Assignment-1
fuzzy logic & fuzzy sets

- Deepti Singh
2K17/MC/037

Q1. $\tilde{A} = \{(2, 0.4), (4, 0.6), (5, 0.7), (6, 1), (7, 1), (8, 0.4), (9, 0.2)\}$

(i) $U = \{1, 2, 3, \cdots 10\}$

$\overline{(\tilde{A})} = \{(1, 1.0), (2, 0.6), (3, 1), (4, 0.4), (5, 0.3), (8, 0.6), (9, 0.8), (10, 1.0)\}$

(ii) $U = \{2, 4, 5, 6, 7, 8, 9\}$

$\overline{(\tilde{A})} = \{(2, 0.6), (4, 0.4), (5, 0.3), (8, 0.6), (9, 0.8)\}$

2. (i) cardinality of $\tilde{A} = |A| = \sum \mu(x)$

$= 0.4 + 0.3 + 0.5 + 0.4 + 0.8$

$= 2.4$

$|U| = 10$

relative cardinality $= \frac{|A|}{|U|} = 0.24$

(ii) $|\tilde{C}| = 10 \times \left(1 - \frac{1}{10}\right) = 9$

rel cardinality, $||\tilde{C}|| = \frac{|\tilde{C}|}{|U|} = \frac{9}{10} = 0.9$

(b) $\tilde{C}$ $\mu_c(x) = \begin{cases} 0 & x \le 10 \\ \dfrac{1}{1+(x-10)^2} & x \ge 10 \end{cases}$

A $\tilde{C}^{\alpha} = \{x \mid x \in R\}$.

at $\alpha = 0$

$C^{0+} = \{x \mid \mu(x) > 0\}$

$\Rightarrow \dfrac{1}{1+(x-10)^2} > 0$

$\Rightarrow x \ge 10$

$= C^{0+} \{x \mid x \ge 10, x \in R\}$

(ii) $\alpha = 0.3$

$C^{0.3} = \{x \mid \cancel{x >} \mu(x) > 0.3, x \in R\}$

$= \dfrac{1}{(1+(x-10)^2)} \ge 0.3$

$\Rightarrow 1 > 0.3(1+(x-10)^2)$

$\Rightarrow 1+(x-10)^2 \le \dfrac{10}{3}$

$(x-10)^2 \le \dfrac{7}{3}$

$-\dfrac{\sqrt{7}}{\sqrt{3}} < x-10 \le \sqrt{\dfrac{7}{3}}$

$-\dfrac{\sqrt{7}}{\sqrt{3}} \le x \le 10+\sqrt{\dfrac{7}{3}}$

$\Rightarrow x \in \left[10-\sqrt{\frac{7}{3}},\ 10+\sqrt{\frac{7}{3}}\right]$

& also $x \geq 10$

$\Rightarrow x \in \left[10,\ 10+\sqrt{\frac{7}{3}}\right]$

~~$x \in [10, 11.8]$~~

$\Rightarrow C^{0.3} = \left[10,\ 10+\sqrt{\frac{7}{3}}\right]$

since,

$C^{0.3+} = \left[10,\ 10+\sqrt{\frac{7}{3}}\right)$

(iii) $\alpha = 0.5$

$C^{0.5} = \{x \mid u(x) \geq 0.5\}$

$u(x) \geq 0.5$

$\Rightarrow \frac{1}{1+(x-10)^2} \geq \frac{1}{2}$ & $x \geq 10$

$\Rightarrow 1+(x-10)^2 \leq 2$

$\Rightarrow (x-10)^2 \leq 3$

$\Rightarrow -\sqrt{3} \leq x-10 \leq \sqrt{3}$

$\Rightarrow x \in [10-\sqrt{3},\ 10+\sqrt{3}]$

since $x \geq 10$

$\Rightarrow x \in [10,\ 10+\sqrt{3}]$

$\Rightarrow C^{0.5} = \{x \mid x \in [10, 10+\sqrt{3}]\}$

$C^{0.5+} = \{x \mid x \in [10, 10+\sqrt{3})\}$

iv) $\alpha = 0.8$

$$C^{\alpha} = \{x \mid \mu(x) \geq 0.8\}$$

$$\mu(x) = \frac{1}{1+(x-10)} \geq 0.8$$

$$\Rightarrow \quad 1+(x-10)^2 \leq \frac{10}{8}$$

$$\Rightarrow \quad (x-10)^2 \leq \frac{1}{4}$$

$$\Rightarrow \quad 10 \leq x \leq 10+\frac{1}{2}$$

$$x \in \left[10, \frac{21}{2}\right]$$

$$\Rightarrow \quad C^{0.8} = \{x \mid x \in [10, 10.5]\}$$

$$C^{0.8+} = \{x \mid x \in [10, 10.5)\}$$

v) $\alpha = 1$

$$C^{1} = \{x \mid \mu(x) \geq 1\}$$

since $\mu(x) \leq 1 \Rightarrow \mu(x) = 1$

$$\Rightarrow \quad \frac{1}{4(x-10)^2} = 1$$

$$\Rightarrow \quad x = 10$$

$$\Rightarrow \quad C^{1} = \{10\}$$

now C^{1+} $\mu(x) > 1$,

$$\Rightarrow \quad C^{1+} = \phi$$

Q.3 (i) Not fair $= (\bar{\tilde{F}}) = \{(1, 1.0), (2, 0.7), (3, 0.4),$
$(4, 0.1), (6, 0.1), (7, 0.5), (8, 0.9)\}.$
$(9, 1.0), (10, 1.0)\}$

(ii) Not Bad $= (\bar{\tilde{B}}) = \{(2, 0.3), (3, 0.6), (4, 0.9), (5, 1.0),$
$(6, 1.0), (7, 1.0), (8, 1), (9, 1), (10, 1)\}.$

(iii) Fair but not bad. $= (\tilde{F}) \cap (\bar{\tilde{B}})$

$\mu_{(\tilde{F} \cap \bar{\tilde{B}})} = \min(\mu(\tilde{F}), \mu(\bar{\tilde{B}}))]$

$= \{(2, 0.3), (3, 0.6), (4, 0.9), (5, 1), (6, 0.9), (7, 0.5), (8, 0.1)\}$

4. $\mu_{(\tilde{A} \cup \tilde{B})} = \max(\mu(\tilde{A}), \mu_{\tilde{B}})$

$= \max\left(\frac{x}{x+2}, 2^{-x}\right)$

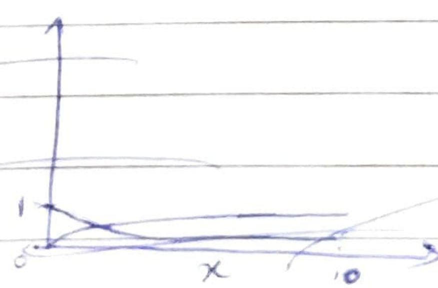
$\mu_{(\bar{\tilde{C}})} = 1 - \frac{1}{x + 10(x-2)^2}$

$\mu_{(\bar{\tilde{C}})} = \frac{x + 10(x-2)^2 - 1}{x + 10(x-2)^2}$

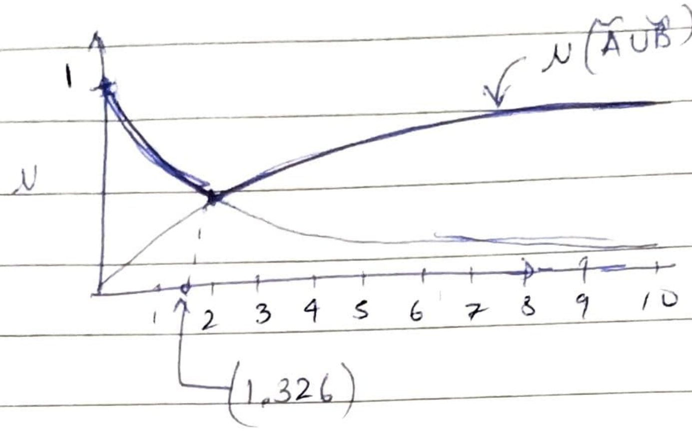
$\mu_{(\tilde{A} \cup \tilde{B})} = \max(\mu_{\tilde{A}}(\tilde{A}), \mu_{\tilde{B}})$

$= \max\left(\frac{x}{x+2}, \frac{1}{2^x}\right)$

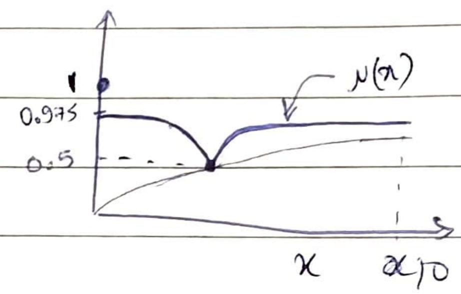
$\Rightarrow \mu(x)$



$\tilde{A} \cup \tilde{B}$

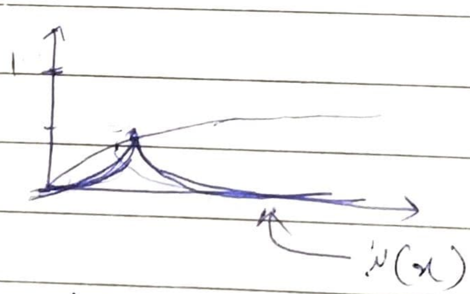


(ii) $\tilde{A} \cup C(\tilde{C})$ $\quad max = \left\{ \frac{x}{x+2}, \; 1 - \frac{1}{x+10(x-2)^2} \right\}$



$$\Rightarrow \mu(x) = 1 - \frac{1}{x+10(x-2)^2}$$

(iii) $\tilde{A} \cap \tilde{C}$ $\quad min = \left\{ \frac{x}{x+2}, \; \frac{1}{x+10(x-2)^2} \right\}$



$$\mu(x) = \frac{1}{x+10(x-2)^2}$$

iv) $c(\tilde{A} \cap \tilde{C})$

$$\mu(x) = 1 - \frac{1}{x + 10(x-2)^2} \quad \text{(from prev part)}$$

5 (i) $\tilde{A} = (2.5, 3, 3.5)$
$\tilde{B} = (3.5, 4, 4.5)$
$\tilde{C} = (1.5, 2, 2.5)$

$$\tilde{A} + \tilde{B} = [2.5+3.5, 3+4, 3.5+4.5)$$
$$= (6, 7, 8)$$

(ii) $\cancel{\tilde{A} \otimes \tilde{C}}$ $\tilde{A} \times \tilde{C} = (2.5 \times 1.5, 2 \times 3, 3.5 \times 2.5)$
$$= (3.25\cancel{3}, 6, 8.75)$$

6. $\bar{A} \otimes \bar{X} = \bar{B}$

$\tilde{A} = (1, 3, 4)$, $\tilde{B} = (2, 12, 48)$

let $\tilde{X} = \{x_1, x_2, x_3)$

$\bar{A}_\alpha = [1 + 2\alpha, 4 - \alpha]$ $\quad \tilde{X} = [x_1 + \alpha(x_2 - x_1)$
$\quad , x_3 + \alpha(x_2 - x_3)]$

$B_\alpha = [2 + 10\alpha, 48 - 36\alpha)$

$\Rightarrow$ min $A_\alpha B_\alpha X_\alpha$ = min B_α

$\Rightarrow [(1+2\alpha)(x_1 + \alpha(x_2 - x_1)), (1+2\alpha)(x_3 + \alpha(x_2 - x_3)),$
$(4-\alpha)(x_1 + \alpha(x_2 - x_1)), (4-\alpha)(x_3 + \alpha(x_2 - x_3))]$
$= [2 + 10\alpha, 48 - 36\alpha)$

applying min, max ~~at~~ for $\alpha = 0$

$\Rightarrow$ $\min[x_1, x_3, 4x_1, 4x_3) = 2 \Rightarrow x_1 = 2$

& $\max(x_1, x_3, 4x_1, 4x_3) = 48 \Rightarrow x_3 = 12$

for $\alpha = 1$ min $3x_2 = 12 \Rightarrow x_2 = 4$

$\Rightarrow \breve{X} = (2, 4, 12)$

7

$\breve{A} = (1, 2, 4, 5)$, $\breve{B} = (2, 3, 5, 6)$

$A \otimes \breve{X} = \breve{B}$

~~$A^\alpha = [1 + 2\alpha, \cancel{4+\alpha}]$~~ 4

$A^\alpha = [1 + 2\alpha, 5 - \alpha]$

$B^\alpha = [2 + \alpha, 6 - \alpha]$

let $\breve{X} = (x_1, x_2, x_3, x_4)$

$X^\alpha = [x_1 + \alpha(x_2 - x_1), x_4 + \alpha(\cancel{x_4 - x_3})]$ $x_3 - x_4$

$A^\alpha_\alpha X^\alpha_\alpha = B_\alpha$

$\Rightarrow [(1+2\alpha)(x_1 + \alpha(x_2 - x_1)), (1+2\alpha)(x_4 + \alpha(x_3 - x_4)),$
$(5-\alpha)(x_1 + \alpha(x_2 - x_1)), (5-\alpha)(x_4 + \alpha(x_3 - x_4))]$
$= [2 + \alpha, 6 - \alpha]$ — 1

taking at $\alpha = 0$

min $[x_1, x_4, 5x_1, 5x_4] = 2$

$\Rightarrow x_1 = 2$

& max $[x_1, x_4, 5x_1, 5x_4] = 6$

$x_4 = \frac{6}{5}$

$\alpha = 1$

~~$[3x_1 + x_2 - x_1, 3x_4 + x_3 - x_4, 4x_1 + x_2 - x_1, 4x_4 +$~~

$[3x_2, 3x_3, 4x_2, 4x_3] = [3, 5]$

$\Rightarrow$ ~~mi~~ taking min

$\Rightarrow 3x_2 = 3$

$\Rightarrow x_2 = \cancel{8}\,1$

& taking max

$x_3 = \frac{5}{4}$

$\Rightarrow \breve{X} = (2, 1, \frac{5}{4}, \frac{6}{5})$

since $2 > 1 \Rightarrow$

$\Rightarrow$ No solⁿ exist

8. $F(x) = (0,2,3) + x = (5,6,7)$

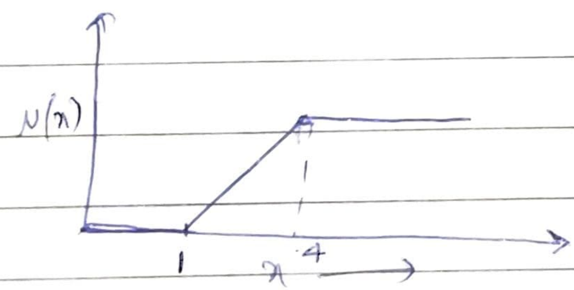
$(0 + x_1, 2 + x_2, 3 + x_3) = (5, 6, 7)$

$\Rightarrow x_1 = 5, x_2 = 4, x_3 = 4$

$(5,4,4) \Rightarrow$ which doesn't exist

hence no solution

~~Q.~~ 11.



9. Let $\hat{A} = \cancel{(1,2,3,4)}\ (-1, 2, 4, 5)$

$\breve{B} = (-3, 1, 4, 5)$

Now $\breve{A} \times \breve{B} = (3, 2, 16, 25)$

since $x_1 > x_2 \Rightarrow \breve{A} \times \breve{B}$ is not a fuzzy set

~~Hence mult~~

Again $\breve{A} = (-2, -1, 3, 4)$

$\breve{B} = (-1, 2, 3, 4)$

$\breve{A} / \breve{B} = (2, \frac{-1}{2}, 1, 1)$

sine ~~B~~: $x_1 > x_2$

$\Rightarrow$ $\breve{A} / \breve{B}$ is not a fuzzy set

Hence Multiplication & division of 2 trapezoidal fuzzy numbers may not be trapezoidal fuzzy numbers.

10 Let $\breve{A} = (-1, 2, 3)$

$\breve{B} = (-2, \frac{1}{2}, 3)$

$\breve{A} \times \breve{B} = (2, 1, 9)$ $\Rightarrow$ not a ~~fuzzy no~~ triangular fuzzy number.

again ~~$\breve{A} / \breve{B} = (\frac{1}{2})$~~

$\breve{B}/A = (2, \frac{1}{4}, 1)$ $\Rightarrow$ not a Δ fuzzy no.

Hence, $\times$ or $\div$ of ~~fuzz~~ Δ fuzzy ~~set~~ members may not be Δ fuzzy numbers