

Cryptanalysis

- Assumption 1. The adversary, Oscar, knows the cryptosystem being used.
2. We will consider ciphertext only attack.
 3. Plaintext is in ordinary English without punctuation and spaces.

1. Cryptanalysis of Affine Cipher:

FMXVEDKAPHFERBNDKRXRSREFMORUDSDKDVSHVUFEDK
APRKDLYEVLRRHHRH

Affine Cipher:

Encryption: $C = (Pk_1 + k_2) \bmod 26$

letter	frequency	letter	frequency
A	2	N	1
B	1	O	1
C	0	P	2
D	7	Q	0
E	5	R	8
F	4	S	3
G	0	T	0
H	5	U	2
I	0	V	4
J	0	W	0
K	5	X	2
L	2	Y	1
M	2	Z	0

The most frequent ciphertext characters are

R - 8 times
D - 7 "
E, H, K - 5 "
F, S, V - 4 "

Guess: $E_K(e) = R$ & $E_K(t) = D$

$E_K(4) = 17$, & $E_K(19) = 3$

$4k_1 + k_2 = 17 \pmod{26}$, — ①

$19k_1 + k_2 = 3 \pmod{26}$ — ②

Solving ① & ② we get $k_1 = 6$, $k_2 = 19$ ✓ X

Q. Is it a valid key? Ans No

We know that $k_1 \in \mathbb{Z}_{26}^* = \{x \in \mathbb{Z}_{26} \mid \gcd(x, 26) = 1\}$
 $k_2 \in \mathbb{Z}_{26}$

$(6, 19)$ can not be a key.

Quest: $E_k(e) = R$, $E_k(t) = K$

i.e. $E_k(4) = 17$, $E_k(19) = 10$

$$4k_1 + k_2 = 17 \pmod{26} \quad \text{--- ③}$$

$$19k_1 + k_2 = 10 \pmod{26} \quad \text{--- ④}$$

Solving ③ & ④ we get $k_1 = 3$ & $k_2 = 5$

Q. Is it a valid key?

A. Yes

$$D_k(c) = [(c - k_2) \times k_1^{-1}] \pmod{26}$$

$$D_k(c_1) = (5 - 5) \times 9 \pmod{26}$$

$$= 0$$

$$\equiv \underline{a}$$

algorithms are quite general definitions of arithmetic processes

Cryptanalysis of Hill Cipher

Hill Cipher :

$$P \equiv (P_1 P_2 \dots P_m) (P_{m+1} P_{m+2} \dots, P_{2m}) \dots$$

$$C \equiv (C_1 C_2 \dots C_m) (C_{m+1}, \dots, C_{2m}) \dots$$

$$K(\text{key}) = \begin{bmatrix} k_{11} & \dots & k_{1m} \\ \vdots & & \vdots \\ k_{m1} & \dots & k_{mm} \end{bmatrix} \quad \left(K \text{ is the key and it is such that } K^{-1} \text{ exists} \right)$$

$$[C_1, C_2, \dots, C_m] = [P_1, P_2, \dots, P_m] \begin{bmatrix} K \end{bmatrix}$$

Ciphertext only attack is very difficult to implement.

Let's assume that m is known to the adversary Oscar.

Suppose Oscar has at least m distinct plaintext-ciphertext pairs.

$$x_j = x_{1,j}, x_{2,j}, \dots, x_{m,j}, \quad 1 \leq j \leq m$$

$$\& \quad y_j = y_{1,j}, y_{2,j}, \dots, y_{m,j}, \quad 1 \leq j \leq m$$

$$\text{such that} \quad y_j = E_K(x_j) \quad 1 \leq j \leq m$$

$$\text{Let} \quad X = [x_{i,j}] \quad \& \quad Y = [y_{i,j}]$$

Then $Y = X K$ - K is the key i.e. it is an $m \times m$ matrix which is unknown.

$$\text{If } X^{-1} \text{ exists then } K = X^{-1} Y$$

Ex: Plaintext : friday
 Ciphertext : PQCFKU } known to Oscar
 $m = 2$
 Hill Cipher is being used

$$E_K(5, 17) = (15, 16) \quad \text{--- ①}$$

$$E_K(8, 3) = (2, 5) \quad \text{--- ②}$$

$$E_K(0, 24) = (10, 20) \quad \text{--- ③}$$

from ① & ②

$$\begin{bmatrix} 15 & 16 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 5 & 17 \\ 8 & 3 \end{bmatrix} K$$

$$K = \begin{bmatrix} 5 & 17 \\ 8 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 15 & 16 \\ 2 & 5 \end{bmatrix}$$

$$K = \begin{bmatrix} 9 & 1 \\ 2 & 15 \end{bmatrix} \begin{bmatrix} 15 & 16 \\ 2 & 5 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 7 & 19 \\ 8 & 3 \end{bmatrix}}}$$

Book: Cryptography: Theory & Practices by Douglas R.
Stinson

Steganography:

This is technique in which we hide a message inside another message.

1. Invisible ink
2. tiny pin punctures ✓
3. minute variations b/w handwritten characters
4. pencil marks on handwritten char. etc.

Take an image file and replace the last two least significant digits of each pixel of that image with two bits of our message.

Resulting image would not look too different