Q7) Consider 3 risky assets with the variance-covariance matrix and expected returns (all the data in %) as follows

Variance-covariance Matrix (C)			Returns (M)
10	4	0	5
4	12	6	6
0	6	10	1

Find 2 efficient portfolios. Also construct portfolio giving 2.8% return with minimum risk. Will this portfolio be efficient?

Taking $\alpha = 0$, $\beta = 1$ we need to solve $\sum \sigma_{ij} v_j^{(1)} = 1$

resulting in the following system of linear equations

$$10 v_1^{(1)} + 4 v_2^{(1)} = 1$$
$$4 v_1^{(1)} + 12 v_2^{(1)} + 6 v_3^{(1)} = 1$$
$$6 v_2^{(1)} + 10 v_3^{(1)} = 1$$

The solution $v^{(1)} = (1/10, 0, 1/10)$

We now take $\alpha = 1$, $\beta = 0$ to solve $\sum \sigma_{ij} v_j^{(2)} = \mu_i \ (i = 1, 2, 3)$

$$10 v_1^{(2)} + 4 v_2^{(2)} = 5$$
$$4 v_1^{(2)} + 12 v_2^{(2)} + 6 v_3^{(2)} = 6$$
$$6 v_2^{(2)} + 10 v_3^{(2)} = 1$$

Solving, we get $v^{(2)} = (3/10, 1/2, -1/5)^T$

Now, normalizing $v^{(1)}$, we get

$w^{(1)} = v^{(1)}_{norm} = (1/2, 0, 1/2)$

Normalizing $v^{(2)}$ we get :-

$w^{(2)} = Norm(v^{(2)}) = (1/2, 5/6, -1/3)^T$

The corresponding returns from the 2 portfolios with weight $w_1^{(1)}$ and $w^{(2)}$ is

$\bar{\mu}_1^{(1)} = m^T w^{(1)} = [5 \ 6 \ 1]^T [1/2, 0, 1/2]$

$= 3\%$

$\boxed{\bar{\mu}^{(1)} = 3\%}$

$\bar{\mu}^{(2)} = m^T w^{(2)} = [5 \ 6 \ 1][1/2 \quad 5/6 \quad -1/3]^T$

$\boxed{\bar{\mu}^{(2)} = 7.16\%}$

Next we consider a case when the investor desires a return of $\mu = 2.8\%$ with minimum risk.

$\lambda \bar{\mu}^{(1)} + (1-\lambda)\bar{\mu}^{(2)} = 2.8$

$\lambda(\bar{\mu}^{(1)} - \bar{\mu}^{(2)}) = 2.8 - \bar{\mu}^{(2)}$

$\lambda = \dfrac{2.8 - \bar{\mu}^{(2)}}{\bar{\mu}^{(1)} - \bar{\mu}^{(2)}} = \dfrac{2.8 - 7.16}{3 - 7.16}$

$\boxed{\lambda = 1.048}$

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Thus by the 2-fund theorem the requisite portfolio is given by :-

$$w = \lambda w^{(1)} + (1-\lambda) w^{(2)}$$

$$= 1.048 \left[1/2, 0, 1/2 \right]^T + (-0.048) \left[1/2 \ 5/6 \ -1/3 \right]^T$$

$$\left[1 \quad -0.04 \quad 27/50 \right]$$

$$(-1/25)$$

$$\boxed{w = \left[1 \quad -1/25 \quad 27/50 \right]}$$

This is not the most efficient portfolio.