \\ -A & 0 & A \end{bmatrix}$$

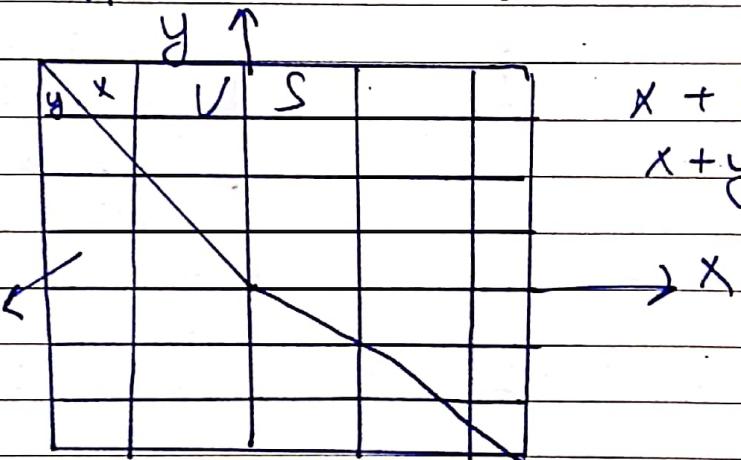
$$45^\circ \quad \begin{bmatrix} 0 & C & D \\ -C & 0 & C \\ -D & -C & 0 \end{bmatrix}$$

Response for 0° mask (0° response)
using 0° mask for step edge
 $= 2A + B$

45° mask using 0° response $= A + B$

0° response $= C + D$

45° response $= 2C + D$



\downarrow - axis response =

$$\begin{aligned} & (1+x+y-w)A + B \\ & = (1+1-w-w)A + B \\ & \approx (1-w)A + B \end{aligned}$$

45° response

$$= (1+V+S-U)C + D$$

Topic: Edge detection

Data vs Mask Generalization

Let the data be

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \quad C = \frac{B}{\sqrt{2}} \quad D = \frac{A}{\sqrt{2}}$$

$$g_0^\circ = A(c+i-a-g) + B(f-d)$$

$$g_90^\circ = A(a+c-g-i) + B(b-h)$$

$$g_{45^\circ} = C(b+f-a-h) + D(c-g)$$

$$g_{45^\circ} = \frac{g_0 + g_{90}}{\sqrt{2}}$$

If we use the step H edge orientation in the neighbourhood of the central pixel then the earlier approach is fine, if this approach gives an error of 6.6° in average

$$\epsilon_{avg} = 6.6$$

Circular operator is one of the way to limit the error and restrict the observations of the circular neighbor.

If we need to increase the number of pixels in neighbourhood then we have to go beyond 4 and 8 connected.

Another replacement of edge detector

tion is known as labeling or
labeling

Segmentation

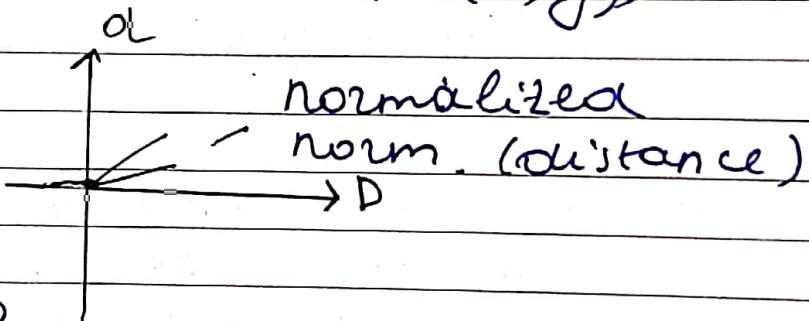
$$\begin{bmatrix} G_1 - \theta_1 \\ G_2 - \theta_2 \\ \vdots \\ G_n - \theta_n \end{bmatrix} - \text{Data matrix}$$

$$[e_1, e_2, e_3]$$

Metric Properties

Distance \Downarrow

$$(d(i, g))$$



$$d(r_i, r_g) > 0$$

$$i \neq 0$$

$$d(r_i, r_j) = d(r_j, r_i)$$

Triangle rule:

$$d(r_i, r_j) + d(r_j, r_k) \geq d(r_i, r_k)$$

Morphological operations

Structuring element

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$