Topic ASSIGNMENT - 2 Date

1. Let $X\ Y \subseteq R$ and

$\tilde{A} = \{ (x, \mu_A(x)) \mid x \in X \}$

$\tilde{B} = \{ (y, \mu_B(y)) \mid y \in Y \}$

be two fuzzy sets then

$\tilde{R} = \{ [(x,y), \mu_R(x,y)] \mid (x,y) \in X \times Y \}$ is a fuzzy relⁿ. on $\tilde{A}$ and $\tilde{B}$ if

$\mu_R(x,y) \leq \min(\mu_A(x), \mu_B(y))$, $\forall (x,y) \in X \times Y$

$\tilde{A} = \{ (a_1, 0.2), (a_2, 0.4), (a_3, 0.6) \}$

$\tilde{B} = \{ (b_1, 0.3), (b_2, 0.4), (b_3, 0.5), (b_4, 0.2) \}$

$\tilde{R} = \{ ((a_1, b_1), 0.2), ((a_1, b_2), .2), ((a_1, b_3), 0.2), ((a_1, b_4), 0.2), ((a_2, b_1), 0.3), ((a_2, b_2), 0.4), ((a_2, b_3), 0.4), ((a_2, b_4), 0.2), ((a_3, b_1), 0.3), ((a_3, b_2), 0.4), ((a_3, b_3), 0.5), ((a_3, b_4), 0.2) \}$

We can represent above relation in matrix form

$$\tilde{R} = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \end{array} \begin{array}{c} \begin{array}{cccc} b_1 & b_2 & b_3 & b_4 \end{array} \\ \begin{bmatrix} .2 & .2 & .2 & .2 \\ .3 & .4 & .4 & .2 \\ .3 & .4 & .5 & .2 \end{bmatrix} \end{array}$$

2. Let $X, Y \subseteq R$

$\tilde{A} = \{ (x, \mu_A(x)) \mid x \in X \}$

$\tilde{B} = \{ (y, \mu_B(y)) \mid y \in Y \}$

be two fuzzy sets

and let $\tilde{R} = \{ [(x,y), \mu_R(x,y)] \mid (x,y) \in X \times Y \}$ be a fuzzy relation on $\tilde{A}$ and $\tilde{B}$

i) Given that $\tilde{R}$ is symmetric prove R^{-1} is symmetric

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Fuzzy Relation $\tilde{R}$ is symmetric iff $\forall (x,y) \in X \times Y$
$\mu_R(x,y) = \mu_R(y,x)$

The inverse of a fuzzy relation $\tilde{R}$ is R^{-1} ~~where~~ $= \{(y,x), \mu_{R^{-1}}(y,x)\}$
$\forall \ ((x,y), \mu_R(x,y)) \in \tilde{R}$

$\because \tilde{R}$ is symmetric, $\mu_R(x,y) = \mu_R(y,x)$ —— ①

$\because \tilde{R}^{-1}$ is inverse of $\tilde{R}$, $\mu_R(x,y) = \mu_{R^{-1}}(x,y)$ —— ②

From 1 and 2, we can infer that

ie $\mu_{R^{-1}}(x,y) = \mu_{R^{-1}}(y,x) \quad \forall ((x,y), \mu_{R^{-1}}(x,y)) \in R^{-1}$

$\Rightarrow R^{-1}$ is symmetric

<u>Proved</u>

ii Given for the fuzzy rel^n $\tilde{R}$, if $\tilde{R} = \tilde{R}^{-1}$
prove that R is symmetric

We know that,

for a rel^n $\tilde{R}$ to be symmetric, $\forall (x,y) \in X \times Y$
$\mu_R(x,y) = \mu_R(y,x)$

Given that $\tilde{R} = \tilde{R}^{-1}$

and $\forall \{((x,y), \mu_R(x,y)), \exists ((y,x), \mu_{R^{-1}}(y,x))$

Let $((x,y), \mu_R(x,y)) \in \tilde{R}$

$\therefore ((x,y), \mu_R(x,y)) \in \tilde{R}^{-1}$ as $\tilde{R} = \tilde{R}^{-1}$

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3. We are given Relation $\tilde{R}_1$

$$\tilde{R}_1 = \begin{array}{c|ccccc} & x_1 & x_2 & x_3 & x_4 & x_5 \\ \hline x_1 & 1 & .8 & 0 & .1 & .2 \\ x_2 & .8 & 1 & .4 & 0 & .9 \\ x_3 & 0 & .4 & 1 & 0 & 0 \\ x_4 & .1 & 0 & 0 & 1 & .5 \\ x_5 & .2 & .9 & 0 & .5 & 1 \end{array}$$

As (x_1, x_1), (x_2, x_2), (x_3, x_3), (x_4, x_4), $(x_5, x_5) = 1$

Above relⁿ is reflexive

Moreover as $\mu_{\tilde{R}_1}(x_1, x_4) = \mu_{\tilde{R}_1}(x_4, x_1) = 0.1$

$\mu_{\tilde{R}_1}(x_2, x_5) = \mu_{\tilde{R}_1}(x_5, x_2) = 0.9$

$\mu_{\tilde{R}_1}(x_1, x_2) = \mu_{\tilde{R}_1}(x_2, x_1) = .8$

$\mu_{\tilde{R}_1}(x_1, x_3) = \mu_{\tilde{R}_1}(x_3, x_1) = 0$

Similarly for all $(x_i, x_j) \in \tilde{R}_1$ ~~such that $i \neq j$~~

$\mu_{\tilde{R}_1}(x_i, x_j) = \mu_{\tilde{R}_1}(x_j, x_i)$

$\therefore$ Above relation is symmetric

To check for transitivity let us assume $\tilde{R}_1(x_1, x_2)$ and $\tilde{R}_1(x_2, x_3)$ with λ_1 and λ_2 their membership values respectively.

From above matrix $\tilde{R}_1$

$\lambda_1 = 0.8$, $\lambda_2 = 0.4$

Now assuming $\tilde{R}_1(x_1, x_3)$ from matrix with λ as its membership value, we see $\lambda = 0$.

The above relⁿ will be transitive if following inequality holds

$$\lambda \geq \min(\lambda_1, \lambda_2)$$

$$\Rightarrow 0 \geq \min(0.8, 0.4)$$

$$\Rightarrow 0 \geq 0.4 \quad \text{which is not true}$$

$\therefore$ Above relⁿ is not an equivalence relⁿ.

But the given relⁿ is a proximity relⁿ.

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4. The membership function of two fuzzy relation $\tilde{R}$ and $\tilde{S}$ are given by

$$\mu_{\tilde{R}} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \begin{bmatrix} .8 & 1 & .1 & .7 \\ 0 & .8 & 0 & 0 \\ .9 & 1 & .7 & .8 \end{bmatrix} \end{array} \qquad \mu_{\tilde{S}} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \begin{bmatrix} .4 & 0 & .9 & .6 \\ .9 & .4 & .5 & .7 \\ .3 & 0 & .8 & .5 \end{bmatrix} \end{array}$$

i) $$\mu_{\tilde{R} \cap \tilde{S}} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \begin{bmatrix} .4 & 0 & .1 & .6 \\ 0 & .4 & 0 & 0 \\ .3 & 0 & .7 & .5 \end{bmatrix} \end{array}$$

ii) $$\mu_{\tilde{R} \cup \tilde{S}} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \begin{bmatrix} .8 & 1 & .9 & .7 \\ .9 & .8 & .5 & .7 \\ .9 & 1 & .8 & .8 \end{bmatrix} \end{array}$$

iii) $$\mu_{\tilde{R}^c} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \begin{bmatrix} .2 & 0 & .9 & .3 \\ 1 & .2 & 1 & 1 \\ .1 & 0 & .3 & .2 \end{bmatrix} \end{array}$$

iv) $$\mu_{\tilde{S}^c} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \begin{bmatrix} .6 & 1 & .1 & .4 \\ .1 & .6 & .5 & .3 \\ .7 & 1 & .2 & .5 \end{bmatrix} \end{array}$$

5. Matrix Representation of R_1 is

$$\begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccccc} y_1 & y_2 & y_3 & y_4 & y_5 \end{array} \\ \begin{bmatrix} .1 & .2 & 0 & 1 & .7 \\ .3 & .5 & 0 & .2 & 1 \\ .8 & 0 & 1 & .4 & .3 \end{bmatrix} \end{array}$$

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Similarly for $\tilde{R}_2$

$$\begin{array}{c|cccc} & z_1 & z_2 & z_3 & z_4 \\ \hline y_1 & .9 & 0 & .3 & .6 \\ y_2 & .2 & 1 & .8 & 0 \\ y_3 & .8 & 0 & .7 & 1 \\ y_4 & .4 & .2 & .3 & 0 \\ y_5 & 0 & 1 & 0 & .8 \end{array}$$

$R_3 = \tilde{R}_1 \circ \tilde{R}_2$ where '$\circ$' is the max-min composition

$$R_3 = \begin{array}{c|cccc} & z_1 & z_2 & z_3 & z_4 \\ \hline x_1 & .4 & .7 & .3 & .7 \\ x_2 & .3 & 1 & .5 & .8 \\ x_3 & .8 & .3 & .7 & 1 \end{array}$$

which can be written as

$$\tilde{R}_1 \circ \tilde{R}_2 = \{((x_1, z_1), 0.4), ((x_1, z_2), 0.7), ((x_1, z_3), 0.3), ((x_1, z_4), 0.7), ((x_2, z_1), 0.3), ((x_2, z_2), 1), ((x_2, z_3), .5), ((x_2, z_4), .8), ((x_3, z_1), .8), ((x_3, z_2), 0.3), ((x_3, z_3), 0.7), ((x_3, z_4), 1)\}$$

6. Let $R_0 =$

$$\begin{array}{c|cccc} & x_1 & x_2 & x_3 & x_4 \\ \hline x_1 & .7 & .5 & 0 & 0 \\ x_2 & 0 & 0 & 0 & .1 \\ x_3 & 0 & .4 & 0 & 0 \\ x_4 & 0 & 0 & .8 & 0 \end{array}$$

$R_1 = R_0 \cup (R_0 \circ R_0)$ ['$\circ$' is the max min composition operator]

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$$R_0 \circ R_0 = \begin{array}{cc} & \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} & \begin{bmatrix} .7 & .5 & 0 & .1 \\ 0 & 0 & .1 & 0 \\ 0 & 0 & 0 & .1 \\ 0 & .4 & 0 & 0 \end{bmatrix} \end{array}$$

$$R_0 \cup (R_0 \circ R_0) = \begin{array}{cc} & \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} & \begin{bmatrix} .7 & .5 & 0 & .1 \\ 0 & 0 & .1 & .1 \\ 0 & .4 & 0 & .1 \\ 0 & .4 & .8 & 0 \end{bmatrix} \end{array}$$

$R_2 = R_1 \cup (R_1 \circ R_1)$

$$\therefore R_2 = \begin{array}{cc} & \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} & \begin{bmatrix} .7 & .5 & .1 & .1 \\ 0 & .1 & .1 & .1 \\ 0 & .4 & .1 & .1 \\ 0 & .4 & .8 & .1 \end{bmatrix} \end{array}$$

$\because R_2 \neq R_1$

$$R_3 = R_2 \cup (R_2 \circ R_2) = \begin{array}{cc} & \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} & \begin{bmatrix} .7 & .5 & .1 & .1 \\ .1 & .1 & .1 & .1 \\ 0 & .4 & .1 & .1 \\ 0 & .4 & .8 & .1 \end{bmatrix} \end{array}$$

$\because R_2 \neq R_3$

$$R_4 = R_3 \cup (R_3 \circ R_3) = \begin{array}{cc} & \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} & \begin{bmatrix} .7 & .5 & .1 & .1 \\ .1 & .1 & .1 & .1 \\ 0 & .4 & .1 & .1 \\ 0 & .4 & .8 & .1 \end{bmatrix} \end{array}$$

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$R_3 = R_4$

Hence we can stop here as R_3 gives us the required transitive max-min closure

7.

$$\tilde{R} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array} \begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\ \begin{bmatrix} 1 & .2 & 1 & .6 & .2 & .6 \\ .2 & 1 & .2 & .2 & .8 & .2 \\ 1 & .2 & 1 & .6 & .2 & .6 \\ .6 & .2 & .6 & 1 & .2 & .8 \\ .2 & .8 & .2 & .2 & 1 & .2 \\ .6 & .2 & .6 & .8 & .2 & 1 \end{bmatrix} \end{array}$$

$\mu_R(x_1, x_3) = 1$ $\quad \mu_R(x_3, x_4) = .6$ $\quad \mu_R(x_1, x_4) = 0.6 \not> \min(1, 0.6)$

But $\mu_R(x_1, x_4) = \min(1, 0.6)$

Similarly we can show the same for all the other values

$\therefore$ $\tilde{R}$ is transitive

To get the equivalence relⁿ, we need to follow these steps

1) $k = 0$

2) $R^{k+1} = R^k \circ R^k$

3) If $R^{k+1} \neq R^k$, then $k = k+1$ and repeat from step 2 ~~else proceed to step 4~~

~~4) R~~

Performing the above steps

$k = 0$

$\tilde{R}^0 = \tilde{R}$

$$\tilde{R}^1 = \tilde{R}^0 \circ \tilde{R}^0 = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array} \begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\ \begin{bmatrix} 1 & .2 & 1 & .6 & .2 & .6 \\ .2 & 1 & .2 & .2 & .8 & .2 \\ 1 & .2 & 1 & .6 & .2 & .6 \\ .6 & .2 & .6 & 1 & .2 & .8 \\ .2 & .8 & .2 & .2 & 1 & .2 \\ .6 & .2 & .6 & .8 & .2 & 1 \end{bmatrix} \end{array}$$

$\tilde{R}_1 = \tilde{R}_0$ $\therefore$ We had an equivalence relⁿ to begin with

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$\tilde{R}_\alpha$ at $\alpha = 1, .8, .6, .2$

$$\tilde{R}_{\alpha=1} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array} \begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$\tilde{R}_{\alpha=.6} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array} \begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

$$\tilde{R}_{\alpha=.8} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array} \begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

$R_{\alpha=.2}$ = Unit Matrix

Similarity Tree :-

$\{x_1, x_2, x_3, x_4, x_5, x_6\}$ $\alpha = .2$

$\{x_1, x_3, x_4, x_5\}$ $\{x_2, x_6\}$ $\alpha = .6$

$\{x_1, x_3\}$ $\{x_2, x_5\}$ $\{x_4, x_6\}$ $\alpha = .8$

$\{x_1, x_3\}$ $\{x_2\}$ $\{x_4\}$ $\{x_5\}$ $\{x_6\}$ $\alpha = 1$

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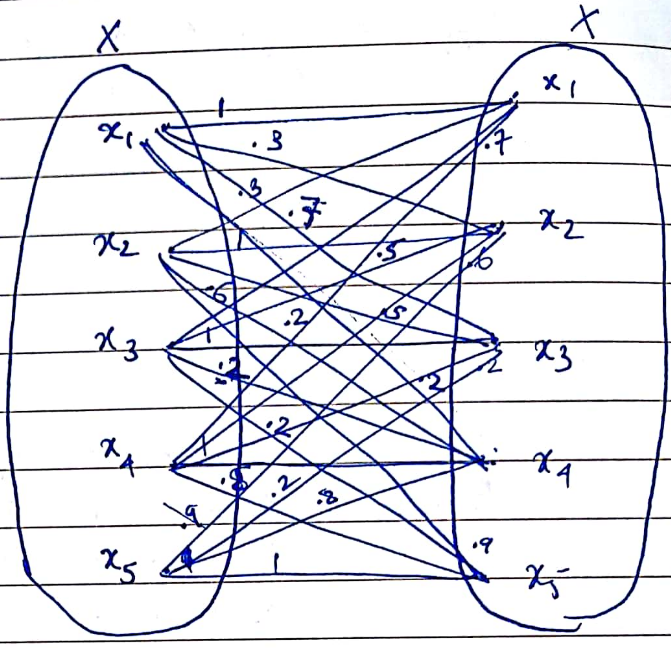
The relation $R(x, x)$ must be reflexive and symmetric to be considered a compatibility rel^n

$$\tilde{R} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{array}\begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \begin{bmatrix} 1 & .3 & .7 & .2 & 0 \\ .3 & 1 & .5 & .6 & .9 \\ .7 & .5 & 1 & .2 & .2 \\ .2 & .6 & .2 & 1 & .8 \\ 0 & .9 & .2 & .8 & 1 \end{bmatrix}\end{array}$$



$x_1 \xrightarrow{1} x_1$ $\quad$ $x_2 \xrightarrow{.3} x_1$

$x_1 \xrightarrow{.3} x_2$ $\quad$ $x_2 \xrightarrow{1} x_2$

$x_1 \xrightarrow{.7} x_3$ $\quad$ $x_2 \xrightarrow{.5} x_2$

$x_1 \xrightarrow{.2} x_4$ $\quad$ $x_2 \xrightarrow{.6} x_2$

$x_1 \xrightarrow{0} x_5$ $\quad$ $x_2 \xrightarrow{.9} x_2$

$x_3 \xrightarrow{.7} x_1$ $\quad$ $x_4 \xrightarrow{.2} x_1$ $\quad$ $x_5 \xrightarrow{0} x_1$

$x_3 \xrightarrow{.5} x_2$ $\quad$ $x_4 \xrightarrow{.6} x_2$ $\quad$ $x_5 \xrightarrow{.9} x_2$

$x_3 \xrightarrow{1} x_3$ $\quad$ $x_4 \xrightarrow{.2} x_3$ $\quad$ $x_5 \xrightarrow{.2} x_3$

$x_3 \xrightarrow{.2} x_4$ $\quad$ $x_4 \xrightarrow{1} x_4$ $\quad$ $x_5 \xrightarrow{.8} x_4$

$x_3 \xrightarrow{.2} x_5$ $\quad$ $x_4 \xrightarrow{.8} x_5$ $\quad$ $x_5 \xrightarrow{1} x_5$

9. $$\tilde{R}_1 = \begin{array}{c} \\ x_1 \\ x_2 \end{array}\begin{array}{c} \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \begin{bmatrix} .3 & 0 & .7 & .3 \\ 0 & 1 & .2 & 0 \end{bmatrix}\end{array} \qquad \tilde{R}_2 = \begin{array}{c} y_1 \\ y_2 \\ y_3 \\ y_4 \end{array}\begin{bmatrix} 1 & 0 & 1 \\ 0 & .5 & 1 \\ .7 & .8 & .6 \\ 0 & 0 & 0 \end{bmatrix}$$

(i) Max Product

$$\tilde{R}_1 \circ \tilde{R}_2 = \begin{bmatrix} 0.49 & 0.56 & 0.42 \\ 0.14 & 0.5 & 0.4 \end{bmatrix}$$

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However

$\mu_R(c,d) \not\leq \mu_g(\beta,\gamma)$

$\therefore$ h is not homomorphic

h is strong homomorphic if

$y_1 = h(x_j)$, $y_2 = h(x_k)$

$$\max_{x_j, x_k} \mu_R(x_j, x_k) = \mu_g(y_1, y_2)$$

Here $\mu_g(\beta,\gamma) = 0$

$h^{-1}(\beta) = c$ $\quad h^{-1}(\gamma) = d$

$\mu_R(c,d) = 0.4 \neq 0$

Therefore h is not strong homomorphic

② $h:$ $a \to \alpha$

$b, c \to \beta$

$d \to \gamma$

Clearly,

$\mu_R(c,d) \leq \mu_g(\beta,\gamma)$ $\quad \because 0.4 \not\leq 0.$

$\therefore$ h is not homomorphic

We have $\mu_g(\beta,\gamma) = 0$

$h^{-1}(\beta) = \{b, c\}$ $\quad h^{-1}(\gamma) = d$

$\max\{\mu_R(b,d), \mu_R(c,d)\} = \max\{.4, 0\} = .4 \neq 0$

$\therefore$ h is not strong homomorphic

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② Max-Avg

$$\tilde{R}_1 \circ \tilde{R}_2 = \begin{bmatrix} .7 & .75 & 0.65 \\ .5 & .75 & .7 \end{bmatrix}$$

10. An element $x \in X$ is undominated iff

$R(x,y) = 0$

for all $y \in X$ and $x \neq y$

An element $x \in X$ is undominating iff

~~$R(x,y) = 0$~~ $R(y,x) = 0$

for all $y \in X$ and $y \neq x$

Here $X = \{a, b, c, d, e\}$

$$\tilde{R} = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} 1 & .7 & 0 & 1 & .7 \\ 0 & 1 & 0 & .9 & 0 \\ .5 & .7 & 1 & 1 & .8 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & .1 & 0 & .9 & 1 \end{bmatrix} \end{array}$$

Undominated elements $= \{d\}$

Undominating elements $= \{c\}$

11.

$$\tilde{R} = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & .5 & 0 & 0 \\ 0 & 0 & .8 & 0 \\ 1 & 0 & 0 & .4 \\ 0 & .4 & 0 & 0 \end{bmatrix} \end{array} \qquad \tilde{g} = \begin{array}{c} \\ \alpha \\ \beta \\ \gamma \end{array} \begin{array}{c} \begin{array}{ccc} \alpha & \beta & \gamma \end{array} \\ \begin{bmatrix} .6 & .8 & 0 \\ 1 & .8 & 0 \\ 1 & 0 & .8 \end{bmatrix} \end{array}$$

1. h : $a, b \to \gamma$

$c \to \beta$

$d \to \gamma$

We can say it is homomorphic if

$\forall (x_1, x_2) \in \tilde{R}$ $(h(x_1), h(x_2)) \in \tilde{g}$

and $\mu_R(x_1, x_2) \leq \mu_g(h(x_1), h(x_2))$

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