

# COMPLEXITY ANALYSIS

## CHEAT SHEET 01 / PORTRAIT SWISS

### 01. NOTATION

$\Theta$	Tight bound. Approx equal.
$O$	Upper bound. Less than or equal.
$\Omega$	Lower bound. Greater than or equal.
$o$	Strict upper. Strictly less.
$\omega$	Strict lower. Strictly greater.

#### Definitions:

**Big-Theta**  $\Theta(g(n))$   $c_1g(n) \leq f(n) \leq c_2g(n)$

**Big-O**  $O(g(n))$   $f(n) \leq c \cdot g(n)$

**Big-Omega**  $\Omega(g(n))$   $c \cdot g(n) \leq f(n)$

### 02. HIERARCHY

**Slow Growth (Fast Algo):**  $O(1), O(\lg n), O(\sqrt{n})$

**Medium Growth:**  $O(n), O(n \lg n)$

**Fast Growth (Slow Algo):**  $O(n^2), O(n^3), O(2^n), O(n!)$

### 03. EXAMPLES

$O(1)$	Access, Push/Pop
$O(\lg n)$	Binary Search
$O(n)$	Linear Scan
$O(n \lg n)$	Merge Sort
$O(n^2)$	Nested Loops
$O(2^n)$	Recursion (Fib)

### 04. PROPERTIES

**Transitivity:** If  $A = O(B)$  and  $B = O(C)$ , then  $A = O(C)$ .

**Additivity:**  $O(f) + O(g) \rightarrow O(\max(f, g))$

**Multiplication:**  $O(f) \times O(g) \rightarrow O(f \cdot g)$

### 05. LIMITS

**Polynomials vs Exps:** Any exponential  $a^n (a > 1)$  beats any polynomial  $n^b$ .

**Logs vs Polynomials:** Any polynomial  $n^a (a > 0)$  beats any poly-log  $\lg^b n$ .

### 06. LOGARITHMS

$$\log(xy) = \log x + \log y$$

$$\log(x^k) = k \log x$$

$$\log_b a = \frac{\log_c a}{\log_c b}$$

### 07. MASTER THEOREM

For recurrence  $T(n) = aT(n/b) + f(n)$ . Compare  $n^{\log_b a}$  vs  $f(n)$ .

**Case 1 (Warrior):** Root heavy.  $T(n) = \Theta(n^{\log_b a})$

**Case 2 (Balanced):** Equal weight.  $T(n) = \Theta(n^{\log_b a} \lg n)$

**Case 3 (Root):** Leaf heavy.  $T(n) = \Theta(f(n))$

### 08. LOOPS

**Simple Loop:**  $N$  iterations  $\times$  Work inside.

**Nested Independent:**  $N \times M$ .

**Nested Dependent:**  $\sum i \approx n^2/2$ .

### 09. MATH

**Arithmetic Series:**  $\sum i = \frac{n(n+1)}{2} = \Theta(n^2)$

**Geometric Series:**  $\sum r^i = \frac{r^{n+1}-1}{r-1}$

**Stirling's:**  $n! \approx (n/e)^n$