Geometrical Transforms

Translation

$x^* = x + \Delta x$
$y^* = y + \Delta y$
$z^* = z + \Delta z$

transformation matrix

$$\begin{bmatrix} x^* \\ y^* \\ z^* \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \Delta x \\ 0 & 1 & 0 & \Delta y \\ 0 & 0 & 1 & \Delta z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Scaling

$$T = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation (Z axis)

$$T = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Along X-axis

$$T'' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Unitary Transforms ⟹ Walsh transforms

$N = 2^n$; $8 = 2^3$, $n = 3$

$$W(x,u) = \frac{1}{N} \prod_{i=0}^{n-1} (-1)^{b_i(x) . b_{n-1-i}(u)}$$

walsh

Kernel of transform

$b_k(z) = b_k(101)$

$z = 5 - (101)$

$b_0(5) = 1$
$b_1(5) = 0$
$b_2(5) = 1$

kth bit of bin repr.. of z

$n = 3$
$N = 8$

$$W(0,0) = \frac{1}{8} . \left[\prod_{i=0}^{2} (-1)^{b_i(0) b_{2-i}(0)} \right.$$

u \ x	0	1	2	3	4	5	6	7
0	+1	+1	+1	+1	+1	+1	+1	+1
1	+1	+1	+1	+1	−1	−1	−1	−1
2								
3								
4								
5								
6								

$$w(0,1) = \frac{1}{8}\left[\prod_{i=0}^{2} (-1)^{b_i(0)\, b_{2-i}(1)}\right]$$

Cosine Transform

$$C(u) = \alpha(u) \sum_{x=0}^{N-1} f(x) \cos\left(\left[\frac{(2x+1)u\pi}{2N}\right]\right)$$

$u = 0, \dots N-1$

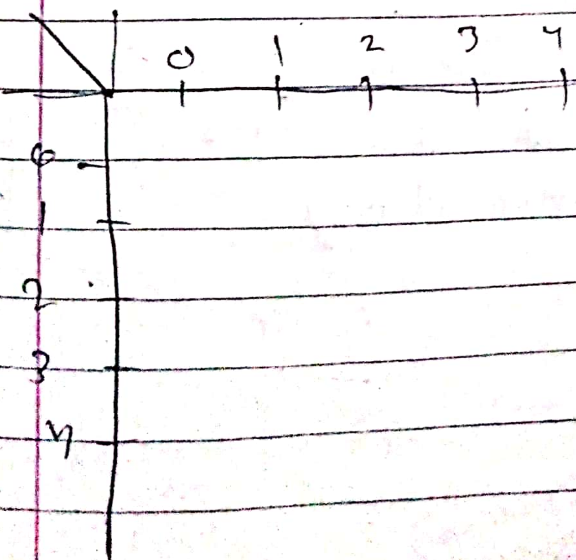
without $f(x)$, it is kernal

$C_K(u) = \alpha(u) \sum \cos(\quad)$

$\alpha(u) = \frac{1}{\sqrt{N}}$, ~~[illegible]~~ $N = 8$

$\alpha(0) = \alpha(u) \sum \cos(0) = 1$

$\alpha(1) =$



Hoteling Transform

$$[X] = \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ \vdots \\ x_{N-1} \end{bmatrix}$$

$$m_x = E\,[X]$$

$$C_K = \left[(x - m_x).\,(x - m_x)^T \right]$$

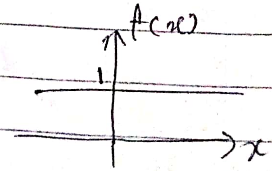
1. Real
2. Symmetrical

x_i & x_j are uncorrelatable

~~[illegible]~~ . $C_{ij} = C_{ji}$

$$m_x = \frac{1}{m} \sum_{i=0}^{m-1} x_i \cdot f(x)$$

If your $f(x)$ is a UDF (Uniformly Distributed Function)



$S.A = E.A$

$y = A\,(x - m_x)$

Addition

4(Standard Deviation)

7W3 GFT

Transformation - normal Mat^x.

$A.\underbrace{\Sigma_x}_{\text{Eigen Value}} A^T =$ Diagonal Matrix

MSE

x & $\bar{x}$

$$\sum_{j=1}^{n} \lambda_j - \sum_{j=1}^{K} \lambda_j \; ; \; K < n$$

$$= \sum_{j=K+1}^{n} \lambda_j$$

K-L Transform

let.

$$C_x = E\left[(x - m_x)(x - m_x)^T\right]$$

$A \rightarrow$ Transformation Matrix.

$A C_x A^T \Rightarrow$ Eigen Vector

$\Downarrow$

considering few top eigen Vectors

Principal components.

Edge Detection

Template matching

Differential Gradient based.

$$g = \max[g_i] ; \ i = 1, \ldots, N$$

$$g = \sqrt{g_x^2 + g_y^2}$$

$$g_x = \max[g_{x_i}] ; \ i = 1 \ldots N$$
$$g_y = \max[g_{y_i}] \ \ i = 1 \ldots N$$

$$\theta = \arctan\left(\frac{g_y}{g_x}\right)$$

Sobel operator

$$S_x = \begin{bmatrix} -1 & 0 & -1 \\ -2 & 0 & 2 \\ -1 & 0 & -1 \end{bmatrix}$$

$$S_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

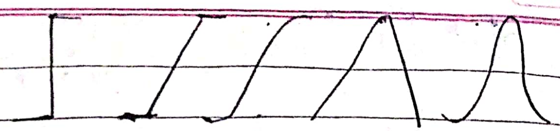
Prewitt operator

0°

$$\begin{bmatrix} -1 & 1 & 1 \\ -1 & 2 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

45°

$$\begin{bmatrix} 1 & 1 & 1 \\ -1 & 2 & 1 \\ -1 & -1 & 1 \end{bmatrix}$$



$$0^\circ \qquad \begin{bmatrix} -A & 0 & A \\ -B & 0 & B \\ -A & 0 & A \end{bmatrix} \qquad\qquad 45^\circ \qquad \begin{bmatrix} 0 & C & D \\ -C & 0 & C \\ -D & -C & 0 \end{bmatrix}$$

Step Edge

0^{th} deg. response ~~for 0° mass~~ using step edge

0° response = $2A + B$

45° response using 0^{th} deg mass for step edge

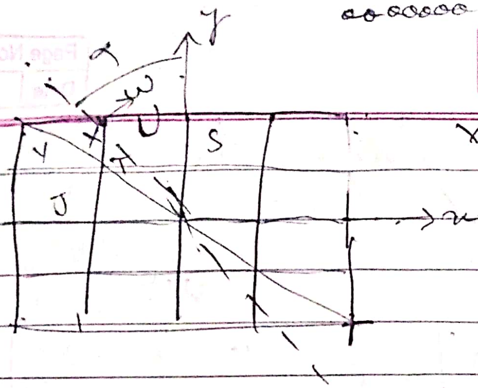
45° response = $A + B$

0° response using 45° mass for step edge

0° response = $C + D$

45° response using 45° ~~deg~~ mass for step edge

45° response = $2C + D$



$x + y + \omega = 1$

$x + y = 1 - \omega$

Q. x° response

$= (1 + x + y - \omega)A + B$

$= 2(1-\omega)A + B$

x° response $= (1 + T - V - U)C + D$

SE: x

APPRAISAL MODEL

SELECTION OF LIFECYCLE MODEL

SELECTION OF MODEL FOR REQT.

Topics covered

CV

Edge detecⁿ

Data Vs Mask Generation

Let the data be $D = A/\sqrt{2}$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \Rightarrow g_{45^\circ} = \frac{g_{0^\circ} + g_{90^\circ}}{\sqrt{2}}$$

$$C = B/\sqrt{2}$$

$$g_{0^\circ} = A(c+i-a-g) + B(f-d)$$

$$g_{90^\circ} = A(a+c-g-i) + B(b-h)$$

$$g_{45^\circ} = C(b+f-d-h) + D(c-g)$$

If we use the step edge orientaⁿ In the neighbourhood of centre pixel then the earlier approach is fine but in practical.

~~If we assume all the~~

Circular op.

1 ~~This is~~ C.O which is one of way to remove the error & restrict Observation of the edge to a circular neighbour.

2 If we need to ↑ the no. of pixels in neigh ~~2~~ we have to go

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beyond

3 Another concept which can replace the ⇒ as labeling ⇒ clustering

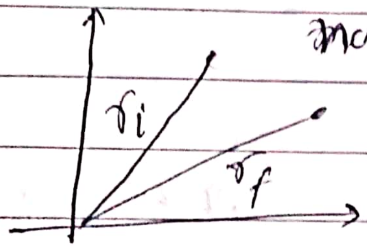
Metric Properties

Distance $d(i, j)$

Euclidean $= d(r_i\ r_j) > 0$

$i \neq j$

normalized norm



$$d(r_i, r_j) = d(r_j, r_i)$$

distances must follow the triangle rule.

$$d(r_i, r_j) + d(r_j, r_k) \geq d(r_i, r_k)$$

Convex hull : convex hull algo.

 ⇒ Morphological operators

 structuring elements.

CV

morphological operators/function/transforms

Dilation

Erosion

opening

closing.

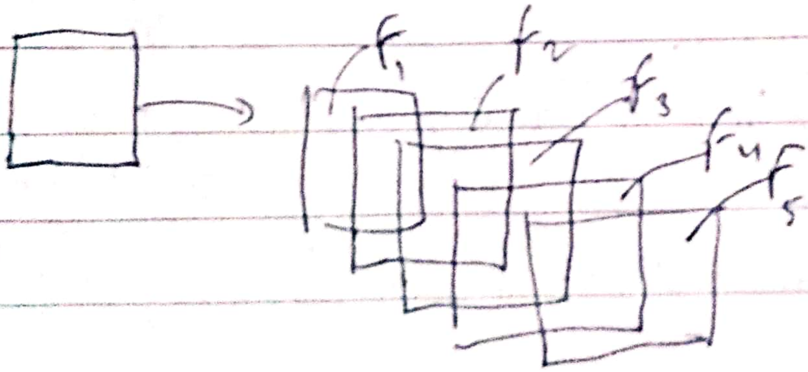
app$^n_2$

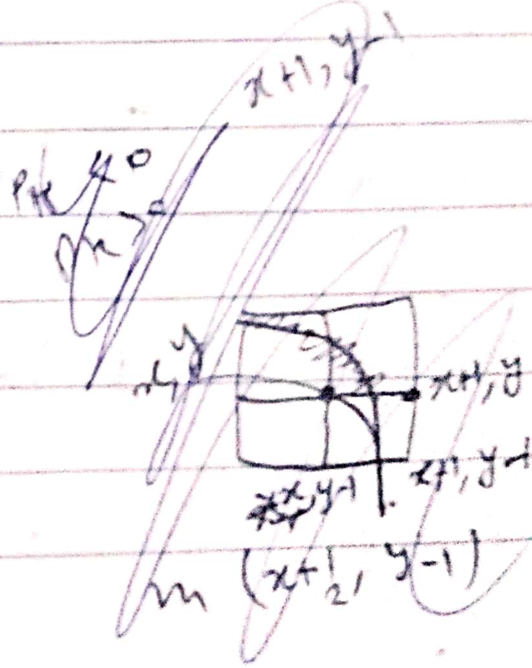
enhancement

Take a GLI

Break the Gray O levels into grps

-10, 11-20, ..)





Enhancement
Histogram - Equilization

g — no. of gray levels of an image

$\Downarrow$

Normalization to $\{0, 1\}$

$T(g) \Rightarrow$ Monotonically $\uparrow$ function

Single valued function

$0 < T(g) < 1, \quad 0 < g < 1$

Let $x = T(g)$ Enhanced image

$$g = T^{-1}(x)$$

T^{-1} .. also produces which the

$T(g)$. 1

T^{-1} & T should follow same condⁿ

$p_g(g) \Rightarrow$ pdf of original image

$p_x(x) \Rightarrow$ pdf of Transformed image

$$p_x(x) = \left| p_g(g) \cdot \frac{dg}{dx} \right| \quad - (1)$$

$\because \; g = T^{-1}(x).$

$$= p_g(g) \cdot \frac{1}{p_g(g)}$$

The cum prob of usual Gray levels. in some books, they might use a dummy variable (eg. a)

$$x = T(g) = \int_0^{g_0} p_g(a)\, da \longrightarrow ②$$

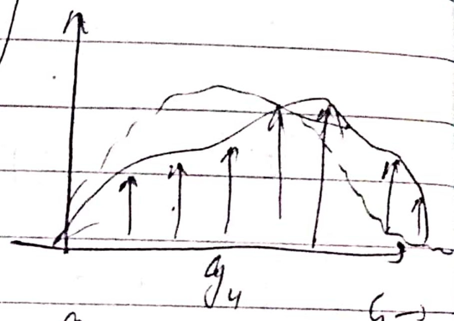
PDF

CDF

$0 < g < 1$

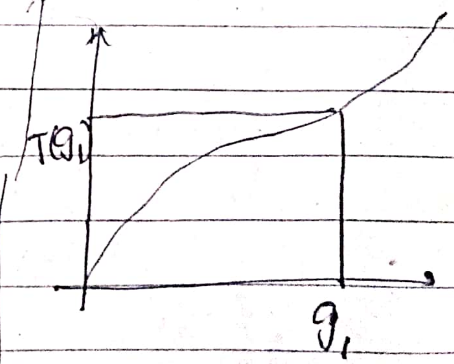
If now if

$$\frac{dx}{dg} = p_g(g) \longrightarrow ③$$



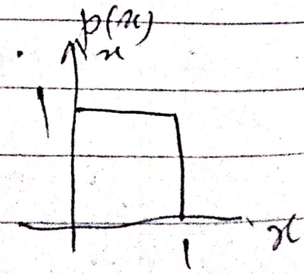
$$\int_0^{g_4} h(g)\, dg = \text{CDF}$$

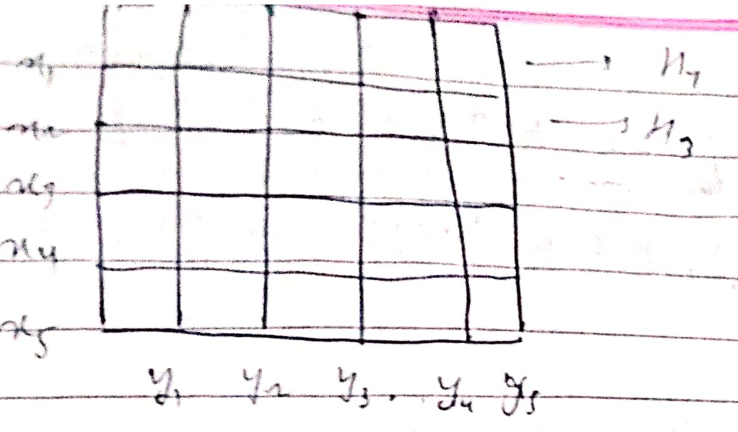
at pdf of histogram equilised



Lateral Histogram is lat. Histo., the way of projecting an image on two or more than 2 axes

UDF .





$H_{y_1}, H_{y_2}, H_{y_3}, \ldots$

- 1 Discrete Analysis
- 2 Complexity
- 3. Lat. Histo. (in general) are used in object identification (recognition).

Let the image be of size $N \times N$

let ~~the~~ object inside image of size $(n \times n)$

Template Matching (TM) = $N \cdot n$.

$$R_{LAT} = 2aN^2 + 2bN \cdot n + cp^2 n^2 \, (g_i)$$

$$a = b = c$$

p: no. of objects in the image

$N \times N = 256 \times 256$, $n \times n = 7 \times 7$

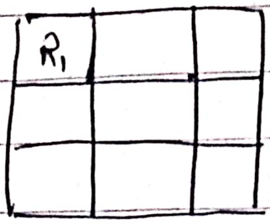
no. of operations using $\overset{\text{lat}}{\text{Histo}}$ equal to

$p = 20$

$$= a\left[2N^2 + 2N.n + p^2 n^2\right]$$

$$= a\left[2.256^2 + 2.256.7 + 26^2.7^2\right]$$

Sub-image Analysis



$= n$ parts

No. of objects, in Sub-image α (Part-Size)

$$= K.(\text{Part-Size})^2$$

Size : $\bar{N} \times \bar{N}$; $\bar{N} < N$

$$\alpha = N/\bar{N} > 1 \;;\; \bar{p} = p/\alpha^2$$

$$\bar{R}_{LAT} = \alpha(2\bar{N}^2 + 2\bar{N}.n + \bar{p}^2 n^2)$$

$$= \alpha\left[\frac{2N^2}{\alpha^2} + \frac{2N.n}{\alpha} + \frac{p^2 n^2}{\alpha^2\alpha^2}\right]$$

$$= \frac{\alpha}{\alpha^2}\left[2N^2 + 2\alpha N.n + \frac{p^2 n^2}{\alpha^2}\right]$$

complexity ↓ because of loss of info. as T become nearly O(1)

1. Index Long → loss of info.
2. [illegible]
3. α should be such that it accomodate the size of the object

Optimum size

$$\bar{R}_{LAT} = \frac{\alpha}{2}\left[2N^2 + 2\alpha N n + \frac{p^2 n^2}{\alpha^2}\right]$$

$$\frac{d\bar{R}_{LAT}}{d\alpha} = 2Nn - \frac{2p^2 n^2}{\alpha^3} = 0$$

$$\Rightarrow \alpha^3 = \frac{2p^2 n^2}{2Nn}$$

If $\alpha = 1$, (+ve)

~~If~~ $\alpha = 1$, then for $p = \sqrt{2}\, N/n$.

Adjust α such that

$\bar{N} = n.\bar{p}$

$\bar{N} = n$

$\alpha = np/N$

(size)