

# COMPLEXITY CHEAT SHEET (BOXED)

## Limits & Comparisons

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### Definitions

$\Theta(g(n))$  - **Tight Bound**  $0 \leq c_1g(n) \leq f(n) \leq c_2g(n)$  for  $n \geq n_0$ .

$O(g(n))$  - **Upper Bound**  $0 \leq f(n) \leq cg(n)$  for  $n \geq n_0$ .

$\Omega(g(n))$  - **Lower Bound**  $0 \leq cg(n) \leq f(n)$  for  $n \geq n_0$ .

$o(g(n))$  - **Strict Upper**  $\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = 0$

$\omega(g(n))$  - **Strict Lower**  $\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \infty$

$$\lim_{n \rightarrow \infty} \frac{n^b}{a^n} = 0 \quad (\forall a > 1)$$

Exp functions > Poly functions.

$$\lim_{n \rightarrow \infty} \frac{\lg^b n}{n^a} = 0 \quad (\forall a > 0)$$

Poly functions > Log functions.

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### Properties

**Transitivity:**  $f = \Theta(g), g = \Theta(h) \implies f = \Theta(h)$  (Applies to all).

**Transpose Symmetry:**  $f = O(g) \iff g = \Omega(f)$   
 $f = o(g) \iff g = \omega(f)$

**Equation Arithmetic:**  $2n^2 + 3n + 1 = 2n^2 + \Theta(n) = \Theta(n^2)$

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### Standard Functions

**Polynomials:**  $\sum a_i n^i = \Theta(n^d)$

**Logarithms:**  $\log_b n = \Theta(\ln n)$

**Factorials:**  $n! = o(n^n), \log(n!) = \Theta(n \lg n)$

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## Stirling's Approx

$$n! \approx \sqrt{2\pi n} \left(\frac{n}{e}\right)^n$$

Useful for analyzing factorial complexities.

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### Recurrences (Master Thm)

$T(n) = aT(n/b) + f(n)$  Depending on  $c = \log_b a$ :  
1.  $f(n) = O(n^{c-\epsilon}) \implies T(n) = \Theta(n^c)$   
2.  $f(n) = \Theta(n^c) \implies T(n) = \Theta(n^c \lg n)$   
3.  $f(n) = \Omega(n^{c+\epsilon}) \implies T(n) = \Theta(f(n))$

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### Analytic Tricks

- Ignore constants.
- Ignore lower order terms.
- $n!$  grows VERY fast.
- $\lg^* n$  (Iterated log) grows VERY slow.