

SIGNMENT-3

1. $\tilde{f}(x) = (f(x), s(x), t(x))$

$$f(x) = x^2 \quad s(x) = \frac{x}{4} \quad t(x) = x/2$$

$$L(x) = \frac{1}{1+x^2} \quad R(x) = \frac{1}{1+2|x|}$$

$L(x)$ and $R(x)$ have no effect on value of integral of such fuzzy function.

$$\int_1^4 \tilde{f}(x) \cdot dx = \int_a^b f(x) dx = \int_1^4 x^2 dx = \left. \frac{x^3}{3} \right|_1^4 = 21$$

$$\int_a^b s(x) dx = \int_1^4 \frac{x}{4} dx = \left. \frac{x^2}{8} \right|_1^4 = 1.875$$

$$\int_a^b t(x) dx = \int_1^4 \frac{x}{2} dx = \left. \frac{x^2}{6} \right|_1^4 = 3.75$$

$$\text{Hence } \tilde{I}(a, b) = (21, 1.875, 3.75)_{LR}$$

2. $\tilde{a} = \{(4, 0.8), (5, 1), (6, 0.4)\}$ $\tilde{b} = \{(6, 0.7), (7, 1), (8, 0.2)\}$

$$f(x) = 2 \quad x \in [2, 4]$$

$[a, b]$	$\int_a^b f(x) dx = \int_a^b 2 dx$	$\min(\mu_A(a), \mu_B(b))$
$[4, 6]$	4	0.7
$[4, 7]$	6	0.8

$[4, 8]$	8	0.2
$[5, 6]$	2	0.7
$[5, 7]$	4	1
$[5, 8]$	6	0.2
$[6, 6]$	0	0.4
$[6, 7]$	2	0.4
$[6, 8]$	4	0.2

$$\tilde{I}(A, B) = \{ (0, 0.4), (2, 0.7), (4, 1), (6, 0.8), (8, 0.2) \}$$

3. $f(x) = 2x - 3$ $g(x) = 2x + 5$

$$\tilde{a} = \{ (1, 0.8), (2, 1), (3, 0.4) \}$$

$$\tilde{b} = \{ (3, 0.7), (4, 1), (5, 0.3) \}$$

$$f \circ g = 2(2x + 5) - 3 = 4x + 10 - 3 = 4x + 7$$

$$g \circ f = 2(2x - 3) + 5 = 4x - 6 + 5 = 4x - 1$$

(i) $[a, b]$	$\int_a^b f(x) dx = \int_a^b 2x - 3 = x^2 - 3x \Big _a^b$	$\min\{\mu_A(a), \mu_B(b)\}$
$[1, 3]$	2	0.7
$[1, 4]$	6	0.8
$[1, 5]$	12	0.3

[2, 3]

2

0.7

[2, 4]

6

1

[2, 5]

12

0.3

[3, 3]

0

0.4

[3, 4]

4

0.4

[3, 5]

10

0.3

$$\tilde{I}(A, B) = \{ (0, 0.4), (2, 0.7), (6, 1), (12, 0.3), (4, 0.4), (10, 0.3) \}$$

(ii) [a, b]

f

$$\int_a^b f(x) dx = \int_a^b 2x + 5$$

$$= x^2 + 5x \Big|_a^b$$

$$\min \{ \mu_A(a), \mu_B(b) \}$$

[1, 3]

18

0.7

[1, 4]

30

0.8

[1, 5]

44

0.3

[2, 3]

10

0.7

[2, 4]

22

~~0.3~~ 1

[2, 5]

36

0.3

[3, 3]

0

0.4

[3, 4]

12

0.4

[3, 5]

26

0.3

$$\tilde{I}(A, B) = \{(0, 0.4), (10, 0.7), (12, 0.4), (18, 0.7), (22, 1), (26, 0.3), (30, 0.2), (36, 0.3), (44, 0.3)\}$$

(iii) & (iv)

$[a, b]$	$\int_a^b 4x+7$ $= 2x^2+7x \Big _a^b$	$\int_a^b 4x-1$ $= 2x^2-x \Big _a^b$	$\min\{\mu_A(a), \mu_B(b)\}$
			0.7
[1, 3]	30	14	0.8
[1, 4]	51	27	0.8
[1, 5]	76	44	0.3
[2, 3]	17	9	0.7
[2, 4]	38	22	0.8 1
[2, 5]	63	39	0.3
[3, 3]	0	0	0.4
[3, 4]	21	13	0.4
[3, 5]	46	30	0.2

$$(iii) \tilde{I}(A, B) = \{(0, 0.4), (17, 0.7), (21, 0.4), (30, 0.7), (38, 1), (46, 0.3), (51, 0.8), (63, 0.3), (76, 0.3)\}$$

$$(iv) \tilde{I}(A, B) = \{(0, 0.4), (9, 0.7), (13, 0.4), (14, 0.7), (22, 1), (30, 0.3), (27, 0.8), (39, 0.3), (44, 0.3)\}$$

4. $f(n) = n^3$ $\tilde{n} = \{(-1, 0.4), (0, 1), (1, 0.6)\}$

$$f'(n) = 3n^2$$

using extension principle:-

$$f'(\tilde{n}) = \{(0, 1), (3, 0.6)\}$$

5. $f(n) = n^2 + 1$, $g(n) = 2 - n$

$$\tilde{a} = (1, 2, 3)$$

$$\tilde{b} = (3, 4, 5)$$

6. $\tilde{x} = \{(-1, 0.4), (0, 0.1), (1, 0.6)\}$

$$f(n) = n^3 + 2$$

$$g(n) = 2n + 3$$

$$f'(n) = 3n^2$$

$$g'(n) = 2$$

$$f'(n) + g'(n) = 3n^2 + 2$$

$$f(n) + g(n) = n^3 + 2n + 5$$

$$f'(\tilde{n}_0) = \{(3, 0.6), (0, 0.1)\}$$

$$g'(n) = \{(2, 0.6)\}$$

$$f'(n) + g'(n) = \{(5, 0.6), (2, 0.1)\}$$

$$(f' + g')(n) = \{(5, 0.6), (2, 0.1)\}$$

Hence, $(f' + g')(\tilde{n}_0) = f'(\tilde{n}_0) + g'(\tilde{n}_0)$

8.7

$$f(x_0) = \{ (-2 + (-1-1), .5), (-1, .8), (2 + (1-1), 1) \\ (16 + (2-1), .8), (54 + (3-1), .4) \}$$

$$= \{ (-4, .5), (-1, .8), (2, 1), (17, .8), (56, .4) \}$$

$$g(x_0) = \{ (-2, .5), (-1, .8), (0, 1), (13, .8), (50, .4) \}$$

$$\text{LHS} = \{ (-6, .5), (-3, .5), (0, .5), (15, .5), (54, .4) \\ (-5, .5), (-2, .8), (1, .8), (16, .8), (55, .4), \\ (-4, .5), (-1, .8), (2, 1), (17, .8), (56, .4), \\ (9, .5), (12, .8), (15, .8), (30, .8), (69, .4), \\ (46, .4), (49, .4), (52, .4), (67, .4), (106, .4) \}$$

$$\text{RHS} = \{ (-6, .5), (-2, .8), (2, 1), (30, .8), (106, .4) \}$$

Since RHS part of LHS hence Proved.

S.8

$$f(x) = \begin{cases} 2x^2 - 3 & -2 \leq x \leq 2 \\ 5 & \text{else where} \end{cases}$$

$$\mu_{\text{new}}(x) = \frac{f(x) - \inf(x)}{\sup(x) - \inf(x)}$$

$$\sup(x) = 5$$

$$\inf(x) = -3$$

So for

$$1) \quad -2 \leq x \leq 2$$

$$\mu_{\text{new}}(x) = \frac{2x^2 - 3 - (-3)}{5 - (-3)} = \frac{2x^2}{8} = \frac{x^2}{4}$$

$$2) \quad \text{else where}$$

$$\mu_{\text{new}}(x) = \frac{5 + 3}{5 + 3} = 1$$

§.9 $f(x) = \sin(\sin x), x \in \mathbb{R}$

$$\sup(x) = \sin 1$$

$$\inf(x) = -\sin 1$$

$$\mu_{\max}(x) = \frac{f(x) - \inf(x)}{\sup(x) - \inf(x)}$$

$$= \frac{\sin(\sin x) + \sin 1}{2 \sin 1}$$

$$\mu_{\max}(0) = \frac{1}{2}$$

$$\mu_{\max}(\pi/2) = 1$$

$$\mu_{\max}(-\pi/2) = 0$$

10. (i) Yes

Classical functions can be defined as a fuzzy function mapping from crisp to crisp set with membership value '1'.

(ii) Yes

We can define fuzzy function, $f: X \rightarrow \tilde{P}(Y)$ where $\tilde{P}(Y)$ is fuzzy power set of Y and $\mu_{f(x)}(y) = \mu_R(x, y) \forall x, y \in X \times Y$ where R is a fuzzy relation.

(iv) Yes

If 'M' is the maximizing set of fuzzy function "f" then we say, we get maximum value of "f" at " x_0 " if

(i) $\mu_M(x_0)$ is maximum

(ii) $\mu_D(x_0)$ is maximum

↪ denotes membership of x_0 in domain

(v) Yes

(vi) Yes

Integration of crisp function over fuzzy domain:

$$\int_{\tilde{a}}^{\tilde{b}} f(x) dx = F(\tilde{b}) - F(\tilde{a})$$

This subtraction is extended subtraction and done with conformation to extension principle.

Integration of fuzzy function over crisp domain:

$$\tilde{I}(a, b) = \left\{ \left(\int_a^b \underline{f}_\alpha(x) dx, \int_a^b \overline{f}_\alpha(x) dx \right) \right\} \text{ where } \underline{f}_\alpha \text{ and } \overline{f}_\alpha \text{ are } \alpha\text{-level curves for fuzzy functions 'f'}$$

⇒ Since this is defined according to α -cuts, it is also in conformity with extension principle.

(vii) No

The condition holds only if $\int_a^b \tilde{f}(x) dx$ and $\int_a^b \tilde{g}(x) dx$ are commutative.

Generally, $\int_a^b (\tilde{f} \oplus \tilde{g}) dx \geq \int_a^b \tilde{f}(x) dx \oplus \int_a^b \tilde{g}(x) dx$ [fuzzy f^n over crisp domain]

(viii) No

The following condition is always there:

$$\int_a^{\tilde{b}} (f + g) x dx \leq \int_a^{\tilde{b}} f(x) dx + \int_a^{\tilde{b}} g(x) dx \quad [\text{crisp } f^n \text{ over fuzzy domain}]$$

(ix) No

The given condition is true for fuzzy f^n over crisp domain but not for crisp f^n over fuzzy domain

$$\int_a^c \tilde{f}(x) dx = \int_a^b \tilde{f}(x) dx + \int_b^c \tilde{f}(x) dx$$

$$\int_a^{\tilde{c}} f(x) dx \leq \int_a^{\tilde{b}} f(x) dx + \int_b^{\tilde{c}} f(x) dx$$

(x) Yes

For question 6, condition was true.

For question 7, condition was not true.

$$11) f(x) = x^4 + x^2 - 1$$

$$\{(1, 0.2), (2, 0.6), (3, 1), (4, 0.6), (5, 0.2)\}$$

$$\{(\text{around } 3) = \{(f(1), 0.2), (f(2), 0.6), (f(3), 1), (f(4), 0.6), (f(5), 0.2)\}$$

$$= \{(1, 0.2), (19, 0.6), (89, 1), (271, 0.6), (649, 0.2)\}$$

$$12) P(\tilde{A})_{\text{true}} = \{(0.5, 0.6), (0.6, 0.7), (0.7, 0.8), (0.8, 0.9), (0.9, 1), (1, 1)\}$$

$$P(\tilde{B})_{\text{true}} = \{(0.2, 0.3), (0.3, 0.8), (0.5, 0.1), (0.7, 0.4), (0.8, 0.9), (0.9, 0.7)\}$$

$$a) P(\tilde{A})_{\text{negation}} = \{(0, 1), (0.1, 1), (0.2, 1), (0.3, 1), (0.4, 1), (0.5, 0.4), (0.6, 0.3), (0.7, 0.2), (0.8, 0.1)\}$$

$$b) P(\tilde{A})_{\text{very true}} = \{(0.5, 0.36), (0.6, 0.49), (0.7, 0.64), (0.8, 0.81), (0.9, 1), (1, 1)\}$$

$$d) P(\tilde{A})_{\text{very very true}} = \{(0.5, 0.216), (0.6, 0.343), (0.7, 0.324), (0.8, 0.729), (0.9, 1), (1, 1)\}$$

$$c) P(\tilde{A}) \wedge P(\tilde{B}) = \{(0.5, 0.1), (0.6, 0), (0.7, 0.4), (0.8, 0.9), (0.9, 0.7)\}$$

$$f) P(\tilde{A}) \cup P(\tilde{B}) = \{(0.2, 0.3), (0.3, 0.8), (0.5, 0.6), (0.6, 0.7), (0.7, 0.8), (0.8, 0.9), (0.9, 1), (1, 1)\}$$

$$g) P(\tilde{A}) \rightarrow P(\tilde{B}) = \{(0, 1), (0.1, 1), (0.2, 1), (0.3, 1), (0.4, 1), (0.5, 0.4), (0.6, 0.3), (0.7, 0.4), (0.8, 0.9), (0.9, 0.7)\}$$

13) using Lukasiewicz implication

$$I(a, b) = \min(1, 1 - a + b)$$

$$\tilde{R} = \begin{matrix} & y_1 & y_2 \\ \begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix} & \begin{bmatrix} 1 & .9 \\ 1 & .4 \\ 1 & .8 \end{bmatrix} \end{matrix}$$

$$B' = \langle (y_1, .9), (y_2, .7) \rangle$$

Using compositional rule of inference

$$\begin{aligned} \mu_B(y_1) &= \sup_{x \in X} \min[\mu_B(x), \mu_R(x, y_1)] \\ &= \max(\min(.6, 1), (.9, 1), \min(.7, 1)) \\ &= 0.9. \end{aligned}$$

44. In this case,

$$\mu_{A'}(x_1) = \sup_{y \in Y} \min[\mu_B(y), \mu_R(x_1, y)] = \max\{\min(.9, 1), \min(.7, .9)\} = .9$$

$$\mu_{A'}(x_2) = \sup_{y \in Y} \min[\mu_B(y), \mu_R(x_2, y)] = .9$$

$$\mu_{A'}(x_3) = \sup_{y \in Y} \min[\mu_B(y), \mu_R(x_3, y)] = .9$$

Hence x is $\tilde{A} = \{(x_1, .9), (x_2, .9), (x_3, .9)\}$

~~Next we consider generalized hypothesis,~~

~~Rule 1: If $x \in \tilde{A}$, then $y \in \tilde{B}$~~

~~Rule 2: If $y \in \tilde{B}$, then $x \in \tilde{A}$~~

~~For each candidate fuzzy proposition in \tilde{A} check a fuzzy relation determined by $\tilde{R}(x, y) = \min[\tilde{A}(x), \tilde{B}(y)]$~~

$$15) \tilde{A} = \{(x_1, 0.5), (x_2, 1), (x_3, 0.6)\}$$

$$\tilde{B} = \{(y_1, 0.1), (y_2, 0.4)\}$$

$$\tilde{C} = \{(z_1, 0.2), (z_2, 0.1)\}$$

$$I(a, b) = \begin{cases} 1 & \text{if } a \leq b \\ b & \text{if } a > b \end{cases}$$

$$\rightarrow \tilde{R}_1 = \begin{matrix} & y_1 & y_2 \\ x_1 & \begin{bmatrix} 1 & .4 \end{bmatrix} \\ x_2 & \begin{bmatrix} 1 & .4 \end{bmatrix} \\ x_3 & \begin{bmatrix} 1 & .4 \end{bmatrix} \end{matrix}$$

$$\tilde{R}_2 = \begin{matrix} & z_1 & z_2 \\ y_1 & \begin{bmatrix} .2 & 1 \end{bmatrix} \\ y_2 & \begin{bmatrix} .2 & 1 \end{bmatrix} \end{matrix}$$

$$\tilde{R}_3 = \begin{matrix} & z_1 & z_2 \\ x_1 & \begin{bmatrix} 1 & .4 \end{bmatrix} \\ x_2 & \begin{bmatrix} 1 & .4 \end{bmatrix} \\ x_3 & \begin{bmatrix} 1 & .4 \end{bmatrix} \end{matrix}$$

$$\tilde{R}_1 \circ \tilde{R}_2 = \begin{matrix} & z_1 & z_2 \\ x_1 & \begin{bmatrix} 1 & .4 \end{bmatrix} \\ x_2 & \begin{bmatrix} 1 & .4 \end{bmatrix} \\ x_3 & \begin{bmatrix} 1 & .4 \end{bmatrix} \end{matrix} = \tilde{R}_3$$

Thus, generalised hypothetical syllogism holds.

17)

i) $(a \wedge b) \Rightarrow c$

a	b	$a \wedge b$	c
0	0	0	1
0	$\frac{1}{2}$	0	1
0	1	0	1
$\frac{1}{2}$	0	0	$\frac{1}{2}$
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}$
1	0	0	0
1	$\frac{1}{2}$	$\frac{1}{2}$	0
1	1	1	0

$(a \wedge b) \Rightarrow c$

1
1
1
1
1
 $\frac{1}{2}$
0
0
0

ii) $(a \vee b) \Leftrightarrow (a \wedge b)$

a	b	\vee	\wedge
0	0	0	0
0	$\frac{1}{2}$	0	$\frac{1}{2}$
0	1	0	1
$\frac{1}{2}$	0	0	$\frac{1}{2}$
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
$\frac{1}{2}$	1	$\frac{1}{2}$	1
1	0	0	1
1	$\frac{1}{2}$	$\frac{1}{2}$	1
1	1	1	1

$(a \vee b) \Leftrightarrow (a \wedge b)$

1
 $\frac{1}{2}$
0
 $\frac{1}{2}$
1
 $\frac{1}{2}$
0
 $\frac{1}{2}$
1

iii) $(a \Rightarrow b) \rightarrow (a \Rightarrow b)$

a	b	\Rightarrow	$(a \Rightarrow b) \Rightarrow (a \Rightarrow b)$
0	0	1	1
0	$\frac{1}{2}$	1	1
0	1	1	1
$\frac{1}{2}$	0	$\frac{1}{2}$	1
$\frac{1}{2}$	$\frac{1}{2}$	1	1
$\frac{1}{2}$	1	1	1
1	0	0	1
1	$\frac{1}{2}$	$\frac{1}{2}$	1
1	1	1	1

18) True = $\{(0.5, 0.3), (0.6, 0.6), (0.7, 0.8), (0.8, 0.9), (0.9, 0.9), (1, 1)\}$

False = $\{(0, 1), (0.1, 0.9), (0.2, 0.8), (0.3, 0.6), (0.4, 0.2), (0.5, 0.1)\}$

Not true = $\overline{\text{True}} = \{(0, 1), (0.1, 0.9), (0.2, 0.8), (0.3, 0.6), (0.4, 0.2), (0.5, 0.1), (0.6, 0.4), (0.7, 0.2), (0.8, 0.1), (0.9, 0.1), (1, 0.9)\}$

Neither True nor false = $(\overline{\text{True} \wedge \text{False}})$

True \cup False = $\{(0, 1), (0.1, 0.9), (0.2, 0.8), (0.3, 0.6), (0.4, 0.2), (0.5, 0.3), (0.6, 0.6), (0.7, 0.8), (0.8, 0.7), (0.9, 0.9), (1, 0.1)\}$

Neither true nor false = $(\overline{\text{True} \cup \text{False}}) = \{(0.1, 0.1), (0.2, 0.2), (0.3, 0.4), (0.4, 0.8), (0.5, 0.7), (0.6, 0.4), (0.7, 0.2), (0.8, 0.1), (0.9, 0.1), (1, 0.9)\}$

$$\text{Very true} = \text{True} - \text{false} = \text{True} \wedge \overline{\text{false}}$$

$$\overline{\text{false}} = \{(0.1, 0.1), (0.2, 0.2), (0.3, 0.4), (0.4, 0.8), (0.5, 0.9), (0.6, 1), (0.7, 1), (0.8, 1), (0.9, 1), (1, 1)\}$$

$$\text{Very True} = \text{True} \cap \overline{\text{false}}$$

$$= \{(0.5, 0.3), (0.6, 0.6), (0.7, 0.8), (0.8, 0.9), (0.9, 0.9), (1, 0.1)\}$$

$$\text{Almost true not false} = \text{True} \cup \overline{\text{false}}$$

$$= \{(0.1, 0.1), (0.2, 0.2), (0.3, 0.4), (0.4, 0.5), (0.5, 0.9), (0.6, 1), (0.7, 1), (0.8, 1), (0.9, 1), (1, 1)\}$$

$$\{(0.1, 0.1), (0.2, 0.2), (0.3, 0.4), (0.4, 0.5), (0.5, 0.9), (0.6, 1), (0.7, 1), (0.8, 1), (0.9, 1), (1, 1)\}$$

$$\{(0.1, 0.1), (0.2, 0.2), (0.3, 0.4), (0.4, 0.5), (0.5, 0.9), (0.6, 1), (0.7, 1), (0.8, 1), (0.9, 1), (1, 1)\}$$

$$\{(0.1, 0.1), (0.2, 0.2), (0.3, 0.4), (0.4, 0.5), (0.5, 0.9), (0.6, 1), (0.7, 1), (0.8, 1), (0.9, 1), (1, 1)\}$$

$$\{(0.1, 0.1), (0.2, 0.2), (0.3, 0.4), (0.4, 0.5), (0.5, 0.9), (0.6, 1), (0.7, 1), (0.8, 1), (0.9, 1), (1, 1)\}$$

$$\{(0.1, 0.1), (0.2, 0.2), (0.3, 0.4), (0.4, 0.5), (0.5, 0.9), (0.6, 1), (0.7, 1), (0.8, 1), (0.9, 1), (1, 1)\}$$

$$\{(0.1, 0.1), (0.2, 0.2), (0.3, 0.4), (0.4, 0.5), (0.5, 0.9), (0.6, 1), (0.7, 1), (0.8, 1), (0.9, 1), (1, 1)\}$$

$$\{(0.1, 0.1), (0.2, 0.2), (0.3, 0.4), (0.4, 0.5), (0.5, 0.9), (0.6, 1), (0.7, 1), (0.8, 1), (0.9, 1), (1, 1)\}$$

$$\{(0.1, 0.1), (0.2, 0.2), (0.3, 0.4), (0.4, 0.5), (0.5, 0.9), (0.6, 1), (0.7, 1), (0.8, 1), (0.9, 1), (1, 1)\}$$

$$\{(0.1, 0.1), (0.2, 0.2), (0.3, 0.4), (0.4, 0.5), (0.5, 0.9), (0.6, 1), (0.7, 1), (0.8, 1), (0.9, 1), (1, 1)\}$$

$$\{(0.1, 0.1), (0.2, 0.2), (0.3, 0.4), (0.4, 0.5), (0.5, 0.9), (0.6, 1), (0.7, 1), (0.8, 1), (0.9, 1), (1, 1)\}$$

$$\{(0.1, 0.1), (0.2, 0.2), (0.3, 0.4), (0.4, 0.5), (0.5, 0.9), (0.6, 1), (0.7, 1), (0.8, 1), (0.9, 1), (1, 1)\}$$

$$\{(0.1, 0.1), (0.2, 0.2), (0.3, 0.4), (0.4, 0.5), (0.5, 0.9), (0.6, 1), (0.7, 1), (0.8, 1), (0.9, 1), (1, 1)\}$$

$$\mu_{RT}(A'/A)(.8) = \mu_A(x_1) = .9$$

$$\mu_{RT}(A'/A)(.6) = \mu_A(x_2) = .8$$

$$\mu_{RT}(A'/A)(.5) = \mu_A(x_3) = .5$$

$$\mu_{RT}(A'/A)(.4) = \mu_A(x_4) = .5$$

USING LUKASIEWICZ FUZZY IMPLICATION

$$I(a, b) = \min(1, 1 - a + b)$$

~~Sketch~~

$$\mu_{RT}(B'/B)(b) = \max \left\{ \begin{array}{l} \min(.9, S(I(.9, b))) \\ \min(.8, S(I(.8, b))) \\ \min(.5, S(I(.5, b))) \\ \min(.5, S(I(.5, b))) \end{array} \right\}$$

$$\mu_B(y_1) = \mu_{RT}(B'/B)(\tilde{B}(y_1)) = \mu_{RT}(B'/B)(.2) = .5$$

$$\mu_B(y_2) = \mu_{RT}(B'/B)(\tilde{B}(y_2)) = \mu_{RT}(B'/B)(.5) = .7$$

$$\mu_B(y_3) = \mu_{RT}(B'/B)(\tilde{B}(y_3)) = \mu_{RT}(B'/B)(.6) = .8$$

$$B = \{ (y_1, .5), (y_2, .7), (y_3, .8) \}$$

Que 21- $X = \{1, 2, 3, 4\}$; $\tilde{A} = \{(1, 0), (2, .2), (3, .6), (4, 1)\}$

$$\tilde{R} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 1 & .5 & 0 & 0 \\ .5 & 1 & .5 & 0 \\ 0 & .5 & 1 & .5 \\ 0 & 0 & .5 & 1 \end{bmatrix} \end{matrix}$$

$$\Rightarrow \tilde{B} = \tilde{A} \circ \tilde{R}$$

$$= x_1 \begin{bmatrix} y_1 & y_2 & y_3 & y_4 \\ 0 & .2 & .6 & 1 \end{bmatrix} \circ y_1 \begin{bmatrix} z_1 & z_2 & z_3 & z_4 \\ 1 & .5 & 0 & 0 \\ .5 & 1 & .5 & 0 \\ 0 & .5 & 1 & .5 \\ 0 & 0 & .5 & 1 \end{bmatrix}$$

for $x = x_1, z = z_1$

$$\mu(x_1, z_1) = \max(\min(0, 1), \min(.2, .5), \min(.6, 0), \min(1, 0)) \\ = \max(0, .2, 0, 0) = .2$$

for $x = x_1, z = z_2$

$$\mu(x_1, z_2) = \max(\min(0, .5), \min(.2, 1), \min(.6, .5), \min(1, 0)) \\ = \max(0, .2, .5, 0) = .5$$

for $x = x_1, z = z_3$

$$\mu(x_1, z_3) = \max(\min(0, 0), \min(.2, .5), \min(.6, 1), \min(1, .5)) \\ = \max(0, .2, .6, .5) = .6$$

for $x = x_1, z = z_4$

$$\mu(x_1, z_4) = \max(\min(0, 0), \min(.2, 0), \min(.6, .5), \min(1, 1)) \\ = \max(0, 0, .5, 1) = 1$$

$$\Rightarrow \tilde{B} = \{(1, .2), (2, .5), (3, .6), (4, 1)\}$$

Que 22-

$$X = \{1, 2, 3, 4, 5\} ; \tilde{A} = \{(1, 1), (2, .5), (3, .4), (4, .2), (5, 0)\}$$

μ of $\tilde{A} \circ \tilde{R}$ be defined by \tilde{B} matrix,

$$\begin{aligned} \tilde{B} &= \tilde{A} \circ \tilde{R} \\ &= x_i \begin{bmatrix} y_1 & y_2 & y_3 & y_4 & y_5 \\ 1 & .5 & .4 & .2 & 0 \end{bmatrix} \circ y_j \begin{bmatrix} z_1 & z_2 & z_3 & z_4 & z_5 \\ 1 & .8 & 0 & 0 & 0 \\ .8 & 1 & .8 & 0 & 0 \\ 0 & .8 & 1 & .8 & 0 \\ 0 & 0 & .8 & 1 & .8 \\ 0 & 0 & 0 & .8 & 1 \end{bmatrix} \end{aligned}$$

for $x = x_1, z = z_1$

$$\begin{aligned} \mu(x_1, z_1) &= \max(\min(1, 1), \min(.5, .8), \min(.4, 0), \min(.2, 0), \min(0, 0)) \\ &= \max(1, .5, 0, 0, 0) = 1 \end{aligned}$$

$$\begin{aligned} \mu(x_1, z_2) &= \max(\min(1, .8), \min(.5, 1), \min(.4, .8), \min(.2, 0), \min(0, 0)) \\ &= \max(.8, .5, .4, 0, 0) = .8 \end{aligned}$$

$$\begin{aligned} \mu(x_1, z_3) &= \max(\min(1, 0), \min(.5, .8), \min(.4, 1), \min(.2, 1), \min(0, .8)) \\ &= \max(0, .5, .4, .2, 0) = .5 \end{aligned}$$

$$\begin{aligned} \mu(x_1, z_4) &= \max(\min(1, 0), \min(.5, 0), \min(.4, .8), \min(.2, 1), \min(0, .8)) \\ &= \max(0, 0, .4, .2, 0) = .4 \end{aligned}$$

$$\begin{aligned} \mu(x_1, z_5) &= \max(\min(1, 0), \min(.5, 0), \min(.4, 0), \min(.2, .8), \min(0, 1)) \\ &= \max(0, 0, 0, .2, 0) = .2 \end{aligned}$$

$$\Rightarrow \tilde{B} = \{ (1, 1), (2, .8), (3, .5), (4, .4), (5, .2) \}$$

Que-23

$$\tilde{A} = \{(100, .5), (120, .7), (140, .8), (160, 1)\}$$

$$\tilde{B} = \{(10, .6), (12, .8), (15, 1)\}$$

Let index of attractive car for diff combinations of mileage and top speed is denoted by relation \tilde{R} .

$$\Rightarrow \tilde{R} = \begin{matrix} & \begin{matrix} 100 \\ 120 \\ 140 \\ 160 \end{matrix} \\ \begin{matrix} 10 \\ 12 \\ 15 \end{matrix} & \begin{bmatrix} .5 \\ .7 \\ .8 \\ 1 \end{bmatrix} \end{matrix}$$

$$\Rightarrow \tilde{R} = \begin{matrix} & \begin{matrix} 100 \\ 120 \\ 140 \\ 160 \end{matrix} \\ \begin{matrix} 10 \\ 12 \\ 15 \end{matrix} & \begin{bmatrix} .6 \\ .8 \\ 1 \end{bmatrix} \end{matrix} \circ \begin{matrix} & \begin{matrix} 100 & 120 & 140 & 160 \end{matrix} \\ \begin{matrix} .5 & .7 & .8 & 1 \end{matrix} \end{matrix}$$

$$= \begin{matrix} & \begin{matrix} 100 & 120 & 140 & 160 \end{matrix} \\ \begin{matrix} 10 \\ 12 \\ 15 \end{matrix} & \begin{bmatrix} .5 & .6 & .6 & .6 \\ .5 & .7 & .8 & .8 \\ .5 & .7 & .8 & 1 \end{bmatrix} \end{matrix}$$

Thus,

Mileage for most attractive car = 15 km/litre

Top speed for most attractive car = 160 km/hr

Ques 24- $\tilde{A} = \{(1, 0.7), (2, 0.4), (3, 0.6), (4, 0.5), (5, 0.8), (6, 0.2)\}$
 $\tilde{B} = \{(1, 0.8), (2, 0.5), (3, 0.6), (4, 0.6), (5, 0.9), (6, 0.1)\}$

① Zadeh's Maximum

$$\mu(x_i \text{ is } \tilde{A} \rightarrow y_j \text{ is } \tilde{B}) = (1 - \mu_A(x_i)) \vee (\mu_A(x_i) \wedge \mu_B(y_j))$$

$\mu =$

		B					
		1	2	3	4	5	6
A	1	0.7	0.5	0.6	0.6	0.7	0.3
	2	0.6	0.6	0.6	0.6	0.6	0.6
	3	0.6	0.5	0.6	0.6	0.6	0.4
	4	0.5	0.5	0.5	0.5	0.5	0.5
	5	0.8	0.5	0.6	0.6	0.8	0.2
	6	0.8	0.8	0.8	0.8	0.8	0.8

② Standard Sequence

$$\mu(x_i \text{ is } \tilde{A} \rightarrow y_j \text{ is } \tilde{B}) = \begin{cases} 1 & \mu_A(x_i) \leq \mu_B(y_j) \\ 0 & \text{else} \end{cases}$$

$\mu =$

		B					
		1	2	3	4	5	6
A	1	1	0	0	0	1	0
	2	0	1	1	1	1	0
	3	1	0	1	1	1	0
	4	1	1	1	1	1	0
	5	1	0	0	0	1	0
	6	1	1	1	1	1	0

③ Gödel's fuzzy Implication

$$t(x_i \text{ is } \tilde{A} \rightarrow y_j \text{ is } \tilde{B}) = \begin{cases} 1 & \mu_A(x_i) \leq \mu_B(y_j) \\ \frac{\mu_B(y_j)}{\mu_A(x_i)} & \text{else} \end{cases}$$

t =

		B					
		1	2	3	4	5	6
A	1	1	7/5	7/6	7/6	1	7
	2	2	1	1	1	1	4
	3	1	6/5	1	1	1	6
	4	1	1	1	1	1	5
	5	1	8/5	4/3	4/3	1	8
	6	1	1	1	1	1	2

Conclusion is not same in all cases.