

Total No. of Pages: 2  
VI<sup>th</sup> SEMESTER  
END SEMESTER EXAMINATION

Roll No. 2K13/MC/064  
B.TECH. [M&C]  
(May: - 2016)

MC-311 ALGORITHMS DESIGN AND ANALYSIS

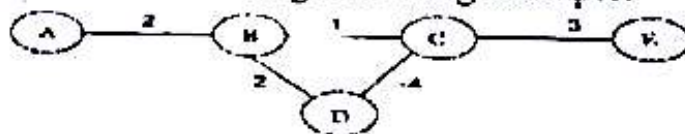
Time: 3:00 Hours

Max. Marks: 70

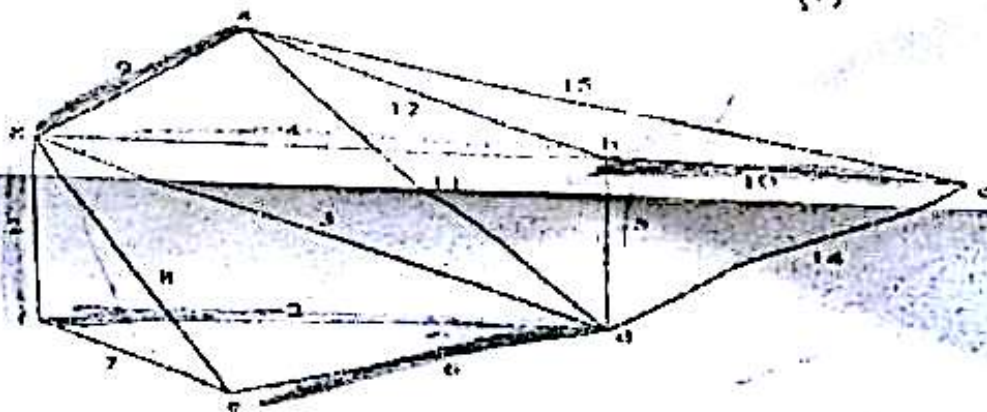
Note: Answer any five questions.  
Assume suitable missing data, if any.

- 1) a) Explain the use of asymptotic notations in the analysis of algorithms with the help of suitable examples. (7)  
b) What is recurrence? Explain different methods for solving a recurrence relation. (7)

- 2) a) Show that the running time of quick sort is  $\theta(n^2)$  when the array A contains distinct elements and elements are sorted in decreasing order. (7)  
b) Write an algorithm to find the single source shortest path when -ve edges are allowed. Solve using following example: (7)



- 3) Define Spanning tree of a graph. Write Prim's algorithm/pseudo code and find the minimum spanning tree for the given graph (Step by Step). (7)



- a) Discuss matrix chain multiplication problem with reference to dynamic programming technique and also apply it on the following array:

(7)

15	5	10	20	25
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- 4) a) Explain backtracking. How backtracking is used for solving 4-queen problem? Show the state space tree.

(7)

- b) Given two sequences  $S=ABAABZDC$  and  $T=EAADCBAD$ , find the longest common subsequence of  $S$  and  $T$  using dynamic programming approach.

(7)

5)

- a) What is LC branch and bound? Also explain, how the branch and bound technique is used to solve 0/1 knapsack problem. Give example.

(3+6)

- b) Find an optimal solution to the fractional knapsack instance  $n = 7$ ,  $M = 15$ .  $(p_1, p_2, p_3, \dots, p_7) = (10, 5, 15, 7, 6, 18, 3)$  and  $(w_1, w_2, w_3, \dots, w_7) = (2, 3, 5, 7, 1, 4, 1)$

(5)

- 6) a) Write short notes on P, NP and NP-complete problems.

(6)

- b) Discuss is string matching? Explain Rabin-Karp method with examples.

(8)

- 7) a) Differentiate between backtracking and branch & bound approach.

(7)

- b) Briefly explain the steps involved to solve a problem in dynamic programming? What are the drawbacks of dynamic programming?

(7)

Total No. of Pages 2  
SIXTH SEMESTER

**B. Tech. Mathematics & Computing**

**END SEMESTER EXAMINATION, May 2016**

**Code & Title: MC 312 Stochastic Processes**

**Time: 3:00 Hours**

**Max. Marks: 70**

**Note :** Answer all question by selecting any two parts from each questions. All questions carry equal marks. Assume suitable missing data, if any.

1[a] What is a Poisson process? Give example of homogeneous and non-homogenous Poisson processes. Prove that sum of two independent Poisson processes is again Poisson. What about their difference?

[b] Describe a renewal process? How does it differ from a Poisson process? Consider a renewal process with renewal function equal to  $at$ . Find the probability distribution of the number of renewals by time unit  $b$ .

[c] Explain the following stochastic processes with suitable examples in each:  
(i) Bernoulli process, (ii) Brownian motion.

2[a] What is a simple random walk. Give examples of random walks with,  
(i) Two reflecting barriers, (ii) One absorbing barrier.

Write the equations depicting the respective walks.

[b] Show that in case of unrestricted simple random walk, if the probability of a jump upward is less than the probability of a jump downward, then the particle will drift to  $-\infty$  with probability one.

[c] Discuss simple random walk with two absorbing barrier.

3[a] Define  $n$ -step transition probability matrix of a Markov chain. By considering an example of your choice demonstrate an application of Chapman Kolmogorov equations.



[b] A man drives a car or catches a train to go office each day. He never goes 2 days in a row by a train but if he drives on a day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week the man tossed a fair coin and drive to work if head appeared. Write the transition probability matrix and find the probability that he takes a train on the second day. Also find the probability that he drives to work in the long run.

[c] A particle performs a random walk with absorbing barriers at 0 and 4. If at present the particle is at position 3 then find the probability that it will revisit 3 after four steps.

4[a] Consider a computer system with Poisson job-arrival stream at an average rate of 60 per hour. Determine the probability that the time interval between successive job arrivals is, (i) Longer than four minutes, (ii) Shorter than eight minutes, (iii) Between two and six minutes.

[b] Explain continuous time Markov chain. Give example. When this chain is said to be (i) regular, (ii) non-regular. Give suitable examples.

[c] Describe death process. Give two examples. Find its probability generating function.

5[a] Describe Erlang loss queuing model. Consider a suitable example to describe its application.

[b] Define reliability. Find the reliability of an  $n$ -components system when the components are in (i) Series, (ii) Parallel, (iii)  $k$  out of  $n$ .

[c] In a railway yard goods trains arrive at the rate of 30 trains per day. Assuming that the inter-arrival time follows an exponential distribution and the service time distribution is also exponential with an average of 36 minutes then find the (i) mean queue size, (ii) probability that the queue size exceeds 10.

**MC- 313, Matrix Computation**

Time: 3.0 Hours

Max. Marks: 70

Note: Attempt **Any** two parts from each questions. All questions carry equal marks.  
Assume suitable missing data, if any. Simple calculators are allowed

1. (a) Discuss Rank deficiency and Numerical rank of a matrix.  
(b) Show if  $A$  is a strictly diagonally dominant matrix, then the Gauss-Seidel iteration scheme converges for any initial starting vector.

(c) Obtain the least square solution of  $Ax = b$ , where  $A = \begin{bmatrix} 0 & 1 \\ -3 & 0 \\ 0 & 2 \\ 4 & 10/3 \end{bmatrix}$   
and  $b = (1, 1, 1, 1)^T$

2. (a) Determine the  $QR$  decomposition of,  $A = \begin{bmatrix} 0 & 3 & 50 \\ 3 & 5 & 25 \\ 4 & 0 & 25 \end{bmatrix}$  using Householder transformation.

- (b) Define the induced matrix norm and determine

(a)  $\|A\|_1$  (b)  $\|A\|_2$  (c)  $\|A\|_\infty$   
when  $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ .

- (c) Discuss Moore Penrose inverse with example.

3. (a) Let  $A(\alpha) = \begin{bmatrix} 0.1\alpha & 0.1\alpha \\ 10 & 15 \end{bmatrix}$ . Determine  $\alpha$  such that condition number of  $A(\alpha)$  is minimized. Use the maximum norm.

(b) Determine the smallest eigenvalue and the corresponding eigenvector of the matrix  $\begin{bmatrix} 4 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 1 & 4 \end{bmatrix}$  correct upto 3 decimal places using the power method.

(c) Define and drive the formula for spectral radius of a matrix  $A$ .

4. (a) Prove that for a system  $Ax = b$ ,  $A$  is a  $m \times n$  matrix,  $(A^T A)$  is non-singular if  $A$  has full rank.

(b) The following system of equations is given

$$3x + 2y = 4.5$$

$$2x + 3y - z = 5$$

$$-y + 2z = -0.5.$$

Set up the SOR iteration scheme for the solution and find the optimal relaxation factor and the rate of convergence.

(c) Find the singular value decomposition of the matrix  $\begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$ .

5. (a) Write all possible Jordan canonical form for the matrix  $\begin{bmatrix} 4 & -2 & 2 \\ 6 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$

(b) Find  $QR$  factorization for the matrix (using Gram-Schmidt process)

$$\begin{bmatrix} 1 & 1 & 4 \\ 1 & 4 & 2 \\ 1 & 4 & 2 \end{bmatrix}$$

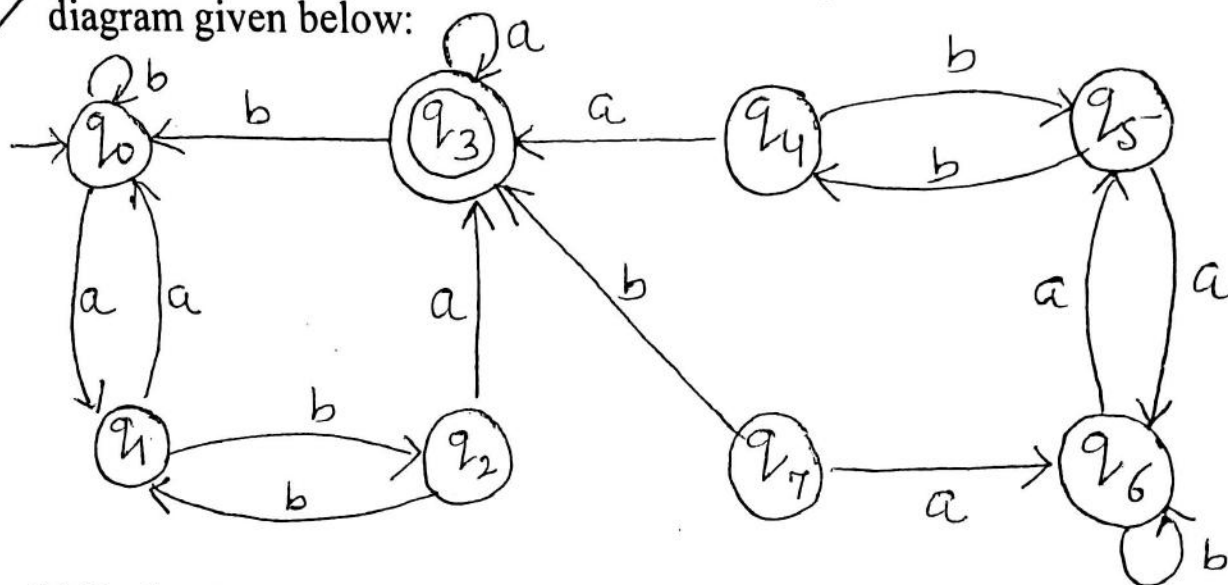
(c) Prove that each eigen value of a square matrix  $A$  lies in at least one Gerschgorin's disk generated by  $A$ .

Time: 3:00 Hours

Max. Marks: 70

**Note:** Answer ALL by selecting any TWO parts from each question.  
All questions carry equal marks.

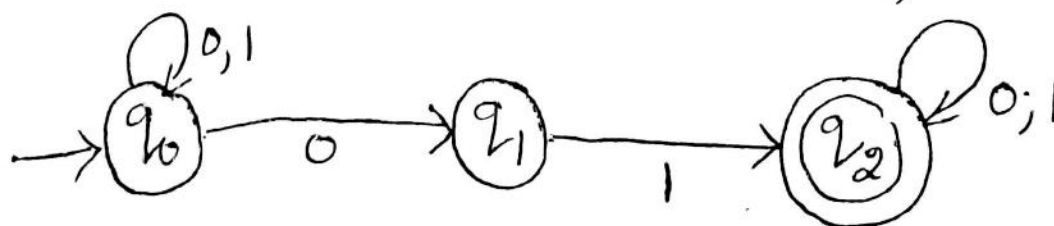
Q1 (a) Construct the minimum state automaton equivalent to the transition diagram given below:



(b) Define Moore machine. Construct a Moore machine equivalent to the Mealy machine defined by the table below:

Present State	Next State			
	a=0		a=1	
	state	output	state	output
→ q <sub>1</sub>	q <sub>1</sub>	1	q <sub>2</sub>	0
q <sub>2</sub>	q <sub>4</sub>	1	q <sub>4</sub>	1
q <sub>3</sub>	q <sub>2</sub>	1	q <sub>3</sub>	1
q <sub>4</sub>	q <sub>3</sub>	0	q <sub>1</sub>	1

(c) Study the automaton 'M' given in figure and state whether the statements given below are true or false (give reasons)





- (i)  $M$  is a nondeterministic automaton
- (ii)  $\delta(q_1, 1)$  is defined
- (iii) 0100111 is accepted by  $M$
- (iv) 010101010 is not accepted by  $M$
- (v)  $\delta(q_0, 01001) = \{q_1\}$
- (vi)  $\delta(q_0, 011000) = \{q_0, q_1, q_2\}$
- (vii)  $\delta(q_2, w) = q_2$  for any string  $w \in \{0,1\}^*$

Q2 (a) Define the language generated by a grammar.

If  $G$  is  $S \rightarrow aS, S \rightarrow bS, S \rightarrow a, S \rightarrow b$ , find  $L(G)$ .

(b) Let  $G = (\{S, A\}, \{0,1,2\}, P, S)$ , where  $P$  consists of

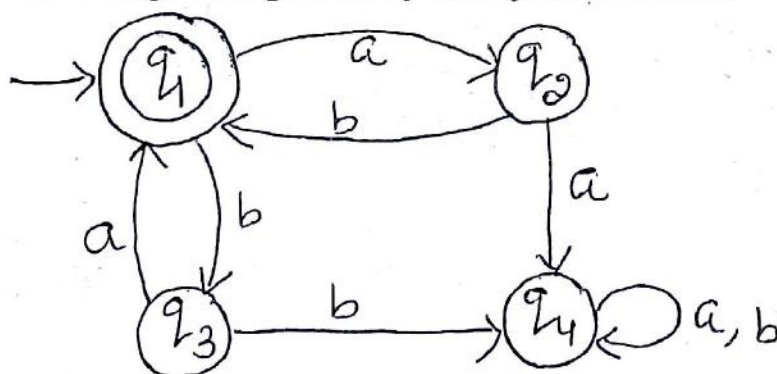
$S \rightarrow 0SA2, S \rightarrow 012, 2A \rightarrow A2, 1A \rightarrow 11$ .

Show that  $L(G) = \{0^n 1^n 2^n : n \geq 1\}$ .

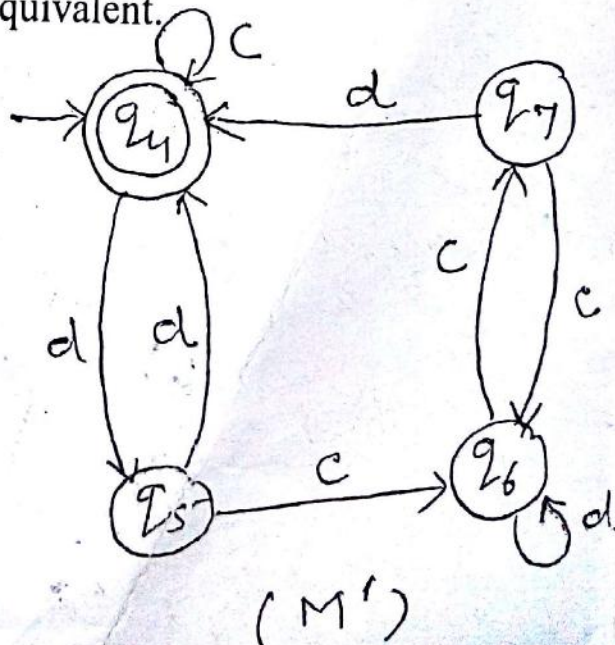
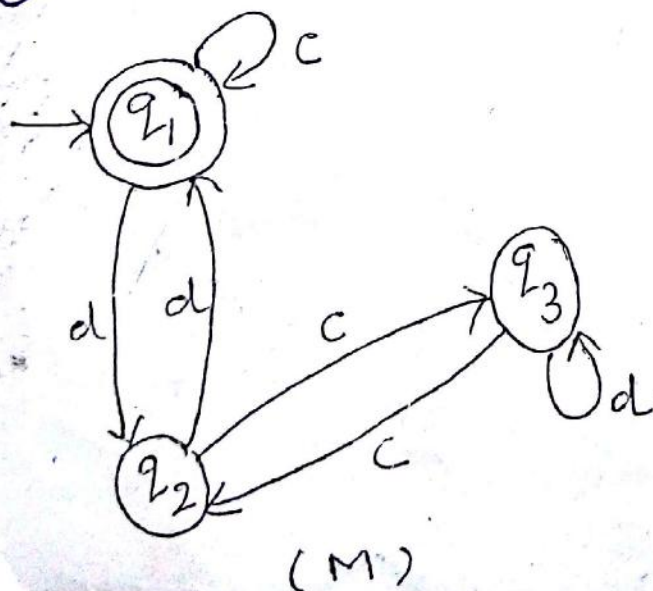
(c) State and prove Arden's theorem. Prove that

$P + PQ^*Q = a^*bQ^*$  where  $P = b + aa^*b$  and  $Q$  is any regular expression.

Q3 (a) Describe the algebraic method using Arden's theorem to find the regular expression recognized by a transition system. Using this method, find the set of strings recognized by the system below:



(b) Define equivalence of two finite automata. Determine whether the following two DFA's  $M$  and  $M'$  are equivalent.





(c) Prove that

(i) If  $L$  is regular then  $L^T$  is also regular.

(ii) If  $L$  is regular set over  $\Sigma$ , then  $\Sigma^* - L$  is also regular over  $\Sigma$ .

Q4 (a) Construct a reduced grammar equivalent to the grammar

$S \rightarrow aAa, A \rightarrow Sb/bCC/DaA, C \rightarrow abb/DD, E \rightarrow aC, D \rightarrow aDA.$

(b) State and prove Pumping lemma for context free language.

(c) Reduce the following grammar to CNF:

$S \rightarrow ASA, S \rightarrow bA, A \rightarrow B, A \rightarrow S, B \rightarrow c.$

Q5 (a) Consider a pda  $A = (\{q_0, q_1, q_f\}, \{a, b, c\}, \{a, b, Z_0\}, \delta, q_0, Z_0, \{q_f\})$  where  $\delta$  is defined as

$\delta(q_0, a, Z_0) = \{(q_0, aZ_0)\}, \delta(q_0, b, Z_0) = \{(q_0, bZ_0)\}$

$\delta(q_0, a, a) = \{(q_0, aa)\}, \delta(q_0, b, a) = \{(q_0, ba)\}$

$\delta(q_0, a, b) = \{(q_0, ab)\}, \delta(q_0, b, b) = \{(q_0, bb)\}$

$\delta(q_0, c, a) = \{(q_1, a)\}, \delta(q_0, c, b) = \{(q_1, b)\}$

$\delta(q_0, c, Z_0) = \{(q_1, Z_0)\}, \delta(q_1, a, a) = \delta(q_1, b, b) = \{(q_1, \Lambda)\}$

$\delta(q_1, \Lambda, Z_0) = \{(q_f, Z_0)\}$

Show that the set  $L = \{wcw^T : w \in \{a, b\}^*\}$  is accepted by final state.

(b) Design the Turing Machine for the following languages:

(i)  $L = \{a^n : n \geq 1\}$  (ii)  $L = \{a^{2n} : n \geq 1\}$

(iii)  $L = [(a + b)^*]$  (iv)  $L = \{a^n b^n : n \geq 1\}$

(v)  $L =$  set of string over  $\{0,1\}$  starting with 00.

(c) In how many ways a Turing machine can be described? Explain with suitable examples.

NOTE: Attempt any 5 Questions. Assume missing data if any.

- Q1. (a) Why is it important for the scheduler to distinguish I/O bound programs from CPU-bound programs? [4]  
(b) Explain the activities of an operating system with regard to memory management. [5]  
(c) What is Banker's algorithm? For what purpose it is used? [5]
- Q2. (a) What do you mean by file access method? Explain sequential and direct access file methods. [5]  
(b) What do you mean by cooperative processes? Why it is necessary to synchronize the activities of concurrent processes? [5]  
(c) In a 64 bit machine, with 256 MB RAM, and a 4KB page size, how many entries will there be in the page table if it is inverted? [4]
- Q3. (a) Differentiate between internal and external fragmentation? How it can be avoided? Does paging suffer from external fragmentation? Explain. [5]  
(b) What is the purpose of interrupts? What are the differences between a trap and an interrupt? [5]  
(c) Explain various operating system services. [4]
- Q4. (a) Under what circumstances do page fault occur? Describe the action taken by operating system when a page fault occurs. [5]  
(b) Consider a logical address space of 64 pages of 1024 words each mapped onto a physical memory of 32 frames. Find the number of bits in the logical as well as in the physical address. [4]  
(c) What is bounded buffer problem? Explain its solution using semaphore. [5]
- Q5. (a) Consider a paging system with the page table stored in memory. If a memory reference takes 200 nanoseconds, how long does a paged memory reference take? [3]  
(b) Consider the following page reference string:  
1,2,3,4,2,1,5,6,2,1,2,3,7,6,3,2,1,2,3,6  
Find the number of page faults for the LRU, FIFO and Optimal page replacement algorithms assuming three free frames. [6]  
(c) Compare paging with segmentation with respect to address translation. [5]
- Q6. (a) Explain following CPU scheduling algorithms: (i) RR (ii) SRTF [4]  
(b) Consider a system with 80% hit ratio, 50 nanoseconds time to search the associative registers, 750 nanoseconds time to access memory.  
(i) Find the time to access a page when the page is in associative memory.  
(ii) Find the time to access a page when a page is not in associative memory.

[10]

[3.5X4]

(iii) Find the effective memory access time.

Q7. Write short note on following:

- (a) Contiguous memory allocation
- (b) Hashed page table
- (c) SCAN and SSTF disk scheduling
- (d) File attributes