

COMPLEXITY CHEAT SHEET (MODERN)

1	DEFINITIONS	RECURSION
$\Theta(g(n))$ - Tight Bound $0 \leq c_1g(n) \leq f(n) \leq c_2g(n)$ for $n \geq n_0$.	$\text{Poly functions} > \text{Log functions}$. $\lim_{n \rightarrow \infty} \frac{\lg^b n}{n^a} = 0 \quad (\forall a > 0)$	$T(n) = aT(n/b) + f(n)$ Depending on $c = \log_b a$: 1. $f(n) = O(n^{c-\epsilon}) \Rightarrow T(n) = \Theta(n^c)$ 2. $f(n) = \Theta(n^c) \Rightarrow T(n) = \Theta(n^c \lg n)$ 3. $f(n) = \Omega(n^{c+\epsilon}) \Rightarrow T(n) = \Theta(f(n))$
$O(g(n))$ - Upper Bound $0 \leq f(n) \leq cg(n)$ for $n \geq n_0$.	$\text{Poly functions} > \text{Log functions}$.	
$\Omega(g(n))$ - Lower Bound $0 \leq cg(n) \leq f(n)$ for $n \geq n_0$.		
$o(g(n))$ - Strict Upper $\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = 0$		
$\omega(g(n))$ - Strict Lower $\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \infty$		
1	PROPERTIES	ANALYSIS
FUNCTIONS	STANDARD	TRICKS
Polynomials: $\sum a_i n^i = \Theta(n^d)$	Transitivity: $f = \Theta(g), g = \Theta(h) \Rightarrow f = \Theta(h)$ (Applies to all).	• Ignore constants.
Logarithms: $\log_b n = \Theta(\ln n)$	Transpose Symmetry: $f = O(g) \iff g = \Omega(f)$ $f = o(g) \iff g = \omega(f)$	• Ignore lower order terms.
Factorials: $n! = o(n^n), \log(n!) = \Theta(n \lg n)$	Equation Arithmetic: $2n^2 + 3n + 1 = 2n^2 + \Theta(n) = \Theta(n^2)$	• $n!$ grows VERY fast.
1	APPROX	STIRLING'S
& COMPARISONS	LIMITS	
$\lim_{n \rightarrow \infty} \frac{n^b}{a^n} = 0 \quad (\forall a > 1)$	$n! \approx \sqrt{2\pi n} \left(\frac{n}{e}\right)^n$ Useful for analyzing factorial complexities.	• $\lg^* n$ (Iterated log) grows VERY slow.