ASSIGNMENT 4 & 5

.

Subject Code : **MC-406** Course Title : **Partial Differential Equations**

Course Coordinator : Dr. Aditya Kaushik
Office : FW4-TF1 (Top Floor, Mechanical Engineering Department)
Contact : Mob. No. : +91-8427580104 Email Id. : apkaushik@gmail.com

Syllabus

Unit - 4 The Heat equation: Fourier series solution of heat equation, Heat conduction in infinite media, The maximum and minimum principles, Duhamel's principle.
Unit - 5 The Laplace equation: Boundary value problems for Laplace and Poisson equations, Maximum and minimum principles, Green's identity and fundamental solution, Poisson integral formula, Dirichlet's problem for upper half plane and cube, Nuemann problem for a rectangle and upper half plane.

Instructions

Write your name and roll number on the first page of your assignment. The assignment should be legibly handwritten and on both sides of the paper. I will follow a zero toleration policy towards copying in any form. The assignment must be submitted as a single pdf file before the due date without fail. For any further query feel free to contact me. Timely submission of the assignment will be appreciated. There will be no credit for late submissions.

× × ×

1. Solve the differential equation $u_t - \alpha^2 u_{xx} = 0$ for the conduction of heat along a rod subject to the following conditions :
   (a) u is not infinite for $t \to \infty$,
   (b) $\frac{\partial u}{\partial x} = 0$ for $x = 0$ and $x = l$,
   (c) $u = lx - x^2$ for $t = 0$, between $x = 0$ and $x = l$.

2. Solve the following initial boundary value problem

$$\begin{cases} \dfrac{\partial u}{\partial t} = K\dfrac{\partial^2 u}{\partial x^2} \quad 0 \le x \le 1,\ 0 \le t \le \infty \\ u_x(0,t) = 0,\ 0 \le t \le \infty \\ u_x(200,t) = \dfrac{-h}{K}[u(200,t) - 20],\ 0 \le t \le \infty \\ u(x,0) = \sin(\pi x),\ 0 \le x \le 1. \end{cases}$$

3. State and prove a maximum principle for solutions of an initial boundary value problem for $u_t = K\Delta u$, where Δ is the laplacian in $\mathbb{R}$.

4. Solve the problem

$$\begin{aligned} &\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \sin(\pi x) \quad 0 \leq x \leq 1,\ 0 \leq t \leq \infty \\ &u(x,0) = 1,\ 0 \leq x \leq 1. \end{aligned}$$

5. Using duhamel's principle find the solution

$$\begin{aligned} &u_t = u_{xx} \quad 0 \leq x \leq 1,\ 0 \leq t \leq \infty \\ &u(1,t) = \sin t,\ 0 \leq t \leq \infty. \end{aligned}$$

6. An infinitely long plane uniform plate is bounded by two parallel edges and an end at right angel to them. The breadth is π, this end is maintained at a temperature u_0 at all points and other edges are at 0 temperature. Determine the temperature at any point of the plate in the steady-state.

7. solve

$$\begin{aligned} &u_{xx} + u_{yy} = 0,\ -\infty \leq x \leq \infty, y \geq 0, \\ &u_y(x,0) = g(x),\ -\infty \leq x \leq \infty, \end{aligned}$$

with the condition that u is bounded as $y \to \infty, u$ and u_x vanish as $|x| \to \infty$, and

$$\int_{-\infty}^{\infty} g(x)dx = 0.$$

8. Solve the Laplace equation

$$u_{xx} + u_{yy} = 0$$

subject to the conditions $u(0,y) = u(l,y) = u(x,0) = 0$ and $u(x,a) = \sin n\pi x/l$.

9. Derive Poisson integral formula of Laplace equation.

10. Solve the following Laplace equation

$$\begin{aligned} &\Delta^2 = 0,\ 0 \leq r \leq 1, \\ &u(1,\theta) = 1 + \sin\theta + \frac{1}{2}\sin(3\theta) + \cos(4\theta). \end{aligned}$$

$\times \times \times$