Contents

1 Introduction

The problem that I have selected for this project is not something that is highly critical in nature or something that concerns our well being very much. It is also not something whose advancement will be very beneficial to us in terms of observable metrics. That is because the problem I have selected is that of creating a sketch composite from any given image I, such that the composite looks as if it was drawn by a sketch artist and not one created by a computer.

This problem is artistic in nature and hence can't be evaluated using a standard metric and the results given below have been produced using Hyperparameters tuned by me (so that the results look akin to something of my liking) and these hyperparameters can be changed and the results can be made to look to something of the users liking easily.

The key thing here are not the hyperparameters or results, but the method. This method gives excellent results not only for this application but is a completely new method of feature extraction from Images and can be used in several Applications. Some examples of other examples, more scientific in nature are given below in section 7 - *Future Scope*. The project proposed there are of too great a length to be shown in all entirety here, hence a simple example of sketching has been selected to display the power of the Orthogonal Feature Extractor.

There are several ways of making a sketch from an Image. Below are examples of 3 different methods to produce a Sketch Composite from a given Image I. We study the different methods below and introduce the Orthogonal Gaussian Lattice Method and discuss trade-offs and benefits.

We observe in Figure 1 that the basic Canny edge Detection Algorithm although correctly identifies edges, doesn't identify different gradients in different regions of the image and the results are too basic to be declared sufficient for a sketch composite.

We observe from Figure 2 given below that the texture based method looks very much like the image itself and not at all like a hand drawn sketch. Furthermore the texture based method has a disadvantage that it changes the original image as it needs to clip away the image to convert it into the same aspect ratio as the texture. The Gaussian Blur Blend method in (c) gives much better results and doesn't need to change the aspect ratio of the method, but the Gaussian Lattice Method in (d) gives considerable better results overall and as this is art, personally looks the best of the bunch.

(a) Lenna: Standard Test Image

(b) Sketch Composite Using Canny Edge Detection Method

Figure 1: Sketch Composite of Standard Lenna Image Using Canny Edge Detection

(a) Lenna: Standard Test Image

(b) Sketch Composite Using Fixed Texture Application Method

(c) Sketch Composite Using Gaussian Blur and Blend Technique Image

(d) Sketch Composite Using Novel Method

Figure 2: Different Methods to Convert Image to Sketch Composite

2 Basic Canny Edge Detection

Canny Edge Detection as the name suggests is an algorithm to detect edges and is a very popular and robust algorithm used heavily in Computer Vision applications. It was developed by John F. Canny in 1986. (2). Canny Edge Detection gives us the edges by computing the Gradient.

Since all edge detection results are easily affected by the noise in the image, it is essential to filter out the noise to prevent false detection caused by it. To smooth the image, a Gaussian filter kernel is convolved with the image.

2.1 Gaussian Filtering

This step will slightly smooth the image to reduce the effects of obvious noise on the edge detector. The equation for a Gaussian filter kernel of size (2k+1)×(2k+1) is given by:

$$H_{ij} = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{(i-(k+1))^2 + (j-(k+1))^2}{2\sigma^2} \right)$$

Here is an example of a 5×5 Gaussian filter, used to create the adjacent image, with $\sigma = 1$. (The asterisk denotes a convolution operation).

$$B = \frac{1}{159} \begin{bmatrix} 2 & 4 & 5 & 4 & 2 \\ 4 & 9 & 12 & 9 & 4 \\ 5 & 12 & 15 & 12 & 5 \\ 4 & 9 & 12 & 9 & 4 \\ 2 & 4 & 5 & 4 & 2 \end{bmatrix} * A$$

It is important to understand that the selection of the size of the Gaussian kernel will affect the performance of the detector. The larger the size is, the lower the detector's sensitivity to noise. Additionally, the localization error to detect the edge will slightly increase with the increase of the Gaussian filter kernel size. A 5×5 is a good size for most cases, but this will also vary depending on specific situations.

2.2 Finding The Intensity Gradient of The Image

An edge in an image may point in a variety of directions, so the Canny algorithm uses four filters to detect horizontal, vertical and diagonal edges in the blurred image. The edge detection operator (such as Roberts, Prewitt, or Sobel) returns a value for the first derivative in the horizontal direction (G_x) and the vertical direction (G_y). From this the edge gradient and direction can be determined:

$$G = \sqrt{G_x^2 + G_y^2}$$
$$\Theta = \arctan \frac{G_y}{G_x}$$

2.3 The Algorithm

The algorithm for performing canny edge Detection is given below. On my machine Intel Core i7-9750H CPU @ 2.6 GHz with 16GB RAM and a Nvidia GeForce GTX 1650 GPU with 4GB of VRAM the following operation performed in under 400 ms.

Algorithm 1 Canny Edge Detection Algorithm for a Given Image I

$kernel \leftarrow$ 5x5 Gaussian Kernel
$I \leftarrow I * kernel$
Create Sobel Kernel to calculate Image Derivatives I_x and I_y
$k_x \leftarrow \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$
$k_y \leftarrow \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}$
Compute the Derivatives I_x and I_y using k_x and k_y
$I_x \leftarrow I * k_x$
$I_y \leftarrow I * k_y$
Compute Magnitude G and angle Θ as
$G = \sqrt{I_x^2 + I_y^2}$
$\Theta = \arctan \frac{I_y}{I_x}$
We now perform Non Maximum Suppression to reduce the variation in Edge Thickness
for all pixels p in I **do**
  for all permitted angles θ in 0°, 45°, 90°, 135°, 180°, 225°, 270°, 315°, 360° **do**
    Threshold all pixels with this gradient angle to 255 and the rest to 0
  end for
end for
$result \leftarrow I$
return $result$

2.4 Results

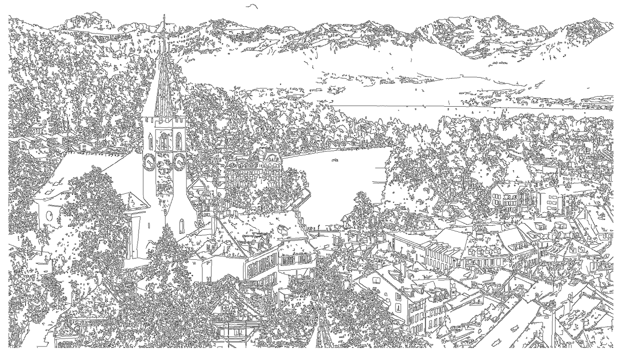
Some results of Canny Edge Detection are given below:

(a) Original Image

(b) Sketch Composite Using Canny Edge Detection

Figure 3: Canny Edge Detection Applied to Macro Image with Blurred Background

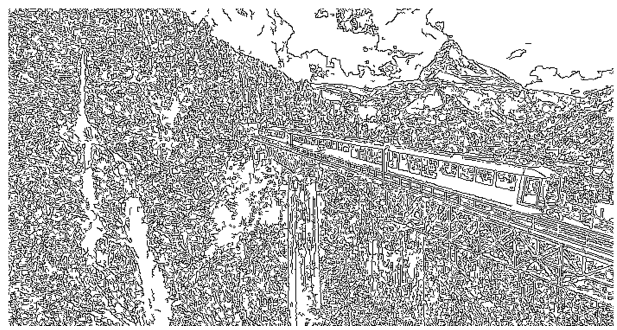
(a) Original Image



(b) Sketch Composite Using Canny Edge Detection

Figure 4: Canny Edge Detection Applied to Large Landscape Shot, with many Objects

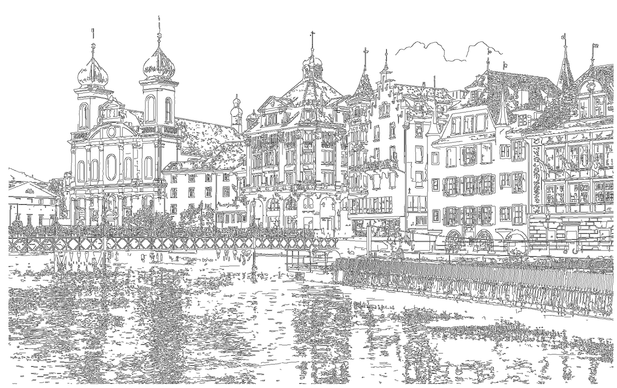
(a) Original Image



(b) Sketch Composite Using Canny Edge Detection

Figure 5: Canny Edge Detection Applied to Image with High Dynamic Range

(a) Original Image

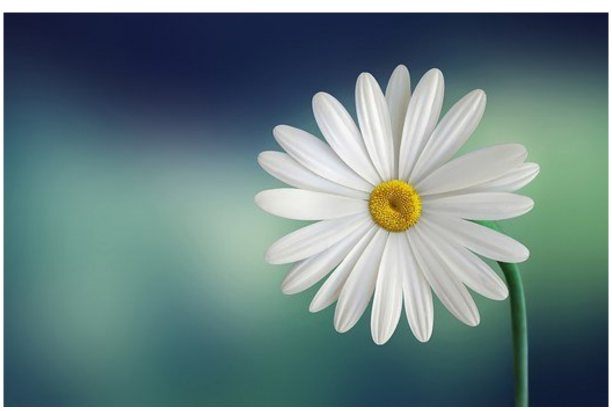


(b) Sketch Composite Using Canny Edge Detection

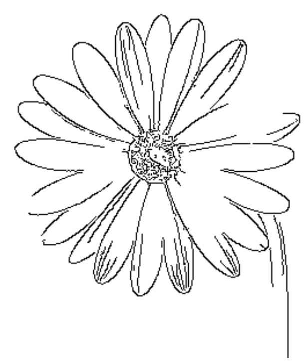
Figure 6: Canny Edge Detection Applied to Image with High Dynamic Range

(a) Original Image

(b) Sketch Composite Using Canny Edge Detection



(a) Original Image



(b) Sketch Composite Using Canny Edge Detection

3 Fixed Texture Application For Creating Sketch Composite

3.1 Method

In this method, we simply take a pre-defined texture file which is basically a mask that contains the pixel intensity values that should exist given an image I. Here the result produced will depend on the mask value of this texture file rather than the image I.

3.2 Algorithm

Algorithm 2 Applying Texture Mask on Image I

Require: The Image I
Require: The mask M
Reshape the Image I to match the ratio of M, crop the image if you must
$I \leftarrow$ Grayscale of I
$result \leftarrow I \cdot M$
return $result$

3.3 Results

A few examples of sketch composites created using the fixed texture Application Method are given below:

(a) Original Image

(b) Sketch Composite Using Simple Texture Application

(a) Original Image

(b) Sketch Composite Using Simple Texture Application

(a) Original Image

(b) Sketch Composite Using Simple Texture Application

(a) Original Image

(b) Sketch Composite Using Simple Texture Application

(a) Original Image

(b) Sketch Composite Using Simple Texture Application

Figure 7: Texture Mask on Standard Lenna Image

(a) Original Image

(b) Sketch Composite Using Simple Texture Application

Figure 8: Texture Mask on Swiss Landscape

3.4 Conclusion

We can clearly see from the above results that the result doesn't depend on any give Image but on the texture that we have selected. This is a computationally good method as it requires negligible time in applying a single texture filter, but the results don't seem very convincing and it doesn't seem as if it were drawn by a human sketch artist.

Another disadvantage of using this method is that the texture may not be the same aspect ratio as the given image and hence we either need to stretch out the texture to fit the image I or we need to crop the image I to fit to the texture. Changing the aspect ratio of the texture doesn't yield good results, hence we always need to change the aspect ratio of the Image as can be seen above in the results. More examples of this can be seen at (3).

The results for some photos such as the Landscape Photograph in Figure 8 seem very good, but when we compare the same method for a portrait image such as Figure 7, we do not get good results and the result seems like a Sepia Photograph rather than a hand drawn sketch, so this method has a hit and miss approach to this problem.

4 Gaussian Blur Blend Technique For Creating Sketch Composite

4.1 Method

This is a relatively straightforward method that borrows some techniques from the Canny Edge Detection Method, but rather than applying a Non Maximum Suppression in the Canny edge Detection, we will use a Dodge and Burn Blend Technique with our Gaussian Blur to obtain a sketch effect. (1)

The steps are very simple,

1. Take the Image I.
2. Take the Grayscale (6) of this Image I.
3. Take the Negative of the grayscale obtained in step 2.
4. Apply a Gaussian Blur (4) to the Negative Obtained in Step 3.
5. Blend the grayscale image from step 2 with the blurred negative from step 4 using a color dodge (5).

4.2 Gaussian Blur

The Gaussian blur is a type of image-blurring filters that uses a Gaussian function (which also expresses the normal distribution in statistics) for calculating the transformation to apply to each pixel in the image. The formula of a Gaussian function in one dimension is

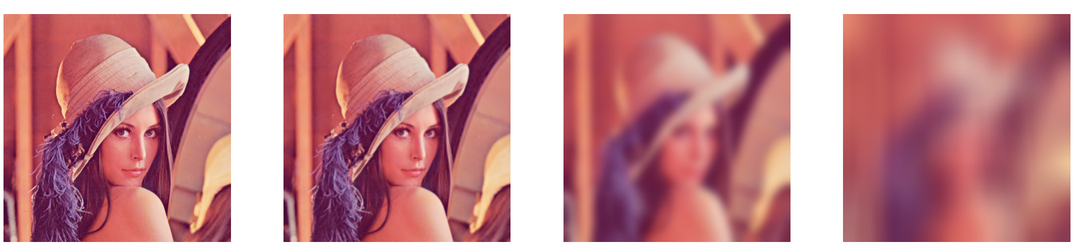
$$G(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

In two dimensions, it is the product of two such Gaussian functions, one in each dimension:

$$G(x,y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$

where x is the distance from the origin in the horizontal axis, y is the distance from the origin in the vertical axis, and σ is the standard deviation of the Gaussian distribution. When applied in two dimensions, this formula produces a surface whose contours are concentric circles with a Gaussian distribution from the center point.

The time complexity of the Gaussian Blur Operation is $\mathcal{O}(w_{kernel}w_{image}h_{image}) + \mathcal{O}(h_{kernel}w_{image}h_{image})$. Gaussian Blur is a low pass filter attenuating high frequency signals.



(a) Standard Lenna Image with No Gaussian Blur

(b) Gaussian Blur with $\sigma = 1$

(c) Gaussian Blur with $\sigma = 10$

(d) Gaussian Blur with $\sigma = 50$

Figure 9: Gaussian Blur Applied to Lenna with Different Standard Deviations σ and Kernel Size $k = (101, 101)$

4.3 Color Dodge and Burn

Dodging and burning are terms used in photography for a technique used during the printing process to manipulate the exposure of a selected area(s) on a photographic print, deviating from the rest of the image's exposure. In a darkroom print from a film negative, dodging decreases the exposure for areas of the print that the photographer wishes to be lighter, while burning increases the exposure to areas of the print that should be darker.

Any material with varying degrees of opacity may be used, as preferred, to cover and/or obscure the desired area for burning or dodging. One may use a transparency with text, designs, patterns, a stencil, or a completely opaque material shaped according to the desired area of burning/dodging.

In both color doge and color burn we receive an Image I and a mask M and we use division operators in images to obtain the dodge or burn using the mask we have. In the case of color doge the image is brightened at the regions specified by the Mask.

In the following example we apply color dodge to standard Lenna Image using different Masks. We take same pixel value masks, where each pixel p in the mask has the same value and the mask has dimensions same as the image.

$$M = \begin{bmatrix} p & p & p & \cdots & p \\ p & p & p & \cdots & p \\ \cdots & & & & \\ \cdots & & & & \\ p & p & p & \cdots & p \end{bmatrix}$$

(a) Standard Lenna Image with No Color Dodge

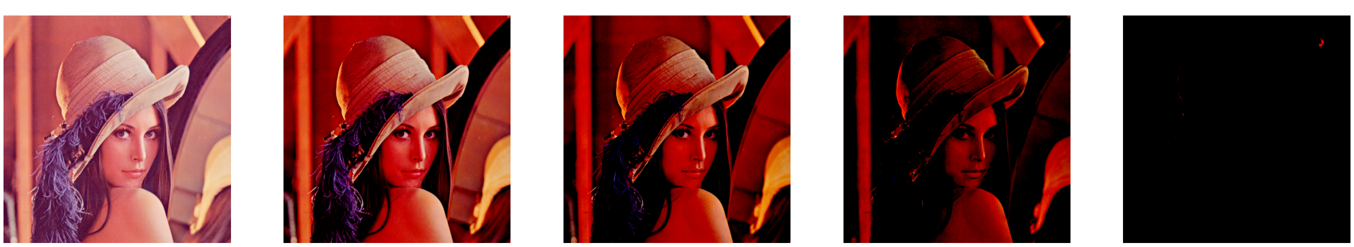
(b) Color Dodge with $p = 100$

(c) Color Dodge with $p = 150$

(d) Color Dodge with $p = 200$

(e) Color Dodge with $p = 254$

Figure 10: Color Dodge Applied to Lenna with Different Masks M of constant dimensions, but different pixel values p.



(a) Standard Lenna Image with No Color Burn

(b) Color Burn with $p = 100$

(c) Color Burn with $p = 150$

(d) Color Burn with $p = 200$

(e) Color Burn with $p = 255$

Figure 11: Color Burn Applied to Lenna with Different Masks M of constant dimensions, but different pixel values p.

4.4 Algorithm

Algorithm 3 Creating the Sketch Composite from I using Gaussian Blur and Blend Method

Require: The Image I

Require: *Sketch Density*: A Hyperparameter which will be used in The Gaussian Blurring Kernel Size and will decide the number of sketch lines to appear in the final result.

We create an Kernel of odd size, for convolution

Kernel Size $\leftarrow 2 * (\textit{Sketch Density}, \textit{Sketch Density}) + 1$

$J \leftarrow$ Grayscale of Image I

$B \leftarrow$ Gaussian Blur of J with kernel of size $k =$ Kernel Size

We divide the Grayscale image with the Gaussian Blur of the Grayscale Image. The following is a pixel by pixel level operation which can be parallelized using a standard numerical matrix package.

$result \leftarrow J/B$

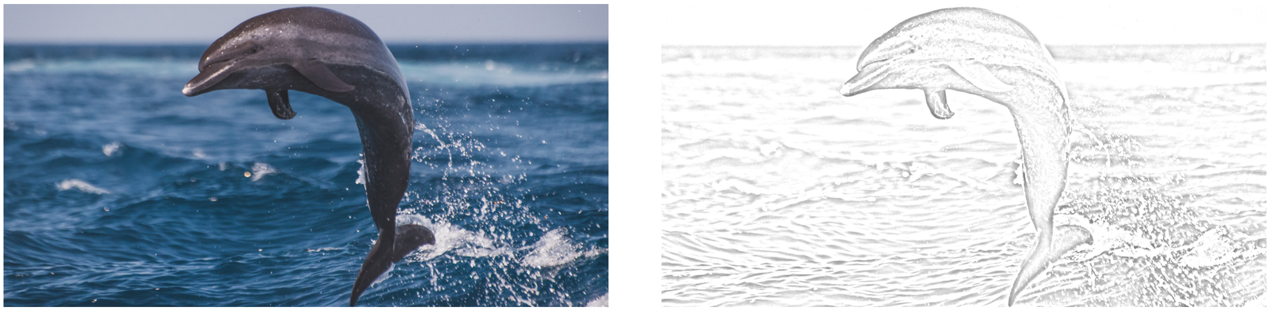
return $result$

4.5 Results

(a) Original Image

(b) Sketch Composite Using Gaussian Blend Method

Figure 12: Gaussian Blur Blend Applied to Macro Butterfly Shot



(a) Original Image (b) Sketch Composite Using Gaussian Blend Method

Figure 13: Gaussian Blur Blend Applied to Dolphin Photo with High Focus Shot

(a) Original Image (b) Sketch Composite Using Gaussian Blend Method

Figure 14: Gaussian Blur Blend Applied to Close Landscape Shot

(a) Original Image (b) Sketch Composite Using Gaussian Blend Method

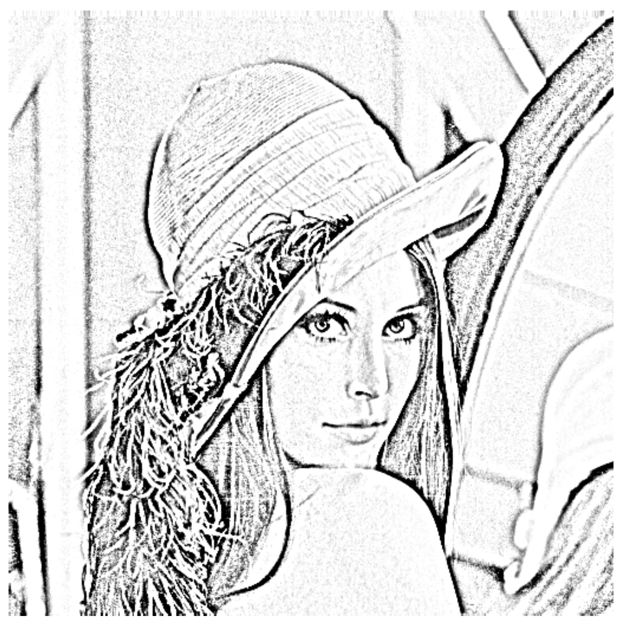
Figure 15: Gaussian Blur Blend Applied to Macro Photo With High Background Blur

(a) Original Image

(b) Sketch Composite Using Gaussian Blend Method

Figure 16: Gaussian Blur Blend Applied to Macro Flower Shot

(a) Original Image



(b) Sketch Composite Using Gaussian Blend Method

Figure 17: Gaussian Blur Blend Applied to Standard Lenna Portrait

(a) Original Image (b) Sketch Composite Using Gaussian Blend Method

Figure 18: Gaussian Blur Blend Applied to Swiss Landscape Shot

(a) Original Image (b) Sketch Composite Using Gaussian Blend Method

Figure 19

(a) Original Image (b) Sketch Composite Using Gaussian Blend Method

Figure 20

4.6 Conclusion

We can see from the above Results that the results for Macro shots with low background noise or in many cases highly reduced blurred out background such as in Figure 12, 13 and 15 the results are very good, especially for Figure 12.

The same can't be said for the portrait photo of Lenna Figure-17. This method does capture the different edges part of the face, but the result seems like a Computer Generated Edge Map rather a hand drawn sketch with different intensities.

Also for the landscape shots such as Figure-18, Figure-19 and Figure-20 the results are not very good. The method has successfully captured all the various edges in the objects present, but there is no gradient in the image. There are no regions of the image where the pencil intensity is waxing and waning. Especially in the Bridge scene in Figure-14 the bridge lines seem very dark as compared to other lines and edges in the image. Furthermore there is a soft shadow emanating from behind all the lines in all the images.

Notice especially Figure-16 where there is a very strong shadow behind the rose petals. This shadow is the result of the Gaussian Blur Blend Method and this shadow makes it appear computer generated rather than hand drawn. We can apply further filters and enhanced smoothing operators such as the Laplacian smoothing operator, but these are specific methods trying to correct for specific bias in a few cases. Also that would still not correct for the very dark regions and strong edge detection in particular regions. We now introduce a novel Feature Extraction Framework that can be used in countless feature extraction tasks including Sketch Composite of an Image.

5 Novel Method Using Orthogonal Gaussian Lattice

View full project and further results at github.com/anishLearnsToCode/image-2-sketch

There are several regions in an Image, initial observation gives us the major features such as cars, people, the road, streets, bicycles, the footpath etc. If we observe closely we can identify softer features such as the lines on a person's face, or the strands of gray hair on the hair of a person. There are many different such features present in an Image and the following method is proposed.

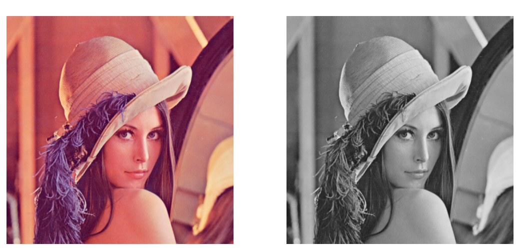
1. We Take 3 different Gaussian with different mean μ and std. deviation σ.
2. We compute the Grayscale of the Image I, therefore reducing the Image pixel value from 3-dimensional to 1 dimensional.
3. We normalize the pixel values in our grayscale Image.
4. We compute 3 Gaussian Inverses using the 3 different Gaussian we took initially from the grayscale Image.
5. Using the Gaussian Inverses we computed, we now take a sliding window of size w and compute deviation spread Vectors (explained later) between a central and surrounding pixel.
6. We take 3 different bounds, denoted in this project by α for the 3 different Gaussian.
7. We compute 3 Simple Graphs from the 3 Gaussian Inverse using the deviation spread vectors and connectivity parameters α we took in step 6.
8. We compute the Different Components in the 3 different Simple Graphs that we calculated in Step 7 and the separate components in a single Frame (Simple Graph) is called a Lattice.
9. We can vary the type of lattices we create, the density of lattices and change the different features we discover by changing our 3 initial Gaussian that we select and also changing the connectivity bound parameter α.

5.1 Grayscale of Image I

An Image I is defined as a function of 2 space coordinates x and y if we assume the image to be a discrete function with 2 axes. This can be denoted as $I = f(x, y)$ and each pixel is denoted by 3 distinct values denoting the amount of red, green and blue in the pixel. So each pixel can be denoted by a 3 dimensional vector (r, g, b). When we convert a RGB Image into grayscale, for every pixel we assign a single gray value between 0 and 255.

We compute Gray Value using the following method (6):

$$\text{Grayscale Image}(x, y) = 0.2126 \cdot R_{linear} + 0.7152 \cdot G_{linear} + 0.0722 \cdot B_{linear}$$



(a) Original Image (b) Grayscale of Image

Figure 21: RGB → Grayscale for Lenna Image

5.2 Gaussian Inverse

A standard Gaussian is defined as

$$G(x;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)$$

where μ is the mean and σ the standard deviation of the curve. The Gaussian Inverse which computes x given y is hence defined as (we are taking the positive x branch in the formula below).

$$G^{-1}(y;\mu,\sigma) = \sigma * \sqrt{-2 * \log(y\sigma\sqrt{2\pi})} + \mu$$

If we are given an image which is represented by a 2-dimensional Matrix I, we can compute the Gaussian Inverse of the image, which implies we calculate Gaussian Inverse of all pixel values separately. This can be parallelized using numerical Matrix library. The maximum value of the Gaussian Curve is at $x = \mu$, where $G(x = \mu;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}}$.

If we take a Gaussian with high standard deviation than that would imply that not all values would have an inverse and any value larger than $\frac{1}{\sigma\sqrt{2\pi}}$ would not have an inverse, so one of our 3 Gaussian should have $\sigma = \frac{1}{\sqrt{2\pi}}$ so that for this Gaussian inverse exists for all $y \in [0,1]$.

This formula is defined as follows for the computer, as we need to deal with imaginary quantities.

$$G^{-1}(y;\mu,\sigma) = \mu + \begin{cases} 0 & y > \frac{1}{\sigma\sqrt{2\pi}} \\ \sigma * \sqrt{-2 * \log(y\sigma\sqrt{2\pi})} & \text{otherwise} \end{cases}$$

Let us take 3 Gaussian as

$$G_1 \leftarrow (\epsilon, \frac{4}{\sqrt{2\pi}})$$
$$G_1 \leftarrow (\epsilon, \frac{2}{\sqrt{2\pi}})$$
$$G_1 \leftarrow (\epsilon, \frac{1}{\sqrt{2\pi}})$$

where ϵ is a very small quantity, 10^{-5} in my program. The purpose of such a small quantity is for deviation spread ratio smoothing, which is explained later on.

We now compute the Gaussian Inverse of Grayscale Images, for the first 2 Gaussian G_1 and G_2, not all values will have an inverse and for such values the pixel values will be represented as 0, for others we will see valid values. So, the Gaussian Inverse will act as a mask for weeding out undesired high pixel values and will also give us a non-polynomial smooth curve for feature extraction.

Pixel values exist in the range $p \in (0,1,2,\cdots,255)$ (8-bit) where $p \in \mathbb{N}$. If we take a very high mean μ, then our inverse function will give many values over 1, resulting in them being thresholded down to 255 and hence having a mean larger than $\mu > 0$ serves no purpose.

(a) Original Image

(b) Grayscale of Image

(c) Gaussian Inverse of Grayscale with $\sigma = \frac{4}{\sqrt{2\pi}}$

(d) Gaussian Inverse of Grayscale with $\sigma = \frac{2}{\sqrt{2\pi}}$

(e) Gaussian Inverse of Grayscale with $\sigma = \frac{1}{\sqrt{2\pi}}$

Figure 22: Gaussian Inverse of Grayscale Lenna Image with different Gaussian Parameters G_1, G_2 and G_3

(a) Original Image

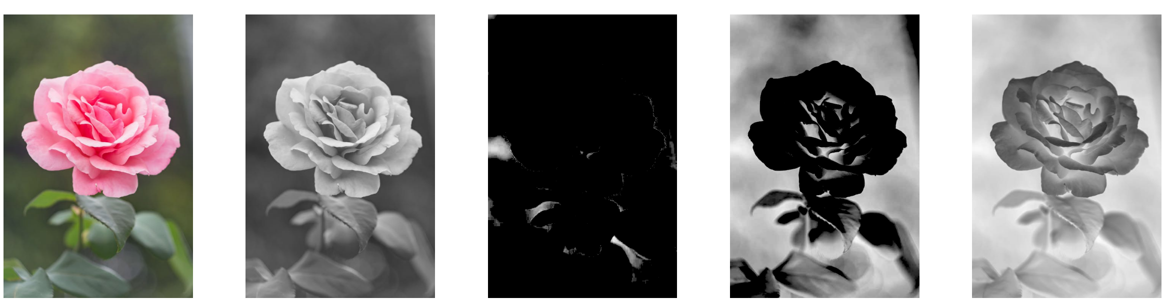
(b) Grayscale of Image

(c) Gaussian Inverse of Grayscale with $\sigma = \frac{4}{\sqrt{2\pi}}$

(d) Gaussian Inverse of Grayscale with $\sigma = \frac{2}{\sqrt{2\pi}}$

(e) Gaussian Inverse of Grayscale with $\sigma = \frac{1}{\sqrt{2\pi}}$

Figure 23: Gaussian Inverse of Grayscale Dolphin Image with different Gaussian Parameters G_1, G_2 and G_3



(a) Original Image

(b) Grayscale of Image

(c) Gaussian Inverse of Grayscale with $\sigma = \frac{4}{\sqrt{2\pi}}$

(d) Gaussian Inverse of Grayscale with $\sigma = \frac{2}{\sqrt{2\pi}}$

(e) Gaussian Inverse of Grayscale with $\sigma = \frac{1}{\sqrt{2\pi}}$

Figure 24: Gaussian Inverse of Grayscale Image with different Gaussian Parameters G_1, G_2 and G_3

5.3 Deviation Spread Ratio Vector

We will now take a window of size w ($w = 3$ in our program implementation) and will convolute the Image I with this window. In our window we will have a central pixel p_x and surrounding pixels (8 in our case), which we denote by p_s.

In every such window we will compute the ratio of surrounding pixels and the central pixel for the 8 possible surrounding pixel and central pixel pair. For 2 pixels in the window we will get 3 such ratios for the different Gaussian. Let us assume we have the following values for a 3×3 window of the 3^d Gaussian.

$$W = \begin{bmatrix} 0.43 & 0.76 & 0.12 \\ 0.18 & 0.90 & 0.44 \\ 0.28 & 0.91 & 0.93 \end{bmatrix}$$

Taking the central pixel value as 0.90, we get the 8 ratios as $\frac{p_s}{p_c}$ as:

$$\begin{bmatrix} 0.47 & 0.84 & 0.13 \\ 0.19 & - & 0.48 \\ 0.31 & 1.01 & 1.03 \end{bmatrix}$$

for every pixel pair (p_s, p_c) we will get 3 such ratios for the 3 different Gaussian, which is what we will call the deviation ratio vector. These deviation ratios tell us different things. They are in fact the ratio of the z-score of the given pixel inverse values. In the Gaussian with smaller standard deviation σ will have a lower ratio for the same amount of change in pixel values as compared to the Gaussian with larger σ, where even a small change in values will trigger a large change in the deviation spread Ratio. So, the Gaussian with a larger σ is being used to extract subtle features whereas the Gaussian with a smaller σ is being used to extract regions of the image where large changes will trigger a change in Ratio.

The 3 Different Gaussian are hence acting as an orthogonal Feature extraction mechanism where from the same pixel pair we are getting different ratios hence using the siding window technique we will get a deviation spread ratio vector for every adjacent pixel pair in the image.

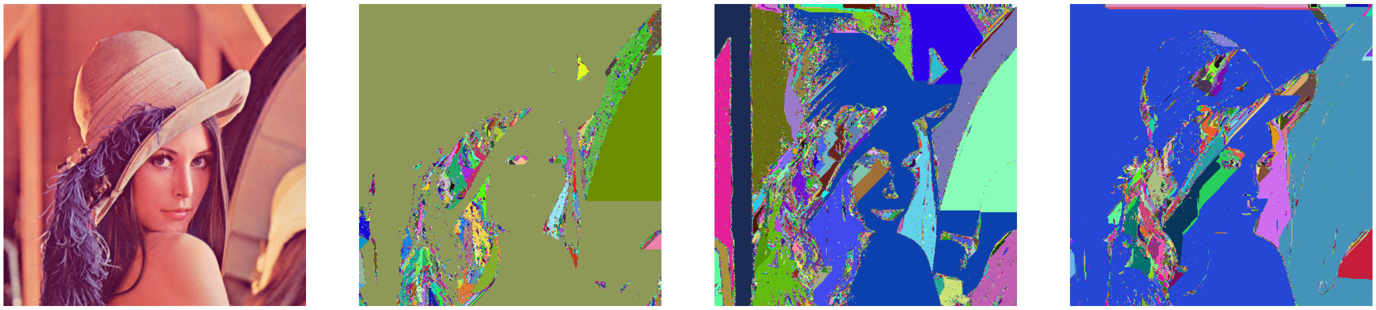
We will now use these deviation spreads to create 3 different Simple Graphs for the 3 Gaussian using the connectivity Parameters $< \alpha_i > i \in \{1, 2, 3\}$. Here $\alpha_i \in (0, 1]$.

5.4 Lattices in Images

We will consider every pixel in our image as a vertex, we will then compare the deviation spread ratio for very adjacent pixel with the connectivity parameter α for that particular Gaussian and if our ratio lies in the range $(\alpha, 1/\alpha)$ then we will add an edge between these vertices. We will hence create 3 Simple Graphs for the 3 Gaussian and by varying the connectivity parameters $< \alpha_i >$, we can make our graph more or less connected and change the density of connected components. In the first 2 Gaussian where not every pixel will have an inverse, if you recall we had chosen a small value ϵ for such pixels and this ϵ value will ensure that when we compute the ratio between any such pixels we do not encounter a Null value, but rather a number the computer can store. This step is what we call **Epsilon Smoothing** and prevents underflow, overflow and Null values in actual program.

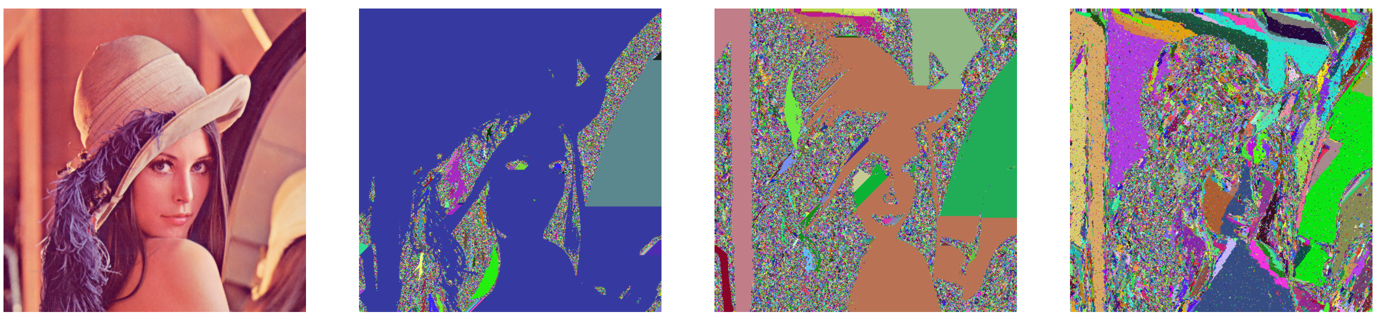
In our application and we will be proceeding by calling a single component in our Graph as a Lattice. Mathematically a Lattice is a connected, unweighted simple graph. Vertices inside a lattice actually represent a single pixel and have maximum degree 8 and minimum degree 0. The largest component possible can be the size of the image and the smallest possible will be a trivial Lattice of size 1 pixel.

We can visualize different Lattices in Images by coloring each Lattice a Different Color and hence visualizing our Lattices. A tighter bound i.e $\alpha \to 1$ will produce many disconnected Lattices whereas a lenient bound $\alpha << 1$ will create a small number of large connected Lattices. Given below are a few examples: ***(The colors are chosen at random for a lattice and have no significance)***



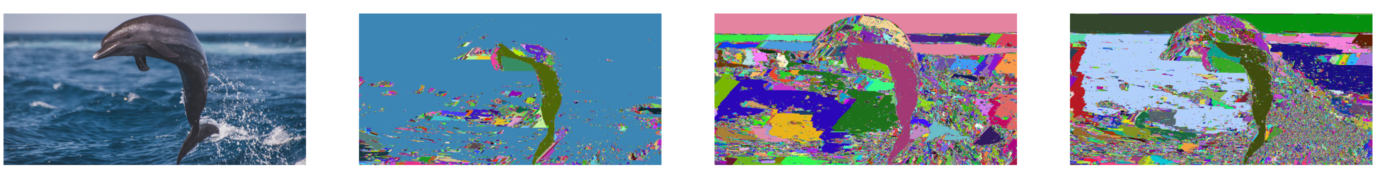
(a) Original Image | (b) Lattices in Image with G_1 and $\alpha = 0.86$ | (c) Lattices in Image with G_2 and $\alpha = 0.9$ | (d) Lattices in Image with G_3 and $\alpha = 0.94$

Figure 25: Lattices in Different Simple Graphs with the connectivity parameter $<\alpha>= (0.86, 0.9, 0.94)$



(a) Original Image | (b) Lattices in Image with G_1 and $\alpha = 0.98$ | (c) Lattices in Image with G_2 and $\alpha = 0.98$ | (d) Lattices in Image with G_3 and $\alpha = 0.98$

Figure 26: Lattices in Different Simple Graphs with the connectivity parameter $<\alpha>= (0.98, 0.98, 0.98)$



(a) Original Image | (b) Lattices in Image with G_1 and $\alpha = 0.86$ | (c) Lattices in Image with G_2 and $\alpha = 0.94$ | (d) Lattices in Image with G_3 and $\alpha = 0.98$

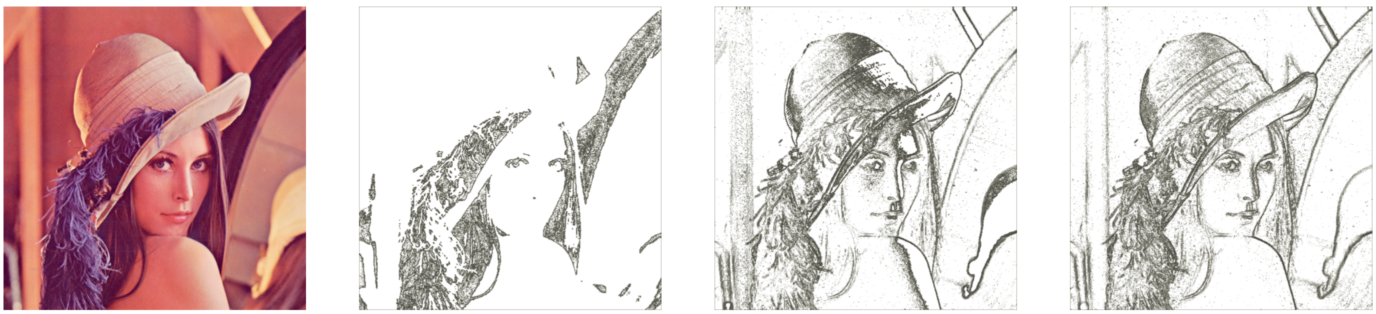
Figure 27: Lattices in Different Simple Graphs with the connectivity parameter $<\alpha>= (0.86, 0.94, 0.98)$

5.5 Lattice Vertex Coloring

The above method to visualize the Lattices in the Images is a good method, but we have another method which will be the foundation for our sketching composite. The major problem we were facing with previous methods was there was no way to identify strong edges from lighter edges, and hence we could not make light and strong strokes as a real Human Sketch Artist will make. This issue can be resolved using the degree of pixels (vertices) in our Simple Graph.

We know that every pixel must have at most a degree of 8 and in the trivial Lattice case, a degree of 0. This gives us 9 different degree values a pixel can have. Pixels that are heavily connected will lie deep inside a Lattice and Pixels that are at the border of the Lattice can at maximum be connected to 7 other pixels and would normally be connected to even less, normally 3-4. And completely isolated lattices will have a degree of 0.

We can assign different colors to pixels on the basis of their degree, where strongly connected pixels (degree 8) will have a white color and as the degree decreases, colors move towards Gray in the color spectrum. This will automatically give stronger edges a bolder color.



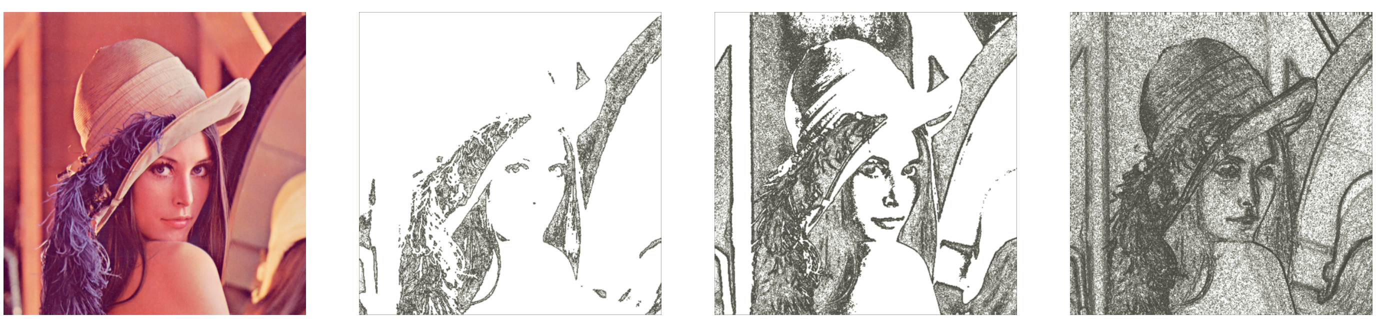
(a) Original Image

(b) Vertex Shading in G_1 with $\alpha = 0.86$

(c) Vertex Shading in G_2 with $\alpha = 0.90$

(d) Vertex Shading in G_3 with $\alpha = 0.94$

Figure 28: Vertex Shading with Gaussian Parameters $G_1 = (\epsilon, \frac{4}{\sqrt{2\pi}}), G_1 = (\epsilon, \frac{1.3}{\sqrt{2\pi}}), G_1 = (\epsilon, \frac{1}{\sqrt{2\pi}})$ and $\alpha = (0.86, 0.9, 0.94)$



(a) Original Image

(b) Vertex Shading in G_1 with $\alpha = 0.86$

(c) Vertex Shading in G_2 with $\alpha = 0.94$

(d) Vertex Shading in G_3 with $\alpha = 0.98$

Figure 29: Vertex Shading with Gaussian Parameters $G_1 = (\epsilon, \frac{4}{\sqrt{2\pi}})$, $G_1 = (\epsilon, \frac{2}{\sqrt{2\pi}})$, $G_1 = (\epsilon, \frac{1}{\sqrt{2\pi}})$ and $\alpha = (0.86, 0.94, 0.98)$

5.6 Linear Fusion of Orthogonal Gaussian Lattice Results with Standard Gaussian Blend

We have seen in the results above that we are obtaining excellent shadings of different regions of the Image using our Vertex Shading Method of the different Gaussian Lattices, but we want only one image where all these different features our prominent. The simplest way to achieve this is taking a simple weighted average of our 3 Gaussian Vertex Shaded results and the Gaussian Blend Image Created in the above section (which is current State of The Art Method). To take a weighted average we assume a Hyperparameter $weight, W = (w_1, w_2, w_3, w_{gb})$ where $< w_i > \forall i \in \{1, 2, 3\}$ are for the 3 Gaussian and w_{gb} is for the Gaussian Blend Method. Also, $\sum_{w^{(i)} \in W} w^{(i)} = 1$.

The weight w_3 is most important as the 3^d Gaussian captures inverses of all Pixel values and acts as the base for the entire image. This weight w_3 is hence being treated as a hyper-parameter **Hand Drawn** which implies to the user how much hand drawn feel the user would like. High values like $[0.7, 1)$ would make it feel very sketchy and completely negate the Blend part, whereas lower values between $[0, 0.2]$ will make it feel more like the Gaussian-Blend result. The parameters *Hand Drawn* and *Sketch Density* are the 2 parameters the user will control to vary results.

$$result = w_1 \cdot VS_1 + w_2 \cdot VS_2 + w_3 \cdot VS_3 + w_{gb} \cdot R_{gb}$$

(a) Original Image

(b) Weighted Result
$W = (0.03, 0.03, 0, 0.94)$

(c) Weighted Result
$W = (0.03, 0.03, 0.15, 0.79)$

(d) Weighted Result
$W = (0.03, 0.03, 0.30, 0.64)$

(e) Weighted Result
$W = (0.03, 0.03, 0.45, 0.49)$

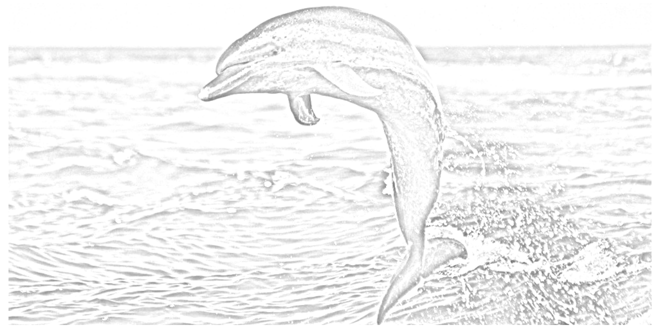
(f) Weighted Result
$W = (0.03, 0.03, 0.60, 0.34)$

Figure 30: Weighted Mean Results with Gaussian Parameters $G_1 = (\epsilon, \frac{4}{\sqrt{2\pi}}), G_1 = (\epsilon, \frac{2}{\sqrt{2\pi}}), G_1 = (\epsilon, \frac{1}{\sqrt{2\pi}})$ and $\alpha = (0.86, 0.94, 0.94)$

We can clearly see as we grow the hyper-parameter w_3, which is the **Hand Drawn** parameter, we get a much more sketch like looking Image. The results in Figure-30 image (b) is from the Gaussian Blur Blend Method, and the following results

; (c), (d), (e) and (f) are from the Orthogonal Gaussian Lattice Method and look distinctly better. A few other examples are:

(a) Original Image



(b) Weighted Result $W = (0, 0, 0, 1)$

(c) Weighted Result $W = (0, 0.1, 0.2, 0.7)$

(d) Weighted Result $W = (0.1, 0.15, 0.3, 0.45)$

(e) Weighted Result $W = (0.2, 0.1, 0, 0.7)$

(f) Weighted Result $W = (0.2, 0.1, 0.2, 0.5)$

Figure 31: Weighted Mean Results with Gaussian Parameters $G_1 = (\epsilon, \frac{4}{\sqrt{2\pi}}), G_1 = (\epsilon, \frac{2}{\sqrt{2\pi}}), G_1 = (\epsilon, \frac{1}{\sqrt{2\pi}})$ with $\alpha = (0.86, 0.94, 0.94)$ and **Sketch Density** $= 17$

5.7 Full Algorithm

Algorithm 4 Calculating the Deviation vectors from a Given image I

Require: Image matrix I

I ← grayscale(I)

I ← I / 255

We create 3 Gaussian

$G_1 \leftarrow (\mu_1, \sigma_1)$

$G_2 \leftarrow (\mu_2, \sigma_2)$

$G_3 \leftarrow (\mu_3, \sigma_3)$

We compute the Inverse Gaussian of the Image with the 3 Gaussian

$IG_1 = InverseGaussian(I, G_1)$

$IG_2 = InverseGaussian(I, G_2)$

$IG_3 = InverseGaussian(I, G_3)$

We now create a 3×3 or $w \times w$ sliding window and slide over our image to compute the Deviation Vectors.

The Deviation Vector of the surrounding and central Pixel Value are the ratio of

the deviation spread if the 2 pixels with the 3 Gaussian.

for all windows w in I **do**

  for all surrounding pixels p_s in window w for central pixel p_c **do**

    deviation(p_c, p_s) = DeviationVector(p_c, p_s, IG_1, IG_2, IG_3)

  end for

end for

Here a single deviation vector is a 3×1 row vector

return DevaiationVectors as D

Algorithm 5 Computing the Simple Graphs from the Deviation Vectors D and Connectivity Bounds α

Require: Deviation vectors D

Require: Connectivity Bounds α

Create 3 Simple Graphs U_1, U_2 and U_3 with no. of vertices = number of pixels and no edges

for all deviation vectors $d^{(i)}$ in D **do**

  for all ratios $d_j^{(i)}$ in $d^{(i)}$ **do**

    if $d_j^{(i)}$ bounded by $(\alpha_j, 1/\alpha_j)$ **then**

      Add an edge between the pixels for this deviation vector $d^{(i)}$ in the Simple Graph U_j

    end if

  end for

end for

return Simple Graphs U_1, U_2 and U_3

Algorithm 6 Computing the Lattices from the Graphs U_1, U_2 and U_3 using Standard Graph Theory Extracting Components List Algorithm

Require: Simple Graphs U_1, U_2 and U_3
  $L_1 \leftarrow$ lattices from U_1
  $L_2 \leftarrow$ lattices from U_2
  $L_3 \leftarrow$ lattices from U_3
  return Lattices L_1, L_2 and L_3

Algorithm 7 Computing Lattice Coloring Graph For the 3 Gaussian Based on The Lattices L_1, L_1 and L_1

Require: Lattices L_1, L_2 and L_3
  Create 3 new images for the results J_1, J_2 and J_3
  for all Gaussian lattices L in L_i **do**
    Each Gaussian Lattices is a list of the several lattices for that particular Gaussian
    for all Lattice l in L **do**
      Each Lattice is a connected component in that particular Graph for a specific Gaussian
      $pixelColor \leftarrow RandomColor()$
      for all pixels p in Lattice l **do**
        Assign pixel p color $pixelColor$ in result image J_i
      end for
    end for
  end for
  return Resulting Images J_1, J_2 and J_3

Algorithm 8 Computing The Vertex Shaded Image Given The Simple Graphs U_i for different Gaussian for image I

Require: Image I
Require: Simple Graphs U_i for $\{1, 2, 3\}$ for the 3 Gaussian
  Create 3 new Images J_1, J_2 and J_3 which will be the result
  for all Simple Graph U in U_i and resulting Image J in J_i **do**
    for all Vertices v in U **do**
      pixel_color $\leftarrow$ *PixelColorFromDegree(v.degree)*
      Assign pixel v in J color *pixel_color*
    end for
  end for
  return Resulting Images J_1, J_2 and J_3

Algorithm 9 Computing The Sketch Composite from Image I

Require: Image I
Require: Gaussian Parameters (μ_1, σ_1), (μ_2, σ_2) and (μ_3, σ_3)
Require: Connectivity Parameters $< \alpha >$
Require: Fusion Weights W
  Compute The Deviation Vectors Using Algorithm-4
  Compute The Simple Graphs from the Deviation Vectors Using Algorithm-5
  $< VS > \leftarrow$ The Vertex Shaded Images From The Simple Graphs using Algorithm-8
  $R_{gb} \leftarrow$ Gaussian Blended Result from Algorithm-3
  return $w_1 \cdot VS_1 + w_2 \cdot VS_2 + w_3 \cdot VS_3 + w_{gb} \cdot R_{gb}$

6 Future Scope

6.1 Novel Feature Extraction Method

The Orthogonal Gaussian Lattice Feature extraction can be used for all Applications that require Feature extraction such as Object Detection, Facial Expression Recognition, Face Detection etc.

This method gives us 3 dimensions from a single adjacent pixel pair and these values can be used as data for the Machine Learning and Deep Learning Models, where extra data about the Image will improve performance and reduce training times.

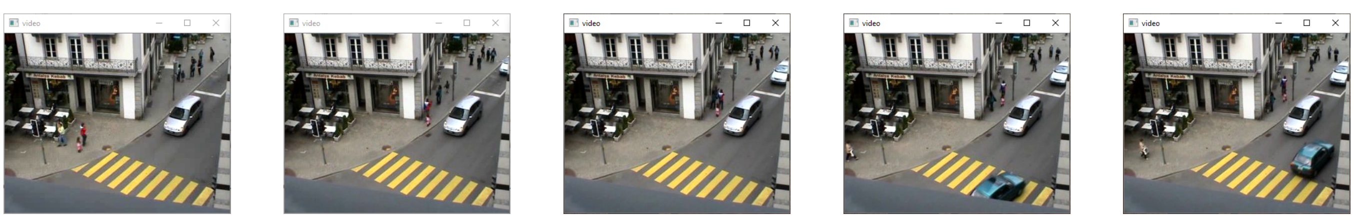
6.2 Object Tracking In Image Sequences

Videos are nothing but Images being represented with a temporal component as well. We can represent Videos mathematically as $V = f(x, y, t)$. There are several objects that are entering and leaving the frames, such as people, cars, trucks etc. and there are several objects that are stable such as the Road, sidewalk etc.

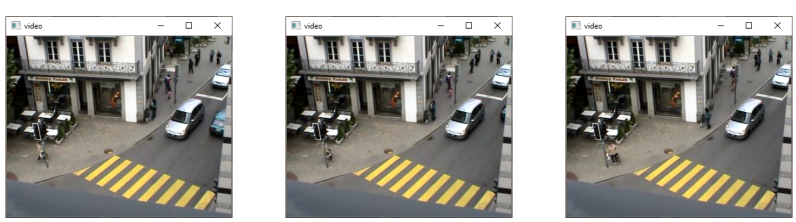
If we extract the Frame level lattices for each Gaussian in the Video, we will obtain fixed lattices for objects, e.g. we might obtain separate Lattices for the windshield, tire, bonnet etc. for a car. For object tracking we need to apply 2 novel methods of inter-frame lattice association and intra-frame lattice association.

Inter-frame Lattice association is the more important and easier to accomplish. Inter-frame association determines a lattice from one frame is associated with which lattice from the next frame. E.g. given a lattice of a car bonnet, which lattice in the next frame represents this lattice. We calculate this as the probability of a given lattice in frame$_i$ being mapped to another lattice in frame$_{i+1}$ as $P(L_{i+1,j_2}|L_{i,j_1})$ where j_1 is the lattice number in Lattice of frame$_i$ and lattice number j_2 is the lattice in frame$_{i+1}$.

We compute this using intersection probability mapping. We compute the Ring Sum of the lattice in frame$_i$ with all lattices in frame$_{i+1}$ and normalize the results with the size of lattice in frame$_i$. The lattice with the highest probability is associated as the next lattice . We can also use their probabilistic inter-frame mapping to perform MCMC (Markov Chain Monte Carlo) and compute the probabilities of different trajectories, where a trajectory here is the motion of a lattice in 3 spatial and 1 temporal dimension, with 3 Gaussian Dimensions. This creates a higher dimension surface $\mathbb{R}^9$ which can be used to track multiple objects parallelly.

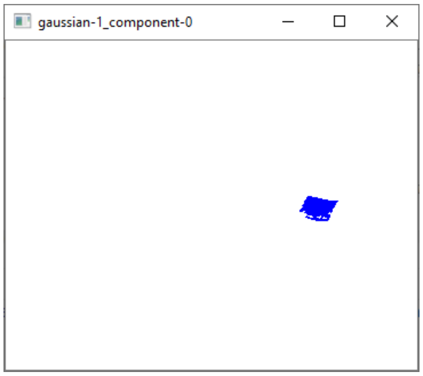


(a) Frame $t = 0$ (b) Frame $t = 5$ (c) Frame $t = 10$ (d) Frame $t = 15$ (e) Frame $t = 20$

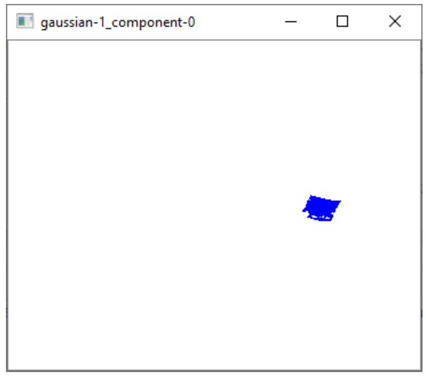


(f) Frame $t = 25$ (g) Frame $t = 25$ (h) Frame $t = 25$

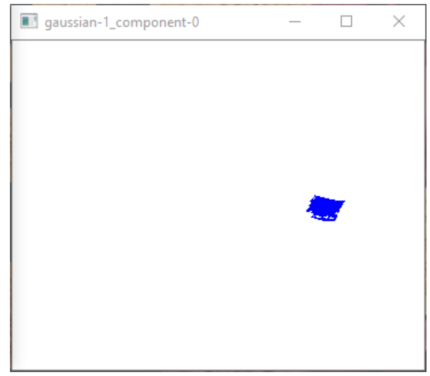
Figure 32: 8 Frames from an anomaly detection standard video (10)



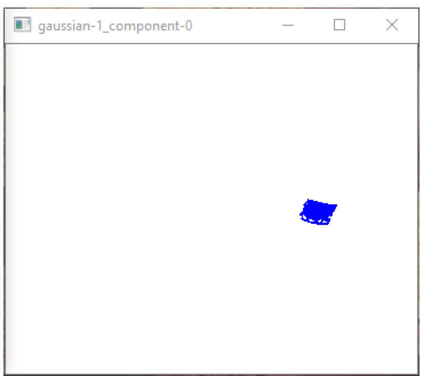
(a) Frame $t = 0$



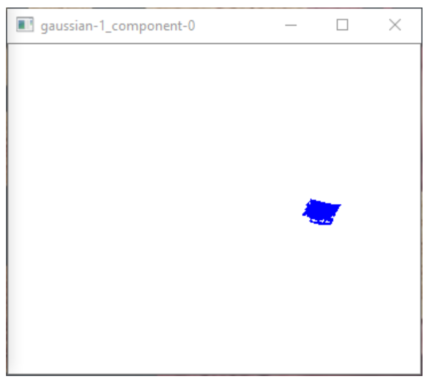
(b) Frame $t = 5$



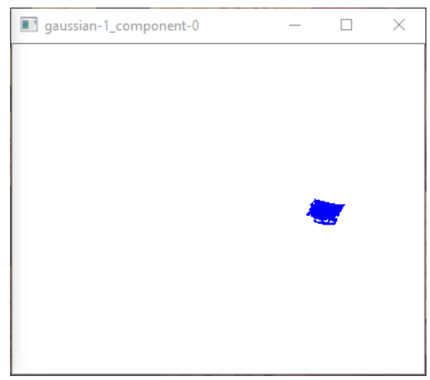
(c) Frame $t = 10$



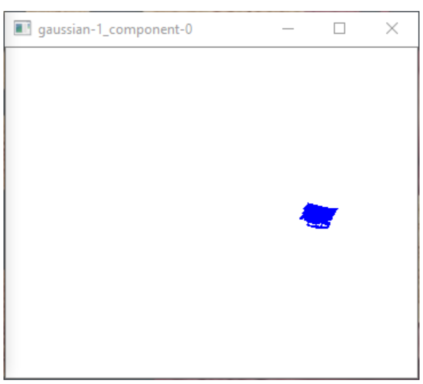
(d) Frame $t = 15$



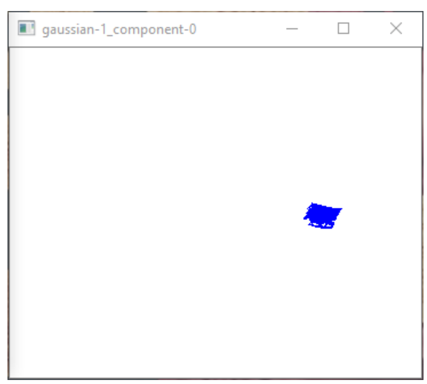
(e) Frame $t = 20$



(f) Frame $t = 25$



(g) Frame $t = 30$



(h) Frame $t = 35$

Figure 33: 8 Frames from an anomaly detection standard video (10) with Lattice based object tracking model to track trajectory of the Silver Car on The Road.

6.3 Anomaly Detection In Image Sequences

We can compute the probability of a lattice trajectory and we can also measure different telemetry from the lattices we have. Such as:

1. **Mass**: Mass is measured as the number of pixels in a Lattice, where each pixel contributes a mass of 1 unit.
2. **Center Of Mass**: Center of Mass is measured as the mean of x and y Coordinates.

$$\text{C.O.M. } (M_\mu) = \frac{1}{\text{Mass}} (\Sigma x_i, \Sigma y_i)$$

3. **Moment Of Inertia**: We are measuring both Moment of Inertia and Mass because 2 different objects with the same size might have similar masses, but very different moment of inertia. E.g. a person with the same mass as a bag will have a higher moment of Inertia than a bag of similar mass and having a person stand at a street corner may be normal behaviour but a bag at a street corner won't be normal and should be considered an anomaly. By recording these separate metrics we can train a model based on both mass and moment of inertia. Moment of inertia is measured as the sum of distance squares of all points with the center of mass.

$$\textit{Moment Of Inertia } (I) = \sum_{r_i \in r} (r_i - r_\mu)^2$$

4. **Displacement**: Displacement is a quantity that is not measured for a lattice per frame independent of the next frame. Displacement requires the probabilistic Inter-frame mappings between lattices and then displacement is measured as the distance between the center of mass of a lattice between 2 frames as it moves along it's trajectory. If the lattice doesn"t map onto any other lattice in the next frame we say that the displacement was very high as the lattice moved completely out of the frame and this is recorded in our experiment as a displacement of ∞.

$$\textit{Displacement } (\nabla L) = \overrightarrow{M_\mu(L(n_i, frame_i))} - \overrightarrow{M_\mu(L(n_j, frame_{i+1}))}$$

   In the above notation M_μ denotes the center of mass and $L(n_i, frame_i)$ denotes the lattice number n_i from $frame_i$ which is mapped onto lattice number n_j in the next frame $frame_{i+1}$.

5. **Birth and Death Rate Factor**: We can measure the amount of new lattices created in a particular portion of the frame. E.g. if cars are entering the street from some corner many new lattices will be created there and if they are leaving the street at some other corner, many lattices will die off at that location. By tracking the lattice births and deaths we can develop a probability distribution of these regions and then measure abnormality when some spurious births or deaths take place.

7 Conclusion

The new method for feature extraction using orthogonal Gaussian Distributions to create Simple Graphs and Lattices from an Image gives us high dimensional data from an image and this can be used for many different tasks and even with learning models utilizing Machine Learning and Deep Learning.

Implementing this method to create a sketch composite from an image was just the first step in showcasing what it can do and it barely scratches the surface. We have also seen another example of object tracking above with very good results right off the bat.

As for creating a sketch composite, the simple method of using a pre-rendered texture gives the same result for all images with similar pattern on it and looks more like a sepia filter than a hand drawn sketch. The Gaussian Blur Blend method is the one which comes the closest and is the current state-of-the-art method, but even in that Method the results lack gradient and in many cases can look just like an image with deep shadows at strong borders.

The Orthogonal Gaussian Lattice Method introduces a mechanism to identify strong edges from weaker edges and also introduces a mechanism for us to understand which vertices lie at the border and which vertices lie in the interior of a region and it also gives us a way of identifying regions within an image using Lattices. Using the telemetry data from these lattices like mass, Moment Of Inertia, Center Of Mass we can then decide on how to shade and color different lattices differently and even when to combine and associate lattices. This method is more computationally expensive than the Gaussian Blur Blend.

We can then combine the orthogonal Lattice Results we have with the Gaussian blend model using a weight vector W in the Linear Fusion Technique and obtain results which look much better than the Gaussian Blur-Blend and highly convincing as a sketch drawn by a real artist.

Figure 34: An Example of Sketch Composite of a landscape where the traditional methods were struggling. $W = (0.1, 0.3, 0, 0.6)$ $\alpha = (0.86, 0.88, 0.92)$ and $G_1 = (\epsilon, \frac{4}{\sqrt{2\pi}})$, $G_1 = (\epsilon, \frac{1.3}{\sqrt{2\pi}})$, $G_1 = (\epsilon, \frac{1}{\sqrt{2\pi}})$

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