DePaul University
College of Computing and Digital Media

CSC 202: Discrete Structures for Computer Science

Midterm Study Guide

April 19, 2010

Exam details

The midterm for in-class students is scheduled for Monday, April 16th, 5:45 pm in Lewis 1207. Online students need to register for an exam period using the Exams tab on the COL site no later than 5 days before taking the exam. You will have 2 hours for the exam. There will be no lecture following the exam.

The exam is closed book. You may use a single, double-sided sheet (8 ½ x 11 inches) or two single-sided (8 ½ x 11 inches) of notes and a calculator. The sheet of notes (with your name at the top) must be handed in with your exam. The calculator should not be programmable and must not be located on a communication device. You may not use any other written materials. In addition, no other electronic devices such as cell phones or laptop computers are allowed.

Note: Any concept that has appeared in the homework or during lecture, including the exercises solved in class, is a possible topic for the midterm. I would recommend reviewing your previous assignments, reading your notes, and if you did not already do so, watching the class recordings. You may want to review relevant sections in your reference textbook, although I will not assume knowledge of any material in the textbook but that does not appear in the class notes.

Important topics

1. **Propositional logic**. Propositions. Compound propositions including connectives (conjunction, disjunction, negation, the conditional proposition, the biconditional), evaluating compound propositions, and precedence of connectives. Truth tables and logical equivalence. Tautologies and falsehoods. DeMorgan's Law. Truth functions and the algorithm for finding a formula equivalent to an arbitrary truth table. Normal forms (disjunctive normal form and conjunctive normal form).
2. **Sets**. Basic definitions (set, element of, cardinality, equality, subset, the empty set). Extensional versus intensional definitions of sets. Basic results about sets (including all the lemmas and corollaries found in the notes). Set operations (union, disjoint union, intersection, complement, difference). The relationship between logic and set definitions.
3. **Pairs and tuples**. Basic definitions (pair, n-tuple, equality, Cartesian product).
4. **Power set**. Basic definitions (power set, size of the power set). Computing power sets.
5. **Relations**. Basic definitions (relation, unary, binary, n-ary). Properties of relations (reflexive, symmetric, transitive, anti-symmetric, anti-reflexive). Equivalence

relations and classes and their relationship to partitions. Ordering relations (partial and total). Strict ordering relations (partial and total). Inverse relations. The join of relations. Intensional versus extensional views of relations. Set-theoretic operations (union, intersection, difference) on relations. Representing relations (tables, matrices, and properties as seen in matrices).

6. **Quantifiers**. Existential quantifier. Universal quantifier. Distribution (or not) over disjunction and conjunction. Multiple quantifiers and order of quantifiers. Negating quantified statements.

The relevant parts of the Epp textbook are Sections 1.1, 1.2, 2.1 – 2.3, 5.1 – 5.3, 10.1 – 10.3, and10.5. As stated above, there will be no questions on material in the textbook that does not also appear in the notes.

Practice problems

The problems given here are exercises. They provide you with additional practice, but are not intended to be representative of the questions found on the midterm. You do not have to submit them. I will post solutions on Wednesday, April 21st.

You should also review all the previous homework questions and make sure you know how to solve them. I also recommend that you work as much as possible on the fourth assignment before starting these exercises.

1) Evaluate the expression $(p \leftrightarrow q) \wedge (\bar{p} \rightarrow r)$ when p = T, q = T, and r = T.
2) Construct a truth table for the compound proposition $(p \vee q) \rightarrow (p \wedge q)$
3) Show that $(p \leftrightarrow q)$ is logically equivalent to $(p \rightarrow q) \wedge (q \rightarrow p)$
4) Show that $(p \wedge q) \rightarrow p$ is a tautology
5) Find a formula in disjunctive normal form that is equivalent to $(p \vee q) \rightarrow (p \wedge q)$
6) For each of the following determine if the statement is true or false:
   a) $2 \in \{\{2\}, \{2, 2\}\}$
   b) $\emptyset \in \{\emptyset, \{\emptyset\}\}$
   c) $\{2\} \subseteq \{\{2\}, \{2, 2\}\}$
   d) $\emptyset \subseteq \{\emptyset, \{\emptyset\}\}$
7) What is $P(P(\emptyset))$?
8) Let A = {1, 2}, B = {b}, and C = {one, two}. Find C x B x A.
9) Show that $A \cap \bar{A} = \emptyset$.
10) Determine if the relation R(x,y) = "x = y + 1 or x = y – 1" is reflexive, anti-reflexive, symmetric, anti-symmetric and/or transitive. Either prove that it has a given property or provide a counterexample that shows it does not have a given property.
11) Indicate whether the relations represented by the following matrices are reflexive or symmetric:
    a) $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$
    b) $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

c) $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

12) Consider the following statement: $(\exists x)\,[x^2 = 2]$ over the real numbers. Which of the following are equivalent ways of expressing this statement?
   a) The square of each real number is 2.
   b) Some real numbers have square 2.
   c) The number x has square 2, for some real number x.
   d) If x is a real number, then $x^2 = 2$.
   e) Some real number has square 2.
   f) There is at least one real number whose square is 2.
13) Let D = {-48, -14, -8, 0, 1, 3, 16, 23, 26, 32, 36} be the set over which quantification occurs in the following statements. Determine which of the following statements are true and which are false. Provide counterexamples for those statements that are false.
   a) $(\forall x)$ [if x is odd then $x > 0$]
   b) $(\forall x)$ [if x is even then $x \leq 0$]
14) Write a negation for the following statements:
   a) $(\forall x)$ [if $x(x+1) > 0$ then $x > 0$ or $x < -1$] quantified over the real numbers
   b) $(\forall x)\,(\forall y)\,(\forall z)$ [if $x – y$ is even and $b – c$ is even, then $a – c$ is even]
15) Indicate whether the following statements are true or false where the statements are quantified over the real numbers. If the statement is false, indicate clearly why the statement is false. If the statement is true, indicate clearly why the statement is true.
   a) $(\forall x)\,(\exists y)[x + y = 0]$
   b) $(\exists y)\,(\forall x)\,[x + y = 0]$