COMPLEXITY CHEAT SHEET (MINIMAL)

DEFINITIONS

$\Theta(g(n))$ **- Tight Bound** $0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n)$ for $n \geq n_0$.

$O(g(n))$ **- Upper Bound** $0 \leq f(n) \leq cg(n)$ for $n \geq n_0$.

$\Omega(g(n))$ **- Lower Bound** $0 \leq cg(n) \leq f(n)$ for $n \geq n_0$.

$o(g(n))$ **- Strict Upper** $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$

$\omega(g(n))$ **- Strict Lower** $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \infty$

STANDARD FUNCTIONS

Polynomials: $\sum a_i n^i = \Theta(n^d)$

Logarithms: $\log_b n = \Theta(\ln n)$

Factorials: $n! = o(n^n)$, $\log(n!) = \Theta(n \lg n)$

LIMITS & COMPARISONS

$$\lim_{n\to\infty} \frac{n^b}{a^n} = 0 \quad (\forall a > 1)$$

Exp functions > Poly functions.

$$\lim_{n\to\infty} \frac{\lg^b n}{n^a} = 0 \quad (\forall a > 0)$$

Poly functions > Log functions.

PROPERTIES

Transitivity: $f = \Theta(g), g = \Theta(h) \implies f = \Theta(h)$ (Applies to all).

Transpose Symmetry: $f = O(g) \iff g = \Omega(f)$ $f = o(g) \iff g = \omega(f)$

Equation Arithmetic: $2n^2 + 3n + 1 = 2n^2 + \Theta(n) = \Theta(n^2)$

STIRLING'S APPROX

$$n! \approx \sqrt{2\pi n} \left(\frac{n}{e}\right)^n$$

Useful for analyzing factorial complexities.

RECURRENCES (MASTER THM)

$T(n) = aT(n/b) + f(n)$ Depending on $c = \log_b a$: 1. $f(n) = O(n^{c-\epsilon}) \implies T(n) = \Theta(n^c)$ 2. $f(n) = \Theta(n^c) \implies T(n) = \Theta(n^c \lg n)$ 3. $f(n) = \Omega(n^{c+\epsilon}) \implies T(n) = \Theta(f(n))$

ANALYTIC TRICKS

- Ignore constants.
- Ignore lower order terms.
- $n!$ grows VERY fast.
- $\lg^* n$ (Iterated log) grows VERY slow.