Q 3) In a coloring of G with $\chi(G)$ colours, removing v trivially produces a coloring of $G-v$ with at most $\chi(G)$ colours.

Hence $\chi(G-v) \leq \chi(G)$

If $\chi(G)$ $\chi(G-v) \leq \chi(G) - 2$, then adding the vertex v back produces a coloring of G with at most $\chi(G)-1$ colors (v may need an extra color),

This is a contradiction $\Rightarrow \Leftarrow$

Hence $\chi(G-v) \geq \chi(G) - 1$

Combining these two inequalities produces the desired result.