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Q6) for given correlation $\rho = -1$ and $\sigma_1 \neq \sigma_2$

So, $\bar{r}_{min} = \dfrac{\sigma_1 \bar{r}_2 + \sigma_2 \bar{r}_1}{\sigma_1 + \sigma_2} = \dfrac{(0.05)(0.08) + (0.02)(0.1)}{0.05 + 0.02}$

$= \dfrac{0.004 + 0.002}{0.07} = \dfrac{0.006}{0.07} = 0.08571$

$\bar{r}_{min} = 8.571\%$

$W_2 = 1 - S_{min}$ $W_2 = S_{min}$

$S_{min} = \dfrac{\sigma_1}{\sigma_1 + \sigma_2} = \dfrac{0.05}{0.05 + 0.02} = \dfrac{0.05}{0.07}$

$\boxed{W_2 = 0.714}$

$W_1 = 1 - W_2 = 1 - 0.714 = 0.286$

for $\rho = -1$

$\boxed{W_1 = 0.286 \quad W_2 = 0.714 \quad \bar{r}_{min} = 8.571\% \quad \sigma_{min} = 0}$

for correlation $\rho = -0.5$

$W_1 = 1 - S_{min}$ $W_2 = S_{min}$

$S_{min} = \dfrac{\sigma_1^2 - \rho \sigma_1 \sigma_2}{\sigma_1^2 + \sigma_2^2 - 2\sigma_1 \sigma_2 \rho}$

$\dfrac{(0.05)^2 - (-0.5)(0.05)(0.02)}{(0.05)^2 + (0.02)^2 - 2(-0.5)(0.05)(0.02)}$

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$$\frac{0.0025 + (0.5)(0.01)}{0.0025 + 0.0004 + (0.01)2}$$

$$\frac{0.0075}{0.0129}$$

$$\boxed{\delta_{min} = 0.5813}$$

$$\boxed{w_1 = 1 - \delta_{min} = 0.4187}$$

$$\bar{r}_{min} = (\bar{r}_2 - \bar{r}_1)\,\delta_{min} + \bar{r}_1$$

$$= (0.08 - 0.1)(0.5813) + 0.2$$

$$\bar{r}_{min} = 0.088374$$

$$\boxed{= 8.83\%}$$

$$\sigma_{min} = \sqrt{\frac{\sigma_1^2 \sigma_2^2 (1-\rho^2)}{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}} \quad 0.001\sqrt{\frac{75.100}{39}}$$

$$\boxed{\sigma_{min} = 0.014 = 1.4\%}$$

for $\rho = 0.5$

$w_1 = 41.84\%$
$w_2 = 58.13\%$
$\bar{r}_{min} = 8.83\%$
$\sigma_{min} = 1.4\%$

for correlation $\rho = 0$

$w_1 = 1 - \delta_{min}$ & $w_2 = \delta_{min}$

$$\delta_{min} = \frac{\sigma_1^2 - \rho\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2} = \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}$$

$$\boxed{\delta_{min} = \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2} = 0.8620}$$

$$\boxed{w_1 = 1 - \delta_{min} = 0.1380}$$

$$\bar{x}_{min} = (\bar{x}_2 - \bar{x}_1)\delta_{min} + \bar{x}_1$$
$$= 0.08276$$
$$\boxed{= 8.276\%}$$

$$\sigma_{min} = \sqrt{\frac{\sigma_1^2 \cdot \sigma_2^2 (1-\rho^2)}{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}} = \frac{\sigma_1\sigma_2}{\sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}}$$

for $\rho = 0$

$w_1 = 13.80\%$

$w_2 = 86.20\%$

$\bar{x}_{min} = 8.276\%$

$\sigma_{min} = 8.56\%$

for $\rho = 0.5$

$w_1 = 1 - \delta_{min}$ $w_2 = \delta_{min}$

$$\delta_{min} = \frac{\sigma_1^2 - 2\rho\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}$$

$$\frac{(0.05)^2 - 2(0.5)(0.05)(0.02)}{0.0029 - 2(0.5)(0.05)(0.02)}$$

$$= \frac{0.0025 - 0.001}{0.0029 - 0.001}$$

$$\frac{0.0015}{0.0019} = \frac{15}{19} = 0.7894$$

$w_1 = 1 - \delta_{min} = 0.2106$

$\bar{x}_{min} = (\bar{x}_2 - \bar{x}_1)\, \delta_{min} + \bar{x}_1$

$$\boxed{= 8.4212\ \%}$$

$$\sigma_{min} = \sqrt{\frac{\sigma_1^2 \sigma_2^2 (1-\rho^2)}{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}}$$

$= 1.98\ \%$

| | |
|---|---|
| for $\rho = 0.5$ | |
| $w_1 =$ | $21.06\ \%$ |
| $w_2 =$ | $78.94\ \%$ |
| $\bar{x}_{min} =$ | $8.42\ \%$ |
| $\sigma_{min} =$ | $1.98\ \%$ |

for $\rho = 1$

$w_1 = 1 - \delta_{min}$ $\quad w_2 = \delta_{min}$

$\delta_{min} = \frac{\sigma_1}{\sigma_1 - \sigma_2} = \frac{0.05}{0.03} > 1$

$w_1 = 1 - \delta_{min} < 0$

∴ The investors will take short forward position with asset 1:

$$x_{min} = \frac{\sigma_1 x_2 - \sigma_2 x_1}{\sigma_1 - \sigma_2}$$

$$= \frac{0.02}{0.03} = 66.67\%$$

for $\rho = 1$

$w_2$ as short forward position $x_{min} = -66.67\%$

$\sigma_{min} = 0$

The minimum standard deviation is zero (0) at $\rho = -1$, at which combination of assets

$w_1 = 28.6\%$

$w_2 = 71.4\%$

$x_{min} = 8.571\%$