1. Consider the polynomial $f(x) = 2024x^{2024} - 2023x + 1 = 0$. Let the roots of the $f(x)$ be $r_1, r_2, \ldots, r_{2024}$. Compute $(2023r_1 - 1)(2023r_2 - 1)\cdots(2023r_{2024} - 1)$.

   Answer: 1

   Solution: We know that $2023r - 1 = 2024r^{2024}$ for any root r, so plugging that in we have

   $$2024^{2024}(r_1 \cdots r_{2024})^{2024}$$

   as the desired product. By Vieta's, we have $r_1 \cdots r_{2024} = \frac{1}{2024}$, and the answer is $\boxed{1}$.

2. Compute

   $$\arcsin\left(\frac{2}{\sum_{k=0}^{\infty} \sin^{2k}(\frac{1}{2024})} - 1\right).$$

   Answer: $\frac{\pi}{2} - \frac{1}{1012}$

   Solution: Let $x = \frac{1}{2024}$. Since $\sin^{2k}(x) < 1$ for all k, then we can convert the problem to $\arcsin\left(\frac{2}{\frac{1}{1-\sin^2(x)}} - 1\right) = \arcsin(1 - 2\sin^2(x)) = \arcsin(\cos(2x))$. We need to find the angle $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ such that $\sin(\theta) = \cos(1/1012)$. Since $\sin(\frac{\pi}{2} - x) = \cos(x)$, we get $\theta = \boxed{\frac{\pi}{2} - \frac{1}{1012}}$.

3. A polynomial f with real coefficients satisfies the functional equation

   $$f(f(x) + y^2) = f(x+y)f(x-y) + 4f(xy)$$

   for all real x, y. What is the sum of all possible values of $|f(1)|$?

   Answer: 4

   Solution: Via comparison of (x, y) and $(x, -y)$, one sees that $4(xy) = 4(-xy)$ for any x, y, so f is even. Checking degrees on both sides, f's degree must be less than or equal to 2. Therefore, we can write $f(x) = ax^2 + b$.

   Plug in $y = 0$, we see that $f(f(x)) = f(x)^2 + 4f(0)$. Plug in $x = 0$, we get $f(f(0) + y^2) = f(y)f(-y) + 4f(0)$. Since f is even, we get $f(f(x)) = f(f(0) + x^2)$. Comparing leading terms forces $a^3 = a$, so $a = \pm 1, 0$. The x^2 terms on each side are $2a^2bx^2$ and $2abx^2$, so $a = -1$ is impossible. Therefore, $a = 1$ or $a = 0$.

   If $a = 0$, $b = 0$ or -3. If $a = 1$, $b = 0$ from the equation $f(f(x)) = (f(x))^2 + 4f(0)$. Therefore the possible solutions for f are $f = 0$, $f = -3$ and $f(x) = x^2$, and the sum of the absolute values at 1 is $0 + 3 + 1 = 4$.