Computer Vision

DATE / /

Computer Vision : Computer Vision is really about interpreting an analysis of the scene. i.e. what is the content of the image of the scene?, who is in there?, what is in the image? and what is happening?.

(1) What is Computer Vision? Why study it?

Goal of Computer Vision is to write Computer programs that can interpret images.

Input	processing	Output.
Image	⟶ computer vision	Some kind of info. extracted from the image.

∴ Understanding of what that image represents.

Image Processing : Manipulation of Images.

Both the input and Output are images.

★ It extracts signals (features) out of the image.

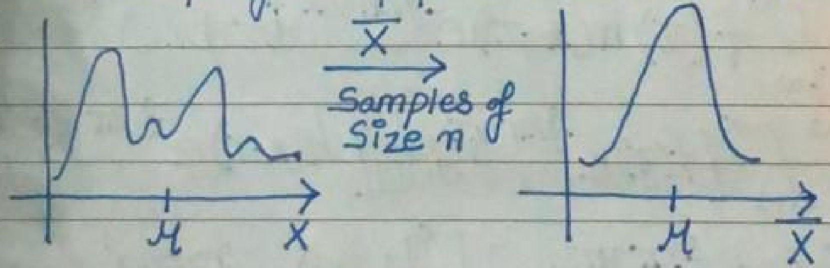
Basics of Image processing : The importance of learning basics of I.P in C.V is

(How)

Q What are the Applications of C.V

✱ CENTRAL LIMIT THEOREM (Statistics)

The Central Limit Theorem states that the sampling distribution of the sample means approaches a normal distribution as the sample size gets larger — no matter what the shape of the population distribution.



∘ population distribution

sampling distribution of the mean.

Convolution

★ It is the process of adding each element of the image to its local neighbors, weighted by the kernel.

mage processing : The importance of
of I.P in C.V is

: Applications of C.V

IMIT THEOREM (Statistics)

Theorem states that the sampling
sample means approaches a normal
sample size gets larger — no matter
the popul

Theorem : The mean of the $\overline{X}$ samples means = mean of the population

$$\mu_{\overline{X}} = \mu_X$$

$\xrightarrow{\overline{X}}$ Samples Size n

μ $\quad$ X

stribution

sampling distribution of the mean.

ess of adding each element of the
neighbors, weighted by the kernel.

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Convolution: the mask is flipped, both Horizontally and Vertically.

$$\begin{bmatrix}1&2&3\\4&5&6\\7&8&9\end{bmatrix} \xrightarrow{H} \begin{bmatrix}7&8&9\\4&5&6\\1&2&3\end{bmatrix} \xrightarrow{V} \begin{bmatrix}9&8&7\\6&5&4\\3&2&1\end{bmatrix}$$

$\boxed{h*f = f*h}$

Notation.

The Fourier transform of the convolution of two functions is the product of their individual FT(s).

Convolution in Spatial domain $\Longleftrightarrow$ Multiplication in frequency domain

Let $g = f*h$

Then $G(u) = \int_{-\infty}^{\infty} g(x) e^{-j2\pi ux} dx$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\tau) h(x-\tau) e^{-j2\pi ux} d\tau dx$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} [f(\tau) e^{-j2\pi u\tau} d\tau][h(x-\tau) e^{-j2\pi u(x-\tau)} dx]$$

$$= \int_{-\infty}^{\infty} [f(\tau) e^{-j2\pi u\tau} d\tau] \int_{-\infty}^{\infty} [h(x') e^{-j2\pi ux'} dx']$$

$$= F(u)H(u)$$

Spatial Domain (x)		Frequency Domain (u)
$g = f*h$	$\longleftrightarrow$	$G = FH$
$g = fh$	$\longleftrightarrow$	$G = F*H$

MON TUE WED THU FRI SAT SUN

$$g = f * h \quad \xrightarrow{}\quad \text{FT},\ \text{FT}$$

$$\text{IFT} \uparrow \qquad G = F \times H$$

 # Computer Vision = Eye + Brain + experience + Intelligence.

$\dfrac{F(u)}{|F(u)|_{max}}$ = optimal Transform Function. (OTF)

F(u) Fourier Transform.

ECE

$$R(u) = \int_{-\infty}^{\infty} \underbrace{f(t)}_{F}\, \underbrace{e^{-jut}}_{A}\, dt$$

or

Digital representation form.

$$\int_{-\infty}^{\infty} \underbrace{f(x)}_{F}\, \underbrace{e^{-jux}}_{A}\, dx$$

Computing representation.

$B = AFA^T$

$A \rightarrow$ Orthogonal Matrix

$B = AFA^T \Rightarrow$ Diagonal Matrix } as a result.

+ve or semi-+ve definite.

Eigen Vectors.

✯ Every transform is a Compression technique.

Papoulis ⟶ Mathematician

$$N(t) = S_1(t) + S_2(t) + S_3(t) + \cdots + S_N(t)$$

Noise

He gave a mathematical model to understand signals independently.

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Mathematics:

☆ Average $= \frac{a+b+c}{3} = \frac{1}{3}a + \frac{1}{3}b + \frac{1}{3}c$

☆ Statistical average: $\boxed{a\,p(a) + b\,p(b) + c\,p(c)}$

provided: $p(a) = p(b) = p(c)$ —— ①

& $p(a) + p(b) + p(c) = 1$ — ②

Now Trying to not follow one condition

Ensemble average $= a\,p(a) + b\,p(b) + c\,p(c)$
E.A

$p(a) \neq p(b) \neq p(c)$

$p(a) + p(b) + p(c) = 1$

Moving Average $= a\,p(a) + b\,p(b) + c\,p(c)$
(M.A)

- $p(a) \neq p(b) \neq p(c)$
- $p(a) + p(b) + p(c) \neq 1$

DT.

EC-353 Computer Vision
Topic: Bayes Law.

$x \rightarrow$ original data
$y \rightarrow$ noisy Data

$$p(x) \cdot p\left(\frac{y}{x}\right) = p(y)\, p\left(\frac{x}{y}\right)$$

$p(x)$ - a-priori
$p(y)$ - a-priori
$p\left(\frac{x}{y}\right)$ - a-posteriori

$p\left(\frac{y}{x}\right)$ - a-priori

MA $\rightarrow$ Filters (Moving Avarage)
$\Downarrow$
Zeros

Denominator - Poles

$$H(z) = \frac{N(z)}{D(z)}$$ - Autoregressive Systems

ARMA Models - Auto Regressive Moving average Models

$AABA^TA^T$

$FT\,(f(t)) = f\,(w)$

$FT\,(f(w) = f(t)$

$$E\left[e^{jwt}\right] = \int_{-\infty}^{\infty} e^{jwt} f(t)\, dt$$

$$E\,[X] = \sum x \cdot f(x)$$

$$= \int_{-\infty}^{\infty} x \cdot f(x)\, dx$$

Spiral

Hadamard Transform

$$H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

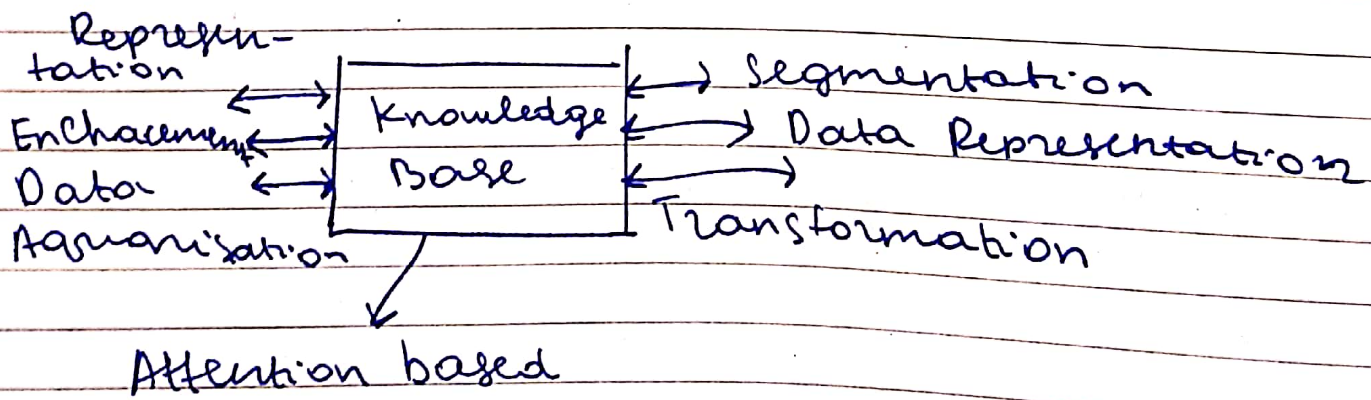
$$H_4 = \begin{bmatrix} H_2 & H_2 \\ H_2 & -H_2 \end{bmatrix}, \quad H_8 = \begin{bmatrix} H_4 & H_4 \\ H_4 & -H_4 \end{bmatrix}$$

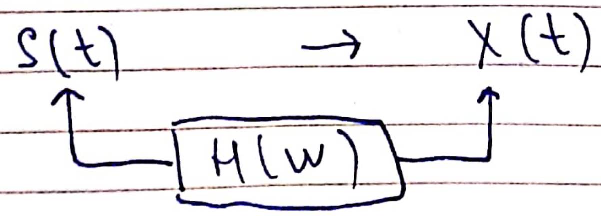
$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & +1 \end{bmatrix}$$

$1 - 1 + 1 - 1 = 0$

$1 + 1 + 1 - 1 = 0$

$1 - 1 - 1 + 1 = 0$





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Topic : Computer vision. Concepts

$$OTF = \frac{F(w)}{|F(w)|_{max}}$$

$$R(w) = \int_{-\infty}^{\infty} \underbrace{f(t)}_{F} \underbrace{e^{-jwt}}_{A} dt$$

$B = AFA^T$ - Diagonal matrix

A - Ortegonal matrix

$B = AFA^T \Rightarrow$ Diagonal matrix $\Downarrow$ eigen vectors

+ve definite.

Every transform is a comprehension technique in itself

$$Avg = \frac{a+b+c}{3} = \frac{1}{3}a + \frac{1}{3}b + \frac{1}{3}c$$

Statistical avg = $ap(a) + bp(b) + cp(c)$

$p(a) = p(b) = p(c)$ - 1

$p(a) + p(b) + p(c) = 1$ - 2

Ensemble avg = $ap(a) + bp(b) + cp(c)$

$p(a) \neq p(b) \neq p(c)$

$p(a) + p(b) + p(c) = 1$

DT.

Probabilities are unequal

$MA = a p(a) + b p(b) + c p(c)$

$; \; p(a) \neq p(b) \neq p(c)$

$\& \; p(a) + p(b) + p(c) \neq 1$

Probability density function

Uniform density function

Gaussian density function

RDF

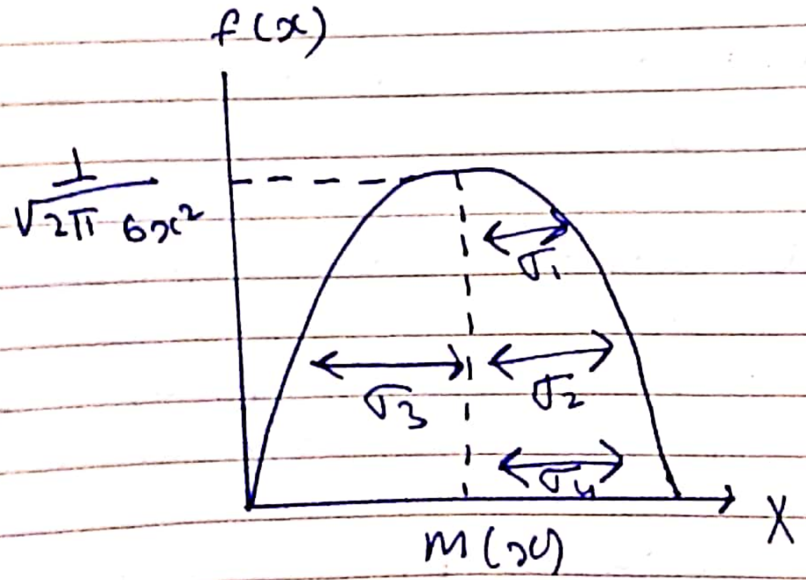
Naragami density function

Binominal density function

GMM $\;$ gdf = $pdf_1 \div pdf_2 \ldots pdf_n$

Gaussian density function

$$f_\mu(x) = \frac{1}{\sqrt{2\pi 6x^2}} e^{-(x-m_x)^2/2\sigma x^2}$$



White noise / colored noise

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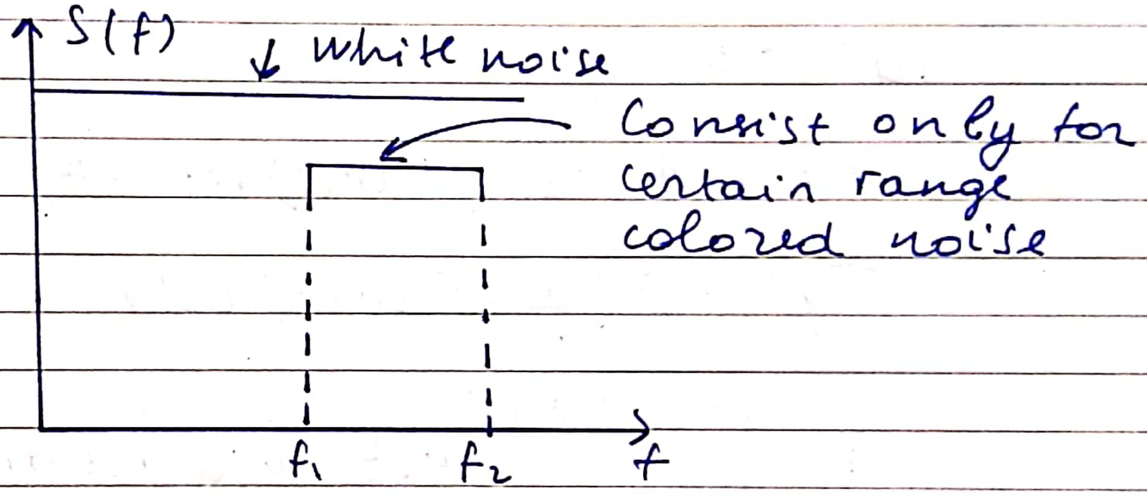
$$\sum_{t=0}^{\infty} x(t)\ x(t-T) = R(t)\ v(t)$$

Weiner - khintchin theorem

$$S(f) = FT(R(T))$$
$$R(T) = F(T)^{-1}(S(f))$$

Power spec density gives behaviour of signal



• Enthropy is a measure of info content Whenever and whatever function manner you can make that enthropy to the max for the given noise that condition when evaluated comes out to be gaussian eqn (noise is gaussian)

$$x(t) = s(t) + w(t)$$
$$s(t) = \left.\begin{matrix} +A \\ -A \end{matrix}\right] \ NR\ 2$$

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$$y(t) = \int_0^T (x(t)) \cdot\cdot 1\, dt$$

$$= \int_0^T (-A)\, dt + \int_0^T w(t)\, dt$$

$$y(t) = -AT + \int_0^T w(t)\, dt$$

$$y(t) = y_{B/T} = T = -A + \frac{1}{T}\int_0^T w(t)\, dt$$

$$m_x = -A$$

$$\sigma_y^2 = E\,(y - m_x)^2$$

$$= E\left(-A + \frac{1}{T}\int_0^T w(t)\, dt + A\right)$$

$$\sigma_y^2 = E\left(\frac{1}{T}\int_0^T w(t)\, dt\right)$$

$$= E\left[\frac{1}{T^2}\int_0^T\int_0^T w(t)\, w(u)\, dt\, du\right]$$

$$= \frac{1}{T^2}\int_0^T\int_0^T E\,(w(t)\, w(u)\, dt\, du))$$

$$= \frac{1}{T^2}\int_0^T\int_0^T R(t-u)\, dt\, du$$

$$= \frac{1}{T^2}\int_0^T\int_0^T \frac{N_2}{2} \delta(t-a)\, dt\, du$$

$$= \frac{N_2}{2T^2}\int_0^T 1 \cdot dt \quad \{t = u\}$$

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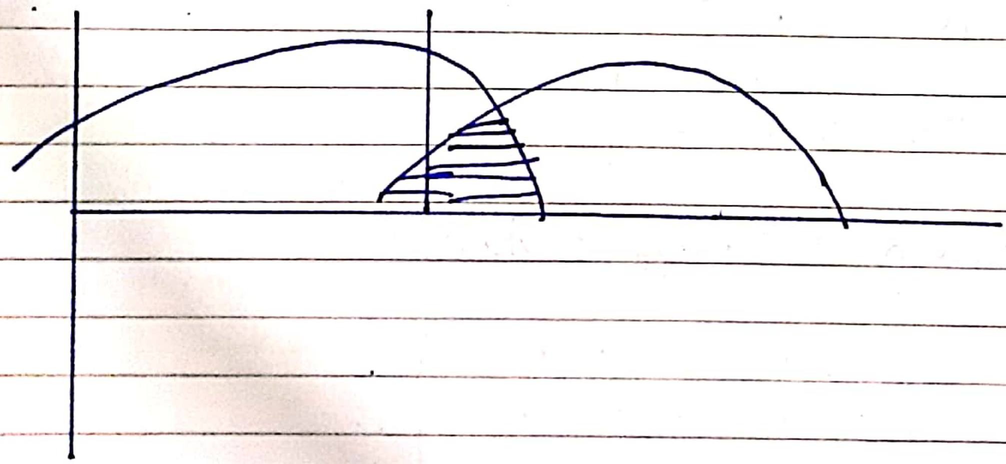
$$= \frac{N_0}{2T^2} \times T = \frac{N_0}{2T}$$

$$f(y/0) = \frac{1}{\sqrt{2\pi \frac{N_0}{2T}}} e^{-(y+A)^2/2 \times \frac{N_0}{2T}}$$

Let $T = 1$

$$f(y/0) = \frac{1}{\sqrt{\pi N_0}} e^{-(y+A)^2 \cdot 1/N_0}$$

$$f(y/1) = \frac{1}{\sqrt{\pi N_0}} e^{-(y-A)^2/N_0}$$



$$P_{01} = \int_0^{\infty} f(y/0)\, dy$$

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Topic: Geometrical Transforms

Translation:

$$x^* = x + \Delta x$$
$$y^* = y + \Delta y$$
$$z^* = z + \Delta z$$

$$\begin{bmatrix} x^* \\ y^* \\ z^* \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \Delta x \\ 0 & 1 & 0 & \Delta y \\ 0 & 0 & 1 & \Delta z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Scaling

$$T = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation (Along z Axis)

$$T = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation (Along x-axis)

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Unitary Transforms

Walsh Transform

$N = 2^n$; $8 = 2^3$ $n = 3$

$W(x, u) = \frac{1}{N} \prod_{i=1}^{n-1} (-1)^{b_i(x) b_{n-1-i}(u)}$

Kernel transform

$b_k(z) = b_k(1\,1\,0)$

$z = 6 - (110)$ kth Bit of Binary representation of 2

$b_0 = 0$ $b_1 = 1$ $b_2 = 1$

$n = 3$ $W(0,0) = \frac{1}{8} \left[\prod_{i=0}^{2} (-1)^{b_i(0) b_{2-i}(0)} \right.$

$N = 8$

y \ x	0	1	2	3	4	5	6	7
0	+	+	+	+	+	+	+	+
1	+	+	+	+	-	-	-	-
2								
3								
4								
5								
6								

HAAR Transform
Slant Transform
Sine Transform

Cosine Transform

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$$\alpha(u) \sum_{x=0}^{n-1} f(x) \cos\left[\frac{(2x+1)u\pi}{2N}\right]$$

$u = 0 \ldots (n-1)$

$$\alpha(u) = \frac{1}{\sqrt{N}} ; u = 0$$

$$= \sqrt{\frac{2}{N}} ; \text{else}$$

u \ x	0	1	2	3	4	5	6	7	8	
0	1	1	1	1	1	1	1	1	1	1
1	1	1								
2										
3										

Hotteling Transform

$$[X] = \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_{n-1} \end{bmatrix} \qquad m_x = E[X]$$

$$C_x = \left[(X - m_x) \cdot (x - m_x)^T \right]$$

1) Real
2) Symmetrical

Toeplitz

$A(i) = A(i-j)$

X_i and X_j are uncorrelated

$C_{ij} = C_{ji}$

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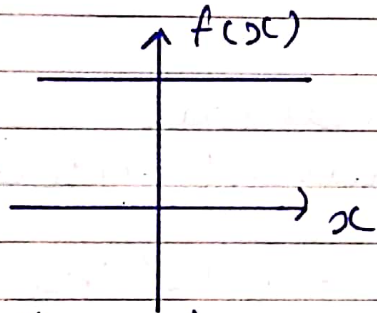
$$m_x = \frac{1}{m}\sum_{i=0}^{m-1} x_i \cdot f(x) \rightarrow \text{UDF}$$

If $f(x)$ is a UDF (uniformly distributed function) then statistical average will be $=$ E.A (Ensamble average)



$$y = A(X - m_x)$$

Transform - ortho-normal Matrix

$$A \cdot \underbrace{\Sigma_x}_{\text{Eigenvalues}} \cdot A^T = \text{Diagonal Matrix}$$

<u>MSE</u>

$x \varepsilon \bar{X}$

$$\sum_{j=1}^{n} l_j - \sum_{j=1}^{k} l_j \text{ ; } k< = \sum_{j=k+1}^{n} l_j$$

K.L. Transform

Let

$$C_x = E\left[(x - m_x)(x - m_x)^T\right]$$

DT.

A → Transformation Matrix

$A C_x A^T$ => Eigen Vectors

⇓

Considering few top eigenvectors

⇓

Principal Components

Edge Detection

↙ ↘

Template Matching | Differential Gradient Based

The template matching uses local edge gradient magnitude by the approximately maximum of the responses of edge mask

$$g = \max [g_i], \quad i = 1 \dots N$$

(gradient mask) 8-12

In differential gradient the local edge magnitude is computed vectorically as a vector using a non-linear transform

$$g_{oc} = \sqrt{g_x^2 + g_y^2}$$

$$g_x = \max [g_x], \quad i = 1 \dots N$$

$$g_y = \max [g_y], \quad i = 1 \dots N$$

$$\theta = \arctan (g_y / g_x)$$

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Scanned by CamScanner

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Sabel operator

$$S_x = \begin{bmatrix} -1 & 0 & -1 \\ -2 & 0 & 2 \\ -1 & 0 & -1 \end{bmatrix}$$

$$S_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

Prewitt Operators

$$P_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

$0°$

$$\begin{bmatrix} -1 & 1 & 1 \\ -1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$P_y = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

$45°$

$$\begin{bmatrix} 1 & 1 & 1 \\ -1 & 2 & 1 \\ -1 & -1 & 1 \end{bmatrix}$$

$$\text{Krish} = \begin{bmatrix} -3 & -3 & 5 \\ -3 & 0 & 5 \\ -3 & -3 & 5 \end{bmatrix}$$

$45°$

$$\begin{bmatrix} -3 & 5 & 5 \\ -3 & 0 & 5 \\ -3 & -3 & -3 \end{bmatrix}$$

Types of edges:

1) Step edge
2) Slant edge
3) signal / Planar edge
4) Triangular edge
5) Impulse Edge

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$0°$

$$\begin{bmatrix} -A & 0 & A \\ -B & 0 & B \\ -A & 0 & A \end{bmatrix}$$

$45°$

$$\begin{bmatrix} 0 & C & D \\ -C & 0 & C \\ -D & -C & 0 \end{bmatrix}$$

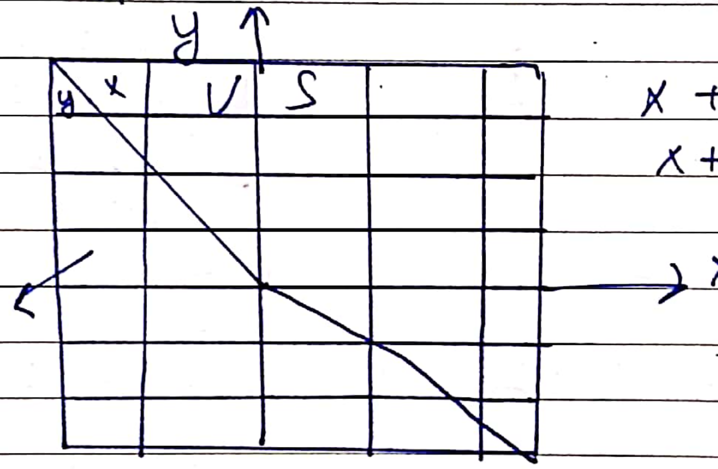
Response for $0°$ mask ($0°$ response) using $0°$ mask for step edge

$= 2A + B$

$45°$ mask using $0°$ response $= A + B$

$0°$ response $- C + D$

$45°$ response $- 2C + D$



$x + y + w = 1$

$x + y = 1 - w$

α-axis response $=$

$$(1 + x + y - w) A + B$$
$$= (1 + 1 - w - w) A + B$$
$$= (1 - w) A + B$$

$\alpha°$ response

$$= (1 + T + S - V)\ C + D$$

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Topic: Edge detection
Data vs Mask Generalization
Let the data be

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \qquad C = \frac{B}{\sqrt{2}} \quad \Big| \quad D = \frac{A}{\sqrt{2}}$$

$$g_{0^\circ} = A((c+i-a-g) + B(f-d)$$
$$g_{90^\circ} = A(a+c-g-i) + B(b-h)$$
$$g_{45^\circ} = C(b+f-d-h) + D(c-g)$$

$$g_{45^\circ} = \frac{g_0 + g_{90}}{\sqrt{2}}$$

If we use the step H edge orientation in the neighbourhood of the central pixel then the earlier approach is fine, if this approach gives an error of 6.6° in average

$$E(avg) = 6.6$$

Circular operator is one of the way to limit the error and restrict the observations of the circular neighbor.

If we need to increase the number of pixels in neighbourhood then we have to go beyond 4 and 8 connected.

Another replacement of edge detec-

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tion is known as labeling or labeling

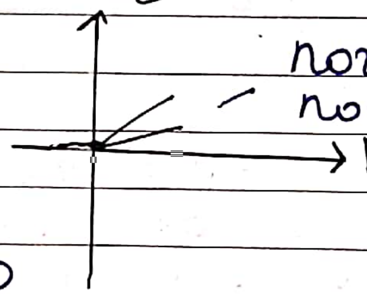
Segmentation

$$\begin{bmatrix} G_1 - \theta_1 \\ G_2 - \theta_2 \\ \vdots \quad \vdots \\ G_n - \theta_n \end{bmatrix}$$ - Data matrix

$[e_1, e_2, e_3]$

Metric Properties
||
Distance

$(d(i,g))$



normalized norm (distance)

$d(r_i, r_g) > 0$

$i \neq 0$

$d(r_i, r_j) = d(r_j, r_i)$

Triangle rule:

$d(r_i, r_j) + d(r_j, r_k) \geq d(r_i, r_k)$

Morphological operations

Structuring element

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Prewitt at 0°

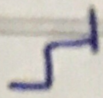
$$\begin{bmatrix} -1 & 1 & 1 \\ -1 & 2 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

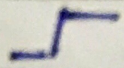
Prewitt at 45°

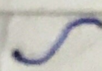
$$\begin{bmatrix} 1 & 1 & 1 \\ -1 & 2 & 1 \\ -1 & -1 & 1 \end{bmatrix}$$

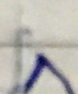
$$\begin{bmatrix} -3 & \overset{-3}{\cancel{0}} & -3 \\ -3 & 0 & 5 \\ -3 & -3 & 5 \end{bmatrix}$$

$$\begin{bmatrix} -3 & 5 & 5 \\ -3 & 0 & 5 \\ -3 & -3 & -3 \end{bmatrix}$$

Step edge : 

Slant edge . 

Signal edge 

Triangle edge 

$$\begin{array}{cc} 0^\circ & 45^\circ \\ \begin{bmatrix} A & 0 & A \\ B & 0 & B \\ A & 0 & A \end{bmatrix} & \begin{bmatrix} 0 & C & D \\ -C & 0 & C \\ -D & -C & 0 \end{bmatrix} \end{array}$$

Step edge

$2A+B$ $\qquad$ $A+B$

$C+D$ $\qquad$ $2C+D$

Computer Vision

Morphological Operators/ function / Transforms

- → Dilation
- → Erosion
- → Opening
- → Closing.

*tools

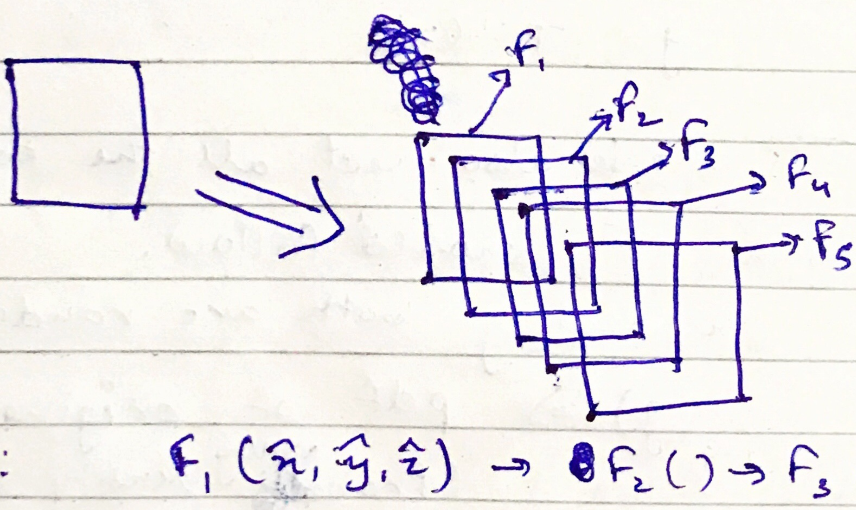
Enhancement

* Application

H - Transform (Dissecting)

→ Take a GLI (Grey level image)

→ Break the grey level into groups
(0-10, 11-20, ...)



Lattice : $f_1(\hat{x}, \hat{y}, \hat{z}) \rightarrow f_2() \rightarrow f_3() \rightarrow$

Tension

and so on

Enhancement

$\Downarrow$

Histogram Equilization

$g \rightarrow$ No. of gray levels of an image

$\Downarrow$

Normalised to $\{0, 1\}$.

$T(g) \Rightarrow$ Monotonically increasing function.
Single valued function.

$0 < T(g) < 1 \quad , \quad 0 < g < 1$

$x = T(g) \longrightarrow$ Enhanced image

$g = T^{-1}(x)$

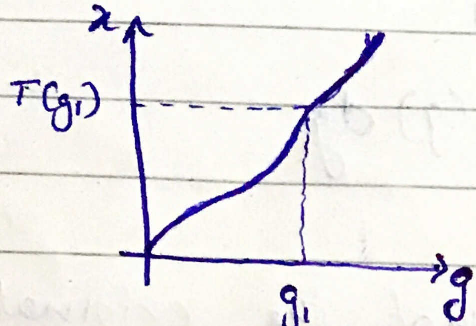
T^{-1} should also meet all the conditions which T should follow.

x and g both are random.

$P_g(g) \Rightarrow$ pdf of original image
(Probability density function)

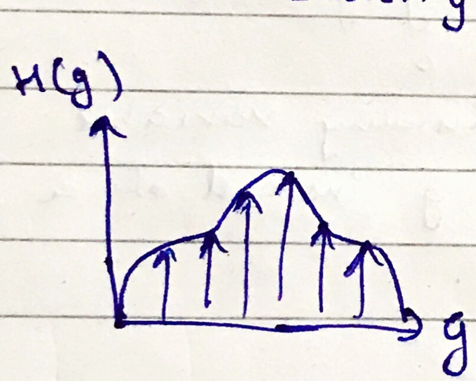
$f_x(x) \Rightarrow$ pdf of Transformed image

$$P_x(x) = \left| P_g(g) \frac{dg}{dx} \right|_{g = T^{-1}(x)}$$



$x = T(g)$

Prob. Density function (PDF) (Derivative)
Cummulative Density function (CDF) (Integral)



$$\int_0^{g_1} H(g)\,dg = \underline{CDF}$$

$$x = T(g) = \int_0^{g_0} P_g(g)\, dg$$

The cummulative probability of the original grey levels

$$x = T(g) = \int_0^{g_0} P_g(a)\, da$$

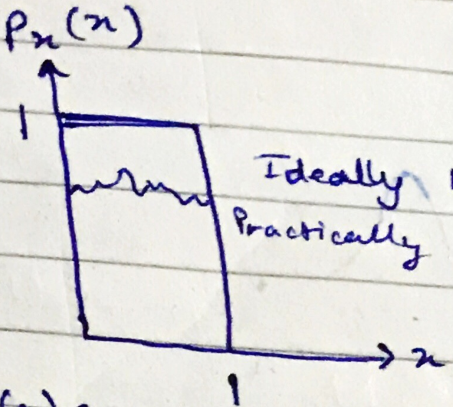
$a \rightarrow$ dummy variable

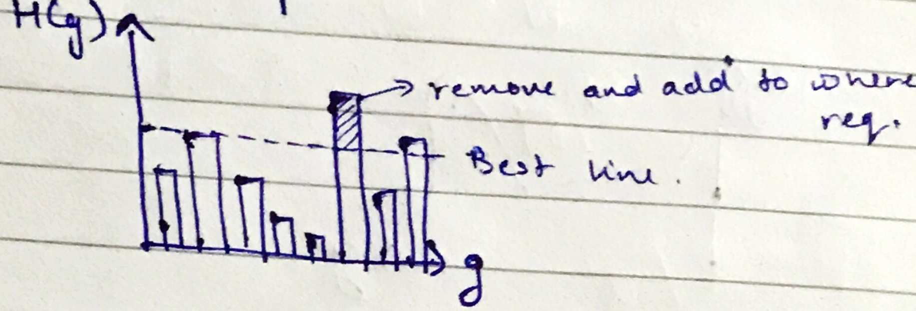
can use g instead of a.

$$0 < g < 1$$

$$\frac{dx}{dg} = P_g(g)$$

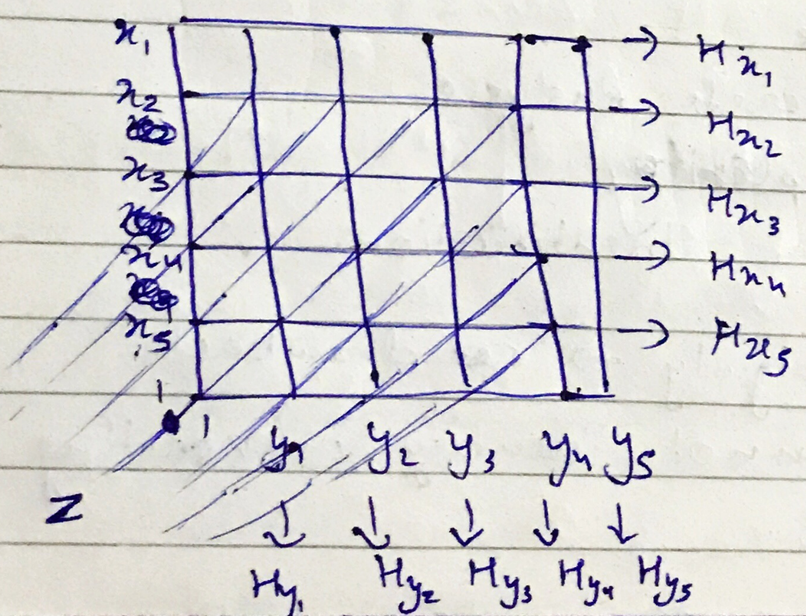
$$\Rightarrow \quad P_x(x) = \left| \cancel{P_g}(g) \frac{1}{\cancel{P_g(g)}} \right|_{g = T^{-1}(x)}$$

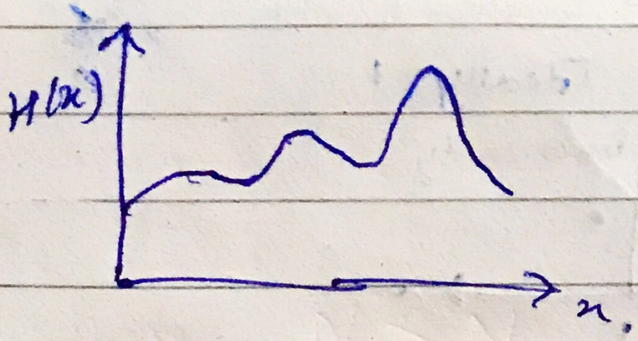


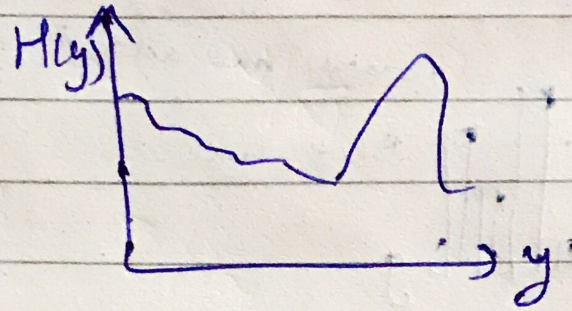


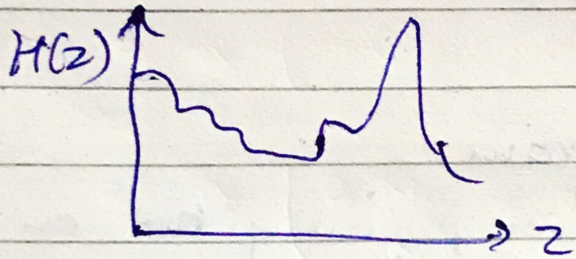
Lateral Histogram:

It is a way of projecting an image on two or more than two axes.









To solve

→ Discrete Analysis

→ complexity

→ Object Identification.

* Ambiguity → drawback

→ It cannot identify intersecting points.

↗

- Let the image be of size $N \times N$
- Object inside image of size $(n \times n)$

Template Matching → Nn complexity

$$R_{LAT} = 2aN^2 + 2bNn + cp^2n^2$$

assuming $a \approx b \approx c$

$p \rightarrow$ No. of objects in the image.

Eg:-

$N = 256$ $p = 20$

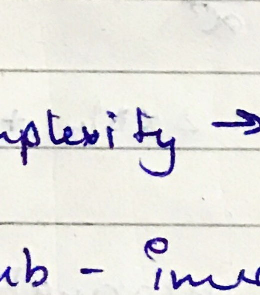
$n = 7$

$$R_{LAT} = a\left[2N^2 + 2Nn + p^2n^2\right]$$

$$= a\left[2 \times (256)^2 + 2 \times 256 \times 7 + (20)^2 (7)^2\right]$$

Complexity → N^2 (template Matching)

Sub - image Analysis



= n parts

No. of objects in Sub image $\propto$ Part size

No. of objects in Sub image $= K\,(\text{Part-size})^2$

Let the Size of Sub image $= \bar{N} \times \bar{N}$

where $\bar{N} < N.$

$$\alpha = N/\bar{N} \Rightarrow \alpha > 1$$

$$\bar{p} = p/\alpha^2$$

$$\bar{R}_{LAT} = \alpha\left(2\bar{N}^2 + 2\bar{N}n + \bar{p}^2 n^2\right)$$

$$= \alpha\left(\frac{2N^2}{\alpha^2} + \frac{2N}{\alpha}n + \frac{p^2}{\alpha^4}n^2\right)$$

$$= \frac{2N^2}{\alpha} + 2Nn + \frac{p^2}{\alpha^3}n^2$$

$$= \alpha\left[2N^2 + 2\alpha Nn + \frac{p^2}{\alpha^2}n^2\right]$$

- Too big $\alpha \Rightarrow$ loss of information.
- Complexity has reduced because of loss of information as $p \to 0$. i.e. p becomes nearly 0, zero.

No. of objects in Subimage $\propto$ Part-size

No. of objects in Sub image = K (Part-size)^2

Let the Size of Sub image = $\bar{N} \times \bar{N}$

where $\bar{N} < N$.

$$\alpha = N/\bar{N} \Rightarrow \alpha > 1$$

$$\bar{P} = P/\alpha^2$$

$$\bar{R}_{LAT} = \alpha\left(2\bar{N}^2 + 2\bar{N}n + \bar{P}^2 n^2\right)$$

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$$= \frac{2N^2}{\alpha} + 2Nn + \frac{P^2}{\alpha^3}n^2$$

$$\cancel{R_{LAT}} = \alpha\left[2N^2 + 2\alpha Nn + \frac{P^2}{\alpha^2}n^2\right]$$

- Too big $\alpha \Rightarrow$ loss of information.
- Complexity has reduced because of loss of information as $P \to 0$. i.e. P becomes nearly 0, zero.

- α should be chosen as such the it accomodates the size of the object.

Optimum Size.

$$\bar{R}_{LAT} = \alpha\left[2N^2 + 2\alpha N n + \frac{p^2}{\alpha^2} n^2\right]$$

$$\boxed{\frac{d\bar{R}_{LAT}}{d\alpha} = 2N^2 + 4\alpha N n - \frac{n^2 p^2}{\alpha^2}}$$

(dividing α in $\bar{R}_{LAT}$)

$$\frac{d\bar{R}_{LAT}}{d\alpha} = 2Nn - \frac{2p^2n^2}{\alpha^3} = 0$$

if $\alpha = 1$

$\Rightarrow p = \sqrt{2}\,.\,N/n$

Adjust α such that

$\bar{N} = n\bar{p}$

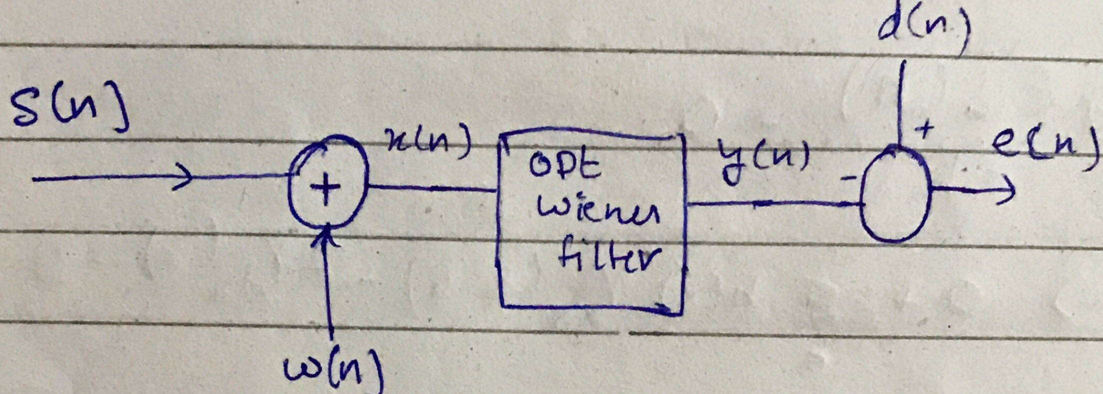
$\bar{N} = n$ (if $p = 1$)

$$\Rightarrow \boxed{\alpha = np/N}$$

Wiener Filter

⇓

Apriori Knowledge



$$x(n) = s(n) + w(n)$$

$$y(n) = \sum_{k=1}^{m} h(k)\, x(n-k)$$

$$e(n) = d(n) - y(n)$$

$$\mathcal{E}_m(n) = E\left[e^2(n)\right]$$

$$= E\left[d(n) - \sum_{k=1}^{m} h(k)\, x(n-k)\right]^2$$

$$= E\left[d(n)\, d(n)\right] + \sum_k \sum_l h(k)\, h(l)\, E\left[x(n-k)\, x(n-l)\right]$$

$$- 2 \sum_k \left[x(n-k)\, d(n)\right]$$

$$= \gamma_{dd}(0) + \sum_k \sum_l h(k)\, h(l)\, \gamma_{xx}(k-l)$$

$$- 2 \sum_k h(k)\, \gamma_{dx}(k)$$

$$\frac{\partial \mathcal{E}_m(m)}{\partial h} = 0$$

$$\Rightarrow \quad 0 + 2\sum_k h(k)\, \gamma_{xx}(k-l) - 2\gamma_{dx}(k) = 0$$

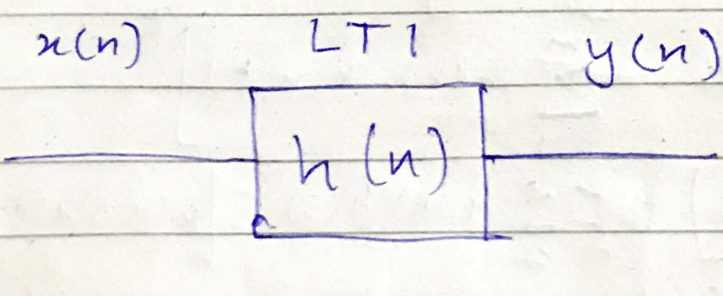
$$\sum_k h(k)\, \gamma_{xx}(k-l) = \gamma_{dx}(k)$$

$$\downarrow$$

$$h_m \Gamma_m = \gamma_d$$

$$\boxed{h_m = \Gamma_m^{-1} \gamma_d}$$

Symmetrical & Toeplitz.



Question: (Task)

Take any foggy image from net
Use wiener filter to see the response?

Bernoulli's Distribution.

$$\text{Trial} \begin{cases} \rightarrow \text{Success} \\ \rightarrow \text{Failure} \end{cases}$$

$$\underset{\text{(random trial)}}{X} = \begin{cases} 1 \Rightarrow \text{Success (S)} \\ 0 \Rightarrow \text{Failure (F)} \end{cases}$$

$P(x=1) = p \qquad P(x=0) = 1-p$

$P(x=i) = p^{i}(1-p)^{1-i}$

$E[x] = p \qquad Var[x] = p(1-p)$

Bionomial Distribution

Let 'M' be the identical independent bernoulli's trials.

Let 'x' be the no. of success out of 'M' trials and let it be bionominally Distributed.

$$P(x=i) = \binom{N}{i} p^i (1-p)^{N-i}$$

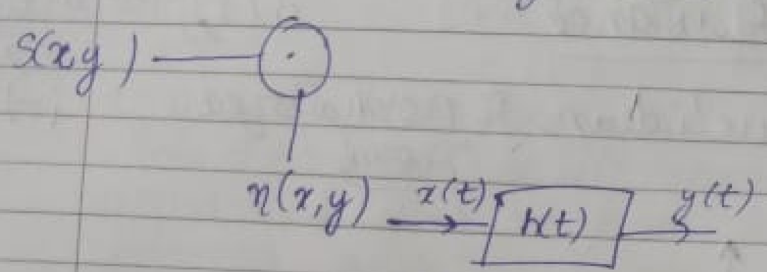
$$E[x] = Np \qquad Var[x] = Np(1-p)$$

04-09-19

Original image → Fog, smog, smoke, dust, fire, blur

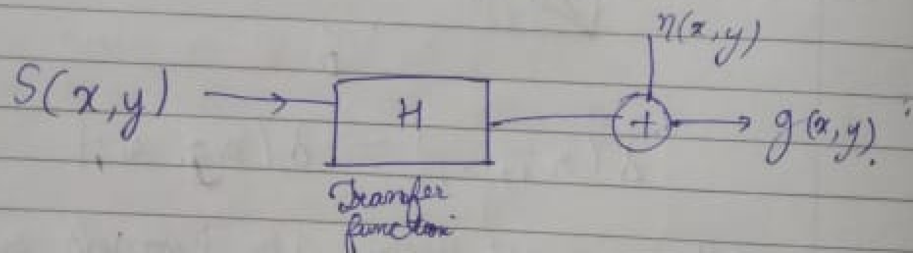
⇓

Degradation model



$$y(t) = x(t) * h(t)$$

convolution



$$g(x,y) = H[S(x,y)] + n(x,y)$$

If H is a linear transfer function,

$$g(x,y) = H[S(x,y)]$$

$$= H[k_1 f_1(x,y) + k_2 f_2(x,y)]$$

$$= k_1 H[f_1(x,y)] + k_2 H[f_2(x,y)]$$

Assume $k_1 = k_2$

$$= H[f_1(x,y)] + H[f_2(x,y)]$$ → Assume to be zero.

$$= H[\delta_1(x,y)]$$

(Property of homogenity)

The given function can be space invariant if $H(\delta_1(x-\alpha, y-\beta)) \Rightarrow g(x-\alpha, y-\beta)$

$$S(x,y) = \left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S(\alpha,\beta)\cdot\delta(x-\alpha, y-\beta)\,d\alpha\,d\beta\right]$$

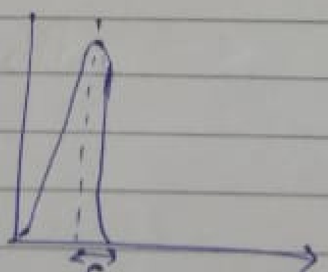
Assume $\eta(x,y)=0$

$$= \int_{-\infty}^{\infty}\int S(\alpha,\beta)\cdot \underbrace{H[\delta(x-\alpha,y-\beta)]}_{\text{Point spread } f^n}\, d\alpha\, d\beta$$

Kronecker

There's nothing such as an IMPULSE

It's called Kronecker Dirac-Δ Function.



Wiener Filter

1. $d(n) = S(n)$
2. $d(n) = S(n+D)$