FUZZY SET THEORY &
FUZZY LOGIC

MC-432

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DTU/2k16/MC/13
10th April' 2021

ASSIGNMENT- III

Q1) $\tilde{J}(x, z)$ $(f(x), s(x), t(x))$

$f(x) = x^2$ $s(x) = \frac{x}{4}$ $t(x) = \frac{x}{2}$

$L(x) = \frac{1}{1+x^2}$ $R(x) = \frac{1}{1+2|x|}$

$L(x)$ and $R(x)$ have no effect on value of integral of such fuzzy function

$$\int_1^4 \tilde{f}(x)dx = \int_a^b f(x)\,dx = \int_1^4 x^2\,dx = \left(\frac{x^3}{3}\right|_1^4 = 21$$

$$\int_a^b s(x)dx = \int_1^4 \frac{x}{4}\,dx = \left(\frac{x^2}{8}\right|_1^4 = 1.875$$

$$\int_a^b t(x)\,dx = \int_1^4 \frac{x}{2}\,dx = \frac{x^3}{6}\Big|_1^4 = 3.75$$

Hence $\tilde{I}(a,b) = (21, 1.875, 3.75)_{LR}$

2) $\tilde{a} = \{(4, 0.8), (5, 1), (6, 0.4)\}$ $\tilde{b} = \{(6, 0.7), (7, 1), (8, 0.2)\}$

$f(x) = 2$ $x \in [2, 4]$

$[a,b]$	$\int_a^b f(x)\,dx = \int_a^b 2\,dx$	$\min(\mu_a(a), \mu_B(b))$
$[4,6]$	4	0.7
$[4,7]$	6	0.8
$[4,8]$	8	0.2
$[5,6]$	2	0.7
$[5,7]$	4	1
$[5,8]$	6	0.2

[6,6]	0	0.4
[6,7]	2	0.4
[6,8]	4	0.2

$\tilde{I}(A,B) = \{(0,0.4), (2,0.7), (4,1), (6,0.8), (8,0.2)\}$

Q3) $f(x) = 2x - 3$ $g(x) = 2x + 5$

$\tilde{a} = \{(1,0.8), (2,1), (3,0.4)\}$

$\tilde{b} = \{(3,0.7), (4,1), (5,0.3)\}$

$f \circ g = 2(2x+5) - 3 = 4x + 10 - 3 = 4x + 7$

$g \circ f = 2(2x-3) + 5 = 4x - 6 + 5 = 4x - 1$

i) [a,b]	$\int_a^b f(x) = \int_a^b 2x - 3 = x^2 - 3x \Big|_a^b$	$\min\{\mu_A(a), \mu_B(b)\}$
[1,3]	2	0.7
[1,4]	6	0.8
[1,5]	12	0.3
[2,3]	2	0.7
[2,4]	6	1
[2,5]	12	0.3
[3,3]	0	0.4
[3,4]	4	0.4
[3,5]	10	0.3

$\tilde{I}(A,B) = \{(0,0.4), (2,0.7), (6,1), (12,0.3), (4,0.4), (10,0.3)\}$

ii) $[a,b]$ | $\int_a^b f(x)\,dx = \int_a^b 2x+5 = x^2+5x \big|_a^b$ | $\min\{\mu_A(a), \mu_B(b)\}$

$[a,b]$	$\int_a^b 2x+5$	$\min\{\mu_A(a), \mu_B(b)\}$
[1,3]	18	0.7
[1,4]	30	0.8
[1,5]	44	0.3
[2,3]	10	0.7
[2,4]	22	0.1
[2,5]	36	0.3
[3,3]	0	0.4
[3,4]	12	0.4
[3,5]	26	0.3

$\tilde{I}(A,B) = \{(0,0.4), (10,0.7), (12,0.4), (18,0.7), (22,1), (26,0.3), (30,0.8), (36,0.3), (44,0.3)\}$

iii) iv)

$[a,b]$	$\int_a^b 4x+7 = 2x^2+7x \big|_a^b$	$\int_a^b 4x-1 = 2x^2-1 \big|_a^b$	$\min\{\mu_A(a), \mu_B(b)\}$
[1,3]	30	14	0.7
[1,4]	51	27	0.8
[1,5]	76	44	0.3
[2,3]	17	9	0.7
[2,4]	38	22	1
[2,5]	63	39	0.3

[3,3]	0	0	0.4
[3,4]	21	13	0.4
[3,5]	46	30	0.3

iii) $\tilde{I}(A,B) = \{(0,0.4),(17,0.7),(21,0.4),(37,0.7),(38,1),(46,0.3),$
$(51,0.8),(63,0.3),(76,0.3)\}$

iv) $\tilde{I}(A,B) = \{(0,0.4),(9,0.7),(13,0.4),(14,0.7),(22,1),(30,0.3),$
$(27,0.8),(39,0.3),(44,0.3)\}$

4) $f(x) = x^3 \quad \tilde{x} = \{(-1,0.4),(0,1),(1,0.6)\}$

$f'(x) = 3x^2$

Using extention principle

$f'(\tilde{x}) = \{(0,1),(3,0.6)\}$

5) $f(x) = x^2+1 \quad g(x) = 2-x$

$\tilde{a} = (1,2,3)$

$\tilde{b} = (3,4,5)$

6) $\tilde{x} = \{(-1,0.4),(0,0.1),(1,0.6)\}$

$f(x) = x^3+2 \quad g(x) = 2x+3$

$f'(x) = 3x^2 \quad g'(x) = 2$

$f'(x) + g'(x) = 3x^2+2$

$f(x) + g(x) = x^2 - \sqrt{3}x + 5$

$f'(\tilde{x}_0) = \{(3, 0.6)(0, 0.1)\}$

$g'(x) = \{(2, 0.6)\}$

$f'(x) + g'(x) = \{(5, 0.6), (2, 0.1)\}$

$(f' + g')(x) = \{(5, 0.6), (2, 0.1)\}$

Hence $(f' + g')(\tilde{x}_0) = f'(\tilde{x}_0) + g'(\tilde{x}_0)$

Q7) $f(x_0) = \{(-2 + (-1 \cdot 1), \cdot 5), (-1, 0.8), (2 + (-1 \cdot 1), 1), ((6 + (2 \cdot 1), 0.8),$
$(54 + (3 \cdot 1), 0.4)\}$

$= \{(-4, 0.5), (-1, 0.8), (2, 1), (17, 0.8), (56, 0.4)\}$

$g(x_0) = \{(-2, 0.5), (-1, 0.8), (0, 1), (13, 0.8), (50, 0.4)\}$

LHS $= \{(-6, 0.5), (-3, 0.5), (0, 0.5), (15, 0.5), (54, 0.4), (-5, 0.5),$
$(-2, 0.8), (1, 0.8), ((6, 0.8), (55, 0.4), (-4, 0.5), (-1, 0.8),$
$(2, 1), (17, 0.8), (56, 0.9), (9, 0.5), (12, 0.8), (15, 0.8), (30, 0.8),$
$(69, 0.4), (46, 0.4), (49, 0.4), (52, 0.4), (67, 0.4), (106, 0.4)\}$

RHS $= \{(-6, 0.5), (-2, 0.8), (2, 1), (30, 0.8), (106, 0.4)\}$

Since RHS $\subseteq$ LHS it is proved.

Q8) $f(x) = \begin{cases} 2x^2 - 3 & -2 \le x \le 2 \\ 5 & \text{otherwise} \end{cases}$

$\mu(x) = \dfrac{f(x) - \inf(x)}{\sup(x) - \inf(x)}$ $\quad$ $\sup(x) = 5$, $\inf(x) = -3$

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So for

i) $2 \geq x \geq -2$

$$\mu_M(x) = \frac{2x^2-3+3}{5-(-3)} = \frac{2x^2}{8} = \frac{x^2}{4}$$

ii) otherwise

$$\mu(x) = \frac{5+3}{5-3} > 1$$

Q9) $f(x) = \sin(\sin x) \quad x \in \mathbb{R}$

$\sup(x) = \sin(1)$

$\inf(x) = -\sin(1)$

$$\mu_{max}(x) = \frac{f(x) - \inf(x)}{\sup(x) - \inf(x)}$$

$$\frac{\sin(\sin x) + \sin(1)}{2\sin(1)}$$

$\mu_{max}(0) = 1/2$

$\mu_{max}(\pi/2) = 1$

$\mu_{max}(-\pi/2) = 0$

Q10) i) Yes

Classical functions can be defined as a fuzzy function mapping from crisp to crisp set with membership value 1.

ii) Yes,

We can define fuzzy functions $f: X \to \tilde{P}(Y)$ where $\tilde{P}(Y)$ is fuzzy power set of Y and $\mu_{f(x)}(y) = \mu_R(x,y) \ \forall x,y \in X \times Y$

where R is a fuzzy relationship.

iii) Yes

If M is the maximising set of fuzzy function "f" then we say we get maximum value of "f" at x_0 if :-

i) $\mu_M(x_0)$ is maximum

ii) $\mu_D(x_0)$ is maximum

↳ denotes membership of x_0 in domain

v) Yes

vi) Yes

Integration of crisp function over fuzzy domain

$\int_{\tilde{a}}^{\tilde{b}} f(x)\,dx = F(\tilde{b}) - F(\tilde{a})$ This subtraction is extended subtraction and done with conformation to extension principle.

Integration of fuzzy function over crisp domain :

$I(a,b) = \int_a^b g_\alpha(x)\,dx - \int_a^b f(x)\,dx$ where f_α^+ and f_α^{-1} are α-level

curves for fuzzy functions f'

⇒ Since this is defined according to α-cuts, it is also in conformity with extension principle.

vii) NO

The condition holds only if $\int_a^b \tilde{f}(x)\,dx$ and $\int_a^b \tilde{g}(x)\,dx$ are commutative

Generally, $\int_a^b (\tilde{f} \oplus \tilde{g})\,dx = \int_a^b f(x)\,dx \oplus \int_a^b g(x)\,dx$

viii) No

The following condition is always true :-

$$\int_a^{\tilde{b}} (f+g)(x)\, dx \subseteq \int_a^{\tilde{b}} f(x)\, dx + \int_a^{\tilde{b}} g(x)\, dx$$ [Crisp f^n over fuzzy domain]

ix) No

The given condition is true for fuzzy function over crisp domain but not for crisp f^n over fuzzy domain

$$\int_a^c \tilde{f}(x)\, dx = \int_a^b \tilde{f}(x)\, dx + \int_b^c \tilde{f}(x)\, dx$$

x) Yes

For question 6, conditions were true

For question 7, condition was not true

11) $f(x) = x^4 + x^2 - 1$

$\{(1, 0.2), (2, 0.6), (3, 1), (4, 0.6), (5, 0.2)\}$

$f(\text{around } 3) = \{(f(1), 0.2), (f(2), 0.6), (f(3), 1), (f(4), 0.6),$
$(f(5), 0.2)\}$

$= \{(1, 0.2), (19, 0.6), (89, 1), (271, 0.6), (649, 0.2)\}$

12) $P(\tilde{A})_{true} = \{(0.5, 0.6), (0.6, 0.7), (0.7, 0.8), (0.8, 0.9), (0.9, 1), (1, 1)\}$

$P(\tilde{B})_{true} = \{(0.2, 0.3), (0.3, 0.8), (0.5, 0.1), (0.7, 0.4), (0.8, 0.9),$
$(0.9, 0.7)\}$

a) $P(\tilde{A})_{negation} = \{(0, 1), (0.1, 1), (0.2, 1), (0.3, 1), (0.4, 1), (0.5, 0.4), (0.6, 0.3),$
$(0.7, 0.2), (0.8, 0.1)\}$

b) $P(\tilde{A})_{\text{very true}} = \{(0.5, 0.36), (0.6, 0.49), (0.7, 0.64), (0.8, 0.81), (0.9, 1), (1,1)\}$

c) $P(\tilde{A}) \wedge P(\tilde{B}) = \{(0.5, 0.1), (0.6, 0), (0.7, 0.4), (0.8, 0.9), (0.9, 0.7)\}$

d) $P(\tilde{A})_{\text{very very true}} = \{(0.5, 0.216), (0.6, 0.343), (0.7, 0.324), (0.8, 0.729), (0.9, 1), (1,1)\}$

f) $P(\tilde{A}) \vee P(\tilde{B}) = \{(0.2, 0.3), (0.3, 0.8), (0.5, 0.6), (0.6, 0.7), (0.7, 0.8), (0.8, 0.9), (0.9, 1), (1,1)\}$

g) $P(\tilde{A}) \rightarrow P(\tilde{B}) = \{(0, 1), (0.1, 1), (0.2, 1), (0.3, 1), (0.4, 1), (0.5, 0.4), (0.6, 0.3), (0.7, 0.4), (0.8, 0.9), (0.9, 0.7)\}$

13) Using Lukasiewicz implication

$I(a,b) = \min(1, 1-a+b)$

$$\tilde{R} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{bmatrix} 1 & 0.9 \\ 1 & 0.4 \\ 1 & 0.8 \end{bmatrix} \end{array}$$

$B' = \{(y_1, 0.9)\ (y_2, 0.7)\}$

Using compensational rules of influence

$$\mu_B(y_1) = \sup_{x \in X} \min[\mu_B(x), \mu_R(x, y_1)]$$

$\max(\min(0.6, 1), (0.9, 1)\ \min(0.7, 1)\}$

$= 0.9$

14) In this case

$$\mu_{A'}(x_1) = \sup_{y \in Y} \min[\mu_B(y), \mu_R(y)] = \max\{\min(0.9, 1), \min(0.7, 0.9)\} = 0.9$$

$$\mu_{A'}(x_2) = \sup_{y \in Y} \min[\mu_B(y), \mu_R(x_2, y)] = 0.9$$

$$\mu_{A'}(x_3) = \sup_{y \in Y} \min[\mu_B(y), \mu_R(x_3, y)] = 0.9$$

Hence x is $\tilde{A} = \{(x_1, 0.9)(x_2, 0.9), (x_3, 0.9)\}$

15) $A = \{(x_1, 0.5)(x_2, 1)(x_3, 0.6)\}$

$\hat{B} = \{(y_1, 0.1), (y_2, 0.4)\}$

$\hat{C} = \{(z_1, 0.2), (z_2, 0.1)\}$

$$I(a,b) = \begin{cases} 1 & \text{if } a \le b \\ b & \text{if } a > b \end{cases}$$

$$\tilde{R}_1 = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{bmatrix} 1 & 0.4 \\ 1 & 0.4 \\ 1 & 0.4 \end{bmatrix} \end{array}$$

$$\tilde{R}_2 = \begin{array}{c} \\ y_1 \\ y_2 \end{array} \begin{array}{c} \begin{array}{cc} z_1 & z_2 \end{array} \\ \begin{bmatrix} 0.2 & 1 \\ 0.2 & 1 \end{bmatrix} \end{array}$$

$$\tilde{R}_3 = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cc} z_1 & z_2 \end{array} \\ \begin{bmatrix} 1 & 0.4 \\ 1 & 0.4 \\ 1 & 0.4 \end{bmatrix} \end{array}$$

$$\hat{R}_1 \circ \tilde{R}_2 = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cc} z_1 & z_2 \end{array} \\ \begin{bmatrix} 1 & 0.4 \\ 1 & 0.4 \\ 1 & 0.4 \end{bmatrix} \end{array}$$

Thus generalized hypothetical syllogism holds.

(7) i) $(a \wedge b) \Rightarrow c$

a	b	$a \wedge b$	c	$a \wedge b \Rightarrow c$
0	0	0	1	1
0	1/2	1/2	1	1
0	1	1	1	1
1/2	0	1/2	1/2	1
1/2	1/2	1/2	1/2	1
1/2	1	1	1/2	1/2
1	1/2	1	0	0
1	1	1	0	0

ii) $(a \vee b) \Leftrightarrow (a \wedge b)$

a	b	$\vee$	$\wedge$	$(a \vee b) \Leftrightarrow (a \wedge b)$
0	0	0	0	1
0	1/2	0	1/2	1/2
0	1	0	1	0
1/2	0	0	1/2	1/2
1/2	1/2	1/2	1/2	1
1/2	1	1/2	1	1/2
1	0	0	1	0
1	1/2	1/2	1	1/2
1	1	1	1	1

iii) $(a \Rightarrow b) \rightarrow (a \Rightarrow b)$

a	b	$\Rightarrow$	$(a \Rightarrow b) \rightarrow (a \Rightarrow b)$
0	0	1	1
0	1/2	1	1
0	1	1	1
1/2	0	1/2	1
1/2	1/2	1	1
1/2	1	1	1
1	0	0	1
1	1/2	1/2	1
1	1	1	1

Q18) True = {(0.5, 0.3), (0.6, 0.6), (0.7, 0.8), (0.8, 0.9), (0.9, 0.9), (1, 1)}

False = {(0, 1), (0.1, 0.9), (0.2, 0.8), (0.3, 0.6), (0.4, 0.2), (0.5, 0.1)}

Not true = $\overline{True}$ = {(0, 1) (0.1, 1), (0.2, 1), (0.3, 1), (0.4, 1), (0.5, 0.7), (0.6, 0.4), (0.7, 0.2), (0.8, 0.1), (0.9, 0.1), (1, 0.9)}

Neither true nor false = $\overline{(True \wedge False)}$

True ∪ False = {(0, 1) (0.1, 0.7) (0.2, 0.8), (0.3, 0.6), (0.4, 0.2), (0.5, 0.3), (0.6, 0.6), (0.7, 0.8), (0.8, 0.9), (0.9, 0.9), (1, 0.1)}

Neither true nor false = $\overline{(True \cup False)}$ =

{(0.1, 0.1) (0.2, 0.2), (0.3, 0.4), (0.4, 0.8), (0.5, 0.7), (0.6, 0.4), (0.7, 0.2), (0.8, 0.1) (0.9, 0.1) (1, 0.9)}

Very true = $\overline{\text{True} - \text{False}}$ = True $\wedge$ $\overline{\text{False}}$

$\overline{\text{False}}$ = $\{(0.1, 0.1)\ (0.2, 0.2)\ (0.3, 0.4)\ (0.4, 0.8)\ (0.5, 0.9), (0.6, 1)$
$(0.7, 1)\ (0.8, 1)\ (0.9, 1)\ (1, 1)\}$

Almost true, not false = True $\vee$ $\overline{\text{False}}$

$= \{(0.1, 0.1), (0.2, 0.2), (0.3, 0.4), (0.4, 0.5), (0.5, 0.9), (0.6, 1)$
$(0.7, 1), (0.8, 1)\ (0.9, 1)\ (1, 1)\}$

20) $\mu_{RT}(A'/A)(0.8) = \mu_A(x_1) = 0.9$

$\mu_{RT}(A'/A)(0.6) = \mu_A(x_2) = 0.8$

$\mu_{RT}(A'/A)(0.5) = \mu_A(x_3) = 0.5$

$\mu_{RT}(A'/A)(0.4) = \mu_A(x_4) = 0.5$

Using Lukasiewicz Fuzzy implication

$I(a, b) = \min(1, 1 - a + b)$

$\mu_{RT}(B'/B)(b) = \max\{\min(0.9, S(I(0.9, b)))$
$\min(0.8, S(I(0.8, b)))$
$\min(0.5, S(I(0.5, b)))$
$\min(0.5, S(I(0.5, b)))\}$

$\mu_B(y_1) = \mu_{RT}(B'/B)(B(y_1)) = 0.5$

$\mu_B(y_2) = \mu_{RT}(B'/B)(B(y_2)) = 0.7$

$\mu_B(y_3) = \mu_{RT}(B'/B)(B(y_3)) = \mu_{RT}(B'/B)(0.6) = 0.8$

$B = \{(y_1, 0.5)\ (y_2, 0.7)\ (y_3, 0.8)\}$

Q2 1) $X = \{1,2,3,4\}$ $\tilde{A} = \{(1,0),(2,0.2),(3,0.6),(4,1)\}$

$$\tilde{R} = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 0.5 & 0 & 0 \\ 2 & 0.5 & 1 & 0.5 & 0 \\ 3 & 0 & 0.5 & 1 & 0.5 \\ 4 & 0 & 0 & 0.5 & 1 \end{array}$$

$\tilde{B} = \tilde{A} \circ \tilde{R}$

$$\begin{array}{c|cccc} & y_1 & y_2 & y_3 & y_4 \\ \hline x_1 & 0 & 0.2 & 0.6 & 1 \end{array} \circ \begin{array}{c|cccc} & z_1 & z_2 & z_3 & z_4 \\ \hline y_1 & 1 & 0.5 & 0 & 0 \\ y_2 & 0.5 & 1 & 0.5 & 0 \\ y_3 & 0 & 0.5 & 1 & 0.5 \\ y_4 & 0 & 0 & 0.5 & 1 \end{array}$$

for $x = x_1$, $z = z_1$

$\mu(x_1, z_1) = \max\{\min(0,1), \min(0.2,0.5), \min(0.6,0), \min(1,0)\}$

$\max\{0, 0.2, 0, 0\} = 0.2$

for $x = x_1$, $z = z_2$

$\mu(x_1, z_2) = \max\{\min(0,0), \min(0.2,0.5), \min(0.6,0.5) \min(1,0)\}$

$\max\{0, 0.2, 0.6, 0.5\} = 0.5$

for $x = x_1$, $z = z_3$

$\mu(x_1, z_3) = \max\{\min(0,0) \min(0.2,0.5) \min(0.6,1) \min(1,0.5)\}$

$\max\{0, 0.2, 0.6, 0.5\} = 0.6$

for $x = x_1$, $z = z_4$

$\mu(x_1, z_4) = \max\{\min(0,0) \min(0.2,0) \min(0.6,0.5) \min(1,1)\}$

$\max\{0, 0, 0.5, 1\} = 1$

$\Rightarrow \tilde{B} = \{(1,0.2),(2,0.5),(3,0.6),(4,1)\}$

Q22) $Y = \{1,2,3,4,5\}$ $\hat{A} = \{(1,1)\ (2,0.5)\ ,(3,0.4),(4,0.2),(5,0)\}$

μ of $\tilde{A} \circ \tilde{R}$ be defined by $\tilde{B}$ matrix

$\tilde{B} = \tilde{A} \circ \tilde{R}$

$$\begin{array}{c} \\ x_1 \end{array} \begin{array}{ccccc} y_1 & y_2 & y_3 & y_4 & y_5 \\ \left[1 \right. & 0.5 & 0.4 & 0.2 & \left. 0 \right] \end{array} \circ \begin{array}{c} \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{array} \begin{array}{ccccc} z_1 & z_2 & z_3 & z_4 & z_5 \\ \left[1 \right. & 0.8 & 0 & 0 & 0 \\ 0.8 & 1 & 0.8 & 0 & 0 \\ 0 & 0.8 & 1 & 0.8 & 0 \\ 0 & 0 & 0.8 & 1 & 0.8 \\ 0 & 0 & 0 & 0.8 & \left. 1 \right] \end{array}$$

for $x = x_1$ $z = z_1$

$\mu(x_1, z_1) = \max\{\min(1,1)\ \min(0.5,0.8)\ \min(0.4,0)\ \min(0.2,0)\ \min(0,0)\}$

$\max\{1, 0.5, 0, 0, 0\} = 1$

$\mu(x_1, z_2) = \max\{\min(1,0.8)\ \min(0.5,1)\ \min(0.4,0.8)\ \min(0.2,0)\ \min(0,1)\}$

$\max\{0.8, 0.5, 0.4, 0, 0\} = 0.8$

$\mu(x_1, z_3) = \max\{\min(1,0)\ \min(0.5,0)\ \min(0.4,0.8), \min(0.4,1)\ \min(0.2,1)$
$\min(0, 0.8)\}$

$\max\{0, 0, 0.4, 0.2, 0\} = 0.4$

$\mu(x_1, z_4) = \max\{\min(1,0)\ \min(0.5,0)\ \min(0.4,0)\ \min(0.2,0.8)\ \min(0,1)\}$

$\max\{0, 0, 0, .2, 0\} = 0.2$

$\Rightarrow \tilde{B} = \{(1,1), (2,0.8)\ (3,0.5)\ (4,0.4)\ (5,0.2)\}$

Q23) $\hat{A} = \{(100, 0.5)\ (120, 0.7)\ (140, 0.8)\ (160, 1)\}$

$\tilde{B} = \{(10, 0.6)\ (12, 0.8)\ (15, 1)\}$

Let index of attractive car for different combinations of mileage and top speed is denoted by relation $\tilde{R}$.

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$$\tilde{R} = \begin{array}{c} 10 \\ 12 \\ 15 \end{array} \begin{bmatrix} 0.6 \\ 0.8 \\ 1 \end{bmatrix} \circ \begin{array}{cccc} 100 & 120 & 140 & 160 \\ \end{array}$$
$$\begin{bmatrix} 0.5 & 0.7 & 0.8 & 1 \end{bmatrix}$$

$$= \begin{array}{c|cccc} & 100 & 120 & 140 & 160 \\ \hline 10 & 0.5 & 0.6 & 0.6 & 0.6 \\ 12 & 0.5 & 0.7 & 0.8 & 0.8 \\ 15 & 0.5 & 0.7 & 0.8 & 1 \end{array}$$

Thus mileage for most attractive car = 15 kmh^{-1}

Top speed for most attractive car = 160 kmh^{-1}

Q24) $\tilde{A} = \{(1, 0.7)(2, 0.4)(3, 0.6)(4, 0.5)(5, 0.8)(6, 0.2)\}$

$\tilde{B} = \{(1, 0.8)(2, 0.5)(3, 0.6)(4, 0.6)(5, 0.4)(6, 0.1)\}$

i) Zadeh's Maximum

$t(x_i \text{ is } \tilde{A} \rightarrow y_j \text{ is } \tilde{B}) = ((1 - \mu_A(x_i)) \vee (\mu_A(x_i) \wedge \mu_B(y_j))$

$$t = A \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} B \\ \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0.7 & 0.5 & 0.6 & 0.6 & 0.7 & 0.3 \\ 0.6 & 0.6 & 0.6 & 0.6 & 0.6 & 0.6 \\ 0.6 & 0.5 & 0.6 & 0.6 & 0.6 & 0.4 \\ 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.8 & 0.5 & 0.6 & 0.6 & 0.8 & 0.2 \\ 0.8 & 0.8 & 0.8 & 0.8 & 0.8 & 0.8 \end{bmatrix} \end{array}$$

2) Standard Sequence

$$t(x_i \text{ is } \tilde{A} \rightarrow y_j \text{ is } \tilde{B}) = \begin{cases} 1 & \mu_A(x_i) \le \mu_B(y_j) \\ 0 & \text{otherwise} \end{cases}$$

$$t = \begin{array}{cc} & \text{B} \\ & \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \text{A} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} & \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

3) Goguen's Fuzzy Implication

$$t(x_i \text{ is } \tilde{A} \rightarrow y_j \text{ is } \tilde{B}) = \begin{cases} 1 & \mu_A(x_i) \le \mu_B(y_j) \\ \dfrac{\mu_B(y_j)}{\mu_A(x_i)} & \text{otherwise} \end{cases}$$

$$t = \begin{array}{cc} & \text{B} \\ & \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \text{A} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} & \begin{bmatrix} 1 & 7/5 & 7/6 & 7/6 & 1 & 7 \\ 2 & 1 & 1 & 1 & 1 & 4 \\ 1 & 6/5 & 1 & 1 & 1 & 6 \\ 1 & 1 & 1 & 1 & 1 & 5 \\ 1 & 8/5 & 4/3 & 4/3 & 1 & 8 \\ 1 & 1 & & 1 & 1 & 2 \end{bmatrix} \end{array}$$

Conclusion is not same in cases.