

COMPLEXITY ANALYSIS

CHEAT SHEET 01 / PORTRAIT SWISS

01. NOTATION

Θ	Tight bound. Approx equal.
O	Upper bound. Less than or equal.
Ω	Lower bound. Greater than or equal.
o	Strict upper. Strictly less.
ω	Strict lower. Strictly greater.

Definitions:

Big-Theta $\Theta(g(n))$ $c_1 g(n) \leq f(n) \leq c_2 g(n)$

Big-O $O(g(n))$ $f(n) \leq c \cdot g(n)$

Big-Omega $\Omega(g(n))$ $c \cdot g(n) \leq f(n)$

02. HIERARCHY

Slow Growth (Fast Algo): $O(1), O(\lg n), O(\sqrt{n})$

Medium Growth: $O(n), O(n \lg n)$

Fast Growth (Slow Algo): $O(n^2), O(n^3), O(2^n), O(n!)$

03. EXAMPLES

$O(1)$	Access, Push/Pop
$O(\lg n)$	Binary Search
$O(n)$	Linear Scan
$O(n \lg n)$	Merge Sort
$O(n^2)$	Nested Loops
$O(2^n)$	Recursion (Fib)

04. PROPERTIES

Transitivity: If $A = O(B)$ and $B = O(C)$, then $A = O(C)$.

Additivity: $O(f) + O(g) \rightarrow O(\max(f, g))$

Multiplication: $O(f) \times O(g) \rightarrow O(f \cdot g)$

05. LIMITS

Polynoms vs Exps: Any exponential $a^n (a > 1)$ beats any polynomial n^b .

Logs vs Polynoms: Any polynomial $n^a (a > 0)$ beats any poly-log $\lg^b n$.

06. LOGARITHMS

$$\log(xy) = \log x + \log y$$

$$\log(x^k) = k \log x$$

$$\log_b a = \frac{\log_c a}{\log_c b}$$

07. MASTER THEOREM

For recurrence $T(n) = aT(n/b) + f(n)$. Compare $n^{\log_b a}$ vs $f(n)$.

Case 1 (Warrior): Root heavy. $T(n) = \Theta(n^{\log_b a})$

Case 2 (Balanced): Equal weight. $T(n) = \Theta(n^{\log_b a} \lg n)$

Case 3 (Root): Leaf heavy. $T(n) = \Theta(f(n))$

08. LOOPS

Simple Loop: N iterations \times Work inside.

Nested Independent: $N \times M$.

Nested Dependent: $\sum i \approx n^2/2$.

09. MATH

Arithmetic Series: $\sum i = \frac{n(n+1)}{2} = \Theta(n^2)$

Geometric Series: $\sum r^i = \frac{r^{n+1}-1}{r-1}$

Stirling's: $n! \approx (n/e)^n$