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Q(1) $Y_n = \exp\left\{\left(\sum_{n=1}^{n} X_n\right) - \frac{n}{2}\right\}$

Let $W(t) = \sum_{n=1}^{n} X_n$, let $n = t$

$\therefore$ we get $Y_n = \exp\left(W(t) - \frac{t}{2}\right)$

Let $0 \le s \le t$. As $X_1, X_2, X_3 \dots$ are random i.i.d

$W(t) - W(s)$ is independent of F_s where F_s is filtration where $s > 0$. $W(s)$ is F_s-measurable

We have : $E[e^{W(t)} / F_s] = E[e^{W(t) - W(s)} e^{W(s)} / F_s]$

$= e^{W(s)} E[e^{W(t) - W(s)} / F_s]$

$e^{W(s)} E[e^{W(t) - W(s)}]$

Given : $W(t) - W(s)$ has normal distribution with mean $= 0$ $(\mu = 0)$ and variance $(\sigma^2 = 1) = 1$

Let $t - s = 1$

We get : $E[e^{W(t) - W(s)}] = e^{(t-s)/2} = e^{1/2}$

or $E[e^{W(t)} | F_s] = e^{W(s)} e^{1/2}$

$E[e^{W(t) - t/2} | F_s] = e^{-t/2} E[e^{W(t)} | F_s]$

$e^{W(s) - \frac{s}{2}}$ Hence Y_n is a martingale