Ans We consider a ~~margin~~ multidimensional random vector x and z having Gaussian distributions:-

$$p(x) = \mathcal{N}(x|\mu_x, \Sigma_x)$$

$$p(z) = \mathcal{N}(z|\mu_z, \Sigma_z)$$

We also have

$$y = x + z$$

$$p(y) = \mathcal{N}(x|\mu_x, \Sigma_x) + \mathcal{N}(z|\mu_z, \Sigma_z)$$

We have

$$\Sigma = \begin{pmatrix} \Sigma_{aa} & \Sigma_{ab} \\ \Sigma_{ba} & \Sigma_{bb} \end{pmatrix} \qquad \Lambda = \begin{pmatrix} \Lambda_{aa} & \Lambda_{ab} \\ \Lambda_{ba} & \Lambda_{bb} \end{pmatrix}$$

Conditional distribution

$$p(x_a|x_b) = \mathcal{N}(x|\mu_{a|b}, \Lambda_{aa}^{-1})$$

$$\mu_{a|b} = \mu_a - \Lambda_{aa}^{-1}\Lambda_{ab}(x_b - \mu_b)$$

Marginal distribution

$$p(x_a) = \mathcal{N}(x_a|\mu_a, \Sigma_{aa})$$

We take marginal and conditional distribution to be:-

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$$P(x) = \mathcal{N}(x \mid \mu, \Lambda^{-1})$$

$$P(y|x) = \mathcal{N}(y \mid Ax+b, L^{-1})$$

To find joint distribution over x and y we define

$$z = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\ln [p(z)] = \ln p(x) + \ln p(y|x)$$

$$-\frac{1}{2}(x-\mu)^T \Lambda (x-\mu)$$

$$-\frac{1}{2}(y-Ax-b)^T L (y-Ax-b) + \text{const}$$

$$-\frac{1}{2} x^T (\Lambda + A^T L A) x - \frac{1}{2} y^T L y + \frac{1}{2} y^T L A x + \frac{1}{2} x^T A^T L y$$

$$= -\frac{1}{2} z^T R z$$

where

$$R = \begin{pmatrix} \Lambda + A^T L A & -A^T L \\ -LA & L \end{pmatrix}$$

$$\text{cov}[z] = R^{-1} = \begin{pmatrix} \Lambda^{-1} & \Lambda^{-1} A^T \\ A\Lambda^{-1} & L^{-1} + A \Lambda^{-1} A^T \end{pmatrix}$$

The marginal distribution of y and the conditional distribution of x are given by :-

$$\boxed{\begin{aligned} p(y) &= \mathcal{N}(y \mid A\mu + b,\ L^{-1} + A\Lambda^{-1}A^T) \\ p(x \mid y) &= \mathcal{N}(x \mid \Sigma\{A^T L(y-b) + \Lambda\mu\},\ \Sigma) \end{aligned}}$$

$$\Sigma = (\Lambda + A^T L A)^{-1}$$