FINANCIAL ENGINEERING [MC-306]

ASSIGNMENT.

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2K16/MC/013.

Q1. $B(0) = 100$: current bond price

$B(1) = 110$: after 1 period

$S(0) = 80$: current stock price

$V(t) = xS(t) + yB(t)$

$V(0) = 10{,}000/-$

Let investors hold x stock shares & y bond shares.

$$x = 10000 \times \frac{3}{5} \times \frac{1}{80} = 75$$

$$y = 10000 \times \frac{2}{5} \times \frac{1}{100} = 40$$

$(75, 40) \rightarrow$ portfolio

$(10000) \rightarrow$ value of portfolio at $t=0$

$$S(1) = \begin{cases} 100 & , p = 0.8 \\ 60 & , p = 0.2 \end{cases}$$

$$V(1) = \begin{cases} 11900 & , p = 0.8 \\ 8900 & , p = 0.2 \end{cases}$$

$$\text{Return} = \frac{V(1) - V(0)}{V(0)}$$

$$E(G) = 0.19 \times 0.08 - 0.11 \times 0.2$$

$$= 0.16 \rightarrow \text{expected return.}$$

Risk $\quad (\sigma_0) = \sqrt{(0.19 - 0.16)^2 \times 0.8 + (-0.11 - 0.16)^2 \times 0.2}$

$$= 0.123$$

Q2 $B(0) = 90$ $S(0) = 25$

$B(1) = 100$

$$S(1) = \begin{cases} 30 & ; p \\ 20 & ; 1-p \end{cases}$$

Portfolio : (10, 15) value of portfolio at $t=0$

$$V(0) = 10 \times 925 + 15 \times 190$$
$$= 250 + 1350$$
$$= 1600$$

$$V(1) = \begin{cases} (10 \times 30 + 55 \times 100) = 1800 \\ (10 \times 20 + 15 \times 100) = 1700 \end{cases}$$

$$K.V = \begin{cases} 0.125 \\ 0.0625 \end{cases}$$

Expected return $= 0.125p + 0.0625q$

Q3. $B(0) = Rs\ 100$

$B(1) = Rs\ 110$

$S(0) = Rs\ 80$

$$S(1) = \begin{cases} 100 & p = 0.8 \\ 60 & p = 0.2 \end{cases}$$

$V(0) = 10000$

$$x = \frac{5000}{80} = 62.5$$

$$y = \frac{5000}{600} = 50$$

$$V(1) = \begin{cases} 62.5 \times 100 + 50 \times 110 = 11750/- \\ 62.5 \times 60 + 50 \times 110 = 9250/- \end{cases}$$

$$K(0) = \begin{cases} 0.175 \\ -0.075 \end{cases}$$

$$E(K(0)) = 0.175 \times 0.8 - 0.075 \times 0.2$$
$$= 0.14 - 0.015$$
$$= 0.125 \text{ (Expected return)}$$

$$Risk(\sigma_0) = \sqrt{(0.175 - 0.125)^2 \times 0.8 + (-0.025 - 0.125)^2 \times 0.2}$$

$$= 0.084$$

Q4. $B(0) = 90$ $\quad S(0) = 25$

$B(1) = 100$

$S(1) = \begin{cases} 30 & \text{with probability } p \\ 20 & \text{" } \quad \text{" } \quad 1-p \end{cases}$

$V(1) = \begin{cases} 1160 & \text{" } \quad \text{" } \quad p \\ 1040 & \text{" } \quad \text{" } \quad 1-p \end{cases}$

portfolio $= (x, y)$

x → no. of shares; y → no. of bonds.

$x \times 30 + y \times 100 = 1160$

$x \times 20 + y \times 100 = 1040$

$10x = 120$

$x = 12$ —①

$y = \frac{1160 - 360}{100} = 8$ —②

from ① & ②,

$V(0) = (12 \times 25 + 8 \times 90)$

$= 1020/-$

Qs. Strike price = Rs 100

$$S(1) = \begin{cases} 100 & , P = 0.8 \\ 60 & , P = 0.2 \end{cases}$$

$$C(1) = \begin{cases} 100 - 100 = 0 \\ 0 \end{cases}$$

Let portfolio be (x, y)

$$V_p(1) = xS(1) + yB(1) = C(1)$$

$$100x + 110y = 0 \quad , P = 0.8$$
$$60x + 110y = 0 \quad , P = 0.2$$

We get $x = 0, y = 0$

Thus payoff = 0 $\qquad C(0) = 0$

$$P(1) = \begin{cases} 0 & , P = 0.8 \\ 40 & , P = 0.2 \end{cases}$$

$$V_p(1) = xS(1) + yB(1) = P(1)$$

$$= 100x + 110y = 0$$
$$60x + 110y = 40$$
$$40x = -40 \qquad \boxed{x = -1}$$

$$\boxed{y = 11/10}$$

$$v(0) = -1(80) + 100$$

Let 450 be invested in pet & 450 in call.

$\frac{450}{30} = 15$ per put. $x = 0$ for call $\qquad C(0) = 0$

$$v(1) = 30(0.00 + 40 \cdot 0.2) = 120$$

Thus, wealth = $450 + 450 + 120 = 1020$.

Ans: **Non-Arbitrage Principle:**

There is no admissible portfolio with initial value $v(0) = 0$ s.t $v(4) \geq 0$ with non-zero prob.

Suppose $v(0) = 0$.

₹ 10000 is borrowed from bank.

① We will buy pounds from dealer B we get,

$\frac{10000}{80}$ = 125 pounds.

② Investing it in bank for 1yr, we get

$(125 + 125 \times 0.06)$ pound = 132.5 pound.

③ We will sell the pound for Rs to dealer A we get,

₹ (132.5×79) = ₹ 10467.5

④ We returned the borrowed amount with interest to the bank i.e ₹ 10000 + ₹ 400

= ₹ 10,400.

⑤ Profit => ₹ 10467.5 − ₹ 10,4000

= ₹ 67.5 > 0

[Arbitrage exist!!].

Q2. current price = ₹30.

price of option = ₹4

Investor is able to make a gain if the price of the Commodity (P) becomes less than ₹34 in future.

as if $P < 34$

then he/she can sell the commodity at ₹30 and buying it again at cheaper price making a profit of $(30+4) - P$

$= ₹ (34 - P)$

Ans 10. Current price of silver : (5000 / 1 gm) = Rs 50/gm

Storage cost : Rs 0.5/gm.

6 months : Rs 0.25/gm = Rs 250/kg.

Growth factor : $\left(1 + \frac{0.09}{2}\right)^2$

Then,

$$V(T) = [F(0.4)] = [S(1-) + A(1-)]\left(1 + \frac{r}{n}\right)^{\wedge}$$

$$= (5000 + 0 + 250)[1 + 0.045)^2$$

$$= (50,200)(1.045)^2$$

$$= \text{Rs } 54,874.26.$$