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DTU/2K16/MC/13

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MC-304 Theory of Computation
Assignment – III

1. Find context free grammars for the following languages

a) $L = \{a^n, b^m : n \neq m, n \geq 0, m \geq 0\}$

$S \to aaAb \mid aBbb \mid A \mid B$
$A \to Aa \mid \varepsilon$
$B \to Bb \mid \varepsilon$

b) $L = \{a^n b^m : 2n \leq m \leq 3m\}$

$S \to aSbb \mid aSbbb \mid \varepsilon$

c) $L = \{w \in \{0,1\}^* : n_a(w) = 2n_b(w) + 1\}$

$S \to aA \mid Aa \mid a$
$A \to Aaab \mid Aaba \mid Abaa \mid \varepsilon$

2) Reduce the following grammar's to Chomsky Normal form

a) $S \to 1A \mid 0B$
$A \to 1AA \mid 0S \mid 0$
$B \to 0BB \mid 1S \mid 1$

Step 1: S has many instances, hence creating new start (initialization) variable S_0

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2) Reduce the following grammar's to Chomsky Normal Form (CNF)

a) ~~S → SS~~ $S \rightarrow 1A \mid 0B$
$A \rightarrow 1AA \mid 0S \mid 0$
$B \rightarrow 0BB \mid 1S \mid 1$

Removing non-terminal symbols that are adjoint to terminal symbols.

$S \rightarrow XA \mid YB$
$A \rightarrow XAA \mid YS \mid 0$
$B \rightarrow \cancel{Y}BB \mid XS \mid 1$
$X \rightarrow 1$
$Y \rightarrow 0$

Now, removing all non-terminal symbols that are present in groups of 3 +.

$S \rightarrow XA \mid YB$
$A \rightarrow \cancel{XA} PA \mid YS \mid 0$
$B \rightarrow QB \mid XS \mid 1$
$X \rightarrow 1$
$Y \rightarrow 0$
$P \rightarrow XA$
$Q \rightarrow YB$

This is now in Chomsky Normal Form.

b) $S \rightarrow a \mid b \mid cSS$

Step 1: Removing terminal symbols that come with non-

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terminal symbol

S → a|b|CSS

C → c

Step 2: Removing all non-terminal symbols that are in groups of 3

S → a|b|XS

C → c

X → CS

This is now in Chomsky normal Form

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~~Step 7: Removing 3 non-terminals coming together 'CSS'~~

$S_0 \to A \mid B \mid XS$
$S \to A \mid B \mid YS$
$A \to a$
$B \to b$
$C \to c$
$X \to CS$
$Y \to CS$

~~This is the Chomsky Normal Form of the context free Grammar.~~

3) Reduce the following Grammars to Greibach Normal Form

a) $S \to SS \mid 0S1 \mid 01$

Assigning non-terminal to terminal '1'

$S \to SS \mid 0SY \mid 0Y$
$Y \to 1$

Removing recursion

$S \to 0SY \mid 0Y \mid 0SYZ \mid 0YZ$
$Y \to 1$
$Z \to S \mid SZ$

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S → 0SY | 0Y | 0SYZ | 0YZ
Y → 1
Z → 0SY | 0Y | 0SYZ | 0YZ | 0SYZ | 0YZ | 0SYZZ | 0YZZ

S → 0SY | 0Y | 0SYZ | 0YZ
Y → 1
Z → 0SY | 0Y | 0SYZ | 0YZ | 0SYZZ | 0YZZ

Removing recursion from Z

S → 0SY | 0Y | 0SYZ | 0YZ
Y → 1
Z → 0SY | 0Y | 0SYZ | 0YZ

This is the resultant Griedelbach normal Form.

ii) S → AB
A → BSB | BB | b
B → aAb | a

S → AB
A → BSB | BB | b
B → aAX | a
X → b

S → AB
A → aAXSB | aSB | aAXB | aB | b
B → aAX | a
X → b

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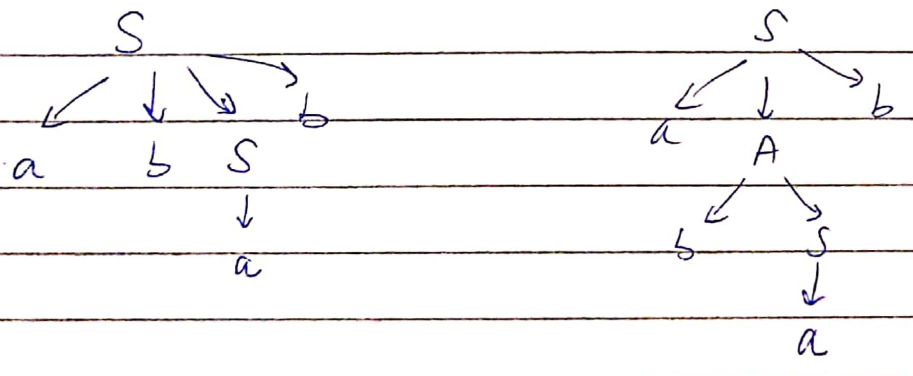
$S \to aAXSBB \mid aSBB \mid aAXSB \mid aBB \mid bB$
$A \to aAXSB \mid aSB \mid aAXB \mid aB \mid b$
$B \to aAX \mid a$
$X \to b$

This is the griedelbach normal form.

4) Show that the following grammars are ambiguous.

a) $S \to a \mid abSb \mid aAb$
$A \to bS \mid aAAb$

Let us consider the string 'abab'. Using the given grammar, there can be more than one left-most derivation tree for the string.



Hence, the grammar is ambiguous.

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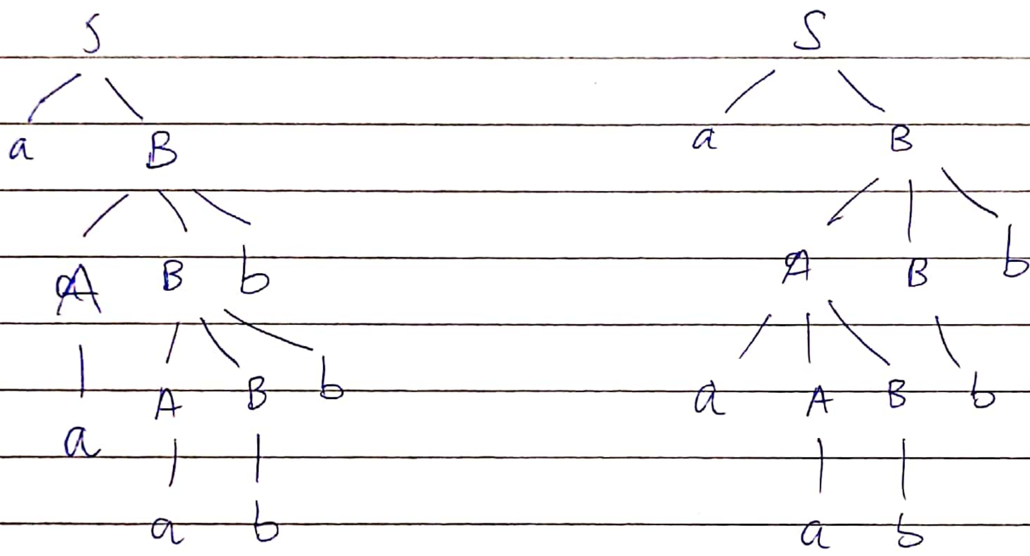
ii) $S \to aB \mid ab$

$A \to aAB \mid a$

$B \to ABb \mid b$

If we consider a string aaabb. Using the given grammar we can have 2 leftmost derivations.



'aaabbb' 'aaabbb'

As we have 2 different leftmost derivations resulting in the same string, this grammar is ambiguous.

5) Use pumping lemma to show that the following are not context free languages.

i) $\{a^{n^2} \mid n \geq 1\}$

Let p be the pumping length of the string a^{p^2} and wil

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be pumpable and a context free grammar.
We will prove this isn't possible by contradiction.

Assume $xyz = a^{p^2}$

$|xyz| = p^2$

i) $|y| > 0$
ii) $|xy| \leq p$
iii) $xy^iz \in L \;\forall\; i \geq 0$

Also $|xyz| = p^2$ $\quad |xy^iz| = |xyz| + (i-1)|y|$

Now, $xy^0z \in L$

$$|xy^0z| = |xyz| - |y|$$
$$p^2 - |y|$$

$$|y| < p$$
$$p^2 - |y| > p^2 - p > (p-1)^2$$

$$|xyz| > (p-1)^2$$

Now, $|xy^2z| \geq |xyz|$
$|xyz| + |y| \geq |xyz|$

$p \quad |xyz| < p^2$

This implies that p^2 must lie strictly between

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$(p-1)^2$ and p^2 which isn't possible. Hence, by contradiction it is not a context free grammar

ii) $\{a^m b^m c^n \mid m \le n \le 2m\}$

Let us take $z = a^m b^m c^n$ where $z = uvwxy$

i) $|vx| \ge 1$
ii) $|vwx| \le n$

vx can't contain all the symbols a, b and c. So, ~~if~~ uv^2wx^2y ~~will~~ contain additional occurences of 2 symbols and the number of occurences hence will not be by the given rule.

Let the pumping length be p and let the string be $a^p b^p c^p$. Now, we can take $z = xy$ $uvxyz$ such that

$uvxy = a^p b^p$ and $z = c^p$. Now. $|uvxyz| = 3p$

$|uv^2xy^2| > |uvxy|$
$> 2p$

$|uv^ixy^i| \to 2ip$ and $n < 2ip$ when $\lim_{i \to \infty} 2ip > n$.

Hence, this is a contradiction and by pumping lemma this is not a context free grammar.

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6) i Construct PDA's that accept the following language on $\Sigma = \{a, b, c\}$

a) $\{w : n_a(w) = 2n_b(w)\}$

We define the PDA as follows:-

$M = (\{q_0, q_1, q_2\}, \{a, b, c\}, \{a, b, z_0\}, \delta, q_0, z_0, \{q_2\})$

Where δ (The transition function is defined as) :-

$\delta(q_0, a, z_0) = (q_0, a, z_0)$
$\delta(q_0, b, z_0) = (q_0, b, z_0)$
$\delta(q_0, b, a) = (q_0, \varepsilon)$
$\delta(q_0, b, b) = (q_1, b)$
$\delta(q_1, a, z_0) = (q_1, a, z_0)$
$\delta(q_1, a, a) = (q_1, a, a)$
$\delta(q_1, a, b) = (q_1, \varepsilon)$
$\delta(q_1, b, a) = (q_1, a)$
$\delta(q_1, c, b) = (q_1, b)$
$\delta(q_1, b, a) = (q_0, a)$
$\delta(q_1, b, b) = (q_0, b, b)$
$\delta(q_1, b, \varepsilon) = (q_0, \varepsilon)$
$\delta(q_1, c, a) = (q_0, a)$
$\delta(q_0, c, b) = (q_0, b)$
$\delta(q_0, c, z_0) = (q_0, c, z_0)$
$\delta(q_0, \varepsilon, z_0) = (q_0, z_0)$

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b) $\{wcw^R : w \in \{a,b\}^*\}$

We define pushdown automata as follows

$M = (\{q_0, q_1, q_2\}, \{a,b,c\}, \{a,b,z_0\}, \delta, q_0, z_0, \{q_2\})$

δ (Transition Function) is as follows:-

$\delta(q_0, a, z_0) = (q_0, az_0)$
$\delta(q_0, a, a) = (q_0, aa)$
$\delta(q_0, b, a) = (q_0, baa)$
$\delta(q_0, b, z_0) = (q_0, bz_0)$
$\delta(q_0, b, b) = (q_0, bb)$
$\delta(q_0, a, b) = (q_0, ab)$
$\delta(q_0, c, z_0) = (q_1, z_0)$
$\delta(q_0, c, a) = (q_1, a)$
$\delta(q_0, c, b) = (q_1, b)$
$\delta(q_1, b, b) = (q_1, \varepsilon)$
$\delta(q_1, a, a) = (q_1, \varepsilon)$
$\delta(q_1, \varepsilon, z_0) = (q_2, z_0)$

c) $\{a^n b^{m+n} c^m : n \geq 0, m \geq 1\}$

We define the pushdown automata as:-

$M = (\{q_0, q_1, q_2, q_3, q_4\}, \{a,b,c\}, \{a,b,c,z_0\}, \delta, q_0, z_0, \{q_4\})$

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Transition Function (S)

$\delta(q_0, a, z_0) = (q_0, az_0)$
$\delta(q_0, a, a) = (q_0, aa$
$\delta(q_0, b, a) = (q_1, \varepsilon)$
$\delta(q_1, b, a) = (q_1, ba)$
$\delta(q_1, \varepsilon, z_0) = (q_2, z_0)$
$\delta(q_2, b, z_0) = (q_2, bz_0)$
$\delta(q_2, c, b) = (q_3, \varepsilon)$
$\delta(q_3, \varepsilon, z_0) = (q_4, z_0)$

7) $(q_0, aacaa, z_0) \rightarrow (q_0, acaa, az_0)$ $[\delta(q_0, a, z_0) = (q_0, az_0)]$

$\delta(q_0, acaa, az_0) = (q_0, caa, aaz_0)$ $[\delta(q_0, a, a) = (q_0, aa)]$
$\delta(q_0, caa, aaz_0) \rightarrow (q_1, aa, aaz_0)$ $[\delta(q_0, ca) = (q_1, a)]$
$\delta(q_1, aa, aaz_0) \rightarrow (q_1, a, az_0)$ $[\delta(q_1, a, a) = (q_1, \varepsilon)]$
$\delta(q_1, a, az_0) \Rightarrow (q, \varepsilon, z_0)$
$\delta(q_1, \varepsilon, z_0) \rightarrow (q_1, \varepsilon, z_0)$ $[\delta(q, \varepsilon, z_0) = (q_4, z_0)]$

i) $\delta(q_0, abcba, z_0) \rightarrow (q_0, bcba, az_0)$
$\delta(q_0, bcba, az_0) \rightarrow (q_0, cba, baz_0)$
$\delta(q_0, cba, baz_0) \rightarrow (q_1, ba, baz_0)$
$\delta(q_1, ba, baz_0) \rightarrow (q_1, a, az_0)$
$\delta(q_1, a, az_0) \rightarrow (q_1, \varepsilon, z_0)$
$\delta(q_1, \varepsilon, z_0) \rightarrow (q_4, \varepsilon, z_0)$

String 'abcba' is accepted by a

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ii) abcb

$\delta(q_0, abcb, Z_0) \rightarrow (q_0, bcb, aZ_0)$

$\delta(q_0, bcb, aZ_0) \rightarrow (q_0, cb, baZ_0)$

$\delta(q_0, cb, baZ_0) \rightarrow (q_1, b\alpha, baZ_0)$

$\delta(q_1, b\alpha, baZ_0) \rightarrow (q_1, \varepsilon, aZ_0)$

Since the remaining string is the null string ε, it will be processed completely by the pushdown automata (PDA) A.

iii) acba

$\delta(q_0, acba, Z_0) \rightarrow (q_0, cba, aZ_0)$

$\delta(q_0, cba, aZ_0) \rightarrow \phi$

Hence, the string will not be processed.

iv) abac

$\delta(q_0, abac, Z_0) \rightarrow (q_0, bac, aZ_0)$

$\delta(q_0, bac, aZ_0) \rightarrow (q_0, ac, baZ_0)$

$\delta(q_0, ac, baZ_0) \rightarrow (q_0, c, abaZ_0)$

$\delta(q_0, c, abaZ_0) \rightarrow (q_1, \varepsilon, abaZ_0)$

Hence, the entire string will be processed.

v) abab

$\delta(q_0, abab, Z_0) \rightarrow (q_0, bab, aZ_0)$

$\delta(q_0, bab, aZ_0) \rightarrow (q_0, ab, baZ_0)$

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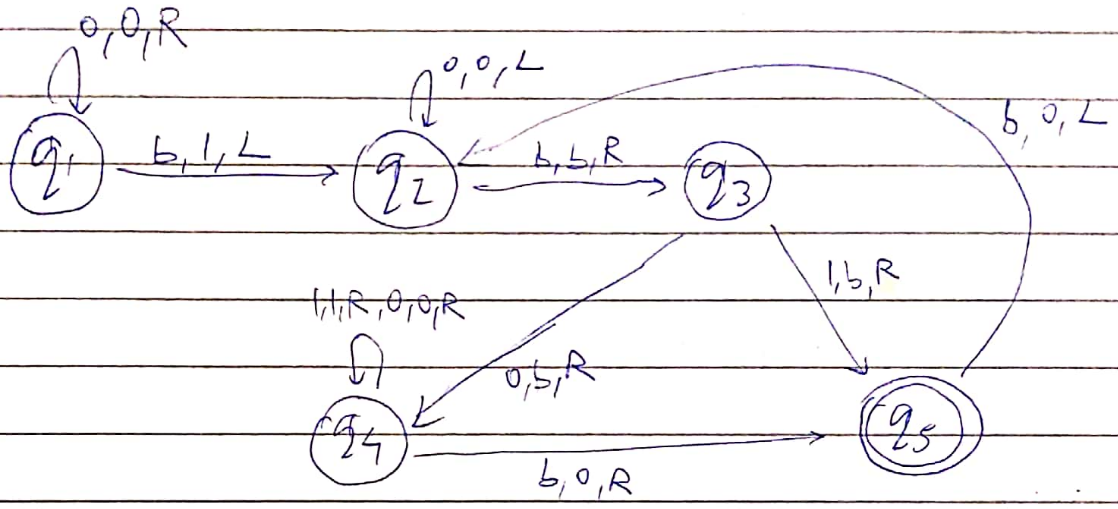
$\delta(q_0, ab, baz_0) \rightarrow (q_0, b, abaz_0)$
$\delta(q_0, b, abaz_0) \rightarrow (q_0, \varepsilon, babaz_0)$

Hence, the string 'abab' will be processed.

8) Draw transition diagram for the turing Machine:



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9) Construct context free grammar G accepting N(M) for the PDA M given below:

Let the CFG $G = (V_N, \{a,b\}, P, S)$

$V_N = \{S,$ $[q_0, a, q_0]$ $[q_0 a q_0]$ $[q_0 a q_f]$
$[q_0 Z_0 q_1]$ $[q_0 Z_0 q_1]$ $[q_0 Z_0 q_f]$ $[q_1 a q_0]$ $[q_1 a q_1]$
$[q_1 a q_f]$ $[q_1 Z_0 q_1]$ $[q_1 Z_0 q_1]$ $[q_1 Z_0 q_f]$ $[q_f a q_0]$
$[q_f a q_1]$ $[q_f a q_f]$ $[q_f Z_0 q_0]$ $[q_f Z_0 q_1]$ $[q_f Z_0 q_f]$
$\}$

Production Rules P

$P_1 : S \to [q_0 Z_0 q_0]$
$P_2 : S \to [q_0 Z_0 q_1]$
$P_3 : S \to [q_0 Z_0 q_f]$

$\delta(q_0, a, Z_0) \to (q_0, a Z_0)$

$P_4 : [q_0 Z_0 q_0] \to a[q_0 a q_0][q_0 Z_0 q_0]$
$P_5 : [q_0 Z_0 q_0] \to a[q_0 a q_1][q_1 Z_0 q_0]$
$P_6 : [q_0 Z_0 q_0] \to a[q_1 a q_f][q_f a q_0]$
$P_7 : [q_0 Z_0 q_1] \to a[q_0 a q_0][q_0 Z_0 q_1]$
$P_7 : [q_0 Z_0 q_1] \to a[q_0 a q_1][q_1 Z_0 q_1]$
$P_8 : [q_0 Z_0 q_1] \to a[q_0 a q_f][q_f Z_0 q_1]$
$P_9 : [q_0 Z_0 q_f] \to a[q_0 a q_0][q_0 Z_0 q_f]$
$P_{10} : [q_0 Z_0 q_f] \to a[q_0 a q_1][q_1 Z_0 q_f]$
$P_{11} : [q_0 Z_0 q_f] \to a[q_0 a q_f][q_f Z_0 q_f]$

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$\delta(q_1, b, a) = (q_1, \varepsilon)$ yields

$P_{12}: [q_2 a q_0] \to b\varepsilon = [q_0 a q_0] \to b$
$P_{13}: [q_1 a q_1] \to b$
$P_{14}: [q_1 a q_3] \to b$

$\delta(q_0, a, a) = (q_0, aa)$ yields

$P_{15}: [q_0 a q_0] \to a [q_0 a q_0][q_0 a q_0]$
$P_{16}: [q_0 a q_0] \to a [q_0 a q_1][q_1 a q_0]$
$P_{17}: [q_0 a q_0] \to a [q_0 a q_f][q_f a q_0]$
$P_{18}: [q_0 a q_1] \to a [q_0 a q_0][q_0 a q_1]$
$P_{19}: [q_0 a q_1] \to a [q_0 a q_1][q_1 a q_1]$
$P_{20}: [q_0 a q_1] \to a [q_0 a q_f][q_f a q_1]$
$P_{21}: [q_0 a q_f] \to a [q_0 a q_0][q_0 a q_f]$
$P_{22}: [q_0 a q_f] \to a [q_0 a q_1][q_1 a q_f]$
$P_{23}: [q_0 a q_f] \to a [q_0 a q_f][q_f a q_f]$

$\delta(q_2, \varepsilon, Z_0) = (q_1, \varepsilon)$ yields

$P_{24}: [q_1 Z_0 q_0] \to \varepsilon$

$\delta(q_0, b, a) \to (q_1, \varepsilon)$ yields

$P_{25}: [q_0 a q_0] \to b\varepsilon$
$P_{26}: [q_0 a q_1] \to b$
$P_{27}: [q_0 a q_f] \to b$

$P_1 \to P_{27}$ gives the production rules of the PDA M in P

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10) a) 1213

$q_1 1213 \rightarrow b q_2 213 \rightarrow b b q_3 13$

From $\delta(q_3, 1)$ is not defined, M stops execution. So, this input string isn't accepted

b) 2133

$q_1 2133 \rightarrow$

As, $\delta(q_1, 2)$ is not defined, the string isn't accepted by the turing machine.

c) 312

$q_1 312 \rightarrow$

As, $\delta(q_1, 3)$ is not defined, M halts so, the string is not accepted by the turing machine