GSMST

APPLICATIONS OF LINEAR ALGEBRA
IN PROGRAMMING

Chapter 3 Assignment

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Infinite Possibilities

GWINNETT SCHOOL OF MATHEMATICS, SCIENCE, AND TECHNOLOGY

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Sets of linear combinations and geometry

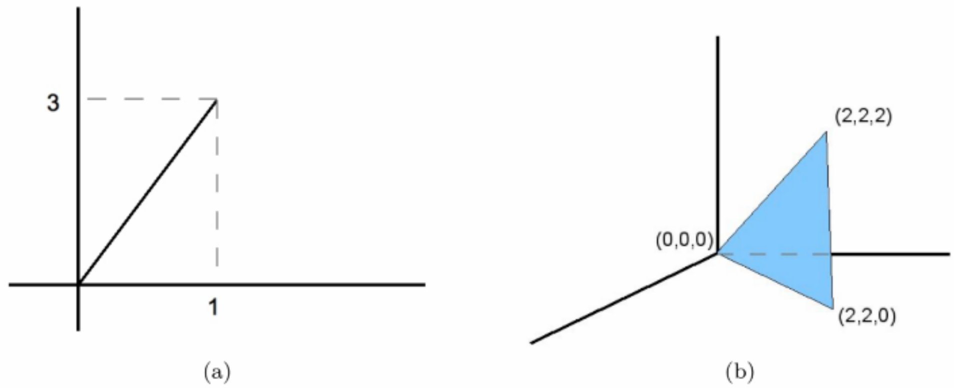

Figure 3.1: Figures for Problem 3.8.6.

Problem 3.8.6

Express the line segment in Figure 3.1(a) using a set of linear combinations. Do the same for the plane containing the triangle in Figure 3.2(b).

3.1(a): $\{\alpha[1,3], \alpha \in \mathbb{R}, 0 \leq \alpha \leq 1\}$

3.1(b): $\{\alpha[2,2,2], \alpha \in \mathbb{R}, 0 \leq \alpha \leq 1\}$

Vector spaces

Problem 3.8.7

Prove or give a counterexample: "$\{[x, y, z] : x, y, z \in \mathbb{R}, x + y + z = 1\}$ is a vector space."

This is not a vector space because it is not closed under scalar multiplication. This is because I could multiply the vector $v = [1, 0, 0]$, which is a member of the set, by 2. The resulting vector $2v = [2, 0, 0]$ is not a member of the set, and therefore it is not a vector space.

Problem 3.8.8

Prove or give a counterexample: "$\{[x, y, z] : x, y, z \in \mathbb{R}, x + y + z = 0\}$ is a vector space."

This is a vector space. The vector contains $v_\varnothing \in \mathbb{R}$, and $0 + 0 + 0 = 0$. This means for any α that we multiply v by, the vector will still be an element of the set because $\alpha(0+0+0) \overset{\checkmark}{=} 0$, and it is therefore closed under scalar multiplication. The set is also closed under vector addition because for any u, v in the set, it will always equal 0, since $u_1 + u_2 + u_3 + v_1 + v_2 + v_3 \overset{\checkmark}{=} 0$.

Problem 3.8.9

Prove or give a counterexample: "$\{[x_1, x_2, x_3, x_4, x_5] : x_1, x_2, x_3, x_4, x_5 \in \mathbb{R}, x_2 = 0 \text{ or } x_5 = 0\}$ is a vector space."

This is not a vector space. If we take any vectors u and v which are both members of the set, it is possible that $u + v \notin \{x_2 = 0 \text{ or } x_5 = 0\}$. An example of this is $v = [3, 0, 5, 1, 8]$ and $v = [-2, 3, 4, 9, 0]$. Although both u and v are members of the set, $u + v$ is not a member of the set because neither the second or fifth element are zero, which means the set is not closed under vector addition.

Problem 3.8.10

Explain your answers.
1. Let V be the set of 5-vectors over $GF(2)$ that have an even number of 1's. Is V a vector space?

V is a vector space. $\forall v \in GF(2), 0v = v_\varnothing$, which has an even number of 1's, and $1v = v$, which also has an even number of 1's per the problem; therefore, it is closed under scalar multiplication. v is also closed under vector addition because two vectors u and v in the set will always have the same number of 1's modulo 2.

2. Let V be the set of 5-vectors over $GF(2)$ that have an odd number of 1's. Is V a vector space?

Any set over $GF(2)$ with an odd number of 1's is NOT closed under scalar multiplication and thus cannot be a vector space. This is because $\forall v \in GF(2), 0v = [0, 0, 0, 0, 0]$, which has an even number of 1's (no 1's at all).