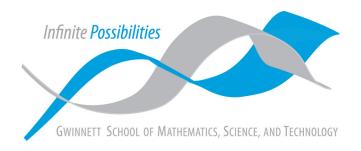
# Gwinnett School of Math, Science, and Technology

# **Multivariable Calculus Yearlong Notes**

Anish Goyal 1st Period Donny Thurston Educator

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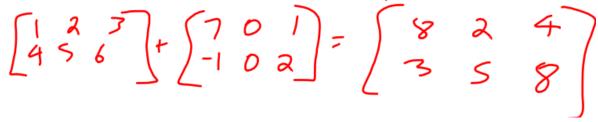
### 1 Systems of Linear Equations and Matrices

#### 1.1 Matrix Operations

- Matrix operations are given as: rows x columns
- ullet Two matrices are equal  $\iff$  they have the same dimensions and values

#### 1.1.1 Addition & Subtraction

Two matrices can be added/subtracted  $\iff$  they have the same dimensions.



#### 1.1.2 Scalar Multiplication

• Scalar multiplication is defined as multiplying each element of a matrix by a number

$$3\begin{bmatrix} 2 & 1 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 3 \\ 15 & 6 \end{bmatrix}$$

#### 1.1.3 Matrix Multiplication

- We can **only** multiply an (m x n) by (n x p) matrix.
- The resulting matrix will be (m x p)

#### 1.1.4 Examples

1.

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \cdot 1 + 2 \cdot 3 & 1 \cdot 2 + 2 \cdot 4 \\ 3 \cdot 1 + 4 \cdot 3 & 3 \cdot 2 + 4 \cdot 4 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$$

2.

$$\begin{bmatrix} 2 & -3 \\ 5 & 0 \\ -2 & 4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \cdot (-1) + (-3) \cdot 3 \\ 5 \cdot (-1) + 0 \cdot 3 \\ -2 \cdot (-1) + 4 \cdot 3 \\ 1 \cdot (-1) + 2 \cdot 3 \end{bmatrix}$$

$$= \begin{bmatrix} -11 \\ -5 \\ 14 \\ 5 \end{bmatrix}$$

3.

$$\begin{bmatrix} 4 & 5 & -1 \end{bmatrix} \begin{bmatrix} 8 \\ 0 \\ 2 \end{bmatrix}$$
$$= \begin{bmatrix} 4 \cdot 8 + 5 \cdot 0 + (-1) \cdot 2 \end{bmatrix}$$
$$= \begin{bmatrix} 30 \end{bmatrix}$$

## 1.2 Transpose of a Matrix

The transpose of an  $(m \times n)$  matrix is the  $(n \times m)$  matrix where the rows and columns are swapped.

If 
$$B = \begin{bmatrix} 4 & 2 \\ -1 & 0 \\ 3 & 5 \end{bmatrix}$$
 ,  $B^T = \begin{bmatrix} 4 & -1 & 3 \\ 2 & 0 & 5 \end{bmatrix}$ 

$$\begin{split} B \cdot B^T &= \begin{bmatrix} 4 & 2 \\ -1 & 0 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} 4 & -1 & 3 \\ 2 & 0 & 5 \end{bmatrix} \\ &= \begin{bmatrix} 4 \cdot 4 + 2 \cdot 2 & 4 \cdot (-1) + 2 \cdot 0 & 4 \cdot 3 + 2 \cdot 5 \\ (-1) \cdot 4 + 0 \cdot 2 & (-1) \cdot (-1) + 0 \cdot 0 & (-1) \cdot 3 + 0 \cdot 5 \\ 3 \cdot 4 + 5 \cdot 2 & 3 \cdot (-1) + 5 \cdot 0 & 3 \cdot 3 + 5 \cdot 5 \end{bmatrix} \\ &= \begin{bmatrix} 20 & -4 & 22 \\ -4 & 1 & -3 \\ 22 & -3 & 34 \end{bmatrix} \end{split}$$

- The transpose of a matrix is **always** multiplicative with the original.
- There is also a main diagonal that is the diagonal from the top left to the bottom right.
- ullet The **trace** of a square matrix A is equal to the sum of all the elements on the main diagonal: tr(A)

#### 1.2.1 Transpose Matrix Properties

- $(A^T)^T = A$
- $(A + B)^T = A^T + B^T$   $(A B)^T = A^T B^T$   $(kA)^T = kA^T$   $(AB)^T = B^TA^T$