GSMST

Applications of Linear Algebra in Programming

# Chapter 1 Assignment


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Infinite Possibilities

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## 1.7 Problems

### Problem 1.7.1:

my_filter(L, num)
*input:* list of numbers and a positive integer
*output:* list of numbers not containing a multiple of num
*example:* given list = [1, 2, 4, 5, 7] and num = 2, return [1, 5, 7]

```
def my_filter(L, num): return [x for x in L if x % num != 0]
print(my_filter([1, 2, 4, 5, 7], 2))
```

```
[1, 5, 7]
```

### Problem 1.7.2:

my_lists(L)
*input:* list L of non-negative integers
*output:* a list of lists: for every element in $x$ in L create a list containing 1, 2,. . . ,$x$
*example:* given [0] return [[]]

```
def my_lists(L): return [[x for x in range(1, y+1)] for y in L]
print(my_lists([0]))
```

```
[[]]
```

### Problem 1.7.3:

my_function_composition(f, g)
*input:* two functions $f$ and $g$, represented by dictionaries, such that $g \circ f$ exists
*output:* dictionary that represents the function $g \circ f$
*example:* given $f$ = {0:'a', 1:'b'} and $g$ = {'a':'apple', 'b':'banana'}, return {0:'apple', 1:'banana}

```
def my_function_composition(f, g): return {x:g[f[x]] for x in f}
print(my_function_composition({0:'a', 1:'b'}, {'a':'apple',
 ↪  'b':'banana'}))
```

```
{0: 'apple', 1: 'banana'}
```

### Problem 1.7.4:

mySum(L)
*input:* list of numbers
*output:* sum of numbers in the list

```
def mySum(L):
    current = 0
    for i in L:
        current += i
    return current
print(mySum([1, 2, 3, 4]))
```

```
10
```

### Problem 1.7.5:

myProduct(L)
*input:* list of numbers
*output:* product of numbers in the list

```
def myProduct(L):
    current = 1
    for i in L:
        current *= i
    return current
print(myProduct([1, 2, 3, 4]))
```

```
24
```

### Problem 1.7.6

myMin(L)
*input:* list of numbers
*output:* minimum number in list

```
import sys
def myMin(L):
    current = sys.maxsize
```

```
    for i in L:
        if i < current:
            current = i
    return current
print(myMin([1, 2, 3, 4]))
```

```
1
```

## Problem 1.7.7

myConcat(L)
*input:* list of strings
*output:* concatenation of all the strings in the L

```
def myConcat(L):
    current = ''
    for i in L:
        current += current.join(i)
    return current
print(myConcat(['a', 'b', 'c', 'd']))
```

```
abcd
```

## Problem 1.7.8

myUnion(L)
*input:* list of sets *output:* the union of all sets in L

```
def myUnion(L):
    current = set()
    for i in L:
        for j in i:
            current.add(j)
    return current
L = [{1, 2, 3}, {2, 3, 4}, {3, 4, 5}]
print(myUnion(L))
```

```
{1, 2, 3, 4, 5}
```

### Problem 1.7.9

Keeping in mind the comments above, what should be the value of each of the following?
1. The sum of the numbers in an empty set
2. The product of the numbers in an empty set
3. The minimum of the numbers in an empty set
4. The concatenation of an empty list of strings
5. The union of an empty list of sets

What goes wrong when we try to apply this reasoning to define the intersection of an empty list of sets?

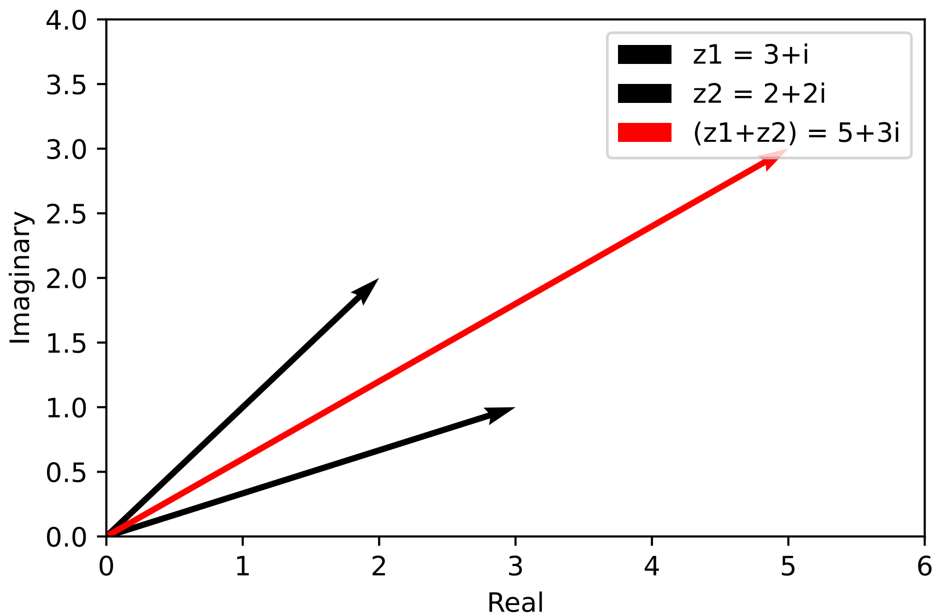
The sum of the numbers in an empty set should be **0**.
The product of the numbers in an empty set should be **1**.
The minimum of the numbers in an empty set should be $\infty$.
The concatenation of an empty list of strings should be **an empty string**.
The union of an empty list of sets should be **an empty set**.
The problem that occurs when we try to apply this reasoning to define the intersection of an empty list of sets is that both sets are empty, so there's nothing to intersect.

### Problem 1.7.10

Each of the following problems asks for the sum of two complex numbers. For each, write the solution and illustrate it with a diagram like that of Figure 1.1. The arrows you draw should (roughly) correspond to the vectors being added.
a. $(3 + 1\mathbf{i}) + (2 + 2\mathbf{i})$

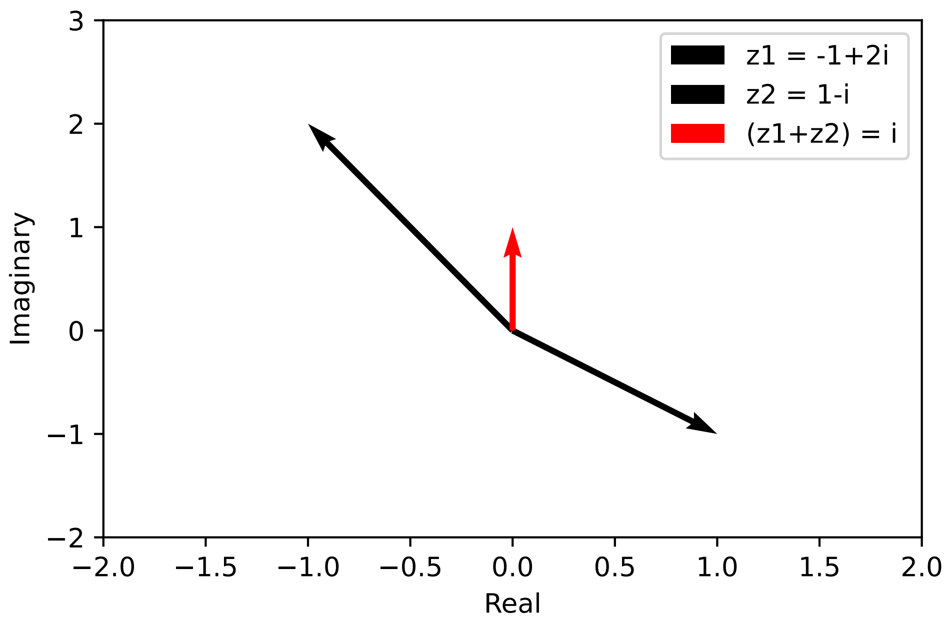
```
import matplotlib.pyplot as plt

# Create a list of the two complex numbers
z1 = 3 + 1j
z2 = 2 + 2j

# Plot the complex numbers as vectors on the complex plane
fig, ax = plt.subplots()
ax.quiver(0, 0, [z1.real], [z1.imag], angles='xy', scale_units='xy', scale=1)
ax.quiver(0, 0, [z2.real], [z2.imag], angles='xy', scale_units='xy', scale=1)

```

```python
# Add a legend and axis labels
ax.legend(['z1 = 3+i', 'z2 = 2+2i'])
ax.set_xlabel('Real')
ax.set_ylabel('Imaginary')

# Add the sum of the complex numbers
z_sum = z1 + z2
ax.quiver(0, 0, [z_sum.real], [z_sum.imag], angles='xy', scale_units='xy', scale=1, color='r')
ax.legend(['z1 = 3+i', 'z2 = 2+2i','(z1+z2) = 5+3i'])

# Set the range of the x-axis and y-axis
ax.set_xlim(0, 6)
ax.set_ylim(0, 4)

# Show the plot
plt.show()
```




b. $(-1 + 2\mathbf{i}) + (1 - 1\mathbf{i})$

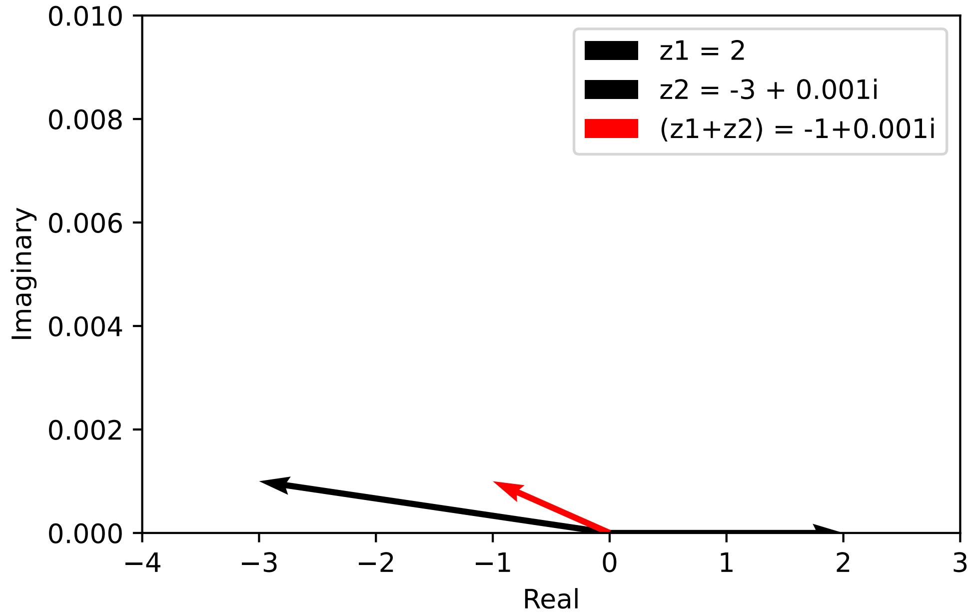
```python
import matplotlib.pyplot as plt

# Create a list of the two complex numbers
z1 = -1 + 2j
z2 = 1 - 1j

# Plot the complex numbers as vectors on the complex plane
fig, ax = plt.subplots()
ax.quiver(0, 0, [z1.real], [z1.imag], angles='xy', scale_units='xy', scale=1)
ax.quiver(0, 0, [z2.real], [z2.imag], angles='xy', scale_units='xy', scale=1)

# Add a legend and axis labels
ax.legend(['z1 = -1+2i', 'z2 = 1-i'])
ax.set_xlabel('Real')
ax.set_ylabel('Imaginary')

# Add the sum of the complex numbers
z_sum = z1 + z2
ax.quiver(0, 0, [z_sum.real], [z_sum.imag], angles='xy', scale_units='xy', scale=1, color='r')
ax.legend(['z1 = -1+2i', 'z2 = 1-i','(z1+z2) = i'])

# Set the range of the x-axis and y-axis
ax.set_xlim(-2, 2)
ax.set_ylim(-2, 3)

# Show the plot
plt.show()
```




c. $(2 + 0\mathbf{i}) + (-3 + .001\mathbf{i})$

```python
import matplotlib.pyplot as plt

# Create a list of the two complex numbers
z1 = 2
z2 = -3 + 0.001j

# Plot the complex numbers as vectors on the complex plane
fig, ax = plt.subplots()
ax.quiver(0, 0, [z1.real], [z1.imag], angles='xy', scale_units='xy',
    scale=1)
ax.quiver(0, 0, [z2.real], [z2.imag], angles='xy', scale_units='xy',
    scale=1)

# Add a legend and axis labels
ax.legend(['z1 = 2', 'z2 = -3+0.001i'])
ax.set_xlabel('Real')
ax.set_ylabel('Imaginary')

# Add the sum of the complex numbers
z_sum = z1 + z2
```

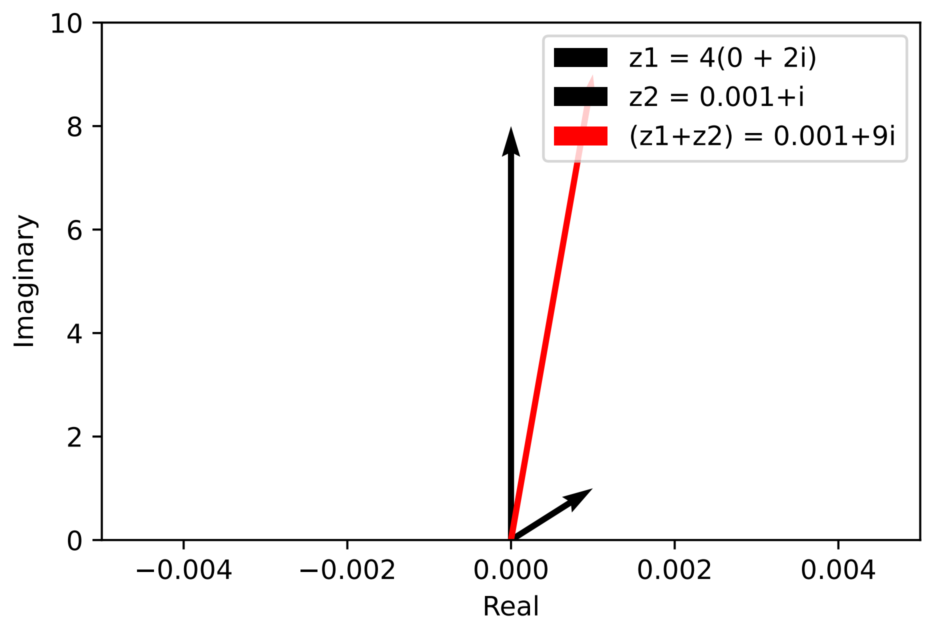
```
ax.quiver(0, 0, [z_sum.real], [z_sum.imag], angles='xy', scale_units='xy', scale=1, color='r')
ax.legend(['z1 = 2', 'z2 = -3 + 0.001i','(z1+z2) = -1+0.001i'])

# Set the range of the x-axis and y-axis
ax.set_xlim(-4, 3)
ax.set_ylim(0, 0.01)

# Show the plot
plt.show()
```




d. $4(0 + 2\mathbf{i}) + (.001 + 1\mathbf{i})$

```
import matplotlib.pyplot as plt

# Create a list of the two complex numbers
z1 = 8j
z2 = 0.001 + 1j

# Plot the complex numbers as vectors on the complex plane
fig, ax = plt.subplots()
```

```
ax.quiver(0, 0, [z1.real], [z1.imag], angles='xy', scale_units='xy',
 ↪  scale=1)
ax.quiver(0, 0, [z2.real], [z2.imag], angles='xy', scale_units='xy',
 ↪  scale=1)

# Add a legend and axis labels
ax.legend(['z1 = 4(0 + 2i)', 'z2 = 0.001+i'])
ax.set_xlabel('Real')
ax.set_ylabel('Imaginary')

# Add the sum of the complex numbers
z_sum = z1 + z2
ax.quiver(0, 0, [z_sum.real], [z_sum.imag], angles='xy',
 ↪  scale_units='xy', scale=1, color='r')
ax.legend(['z1 = 4(0 + 2i)', 'z2 = 0.001+i','(z1+z2) = 0.001+9i'])

# Set the range of the x-axis and y-axis
ax.set_xlim(-0.005, 0.005)
ax.set_ylim(0, 10)

# Show the plot
plt.show()
```



## Problem 1.7.11

Use the First Rule of Exponentiation (Section 1.4.9) to express the product of two exponentials as a single exponential. For example, $e^{(\pi/4)\mathbf{i}}e^{(\pi/4)\mathbf{i}} = e^{(\pi/2)\mathbf{i}}$.
a. $e^{1\mathbf{i}}e^{2\mathbf{i}} = e^{-2}$
b. $e^{(\pi/4)\mathbf{i}}e^{(2\pi/3)\mathbf{i}} = e^{-\dfrac{\pi^2}{6}}$
c. $e^{-(\pi/4)\mathbf{i}}e^{(2\pi/3)\mathbf{i}} = e^{\dfrac{\pi^2}{6}}$

## Problem 1.7.12

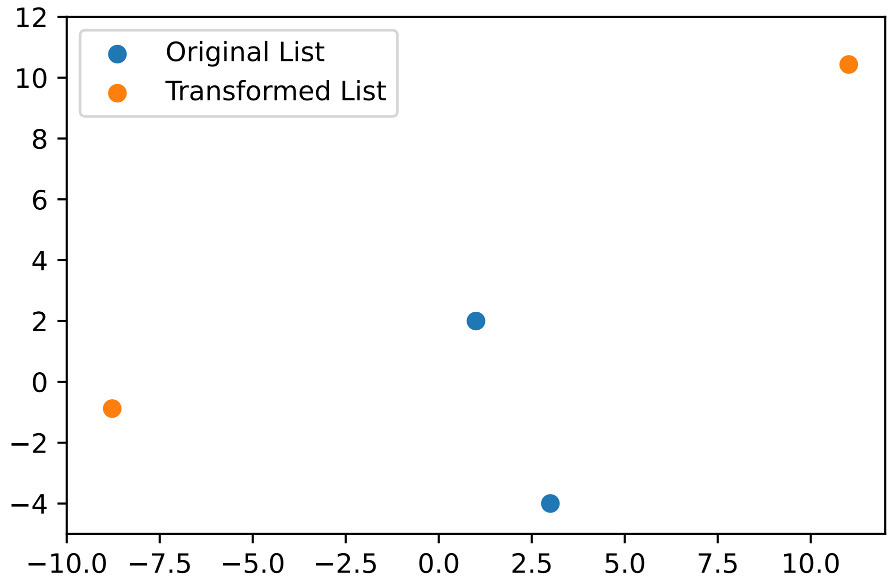
Write a procedure transform(a, b, L) with the following spec:
- *input:* complex numbers $a$ and $b$, and a list $L$ of complex numbers
- *output:* the list of complex numbers obtained by applying $f(z) = \mathrm{a}z + b$ to each complex number in $L$

Next, for each of the following problems, explain which value to choose for a and b in order to achieve the specified transformation. If there is no way to achieve the transformation, explain.
a. Translate $z$ one unit up and one unit to the right, then rotate ninety degrees clockwise, then scale by two.
b. Scale the real part by two and the imaginary part by three, then rotate by forty-five degrees counterclockwise, and then translate down two units and left three units.

a. There is no way to achieve this transformation because the scaling and rotation will occur before the translation due to order of operations.
b. This is possible by choosing $a = (2 + 3i)e^{(\frac{\pi}{4})i}$ and $b = -1 - 3i$, as shown below.

```
from math import e, pi
import matplotlib.pyplot as plt
def transform(a, b, L): return [a*z + b for z in L]
L=[1+2j, 3-4j]
transformed = transform((2+3j)*(e**((pi/4)*1j)), -1-3j, L)
def plot_complex_points(complex_points, xlim=(0, 10), ylim=(0, 10)):
    plt.scatter([p.real for p in complex_points], [p.imag for p in
    complex_points])
    plt.xlim(xlim)
    plt.ylim(ylim)
plot_complex_points(L, xlim=(-15, 12), ylim=(-5, 15))
plot_complex_points(transformed, xlim=(-10, 12), ylim=(-5, 12))
plt.legend(['Original List', 'Transformed List'])
```

```
<matplotlib.legend.Legend at 0x1d991bcbd00>
```




### Problem 1.7.13

For each of the following problems, calculate the answer over $GF(2)$.
a. $1+1+1+0$
b. $1\cdot 1+0\cdot 1+0\cdot 0+1\cdot 1$ c. $(1+1+1)\cdot(1+1+1+1)$

a. $1+1+1+0$
$=1$

b. $1\cdot 1+0\cdot 1+0\cdot 0+1\cdot 1 \quad =1+0+0+1$
$=0$

c. $(1+1+1)\cdot(1+1+1+1)$
$=1\cdot 0=0$

### Problem 1.7.14

Copy the example network used in Section 1.5.2. Suppose the bits that need to be transmitted in a given moment are $b_1 = 1$ and $b_2 = 1$. Label each link of the network with the bit transmitted across it according to the network-coding scheme. Show how the customer nodes $c$ and $d$ can recover $b_1$ and $b_2$.