Class Fun

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Q) Q1) Denote distance in G by d and distance in $\bar{G}$ by $\bar{d}$. Define u, v as you did, so that $d(u,v) \geq 4$. We wish to show for any vertex x, y that $\bar{d}(x,y) \leq 2$

Among x, y, u, v there are atmost 4 distinct vertices. In particular since $d(u,v) \geq 4$, u and v cannot be connected by a path among x, y, u, v. So, it is possible to split x, y, v, u into two components which are not connected in G.

One containing u and one containing v. Without any loss of generality we may assume the components are $\{u, x\}$, $\{v, x\}$ or $\{u, x, y\}, \{v\}$

→ In the first case x and y have no edge in G, so they have an edge in $\bar{G}$ and $\bar{d}(x,y) = 1$

→ In the second case x and y are both connected to v in $\bar{G}$, so $\bar{d}(x,y) \leq \bar{d}(x,v) + \bar{d}(y,v) = 2$

Thus $\bar{d}(x,y) \leq 2$ for x, y, proving that $\operatorname{diam} \bar{G} \leq 2$.