Q1 We have from (2.115) and (2.116)

$$p(y) = \mathcal{N}(y \mid A\mu + b, L^{-1} + A\Lambda^{-1}A^T)$$
$$p(x|y) = \mathcal{N}(x \mid \Sigma\{A^T L(y-b) + A\mu\}, \Sigma)$$

$$\Sigma = (\Lambda + A^T L A)^{-1}$$

The predictive distribution is given by :-

$$p(t|t, \alpha, \beta) = \int p(t|w, \beta) p(w|t, \alpha, \beta)\, dw$$

Substituting value of $p(y)$ in $p(t|t, \alpha, \beta)$:-

$$p(t|t, \alpha, \beta) = \int \mathcal{N}(t \mid A\mu + b, L^{-1} + A\Lambda^{-1}A^T)$$
$$\mathcal{N}(t \mid \Sigma\{A^T L(y-b) + A\mu\}, \Sigma)$$

$$= \int \mathcal{N}(t \mid \Sigma\{A^T L(y-b) + A\mu\}, \Sigma)\, dw$$

We see $m_N = \Sigma\{A^T L(y-b) + A\mu\}$

$$p(t|x,w,\beta) = \mathcal{N}(t|y(x,w),\beta^{-1})$$

$$p(t|x,w,\beta) = \prod_{n=1}^{N} \mathcal{N}(t_n|w^T\phi(x_n),\beta^{-1})$$

$$p(w) \sim \mathcal{N}(w|m_0, S_0)$$

$$p(t|X,w,\beta) = \prod_{n=1}^{N} \mathcal{N}(t_n|w^T\phi(x_n),\beta^{-1})$$

Let $p(w) = \mathcal{N}(w|\mu, \Lambda^{-1})$

$$p(t|w) = \mathcal{N}(t|Aw+b, L^{-1})$$
$$p(t) = \mathcal{N}(t|A\mu+b, L^{-1}+A\Lambda^{-1}A^T)$$

$$p(w|t) = \mathcal{N}(w|\Sigma\{A^T L(t-b)+\Lambda\mu\}, \Sigma)$$

$$p(w) = \mathcal{N}(w|m_0, S_0)$$

$$p(t|X,w,\beta) = \prod_{n=1}^{N} \mathcal{N}(t_n|w^T\phi(x_n),\beta^{-1})$$

$$p(w|t) = p(w|m_N, S_N)$$

$$m_N = S_N(S_0^{-1}m_0 + \beta\phi^T t)$$

given by $S_N^{-1} = S_0^{-1} + \beta\phi^T\phi$

$$\boxed{p(t|x,t,\alpha,\beta) = \mathcal{N}(t|m_N^T\phi(x), \sigma_N^2(x))}$$

$$\boxed{\sigma_N^2(x) = \frac{1}{\beta} + \phi(x)^T S_N \phi(x)}$$