**Delhi Technological University**

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# Graph Theory Lab File

**Graph Theory (MC-405)**

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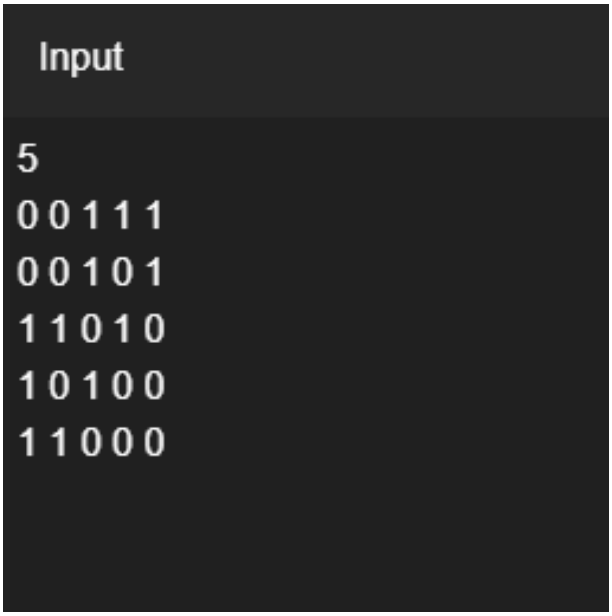
# Contents

# 1 Program to Find the Number of Vertices, Even Vertices, Odd Vertices and Number of Edges in a Graph

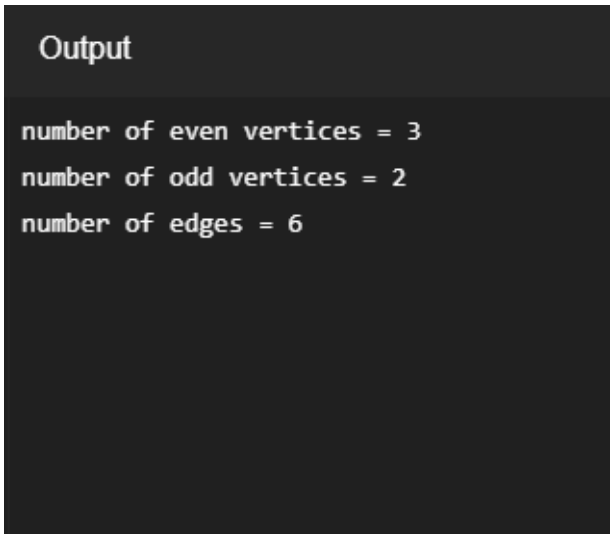
## 1.1 Code

```
// Program to Find the Number of Vertices, Even Vertices, Odd Vertices and
// Number of Edges in a Graph

#include<bits/stdc++.h>

using namespace std;

int main() {
  int n;
  cin >> n;

  int a[n][n];

  for (int i = 0; i < n; i++) {
    for (int j = 0; j < n; j++) {
      cin >> a[i][j];
    }
  }

  int degree[n] = {};

  for (int i = 0; i < n; i++) {
    for (int j = 0; j < n; j++) {
      degree[i] += a[i][j];
    }
  }

  int even = 0, odd = 0, edges = 0;

  for (int i = 0; i < n; i++) {
    even += (degree[i] % 2 == 0);
    odd += (degree[i] % 2 != 0);
    edges += degree[i];
  }

  cout << "number of even vertices" << " = " << even << '\n';
  cout << "number of odd vertices" << " = " << odd << '\n';
  cout << "number of edges" << " = " << edges / 2 << '\n';
}
```

## 1.2 Input/Output




(a) Input




(b) Output

Figure 1: Input and Output for Program 1

# 2 Program to Find Union, Intersection and Ring-Sum of 2 Graphs

## 2.1 Code

```
#include <iostream>
#include <vector>

using namespace std;

bool IsValid(vector<vector<int> >&adj){
    int n=adj.size();
    for(int i=0;i<n;i++){
        for(int j=0;j<n;j++){
            if(adj[i][j]!=adj[j][i]){
                return false;
            }
        }
    }
    return true;
}

void Union(vector<vector<int> >&adj1,vector<vector<int> >&adj2) {
    cout<<"Union ...."<<endl;
    int n1=adj1.size();
    int n2=adj2.size();
    int n=max(n1,n2);
    vector<vector<int> >adj(n,vector<int>(n));
    for(int i=0;i<n;i++){
        for(int j=0;j<n;j++){
            adj[i][j]=((i<n1 && j<n1)?adj1[i][j]:0) || ((i<n2 && j<n2)?adj2[i][j]:0);
            cout<<adj[i][j]<<" ";
        }
        cout<<endl;
    }
    cout<<endl;
}

void Intersection(vector<vector<int> >&adj1,vector<vector<int> >&adj2){
    cout<<"Intersection ..."<<endl;
    int n1=adj1.size();
    int n2=adj2.size();
    int n=min(n1,n2);
    vector<vector<int> >adj(n,vector<int>(n));
    for(int i=0;i<n;i++){
        for(int j=0;j<n;j++){
            adj[i][j]=adj1[i][j] && adj2[i][j];
            cout<<adj[i][j]<<" ";
        }cout<<endl;
    }
    cout<<endl;
}

void RingSum(vector<vector<int> >&adj1,vector<vector<int> >&adj2){
    cout<<"RingSum ..."<<endl;
    int n1=adj1.size();
```

```
    int n2=adj2.size();
    int n=max(n1,n2);
    vector<vector<int> >adj(n,vector<int>(n));
    for(int i=0;i<n;i++){
        for(int j=0;j<n;j++){
            adj[i][j]=((i<n1 && j<n1)?adj1[i][j]:0) ^ ((i<n2 && j<n2)?adj2[i][j]:0);
            cout<<adj[i][j]<<" ";
        }cout<<endl;
    }
    cout<<endl;
}

int main(){
#ifndef ONLINE_JUDGE
    freopen("input.txt", "r", stdin);
    freopen("output.txt", "w", stdout);
#endif
    int n1;
    cin>>n1;
    vector<vector<int> >adj1(n1,vector<int>(n1));
    for(int i=0;i<n1;i++){
        for(int j=0;j<n1;j++){
            cin>>adj1[i][j];
        }
    }
    int n2;
    cin>>n2;
    vector<vector<int> >adj2(n2,vector<int>(n2));
    for(int i=0;i<n2;i++){
        for(int j=0;j<n2;j++){
            cin>>adj2[i][j];
        }
    }
    if(IsValid(adj1) && IsValid(adj2)){
        Union(adj1,adj2);
        Intersection(adj1,adj2);
        RingSum(adj1,adj2);
    }else{
        cout<<"Entered Graph is InValid...."<<endl;
    }
    return 0;
}
```

## 2.2 Input/Output

```
int[][] matrix1 = {
        {0, 1, 0, 0},
        {1, 0, 0, 0},
        {0, 0, 0, 0},
        {0, 0, 0, 0}
};

int[][] matrix2 = {
        {0, 1, 1, 1},
        {1, 0, 1, 0},
        {1, 1, 0, 0},
        {1, 0, 0, 0}
};
```

(a) Input

```
Union of 2 Graphs
Graph{
adjacencyMatrix=
[0, 1, 1, 1]
[1, 0, 1, 0]
[1, 1, 0, 0]
[1, 0, 0, 0]
, numberOfVertices=4, degree=16
}
```

(b) Union of 2 Graphs

```
Intersection of 2 Graphs
Graph{
adjacencyMatrix=
[0, 1, 0, 0]
[1, 0, 0, 0]
[0, 0, 0, 0]
[0, 0, 0, 0]
, numberOfVertices=4, degree=4
}
```

(c) Intersection of 2 Graphs

```
Ring Sum of 2 Graphs
Graph{
adjacencyMatrix=
[0, 0, 1, 1]
[0, 0, 1, 0]
[1, 1, 0, 0]
[1, 0, 0, 0]
, numberOfVertices=4, degree=12
}
```

(d) Ring Sum of 2 Graphs

```
Complement of Graph
Graph{
adjacencyMatrix=
[1, 0, 1, 1]
[0, 1, 1, 1]
[1, 1, 1, 1]
[1, 1, 1, 1]
, numberOfVertices=4, degree=28
}
```

(e) Complement of 2 Graphs

Figure 2: Input and Output for Program 2

# 3 Program to Find Minimum Spanning Tree Using Prim's Algorithm

## 3.1 Code

```
#include <bits/stdc++.h>
using namespace std;

int size;

int minKey(int key[], bool mstSet[]) {
    int min = INT_MAX, min_index;

    for (int v = 0; v < size; v++)
        if (mstSet[v] == false && key[v] < min)
            min = key[v], min_index = v;

    return min_index;
}

void printMST(int parent[], vector<vector<int>>& graph) {
    cout<<"Edge \tWeight\n";
    for (int i = 0; i < graph.size()-1; i++)
        cout<<parent[i]<<" - "<<i<<" \t"<<graph[i][parent[i]]<<" \n";
}

void primMST(vector<vector<int>>& graph) {
  size = graph.size();
  int parent[graph.size()];
  bool mstSet[graph.size()];
  int key[graph.size()];

  for(int i=0;i<graph.size();i++){
    mstSet[i]=false;
    key[i]=INT_MAX;
  }

  key[5]=0;
  parent[5]=-1;
  for(int c=0;c<graph.size()-1;c++){
    int u=minKey(key, mstSet);
    mstSet[u]=true;
    for(int v=0;v<graph.size();v++){
      if(graph[u][v] && key[v]>graph[u][v] && u!=v){
        parent[v]=u, key[v]=graph[u][v];
      }
    }
  }
  printMST(parent, graph);
}

int main()  {
#ifndef ONLINE_JUDGE
    freopen("input.txt", "r",stdin);
    freopen("output.txt", "w",stdout);
#endif
```

```
    int vertex;
    cin>>vertex;
    vector<vector<int>> graph(vertex, vector<int>(vertex, 0));

    for(int i=0;i<vertex;i++){
      for(int j=0;j<vertex;j++){
        cin>>graph[i][j];
      }
    }

    //Parallel Edges
    for(int i=0;i<graph.size();i++){
        for(int j=0;j<graph.size();j++){
            graph[i][j] = min(graph[i][j], graph[j][i]);
        }
    }

  primMST(graph);

    return 0;
}
```

### 3.2 Input/Output

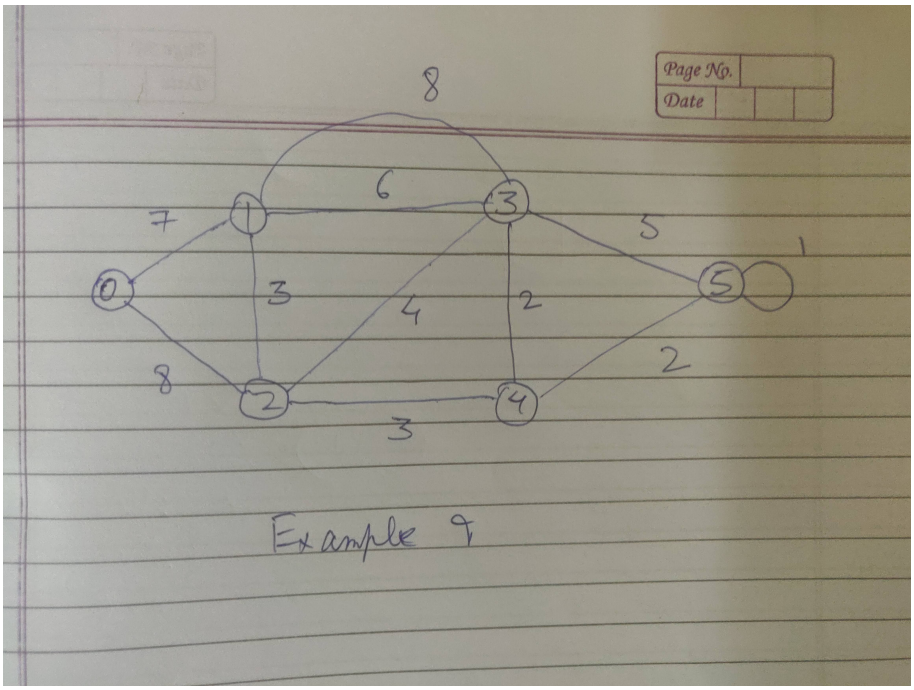



Figure 3: Un-directed Graph with Parallel Edges ad Self Loops Used In Prim's Algorithm

```
The Original Graph
Graph{order=6, size=11}
Edge Sum: 49
Vertex{data=0, degree=2, edges= [1 2 ]}
Vertex{data=1, degree=4, edges= [2 0 3 3 ]}
Vertex{data=2, degree=4, edges= [4 3 1 0 ]}
Vertex{data=3, degree=5, edges= [2 4 1 5 1 ]}
Vertex{data=4, degree=3, edges= [2 3 5 ]}
Vertex{data=5, degree=4, edges= [4 3 5 ]}

The new Graph After Applying Prim's Algorithm
Graph{order=6, size=5}
Edge Sum: 17
Vertex{data=0, degree=1, edges= [1 ]}
Vertex{data=1, degree=2, edges= [0 2 ]}
Vertex{data=2, degree=2, edges= [4 1 ]}
Vertex{data=3, degree=1, edges= [4 ]}
Vertex{data=4, degree=3, edges= [5 2 3 ]}
Vertex{data=5, degree=1, edges= [4 ]}

Process finished with exit code 0
```

Figure 4: Output for Prim's Algorithm

# 4 Program to Find Minimum Spanning Tree Using Kruskal's Algorithm

## 4.1 Code

```
#include <bits/stdc++.h>
using namespace std;

// a structure to represent a
// weighted edge in graph
class Edge {
public:
  int src, dest, weight;
};

// a structure to represent a connected,
// undirected and weighted graph
class Graph {
public:

  // V-> Number of vertices, E-> Number of edges
  int V, E;

  // graph is represented as an array of edges.
  // Since the graph is undirected, the edge
  // from src to dest is also edge from dest
  // to src. Both are counted as 1 edge here.
  Edge* edge;
};

// Creates a graph with V vertices and E edges
Graph* createGraph(int V, int E)
{
  Graph* graph = new Graph;
  graph->V = V;
  graph->E = E;

  graph->edge = new Edge[E];

  return graph;
}

// A structure to represent a subset for union-find
class subset {
public:
  int parent;
  int rank;
};

// A utility function to find set of an element i
// (uses path compression technique)
int find(subset subsets[], int i)
{
  // find root and make root as parent of i
  // (path compression)
  if (subsets[i].parent != i)
```

```
    subsets[i].parent
      = find(subsets, subsets[i].parent);

  return subsets[i].parent;
}

// A function that does union of two sets of x and y
// (uses union by rank)
void Union(subset subsets[], int x, int y)
{
  int xroot = find(subsets, x);
  int yroot = find(subsets, y);

  // Attach smaller rank tree under root of high
  // rank tree (Union by Rank)
  if (subsets[xroot].rank < subsets[yroot].rank)
    subsets[xroot].parent = yroot;
  else if (subsets[xroot].rank > subsets[yroot].rank)
    subsets[yroot].parent = xroot;

  // If ranks are same, then make one as root and
  // increment its rank by one
  else {
    subsets[yroot].parent = xroot;
    subsets[xroot].rank++;
  }
}

// Compare two edges according to their weights.
// Used in qsort() for sorting an array of edges
int myComp(const void* a, const void* b)
{
  Edge* a1 = (Edge*)a;
  Edge* b1 = (Edge*)b;
  return a1->weight > b1->weight;
}

// The main function to construct MST using Kruskal's
// algorithm
void KruskalMST(Graph* graph)
{
  int V = graph->V;
  Edge result[V]; // Tnis will store the resultant MST
  int e = 0; // An index variable, used for result[]
  int i = 0; // An index variable, used for sorted edges

  // Step 1: Sort all the edges in non-decreasing
  // order of their weight. If we are not allowed to
  // change the given graph, we can create a copy of
  // array of edges
  qsort(graph->edge, graph->E, sizeof(graph->edge[0]),
    myComp);

  // Allocate memory for creating V ssubsets
  subset* subsets = new subset[(V * sizeof(subset))];
```

```
  // Create V subsets with single elements
  for (int v = 0; v < V; ++v)
  {
    subsets[v].parent = v;
    subsets[v].rank = 0;
  }

  // Number of edges to be taken is equal to V-1
  while (e < V - 1 && i < graph->E)
  {
    // Step 2: Pick the smallest edge. And increment
    // the index for next iteration
    Edge next_edge = graph->edge[i++];

    int x = find(subsets, next_edge.src);
    int y = find(subsets, next_edge.dest);

    // If including this edge does't cause cycle,
    // include it in result and increment the index
    // of result for next edge
    if (x != y) {
      result[e++] = next_edge;
      Union(subsets, x, y);
    }
    // Else discard the next_edge
  }

  // print the contents of result[] to display the
  // built MST
  cout << "Following are the edges in the constructed "
       "MST\n";
  int minimumCost = 0;
  for (i = 0; i < e; ++i)
  {
    cout << result[i].src << " -- " << result[i].dest
      << " == " << result[i].weight << endl;
    minimumCost = minimumCost + result[i].weight;
  }
  // return;
  cout << "Minimum Cost Spanning Tree: " << minimumCost
    << endl;
}

// Driver code
int main()
{
  /* Let us create following weighted graph
       10
    0--------1
    | \ |
  6| 5\ |15
    | \ |
    2--------3
       4 */
  int V = 4; // Number of vertices in graph
  int E = 5; // Number of edges in graph
```

```
  Graph* graph = createGraph(V, E);

  // add edge 0-1
  graph->edge[0].src = 0;
  graph->edge[0].dest = 1;
  graph->edge[0].weight = 10;

  // add edge 0-2
  graph->edge[1].src = 0;
  graph->edge[1].dest = 2;
  graph->edge[1].weight = 6;

  // add edge 0-3
  graph->edge[2].src = 0;
  graph->edge[2].dest = 3;
  graph->edge[2].weight = 5;

  // add edge 1-3
  graph->edge[3].src = 1;
  graph->edge[3].dest = 3;
  graph->edge[3].weight = 15;

  // add edge 2-3
  graph->edge[4].src = 2;
  graph->edge[4].dest = 3;
  graph->edge[4].weight = 4;


  // Function call
  KruskalMST(graph);

  return 0;
}
```

## 4.2 Input/Output

```
int V = 4; // Number of vertices in graph
int E = 5; // Number of edges in graph
Graph* graph = createGraph(V, E);

// add edge 0-1
graph->edge[0].src = 0;
graph->edge[0].dest = 1;
graph->edge[0].weight = 10;

// add edge 0-2
graph->edge[1].src = 0;
graph->edge[1].dest = 2;
graph->edge[1].weight = 6;

// add edge 0-3
graph->edge[2].src = 0;
graph->edge[2].dest = 3;
graph->edge[2].weight = 5;

// add edge 1-3
graph->edge[3].src = 1;
graph->edge[3].dest = 3;
graph->edge[3].weight = 15;

// add edge 2-3
graph->edge[4].src = 2;
graph->edge[4].dest = 3;
graph->edge[4].weight = 4;
```

Figure 5: Input for Minimum Spanning Tree Kruskal's Algorithm

```
Following are the edges in the constructed MST
0 -- 2 == 6
0 -- 1 == 10
0 -- 3 == 5
Minimum Cost Spanning Tree: 21
```

Figure 6: Output for Minimum Spanning Tree Kruskal's Algorithm

# 5 Program to find Shortest Path between 2 Vertices using Dijkstra Algorithm

## 5.1 Code

```
// A C++ program for Dijkstra's single source shortest path algorithm.
// The program is for adjacency matrix representation of the graph

#include <limits.h>
#include <stdio.h>

// Number of vertices in the graph
#define V 9

// A utility function to find the vertex with minimum distance value, from
// the set of vertices not yet included in shortest path tree
int minDistance(int dist[], bool sptSet[])
{
  // Initialize min value
  int min = INT_MAX, min_index;

  for (int v = 0; v < V; v++)
    if (sptSet[v] == false && dist[v] <= min)
      min = dist[v], min_index = v;

  return min_index;
}

// A utility function to print the constructed distance array
void printSolution(int dist[])
{
  printf("Vertex \t\t Distance from Source\n");
  for (int i = 0; i < V; i++)
    printf("%d \t\t %d\n", i, dist[i]);
}

// Function that implements Dijkstra's single source shortest path algorithm
// for a graph represented using adjacency matrix representation
void dijkstra(int graph[V][V], int src)
{
  int dist[V]; // The output array. dist[i] will hold the shortest
  // distance from src to i

  bool sptSet[V]; // sptSet[i] will be true if vertex i is included in shortest
  // path tree or shortest distance from src to i is finalized

  // Initialize all distances as INFINITE and stpSet[] as false
  for (int i = 0; i < V; i++)
    dist[i] = INT_MAX, sptSet[i] = false;

  // Distance of source vertex from itself is always 0
  dist[src] = 0;

  // Find shortest path for all vertices
  for (int count = 0; count < V - 1; count++) {
    // Pick the minimum distance vertex from the set of vertices not
```

```
    // yet processed. u is always equal to src in the first iteration.
    int u = minDistance(dist, sptSet);

    // Mark the picked vertex as processed
    sptSet[u] = true;

    // Update dist value of the adjacent vertices of the picked vertex.
    for (int v = 0; v < V; v++)

      // Update dist[v] only if is not in sptSet, there is an edge from
      // u to v, and total weight of path from src to v through u is
      // smaller than current value of dist[v]
      if (!sptSet[v] && graph[u][v] && dist[u] != INT_MAX
        && dist[u] + graph[u][v] < dist[v])
        dist[v] = dist[u] + graph[u][v];
  }

  // print the constructed distance array
  printSolution(dist);
}

// driver program to test above function
int main() {
  /* Let us create the example graph discussed above */
  int graph[V][V] = { { 0, 4, 0, 0, 0, 0, 0, 8, 0 },
            { 4, 0, 8, 0, 0, 0, 0, 11, 0 },
            { 0, 8, 0, 7, 0, 4, 0, 0, 2 },
            { 0, 0, 7, 0, 9, 14, 0, 0, 0 },
            { 0, 0, 0, 9, 0, 10, 0, 0, 0 },
            { 0, 0, 4, 14, 10, 0, 2, 0, 0 },
            { 0, 0, 0, 0, 0, 2, 0, 1, 6 },
            { 8, 11, 0, 0, 0, 0, 1, 0, 7 },
            { 0, 0, 2, 0, 0, 0, 6, 7, 0 } };

  dijkstra(graph, 0);
  return 0;
}
```

## 5.2 Input/Output

```
{
    { 0, 4, 0, 0, 0, 0, 0, 8, 0 },
    { 4, 0, 8, 0, 0, 0, 0, 11, 0 },
    { 0, 8, 0, 7, 0, 4, 0, 0, 2 },
    { 0, 0, 7, 0, 9, 14, 0, 0, 0 },
    { 0, 0, 0, 9, 0, 10, 0, 0, 0 },
    { 0, 0, 4, 14, 10, 0, 2, 0, 0 },
    { 0, 0, 0, 0, 0, 2, 0, 1, 6 },
    { 8, 11, 0, 0, 0, 0, 1, 0, 7 },
    { 0, 0, 2, 0, 0, 0, 6, 7, 0 }
}
```

Figure 7: Input for Dijkstra's Algorithm

```
Vertex      Distance from Source
0       0
1       4
2       12
3       19
4       21
5       11
6       9
7       8
8       14
```

Figure 8: Output for Dijkstra's Algorithm

# 6 Program to Find Shortest Path Between Every Pair of vertices in a Graph Using Floyd-Warshall's Algorithm

## 6.1 Code

```
// C++ Program for Floyd Warshall Algorithm
#include <bits/stdc++.h>
using namespace std;

// Number of vertices in the graph
#define V 4

/* Define Infinite as a large enough
value.This value will be used for
vertices not connected to each other */
#define INF 99999

// A function to print the solution matrix
void printSolution(int dist[][V]);

// Solves the all-pairs shortest path
// problem using Floyd Warshall algorithm
void floydWarshall (int graph[][V])
{
  /* dist[][] will be the output matrix
  that will finally have the shortest
  distances between every pair of vertices */
  int dist[V][V], i, j, k;

  /* Initialize the solution matrix same
  as input graph matrix. Or we can say
  the initial values of shortest distances
  are based on shortest paths considering
  no intermediate vertex. */
  for (i = 0; i < V; i++)
    for (j = 0; j < V; j++)
      dist[i][j] = graph[i][j];

  /* Add all vertices one by one to
  the set of intermediate vertices.
  ---> Before start of an iteration,
  we have shortest distances between all
  pairs of vertices such that the
  shortest distances consider only the
  vertices in set {0, 1, 2, .. k-1} as
  intermediate vertices.
  ----> After the end of an iteration,
  vertex no. k is added to the set of
  intermediate vertices and the set becomes {0, 1, 2, .. k} */
  for (k = 0; k < V; k++)
  {
    // Pick all vertices as source one by one
    for (i = 0; i < V; i++)
    {
      // Pick all vertices as destination for the
```

```
      // above picked source
      for (j = 0; j < V; j++)
      {
        // If vertex k is on the shortest path from
        // i to j, then update the value of dist[i][j]
        if (dist[i][k] + dist[k][j] < dist[i][j])
          dist[i][j] = dist[i][k] + dist[k][j];
      }
    }
  }

  // Print the shortest distance matrix
  printSolution(dist);
}

/* A utility function to print solution */
void printSolution(int dist[][V])
{
  cout<<"The following matrix shows the shortest distances"
      " between every pair of vertices \n";
  for (int i = 0; i < V; i++)
  {
    for (int j = 0; j < V; j++)
    {
      if (dist[i][j] == INF)
        cout<<"INF"<<"   ";
      else
        cout<<dist[i][j]<<"  ";
    }
    cout<<endl;
  }
}

// Driver code
int main()
{
  /* Let us create the following weighted graph
      10
  (0)-------->(3)
    |  /|\
  5 |  |
    |  | 1
  \|/  |
  (1)-------->(2)
      3  */
  int graph[V][V] = { {0, 5, INF, 10},
            {INF, 0, 3, INF},
            {INF, INF, 0, 1},
            {INF, INF, INF, 0}
          };

  // Print the solution
  floydWarshall(graph);
  return 0;
}
```

## 6.2 Input/Output

```
{
    {0, 5, INF, 10},
    {INF, 0, 3, INF},
    {INF, INF, 0, 1},
    {INF, INF, INF, 0}
}
```

Figure 9: Input for Floyd-Warshall's Algorithm

```
The following matrix shows the shortest distances between every pair of vertices
0   5      8   9
INF   0      3   4
INF   INF   0   1
INF   INF   INF   0
```

Figure 10: Output for Floyd-Warshall's Algorithm

# 7 Program to find Shortest Path between Every Pair of Vertices using Bellman Ford's Algorithm

## 7.1 Code

```
// A C++ program for Bellman-Ford's single source
// shortest path algorithm.
#include <bits/stdc++.h>

// a structure to represent a weighted edge in graph
struct Edge {
  int src, dest, weight;
};

// a structure to represent a connected, directed and
// weighted graph
struct Graph {
  // V-> Number of vertices, E-> Number of edges
  int V, E;

  // graph is represented as an array of edges.
  struct Edge* edge;
};

// Creates a graph with V vertices and E edges
struct Graph* createGraph(int V, int E)
{
  struct Graph* graph = new Graph;
  graph->V = V;
  graph->E = E;
  graph->edge = new Edge[E];
  return graph;
}

// A utility function used to print the solution
void printArr(int dist[], int n)
{
  printf("Vertex Distance from Source\n");
  for (int i = 0; i < n; ++i)
    printf("%d \t\t %d\n", i, dist[i]);
}

// The main function that finds shortest distances from src to
// all other vertices using Bellman-Ford algorithm. The function
// also detects negative weight cycle
void BellmanFord(struct Graph* graph, int src)
{
  int V = graph->V;
  int E = graph->E;
  int dist[V];

  // Step 1: Initialize distances from src to all other vertices
  // as INFINITE
  for (int i = 0; i < V; i++)
    dist[i] = INT_MAX;
```

```
  dist[src] = 0;

  // Step 2: Relax all edges |V| - 1 times. A simple shortest
  // path from src to any other vertex can have at-most |V| - 1
  // edges
  for (int i = 1; i <= V - 1; i++) {
    for (int j = 0; j < E; j++) {
      int u = graph->edge[j].src;
      int v = graph->edge[j].dest;
      int weight = graph->edge[j].weight;
      if (dist[u] != INT_MAX && dist[u] + weight < dist[v])
        dist[v] = dist[u] + weight;
    }
  }

  // Step 3: check for negative-weight cycles. The above step
  // guarantees shortest distances if graph doesn't contain
  // negative weight cycle. If we get a shorter path, then there
  // is a cycle.
  for (int i = 0; i < E; i++) {
    int u = graph->edge[i].src;
    int v = graph->edge[i].dest;
    int weight = graph->edge[i].weight;
    if (dist[u] != INT_MAX && dist[u] + weight < dist[v]) {
      printf("Graph contains negative weight cycle");
      return; // If negative cycle is detected, simply return
    }
  }

  printArr(dist, V);

  return;
}

// Driver program to test above functions
int main()
{
  /* Let us create the graph given in above example */
  int V = 7; // Number of vertices in graph
  int E = 10; // Number of edges in graph
  struct Graph* graph = createGraph(V, E);

  // add edge 0-1 (or A-B in above figure)
  graph->edge[0].src = 0;
  graph->edge[0].dest = 1;
  graph->edge[0].weight = 6;

  graph->edge[1].src = 0;
    graph->edge[1].dest = 2;
    graph->edge[1].weight = 5;

    graph->edge[2].src = 0;
    graph->edge[2].dest = 3;
    graph->edge[2].weight = 5;

    graph->edge[3].src = 1;
```

```
    graph->edge[3].dest = 4;
    graph->edge[3].weight = -1;

    graph->edge[4].src = 2;
    graph->edge[4].dest = 1;
    graph->edge[4].weight = -2;

    graph->edge[5].src = 2;
    graph->edge[5].dest = 4;
    graph->edge[5].weight = 1;

    graph->edge[6].src = 3;
    graph->edge[6].dest = 2;
    graph->edge[6].weight = -2;

    graph->edge[7].src = 3;
    graph->edge[7].dest = 5;
    graph->edge[7].weight = -1;

    graph->edge[8].src = 4;
    graph->edge[8].dest = 6;
    graph->edge[8].weight = 3;

    graph->edge[9].src = 5;
    graph->edge[9].dest = 6;
    graph->edge[9].weight = 3;

  BellmanFord(graph, 0);

  return 0;
}
```

## 7.2 Input/Output

```
int V = 7; // Number of vertices in graph
int E = 10; // Number of edges in graph
struct Graph* graph = createGraph(V, E);

// add edge 0-1 (or A-B in above figure)
graph->edge[0].src = 0;
graph->edge[0].dest = 1;
graph->edge[0].weight = 6;

graph->edge[1].src = 0;
graph->edge[1].dest = 2;
graph->edge[1].weight = 5;

graph->edge[2].src = 0;
graph->edge[2].dest = 3;
graph->edge[2].weight = 5;

graph->edge[3].src = 1;
graph->edge[3].dest = 4;
graph->edge[3].weight = -1;

graph->edge[4].src = 2;
graph->edge[4].dest = 1;
graph->edge[4].weight = -2;

graph->edge[5].src = 2;
graph->edge[5].dest = 4;
graph->edge[5].weight = 1;

graph->edge[6].src = 3;
graph->edge[6].dest = 2;
graph->edge[6].weight = -2;

graph->edge[7].src = 3;
graph->edge[7].dest = 5;
graph->edge[7].weight = -1;

graph->edge[8].src = 4;
graph->edge[8].dest = 6;
graph->edge[8].weight = 3;

graph->edge[9].src = 5;
graph->edge[9].dest = 6;
graph->edge[9].weight = 3;
```

Figure 11: Input for Bellman Ford's Algorithm

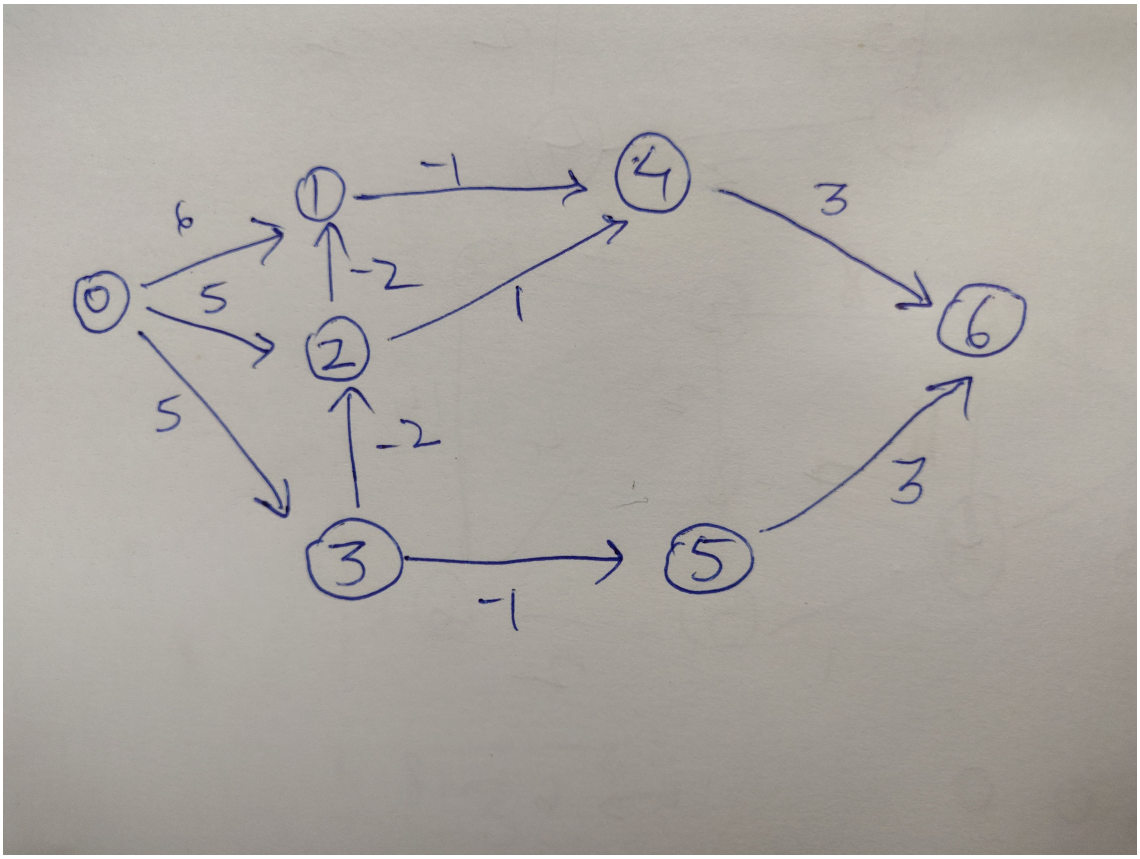



Figure 12: Directed Weighted Graph Taken As Input For Bellman Ford's Algorithm

```
Vertex Distance from Source
0       0
1       1
2       3
3       5
4       0
5       4
6       3
```

Figure 13: Output for Bellman Ford's Algorithm

# 8 Program For Finding Maximal Matching for Bipartite Graph

## 8.1 Code

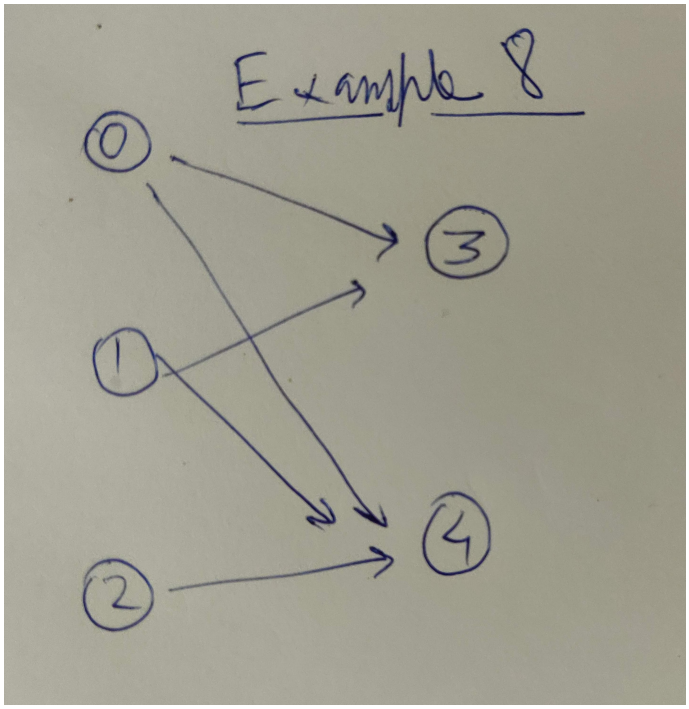
```
// A C++ program to find maximal
// Bipartite matching.
#include <iostream>
#include <string.h>
using namespace std;

// M is number of applicants
// and N is number of jobs
#define M 6
#define N 6

// A DFS based recursive function
// that returns true if a matching
// for vertex u is possible
bool bpm(bool bpGraph[M][N], int u,
    bool seen[], int matchR[])
{
  // Try every job one by one
  for (int v = 0; v < N; v++)
  {
    // If applicant u is interested in
    // job v and v is not visited
    if (bpGraph[u][v] && !seen[v])
    {
      // Mark v as visited
      seen[v] = true;

      // If job 'v' is not assigned to an
      // applicant OR previously assigned
      // applicant for job v (which is matchR[v])
      // has an alternate job available.
      // Since v is marked as visited in
      // the above line, matchR[v] in the following
      // recursive call will not get job 'v' again
      if (matchR[v] < 0 || bpm(bpGraph, matchR[v],
                  seen, matchR))
      {
        matchR[v] = u;
        return true;
      }
    }
  }
  return false;
}

// Returns maximum number
// of matching from M to N
int maxBPM(bool bpGraph[M][N])
{
  // An array to keep track of the
  // applicants assigned to jobs.
```

```
  // The value of matchR[i] is the
  // applicant number assigned to job i,
  // the value -1 indicates nobody is
  // assigned.
  int matchR[N];

  // Initially all jobs are available
  memset(matchR, -1, sizeof(matchR));

  // Count of jobs assigned to applicants
  int result = 0;
  for (int u = 0; u < M; u++)
  {
    // Mark all jobs as not seen
    // for next applicant.
    bool seen[N];
    memset(seen, 0, sizeof(seen));

    // Find if the applicant 'u' can get a job
    if (bpm(bpGraph, u, seen, matchR))
      result++;
  }
  return result;
}

// Driver Code
int main()
{
  // Let us create a bpGraph
  // shown in the above example
  bool bpGraph[M][N] = {{0, 0, 0, 1, 1},
            {0, 0, 0, 1, 1},
            {0, 0, 0, 0, 1},
            {0, 0, 0, 0, 0, 0},
            {0, 0, 0, 0, 0, 0}};

  cout << "Maximum number of applicants that can get job is "
    << maxBPM(bpGraph);

  return 0;
}
```

## 8.2 Input/Output

```
{
    {0, 0, 0, 1, 1},
    {0, 0, 0, 1, 1},
    {0, 0, 0, 0, 1},
    {0, 0, 0, 0, 0, 0},
    {0, 0, 0, 0, 0, 0}
}
```

Figure 14: Input for Maximum Matching for Bipartite Graph Algorithm




(a) Directed Un-weighted Graph Taken As Input For Maximum Matching for Bipartite Graph Algorithm

Figure 15: Input for Program 8

```
Maximum number of applicants that can get job is 2
```

Figure 16: Output for Maximum Matching for Bipartite Graph Algorithm

# 9 Program For Finding Maximal Matching for General Path

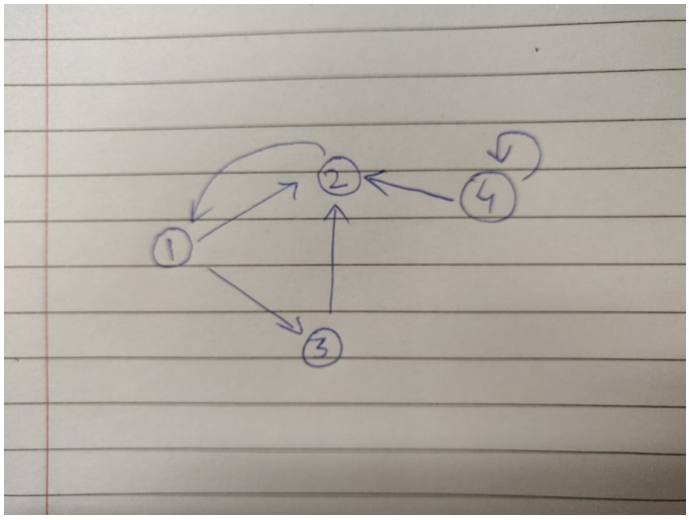
## 9.1 Code

```
// C++ implementation of Hopcroft Karp algorithm for
// maximum matching
#include<bits/stdc++.h>
using namespace std;
#define NIL 0
#define INF INT_MAX

// A class to represent Bipartite graph for Hopcroft
// Karp implementation
class BipGraph {
    // m and n are number of vertices on left
    // and right sides of Bipartite Graph
    int m, n;
    // adj[u] stores adjacents of left side
    // vertex 'u'. The value of u ranges from 1 to m.
    // 0 is used for dummy vertex
    list<int> *adj;
    // These are basically pointers to arrays needed
    // for hopcroftKarp()
    int *pairU, *pairV, *dist;
    public:
    BipGraph(int m, int n); // Constructor
    void addEdge(int u, int v); // To add edge
    // Returns true if there is an augmenting path
    bool bfs();
    // Adds augmenting path if there is one beginning
    // with u
    bool dfs(int u);
    // Returns size of maximum matcing
    int hopcroftKarp();
};

// Returns size of maximum matching
int BipGraph::hopcroftKarp() {
    // pairU[u] stores pair of u in matching where u
    // is a vertex on left side of Bipartite Graph.
    // If u doesn't have any pair, then pairU[u] is NIL
    pairU = new int[m+1];
    // pairV[v] stores pair of v in matching. If v
    // doesn't have any pair, then pairU[v] is NIL
    pairV = new int[n+1];
    // dist[u] stores distance of left side vertices
    // dist[u] is one more than dist[u'] if u is next
    // to u'in augmenting path
    dist = new int[m+1];

    // Initialize NIL as pair of all vertices
    for (int u=0; u<m; u++)
        pairU[u] = NIL;
    for (int v=0; v<n; v++)
        pairV[v] = NIL;
```

```
    // Initialize result
    int result = 0;
    // Keep updating the result while there is an
    // augmenting path.
    while (bfs()) {
        // Find a free vertex
        for (int u=1; u<=m; u++)
        // If current vertex is free and there is
        // an augmenting path from current vertex
        if (pairU[u]==NIL && dfs(u))
        result++;
    }
    return result;
}

// Returns true if there is an augmenting path, else returns
// false
bool BipGraph::bfs() {
    queue<int> Q; //an integer queue
    // First layer of vertices (set distance as 0)
    for (int u=1; u<=m; u++) {
        // If this is a free vertex, add it to queue
        if (pairU[u]==NIL) {
            // u is not matched
            dist[u] = 0;
            Q.push(u);
        }

        // Else set distance as infinite so that this vertex
        // is considered next time
        else dist[u] = INF;
    }

    // Initialize distance to NIL as infinite
    dist[NIL] = INF;
    // Q is going to contain vertices of left side only.
    while (!Q.empty()) {
        // Dequeue a vertex
        int u = Q.front();
        Q.pop();
        // If this node is not NIL and can provide a shorter path to NIL
        if (dist[u] < dist[NIL]) {
            // Get all adjacent vertices of the dequeued vertex u
            list<int>::iterator i;
            for (i=adj[u].begin(); i!=adj[u].end(); ++i) {
                int v = *i;
                // If pair of v is not considered so far
                // (v, pairV[V]) is not yet explored edge.
                if (dist[pairV[v]] == INF) {
                    // Consider the pair and add it to queue
                    dist[pairV[v]] = dist[u] + 1;
                    Q.push(pairV[v]);
                }
            }
        }
```

```
    }
    // If we could come back to NIL using alternating path of distinct
    // vertices then there is an augmenting path
    return (dist[NIL] != INF);
}

// Returns true if there is an augmenting path beginning with free vertex u
bool BipGraph::dfs(int u) {
    if (u != NIL) {
        list<int>::iterator i;
        for (i=adj[u].begin(); i!=adj[u].end(); ++i) {
            // Adjacent to u
            int v = *i;
            // Follow the distances set by BFS
            if (dist[pairV[v]] == dist[u]+1) {
                // If dfs for pair of v also returns
                // true
                if (dfs(pairV[v]) == true) {
                    pairV[v] = u;
                    pairU[u] = v;
                    return true;
                }
            }
        }

        // If there is no augmenting path beginning with u.
        dist[u] = INF;
        return false;
    }
    return true;
}

// Constructor
BipGraph::BipGraph(int m, int n) {
    this->m = m;
    this->n = n;
    adj = new list<int>[m+1];
}

// To add edge from u to v and v to u
void BipGraph::addEdge(int u, int v) {
    adj[u].push_back(v); // Add u to  v s  list.
}

// Driver Program
int main() {
    BipGraph g(4, 4);
    g.addEdge(1, 2);
    g.addEdge(1, 3);
    g.addEdge(2, 1);
    g.addEdge(3, 2);
    g.addEdge(4, 2);
    g.addEdge(4, 4);
    cout << "Size of maximum matching is " << g.hopcroftKarp();
    return 0;
}
```

## 9.2 Input/Output




(a) Directed Un-weighted Graph Taken As Input For Maximal Matching for General Graph Algorithm

Figure 17: Input for Program 9

```
Size of Maximum Matching is 4
```

Figure 18: Output for Maximal Matching for General Path Algorithm

# 10 Program to Find Maximum Flow From Source Node to Sink Node Using Ford-Fulkerson Algorithm

## 10.1 Code

```
// C++ program for implementation of Ford Fulkerson algorithm
#include <iostream>
#include <limits.h>
#include <string.h>
#include <queue>
using namespace std;

// Number of vertices in given graph
#define V 6

/* Returns true if there is a path from source 's' to sink 't' in
residual graph. Also fills parent[] to store the path */
bool bfs(int rGraph[V][V], int s, int t, int parent[])
{
  // Create a visited array and mark all vertices as not visited
  bool visited[V];
  memset(visited, 0, sizeof(visited));

  // Create a queue, enqueue source vertex and mark source vertex
  // as visited
  queue <int> q;
  q.push(s);
  visited[s] = true;
  parent[s] = -1;

  // Standard BFS Loop
  while (!q.empty())
  {
    int u = q.front();
    q.pop();

    for (int v=0; v<V; v++)
    {
      if (visited[v]==false && rGraph[u][v] > 0)
      {
        q.push(v);
        parent[v] = u;
        visited[v] = true;
      }
    }
  }

  // If we reached sink in BFS starting from source, then return
  // true, else false
  return (visited[t] == true);
}

// Returns the maximum flow from s to t in the given graph
int fordFulkerson(int graph[V][V], int s, int t)
{
```

```
  int u, v;

  // Create a residual graph and fill the residual graph with
  // given capacities in the original graph as residual capacities
  // in residual graph
  int rGraph[V][V]; // Residual graph where rGraph[i][j] indicates
          // residual capacity of edge from i to j (if there
          // is an edge. If rGraph[i][j] is 0, then there is not)
  for (u = 0; u < V; u++)
    for (v = 0; v < V; v++)
      rGraph[u][v] = graph[u][v];

  int parent[V]; // This array is filled by BFS and to store path

  int max_flow = 0; // There is no flow initially

  // Augment the flow while tere is path from source to sink
  while (bfs(rGraph, s, t, parent))
  {
    // Find minimum residual capacity of the edges along the
    // path filled by BFS. Or we can say find the maximum flow
    // through the path found.
    int path_flow = INT_MAX;
    for (v=t; v!=s; v=parent[v])
    {
      u = parent[v];
      path_flow = min(path_flow, rGraph[u][v]);
    }

    // update residual capacities of the edges and reverse edges
    // along the path
    for (v=t; v != s; v=parent[v])
    {
      u = parent[v];
      rGraph[u][v] -= path_flow;
      rGraph[v][u] += path_flow;
    }

    // Add path flow to overall flow
    max_flow += path_flow;
  }

  // Return the overall flow
  return max_flow;
}

// Driver program to test above functions
int main()
{
  // Let us create a graph shown in the above example
  int graph[V][V] = {
          {0, 3, 0, 3, 0, 0, 0},
          {0, 0, 4, 0, 0, 0, 0},
          {0, 0, 0, 1, 2, 0, 0},
          {0, 0, 0, 0, 2, 6, 0},
          {0, 1, 0, 0, 0, 0, 1},
```

```
            {0, 0, 0, 0, 0, 0, 9},
            {0, 0, 0, 0, 0, 0, 0}
    };

  cout << "The maximum possible flow is " << fordFulkerson(graph, 0, 5);

  return 0;
}
```

## 10.2 Input/Output

```
int graph[V][V] = {
      {0, 3, 0, 3, 0, 0, 0},
      {0, 0, 4, 0, 0, 0, 0},
      {0, 0, 0, 1, 2, 0, 0},
      {0, 0, 0, 0, 2, 6, 0},
      {0, 1, 0, 0, 0, 0, 1},
      {0, 0, 0, 0, 0, 0, 9},
      {0, 0, 0, 0, 0, 0, 0}
};
```

Figure 19: Input for Maximum Matching for Bipartite Graph Algorithm

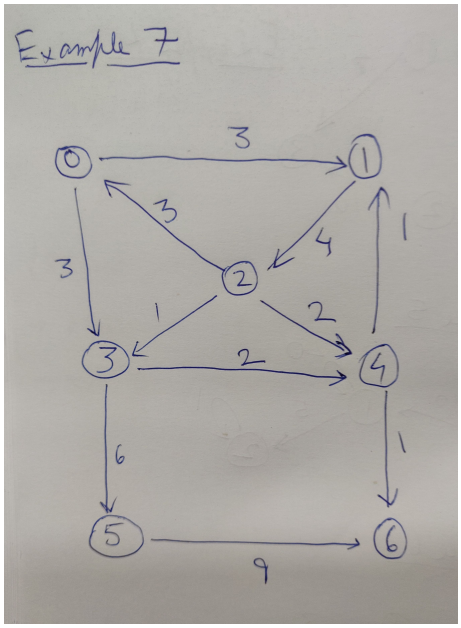



Figure 20: Directed Weighted Graph Taken As Input For Ford-Fulkerson Algorithm

```
The maximum possible flow is 4
```

Figure 21: Output for Ford-Fulkerson Algorithm