Decoding Insertions/Deletions via List Recovery

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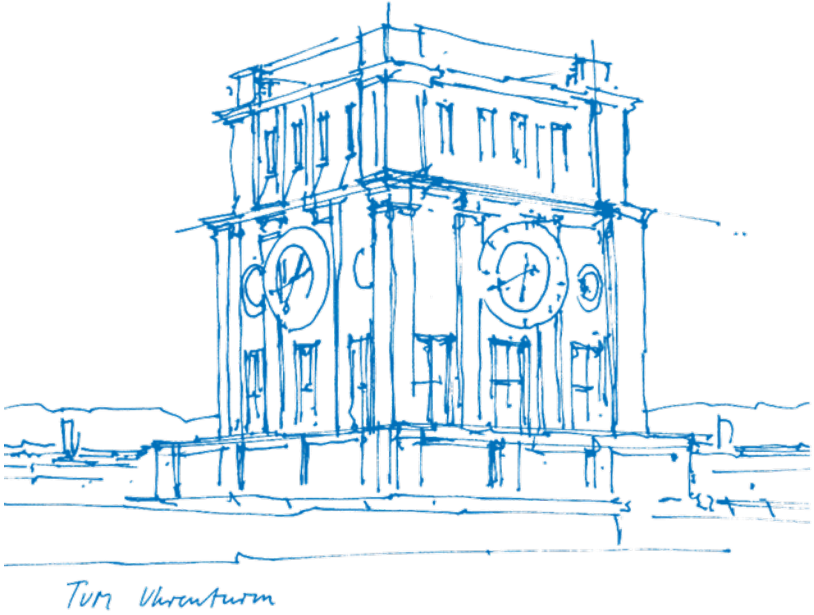
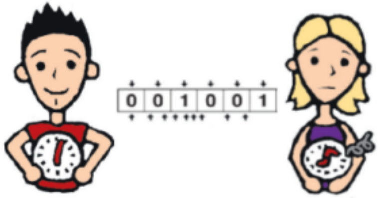


Outline

Motivation

- Insertions & deletions occur
  - due to improper synchronization



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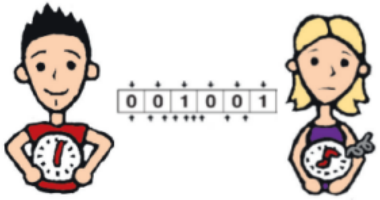

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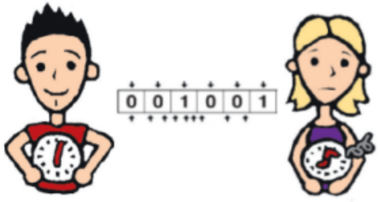

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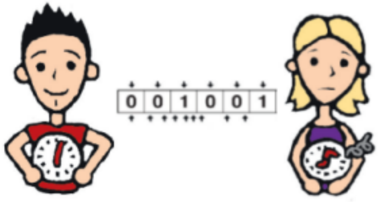
- Hard to perform decoding efficiently.
- *Trick:* reduce the insdel-decoding problem to *list recovery*!



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What is List Recovery?

- Consider $\mathcal{C} \subseteq \mathbb{F}_q^n$ that corrects t substitutions.
- Assume integers $t' > t$ and $L, \ell > 1$.

V. Guruswami, "Algorithmic results in list decoding," *Foundations and Trends® in Theoretical Computer Science*, vol. 2, no. 2, pp. 107–195, 2006

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$$\boldsymbol{y} = \boxed{1 \mid 3 \mid \cdots\cdots \mid 0} \in \mathbb{F}_{q=4}^n$$

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- List Decoding:
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List-Recoverable Codes

Definition (List-Decodable Codes)

For $\rho \in [0, 1]$ and integer L, a code $\mathcal{C} \subseteq \mathbb{F}_q^n$ is (ρ, L)-*list-decodable* if for any $\boldsymbol{y} \in \mathbb{F}_q^n$ there are $\leq L$ codewords $\boldsymbol{c} \in \mathcal{C}$ that satisfy $|\{i \in [n] : c_i \neq y_i\}| \leq \rho n$.

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- Example: RS codes are efficiently list-recoverable by the Guruswami-Sudan (GS) decoder.

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- Only efficient non-trivial insdel decoder for $[n, k = 2]_q$ RS codes [Singhvi '24]
  - for $[n, k = 2]_q$ RS codes for insdels in [Con *et al.* '24]
  - corrects $n - 3$ deletions in linear time.

Overview of Results

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  - First efficient insdel-list-decoder for $[n, k > 2]$ RS codes!
  - Rate-error tradeoffs for adversarial & probabilistic insdel channels
- Adapted the Koetter-Vardy algorithm for probabilistic insdel channel [Davey and MacKay '01]
  - also for jointly decoding multiple received sequences

Correct One Insertion via List Recovery

Example

Consider a $(\rho, \ell = 2, L)$-list-recoverable code $\mathcal{C} \subseteq \mathbb{F}_{q=3}^{n=5}$.

$$\boldsymbol{c} \in \mathcal{C} \xrightarrow{\text{1 insertion}} \boldsymbol{y} = (0, 2, 1, 1, 0, 2)$$

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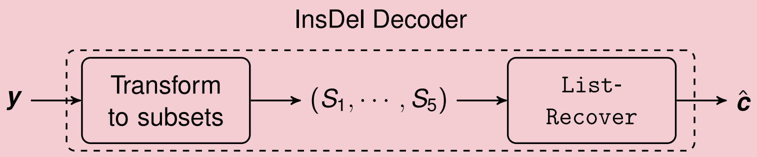

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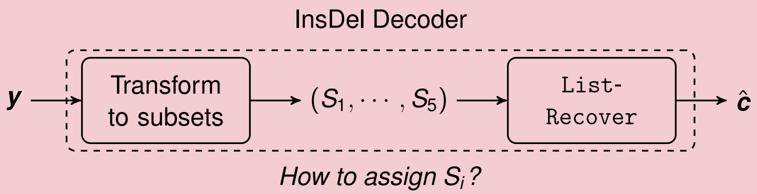

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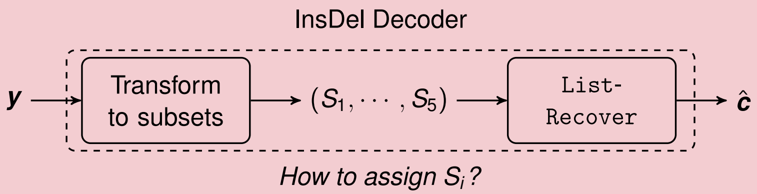

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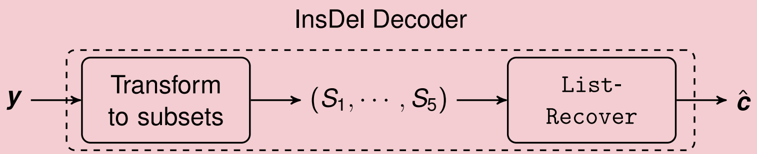
$$\begin{aligned} c_1 &\in \{0, 2\} \\ c_2 &\in \{2, 1\} \\ &\vdots \\ c_5 &\in \{0, 2\} \end{aligned}$$



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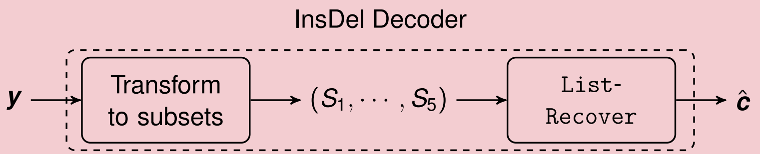

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Decoding Algorithm

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Algorithm: `Decode`

Input: $(y_1, \ldots, y_m)$, ℓ
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3 Execute `List-Recover` with the lists $S_1, \ldots, S_n$ to get $\mathcal{L} \subseteq \mathcal{C}$.

4 return $\{\boldsymbol{c} \in \mathcal{L} | d_{\mathrm{ed}}(\boldsymbol{c}, \boldsymbol{y}) \leq \rho n\}$.

Decoding Adversarial Insdels

Theorem

Let $\rho \in [0, 1]$ and set $\ell = 2\rho n + 1$. Assume that $\mathcal{C} \subseteq \mathbb{F}_q^n$ is a (ρ, ℓ, L)-list-recoverable code with an algorithm `List-Recover` that runs in time T.

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1. Then, running alg. `Decode` on input $\boldsymbol{y} \in \mathbb{F}_q^m$ returns a list $\mathcal{L}' \subseteq \mathcal{C}$ such that
   - for every $\boldsymbol{c} \in \mathcal{L}'$, $d_{\mathrm{ed}}(\boldsymbol{c}, \boldsymbol{y}) \leq \rho n$, and
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2. The running time of Alg. `Decode` is $O(T + L \cdot n^2)$.
3. Moreover, if $\rho n \leq \lfloor \frac{d_{\mathrm{ed}}(\mathcal{C})-1}{2} \rfloor$, then $|\mathcal{L}'| \leq 1$.

Reed-Solomon Codes

- Efficiently list-recoverable!

Definition

Let $\alpha_0, \ldots, \alpha_{n-1} \in \mathbb{F}_q$ be distinct. An $[n, k]_q$ *Reed-Solomon (RS) code* is defined as

$$\mathcal{RS}(n, k)_q = \{(f(\alpha_0), \ldots, f(\alpha_{n-1})) \mid f \in \mathbb{F}_q[x], \deg(f) < k\}.$$

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Corollary (Guruswami-Sudan Decoder)

Let $\varepsilon > 0$ and $\mathcal{C}$ be an $[n,k]_q$ RS code that corrects from t insdels where

$$t \leq n - \sqrt{(1+\varepsilon) \cdot kn \cdot (2t+1)}\,. \tag{1}$$

Then, $\mathcal{C}$ has a deterministic unique-decoding algorithm that corrects t insdels in time $O(n^3\varepsilon^{-6})$.

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- Efficiently list-recoverable!

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Let $\alpha_0, \ldots, \alpha_{n-1} \in \mathbb{F}_q$ be distinct. An $[n, k]_q$ *Reed-Solomon (RS) code* is defined as

$$\mathcal{RS}(n, k)_q = \{(f(\alpha_0), \ldots, f(\alpha_{n-1})) \mid f \in \mathbb{F}_q[x], \deg(f) < k\}.$$

Corollary (Guruswami-Sudan Decoder)

Let $\varepsilon > 0$ and $\mathcal{C}$ be an $[n, k]_q$ RS code that corrects from t insdels where

$$t \leq n - \sqrt{(1+\varepsilon) \cdot kn \cdot (2t+1)}\ . \tag{1}$$

Then, $\mathcal{C}$ has a deterministic unique-decoding algorithm that corrects t insdels in time $O(n^3\varepsilon^{-6})$.

- ***Note***: (1) requires $k \cdot t = O(n)$.

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for i ∈ [n] do
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2 $\quad S_i \leftarrow \{y_{\max\{1,(1-P_{\mathrm{d}})i-\ell/2}, \ldots, y_{\min\{m,(1-P_{\mathrm{d}})i+\ell/2}\}$
3 Execute `List-Recover` with the lists $S_1, \ldots, S_n$ to get $\mathcal{L} \subseteq \mathcal{C}$.
4 return $\{\boldsymbol{c} \in \mathcal{L} | d_{\mathrm{ed}}(\boldsymbol{c}, \boldsymbol{y}) \leq \rho n\}$.

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Theorem

Let $P_{\mathrm{d}} \in (0,1)$, $\varepsilon > 0$ and $\mathcal{C} \subseteq \mathbb{F}_q^n$ be a $(P_{\mathrm{d}} + \varepsilon, n^{1/2+0.001}, L)$-list-recoverable code with an efficient list recoverable algorithm `List-recover`.

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  - Similar result for a probabilistic insdel channel! [Davey and MacKay '01]

Introduction

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- Adversarial Insdels
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Adapting Koetter-Vardy Algorithm

Conclusion

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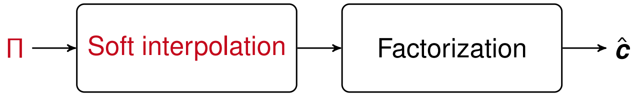

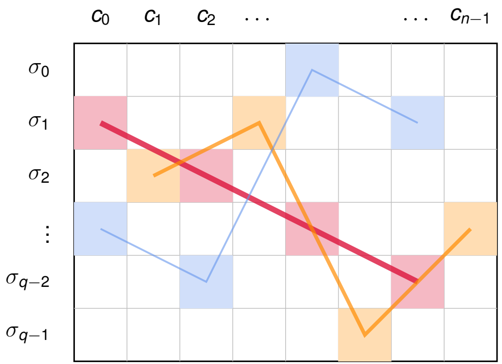

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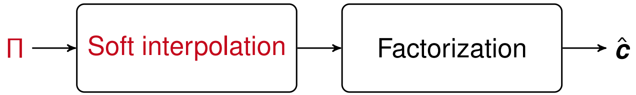

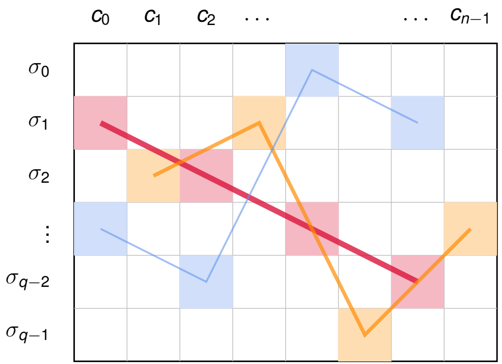

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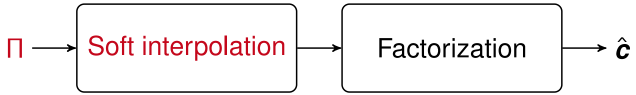

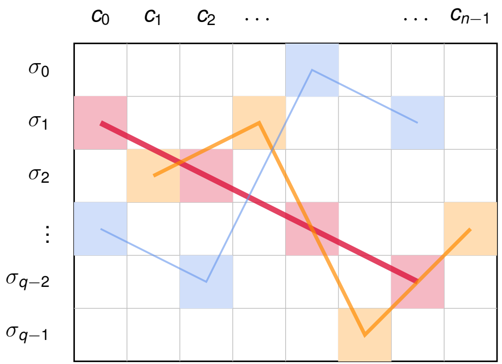

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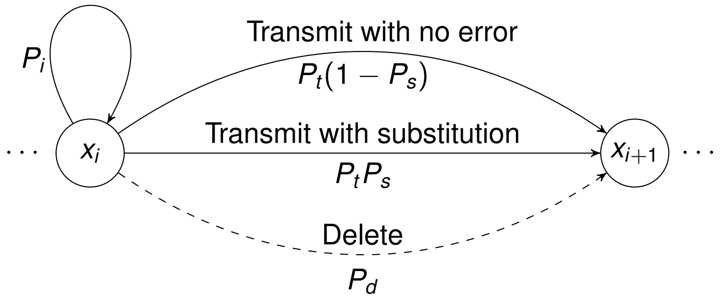

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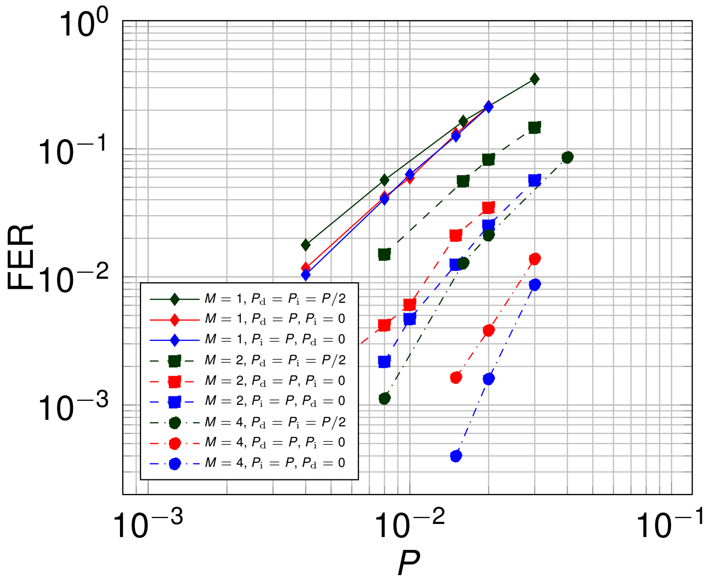

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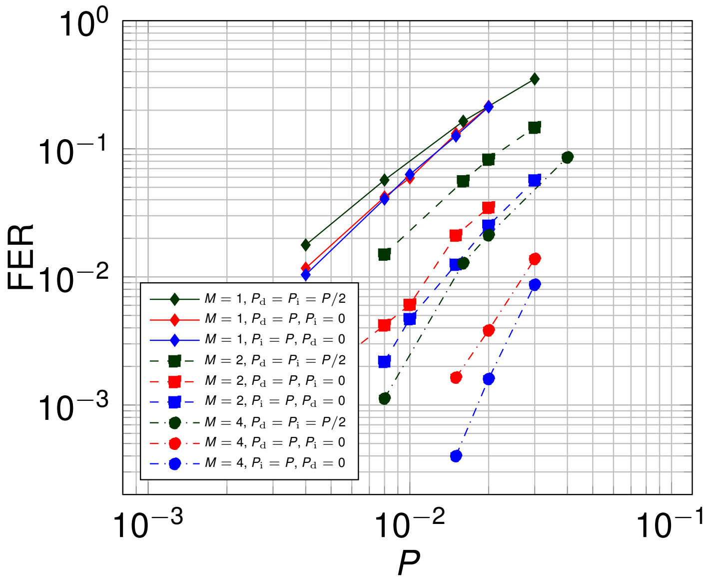

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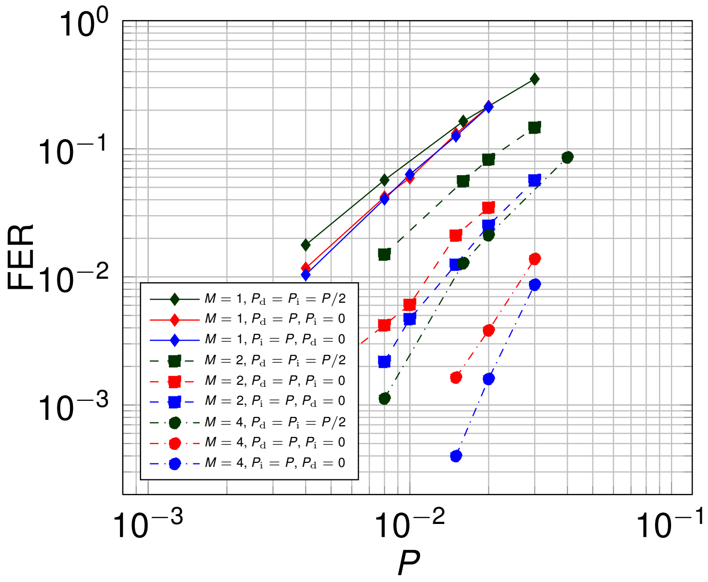

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DNA DATA APPLICATIONS
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References I

[1] G. M. Church, Y. Gao, and S. Kosuri, "Next-generation digital information storage in dna," *Science*, vol. 337, no. 6102, pp. 1628–1628, Sep. 2012.

[2] N. Goldman *et al.*, "Towards practical, high-capacity, low-maintenance information storage in synthesized dna," *Nature*, vol. 494, no. 7435, pp. 77–80, Feb. 2013.

[3] V. Guruswami, "Algorithmic results in list decoding," *Foundations and Trends® in Theoretical Computer Science*, vol. 2, no. 2, pp. 107–195, 2006.

[4] B. Haeupler and A. Shahrasbi, "Synchronization strings: Codes for insertions and deletions approaching the Singleton bound," in *Proceedings of the 49th Annual ACM SIGACT Symposium on Theory of Computing (STOC)*, ACM, 2017, pp. 33–46.

[5] R. Safavi-Naini and Y. Wang, "Traitor tracing for shortened and corrupted fingerprints," in *ACM workshop on Digital Rights Management*, Springer, 2002, pp. 81–100.

[6] Y. Wang, L. McAven, and R. Safavi-Naini, "Deletion correcting using generalized Reed-Solomon codes," in *Coding, Cryptography and Combinatorics*, Springer, 2004, pp. 345–358.

References II

[7] D. Tonien and R. Safavi-Naini, "Construction of deletion correcting codes using generalized Reed–Solomon codes and their subcodes," *Designs, Codes and Cryptography*, vol. 42, no. 2, pp. 227–237, 2007.

[8] T. D. Duc, S. Liu, I. Tjuawinata, and C. Xing, "Explicit constructions of two-dimensional Reed-Solomon codes in high insertion and deletion noise regime," *IEEE Transactions on Information Theory*, vol. 67, no. 5, pp. 2808–2820, 2021.

[9] S. Liu and I. Tjuawinata, "On 2-dimensional insertion-deletion Reed-Solomon codes with optimal asymptotic error-correcting capability," *Finite Fields and Their Applications*, vol. 73, p. 101 841, 2021.

[10] R. Con, A. Shpilka, and I. Tamo, "Reed–Solomon codes against adversarial insertions and deletions," *IEEE Transactions on Information Theory*, 2023.

[11] J. Liu, "Optimal RS codes and GRS codes against adversarial insertions and deletions and optimal constructions," *IEEE Transactions on Information Theory*, 2024.

[12] R. Con, Z. Guo, R. Li, and Z. Zhang, "Random reed-solomon codes achieve the half-singleton bound for insertions and deletions over linear-sized alphabets," *arXiv preprint arXiv:2407.07299*, 2024.

[13] S. Singhvi, "Optimally decoding two-dimensional reed-solomon codes up to the half-singleton bound," *arXiv preprint arXiv:2412.20771*, 2024.

References III

[14] R. Con, A. Shpilka, and I. Tamo, "Optimal two-dimensional reed–solomon codes correcting insertions and deletions," *IEEE Transactions on Information Theory*, 2024.

[15] M. Davey and D. MacKay, "Reliable communication over channels with insertions, deletions, and substitutions," *IEEE Transactions on Information Theory*, vol. 47, no. 2, pp. 687–698, Feb. 2001.

[16] R. Koetter and A. Vardy, "Algebraic soft-decision decoding of reed-solomon codes," *IEEE Transactions on Information Theory*, vol. 49, no. 11, pp. 2809–2825, Nov. 2003.

[17] P. Beelen, R. Con, A. Gruica, M. Montanucci, and E. Yaakobi, *Reed-solomon codes against insertions and deletions: Full-length and rate-$1/2$ codes*, Jan. 2025. arXiv: 2501.11371 [cs].