

11)

It is given that  $Z(t)$  is a normally distributed random variable with

$$E[Z] = \mu = 0 \text{ and } \text{Var}(Z) = \sigma^2 = 1$$

$$\text{Var}(Z) = E[Z^2] - (E[Z])^2$$

$$E[Z]^2 - \mu^2$$

$$E[Z^2] - 0$$

$$E[Z^2] = \sigma^2 = 1$$

$$E[Z^2] = 1$$

Now, we are given another distribution

$$X = \sqrt{t} Z$$

$$E[X] = E[\sqrt{t} Z]$$

$$\text{we know that } E[aY] = a E[Y], \text{ so}$$

$$E[X] = \sqrt{t} E[Z] = 0$$

$$\mu_X = 0$$

$$\text{Now, } \text{Var}(X) = \text{Var}(\sqrt{t} Z)$$

$$\text{We know } \text{Var}(aY + b) = a^2 \text{Var}(Y)$$

$$\text{Var}(X) = \text{Var}(\sqrt{t} Z)$$

$$= (\sqrt{t})^2 \text{Var}(Z)$$

$$= t (1) = t$$

$$\boxed{\sigma_X^2 = t}$$

So, the random variable  $X$  has mean  $(\mu_X) = 0$  and

with variance  $(\sigma_x^2) = t$

Now, we analyze  $x(t)$  and discover that it is not a brownian motion.

$$t_1 < t_2 \leq t_3 < t_4$$

$x(t_2) - x(t_1) = (\sqrt{t_2} - \sqrt{t_1}) Z$  is not independent

of  $x(t_3) - x(t_2) = (\sqrt{t_3} - \sqrt{t_2}) Z$  as both

are multiples of same sample value  $Z$  drawn from  $N(0)$  population

Secondly, the distribution of  $(\sqrt{t_2} - \sqrt{t_1}) Z$  is normal with  $t$ .

$x(t)$  is not a brownian motion