Q3) Prove that if short sales are not allowed then the risk of the portfolio can not exceed the greater of the risks of the individual components of the portfolio.

Let us assume $\sigma_1^2 \leq \sigma_2^2$. If short sales are not allowed then w_1 and $w_2 \geq a$ and

$$w_1 \sigma_1 + w_2 \sigma_2 \leq (w_1 + w_2) \sigma_2 = \sigma_2$$

Since the correlation coefficient satisfies

$$\rho_1 \leq \rho_{12} < 1 \quad \text{it follows that}$$

$$w \sigma_2^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2$$

$$\leq w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \sigma_1 \sigma_2$$

$$(w_1^2 \sigma_1^2 + w_2 \sigma_2)^2 \leq \sigma_2^2$$

If $\sigma_1^2 \geq \sigma_2^2$, this is a contradiction, and hence our initial of way false.

Hence, the risk of portfolio can never exceed the individual components.