DePaul University
College of Computing and Digital Media

CSC 202: Discrete Structures for Computer Science

Practice Problem Solutions

April 21, 2010

Problem 1

$(p \leftrightarrow q) \wedge (\bar{p} \rightarrow r) \equiv (T \leftrightarrow T) \wedge (F \rightarrow T) \equiv T$ when $p = T$, $q = T$, and $r = T$.

Problem 2

Construct a truth table for the proposition $(p \vee q) \rightarrow (p \wedge q)$

p	q	$(p \vee q)$	$(p \wedge q)$	$(p \vee q) \rightarrow (p \wedge q)$
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

Problem 3

Show that $(p \leftrightarrow q)$ is logically equivalent to $(p \rightarrow q) \wedge (q \rightarrow p)$

p	q	$p \leftrightarrow q$	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \wedge (q \rightarrow p)$
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

Problem 4

Show that $(p \wedge q) \rightarrow p$ is a tautology

p	q	$(p \wedge q)$	$(p \wedge q) \rightarrow p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

Problem 5

Find a formula in disjunctive normal form that is equivalent to $(p \vee q) \rightarrow (p \wedge q)$

The truth table for this formula is given above. A formula in DNF that is equivalent to it is $(p \wedge q) \vee (\bar{p} \wedge \bar{q})$

Problem 6

a) False
b) True
c) True
d) True

Problem 7

$P(P(\emptyset)) = P(\{\emptyset\}) = \{\emptyset, \{\emptyset\}\}$

Problem 8

If A = {1, 2}, B = {b}, and C = {one, two}, then C x B x A = {(one, b, 1}, {one, b, 2}, {two, b, 1}, {two, b, 2}}.

Problem 9

$A \cap \overline{A} = \{x : x \in A \wedge x \notin A\}$. But an element cannot both be in the set A and not be in the set A. So $\{x : x \in A \wedge x \notin A\}$ must be empty.

Problem 10

R(x,y) = "x = y + 1 or x = y – 1" is not reflexive since $x \neq x+1$ and $x \neq x -1$ for all integers x.
R(x,y) = "x = y + 1 or x = y – 1" is anti-reflexive since since $x \neq x+1$ and $x \neq x -1$ for all integers x.
R(x,y) = "x = y + 1 or x = y – 1" is symmetric since if x = y+1 or x = y -1, then y = x – 1 or y = x + 1 using basic algebra.
R(x,y) = "x = y + 1 or x = y – 1" is not anti-symmetric. Suppose x = y+1 or x = y-1 and y = x+1 or y = x-1. There are four cases:

1. x = y + 1 and y = x + 1. This isn't possible since it would mean that x = y + 1 = (x + 1) + 1 = x + 2.
2. x = y + 1 and y = x – 1. While this is possible, it does mean that $x \neq y$.
3. x = y – 1 and y = x + 1. Again this is possible, but it again means that $x \neq y$.
4. x = y – 1 and y = x – 1. This isn't possible since it would mean that x = y – 1 = (x – 1) – 1 = x – 2.

R(x,y) = "x = y + 1 or x = y – 1" is not transitive since for x = 3, y = 2, and z = 1, we have that x = y – 1 and y = z – 1 but $x \neq z + 1$ and $x \neq z – 1$.

Problem 11

a) The relation is reflexive and symmetric.
b) The relation is neither reflexive nor symmetric.
c) The relation is not reflexive but is symmetric.

Problem 12

(b), (c), (e), and (f)

Problem 13

a) True because all the odd numbers in D are positive.
b) False. The counterexamples are $x = 16$, $x = 26$, $x = 32$, and $x = 36$.

Problem 14

a) $(\exists x)[x(x+1) > 0 \land x \leq 0 \land x \geq -1]$
b) $(\exists x)\ (\exists y)\ (\exists z)$ [$x - y$ is even and $b - c$ is even, but $a - c$ is odd]

Problem 15

a) True. If you choose $-x$, then $x + (-x) = 0$.
b) False. There is no real number that when added to all real numbers, produces 0.