Complexity Cheat Sheet

Asymptotic Analysis Reference - Portrait Boxed

The 5 Notations

Sym	Term	Analogy
Θ	Tight	$a = b$
O	Upper	$a \leq b$
Ω	Lower	$a \geq b$
o	Strict Upper	$a < b$
ω	Strict Lower	$a > b$

Definitions

$\Theta(g(n))$: $0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n)$

$O(g(n))$: $0 \leq f(n) \leq c \cdot g(n)$

$\Omega(g(n))$: $0 \leq c \cdot g(n) \leq f(n)$

$o(g(n))$: $\lim(f/g) = 0$

$\omega(g(n))$: $\lim(f/g) = \infty$

Growth Hierarchy

$$O(1) < O(\lg n) < O(\sqrt{n}) < O(n) < O(n \lg n) < O(n^2) < O(2^n)$$

Common Complexities

$O(1)$	Array access
$O(\lg n)$	Binary Search
$O(n)$	Linear Scan
$O(n \lg n)$	Merge Sort
$O(n^2)$	Bubble Sort
$O(2^n)$	Power Set

Key Theorems

- $f = \Theta(g) \iff O(g) \wedge \Omega(g)$
- Polynomials: Degree determines growh.
- Transpose: $f = O(g) \iff g = \Omega(f)$.

Properties

Transitivity: $f = \Theta(g) \wedge g = \Theta(h) \implies f = \Theta(h)$. **Additivity**: $f + g = \max(f, g)$.

Critical Limits

Exp vs Poly: $n^b = o(a^n)$ $(a > 1)$. **Poly vs Log**: $\lg^b n = o(n^a)$ $(a > 0)$. **Factorial**: $n!$ beats 2^n but $n! < n^n$.

Logarithms

- $\log(ab) = \log a + \log b$
- $\log(a^k) = k \log a$
- Change of base: $\log_b a = \frac{\ln a}{\ln b}$
- Base doesn't matter for Big-O!

Master Theorem

$T(n) = aT(n/b) + f(n)$, $c_{crit} = \log_b a$.

1. $f(n)$ implies $O(n^{c-\epsilon}) \to T(n) = \Theta(n^{c_{crit}})$
2. $f(n)$ implies $\Theta(n^{c_{crit}}) \to T(n) = \Theta(n^{c_{crit}} \lg n)$
3. $f(n)$ implies $\Omega(n^{c+\epsilon}) \to T(n) = \Theta(f(n))$

Common Mistakes

Avoid These errors

[X] Saying "at least O(n)" [X] Confusing 2^n with n^2 [X] Counting input size in space complexity

Loop Analysis

- **Linear**: for i:1..n $\to O(n)$
- **Nested**: for i:1..n, j:1..n $\to O(n^2)$
- **Dependent**: $\sum i \to O(n^2)$
- **Halving**: while n¿1 (n/=2) $\to O(\lg n)$

Useful Series

$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} = \Theta(n^2)$

$\sum_{i=0}^{n} r^i = \frac{r^{n+1}-1}{r-1}$ (Geom)