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University of Delhi, Delhi-110007

M.Sc. Physics, Semester 3

Dissertation Thesis

On

Neutron Star as a signal for Quark-Gluon Plasma formation

Under the supervision of

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ACKNOWLEDGEMENT

I extend my heartfelt gratitude to my supervisor, Prof. S. Somorendro Singh, for his invaluable guidance, constructive feedback, and unwavering support throughout this project. His expertise and insightful mentorship have played a crucial role in shaping the direction and depth of this work.

I would also like to thank my friends and family for their encouragement and constant support during this journey.

Finally, I am sincerely grateful to the Department of Physics and Astrophysics, University of Delhi, for providing me the opportunity and resources to undertake this dissertation.

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CERTIFICATE

It is certified that the dissertation entitled "Neutron star as a signal for Quark-Gluon Plasma formation" has been carried out by Anisha Khatri(MSc.2023-2025, Roll No.-23026762005) in the University of Delhi towards partial fulfillment of requirements for the award of degree of Master Of Science in Physics is a record of bonafide work carried out by her under my supervision and guidance during third semester. The content of this project report has not been submitted elsewhere for the award of any academic and professional degree.

Date: 5/12/2024

Place: New Delhi, India

Signature of the Supervisor
Prof. S. Somorendro Singh
Project Supervisor

DECLARATION

I declare that this thesis entitled is the record of the bonafide work carried out by me under the supervision Prof. S. Somorendro Singh, Professor, Department of Physics and Astrophysics, University of Delhi, Delhi, India.

I further declare that this thesis has not previously formed the basis for the award of any degree, diploma, associateship, fellowship or other similar title of recognition.

Anisha Khatri

Date: 5/12/2024

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ABSTRACT

Studying neutron stars can help us understand the early formation of the universe. To understand and describe neutron stars, an appropriate equation of state that satisfies bulk nuclear matter properties is necessary. This thesis studies the quark matter EoS by applying the Density Dependent Quark Mass model which is an extension of the MIT Bag model. The EoS such as pressure, energy density, entropy, specific heat and speed of sound are calculated for a range of temperatures. The model results provide QGP EoS which are then compared with Lattice QCD data.

Contents

Chapter 1

Introduction

The physics of compact stars—such as white dwarfs, neutron stars, and potential quark or hybrid stars—presents a fascinating interplay between nuclear processes and astrophysical observables. These stars are the remnants of massive stars that have undergone supernova explosions, marking the final stages of stellar evolution.

The discovery of *pulsars* in 1968–69, identified as rotating neutron stars, sparked significant interest in neutron star physics. Neutron stars, have typical masses of $\sim 1 - 2M_{\odot}$ and radii of $\sim 10 - 15$ km, are primarily composed of neutrons, along with smaller amounts of protons and electrons, governed by beta-stability and charge neutrality. Their surface matter density is around $\rho \sim 10^4\,\mathrm{g\ cm^{-3}}$, while their cores can reach densities exceeding the normal nuclear matter density, $\rho_0 \approx 2.8 \times 10^{14}\,\mathrm{g\ cm^{-3}}$. At such extreme densities, exotic matter like hyperons, quark-hadron mixed phases, and Bose-Einstein condensates of kaons may emerge.

Neutron stars not only exhibit extreme density but also rotate at incredibly high speeds, with periods in the millisecond range. Some possess huge magnetic fields, referred to as magnetars, with intensities up to 10^{15} Gauss. These unique conditions, unattainable in terrestrial laboratories, provide a natural platform for investigating matter in its most extreme states and brings together all four fundamental forces of nature: gravitational, weak, strong, and electromagnetic forces, making them unique in astrophysical research.

Studying neutron stars involves multiple branches of physics, such as nuclear physics, general relativity, and astrophysics, bridging length scales from femtometers to kilometers. This complexity has fueled extensive research over the past few decades. From a theoretical standpoint, the interior composition of neutron stars presents a fundamental challenge, requiring the understanding of particle interactions under extreme densities. While quantum chromodynamics (QCD) is the standard theory for such interactions, many-body calculations remain computationally challenging.

In this context, a wide variety of theoretical models exist to describe neutron star matter. This thesis solely focuses on studying the equation of state (EoS) of quark matter formed in the cores of neutron stars, using the Density Dependent Quark Mass (DDQM) model. We begin by providing the necessary background to ensure a comprehensive understanding of the results presented later. This section offers a brief introduction to the Standard Model of particle physics, covering the fundamental forces, distribution, and partition functions. Additionally, it includes an explanation of four-vector algebra and the use of natural units employed in our study.
Chapter 2 presents a comprehensive review of the structure and composition of neutron stars. It introduces the various models used to describe the equation of state for neutron star matter. A brief discussion on quarks and strange stars is also presented.
In the subsequent Chapter 3, we delve into the formalism of Quantum Chromodynamics (QCD) by presenting the QCD Lagrangian and exploring one of its key properties, asymptotic freedom, which plays a crucial role in the behavior of quarks and gluons at high energies. We then introduce the Bag Model, which offers a framework to understand the confinement of quarks and gluons within hadrons and serves as the foundation for our study.

Chapter 4 focuses on the phenomenological MIT-bag model, deriving the pressure from the Lagrangian density. We then explore the theoretical framework of the density-dependent quark mass model (DDQM). In Chapter 5, we calculate the thermodynamic properties and speed of sound, comparing our results with lattice QCD data. Finally, we conclude with a summary of the findings and discuss potential future research work.

1.1 Background Theory

Here the reader is given the elementary knowledge needed to understand the result and discussion sections of the thesis. The different concepts are only briefly explained and for further information regarding each subject, we refer to the mentioned references.

1.1.1 Particles and the Standard Model

The nature of matter has intrigued humanity for centuries. Initially, the atom was considered the fundamental building block of matter. However, further investigation revealed that atoms themselves are composed of subatomic particles—protons, neutrons, and electrons. As research progressed, it became evident that even these subatomic particles were made up of smaller, more fundamental components. These elementary particles, governed by the Standard Model are referred to as the fundamental building blocks of matter and they define the interactions that govern the universe.

The term ”particle zoo” informally refers to the classification of particles in particle physics. Just as animals in a zoo are categorized into different zones based on their physical characteristics, particles are grouped into two broad categories based on their properties, depending on their spin and the statistics they obey. These categories help us organize the numerous particles discovered so far, allowing for a clearer understanding of their behavior and interactions.

Fermions

These are the particles which have half integers spin. These particles follow the Fermi Dirac statistics as they follow the Pauli exclusion principle. According to this principle no two fermions can be in the same energy state. They are divided into the following categories:

- **Quarks:** The fundamental constituents of protons and neutrons are spin $+\frac{1}{2}$ particles that cannot exist freely and interact via the strong force through gluon exchange. There are six flavors of quarks: up, down, charm, strange, top, and bottom. Up and charm quarks have a charge of $+\frac{2}{3}$, while down and strange quarks have $-\frac{1}{3}$. Charm and top quarks are heavier versions of up, and bottom quarks are heavier versions of down and strange. Quarks possess a color charge (the term color is arbitrary and has nothing to do with visual colors), with each flavor corresponding to one of three colors: red, green, or blue, as described by quantum chromodynamics (QCD). The corresponding antiquarks have anticolors. Each quark flavor has an associated quantum number (e.g., strangeness, charmness), which is conserved in reactions. We shall discuss QCD in Chapter 3.
- **Leptons:** These are elementary particles that carry integral charge and exist freely in nature. They interact with all fundamental forces except the strong force. The

Quarks	Q	I_z	C	S	T	B	mass [7] (MeV)
u	$\frac{2}{3}$	$\frac{1}{2}$	0	0	0	0	5.6 ± 1.1
d	$-\frac{1}{3}$	$-\frac{1}{2}$	0	0	0	0	9.9 ± 1.1
c	$\frac{2}{3}$	0	1	0	0	0	1350 ± 50
s	$-\frac{1}{3}$	0	0	-1	0	0	199 ± 33
t	$\frac{2}{3}$	0	0	0	1	0	> 90000
b	$-\frac{1}{3}$	0	0	0	0	-1	$\simeq 5000$

FIGURE 1.1: Table showing the different types of quarks along with their mass, charge, quantum numbers (isospin, strangeness, charmness, bottomness, and topness)

lepton family includes three charged particles—electron, muon, and tau—referred to as the three flavors of charged leptons. Each of these charged leptons has an associated neutrino: the electron neutrino, muon neutrino, and tau neutrino. While tau and muon particles share properties with the electron, they are significantly heavier. Neutrinos, on the other hand, are nearly massless, though they possess a small but nonzero mass. Each lepton flavor is characterized by a corresponding quantum number that is conserved in interactions.

Lepton name	*Mass* $[MeV/c^2]$	*Electric charge* $[e]$	*Spin* (J)	*Lepton number* (L)	*Electron lepton number* (L_e)	*Muon lepton number* (L_μ)	*Tau lepton number* (L_τ)
electron (e^-)	0.511	- 1	1/2	+1	+1	0	0
electron neutrino (ν_e)	$<2\times10^{-6}$	0	1/2	+1	+1	0	0
muon (μ^-)	105.66	- 1	1/2	+1	0	+1	0
muon neutrino (ν_μ)	$<2\times10^{-6}$	0	1/2	+1	0	+1	0
tau (τ^-)	1776.86	- 1	1/2	+1	0	0	+1
tau neutrino (ν_τ)	$<2\times10^{-6}$	0	1/2	+1	0	0	+1

FIGURE 1.2: Table showing the different types of leptons along with their mass, charge and quantum numbers (spin, lepton numbers)

Bosons

Bosons are the particles that are responsible for the mediation of the particle interactions. They possess integral spin and obey Bose-Einstein statistics, allowing multiple particles to occupy the same quantum state, as they do not adhere to the Pauli exclusion principle. Bosons include the exchange particles of the fundamental forces, which are as follows:

- **The Photon:** It mediates electromagnetic interactions, which do not involve charge transfer. As a result, the photon, being the exchange particle of these interactions, is both chargeless and massless. Due to this nature they participate in interaction mediation.

- **The $W_{\pm}$ and Z Bosons:** These are the mediators of the weak interaction. These exchange particles are characterized by their significant masses. Since weak interactions involve both charge transfer and non-charge transfer processes, these particles act as charged mediators ($W_{\pm}$) as well as a neutral mediator(Z).

- **The Gluons:** These are the mediators of the strong interaction. They carry a color–anticolor pair and are of eight distinct types, each associated with a specific combination of color and anticolor charges.

- **The Mesons:** These are composite particles made up of a quark and an antiquark. They are not fundamental in nature. Examples include the pion (π-meson) and kaon (K-meson).

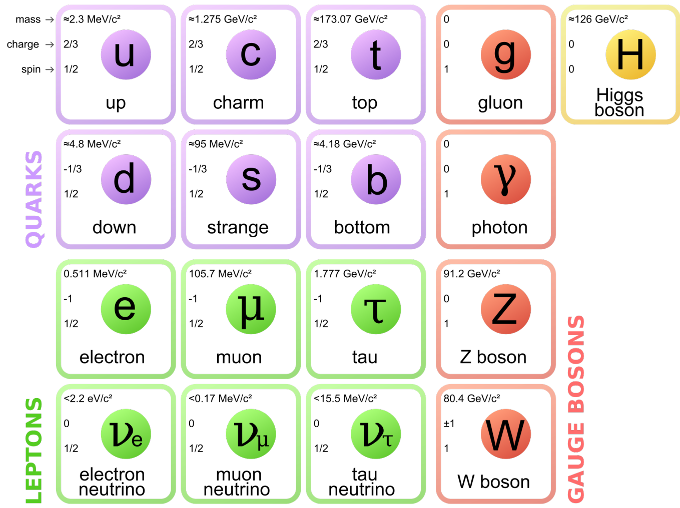

FIGURE 1.3: The elementary particles according to the SM. The values in the upper left corner of each box tells the mass, electrical charge and spin of the particle.

1.1.2 The Fundamental Forces

To understand the forces of nature, it is essential to study the Standard Model, a unified framework that explains three of the four fundamental forces in nature. The four types of particle interactions are the strong interaction, electromagnetic interaction, weak interaction, and gravitational interaction.

- **Strong Interaction:** It is the strongest of the four interactions, being approximately 137 times stronger than the electromagnetic force, 10^6times stronger than the weak force, and 10^{38} times stronger than the gravitational force. It acts on color charges, which are carried by quarks. Just as the electromagnetic force attract particles of different electric charge, so does the strong interaction attract particles of different color. According to quantum chromodynamics (QCD), quarks possess three 'colors' red, blue, and green. The mediator of this interaction is the gluon, a gauge boson that carries both a color and an anti-color, facilitating color transfer between quarks. Unlike the Coulomb force, where the strength decreases with distance, the strength of the color force increases with distance. The range of this force is approximately 10^{-15} meters.

- **Electromagnetic Interaction:** It is the second strongest fundamental force, being 137 times weaker than the strong force, 10^4times stronger than the weak force, and 10^{36} times stronger than the gravitational force. It acts on all particles with electric charge. The strength of this force is inversely proportional to the square of the distance, and its range is infinite. The mediator of the electromagnetic interaction is the photon, which is both chargeless and massless.

- **Weak Interaction:** It is weaker than both the electromagnetic and strong forces, but stronger than the gravitational force. It is the only interaction capable of changing the flavor of quarks. The weak force is mediated by the W and Z bosons, which have significant masses, on the order of 80 GeV. Due to their mass, the uncertainty principle dictates a range of about 10^{-18} m, which is approximately 1% of the proton's diameter. The weak force plays a crucial role in changing the flavors of quarks.

- **Gravitational Interaction:** It is the weakest of the four fundamental forces. It is 10^{38} times weaker than the strong force, 10^{36} times weaker than the electromagnetic force, and 10^2 times weaker than the weak force. As a result, gravity has a negligible

	Gravitation	Electromagnetic	Weak	Strong
Acts on	particles with mass and energy	particles with charge	quarks and leptons (decay)	quarks
Exchange particle	graviton (not yet observed)	photon, γ	W^+, W^- and Z^0	gluons, g, and mesons
Exchange particle mass	massless	massless	$M_{\mathrm{W}^\pm} = 80\,\mathrm{GeVc}^{-2}$, $M_{\mathrm{Z}} = 91\,\mathrm{GeVc}^{-2}$	gluons are massless
Relative strength	negligible, predicted about 10^{-41}	$\frac{1}{137}$	10^{-6}	1
Range	∞ decreasing $\propto \frac{1}{r^2}$	∞ decreasing $\propto \frac{1}{r^2}$	10^{-18} decreasing $\propto \frac{1}{r}$	10^{-15} increasing $\propto r$

FIGURE 1.4: Properties of the four fundamental interactions

influence on the behavior of subatomic particles and does not play a role in determining the internal properties of everyday matter. However, it causes all masses to attract each other, including planets, stars, and galaxies. The range of the gravitational interaction is infinite.

1.2 The Distribution and the Partition Functions

In statistical mechanics, observing the interaction between a large number of objects is key to understanding the behaviour of materials. Such a collection of particles is referred to as a system of particles. A thermodynamic system consists of matter or fields, such as molecules, atoms, or magnetic fields, confined within a defined boundary.
In a classical system, a *microstate* specifies the exact position and momentum of each particle. For a single particle in one dimension, *microstates* represent the various possible positions of the particle. Similarly, in a system composed of multiple particles, *microstates* correspond to the different possible arrangements of those particles.

The **Landau potential**, or simply the **grand potential** Ω, is the thermodynamic potential expressed in terms of the variables T (temperature), V (volume), and μ (chemical potential). It is defined as:

$$\Omega(T, V, \mu) = F - \mu N = F - G \tag{1.1}$$

where G is the Gibbs free energy, given by:

$$G = F + PV \tag{1.2}$$

Substituting for G, the grand potential becomes:

$$\Omega(T, V, \mu) = F - \mu N = -PV \tag{1.3}$$

This relation will be very useful for obtaining the equation of state of various systems.

- **Maxwell Boltzmann Statistics:** It is a key result from statistical mechanics and describes the distribution of identical distinguishable particles among the available energy states in a physical system. It states that the probability of finding a physical system in a given microstate is,

$$p_i = \frac{1}{Z} \exp(-\beta E_i) \tag{1.4}$$

where $\beta = \frac{1}{k_B T}$, where k_B is the Boltzmann constant ($1.38 \times 10^{-23}\,\mathrm{JK}^{-1}$), T is the absolute temperature of the system, and E_i is the energy of state i.

Z is the partition function, defined as the sum over all states i:

$$Z = \sum_i \exp(-\beta E_i) \tag{1.5}$$

The partition function Z normalizes the probabilities, ensuring that the total probability of all possible states sums to 1. It serves as a fundamental pillar as all thermodynamic properties of a system can be calculated from it.

- **Bose and Fermi Statistics:** For these distributions we have ideal quantum systems. Hence instead of determining the energy levels E_s of the gas as a whole we need to determine ϵ_k the energy levels of a single particle. Because these particles are indistinguishable, we cannot specify the microstates of each particle. Instead it will be specified by the occupation number n_k, the number of particles in each of the single particle energies ϵ_k. The total energy of the system can be written as:

$$E_s = \sum_k n_k \epsilon_k \tag{1.6}$$

The set of n_k completely specifies the microstate of the system. The partition function for an ideal gas can be expressed in terms of the occupation numbers as

$$Z(V,T,N) = \sum_k e^{-\beta \sum_k n_k \epsilon_k} \tag{1.7}$$

where the occupation numbers n_k satisfy the condition

$$N = \sum_k n_k \tag{1.8}$$

From the results of relativistic quantum mechanics, we know that the particles can be classified into two groups. Particles with zero or integral spin are *bosons* and have wave functions that are symmetric under the exchange of any pair of particles. Particles with half-integral spin such as electrons, protons and neutrons are *fermions* and have wave functions that are antisymmetric under particle exchange.
The difference between fermions and bosons is specified by the possible values of n_k. For fermions we have

$$n_k = 0 \,\mathrm{or}\, 1 \tag{1.9}$$

The restriction (1.9) states the Pauli exclusion principle, no two identical fermions cannot be in the same single particle state. In contrast, the occupation numbers n_k for identical bosons can take any positive integer value:

$$n_k = 0, 1, 2, \ldots \tag{1.10}$$

Here we choose the system to be the set of all particles that are in a given single particle state. Because the number of particles in a given quantum state varies, we need to use the grand canonical ensemble and assume that each system is populated from a particle reservoir independently of the other single particle states. The connection of thermodynamics to statistical mechanics is given by $\Omega = -k_B T \ln \mathcal{Z}$, where the grand partition function $\mathcal{Z}_k$ is given by,

$$\mathcal{Z}_k = \sum_{n_k} e^{-\beta n_k (\epsilon_k - \mu)} \tag{1.11}$$

For fermions since $n_k = 0$ or 1, hence

$$\mathcal{Z}_k = 1 + e^{-\beta(\epsilon_k - \mu)} \tag{1.12}$$

The corresponding thermodynamic or landau potential, Ω_k is given by

$$\Omega_k = -k_B T ln \mathcal{Z}_k = -k_B T ln[1 + e^{-\beta(\epsilon_k - \mu)}] \tag{1.13}$$

Using the relation $\bar{n}_k = -\frac{\partial \Omega_k}{\partial \mu}$, the mean number of particles in the k-th quantum state is given as:

$$\bar{n}_k = -\frac{\partial \Omega_k}{\partial \mu} = \frac{e^{-\beta(\epsilon_k - \mu)}}{1 + e^{-\beta(\epsilon_k - \mu)}}. \tag{1.14}$$

or

$$\bar{n}_k = \frac{1}{e^{\beta(\epsilon_k - \mu)} + 1} \tag{1.15}$$

The result (1.15) for the mean number of particles in the kth state is known as the *Fermi-Dirac distribution.*

For bosons, the integer values of n_k are unrestricted. Therefore,

$$\mathcal{Z}_k = 1 + e^{-\beta(\epsilon_k - \mu)} + e^{-2\beta(\epsilon_k - \mu)} + ... = \sum_{n_k=0}^{\infty} [e^{-\beta(\epsilon_k - \mu)}]^{n_k} \tag{1.16}$$

The geometric series in (1.16) is convergent for $e^{-\beta(\epsilon_k - \mu)} < 1$. Because this condition must be satisfied for all values of ϵ_k, we require that $e^{\beta\mu} < 1$, or

$$\mu < 0 \tag{1.17}$$

The summation of the geometrical series in (1.16) gives

$$\mathcal{Z}_k = \frac{1}{1 - e^{-\beta(\epsilon_k - \mu)}} \tag{1.18}$$

and hence we obtain

$$\Omega_k = k_B T ln[1 - e^{-\beta(\epsilon_k - \mu)}] \tag{1.19}$$

The mean number of particles in the kth state is given by

$$\bar{n}_k = -\frac{\partial \Omega_k}{\partial \mu} = \frac{e^{-\beta(\epsilon_k - \mu)}}{1 - e^{-\beta(\epsilon_k - \mu)}}. \tag{1.20}$$

or

$$\bar{n}_k = \frac{1}{e^{\beta(\epsilon_k - \mu) - 1}} \tag{1.21}$$

The form (1.21) is known as the *Bose-Einstein distribution.*

1.3 Four vector algebra

Throughout this study, we will make common use of the four-vector notation employed in special relativity. A four-vector is denoted with a Greek letter, usually μ or ν, as a superscript or subscript:

$$x^\mu = (x^0, x) = (t, x, y, z), \quad x_\mu = (x^0, -x) = (t, -x, -y, -z), \tag{1.22}$$

where t is the time component and x, y, z are the ordinary spatial components. Observe that four-vectors are indeed vectors but are not written in bold as ordinary vectors are. Above equation yields the scalar product:

$$x \cdot x = x^\mu x_\mu = x_\mu x^\mu = t^2 - x \cdot x = t^2 - x^2 - y^2 - z^2. \tag{1.23}$$

The four-derivatives are defined as:

$$\partial_\mu \equiv \frac{\partial}{\partial x^\mu} = \left(\frac{\partial}{\partial t}, \nabla\right), \quad \partial^\mu \equiv \frac{\partial}{\partial x_\mu} = \left(\frac{\partial}{\partial t}, -\nabla\right). \tag{1.24}$$

Defining the Minkowski tensor $g^{\mu\nu}$ as a 4×4 matrix that is only non-zero on the diagonal:

$$g^{\mu\nu} = g_{\mu\nu} \equiv \mathrm{diag}(1, -1, -1, -1). \tag{1.25}$$

Note the special property $g^{\mu\nu} = g_{\mu\nu}$. Following the rules of Einstein notation, any index appearing twice indicates a summation over that index. An example of how the Minkowski metric tensor works is:

$$x_\mu = g_{\mu\nu} x^\nu = (t, -x, -y, -z), \quad x^\mu = g^{\mu\nu} x_\nu = (t, x, y, z). \tag{1.26}$$

1.4 Natural Units

Theoretical physicists often use what is called *natural units*. In natural units, the reduced Planck constant ($\hbar$), the speed of light (c), and other physical constants are set to 1. This leads to some unconventional relationships between ordinary units. As the speed of light equals 1, we have:

$$1\,\mathrm{s} = 2.998 \times 10^8\,\mathrm{m}. \tag{1.27}$$

With the relation above, $\hbar = 6.582 \times 10^{-16}\,\mathrm{eV} \cdot \mathrm{s} = 1.973 \times 10^{-7}\,\mathrm{eV} \cdot \mathrm{m}$. Setting $\hbar = 1$ then gives:

$$1\,\mathrm{m} = \frac{1}{1.973 \times 10^{-7}\,\mathrm{eV}}, \tag{1.28}$$

meaning that the reciprocal of length has the unit of energy. Einstein's famous formula,

$$E = mc^2, \tag{1.29}$$

also simplifies to $E = m$, implying that mass has the unit of energy. Using the Planck-Einstein relation $E = \hbar\omega$, this further reduces to $E = \omega$. Natural units will be used throughout this study.

Chapter 2

Neutron Star

2.1 Birth of a Neutron star

Neutron stars measure roughly 20 km in diameter and possess a mass approximately 1.4 times that of the Sun. This indicates that a neutron star is so dense that one teaspoonful would weigh a billion tons on Earth. Due to its compact size and high density, a neutron star has a surface gravitational field that is approximately 2×10^{11} times stronger than that of Earth. Neutron stars can possess magnetic fields that are a million times more powerful than the strongest magnetic fields generated on Earth.

Stars are typically formed in a dense medium of gas and molecular clouds, known as nebulae. In young stars, nuclear fusion occurring at their core produces enough outward pressure to balance the inward gravitational force, maintaining their structure in a state known as hydrostatic equilibrium. Due to the nuclear reactions they shine brightly over the years. A star's lifespan is determined by its size. Massive stars consume their nuclear fuel rapidly and may only survive for a few million years. Less massive stars, such as our Sun, burn their fuel slowly and continue to shine for billions of years before they die.

As the stars exhaust their nuclear fuel, they begin to collapse due to their own gravity until some other processes come into play to counteract this collapse. For the low-mass stars (below $\sim 8M_{\odot}$), their death is more peaceful as they transition from the red giant stage to the planetary nebulae. Ultimately, they become a white dwarf, where electron degeneracy pressure arising from the electron gas surrounding the nuclei counteracts gravity. Massive stars meet a more energetic and violent end as they explode into supernova. Stars ranging from $8-30M_{\odot}$, after the supernova will leave behind a dense remnant called as neutron

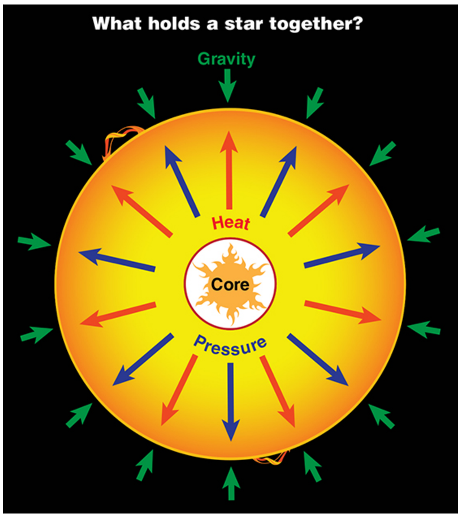

FIGURE 2.1: Hydrostatic equilibrium in a star. This schematic illustrates the balance between the outward thermal pressure generated by nuclear fusion in the core and the inward pull of gravitational forces.

star (NS). NSs can often be rapidly rotating and when they possess a magnetic field and beam, they are referred to as pulsars. Pulsars, so named because they are sources of periodic signals of great timing stability, are the astrophysical entities that are believed to be associated with neutron stars. Following the discovery of the first Pulsar in late 1967 by graduate student Jocelyn Bell Burnett, they were characterized as a highly magnetized neutron star[1]. They are observed under different circumstances, usually as isolated sources, but occasionally in binary orbit with another star (white dwarf or neutron star), in x-ray binary systems, and are identified as soft gamma-ray repeaters [2]. Pulsars are believed to be neutron stars for very good reasons. The term neutron star is designated for the theoretical object, regardless of whether it is observed as a pulsar or identified as a very compact star through means other than its pulsed radiation.

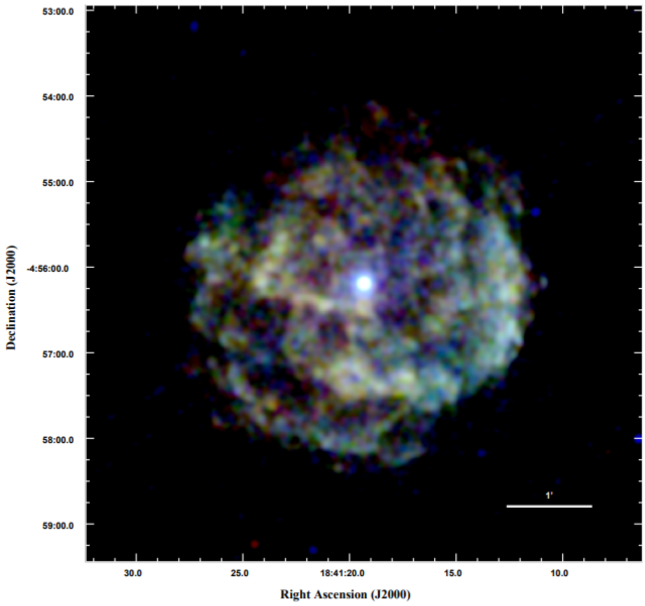

FIGURE 2.2: A young supernova remnant SNR Kes 73 containing a neutron star [3].

The neutrons themselves exert degenerate pressure that prevents a neutron star from collapsing under gravity. The resulting fluid of mostly neutrons may constitute the core of a neutron star. Nonetheless, neutron degeneracy pressure alone is insufficient to support an object exceeding $0.07M_{\odot}$ [4]. Additional forms of repulsive nuclear forces are required to support more massive NSs [5]. Conversely, stars that are more massive than $30M_{\odot}$ primarily collapse to form Black holes.

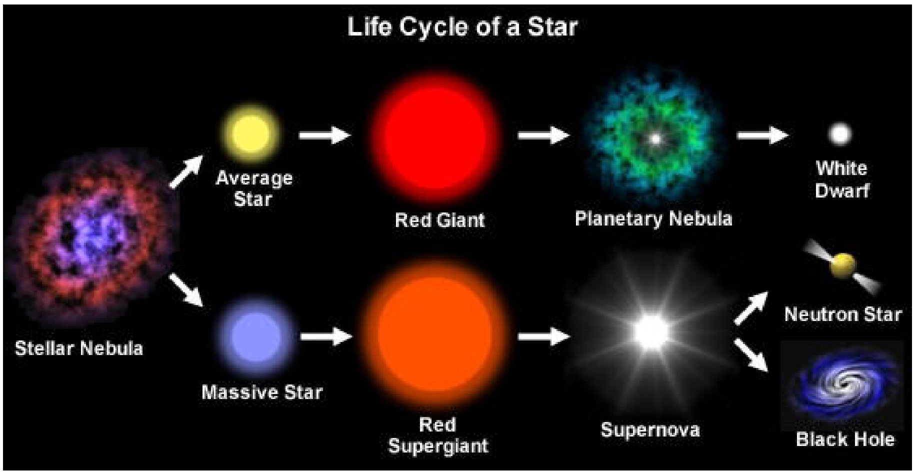

FIGURE 2.3: Lifecycle of a star illustrating the key phases of stellar evolution. The figure shows the initial formation of a star from a nebula, its main sequence phase powered by hydrogen fusion, and the subsequent paths based on its mass.

2.2 Structure and Composition

2.2.1 The Structure of a Neutron Star

A neutron star's cross-section can be divided into four distinct regions:

1. The atmosphere which is relatively thin, being only a few centimetres thick.

2. The outer crust comprises a lattice structure of atomic nuclei, alongside a Fermi liquid of relativistic degenerate electrons. This matter is essentially that of a white dwarf.

3. The inner crust is surrounded by the outer crust, which extends from the neutron drip density $\sim 10^{11}\,\mathrm{g\,cm^{-3}}$ to a transition density $\rho_{\mathrm{tr}} = 1.7 \times 10^{14}\,\mathrm{g\,cm^{-3}}$.

4. Beyond the point of transition density, one enters the core, where all atomic nuclei are broken down into their constituent neutrons and protons. The core could also contain hyperons (baryon made up of u,d,s quarks), more massive baryon resonances, and a gas of free up, down, and strange quarks due to the high Fermi pressure [6]. In addition, π and K-meson condensates are also likely to be present.

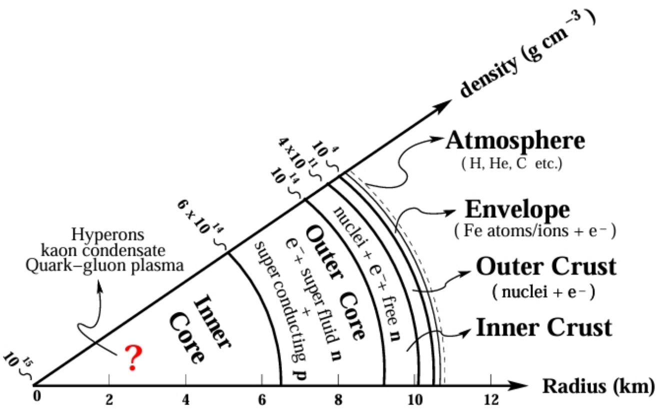

FIGURE 2.4: Schematic diagram for different layers composing a neutron star.

Neutron stars are classified into different types based on the composition of their central core. Here, we are referring to the traditional type of neutron stars (or hadronic stars), where the core primarily comprises neutrons, protons and electrons. In subsequent sections, we shall discuss other types of neutron stars in detail. At high densities, heavier baryons are also excited, and the neutron star transforms into a hyperon star. Densely packed baryons may facilitate the formation of a quark bag, in which quarks are likely to be in a color-superconducting state. The composition and equation of state for this dense region are heavily model dependent. Pion and K meson Bose condensates could also potentially exist. These varying internal structures result in distinct mass–radii relationships. In comparison to neutron stars containing quark cores, a traditional neutron star, by definition, has the largest radius. Strange stars will have the smallest radii.

2.2.2 The Composition of a Neutron Star

In their cold state, neutron stars represent matter in its absolute ground state, with energy minimized by all strong, weak, or electromagnetic processes. The matter density peaks at the center and decreases toward the surface, resulting in different compositions and phases across regions.

1. **Inner Crust:** Complex structures, collectively known as "nuclear pasta" (e.g., rods, slabs, bubbles), form as matter transitions from a crystalline to a homogeneous phase [7, 8].

2. **Outer Core:** Uniform nuclear matter consisting of neutrons, protons, electrons, and muons, governed by charge neutrality and beta equilibrium. This layer is $\sim 10\,\mathrm{km}$ thick, constitutes most of the star's volume.

3. **Inner core:** At extreme densities, the inner core may host superfluid neutrons, hyperons, meson condensates, or quark matter [6]. This region, though poorly understood, accounts for most of the neutron star's mass.

The *charge neutrality* condition states that the net charge per nucleon (and therefore the average charge per nucleon on any star) must be very small, essentially zero. The equation of state will describe a gas of non interacting neutrons, protons, and electrons in such proportions at each baryon number density that the gas has its lowest possible energy. Such a situation is referred to as *beta equilibrium* or simply equilibrium.

Despite their extreme density, neutron stars are relatively cold for most of their lifespan. Initially very hot, they cool rapidly to $\sim 10^7\,\mathrm{K}$ via neutrino emissions. The Fermi temperature of neutrons at such densities, however, is $\sim 10^{11} - 10^{12}\,\mathrm{K}$, making neutron star matter cold relative to its density. Such dense but cold matter cannot be replicated in terrestrial laboratories. Even facilities like the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) achieve high densities but produce hot, out-of-equilibrium states.

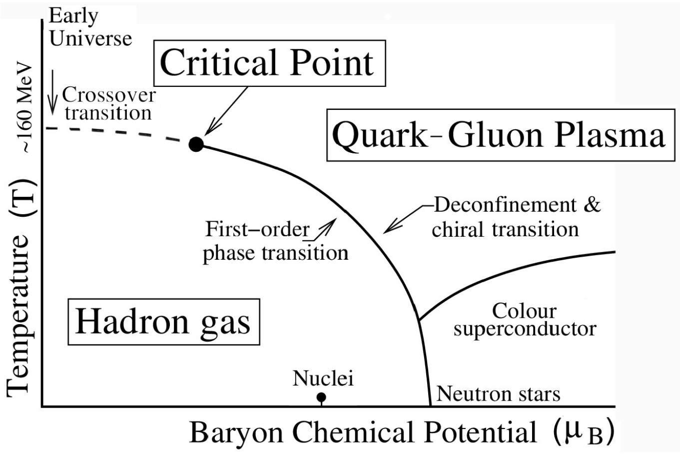

FIGURE 2.5: A schematic QCD phase diagram indicating possible phases of matter.[9]

From a theoretical perspective, the composition of neutron stars poses a fundamental question in physics. Modeling the interior requires understanding its constituents and their interactions under extreme densities. While quantum chromodynamics (QCD) is the standard theory for such interactions, many-body calculations are computationally intractable. Exotic phases like hyperons, mesons, or quark matter may emerge at the inner core. Neutrons and protons might break into a soup of quark matter, depicted in the QCD phase diagram (Figure 2.5). Since probing the physics of NS matter is inaccessible by our earth based experiments, astrophysical observations remain the primary method for investigating neutron star interiors. Measurements of macroscopic properties such as mass, spin, and radius can be used to infer the equation of state (EoS), which describes the relationship between pressure and energy density inside a NS.

2.3 Equations of State

This section introduces the different models used to describe the equation of state (EoS) of neutron star matter. It spans an extensive range of densities, from the surface density of iron ($\rho \approx 8\,\mathrm{g/cm^3}$) to several times the normal nuclear matter density ($\rho_0 = 140\,\mathrm{MeV/fm^3} \approx 2.5 \times 10^{14}\,\mathrm{g/cm^3}$) at the core. Unfortunately, no single theory is adequate to describe the various degrees of freedom encountered at different density regimes. As a result, different models are employed to describe the equation of state (EOS) at different density ranges.

2.3.1 Subnuclear and Nuclear Densities

At subnuclear densities, the Baym–Pethick–Sutherland EoS [10] describes the crust of the neutron star, where matter consists of a Coulomb lattice of ^{56}Fe nuclei, with pressure dominated by degenerate electrons. For nuclear densities ($\rho \approx \rho_0$), the relativistic mean-field (RMF) model [11, 12, 13, 14, 15] describes the transition from nuclei to nucleons as the dominant degrees of freedom.
This study, however, focuses solely on the quark matter EoS, which becomes relevant at densities exceeding ρ_0. While subnuclear and nuclear density regimes are crucial to understanding the complete structure of neutron stars, they fall outside the scope of this dissertation.

2.3.2 Quark Matter

At densities exceeding ε_0, the hadronic phase undergoes a transition to a deconfined quark matter phase (QP). This section describes how we model the QP in weak equilibrium, comprising u, d, and s quarks and electrons i.e., the weak reactions

$$d \rightarrow u + e^- + \bar{\nu}_e \tag{2.1}$$

$$s \rightarrow u + e^- + \bar{\nu}_e \tag{2.2}$$

$$s + u \leftrightarrow d + u \tag{2.3}$$

This implies the relations between four chemical potentials μ_u μ_d μ_s μ_e to be

$$\mu_s = \mu_d = \mu_u + \mu_e \tag{2.4}$$

Since the neutrinos can diffuse out of the star their chemical potentials are taken to be zero. The number of chemical potentials necessary for the description of the QP in weak equilibrium (the number of components) is therefore reduced to two independent ones. We choose the pair (μ_n, μ_e) with the neutron chemical potential

$$\mu_n \equiv \mu_u + 2\mu_d \tag{2.5}$$

In a pure QP we have to require the QP to be charge neutral. This gives us an additional constraint on the chemical potentials

$$\rho_c^{QP} = \frac{2}{3}\rho_u - \frac{1}{3}\rho_d - \frac{1}{3}\rho_s - \rho_e = 0 \tag{2.6}$$

ρ_c^{QP} denotes the charge density of the QP, and ρ_f $(f \in u, d, s)$, ρ_e are the particle densities of the quarks and the electrons, respectively. The EOS can now be parametrized by only one chemical potential, say μ_n.

To calculate the EOS of the QP, we apply the "effective mass bag model". In this model, medium effects are taken into account in the framework of the MIT bag model by introducing density-dependent effective quark masses. we shall discuss this in detail in Chapter 4.

2.4 Exotic Stars

Exotic stars, including quark stars and strange quark stars, are astrophysical objects composed of exotic forms of matter, where the traditional structure of protons and neutrons is replaced by quarks, which may exist in deconfined or superconducting states. These stars provide critical insight into nuclear matter and the strong interaction under extreme conditions of density.

2.4.1 Quark Stars

Quark stars, or hybrid neutron-quark stars, are compact objects composed partly or entirely of quark matter. At extremely high densities, quarks become asymptotically free, losing their confinement within baryons, and form a *quark-gluon plasma*, where nucleons dissolve into their constituent quarks, creating a colorless region of quark matter.
This occurs when the pressure exceeds nuclear saturation density $(5 - 10\,\rho_{\text{sat}})$, leading to a star that is more compact and denser than typical neutron stars. Quark stars are thought to have different observational signatures, including distinct mass-radius relations and thermal emissions, though their existence remains theoretical.

Quark stars provide insights into matter at extreme conditions that cannot be replicated in labs. They may form in the cores of neutron stars, where high pressure could convert ordinary hadronic matter into quark matter, forming a hybrid star with a quark matter core and a nuclear matter mantle.

2.4.2 Strange Stars

At slightly higher densities, the system might further transition into strange quark matter, a hypothesized state consisting of up, down, and strange quarks. This state could occur in the range of $(5-15)\,\rho_{\rm sat}$, depending on the equation of state. It is even possible that the entire star might convert into a lower energy self-bound state of strange quark matter, known as a strange quark star [16, 17, 18].

Several strange matter possibilities include hyperons (strange baryons) [19, 20], deconfined quarks forming hybrid stars [21], and color superconducting phases [22]. These transitions and the potential formation of strange quark stars remain areas of ongoing research, strongly influenced by the equation of state and densities in the star's core.

Chapter 3

Quark-Gluon Plasma State

3.1 Introduction

The only noticeable effects in our daily world are those of electromagnetism and gravity, since these forces have an infinite range. Thinking of a force, we imagine something able to change an object's velocity, be it speed or direction. When looking at the extremely small scales of elementary particles, it is necessary to leave macroscopic Newtonian mechanics behind and work with quantum mechanics instead. In quantum mechanics, a force is rather an interaction between particles. Only gravity has not yet been studied in the Standard Model (SM) because it is so much weaker than the other forces; on a subatomic level, its effects are minuscule.

The other three forces are described using gauge theories. A gauge theory is a mathematical description of how the force behaves. In gauge theory, the concept of symmetry is essential. Symmetry arises when the solutions to a given set of equations remain the same even though some characteristic of the system is changed. If a characteristic can be changed in a varying amount across all points in space and the equations remain valid, the system is said to have local symmetry. According to gauge theories, forces arise to ensure local symmetry still holds when characteristics are changed. As an analogy, one might think of a rubber ball: it may deform (i.e., a change of a characteristic), but that deformation gives rise to forces attempting to restore the ball to its original shape.

The gauge theory of the strong force is called Quantum Chromodynamics (QCD). Just as the gauge theories for the other elementary forces, QCD has force carriers. These are called gluons and are equivalent to the photons in electromagnetic interactions. Colored quarks interact through the exchange of gluons. There are eight different types of gluons, and they themselves are color-charged.

Making QCD match all experimental observations has proven quite difficult, primarily due to the nature of the color charge. While gravity and electromagnetic interactions decrease with distance, the strong interaction behaves differently. As no single quark has ever been observed in isolation, it is believed that the strong interaction increases with greater distance. This property makes detecting an unpaired quark extremely challenging. Since the laws of motion for quarks are not fully understood, several models have been proposed to describe their behavior. One of the most prominent model is discussed later in this chapter.

3.2 Formalism of QCD

It is generally held that the field theory for quarks and gluons belongs to a special class of field theories known as *gauge field theories*. That is, the interaction of the field theory can be represented as arising from the requirement that the Lagrangian is invariant under a local gauge transformation. This invariance is called as local gauge invariance or simply 'gauge invariance'. If the quanta of the gauge field have the rest mass, then the Lagrangian will not be invariant under a local gauge transformation. To maintain gauge invariance, the quanta of the gauge field, the gluons, must be massless.

In a quark-antiquark interaction, a particle with three types of color charges interacts with another particle with three types of color charges (red,green,blue). There are eight gluons as members of the color octet, all of which carry color charges. In the color space, the internal symmetry group which describes the gluons and quarks is that of the $SU(3)_c$ group, with the quarks residing in the fundamental representation of the group. The subscript c denotes the color degree of freedom. Because the gluons also carry color charge, they interact with quarks and gluons with the exchange of other gluons. The theory which describes the interaction of the color charges of quarks and gluons is called the quantum chromodynamics(QCD). Since the gluons do not carry flavor and the interaction does not depend on the flavor degree of freedom, the flavor labels and the flavor group are often not explicitly written out.

We start out by writing down the lagrangian for QCD. The derivation will follow the reasoning in [23]. It is a **non-Abelian gauge field theory** and has an additional internal 3-dimensional coordinate frame known as the color coordinate frame. The term "non-Abelian" refers to the property that the symmetry group which is associated with QCD in color space is a non-commutative group. The group elements do not commute and the generators of the groups do not commute.

The symmetry group for QCD in color space is generally taken to be the $SU(3)_c$ group so that a rotation in color space is carried out by using a unitary 3 X 3 SU(3) matrix. The field theory of QCD is a gauge field theory. This means that in the color space, QCD possesses the property that its Lagrangian is invariant under a rotation of the color coordinate frame of reference, with the amount of color frame rotation allowed to be arbitrary at different space-time points. The property that the Lagrangian is invariant under arbitrary color frame rotations at different space-time points is called *local gauge invariance*. The term 'local' here refers to different color frame rotations at different space-time points, in contrast to 'global' rotations in which the amount of rotations are the same at all space-time points.

We fix our attention at the space-time point n and consider the consequence of a gauge transformation, which is an arbitrary rotation of the color coordinate frame at n. As the gauge theory of QCD possess $SU(3)_c$ symmetry, an arbitrary rotation in color space can be carried out by the operation of a unitary 3X3 $SU(3)$ matrix G(n). The unitary matrix G(n) can be generated by using generators λ_i and an ordered set of eight real numbers $\chi_i(n)$ with i=1,2,...8 [23],

$$G(n) = exp[i\frac{\lambda_i}{2}\chi_i(n)] \tag{3.1}$$

where λ_i are the eight 3×3 Gell-Mann matrices given by

$$\begin{aligned}
&\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\
&\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \\
&\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}
\end{aligned} \tag{3.2}$$

Treating the set of eight generators λ_i as components of a vector λ and the set of eight real numbers $\chi_i(n)$ as components of a vector $\chi(n)$, the (3.1) equation can be written in the form:

$$G(n) = exp[i\frac{\lambda}{2}\chi(n)] \tag{3.3}$$

Under the rotation of the color coordinate frame by an arbitrary 'angle' $\chi(n)$, a vector V in color space will be transformed into $G(n)V$.

The fermion field $\psi(n)$ at the space time n is a vector in color space which can be represented as a three component column matrix. Accordingly, under the rotation of the color coordinate frame by an arbitrary angle $\chi(n)$, the fermion field $\psi(n)$ is transformed to $\psi'(n)$ according to

$$\psi(n) \to \psi'(n) = G(n)\psi(n) \tag{3.4}$$

The fermion field $\bar{\psi}(n)$ is transformed to $\bar{\psi}'(n)$

$$\bar{\psi}(n) \to \bar{\psi}'(n) = \bar{\psi}(n)G^{-1}(n) \tag{3.5}$$

Equations (3.1) - (3.5) give the rules of gauge transformation for the gauge field $A_\mu(n)$ which represents gluons, we follow the procedure familiar with Abelian gauge field theory. We define a gauge covariant derivative D_μ to be applied to the fermion field ψ:

$$D_\mu\psi = (\partial_\mu - igA_\mu)\psi \tag{3.6}$$

The 'minimal coupling' term $i\bar{\psi}\gamma^\mu D_\mu$ can be introduced in the Lagrangian to represent the kinetic energy part of the fermion field and the coupling of the fermion field ψ with the gauge field A_μ. γ^μ represent the Dirac matrices which are 4×4 matrices in spin space. The components of the Dirac matrices are defined as follows:

$$\gamma^0 = -i\begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \quad \gamma^i = -i\begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} \tag{3.7}$$

where I is the 2×2 identity matrices, σ_i are the 2×2 Pauli spin matrices. γ^0 and γ^i represents the time and spatial components. We require that this term $i\bar{\psi}\gamma^\mu D_\mu$ be invariant under local gauge transformations (3.1) - (3.5). This requirement leads to the rule of gauge transformation for the gauge field A_μ given by

$$A_\mu(n) \to A'\mu(n) = G(n)A_\mu(n)G^{-1}(n) - \frac{i}{g}[\partial_\mu G(n)]G^{-1}(n) \tag{3.8}$$

Besides the term $i\bar{\psi}\gamma^\mu D_\mu$, the QCD Lagrangian should contain a term which depends on the mass m of the fermion. The fermion mass term is simply $-m\bar{\psi}\psi$, which is gauge invariant as one can easily see from the definition of the gauge transformation (3.4). (when the flavor degrees of freedom are taken into account explicitly, this term should be generalized to include different masses m_f for fermions with different flavors) The fermion part of the (Minkowski) Lagrangian is therefore

$$\mathcal{L}_{Mq} = i\bar{\psi}\gamma^{\mu}D_{\mu}\psi - m\bar{\psi}\psi \tag{3.9}$$

In addition to $\mathcal{L}_{Mq}$, the lagrangian should contain a term which depends on the field strength tensor $F_{\mu\nu}$. To express $F_{\mu\nu}$ as a function of A_{μ}, we again follow a procedure similar to that in Abelian gauge field theory. We construct the operator $F_{\mu\nu}$ as the commutator of the gauge-covariant derivatives D_{μ} and D_{ν}:

$$-igF_{\mu\nu} = D_{\mu}D_{\nu} - D_{\nu}D_{\mu} \tag{3.10}$$

The type of gauge field theory we wish to construct for QCD is a non- Abelian gauge field theory. The gauge field operators A_{μ} and A_{ν} do not commute when $\mu \neq \nu$. After substituting the gauge-covariant derivatives into the above equation, we obtain

$$F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu} - ig[A_{\mu}, A_{\nu}] \tag{3.11}$$

The field strength tensor $F_{\mu\nu}$ at a point n transforms as [23]

$$F_{\mu\nu}(n) \to F'_{\mu\nu} = G(n)F_{\mu\nu}G^{-1}(n) \tag{3.12}$$

and the quantity $tr\left\{F_{\mu\nu}F^{\mu\nu}\right\}$ transforms as

$$\begin{aligned}
\mathrm{tr}\left\{F_{\mu\nu}F^{\mu\nu}\right\} \to \mathrm{tr}&\left\{F'_{\mu\nu}F^{\mu\nu'}\right\} \\
&= \mathrm{tr}\left\{G(n)F_{\mu\nu}G^{-1}(n)G(n)F^{\mu\nu}G^{-1}(n)G(n)\right\} \\
&= \mathrm{tr}\left\{F_{\mu\nu}G^{-1}(n)G(n)F^{\mu\nu}G^{-1}(n)G(n)\right\} \\
&= \mathrm{tr}\left\{F_{\mu\nu}F^{\mu\nu}\right\} \quad \text{(no sum over } \mu \text{ or } \nu\text{)},
\end{aligned}$$

where we have used the cyclic property of the trace of matrices. Thus, the quantity $tr\left\{F_{\mu\nu}F^{\mu\nu}\right\}$ is invariant under a local gauge transformation. It can be used for the gauge field strength term in the QCD Lagrangian.

We expand $F_{\mu\nu}$ as a linear combination of the generators of the $SU(3)_c$ group with the coefficients $F^i_{\mu\nu}$ by the definition:

$$F_{\mu\nu} = \frac{\lambda_i}{2}F^i_{\mu\nu} \tag{3.13}$$

Then, in terms of A^i_μ, the field strength $F^i_{\mu\nu}$ is

$$F^i_{\mu\nu} = \partial_\mu A^i_\mu + g f_{ijk} A^j_\mu A^k_\nu \tag{3.14}$$

where we have used the commutator for the Gell-Mann λ matrices:

$$[\frac{\lambda_j}{2}, \frac{\lambda_k}{2}] = i f_{ijk} \frac{\lambda_i}{2}, \tag{3.15}$$

and f_{ijk}'s are the structure constants of SU(3). By using the relation for the trace of the λ matrices:

$$\mathrm{tr}\left\{\frac{\lambda_i}{2}\frac{\lambda_j}{2}\right\} = \frac{1}{2}\delta_{ij}, \tag{3.16}$$

the quantity $\mathrm{tr}\{F_{\mu\nu}F^{\mu\nu}\}$ can be rewritten as

$$\mathrm{tr}\{F_{\mu\nu}F^{\mu\nu}\} = \frac{1}{2} F^i_{\mu\nu} F^{\mu\nu,i} (\text{no sum over } \mu \text{ or } \nu), \tag{3.17}$$

As the left hand quantity if a gauge invariant quantity, the right-hand side quantity is also gauge-invariant. With the fermion part of the (Minkowski) Lagrangian as given by (3.9), the gauge invariant QCD (Minkowski) Lagrangian is therefore

$$\mathcal{L}_\mathcal{M} = i\bar{\psi}\gamma^\mu D_\mu \psi - m\bar{\psi}\psi - \frac{1}{4} F^i_{\mu\nu} F^{\mu\nu,i} \quad (\text{sum over repeated indices or } \nu) \tag{3.18}$$

where the coefficient of the $F^i_{\mu\nu}F^{\mu\nu,i}$ term is chosen by analogy with the Abelian gauge theory of QED.

3.3 Gluons and Asymptotic Freedom

Quarks have spin 1/2. We have discussed the properties of quarks previously. They are characterized by flavors. The discovery of the flavor degree of freedom of quarks led to the concept that quarks are the basic fundamental particles out of which many other particles can be built up. Quarks comes in three colors: red, green and blue. They are represented as

$$|r\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad |g\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad |b\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{3.19}$$

Particles that experience the strong force are called **hadrons**. Amongst the hadrons are the proton, the neutron and the mesons. The strong interaction acts in such a way as to make all hadrons color neutral or "white". That is why the only possible combination of hadrons are the mesons with one quark and one antiquark of the same color, or the baryons and antibaryons with three quarks or antiquarks of different colors

The mass of the quark in the absence of confinement is known as the *current mass* of the quark. When the quark is confined in a hadron, the quark may acquire an effective mass which includes the effect of zero-point energy of the quark in the confining potential. The effective mass of a confined quark in a hadron is known as the *constituent mass* of the quark and is typically a few hundred MeV in magnitude.

Each quark carries a baryon number (quantum number) 1/3 and a color. There are three different colors a quark can carry. The interaction between the quarks depends on the colors of the interacting quarks, similar to the interaction between electric charges. For this reason, the color of the quark is sometimes called its *color charge*. By the exchange of a **gluon**, a quark with one color can interact with another quark of any other color.

Gluons are the gauge bosons of QCD. They binds quarks together to form hadrons, such as protons and neutrons, and operates within the framework of the SU(3) color symmetry group. Unlike photons, which mediate the electromagnetic force, gluons carry color charge and therefore interact with one another, making QCD a highly non-linear and complex field theory.

In Feynman diagrams, gluons are typically represented as curly lines, symbolizing their unique role in mediating the strong interaction:

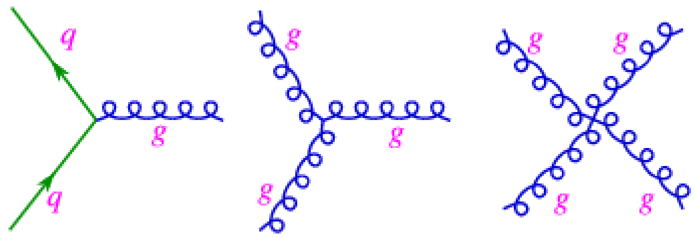

FIGURE 3.1: Illustration of gluon interactions: (a) Gluon exchange between two quarks, mediating the strong interaction, and (b) Gluon-gluon scattering, demonstrating the self-interaction property of gluons.

- **Quark-Gluon Vertex:** Gluons are emitted or absorbed by quarks at interaction vertices, ensuring the conservation of color charge. The quark confinement mechanism within hadrons is built upon this mechanism.
- **Self-Interaction:** A key characteristic of gluons is their capacity to interact with each other as a result of their color charge. It is illustrated in Feynman diagrams as

three-gluon or four-gluon vertices, which contribute to the non-Abelian properties of QCD.

- **Color Charge Exchange:** Gluons play a crucial role in facilitating the exchange of color charges during processes like quark-antiquark annihilation or scattering, which helps to maintain an overall color neutrality.

The nature of interaction of quarks and gluons on short distance scales was provided by deep-inelastic scattering experiments. In these experiments , an incident electron interacts with a quark within a hadron and is accompanied by a transfer of momentum of the electron before and after the collision allows a probe of the momentum distribution of the quarks inside the nucleon. It was found that with very large momentum transfers, the quarks inside the hadron behave as if they were *almost free* as demonstrated by the success of Bjorken scaling and the parton model.

A non-Abelian gauge theory can describe a system which is weakly interacting on short distance scales but very strongly interacting on a large distance scale. It was found that non-Abelian gauge field theories possess the property of *'asymptotic freedom'*. The effective strength of the interaction between quarks and gluons depends on the conditions of their interaction. The QCD coupling constant α is related to the scale of momentum transfer q by the relation

$$\alpha(q^2) = \frac{\alpha_0}{1 + \alpha_0 \frac{(32-n_f)}{12\pi} ln(\frac{-q^2}{\mu^2})} \tag{3.20}$$

where α_0 is the coupling constant for the momentum transfer μ and n_f is the number of flavors. When the distance scale of the interaction is small, for example when one probes the high momentum component of the distribution of quarks, the coupling constant is small. This is the case of 'asymptotic freedom'.

Often when modeling complicated systems physicists use a method called perturbation theory. Perturbation theory is a technique where one first uses an easier system with a known solution. Then to model the real system one adds small disturbances, so called perturbations. The problems simplifies significantly using such a technique.

For the case, when the coupling constant is small, a perturbative treatment is good description of the process. On the other hand, when the distance scale is large, as for example in the study of the structure of the ground state of a hadron, then the interaction strength is large. A perturbative treatment, based on an expansion in powers of the coupling constant, is no longer applicable. The constituent quarks are subject to confining forces and a non perturbative treatment is needed. The investigation of quarks and gluons systems can be classified into *perturbative QCD* and *nonperturbative QCD* categories. Investigating the new phase is a challenge in nonperturbative QCD, as it requires characterizing the system over a large spatial extent.

Well-developed techniques for studying perturbative QCD exist, but finding analytical and numerical solutions to nonperturbative QCD problems is quite challenging. Phenomenological models, like the bag model, can serve as a useful qualitative guide for understanding certain aspects of the strong interaction phenomena. Our discussion of a system of quarks and gluons will be based on the bag model, particularly under extreme conditions of high temperature and density.

3.4 Bag model for hadrons

This model was suggested by the Russian theoretical physicist Nikolay Bogoliubov. His idea was to give the quarks an enormous mass, thereby confining them by making them unable to move. However, this approach seemed to contradict the asymptotic freedom observed at very short distances. He resolved this issue by confining the quarks within a spherical cavity of radius R, where they experience an attractive field of strength m. The quarks' masses were also set to m, and then the limit $m \to \infty$ was taken. This led to Bogoliubov's bag model, where the quarks could move freely inside the bag but were completely confined within it.

Although Bogoliubov's model was quite simple, it still provided some accurate predictions. The MIT bag model is an extension of this simple bag model and will be explained in detail in the Chapter 4.

3.5 Quark Gluon Plasma

We have outlined the essence of bag model for hadrons. For our purposes, it is useful to use it as a heuristic model to discuss matter under extreme conditions. We can interpret the bag model as indicating that the essential effect of nonperturbative QCD is to give rise to an inward bag pressure of magnitude B. For a hadron in which the quarks can be considered to be confined to be in the lowest $S_{1/2}$ state inside the bag, the inward bag pressure is balanced by the quantum stress arising from the wave function of the quarks.

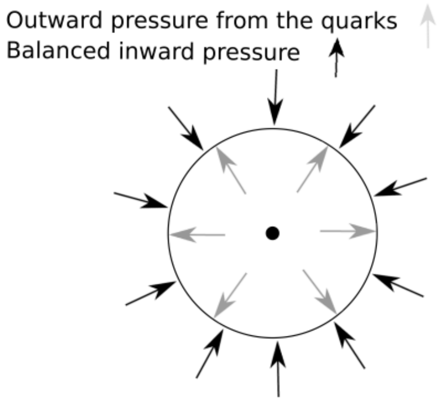

FIGURE 3.2: Pressure balance of the bag

This balance of opposing pressures allows a simple intuitive description of the competing forces leading to a stable system. It also provides a simple intuitive understanding of why new phases of quark matter are expected. It is clear that if the pressure of the quark matter inside the bag is increased, there will be a point when the pressure directing outward is greater than the inward bag pressure. When that happens, the bag pressure cannot balance the outward quark matter pressure and the bag cannot confine the quark matter contained inside. A new phase of matter containing the quarks and gluons in an unconfined state is then possible. It is this situation which leads to the possible existence of different phases of the quark matter. The main condition for a new phase of quark matter is the occurrence of a large pressure exceeding the bag pressure B. A large pressure arises only when either the temperature is high or baryon no. density is high [24]. The critical temperature at which Bag pressure is equal to Quark Gluon pressure is:

$$T_c = (\frac{90}{37\pi^2})^{1/4} B^{1/4} \tag{3.21}$$

As $B^{1/4}$= 206 MeV as estimated in [24] we have $T_c \approx 144$ MeV. If the quark matter in a bag are heated up to a high temperature greater than the critical temperature, the quark matter inside the bag will have a pressure which is greater than that of the bag pressure. When this happens, the bag will not be able to hold the quark matter in the bag and the quark matter will be deconfined. The deconfined phase of the quark matter is given the general name '*quark-gluon plasma*'. Thus, quark-gluon plasma may arise when the temperature of the quark matter is very high.

Chapter 4

QGP Phenomenological Models

4.1 MIT Bag model for hadrons

Derivation of the MIT Bag model equations

This section will derive important equations for the quark fields using the Lagrangian density of the MIT bag-model [25]. The derivation will follow the reasoning in Bhaduri's Models of the Nucleon [26]. These derivations will be pretty tedious so we will begin by briefly outlining the approach. The derivations have been divided into three different subsections:

- Starting with minimizing the action S given from field theory as the integral over the Lagrangian density, the Euler-Lagrange equations of motion are obtained.
- The specific Lagrangian density of the MIT bag-model is inserted. This yields the boundary conditions for the quark fields in the MIT bag-model.
- Using the energy momentum tensor and the conservation of energy and momentum, the value of the constant B in the Lagrangian density is derived.

4.1.1 Derivation of the Euler-Lagrange equation

As in classical mechanics, the equations of motion are found when minimizing the action S. In field theory, S is given by

$$S = \int L(\phi_i(x), \partial_\mu \phi_i(x))\, d^4x. \tag{4.1}$$

Here L is the Lagrangian density, dependent on the field $\phi_i(x)$ and its first derivatives. The Lagrangian is a function summarizing the dynamics of a system. The Lagrangian density is like an ordinary Lagrangian that can vary in both space and time, and it must therefore be integrated over all space-time. By the principle of least action we want to minimize S. Setting $\delta S = 0$, i.e. minimizing S, leaves the following equation

$$\delta S = \int \left(\frac{\partial L}{\partial \phi_i} \delta \phi_i + \frac{\partial L}{\partial(\partial_\mu \phi_i)} \delta(\partial_\mu \phi_i) \right) d^4x = 0. \tag{4.2}$$

Using integration by parts on the second term in the equation above yields

$$\int \frac{\partial L}{\partial(\partial_\mu \phi_i)} \delta(\partial_\mu \phi_i) d^4x = \left(\frac{\partial L}{\partial(\partial_\mu \phi_i)} \delta \phi_i \right) - \int \left(\partial_\mu \left(\frac{\partial L}{\partial(\partial_\mu \phi_i)} \right) \delta \phi_i \right) d^4x. \tag{4.3}$$

Assuming that $\delta \phi_i$ vanishes at the endpoints, the first part of the right hand side is zero. Equation (4.3) is then reduced to

$$\delta S = \int \delta \phi_i \left(\frac{\partial L}{\partial \phi_i} - \partial_\mu \left(\frac{\partial L}{\partial(\partial_\mu \phi_i)} \right) \right) d^4x = 0. \tag{4.4}$$

Ignoring the trivial field $\phi_i \equiv 0$, the expression in the brackets in the equation above has to equal zero at all points in space-time, hence

$$\frac{\partial L}{\partial \phi_i} - \partial_\mu \left(\frac{\partial L}{\partial(\partial_\mu \phi_i)} \right) = 0, \tag{4.5}$$

which are the Euler-Lagrange equations of motion that hold for all independent fields ϕ_i.

4.1.2 Using the Lagrangian density of the MIT bag-model

Now introducing the specific Lagrangian for the MIT bag-model as the following expression [26]

$$L = \left[\frac{i}{2} \left(\psi \gamma^\mu \partial_\mu \psi - (\partial_\mu \bar{\psi}) \gamma^\mu \psi \right) - B \right] \theta_v(x) - \frac{1}{2} \psi \bar{\psi} \Delta_s, \tag{4.6}$$

Here ψ and $\bar{\psi}$ are the wave function and the adjoint wave function of the quarks and γ^μ is the gamma matrix, also called Dirac matrix(3.7). θ_v is a step-function that equals one inside the bag and zero outside the bag. Δ_s is given by the derivative of θ_v as is shown below

$$\theta_v = \theta(R - r), \quad \frac{\partial \theta_v}{\partial x^\mu} = n_\mu \Delta_s, \quad \Delta_s = \delta(R - r). \tag{4.7}$$

Here θ is the Heaviside function, δ is the Kronecker delta, n_μ is a unit vector normal to the surface, and R is the radius of the bag. Inserting $\bar{\psi}$ as the field ϕ_i in equation (4.5) gives

$$\frac{\partial L}{\partial \bar{\psi}} - \partial_\mu \left(\frac{\partial L}{\partial(\partial_\mu \bar{\psi})} \right) = 0. \tag{4.8}$$

(Using $\bar{\psi}$ instead of ψ yields the boundary condition for ψ). Inserting the Lagrangian density L from equation (4.6) into equation (4.8) and calculating each part separately yields the following expressions

$$\frac{\partial L}{\partial \bar{\psi}} = \frac{\partial}{\partial \bar{\psi}} \left(\frac{i}{2} \left(\psi \gamma^\mu \partial_\mu \psi - (\partial_\mu \bar{\psi}) \gamma^\mu \psi \right) - B\theta_v(x) - \frac{1}{2} \psi \bar{\psi} \Delta_s \right) = \frac{i}{2} \gamma^\mu \partial_\mu \psi \, \theta_v(x) - \frac{1}{2} \psi \Delta_s, \tag{4.9}$$

and similarly

$$\partial_\mu \left(\frac{\partial L}{\partial(\partial_\mu \bar{\psi})} \right) = \partial_\mu \left(\frac{-i}{2} \gamma^\mu \psi \right) \theta_v(x) = -\frac{i}{2} \gamma^\mu \partial_\mu \psi \, \theta_v(x) - \frac{i}{2} \gamma^\mu n_\mu \psi \Delta_s. \tag{4.10}$$

Taking (4.9) and (4.10) and inserting them into (4.8) yields

$$\frac{i}{2} \gamma^\mu \partial_\mu \psi \, \theta_v(x) - \frac{1}{2} \psi \Delta_s - \left(-\frac{i}{2} \gamma^\mu \partial_\mu \psi \, \theta_v(x) - \frac{i}{2} \gamma^\mu n_\mu \psi \Delta_s \right) = 0, \tag{4.11}$$

which simplifies to

$$\frac{i}{2} \gamma^\mu \partial_\mu \psi \, \theta_v(x) + \frac{1}{2} \left(i\gamma^\mu n_\mu \psi - \psi \right) \Delta_s = 0. \tag{4.12}$$

From this, two different conditions are derived:

- Inside the bag, $\Delta_s = 0$ and $\theta_v = 1$ results in

$$i\gamma^\mu \partial_\mu \psi = 0. \tag{4.13}$$

- On the surface of the bag, $\Delta_s = \infty$ and $\theta_v = 0$ yields

$$\frac{1}{2} \left(i\gamma^\mu n_\mu \psi - \psi \right) \Delta_s = 0. \tag{4.14}$$

  Since $\Delta s = \infty$, the expression inside the parenthesis must be zero. This yields the boundary condition

$$i\gamma^\mu n_\mu \psi = \psi \tag{4.15}$$

  An identical calculation for the adjoint wave function ($\bar{\psi}$) shows that the surface boundary condition is given by

$$-i\gamma^\mu n_\mu \bar{\psi} = \bar{\psi} \tag{4.16}$$

The probability density is obtained from $\psi\bar{\psi}$. By substituting first ψ and then $\bar{\psi}$ while using the boundary conditions above, we get

$$\psi\bar{\psi} = i\bar{\psi}(\gamma^\mu n_\mu \psi) = -i(\bar{\psi}\gamma^\mu n_\mu)\psi \tag{4.17}$$

on the bag surface. As the left- and right-hand sides are exactly the same but of opposite signs, this can only be true if $\psi\bar{\psi} = 0$. Since the probability density is zero at the bag surface, there is no quark current across the surface, which means that the quarks are confined within the hadron.

In the next section, the value of the bag constant B will be determined.

Determination of the Bag constant B

The purpose of this section is to determine the value of the bag constant B in the Lagrangian density. This will be done by starting with the energy-momentum tensor, given by the expression below, and then utilizing the conservation of it. The energy-momentum tensor $T^{\mu\nu}$ is

$$T^{\mu\nu} = -g^{\mu\nu} L + \left(\frac{\partial L}{\partial(\partial_\mu \psi)} \right) (\partial^\nu \psi) + (\partial^\nu \bar{\psi}) \left(\frac{\partial L}{\partial(\partial_\mu \bar{\psi})} \right). \tag{4.18}$$

The big parenthesis in the second term in the brackets has already been calculated in the first step in (4.10). The same method also gives

$$\left(\frac{\partial L}{\partial(\partial_\mu \psi)} \right) = \frac{i}{2}(\bar{\psi}\gamma^\mu)\theta_v. \tag{4.19}$$

Inserting these expressions into the energy-momentum tensor results in

$$T^{\mu\nu} = -g^{\mu\nu} L + \frac{i}{2}(\bar{\psi}\gamma^\mu \partial^\nu \psi - (\partial^\nu \bar{\psi})\gamma^\mu \psi)\theta_v. \tag{4.20}$$

The conservation of energy and momentum says that $\partial_\mu T^{\mu\nu} = 0$. Simplifying the first term in the energy-momentum tensor $-g^{\mu\nu}L$ according to the four-vector algebra from section (1.3) yields the following expression

$$-g^{\mu\nu}L = -\frac{i}{2}\left(\bar{\psi}\gamma^\mu\partial^\nu\psi - (\partial^\nu\bar{\psi})\gamma^\mu\psi\right)\theta_v + g^{\mu\nu}B\theta_v + \frac{1}{2}g^{\mu\nu}\psi\bar{\psi}\Delta_s, \tag{4.21}$$

where the property $g^{\mu\nu}\partial_\mu = \partial^\nu$ has been used. The term in brackets on the right-hand side in the equation above is exactly the same as the term in parentheses in $T^{\mu\nu}$, hence they cancel out. This leaves the conservation equation

$$\partial_\mu T^{\mu\nu} = \partial_\mu\left(g^{\mu\nu}B\theta_v + \frac{1}{2}g^{\mu\nu}\psi\bar{\psi}\Delta_s\right) = 0. \tag{4.22}$$

Once again, using the properties of the Minkowski tensor $\partial_\mu g^{\mu\nu} = \partial_\nu$, equation (4.22) is reduced to

$$\partial^\nu(B\theta_v) + \frac{1}{2}\partial^\nu(\psi\bar{\psi}\Delta_s) = 0. \tag{4.23}$$

As B is the bag constant, its derivative is zero. Equation (1.24) and feq:derofheavisideyield $\partial_\nu\theta_v = -n_\nu\Delta s$, and hence

$$-Bn^\nu\Delta_s + \frac{1}{2}\partial^\nu(\psi\bar{\psi})\Delta_s = 0, \tag{4.24}$$

and as $\partial^\nu\Delta_s = 0$, we are left with the following equation

$$Bn^\nu\Delta s = \frac{1}{2}\partial^\nu(\psi\bar{\psi})\Delta_s. \tag{4.25}$$

The quantity Δ_s is only non-zero at the boundary of the bag and thus

$$Bn^\nu = \frac{1}{2}\partial_\nu(\psi\bar{\psi}) \text{ on the bag surface.} \tag{4.26}$$

The normal n^ν is space-like, meaning that its time component is zero and therefore $n_\nu n^\nu = -1$ holds. Finally, we arrive at

$$B = -n_\nu \frac{1}{2}\partial^\nu(\psi\bar{\psi}), \tag{4.27}$$

as the value for B on the bag's surface. In chapter 2.5 in [26], it is shown that the outward pressure exerted by a particle in an infinitely deep spherical cavity is

$$P = -\frac{1}{2}\frac{d}{dr}(\psi\bar{\psi}) \text{ at } r = R. \tag{4.28}$$

If we approximate the bag as spherically symmetric, then $n_\nu \partial^\nu = \frac{d}{dr}$ and we see that

$$B = -\frac{1}{2}\frac{d}{dr}(\psi\bar{\psi}), \tag{4.29}$$

meaning that $B = P$, the pressure on the surface of the bag. In the specific Lagrangian density of the MIT bag model, B is negative. This means that the outward pressure on the boundary exerted by the confined particles within the bag is balanced by the inward vacuum pressure B.

Now, the bag constant is not totally arbitrary. In fact, B needs to assure that two-flavored quark matter is unstable and that it is not the ground state of the hadrons, i.e., its energy per baryon must be higher than 930 MeV at zero pressure, otherwise, the protons and neutrons would decay into u and d quarks. On the other hand, if the Bodmer-Witten conjecture [16, 17], that states that the true ground state of hadronic matter is not composed of baryons, but strange matter consisting of $\mu_u = \mu_d = \mu_s$, is true, then the three-flavored quark matter needs to be stable (energy per baryon lower than 930 MeV), while the two-flavored quark matter is unstable. Therefore, the bag pressure value B can only assume a range of values, known as the stability window [16, 17, 27]. These values depend on the quark masses.

4.2 Density-Dependent Quark Mass Model

The quark-gluon plasma, composed of nearly equal numbers of up (u), down (d), and strange (s) quarks, along with a gas of electrons to maintain charge neutrality, is hypothesized to constitute the true ground state of matter [17, 28]. In this model we have assumed SM as a free Fermi gas.

We study the general properties of strange quark matter in the framework of a new equation of state in which the quark masses are parametrized as functions of the baryon density n_B as follows [29]:

$$m_u = m_d = \frac{c}{3n_B}, \quad m_s = m_{s0} + \frac{c}{3n_B}. \tag{4.30}$$

Here, m_{s0} is the strange quark current mass and c is a constant. The range of both these parameters is to be constrained by a stability argument. It has to be noted that in the bag model, confinement is independent of density. Here we have not treated the change of mass as a phase transition.

The system is in the presence of a magnetic field $\mathbf{B}$ which is directed along the z-axis. The energy of the charged particle of mass m_i and charge q_i in the presence of the magnetic field is given by [30]

$$\epsilon_i^{\pm} = \left[m_i^2 + p_{z,i}^2 + 2q_i Bn\right]^{1/2} \tag{4.31}$$

where +(-) refers to spin-up(-down) states of the particles and p_z is the momentum along the z-axis. The thermodynamic potential is given by [29]

$$\Omega = \sum_i \Omega_i - \frac{8}{45}\pi^2 T^4 \tag{4.32}$$

where the second term is contribution due to gluons. The general expression for for the thermodynamic potential Ω_i in the presence of magnetic field is

$$\Omega_i = -T\frac{g_i q_i}{2\pi^2}B \int dp_z \, \ln\left[1 - e^{-\beta(\epsilon_i - \mu_i)}\right] \tag{4.33}$$

where i = (u, d, s, e), g_i is the degeneracy factor ($g_i = 2 \times 3 = 6$ for quarks, $g_i = 2$ for electrons). From the chemical potential μ_i the number density n_i of various species can be obtained from the well-known relation:

$$n_i = \left(\frac{\partial \Omega_i}{\partial \mu_i}\right)_{T,n_B} \tag{4.34}$$

Since the system is in beta-equilibrium, we have

$$\mu_d = \mu_s \tag{4.35}$$

and assuming that neutrinos or antineutrinos produced stream out freely, μ_ν=0

$$\mu_d = \mu_u + \mu_e \tag{4.36}$$

The charge neutrality condition gives

$$2n_u - n_d - n_s - 3n_e = 0 \tag{4.37}$$

The baryon number density of the system is

$$n_B = (n_u + n_d + n_s)/3 \tag{4.38}$$

Thus for a given value of n_B equation (4.35)-(4.38) can be solved for μ_u, μ_d, μ_s, μ_e. The thermodynamical properties, pressure P_i, energy density E_i, entropy S, and specific heat C_v are discussed and numerically computed in upcoming Chapter 5 which provides us the equation of state for the system.

Chapter 5

Deriving the EoS for Neutron Star

Thermodynamic Properties and Speed of Sound

The thermodynamic parameters like pressure, energy density, entropy and specific heat of the system can be calculated from the total thermodynamic potential. The standard thermodynamics gives the following relations:

$$\text{Pressure } P(k,T) = -\Omega_{\text{total}} \tag{5.1}$$

$$\text{Energy Density } E(k,T) = -T^2 \left[\frac{\partial}{\partial T}\left(\frac{\Omega_{\text{total}}}{T}\right)\right] \tag{5.2}$$

$$\text{Entropy } S(k,T) = -\frac{4}{3}T \left[\frac{\partial}{\partial T}\left(\frac{\Omega_{\text{total}}}{T}\right)\right] \tag{5.3}$$

$$\text{Specific Heat } C_v(k,T) = T\left[\frac{\partial S}{\partial T}\right] \tag{5.4}$$

The **Interaction term** refers to the force or potential that governs the interactions between the constituent quarks and gluons within the plasma and is defined as

$$E - 3P \quad , (E = \text{Energy Density}, P = \text{Pressure}) \tag{5.5}$$

Unlike in ideal gas, where $E = 3P$, the interaction term indicates deviations from this relationship, which are crucial for understanding the thermodynamic properties of the system.

The behavior of entropy and specific heat with temperature indicates the nature of phase transition of the system. So, we calculate these parameters for a range of temperatures at μ=300 MeV. At finite temperature the μ_e varies between 6 and 50 MeV [31].
The values of the parameters c and m_{s0} are constrained by requiring that

$$\begin{aligned} \frac{E}{n_B} &< 930\,\text{MeV} \quad \text{for SQM,} \\ \frac{E}{n_B} &> 930\,\text{MeV} \quad \text{for two-flavor quark matter.} \end{aligned} \tag{5.6}$$

From the stability window of SQM [32], the range for c is from 70 to 110 MeV fm^{-3} and for m_{s0} is within 50-180 MeV. In our study we have considered c=75 MeV fm^{-3} and m_{s0}= 140 MeV fm^{-3}. The calculated results are presented in the following plots:

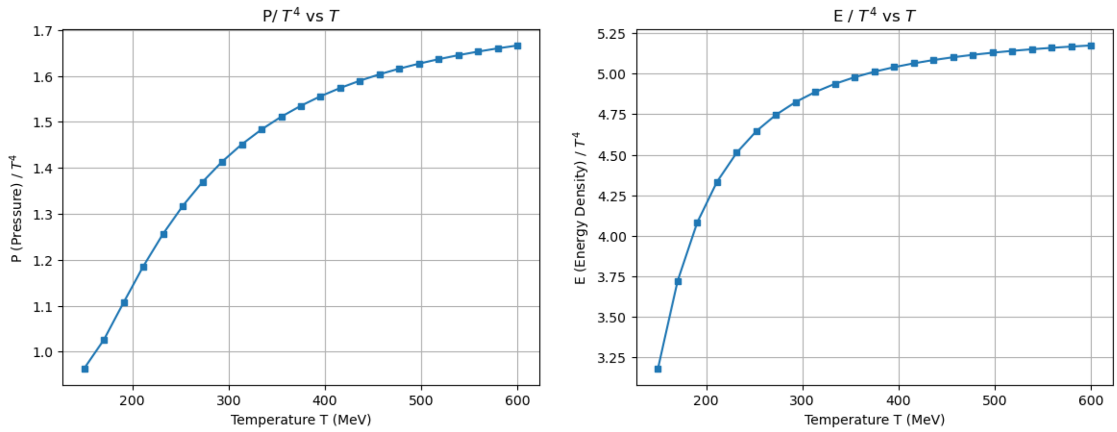

FIGURE 5.2: The variation of P/T^4 on the left-hand side (LHS) and E/T^4 on the right-hand side (RHS) with Temperature T (MeV). Scaling by T^4 allows for a comparison with the Stefan-Boltzmann limit, highlighting deviations due to interactions and medium effects.

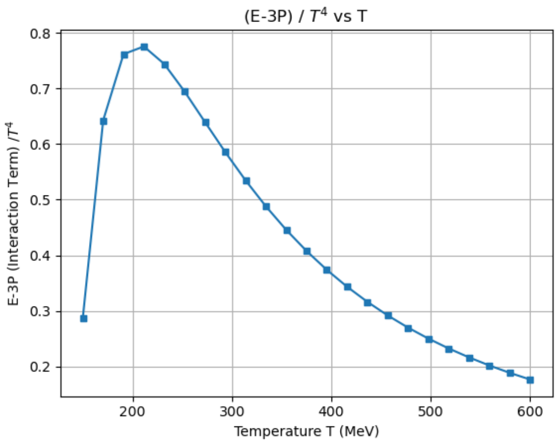

FIGURE 5.5: The variation of Interaction Term ($E - 3P/T^4$) with Temperature T(MeV)

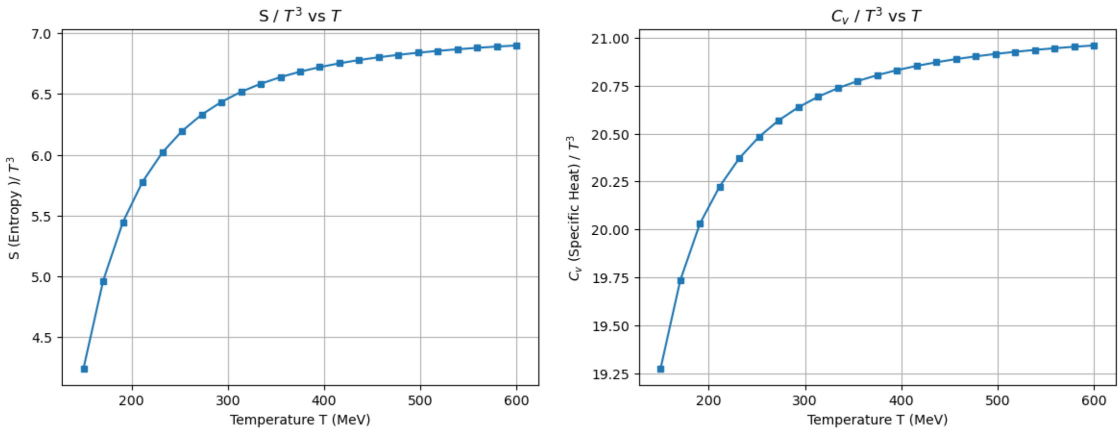

FIGURE 5.4: The variation of S/T^3 on the left-hand side (LHS) and C_v/T^3 on the right-hand side (RHS) with Temperature T (MeV). Here also, scaling by T^3 allows for a comparison with the Stefan-Boltzmann limit.

Now using the entropy and specific heat we further calculate the speed of sound of QGP. The speed of sound is given as the ratio of these two thermodynamic parameters:

$$C_s^2 = \frac{S}{C_v} \tag{5.7}$$

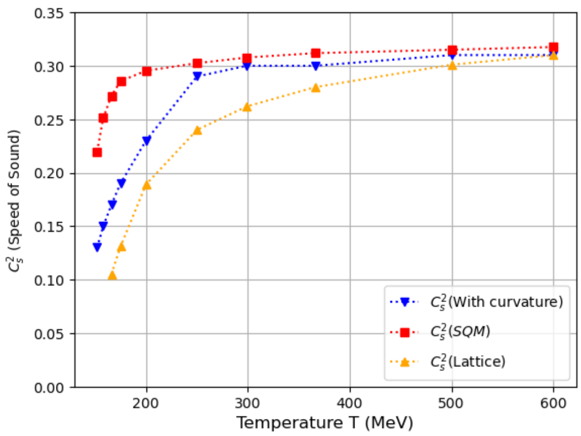

FIGURE 5.6: Speed of sound calculated from our theoretical Model (SQM) is compared with Lattice data and with curvature data.

From the plot, we can infer that at higher temperatures $T \sim 600$ (MeV), the speed of sound values for strange quark matter, as calculated using the DDQM model, begin to overlap with the speed of sound values from Lattice QCD data, indicating a convergence of the theoretical and lattice results at elevated temperatures.

Chapter 6

Conclusion and Future Works

In this thesis, we investigated various aspects of strange quark matter (SQM), which may be found in the cores of neutron stars, using the DDQM model. We focused on thermodynamic quantities such as pressure, energy density, entropy, and specific heat, which provided us with the equation of state of the system. We also calculated the speed of sound and compared it to lattice and curvature data, showing agreement at higher temperatures. Additionally, we discussed QCD formalism and QGP phenomenological models, such as the MIT Bag Model.

For future work, a new model can be employed where quark masses can be made both density- as well as temperature-dependent, with comparisons to the DDQM model to account for corrections arising from the DDQM model. Further, while some important aspects of neutron stars were addressed, future studies will focus on calculating the radial pulsations for neutron or quark stars to better understand their dynamics and stability.

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