Neutron Star as a signal for Quark-Gluon Plasma formation

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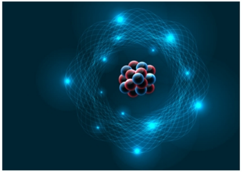
Outline

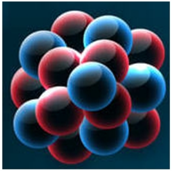
Particles and the Standard Model

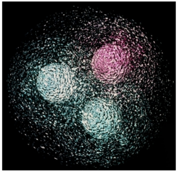
What are elementary particles?

- Smallest constituents of matter (no substructure)
- Examples: electrons (in the atoms) and quarks (in the protons)


DNA: 10^{-8} m


Atom: 10^{-10} m


Nucleus: 10^{-15} m


Quarks: 10^{-18} m

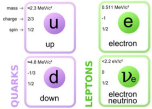
Particles and the Standard Model

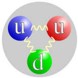
Elementary particles and their basic properties

- These particles are characterized by: their **mass**, **electric charge**, **spin**, color charge, flavor
- Charge dictates which particles participate in which interaction

Ordinary Particles

- Electrons e^- (Thomson, 1897)
- Neutrinos ν (Pauli 1930, Fermi 1932)
- Quarks u, d (Gell-Mann 1964)

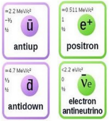


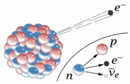


Proton

Antiparticles

- Positrons e^+
- Anti-neutrino $\bar{\nu}$
- Anti-quarks $\bar{u}, \bar{d}$





β^- Decay

$$n \to p + e^- + \bar{\nu}$$

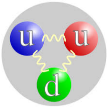
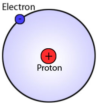
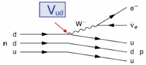
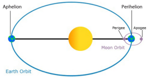
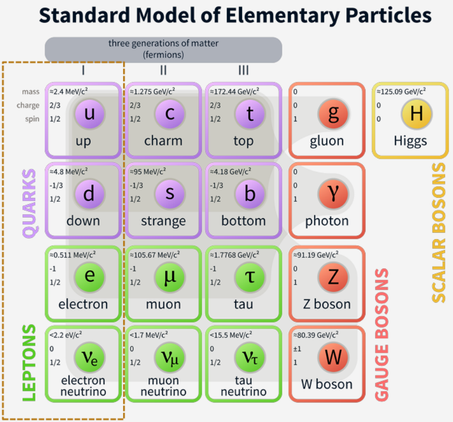
	Strong	**Electromagnetic**	**Weak**	**Gravity**
Current Theory	QCD	QED	EWT	GR
Acts on	Hadrons	Charged particles	All particles	Particles with mass
Mediators	Gluons (g)	Photons (γ)	$W^{\pm}$, Z bosons	Graviton
Relative Strength	10^{2}	1	10^{-4}	10^{-33}
Range (in m)	10^{-15}	∞	10^{-18}	∞
Example:				

Table: Fundamental forces and their properties

Particles and the Standard Model

- Fermions (spin ½)
  - Divided into three generations of:
    - quarks
    - leptons
  - First generation : lightest and most stable particles → the constituents of all stable matter in the universe
- Gauge Bosons (spin 1) → the force carriers
- Higgs boson (spin 0)→ gives mass to elementary particles of the standard model
- Antiparticles : same mass as particle but opposite charge
- Hadrons are built from the elementary blocks of matters:
  - Mesons : 1 quark + antiquark (eg. pion)
  - Baryons : 3 quarks (eg. proton, neutron)



Distribution and Partition Functions

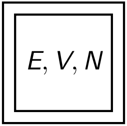
Statistical mechanics helps us relate the behavior of various macroscopic quantities to the underlying microscopic behavior of individual atoms or other constituents. It does so by using the concept of probability theory.

A **thermodynamic system** is defined by its macroscopic properties, such as pressure (P), volume (V), and temperature (T)

Distribution and Partition Functions

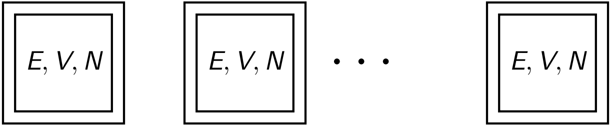
Consider an isolated thermodynamic system of N particles in a volume V with total energy E.

(E, V, N) specify a macrostate of the system $= M$



M

Ensemble is a set of N copies of the system, all in the same macrostate:



M_1 M_2 M_n

Distribution and Partition Functions

Canonical Partition Function (Z):

$$Z = \sum_i e^{-\beta E_i}, \quad \beta = \frac{1}{k_B T}$$

Grand Canonical Partition Function (Ξ):

$$\Xi = \sum_i e^{-\beta(E_i - \mu N_i)}$$

Key Concept: The partition function Z normalizes the probabilities, ensuring that the total probability of all possible states sums to 1. Here k_B is the Boltzmann constant, $k_B = 1.38 \times 10^{-23}$ J/K.

Ensemble	Macrostate	Probability Distribution	Thermodynamics
Microcanonical	E, V, N	$p_i = \frac{1}{\Omega}$	$S(E, V, N) = k_B \ln \Omega$
Canonical	T, V, N	$p_i = \frac{1}{Z} e^{-\beta E_i}$	$F(T, V, N) = -k_B T \ln Z$
Grand Canonical	T, V, μ	$p_i = \frac{1}{\Xi} e^{-\beta(E_i - \mu N_i)}$	$\Omega(T, V, \mu) = -k_B T \ln \Xi$

Table: The three ensembles. Ω is the number of accessible microstates in the microcanonical ensemble, and the thermodynamic potential in the grand canonical ensemble.

Quantum Statistical Mechanics deals with four different types of particles:

1. Distinguishable particles
2. Classically indistinguishable particles
3. Quantum indistinguishable particles:
    - **Bosons**
    - **Fermions**

Bosons

1. '0' or integral spin
2. Do not obey Pauli Exclusion Principle
3. Symmetric wave function
4. Any number of bosons can exist in the same quantum state of the system

Fermions

1. Odd half-integral spin
2. Obey Pauli Exclusion Principle
3. Anti-Symmetric wave function
4. Only one fermion can exist in a particular quantum state of the system

Distribution and Partition functions

For Quantum indistinguishable particles:

1. cannot ask which particle is in which state
2. can only ask how many particle there are in each state
3. leads to occupation number representation of multiparticle states

Distribution and Partition Functions

A system microstate i is specified by giving the **occupation numbers** n_k for all single-particle states $|k\rangle$, where $k = 1, 2, 3, \ldots$.

The set $(n_1, n_2, n_3, \ldots) = \{n_k\}$ defines the **multiparticle state** of indistinguishable particles.

For a given microstate i, the total number of particles is:

$$\sum_k n_k = N_i$$

The total energy in microstate i is:

$$\sum_k \epsilon_k n_k = E_i$$

where ϵ_k is the energy of state k, and n_k is the occupation number.

The grand canonical partition function for an ideal gas is given by:

$$\Xi(T, V, \mu) = \sum_{n_k} e^{-\beta n_k(\epsilon_k - \mu)}$$

Distribution and Partition functions

For **fermions**, the occupation number n_k can only be 0 or 1:

$$\Xi_k = 1 + e^{-\beta(\epsilon_k - \mu)}$$

The corresponding Landau potential Ω_k is:

$$\Omega_k = -k_B T \ln \Xi_k = -k_B T \ln \left(1 + e^{-\beta(\epsilon_k - \mu)}\right)$$

Using the relation $\bar{n}_k = -\frac{\partial \Omega_k}{\partial \mu}$, the mean number of particles in the k-th quantum state is:

$$\bar{n}_k = \frac{1}{e^{\beta(\epsilon_k - \mu)} + 1}$$

This result is known as the ***Fermi-Dirac distribution***.

For **bosons**, the occupation number n_k can take any non-negative integer values $\mathrm{n}_k = 0, 1, 2, \ldots$

$$\Xi_k = 1 + e^{-\beta(\epsilon_k - \mu)} + e^{-2\beta(\epsilon_k - \mu)} + \cdots = \sum_{n_k=0}^{\infty} \left[e^{-\beta(\epsilon_k - \mu)} \right]^{n_k} = \frac{1}{1 - e^{-\beta(\epsilon_k - \mu)}}$$

The corresponding Landau potential Ω_k is:

$$\Omega_k = k_B T \ln \left[1 - e^{-\beta(\epsilon_k - \mu)} \right]$$

The mean number of particles in the k-th state is:

$$\bar{n}_k = -\frac{\partial \Omega_k}{\partial \mu} = \frac{1}{e^{\beta(\epsilon_k - \mu)} - 1}$$

This result is known as the ***Bose-Einstein distribution***.

Motivation

- Neutron stars allow us to investigate matter in its most extreme states, offering insights into fundamental physics
- They serve as cosmic probes to understand the early formation and evolution of the universe

Birth of a Neutron Star

- Stars are typically formed in dense regions of gas and molecular clouds, known as **nebulae**

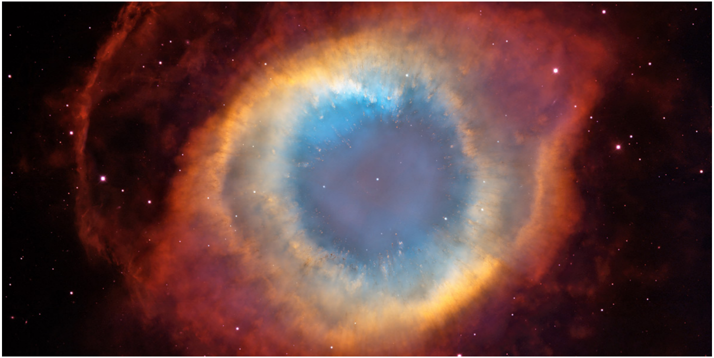
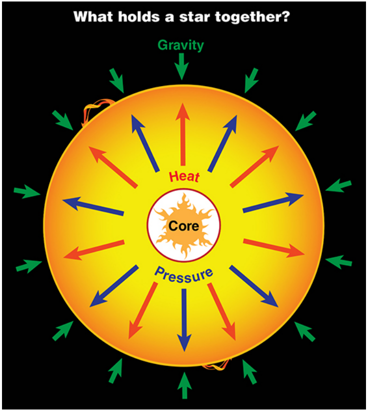


Figure: Formation of stars in a nebula

Birth of a Neutron Star

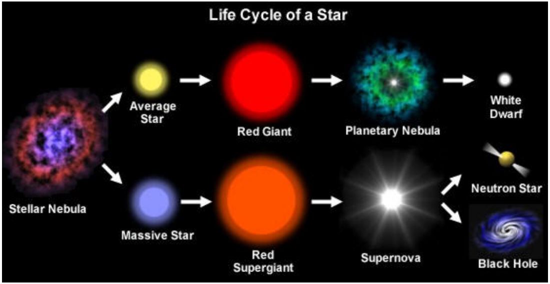


In young stars, nuclear fusion produces enough outward pressure to balance the inward gravitational force, maintaining their structure in a state known as **hydrostatic equilibrium**

Birth of a Neutron Star

As stars exhaust their nuclear fuel,

- Stars below $\sim 8M_{\odot}$ transition from the red giant stage to the planetary nebulae ultimately becoming white dwarf
- Stars $> 8M_{\odot}$ explode into Supernova which leaves neutron stars or black holes as remnants
- Stars that are more massive than $30M_{\odot}$ primarily collapse to form black holes



Neutron Star

- Compact remnants of massive stars with a radius ~ 20 km and mass ~ 1.4 $M_{\odot}$
- Their surface matter density is $\rho \sim 10^4$ g cm^{-3}, while their cores can reach densities exceeding the normal nuclear matter density, $\rho_0 \sim 2.8 \times 10^{14}$ g cm^{-3}

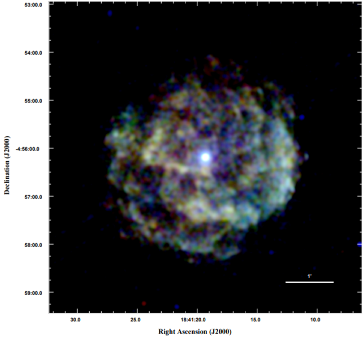
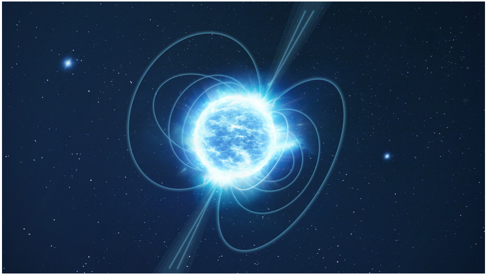


Figure: A young supernova remnant SNR Kes 73 containing a neutron star [1]

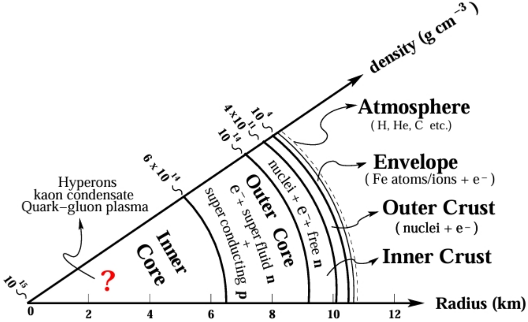
Neutron Star

- Surface gravity $\sim 2 \times 10^{11}$ times Earth's gravity
- Extremely **strong magnetic fields**, up to 10^{15} Gauss
- **Pulsars**: Rotating neutron stars emitting periodic radio signals
- The neutrons themselves exert degenerate pressure that prevents a neutron star from collapsing under gravity
- The resulting fluid of mostly neutrons may constitute the core of a neutron star



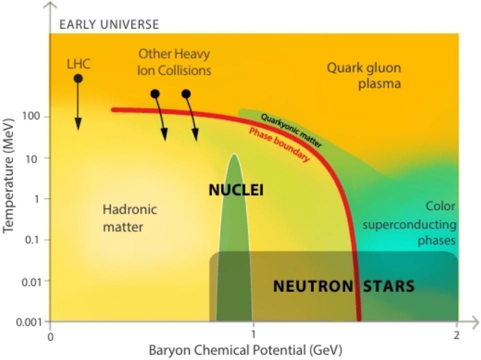
Structure of a Neutron Star

- **Atmosphere:** Thin plasma layer
- **Outer Crust:** Lattice structure of atomic nuclei and fermi liquid of relativistic degenerate electrons
- **Inner Crust:** Free neutrons and neutron-rich nuclei. It which extends from the neutron drip density $\sim 10^{11}\,\mathrm{g\,cm^{-3}}$ to a transition density $\rho_{\mathrm{tr}} = 1.7 \times 10^{14}\,\mathrm{g\,cm^{-3}}$



Structure of a Neutron Star

- **Outer Core:** Uniform nuclear matter consisting of neutrons, protons, electrons, and muons, governed by charge neutrality and beta equilibrium. This layer is $\sim 10\,\text{km}$ thick, constitutes most of the star's volume
- **Inner core:** At extreme densities, the inner core may host exotic matter ,mesons, hyperons, or quark matter [2]. This region, though poorly understood, accounts for most of the neutron star's mass



Neutron Star

The equation of state (EoS) of neutron star matter is described by different models at different density ranges

- At Subnuclear densities($\rho \approx 10^4$ g/cm^3): Bayn-Pethick-Sutherland EoS describes the crust of neutron star
- At Nuclear densities($\rho_0 \approx 2.5 \times 10^{14}$ g/cm^3): The relativistic mean-field (RMF) model is used

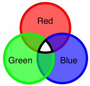
Our study focuses on the strange quark matter EoS, which becomes relevant at densities exceeding ρ_0

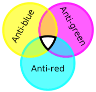
- At extreme densities, nucleons dissolve into a quark-gluon plasma
- Composed of up (u), down (d), strange (s) quarks and electrons
- Possible existence of quark cores within massive neutron stars

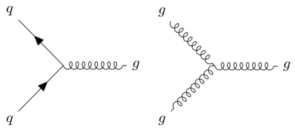
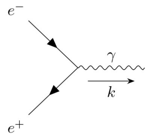
To calculate the EoS, we use the **Density Dependent Quark Mass Model** (DDQM). We shall discuss this in the coming slides

Basics of Quantum Chromodynamics (QCD)

- QCD is a quantum field theory with similarities and differences with respect to QED, which describes electromagnetic interaction
- Quarks have a new type of charge called **color** (red, blue, green). Each color has an anti-color
- Quarks combine to form colorless **hadrons**





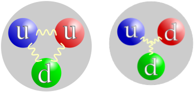
	Strong	**Electromagnetic**
Current Theory	QCD	QED
Charge types	3 color charges	Electric charge (e)
Acts on	Color-charged objects: quarks and gluons	Electrically charged particles
Mediators	Gluons (g)	Photons (γ)
Relative Strength	10^2	1
Range (in m)	10^{-15}	∞
Fundamental vertices:		

Forming Hadrons

Particles that experience the strong force are called **hadrons**. Hadrons are divided into two types:

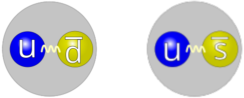
Baryons

- Combination of three quarks
- Examples:
  - Proton (uud)
  - Neutron (udd)



Mesons

- Combination of quark and anti-quark pairs
- Examples:
  - Pion (π^+: $u\bar{d}$)
  - Kaon (K^+: $u\bar{s}$)



The strong force ensures hadrons are color neutral or "white".

Important features of QCD

- **Current mass:** Mass of the quark in the absence of confinement
- **Constituent mass:** Effective mass of a confined quark in a hadron (few hundred MeV in magnitude)
- QCD is **weakly** interacting at **short** distance scales but very **strongly** interacting at **large** distance scales

The **QCD coupling constant** α is related to the scale of momentum transfer q by the relation [4]:

$$\alpha(q^2) = \frac{\alpha_0}{1 + \alpha_0 \frac{(32 - n_f)}{12\pi} \ln\left(\frac{-q^2}{\mu^2}\right)}$$

where α_0 is the initial value of the coupling constant, and n_f is the number of flavors. μ serves as a reference point against which other energy scales (the momentum transfer q) are compared.

Important features of QCD

Asymptotic Freedom at high energies:

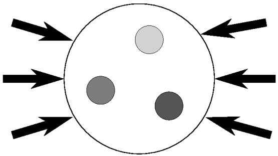
- At short distances (perturbative QCD regime, $Q \gg 1\,\text{GeV}$):
- In such regime quarks and gluons appear to be **"quasi-free"**

Confinement at low energies:

- At large distances ($Q < 1\,\text{GeV}$)
- Gluons bind quarks together to form the hadrons (the particles we observe in nature)
- **Perturbation theory** in α at low energies is not applicable!

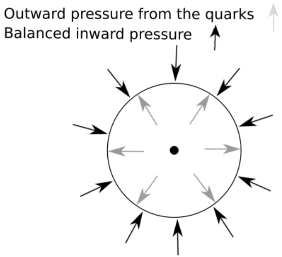
MIT Bag Model

- Developed in 1974 at Massachusetts Institute of Technology (MIT)
- It models spatial confinement only
- Quarks are forced by a fixed external pressure to move only inside a given spatial region
- The shape of the bag is spherical if all the quarks are in the ground state
- Inside the bag, quarks are allowed to move quasi-free
- **Main Assumption**: Quarks are massless inside a bag of radius R



Quark Gluon Plasma

- An inward bag pressure B confines the quarks inside a bag
- When the pressure of quark matter inside exceeds the bag pressure, the bag can no longer confine the quarks
- A new phase of unconfined quarks and gluons i.e. **quark-gluon plasma** is possible at high pressures, temperature, or baryon density
- The **quark-gluon plasma** is composed of nearly equal numbers of up (u), down (d), and strange (s) quarks along with a gas of electrons



Density-Dependent Quark Mass Model

- The quark-gluon plasma is referred to as **strange quark matter** (SQM)

We study the general properties in the framework of a new EoS in which the quark **masses are parametrized** as functions of the baryon density n_B as follows [5]:

$$m_u = m_d = \frac{c}{3n_B}, \quad m_s = m_{s0} + \frac{c}{3n_B}.$$

Here, m_{s0} is the strange quark current mass and c is a constant.

- The system is in the presence of a magnetic field **B** which is directed along the z-axis
- The **energy** of the charged particle of mass m_i and charge q_i in the presence of the B is given by [6]

$$\epsilon_i = \left[m_i^2 + p_{z,i}^2 + 2q_iBn\right]^{1/2}$$

where p_z is the momentum along the z-axis,
n is represents the Landau level.

Density-Dependent Quark Mass Model

The **Thermodynamic potential** is given by [5]

$$\Omega = \sum_i \Omega_i - \frac{8}{45}\pi^2 T^4$$

where the second term is contribution due to gluons.

The general expression for the thermodynamic potential Ω_i in the presence of B is

$$\Omega_i = -T\frac{g_i q_i}{2\pi^2} B \int dp_z \ \ln\left[1 - e^{-\beta(\epsilon_i - \mu_i)}\right]$$

where i = (u, d, s, e), g_i is the degeneracy factor ($g_i = 2 \times 3 = 6$ for quarks, $g_i = 2$ for electrons).

- The thermodynamical properties, pressure P_i, energy density E_i, entropy S, and specific heat C_v can be calculated from Ω which provides us the equation of state (EoS) for the system.

Computing the EoS for Neutron Star

The standard thermodynamics gives the following relations:

$$P(k, T) = -\Omega_{\text{total}}$$

$$E(k, T) = -T^2 \left[\frac{\partial}{\partial T} \left(\frac{\Omega_{\text{total}}}{T} \right) \right]$$

$$S(k, T) = -\frac{4}{3} T \left[\frac{\partial}{\partial T} \left(\frac{\Omega_{\text{total}}}{T} \right) \right]$$

$$C_v(k, T) = T \left[\frac{\partial S}{\partial T} \right]$$

Results

- We numerically computed these parameters for a range of temperatures at $\mu = 300\,\mathrm{MeV}$
- Since the system is in beta-equilibrium, we have

$$\mu_d = \mu_s$$

and assuming that neutrinos or antineutrinos produced stream out freely $\mu_\nu = 0$,

$$\mu_d = \mu_u + \mu_e$$

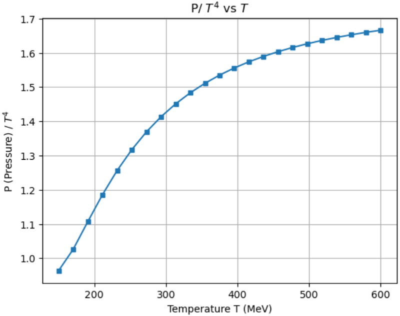
- At finite temperature, the chemical potential μ_e varies between 6 and 50 MeV [7]. We considered $\mu_e = 7$ MeV
- From the stability window of SQM [8], the range for c is from 70 to 110 MeV fm^{-3} and for m_{s0} is within 50-180 MeV. In our study, we have considered:

$$c = 75\,\mathrm{MeV\,fm}^{-3}, \quad m_{s0} = 140\,\mathrm{MeV}$$

Results

The computed results for the thermodynamic properties of the system are presented as:

Pressure



Energy Density

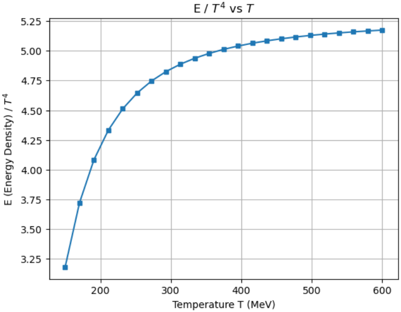


Figure: The variation of pressure and energy density. The left-hand side (LHS) shows the pressure, while the right-hand side (RHS) shows the variation of P/T^4 and E/T^4 with Temperature T (MeV). Scaling by T^4 allows for a comparison with the Stefan-Boltzmann limit, highlighting deviations due to interactions and medium effects.

Results

Interaction Term

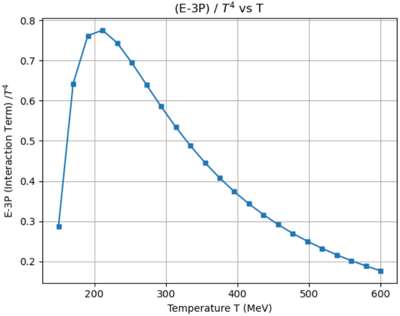
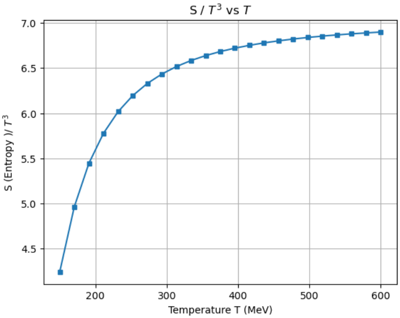


Figure: The variation of Interaction Term ($E - 3P/T^4$) with Temperature T(MeV).

The **Interaction term** refers to the force or potential that governs the interactions between the constituent quarks and gluons within the plasma and is defined as **E - 3P** ($E =$ Energy Density, $P =$ Pressure)

Unlike in ideal gas, where $E = 3P$, the interaction term indicates deviations from this relationship which are crucial for understanding the thermodynamic properties of the system.

Entropy



Specific Heat

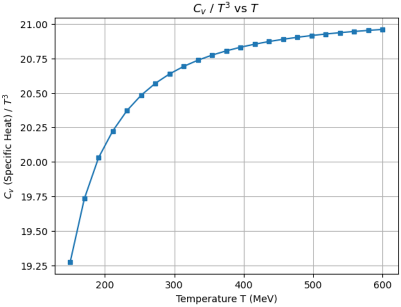


Figure: The variation of S/T^3 on the left-hand side (LHS) and C_v/T^3 on the right-hand side (RHS) with Temperature T (MeV). Here also, scaling by T^3 allows for a comparison with the Stefan-Boltzmann limit.

Results

Now using the entropy and specific heat we further calculated the speed of sound of QGP. The **Speed of Sound** is given as:

$$C_s^2 = \frac{S}{C_v}$$

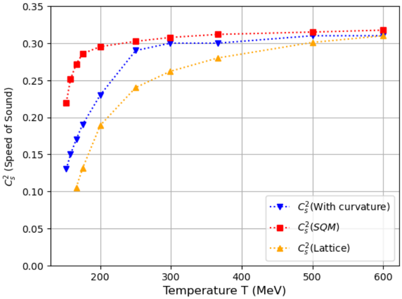


Figure: Speed of sound calculated from our theoretical Model (SQM) is compared with Lattice data and with curvature data.

Conclusion and Future Works

- In our work, we investigated various aspects of strange quark matter (SQM), which may be found in the cores of neutron stars, using the DDQM model
- We focused on thermodynamic quantities such as **pressure, energy density, entropy, and specific heat**, which provided us with the EoS for the system
- We also calculated the **speed of sound** and compared it to lattice and curvature data, showing agreement at higher temperatures
- For future work, a new model can be employed where quark masses can be made both density-as well as temperature-dependent, with comparisons to the DDQM model
- Future studies will also focus on calculating the radial pulsations of neutron stars to better understand their dynamics and stability

Thank you for your attention

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