borrowing rates in portfolio optimization theory has also been analyzed in literature. Some of the books mentioned above contain subject matter on this issue.

- Some researchers have re-visited Markowitz's mean- variance model so as to simplify the analysis and computations in the determination of efficient frontier. The interested readers may refer to Chawla [27] and Steinbach [129].

5.9 Exercises

Exercise 5.1 *Suppose there are three financial market scenarios* $\Omega = \{w_1, w_2, w_3\}$ *with different probabilities of occurrence. Consider the following table showing the returns on two different stocks in these three scenarios*

scenario	prob	return k_1%	return k_2%
w_1	$0 \cdot 2$	−10	−30
w_2	$0 \cdot 5$	0	20
w_3	$0 \cdot 3$	20	15

(a) What are the expected returns on the stocks?

(b) Suppose 60% *of the available fund is invested in stock 1 and the remaining is invested in stock 2, then what is the expected return of the portfolio?*

(c) Compute the weights if the expected return on a portfolio is 20%.

Exercise 5.2 *Consider the following data for two different stocks*

scenario	prob	return k_1%	return k_2%
w_1	$0 \cdot 4$	−10	20
w_2	$0 \cdot 2$	0	20
w_3	$0 \cdot 4$	20	10

Suppose a portfolio comprises of 40% *of total investment in stock 1 and* 60% *in stock 2. Compare the risk of the portfolio with the risks of its individual components. What will be the risk situation if a portfolio is designed with investment of* 80% *in stock 1 and the remaining in stock 2.*

Exercise 5.3 *Prove that if short sales are not allowed then the risk of the portfolio can not exceed the greater of the risks of the individual components of the portfolio.*

Exercise 5.4 *Let a portfolio be designed with investment of* 50% *in stock 1 and the remaning* 50% *in stock 2. Further let short sale be allowed in stock 1 and all the other data being the same as in Exercise 5.2. Does the conclusion of Exercise 5.3 hold.*

Exercise 5.5 *Suppose the portfolios are constructed using three securities* a_1, a_2, a_3 *with expected returns,* $\mu_1 = 20\%$, $\mu_2 = 13\%$, $\mu_3 = 4\%$, *standard deviations of returns,* $\sigma_1 = 25\%$, $\sigma_2 = 28\%$, $\sigma_3 = 20\%$, *and the correlation between returns,* $\rho_{12} = 0 \cdot 3$, $\rho_{13} = 0 \cdot 15$ *and* $\rho_{23} = 0 \cdot 4$. *Among all the attainable portfolios, find the one with minimum variance. What are the weights of the three securities in this portfolio? Also compute the expected return and standard deviation of this portfolio.*

Exercise 5.6 *Among all attainable portfolios with expected return* 20% *constructed using the data provided in Exercise* **5.5**, *find the portfolio with minimum variance. Compute the weights of individual assets in this portfolio.*

Exercise 5.7 *Consider the following data*

	μ	σ
asset 1	10%	5%
asset 2	8%	2%

For each correlation coefficient $\rho = -1, -0 \cdot 5, 0, 0 \cdot 5, 1$, *what is the combination of the two assets that yields the minimum standard deviation and what is the minimum value of the standard deviation?*

Exercise 5.8 *Compute the minimum risk portfolio for the following rate return (%) data*

	Jan	Feb	Mar	Apr	May	June
asset 1	12	10	5	7	15	12
asset 2	7	12	10	10	12	15

Also compute the expected return for the optimal portfolio.

Exercise 5.9 *Consider three risky assets with the variance-covariance matrix and expected returns (all data in %) as follows.*

variance - covariance matrix(C)			return(M)
10	4	0	5
4	12	6	6
0	6	10	1

Find two efficient portfolios. Also construct the portfolio giving the return of $2 \cdot 8\%$ *with minimum risk. Will this portfolio be also efficient?*
(Hint : use two fund theorem).

Exercise 5.10 *Suppose an investor is interested in constructing a portfolio with one risk-free asset* a_1, *and three risky assets* a_2, a_3 *and* a_4. *Let the expected returns of* a_1, a_2, a_3 *and* a_4 *be* 6%, 10%, 12% *and* 18% *respectively. Let the variance-covariance matrix* C *of the three risky assets be*

$$\mathrm{C} = \begin{pmatrix} 4 & 20 & 40 \\ 20 & 10 & 70 \\ 40 & 70 & 14 \end{pmatrix}.$$

Determine all efficient portfolios for the investor.

Exercise 5.11 *Consider the data of two risky assets* a_1, a_2 *with* $\mu_1 = 12 \cdot 5\%$, $\mu_2 = 10 \cdot 5\%$, $\sigma_1 = 14 \cdot 9\%$, $\sigma_2 = 14\%$, $\rho = 0 \cdot 33$.

(a) *Is it advisable to diversify the investment? If so then what composition of the assets will minimize the risk?*
(b) *What is the minimum value of the risk?*
(c) *If the risk-free rate of return is* 5% *then derive the equation of the capital market line?*

Exercise 5.12 *Given the following information about the one risk-free asset and three risky assets, find the expected return and standard deviation of the market portfolio. Also determine the equation of the capital market line.*

$$\mu_{rf} = 5\%, \ \mu_1 = 14\%, \ \mu_2 = 8\%, \ \mu_3 = 20\%;$$
$$\sigma_1 = 6\%, \ \sigma_2 = 3\%, \ \sigma_3 = 15\%; \ \sigma_{12} = 0 \cdot 5\%, \ \sigma_{13} = 0 \cdot 2\%, \ \sigma_{23} = 0 \cdot 4\%.$$

Exercise 5.13 *Assume that the following assets are correctly priced according to the security market line. Derive the security market line.*

$$\mu_1 = 6\%, \ \beta_1 = 0 \cdot 5; \quad \mu_2 = 12\%, \ \beta_2 = 1 \cdot 5.$$

What is the expected return on an asset with $\beta = 2$?

Exercise 5.14 *If the following two assets are correctly priced according to the security market line, what is the return of the market portfolio? What is the risk-free return?*

$$\mu_1 = 9 \cdot 5\%, \ \beta_1 = 0 \cdot 8; \quad \mu_2 = 13 \cdot 5\%, \ \beta_2 = 1 \cdot 3.$$

Exercise 5.15 *Let the expected rate of return on the market portfolio be 23% and that of the risk free asset be 7%. Also let the standard deviation of the market portfolio be 32% and let us assume that the market is efficient.*

(a) What is the equation of capital market line?
(b) If Rs 300 is invested in the risk free asset, and Rs 700 in the market portfolio then what is the expected return at the end of the year?
(c) If an investor has Rs 1000 to invest and he/she desires a return of 39%, then what should be his/her portfolio?