

Q2) Prove or disprove, Every Tree has at most one perfect matching.

Let M, M' be perfect matchings in the tree $T = (V, E)$ and consider the graph on V with edge set $M \cup M'$.

Since M and M' both cover all the vertices, every component of this new graph is either a single edge (common to both M and M') or a cycle.

Since T is a tree, it is an acyclic connected graph and contains no cycles, so we conclude that $M = M'$.

Hence proved. 