Q2) We know that :-

X and Y are i.i.d random variables each having uniform distribution on the interval $(-\pi, \pi)$

$E[X] = 0$
$E[Y] = 0$
$Var(X) = 1$
$Var(Y) = 1$
$E[X^2] = Var(X) + (E[X])^2 = 1$
$E[Y^2] = Var(Y) + (E[Y])^2 = 1$
$E[XY] = E[X]E[Y] = 0$ (as they are i.i.d)

Now, $Z(t) = \cos(Xt+Y)$

$E[Z(t)] = E[\cos(Xt+Y)]$

$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \frac{\theta8}{8!}, \ldots$

$E[Z(t)] = E\left[1 - \frac{(Xt+Y)^2}{2!} + \frac{(Xt+Y)^4}{4!} + \ldots\ldots\right]$

$1 - \frac{1}{2!} E[(Xt+Y)^2] + \frac{1}{4!} E[(X t+Y)^4] + \ldots\ldots$

$1 - \frac{1}{2} E[X^2t^2 + Y^2 + 2XYt] + \frac{1}{4} E[(Xt+Y)^4] + \ldots$

$1 - \frac{1}{2} \left(t^2E[X^2] + E[Y^2] + 2tE(XY)\right) + \ldots$

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$1 - \frac{1}{2}(t^2 + 1 + 0) + \dots$

$1 - \frac{1}{2}(t^2 + 1) + \frac{1}{4!}E[(Xt+Y)^4] + \dots$

$1 - \frac{t^2}{2} - \frac{1}{2} + \frac{1}{4!}E[(Xt+Y)^4] + \dots$

$\frac{1}{2} - \frac{t^2}{2} + \frac{1}{4!}E[(Xt+Y)^4] + \dots \quad \text{——} \quad (1)$

Now, the conditions that a Wide Sense Stationary process must hold are:-

i) $E[Z(t)]$ must be independent of t
ii) $Cov[Z(t), Z(s)]$ depends only on the time difference $|t-s|$ for all t, s
iii) $E[X^2(t)] < \infty$ (finite second order moment)

But we can see clearly from (1) that the first condition isn't satisfied as

$E[Z(t)] \propto t^2$ (depends on t^2)

Hence, this is not a Wide Sense Stationary process.