GSMST

Applications of Linear Algebra in Programming

Spanday 1

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Infinite Possibilities

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Let $u = \begin{bmatrix} 2 \\ -3 \\ 2 \end{bmatrix}$ **and** $A = \begin{bmatrix} 5 & 8 & 7 \\ 0 & 1 & -1 \\ 1 & 3 & 0 \end{bmatrix} = [\boldsymbol{a}_1, \boldsymbol{a}_2, \boldsymbol{a}_3]$**. Is** u **spanned by the columns of** A**?**

We need to see if there is a linear combination that exists for $A \overset{\checkmark}{=} \boldsymbol{u}$. Therefore, we use the following augemented matrix:

$$[A|\boldsymbol{u}] = \left[\begin{array}{ccc|c} 5 & 8 & 7 & 2 \\ 0 & 1 & -1 & -3 \\ 1 & 3 & 0 & -2 \end{array}\right] \begin{array}{c} \xrightarrow{R_3} \\ \\ \xrightarrow{R_1} \end{array} \left[\begin{array}{ccc|c} 1 & 3 & 0 & -2 \\ 0 & 1 & -1 & -3 \\ 5 & 8 & 7 & 2 \end{array}\right]$$

$$\xrightarrow{-5R_1+R_3} \left[\begin{array}{ccc|c} 1 & 3 & 0 & 2 \\ 0 & 1 & -1 & -3 \\ 0 & -7 & 7 & -8 \end{array}\right] \xrightarrow{7R_2+R_3} \left[\begin{array}{ccc|c} 1 & 3 & 0 & 2 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 0 & -29 \end{array}\right]$$

Since $0 \neq -29$, there is **no solution**. Therefore, $\boldsymbol{u}$ isn't spanned by the columns of A.

Let $B = \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 1 & 2 & -3 & 7 \\ -2 & -8 & 2 & -1 \end{bmatrix}$ Do the columns of B span $\mathbb{R}^4$? Does the equation $Bx = \boldsymbol{y}$ have a solution for each $\boldsymbol{y}$ in $\mathbb{R}^4$?

This question is surprisingly trivial. Just put B into row echelon form and see if each row contains a pivot:

$$B = \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 1 & 2 & -3 & 7 \\ -2 & -8 & 2 & -1 \end{bmatrix} \xrightarrow{-R_1+R_3} \xrightarrow{2R_1+R_4} \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 0 & -1 & -1 & 5 \\ 0 & -2 & -2 & 3 \end{bmatrix} \xrightarrow{R_2+R_3} \xrightarrow{2R_2+R_4} \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix}$$

$$\xrightarrow{R_4} \xrightarrow{R_3} \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 0 & 0 & 0 & -7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since not every row in B_{REF} contains a pivot—that is, the last row has all zeros—we know that the equation $B\boldsymbol{x} = \boldsymbol{y}$ does not have a solution for each $\boldsymbol{y} \in \mathbb{R}^4$.

Could a set of three vectors in $\mathbb{R}^4$ span all of $\mathbb{R}^4$? Explain. What about n vectors in $\mathbb{R}^m$ if $n < m$?

A set of three vectors in $\mathbb{R}^4$ cannot span all of $\mathbb{R}^4$ because it would have at most three pivots with the last row being all zeros. This means that the columns of the set of those three vectors cannot span $\mathbb{R}^4$.

Describe all solutions of $Ax = 0$ in parametric vector form where A is row equivalent to the matrix $U = \begin{bmatrix} 1 & -2 & -9 & 5 \\ 0 & 1 & 2 & -6 \end{bmatrix}$.

This question basically wants us to write $\boldsymbol{Ax} = \mathbf{0}$ as a vector-valued equation. This basically just means we need to find the solutions of $\boldsymbol{x}$ that, when multiplied with A, equal zero. It's row reduction time:

$$[A|0] = \left[\begin{array}{cccc|c} 1 & -2 & -9 & 5 & 0 \\ 0 & 1 & 2 & -6 & 0 \end{array}\right] \xrightarrow{2R_2+R1} \left[\begin{array}{cccc|c} 1 & 0 & -5 & -7 & 0 \\ 0 & 1 & 2 & -6 & 0 \end{array}\right]$$

Transposing the resultant matrix and treating each of the columns as vectors gives us the solution in vector-valued form, which is:

$$\boldsymbol{x}z = \begin{bmatrix} 5 \\ -2 \\ 1 \\ 0 \end{bmatrix} s + \begin{bmatrix} 7 \\ 6 \\ 0 \\ 1 \end{bmatrix} t, \{s, t \in \mathbb{R}\}$$

Suppose $Ax = \boldsymbol{b}$ has a solution. Explain why the solution is unique if and only if $Ax = 0$ has only the trivial solution.

A homogenous system of linear equations either has a single solution or infinite solutions (the zero vector). Per the problem, we know that this homogenous system has a single solution. This means that A has a pivot in each column and therefore $A\boldsymbol{x} = \boldsymbol{b}$ must also only have a single, unique solution.

For this problem you should use a computer algebra program like Wolfram Alpha to find the solutions. Unlike in other problems you do not have to show your steps.

Construct the exchange table for the given economy involving the sectors Agriculture (A), Energy (E), Manufacturing (M), Transportation (T). Here is the distribution of the output of goods in percent %:

To \ From	A	E	M	T
A	65	30	30	20
E	10	10	15	10
M	25	35	15	30
T	0	25	40	40

Looks like the solution for this problem has already been given to us. Thanks Ms. Stiffler!

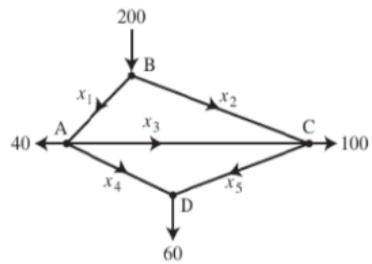
Find a set of equilibrium prices for the economy.

$$\begin{array}{ccccccccc} A & = & 0.65p_A & + & 0.3p_E & + & 0.3p_M & + & 0.2p_T \\ E & = & 0.1p_A & + & 0.1p_E & + & 0.15p_M & + & 0.1p_T \\ M & = & 0.25p_A & + & 0.35p_E & + & 0.15p_M & + & 0.3p_T \\ T & = & & + & 0.25p_E & + & 0.4p_M & + & 0.4p_T \end{array}$$

Using Wolfram Alpha to solve this system of linear equations gives us the following solution:

$$(p_A, p_E, p_M, p_T) = (2.03, 0.53, 1.17, 1)s, \{s \in \mathbb{R}\}$$

1.6.12: We want to analyze the traffic flow of the freeway network below where the numbers are given by cars per minute.



Find the flow in each street of the freeway network.

We can create an augmented matrix reperesenting a system of linear equations with the variables x_1, x_2, x_3, x_4, x_5 and their varying outputs, respectively:

$$\left[\begin{array}{ccccc|c} 1 & 0 & -1 & -1 & 0 & 40 \\ 1 & 1 & 0 & 0 & 0 & 200 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{array}\right]$$

Notice how both x_3 and x_5 are free systems! And since the system has four pivots, it can actually be reduced and solved into:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 100 \\ 100 \\ 0 \\ 60 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} -1 \\ 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

Describe the traffic pattern if the road whose flow is x_4 is closed.

Per the solution from the last part, $x_5 = 60$ if $x_4 = 0$. This means the solution simplifies further to:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 40 \\ 160 \\ 0 \\ 0 \\ 60 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} x_3$$