Gwinnett School of Math, Science, and Technology

AP Physics: Mechanics/Electricity & Magnetism Notes

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1 Kinematics

1.1 Variables

1.1.1 Position

- Typically given by the variable x

1.1.2 Time

- Typically given by the variables t

1.1.3 Displacement

- Defined as the change in position ($X_f - X_i$)
- Given by the variable Δx

1.1.4 Distance

- You have to take the magnitude of vectors for every time you change position
-
- Defined as the change in displacement over time
- $\frac{\Delta x}{\Delta t} = V_{\text{avg}} = \bar{V}$

1.1.5 Velocity

- Defined as the change in displacement as time approaches 0
- $\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} = V$

1.1.6 Average Acceleration

- Defined as the change in velocity over time
- $\frac{\Delta V}{\Delta t} = A_{\text{avg}} = \bar{A}$

1.1.7 Acceleration

- Defined as the change in velocity as time approaches 0
- $a = \lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t} = \frac{dV}{dt} = \frac{d^2x}{dt^2}$

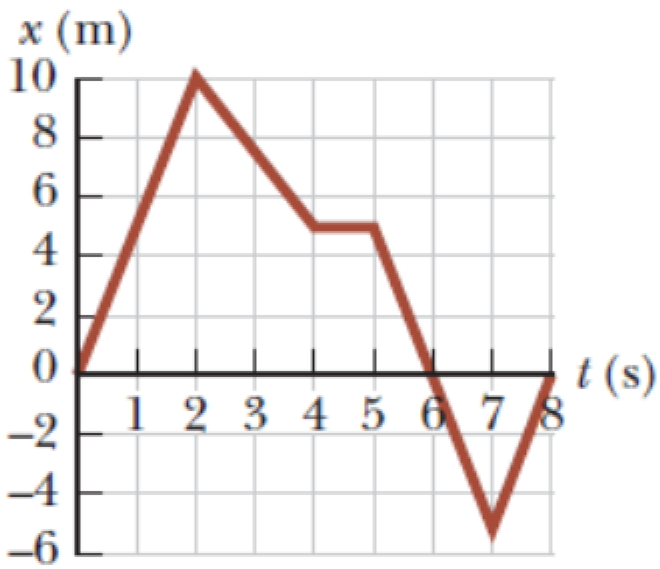
1.1.8 Speed

- Speed = $\frac{s}{t}$
- Alternatively, when referring to vectors: Speed = $||\vec{V}||$

1.2 Velocity and Position Definitions (08/03 Homework)

1.2.1 Problem 1

The position versus time for a certain particle moving along the x axis is shown in the figure below. Find the average velocity in the following time intervals.



(a) 0 to 2s

- $\frac{10-0}{2-0} = \frac{10}{2}$ = 5.000 m/s

(b) 0 to 4s

- $\frac{5-0}{4-0} = \frac{5}{4}$ = 1.250 m/s

(c) 2 to 6s

- $\frac{0-10}{6-2}$ = $\frac{-10}{4}$ = −2.500 m/s

(d) 1 to 7s

- $\frac{-5-5}{7-1}$ = $\frac{-10}{6}$ = −1.667 m/s

(e) 0 to 7s

- $\frac{-5-0}{7-0}$ = $\frac{-5}{7}$ = −0.714 s

1.2.2 Problem 2

A person walks first at a constant speed of 4.80 m/s along a straight line from point A to point B and then back along the line from B to A at a constant speed of 2.90 m/s.

(a) What is her average speed over the entire trip?

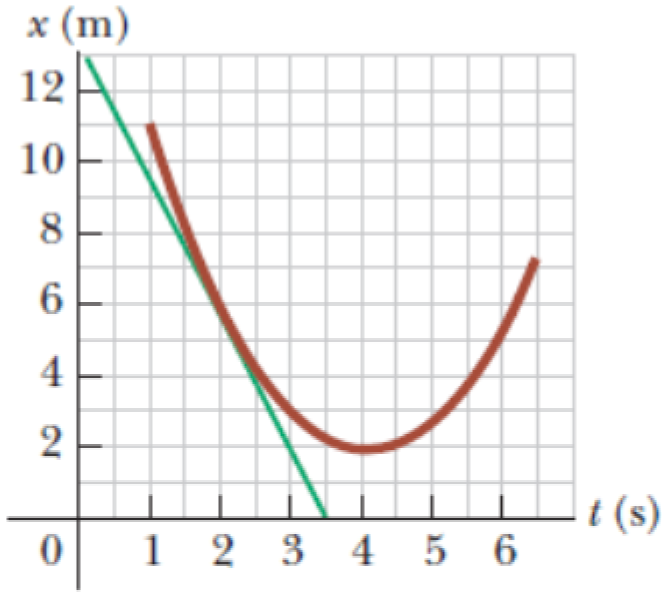
$$
\begin{aligned}
\text{Average speed} &= \frac{d_{AB} + d_{BA}}{t_{AB} + t_{BA}} \\
d &= d_{AB} = d_{BA} \\
t_{AB} &= \frac{d}{V_{AB}} \\
tBA &= \frac{d}{V_{BA}} \\
\therefore \text{Average speed} &= \frac{d + d}{\frac{d}{V_{AB}} + \frac{d}{V_{BA}}} \\
&= \frac{2d(V_{AB})(V_{BA})}{2d} \\
&= \frac{2(V_{AB})(V_{BA})}{V_{AB} + V_{BA}} \\
&= 2\left[\frac{(4.80)(2.90)}{4.80 + 2.90}\right] \\
&= 3.615
\end{aligned}
$$

(b) What is her average velocity over the entire trip?

- Since her displacement is 0, her average velocity is also 0.

1.2.3 Problem 3

A position-time graph for a particle moving along the x axis is shown in the figure below.



(a) Find the average velocity in the time interval t = 2.00 s to t = 3.50 s.

- $\frac{2-6}{3.5-2} = \frac{-4}{1.5}$ = −2.667 m/s

(b) Determine the instantaneous velocity at t = 2 s (where the tangent line touches the curve) by measuring the slope of the tangent line shown in the graph.

- $\frac{2-12}{3.5-0} = \frac{-10}{3.5}$ = −3.714 m/s

(c) At what value of t is the velocity zero?

- 4.000 s

1.2.4 Problem 4

A person takes a trip, driving with a constant speed of 91.5 km/h, except for a 24.0-min rest stop. The person's average speed is 71.6 km/h.

(a) How much time is spent on the trip?

$$\bar{V} = \frac{(\text{Time driving}) * V_{\text{driving}}}{\text{Total time}}$$
$$71.6 = \frac{(t - \frac{24}{60}) \cdot 91.5}{t}$$
$$71.6t = (t - \frac{24}{60}) \cdot 91.5$$
$$\frac{71.6}{91.5}t = t - \frac{24}{60}$$
$$\frac{71.6}{91.5}t - t = -\frac{24}{60}$$
$$\frac{71.6}{91.5}t - \frac{91.5}{91.5}t = -\frac{24}{60}$$
$$-\frac{19.9}{91.5}t = -\frac{24}{60}$$
$$t = -\frac{24}{60} \cdot -\frac{91.5}{19.9}$$
$$t = 1.839 \text{ hours}$$

(b) How far does the person travel?

- $d = \bar{V} \cdot t = (71.6)(1.839) = 131.6$ km

1.3 Derivative Relationships

1.3.1 Acceleration and Velocity

- $a = \frac{dV}{dt}$
- $V = \frac{dx}{dt}$
- $V = at + V_0$

1.3.2 Acceleration and Position

- $V = \frac{dx}{dt}$
- $x = \frac{1}{2}at^2 + V_0 t$

If initial time or position is not zero:

- $x - x_0 = V_0(t - t_0) + \frac{1}{2}a(t - t_0)^2$

1.3.3 Given $x = 4t^4 - 6t$, find a when $t = 2$.

$$
\begin{aligned}
\frac{dx}{dt} &= 16t^3 - 6 \\
\frac{d^2x}{dt^2} &= 48t^2 \\
a &= 48(2)^2 \\
a &= 192
\end{aligned}
$$

1.4 Practice with 1-D Kinematic Equations (moderate)

1.4.1 Problem 1

A world-class sprinter can burst out of the blocks to essentially top speed (of about 11.5 m/s) in the first 15.0m of the race. What is the average acceleration of this sprinter and how long does it take her to reach that speed (she accelerates uniformly)?

1.4.2 Problem 2

A car slows down from a speed of 25.0m/s to rest in 5.0s. How far did it travel in that time?

1.4.3 Problem 3

In coming to a stop, a car leaves skid marks 80m long on the highway. Assuming a deceleration of $7.00 m/s^2$, estimate the speed of the car just before braking.

1.4.4 Problem 4

A car traveling 45km/h slows down at a constant $0.50 m/s^2$ just by "letting up on the gas." Calculate:

(a) the distance the car coasts before it stops
(b) the time it takes to stop
(c) the distance it travels during the first and fifth seconds

1.4.5 Problem 5

A car traveling at 90km/h strikes a tree. The front end of the car compresses and the driver comes to rest after traveling 0.80m. What was the average acceleration of the driver during the collision? Express the answer in terms of "g's," where 1.00g = $9.80 m/s^2$

1.4.6 Problem 6

Determine the stopping distances for an automobile with ana initial speed of 90km/h and human reaction time of 1.0s:

(a) for a = −4.0m/s^2
(b) for a = −8.0m/s^2

1.5 Acceleration Definitions (08/04 Homework)

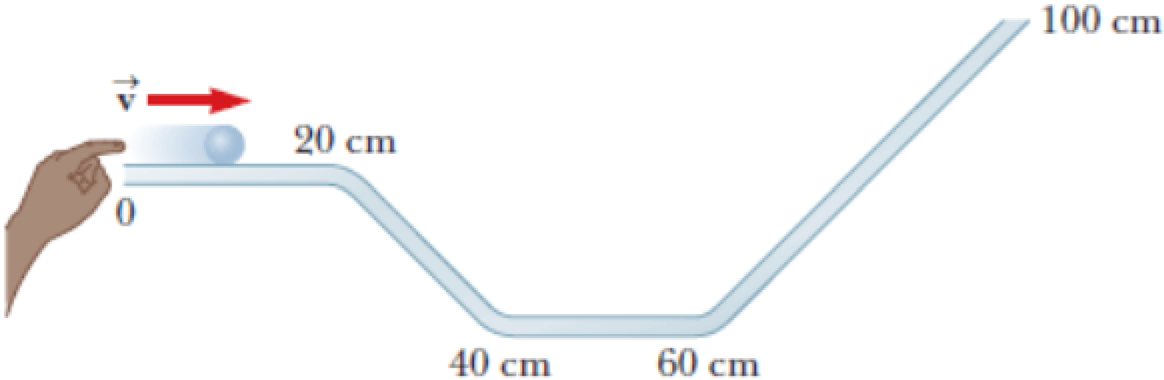
1.5.1 Problem 1

A 48.0 g Super Ball traveling at 29.0 m/s bounces off a brick wall and rebounds at 18.0 m/s. A high-speed camera records this event. If the ball is in contact with the wall for 4.05 ms, what is the magnitude of the average acceleration of the ball during this time interval?

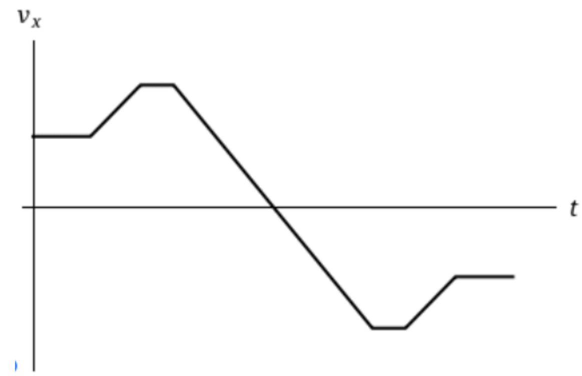
- $a = \left|\frac{-18-29}{0.00405} = 11604.938\right|$ m/s^2

1.5.2 Problem 2

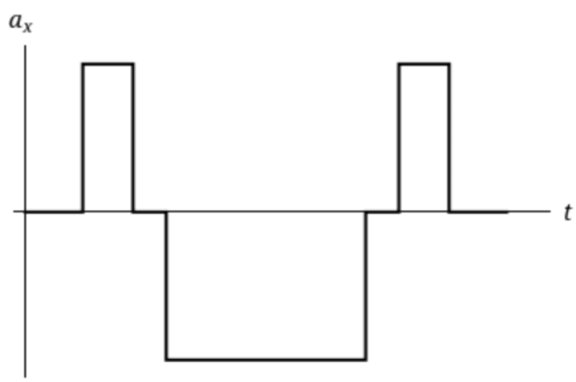
A child rolls a marble on a bent track that is 100 cm long as shown in the figure below. We use x to represent the position of the marble along the track. On the horizontal sections from $x = 0$ to $x = 20$ cm and from $x = 40$ to $x = 60$ cm, the marble rolls with constant speed. On the sloping sections, the marble's speed changes steadily. At the places where the slope changes, the marble stays on the track and does not undergo any sudden changes in speed. The child gives the marble some initial speed at $x = 0$ and $t = 0$ and then watches it roll to $x = 90$ cm, where it turns around, eventually returning to $x = 0$ with the same speed with which the child released it. Prepare graphs of x versus t, v_x versus t, and a_x versus t, with their time axes identical, to show the motion of the marble. You will not be able to place any numbers other than zero on the horizontal axis or on the velocity and acceleration axes, but show the correct graph shapes.



x versus *t*



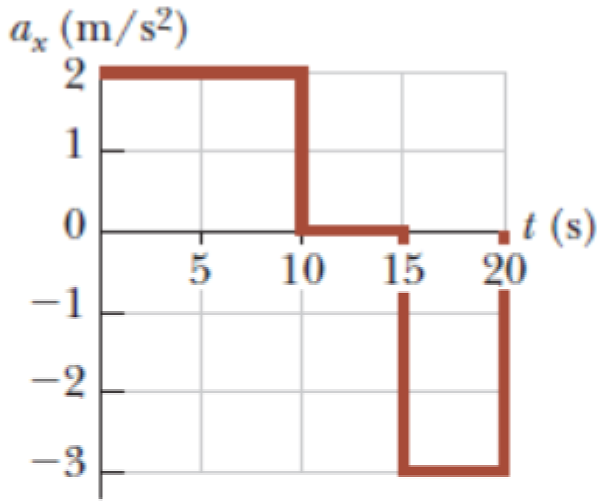
V_x versus *t*



a_x versus *t*

1.5.3 Problem 3

A particle starts from rest and accelerates as shown in the figure below.



(a) Determine the particle's speed at $t = 10.0$ s.

- $\int_0^{10} a\, dt = 10 \cdot 2 = 20$ m/s

(b) Determine the particle's speed at $t = 20.0$ s?

- $\int_0^{20} a\, dt = 10 \cdot 2 + 5 \cdot (-3) = 20 - 15 = 5$ m/s

(c) Determine the distance traveled in the first 20.0 s. (Enter your answer to one decimal places.)

$$\Delta x \text{ from 0-10 seconds: } \frac{1}{2} \cdot 2 \cdot 10^2 = 100$$
$$\Delta x \text{ from 10-15 seconds: } 20 \cdot 5 = 100$$
$$\Delta x \text{ from 15-20 seconds: } \frac{1}{2} \cdot -3 \cdot 5^2 + 20 \cdot 5 = 62.5$$
$$\therefore \Delta x = 262.5 \text{ m}$$

1.5.4 Problem 4

An object moves along the x axis according to the equation $x = 3.25t^2 - 2.00t + 3.00$, where x is in meters and t is in seconds.

(a) Determine the average speed between $t = 1.70$ s and $t = 3.30$ s.

- $\bar{V} = \frac{x(3.30)-x(1.70)}{t_1-t_0} = \frac{(3.25(3.30)^2-2(3.30)+3)-(3.25(1.70)^2-2(1.70)+3)}{3.30-1.70}$ = 14.25 m/s

(b) Determine the instantaneous speed at t = 1.70 s.

- $V = \frac{dx}{dt}$ = 6.5(1.7) – 2 = 9.05 m/s

(c) Determine the instantaneous speed at t = 3.30 s.

- $V = \frac{dx}{dt}$ = 2(3.25)(3.3) – 2 = 19.45 m/s

(d) Determine the average acceleration between t = 1.70s and t = 3.30 s.

- $\bar{a} = \frac{V(3.30)-V(1.70)}{t_1-t_0} = \frac{(2(3.25)(3.30)-2)-(2(3.25)(1.70)-2)}{3.30-1.70}$ = 6.5 m/s^2

(e) Determine the instantaneous acceleration at t = 1.70 s.

- $a = \frac{dV}{dt}$ = 2(3.25) = 6.5 m/s^2

(f) Determine the instantaneous acceleration at t = 3.30 s.

- $a = \frac{dV}{dt}$ = 2(3.25) = 6.5 m/s^2

(g) At what time is the object at rest?

- $0 = 6.5t - 2 \implies 0.308$ s

1.6 Velocity-Acceleration Relationships

- If acceleration is constant, $\frac{da}{dt} = 0$
  - And as long as acceleration remains constant, and V_0 and t_0 are 0, $\bar{V} = \frac{1}{2}(V_0 + V_f)$
- If acceleration is constant, but either V_0 or t_0 are not 0, $X = V_0 t + \frac{1}{2}V_f t - \frac{1}{2}V_0 t_0$

1.6.1 Phab Four

1. $V_f = V_0 + at$
2. $t = \frac{1}{2}t(V_0 + V_f)$
3. $x = V_0 t + \frac{1}{2}at^2$
4. $V_f^2 = V_0^2 + 2ax$

Phab Four	V_0	V_f	a	t	x
$V_f = V_0 + at$	✓	✓	✓	✓	
$t = \frac{1}{2}(V_0 + V_f)$	✓	✓		✓	
$x = V_0 t + \frac{1}{2}at^2$	✓		✓	✓	✓
$V_f^2 = V_0^2 + 2ax$	✓	✓	✓		✓

1.6.2 What if V_0 isn't given?

- Simply use the equation $x = V_f t - \frac{1}{2}at^2$
  - This will rarely show up, if ever.

1.6.3 What if time and/or starting distance are not 0?

- Time and distance are actually *deltas* (i.e. Δt and Δx)
- So you have to subtract the initial time and distance from the final time and distance
  - Example: $x - x_0 = V_0(t - t_0) + \frac{1}{2}a(t - t_0)^2$

1.7 Freefall and Calculus (08/08 Homework)

1.7.1 Problem 1

An attacker at the base of a castle wall 3.60 m high throws a rock straight up with speed 8.00 m/s from a height of 1.60 m above the ground.

(a) Will the rock reach the top of the wall?

- Yes, because the result from part b is not imaginary.

(b) If so, what is its speed at the top? If not, what initial speed must it have to reach the top?

$$V_f^2 = 8^2 + 2(-9.81)(3.6 - 1.6)$$
$$V_f^2 = 64 - 39.24$$
$$V_f = \sqrt{24.76}$$
$$= 4.976 \text{ m/s}$$

(c) Find the change in speed of a rock thrown straight down from the top of the wall at an initial speed of 8.00 m/s and moving between the same two points.

$$V_f^2 = 8^2 + 2(-9.81)(1.6 - 3.6)$$
$$V_f^2 = 64 + 39.24$$
$$V_f = \sqrt{103.24}$$
$$= 10.161 \text{ m/s}$$

(d) Does the change in speed of the downward-moving rock agree with the magnitude of the speed change of the rock moving upward between the same elevations?

- No.

(e) Explain physically why it does or does not agree.

- The initial velocity when the ball up is sent up is acting against acceleration due to gravity, whereas the initial velocity when you send the ball back down is acting with acceleration due to gravity. And this is important to note because the ball spends less time to reach the ground than the time it takes to halt in the air when thrown upward. And therefore, the delta V's for both are not going to remain consistent since the times are not the same.

1.7.2 Problem 2

It is possible to shoot an arrow at a speed as high as 122 m/s.

(a) If friction is neglected, how high would an arrow launched at this speed rise if shot straight up?

$$
\begin{aligned}
0 &= 122^2 + 2(-9.81)x \\
-14884 &= -19.62x \\
x &= 758.614 \text{ m}
\end{aligned}
$$

(b) How long would the arrow be in the air?

$$
\begin{aligned}
758.614 &= 122t + \frac{1}{2}(9.81)t^2 \\
0 &= -4.905t^2 + 122t - 758.614 \\
t &= 24.873 \text{ s}
\end{aligned}
$$

1.7.3 Problem 3

The height of a helicopter above the ground is given by $h = 3.10t^3$, where h is in meters and t is in seconds. At $t = 1.90$ s, the helicopter releases a small mailbag. How long after its release does the mailbag reach the ground?

$$
\begin{aligned}
x &= 3.10 \cdot 1.9^3 \\
&= 21.2629 \\
V_0 &= 9.30 \cdot 1.9^2 \\
&= 33.573 \\
-21.2629 &= 33.573t + \frac{1}{2}(-9.81)t^2 \\
0 &= -4.905t^2 + 33.573t + 21.2629 \\
t &= 7.428 \text{ s}
\end{aligned}
$$

1.7.4 Problem 4

The speed of a bullet as it travels down the barrel of a rifle toward the opening is given by $v = (-4.20 \cdot 10^7)t^2 + (2.30 \cdot 10^5)t$, where v is in meters per second and t is in seconds. The acceleration of the bullet just as it leaves the barrel is zero.

(a) Determine the acceleration and position of the bullet as a function of time when the bullet is in the barrel. (Use t as necessary and round all numerical coefficients to exactly 3 significant figures.)

- $a = \left(-8.40 \cdot 10^7\right) t + \left(2.30 \cdot 10^5\right)$ m/s^2
- $x = \frac{\left(-4.20 \cdot 10^7\right) t^3}{3} + \frac{\left(2.30 \cdot 10^5\right) t^2}{2}$ m

(b) Determine the length of time the bullet is accelerated.

$$0 = \left(-8.40 \cdot 10^7\right) t + \left(2.30 \cdot 10^5\right)$$
$$= 0.002738 \text{ s}$$

(c) Find the speed at which the bullet leaves the barrel.

$$v = \left(-4.20 \cdot 10^7\right)(0.002738)^2 + \left(2.30 \cdot 10^5\right)(0.002738)$$
$$= 312 \text{ m/s}$$

(d) What is the length of the barrel?

$$x = \frac{\left(-4.20 \cdot 10^7\right)(0.002738)^3}{3} + \frac{\left(2.30 \cdot 10^5\right)(0.002738)^2}{2}$$
$$= 0.575 \text{ m}$$

1.8 Free Fall Review

- Allegedly discovered by Galileo on the Leaning Tower of Pisa
- An object is considered “in free fall” if the only force acting on it is gravity
- The phab four equation that you will probably use the most for free fall equations, given time and initial velocity, is $x = V_0 t + \frac{1}{2} a t^2$
  - If time isn’t given, you would use $V_f^2 = V_0^2 + 2ax$ instead

1.9 Vectors Review

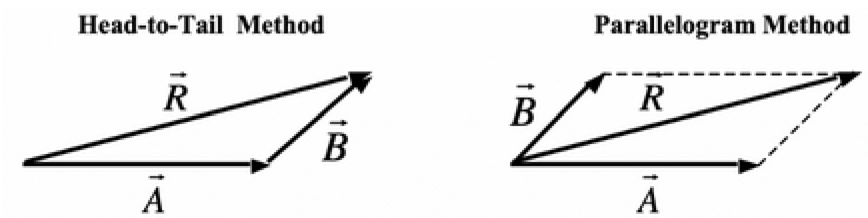
Vectors have magnitude and direction

1.9.1 Vector-Scalar Multiplication

- You can multiply scalar values to vectors, which changes the vector's magnitude by a factor of the scalar value, but not its direction
- Example: $2\vec{V}$ would double the magnitude of the resultant vector

1.9.2 Vector Addition

- Vector addition is commutative
- Two methods:
  - Head to tail method
    * Lining up the head of the first vector to the tail of the second vector and drawing the resultant vector from the tail of the first vector to the head of the second vector
  - Parallelogram method
    * Drawing the two vectors as a parallelogram and drawing the resultant vector from the intersecting points



1.9.3 Vector Subtraction

- The same as vector addition, but you flip the direction of the vector you are subtracting

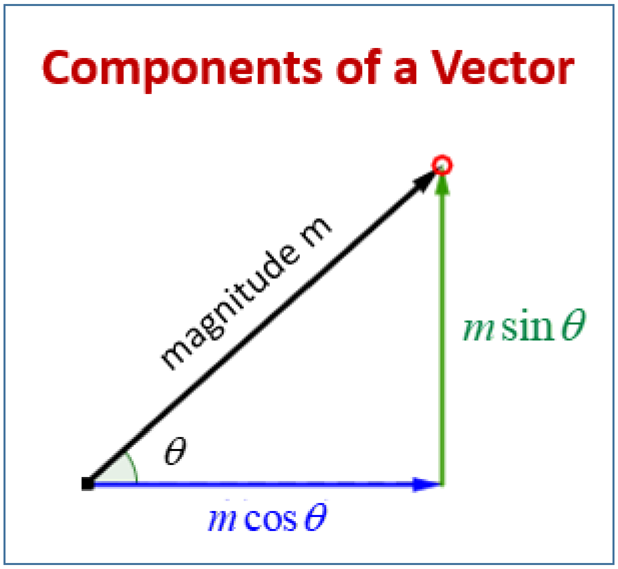
1.9.4 Components of Vectors

- You can easily break down 2D vectors into their x and y components
  - $\vec{V}_x = ||\vec{V}|| \cos\theta$
  - $\vec{V}_y = ||\vec{V}|| \sin\theta$

1.9.4.1 What if I don't know θ or $||\vec{V}||$?

Well, you can easily figure that out with some trig:

- $||\vec{V}|| = \sqrt{\vec{V}_x^2 + \vec{V}_y^2}$
- $\theta = \tan^{-1}\left(\frac{\vec{V}_y}{\vec{V}_x}\right)$



1.9.5 Unit Vectors

- A **unit vector** is any vector with a magnitude of 1
- If you take the vector (cross) product of the unit vectors in the *x* and *y* direction, the resultant vector will be the same in the *z* direction
  - $\hat{i} \times \hat{j} = \hat{k}$

1.9.6 Unit Vector Notation

- You can represent a vector in unit vector notation by multiplying the unit vectors by their respective components:
  - $\vec{V} = V_x\hat{i} + V_y\hat{j} + V_z\hat{k}$
- You can also add two vectors in unit vector notation as follows:

- $\vec{V} + \vec{W} = (V_x + W_x)\hat{i} + (V_y + W_y)\hat{j} + (V_z + W_z)\hat{k}$

- You can also represent a vector in unit vector notation by multiplying the unit vectors by their respective magnitudes and the cosine of the angle between the vector and the unit vector
  - For example: $\vec{V} = ||\vec{V}|| \cos\theta\hat{i} + ||\vec{V}|| \cos\theta\hat{j} + ||\vec{V}|| \cos\theta\hat{k}$

1.9.7 Unit Vectors to Find Position

You can use unit vectors to find the position of a vector:

$$\begin{aligned}
\vec{r} &= r_x\hat{i} + r_y\hat{j} + r_z\hat{k} \\
\vec{v} &= \frac{\mathrm{d}\vec{r}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}(r_x\hat{i}) + \frac{\mathrm{d}}{\mathrm{d}t}(r_y\hat{j}) + \frac{\mathrm{d}}{\mathrm{d}t}(r_z\hat{k}) \\
&= \frac{\mathrm{d}r_x}{\mathrm{d}t}\hat{i} + \frac{\mathrm{d}r_y}{\mathrm{d}t}\hat{j} + \frac{\mathrm{d}r_z}{\mathrm{d}t}\hat{k}
\end{aligned}$$

1.9.7.1 Why does this work?

Remember that the derivative of a constant is 0, and $\hat{i}, \hat{j}, \hat{k}$ are constants:

$$\frac{\mathrm{d}}{\mathrm{d}t}(r_x\hat{i}) = \frac{\mathrm{d}r_x}{\mathrm{d}t}\hat{i} + r_x\frac{\mathrm{d}\hat{i}}{\mathrm{d}t}$$

1.10 Kinematics in Two Dimensions

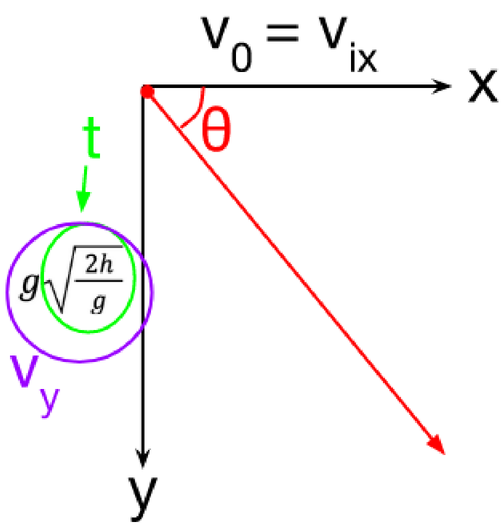
- Horizontal and vertical motion are completely independent of each other
  - However, they share the same **time**

1.10.1 Projectile Vectors

- The horizontal component of a projectile's velocity is constant, equal to the initial velocity
  - $v_x = v_0$
- The vertical component of a projectile's velocity is constantly changing due to gravity

- $v_y = v_0 + gt$

- The overall velocity of a projectile is the resultant vector of the horizontal and vertical components
  - $||\vec{v}|| = \sqrt{{v_{0x}}^2 + (v_{0y}gt)^2}$
- The angle of the projectile is the angle between the resultant vector and the horizontal component
  - $\theta = \tan^{-1}\left(\frac{gt}{v_0}\right)$



1.10.2 Horizontal Projectile Motion

- Acceleration is always 0
- $V_f = V_0 = v\cos\theta$
- $t = \frac{2v\sin\theta}{g}$
- $x = V_0 t + \frac{1}{2}at^2$
- $x = \frac{v^2 \cdot 2\sin(\theta)\cos(\theta)}{g}$
  - Therefore, per the *Double Angle Identity*, $x = \frac{v^2\sin(2\theta)}{g}$

1.10.3 Vertical Projectile Motion

- Horizontal displacement (x) is always 0
- $v_0 = v \sin\theta$
- $a = -g$
- $x = v_0 t + \frac{1}{2}at^2$
    - Therefore, $0 = (v \sin\theta)t - \frac{1}{2}gt^2$
- $t = \frac{2v \sin\theta}{g}$
- $h = \frac{v^2}{g}\sin^2\theta$