FUZZY LOGIC & FUZZY SETS

MC-432

ASSIGNMENT-1

ANISH SACHDEVA
DTU/2K16/MC/13

Q1) Write the complement of the fuzzy set {(2, 0.4), (4, 0.6) (5, 0.7) (6, 1) (7, 1), (8, 0.4), (9, 0.2)} defined over the universe of discourse

i) $X = \{1, 2, 3 \ldots 10\}$

$\tilde{A} = \{(1, 1.0), (2, 1.6), (3, 1), (4, 0.4), (5, 0.3), (8, 0.6), (9, 0.8), (10, 1)\}$

ii) $X = \{2, 4, 5, 6, 7, 8, 9\}$

$\tilde{A} = \{(2, 0.6), (4, 0.4), (5, 0.3), (8, 0.6), (9, 0.8)\}$

Q2) a) Compute the cardinality and relative cardinality of the following fuzzy sets

i) $\bar{A} = \frac{0.4}{1} + \frac{0.3}{3} + \frac{0.5}{4} + \frac{0.4}{7} + \frac{0.8}{8}$ defined on universe of discourse $U = \{1, 2, 3 \ldots 10\}$

Cardinality $|\tilde{A}| = \Sigma \mu(x)$

$= 0.4 + 0.3 + 0.5 + 0.4 + 0.8$

$= 2.4$

$|U| = 10$

Relative Cardinality $= \frac{|A|}{|U|} = \frac{2.4}{10} = 0.24$

ii) $\tilde{C} = \{ \{x, \mu_c(x)\} \ \mu_c = 1 \frac{1}{10}, \ x \in \{0,1,2 \ldots 10\}\}$

$$|\tilde{C}| = 10\left(1 - \frac{1}{10}\right) = 10 \cdot \frac{9}{10} = 9$$

$$\text{Relative Cardinality} = \frac{|\tilde{C}|}{|U|} = \frac{9}{10} = 0.9$$

b) Determine α-level and strong α-level set for the following fuzzy sets

$\tilde{C} = \{ (x, \mu_c(x)) \mid x \in \mathbb{R} \}$ where

$$\mu_c(x) = \begin{cases} 0 & x \leq 10 \\ \dfrac{1}{1+(x-10)^2} & x \geq 10 \end{cases}$$

for $\alpha = 0, 0.3, 0.5, 0.8, 1$

i) $\alpha = 0$

$\tilde{C}^{\alpha} = \{x \mid x \in \mathbb{R}\}$ at $\alpha = 0$

$\tilde{C}^{0+} = \{ x \mid \mu_x(x)\} > 0$

$$\frac{1}{1+(x-10)^2} > 0$$

$$\Rightarrow x \geq 10$$

$\tilde{C}^{0+} = \{ x \mid x \geq 10, \ x \in \mathbb{R}\}$

ii) $\alpha = 0.3$

$$\tilde{C}^{0.3} = \{ x \mid \mu_C(x) > 0.3 ,\ x \in \mathbb{R} \}$$

$$\frac{1}{1+(x-10)^2} \geq 0.3$$

$$1 + (x-10)^2 \leq \frac{10}{3}$$

$$(x-10)^2 \leq \frac{7}{3}$$

$$-\frac{\sqrt{7}}{\sqrt{3}} \leq (x-10) \leq \sqrt{\frac{7}{3}}$$

$$10 - \sqrt{\frac{7}{3}} \leq x \leq 10 + \sqrt{\frac{7}{3}}$$

also $x \geq 10$

$$x \in \left[10,\ 10 + \sqrt{\frac{7}{3}}\right]$$

$$\therefore \tilde{C}^{0.3} = \left[10,\ 10 + \sqrt{\frac{7}{3}}\right]$$

Similarly $\tilde{C}^{0.3+} = \left[10,\ 10 + \sqrt{\frac{7}{3}}\right)$

iii) $\alpha = 0.5$

$$\tilde{C}^{0.5} = \{ x \mid \mu_C(x) \geq 0.5 \}$$

$$\mu(x) \geq 0.5$$

$$\frac{1}{1+(x-10)^2} \geq \frac{1}{2}$$ and $x \geq 10$

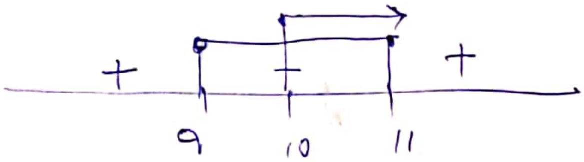
$$1 + (x-10)^2 \leq 2$$

$$(x-10)^2 - 1 \leq 0$$

$(x-10)^2 - 1^2 \le 0$

$(x-10-1)(x-10+1) \le 0$

$(x-11)(x-9) \le 0$



$x \in [10, 11]$

$C^{0.5} \in [10, 11]$ $\therefore$ $C^{0.5+} \in [10, 11)$

iv) $\alpha = 0.8$

$\tilde{C}^{\alpha} = \{x \mid \mu_{\tilde{C}}(x) \ge 0.8\}$

$\mu(x) = \dfrac{1}{1+(x-10)^2} \ge 0.8$

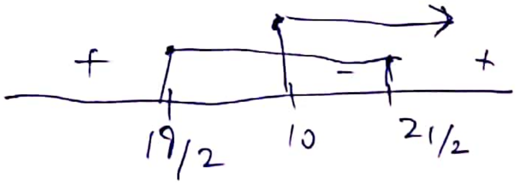
$1+(x-10)^2 \le \dfrac{10}{8} = \dfrac{5}{4}$

$(x-10)^2 \le \dfrac{1}{4}$

$(x-10)^2 - \dfrac{1}{4} \le 0$

$\left(x-10-\tfrac{1}{2}\right)\left(x-10+\tfrac{1}{2}\right) \le 0$

$\left(x-\tfrac{21}{2}\right)\left(x-\tfrac{19}{2}\right) \le 0$



$c^{0.8} \in [10, 10.5]$ $\therefore$ $c^{0.8+} \in [10, \frac{21}{2})$

v) $\alpha = 1$

$\tilde{c}^1 = \{x \mid \mu_c(x) \geq 1\}$

We know that $\mu_c(x) \leq 1$ So, $\mu_c(x) = 1$

$\tilde{c}^1 = \{x \mid \mu_c(x) = 1\}$

$$\frac{1}{1 + (x-10)^2} = 1$$

$$1 + (x-10)^2 = 1$$

$$(x-10)^2 = 0$$

$$x = 10$$

$\tilde{c}^1 = \{10\}$

$\tilde{c}^{1+} = \phi$ as $\mu_c(x) \leq 1$

Q3) Let the fuzzy sets :

Fair $\tilde{F} = \{(2, 0.3), (3, 0.6), (4, 0.9), (5, 1), (6, 0.9), (7, 0.5), (8, 0.1)\}$

Bad $\tilde{B} = \{(1, 1), (2, 0.7), (3, 0.4), (4, 0.1)\}$

be defined on the universe $X = \{1, 2 \ldots\ldots 10\}$

construct membership functions for the following

Compound sets i) Not Fair ii) Not bad iii) Fair but not bad

i) Not Fair

$$\tilde{\tilde{F}} = \{ (1, 1.0), (2, 0.7), (3, 0.4), (4, 0.1), (6, 0.1), (7, 0.5), (8, 0.9), (9, 1.0), (10, 1.0) \}$$

ii) Not Bad

$$\tilde{\tilde{B}} = \{ (2, 0.3), (3, 0.6), (4, 0.9), (5, 1.0), (6, 1), (7, 1), (8, 1), (9, 1), (10, 1) \}$$

iii) Fair but not bad

$$\tilde{F} \cap \tilde{B}$$

$$\mu_{\tilde{F} \cap \tilde{B}} = \min(\mu_{\tilde{F}}, \mu_{\tilde{B}})$$

$$= \{ (2, 0.3), (3, 0.6), (4, 0.9), (5, 1), (6, 0.9), (7, 0.5), (8, 0.1) \}$$

Q4) Consider the fuzzy sets $\tilde{A}, \tilde{B}, \tilde{C}$ defined on the universe $[0, 10]$ of real numbers by membership functions

$$\mu_A(x) = \frac{x}{x+2} \qquad \mu_B(x) = 2^{-x} \qquad \mu_C(x) = \frac{1}{x + 10(x-2)^2}$$

Determine the membership function of $c(\tilde{C})$

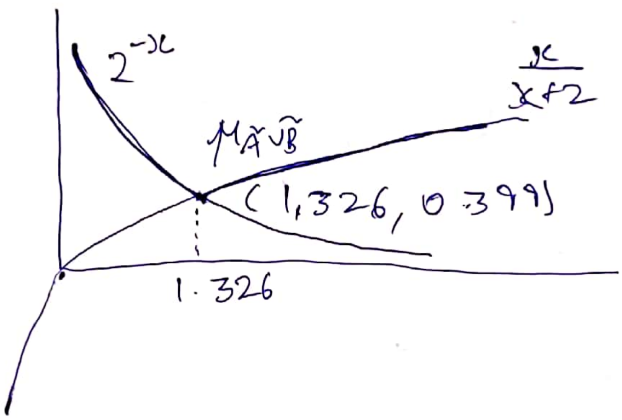
$\tilde{A} \cup \tilde{B}$, $\tilde{A} \cup c(\tilde{C})$, $\tilde{A} \cap \tilde{C}$,

$$\mu(\tilde{C}) = 1 - \frac{1}{x + 10(x-2)^2}$$

$$\frac{x + 10(x-2)^2 - 1}{x + 10(x-2)^2}$$

$$\mu_{\tilde{A} \vee \tilde{B}} = \max(\mu_{\tilde{A}}, \mu_{\tilde{B}})$$

$$\max\left(\frac{x}{x+2}, \frac{1}{2^x}\right)$$



$$\tilde{A} \cup C(\tilde{C}) \quad \max\left\{\frac{x}{x+2}, 1 - \frac{1}{x + 10(x-2)^2}\right\}$$

Seeing point of crossing

$$\frac{x}{x+2} = 1 - \frac{1}{x + 10(x-2)^2}$$

$$\frac{x}{x+2} = \frac{x + 10(x-2)^2 - 1}{x + 10(x-2)^2}$$

$$x[x + 10(\cancel{2} - x)^2] = [x + 10(x-2)^2 - 1](x+2)$$

$$x^2 + 10x(\cancel{2} - x)^2 = x^2 + 2x + 10(x-2)^2 x + 20(x-2)^2 - x - 2$$

$$0 = x + 20(x-2)^2 - 2$$

$$20\left[x^2 - 4x + 4\right] + x - 2 = 0$$

$$20x^2 - 80x + 80 + x - 2 = 0$$

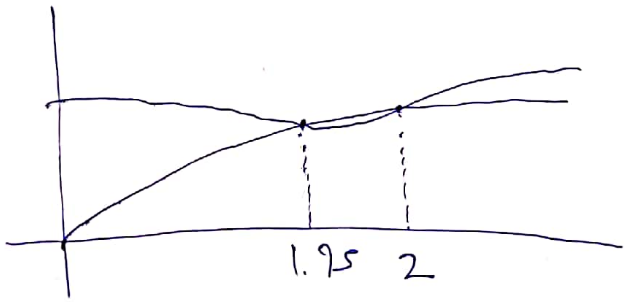
$$20x^2 - 79x + 78 = 0$$

$$x = \frac{79 \pm \sqrt{(79)^2 - 4 \cdot 20 \cdot 78}}{2 \cdot 20}$$

$$= \frac{79 \pm \sqrt{5681}}{40}$$

$$= \cancel{2.29}\ 1.95, 2$$

So, the curves intersect at 2 points :-



So, the characteristic function will be

$$\mu(x) = \begin{cases} \dfrac{x}{x+2} & \cancel{x \in [0, 1.95] \cup [} \\ & x \in [1.95, 2] \\ 1 - \dfrac{1}{x + 10(x-2)^2} & x \in [0, 1.95] \cup [2, 10] \end{cases}$$

iii) $\tilde{A} \cap \tilde{C}$ $\quad \min\left\{ \frac{x}{x+2}, \frac{1}{x+10(x-2)^2} \right\}$

Finding points of intersection :-

$$\frac{x}{x+2} = \frac{1}{x+10(x-2)^2}$$

$$\cancel{\frac{x}{x+2}} = \quad x[x+10(x-2)^2] = x+2$$

$$x^2 + 10x(x-2)^2 = x+2$$

$$x^2 + 10x[x^2 - 4x + 4] = x+2$$

$$x^2 + 10x^3 - 40x^2 + 40x - x - 2 = 0$$

$$10x^3 - 39x^2 + 39x - 2 = 0$$

The points of intersection are:-

$$x = \{0.054, 1.846, 2\}$$

$$\mu_{\tilde{A}\cap\tilde{C}} = \begin{cases} \frac{x}{x+2} & x \in [0, 0.054] \cup [1.846, 2] \\ \frac{1}{x+10(x-2)^2} & x \in [0.054, 1.846] \cup [2, 10] \end{cases}$$

iv) $c(\tilde{A} \cap \tilde{C}) = \begin{cases} 1 - \frac{x}{x+2} & x \in [0, 0.054) \cup [1.1846, 2) \\ 1 - \frac{1}{x+10(x-2)^2} & x \in [0.054, 1.846) \cup [2, 10] \end{cases}$

Scanned with CamScanner

Q5) Compute i) $\tilde{A}+\tilde{B}$ and ii) $\tilde{A}\times\tilde{C}$ where $\tilde{A}, \tilde{B}, \tilde{C}$ are triangular fuzzy numbers defined as :-

$\tilde{A} = (2.5, 3, 3.5)$ $\tilde{B} = (3.5, 4, 4.5)$

$\tilde{C} = (1.5, 2, 2.5)$

i) $\tilde{A}+\tilde{B} = (2.5+3.5, 3+4, 3.5+4.5)$

$= (6, 7, 8)$

ii) $\tilde{A}\times\tilde{C} = (2.5\times 1.5, 2\times 3, 3.5\times 2.5)$

$= (3.25, 6, 8.75)$

Q6) Find $\tilde{x}$ such that $\tilde{A} \otimes \tilde{X} = \tilde{B}$ where $\tilde{A} = (1, 3, 4)$ and $\tilde{B} = (2, 12, 48)$

$\tilde{A} \otimes \tilde{x} = \tilde{B}$

Let $\tilde{x} = (x_1, x_2\ x_3)$

$\tilde{A}_\alpha = [1+2\alpha, 4-\alpha]$ $\tilde{x} = [x_1 + \alpha(x_2 - x_1)]$

$\tilde{B}_\alpha = [2+10\alpha, 48-36\alpha]$

$x_3 + \alpha(x_2 - x_3)$

$\min A_\alpha \circ X_\alpha = \min B_\alpha$

$[(1+2\alpha)(x_1+\alpha(x_2-x_1))(1+2\alpha)(x_3+\alpha(x_2-x_3)),$

$(4-\alpha)(x_1+\alpha(x_2-x_1)), (4-\alpha)(x_3+\alpha(x_2-x_3))]$

$= [2+10\alpha, 48-36\alpha]$

Applying min, max at $\alpha = 0$

$\min[x_1 x_3\ 4x_1, 4x_3] = 2 \Rightarrow x_1 = 2$

$\max[x_1 x_3\ 4x_1\ 4x_3] = 48 \Rightarrow x_3 = 12$

For $\alpha = 1$, $\min 3x_2 = 12 \Rightarrow x_2 = 4$

$\Rightarrow \therefore \tilde{x} = (2, 4, 12)$

Q7) If $\tilde{A} \oplus \tilde{X} = \tilde{B}$ Find $\tilde{x}$ where $\tilde{A} = (1, 2, 4, 5)$ and $\tilde{B} = (2, 3, 5, 6)$

$A^\alpha = [1 + 2\alpha, 5 - \alpha]$

$B^\alpha = [2 + \alpha, 6 - \alpha]$

Let $\tilde{x} = [x_1, x_2\ x_3\ x_4]$

$x^\alpha = [x_1 + \alpha(x_2 - x_1),\ x_4 + \alpha(x_3 - x_4)]$

$A^\alpha \otimes x^\alpha = B^\alpha$

$[(1+2\alpha)(x_1 + \alpha(x_2 - x_1)),\ (1+2\alpha)(x_4 + \alpha(x_3 - x_4)),$

$(5-\alpha)(x_1 + \alpha(x_2 - x_1)),\ (5-\alpha)(x_4 + \alpha(x_3 - x_4))$

$(2+\alpha, 6-\alpha)$ —— (1)

$\min(x_1, x_4, 5x_1, 5x_4) = 2$

$x_1 = 2$

$\max(x_1\ x_4\ 5x_1\ 5x_4) = 6$

$x_4 = \frac{6}{5}$

Scanned with CamScanner

$(3x_2, 3x_3, 4x_2, 4x_3) = (3, 5)$

taking minimum

$3x_2 = 3$

$\boxed{x_2 = 1}$

taking maximum

$\boxed{x_3 = \frac{5}{4}}$

$\Rightarrow \tilde{x} = (2, 1, \frac{5}{4}, \frac{6}{5})$

since $2 > 1$, no solution exists

Q8) Find the best approximate real numbers x for fuzzy equation

$F(+) = (0,2,3) + x = (5,6,7)$

$F(x) = (0,2,3) + x = (5,6,7)$

$(0+x_1, 2+x_2, 3+x_3) = (5,6,7)$

$\Rightarrow x_1 = 5, \; x_2 = 4 \;\; x_3 = 4$

$(5,4,4) \Rightarrow$ No solution exists for this.

Q9) Prove that multiplication and division of 2 trapezoidal Fuzzy numbers may not be trapezoidal Fuzzy number. Discuss with example

Let $\tilde{A} = (-1, 2, 4, 5)$

$\tilde{B} = (-3, 1, 4, 5)$

Now $\tilde{A} \times \tilde{B} = (3, 2, 16, 25)$

Since $x_1 > x_2 \Rightarrow \tilde{A} \times \tilde{B}$ is not a fuzzy set

Again $\tilde{A} = (-2, -1, 3, 4)$

$\tilde{B} = (-1, 2, 3, 4)$

$\tilde{A} / \tilde{B} = (2, -\frac{1}{2}, 1, 1)$

Since $x_1 > x_2$

$\Rightarrow \tilde{A} / \tilde{B}$ is not a fuzzy set

Hence multiplication ~~ad~~ and division of a trapezoidal fuzzy numbers may not be trapezoidal fuzzy numbers

Q10) Prove that multiplication and division of 2 triangular fuzzy numbers may not be triangular fuzzy numbers. Discuss with an example also.

Let $\tilde{A} = (-1, 2, 3)$

$\tilde{B} = (-2, \frac{1}{2}, 3)$

$\tilde{A} \times \tilde{B} = (2, 1, 9)$

This is not a triangular Fuzzy number

Now, $\tilde{B}/\tilde{A} = (2, \frac{1}{4}, 1)$

This is also not a triangular fuzzy number.

Hence multiplication and division of triangular fuzzy numbers may not be a fuzzy number.

Q11) Draw the graphs of fuzzy sets whose membership functions are defined as follows:-

i) $$\begin{cases} 0 & x \leq 1 \\ \frac{x-1}{3} & 1 < x \leq 4 \\ 1 & x \geq 4 \end{cases}$$