

FINANCIAL ENGINEERING

ASSIGNMENT - 3

1. Consider the binomial model for trading in stock, $t = 1, 2$ where at each time the stock can go up by the factor u or down by the factor d . The sample space $Q = \{(u, u), (u, d), (d, u), (d, d)\}$. Create one non-trivial σ -field and the largest σ -field on Q .
2. Let X and Y be i.i. d. random variables each having uniform distribution on the interval $(-\pi, \pi)$. Let $Z(t) = \cos(tX + Y)$, $t \geq 0$. Is $\{Z(t), t \geq 0\}$ wide sense stationary process?
3. Let $\{W(t), t \geq 0\}$ be a Brownian motion. Prove that $\{tW(1/t), t \geq 0\}$ where $tW(1/t)$ is taken to be zero when $t = 0$, is a Brownian motion.
4. Consider two i.i.d random variables X and Y each having uniform distribution between the intervals 0 and 1. Define $Z = X + Y$. Prove that $E(X/Z) = Z/2$
5. Consider $\Omega = \{a, b, c, d\}$. Construct 4 distinct σ -field $\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3, \mathcal{F}_4$ such that $\mathcal{F}_1 \subset \mathcal{F}_2 \subset \mathcal{F}_3 \subset \mathcal{F}_4$.
6. Let X_1, X_2, \dots be i.i. d random variables each having normal distribution with zero mean and unit variance. Show that the sequence $Y_n = \exp\{(\sum_{i=1}^n X_i) - \frac{n}{2}\}$ forms a martingale.
7. Let $\{N(t), t \geq 0\}$ be a Poisson process with parameter 1. Which of the following are martingales.
 - (i) $\{N(t) - t, t \geq 0\}$.
 - (ii) $\{N^2(t) - t, t \geq 0\}$.
 - (iii) $\{N(t) - t\}^2 - t, t \geq 0\}$.
8. Let $\{W(t), t \geq 0\}$ be a Wiener process. Is $\exp\{\sigma W(t) - \frac{\sigma^2}{2}t\}$ a martingale where σ is a positive constant?
9. Find the stochastic differential of $W^2(t)$.
10. Consider a stock whose value $S(t)$ follows sde $dS = r.Sdt + \sigma.SdW$ and has a current price $S(0)$. What is the probability that a call option is in the money based on a strike price $K = 1.25 S(0)$ at time of expiration T ? Given that $T = 0.5, r = 0.04$ and $\sigma = 0.10$.
11. Let Z be a normally distributed random variable, with mean 0 and variance 1, $Z \sim N(0, 1)$. Then consider the continuous time stochastic process $X = \sqrt{t}Z$, Show that the distribution of X is normal with mean 0 and variance t . Is $X(t)$ a Brownian motion?
12. What is the distribution of $W(s) + W(t)$, for $0 \leq s \leq t$?
13. A stock price is currently Rs.50. Assume that the expected return from the stock is 18% per annum and its volatility is 30% per annum. What is the probability distribution for the stock price in two years? Calculate the mean and standard deviation of the distribution. Determine the 95% confidence interval.

