

b) x2r + 2yxs + y2t = 0

R=n2, S=2ny, T=y2 => S2-4RT =0 (parabolic eurywhere)

à quadratic → R12+81+T=0 22+2xyx+y2=0

solving (ndty)=0 me get d=-y/n, -y/n The characturistic equation are

dylan+ (-yln) = 0 or 1/ydy - 1/ndn=0

guing yln=G or y=c/n

Characteric equation represents family of straighthine passing through origin

a) 32/8x2 - 22/8y=0

R=1, S=0, T=-1 2 quadrate > 22-1-0 hence  $\lambda = 1, -1 \rightarrow \lambda_1 = 1, \lambda_2 = -1$  (Real and distinct)

dy/dx+1=0 and dy/dn-1=0

y-1= 62 y+n=4

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$$y=x+y$$
 and  $y=y-x \longrightarrow 0$ 

$$P = \frac{\partial z}{\partial n} = \frac{\partial z}{\partial n} \cdot \frac{\partial u}{\partial n} + \frac{\partial z}{\partial n} \cdot \frac{\partial v}{\partial n} = \frac{\partial z}{\partial u} \cdot \frac{\partial z}{\partial v}$$
(using 1)

$$r = \frac{\partial^2 z}{\partial u^2} = \frac{\partial}{\partial u} \left( \frac{\partial z}{\partial u} \right) = \left( \frac{\partial}{\partial u} - \frac{\partial}{\partial v} \right) \left( \frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} \right)$$

$$= \frac{\partial}{\partial x} \left( \frac{\partial z}{\partial y} - \frac{\partial z}{\partial y} \right) - \frac{\partial}{\partial y} \left( \frac{\partial z}{\partial y} - \frac{\partial z}{\partial y} \right).$$

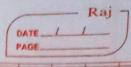
$$= \frac{\partial^2 z}{\partial u} - \frac{2\partial^2 z}{\partial u\partial v} + \frac{\partial^2 z}{\partial v^2} \longrightarrow 2$$

$$t = \frac{\partial^2 z}{\partial y^2} = \left(\frac{\partial}{\partial u} + \frac{\partial}{\partial v}\right) \left(\frac{\partial z}{\partial u} + \frac{\partial z}{\partial v}\right)$$

uling 3 & 8 our canonical form is

$$\frac{\partial^2 z}{\partial u^2} - 2\frac{\partial^2 z}{\partial u \partial v} + \frac{\partial^2 z}{\partial v^2} - \left(\frac{\partial^2 z}{\partial u^2} + 2\frac{\partial^2 z}{\partial u \partial v} + \frac{\partial^2 z}{\partial u^2}\right) = 0$$

$$\Rightarrow \frac{\partial^2 z}{\partial u \partial v} = 0$$



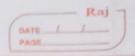
by yo(82/8x2) + n2(82/842) =0 R=y2,S=0, T= n2 => S2-4RT = -4n2y210 for a quedratic equil is  $y^2 / 2 + n^2 = 0$  or  $\chi^2 = -n^2 / y^2$  effiptic > A = ixly, -ixly -> 0 The corresponding characteristic equi an dy +in -o and dy -in -o + y2+in2 = cy and - y2-in2-cz u= y+in =x+ip and v=y-in=x-ip User fration of posterior to converse of mornal where d=ye & B=ye -> 3 p=dz = dz.da+dz.dß = 2ndz by 3 9 - 22 = 22.2x + 22.2k = 2y 2z by 3  $F = \frac{\partial^2 z}{\partial u} = \frac{\partial}{\partial u} \left( \frac{\partial z}{\partial u} \right) = \frac{\partial}{\partial u} \left( \frac{\partial z}{\partial u} \right) = \frac{\partial^2 z}{\partial u} + \frac{\partial z}{\partial u} \frac{\partial}{\partial u} \left( \frac{\partial z}{\partial u} \right)$ 

and

$$\frac{1}{1+\frac{3^2}{3^2}} = \frac{\partial}{\partial y} \left(\frac{32}{3y}\right) = \frac{\partial}{\partial y} \left(\frac{2y}{3z}\right) = 2\frac{\partial z}{\partial x} + 2y\frac{\partial}{\partial y} \left(\frac{\partial z}{\partial x}\right)$$

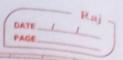
using (4) AG

$$\frac{3}{3} + \frac{3}{4} + \frac{3}$$



and q (yq +z)r-p(2yq+z)s+ypet+peq =0 ming 10 Monge's subsidiary equations are 9(49+2) dpdy + yp2dqdn + pqdndy=0 -0 q(49+2) (dy)2 + p(249+2) dndy + yp2dn)2=0 -> @ on factorizing @ gives (qdy+pdn) {(yq+z)dy +ypdn}=0 Hence 2 systems to be considered are  $\frac{q \left( yq + z \right) dpdy + yp^2 dqdn + p^2qdndy = 0}{q \left( yq + z \right) dpdy + yp^2 dqdn + p^2qdndy = 0} \frac{q dy + p dn = 0 \rightarrow 0}{(yq + z) dy + q dndy = 0}$ using de = pan +qdy she second equation of (3) reduces to dz=0 so that z=4 ->5 From second equity 3 gdy = -pdm, Hence 1st-equation of 3 reduces to or (41+2) dp-pd(49)=0 ( y2+2)dp - ypdq - pqdy=0 00d dz=0 by B or (49+2) dp -pd (49+2)=0 so that log (yq+2) - logp = loge or dlyg+2) - dp =0

yq+2 P



or (9+2)/p = cz, cz being arbitary cometan from 0 + 6 , the intermediate enliged correspond to ( yq+2) p= p,(2) or yq+2=pp,(2) -> @ Using d2 = p dn + qdy, the second equi of (9) become y (ady +pda) tedy 20 or ydz tedy 20 or drysto Integrating it, y= (3, (3 bing arbitrary and. for this fact Using this fact, first equation gas 9 9p - pdg - (pg/y)dy -0 or - (1/p) dp + (1/9) dq + 1/ydy =0 Integrating, - logp + log q + logy = wgc, or 49/p=4 -> 0 from & 40 another intermedial integral corresponding 941 p = 1/2(42), where p2 is arrivary of n -> (10) Solving  $\oplus$  & (10) for p Lq, we have  $P = \frac{2}{\sqrt{q_1(2)} - \varphi_2(y_2)} = \frac{2}{\sqrt{q_1(2)} - \varphi_2(y_2)}$ 

