Gwinnett School of Math, Science, and Technology

Multivariable Calculus Yearlong Notes

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1 Systems of Linear Equations and Matrices

1.1 Matrix Operations

- Matrix operations are given as: rows x columns
- Two matrices are equal $\iff$ they have the same dimensions and values

1.1.1 Addition & Subtraction

Two matrices can be added/subtracted $\iff$ they have the same dimensions.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 1 \\ -1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 2 & 4 \\ 3 & 5 & 8 \end{bmatrix}$$

1.1.2 Scalar Multiplication

- Scalar multiplication is defined as multiplying each element of a matrix by a number

$$3\begin{bmatrix} 2 & 1 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 3 \\ 15 & 6 \end{bmatrix}$$

1.1.3 Matrix Multiplication

- We can **only** multiply an (m x n) by (n x p) matrix.
- The resulting matrix will be (m x p)

1.1.4 Examples

1.

$$\begin{aligned}
&\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\\
&= \begin{bmatrix}1\cdot1+2\cdot3 & 1\cdot2+2\cdot4\\3\cdot1+4\cdot3 & 3\cdot2+4\cdot4\end{bmatrix}\\
&= \begin{bmatrix}7 & 10\\15 & 22\end{bmatrix}
\end{aligned}$$

2.

$$\begin{aligned}
&\begin{bmatrix}2 & -3\\5 & 0\\-2 & 4\\1 & 2\end{bmatrix}\begin{bmatrix}-1\\3\end{bmatrix}\\
&= \begin{bmatrix}2\cdot(-1)+(-3)\cdot3\\5\cdot(-1)+0\cdot3\\-2\cdot(-1)+4\cdot3\\1\cdot(-1)+2\cdot3\end{bmatrix}\\
&= \begin{bmatrix}-11\\-5\\14\\5\end{bmatrix}
\end{aligned}$$

3.

$$\begin{aligned}
&\begin{bmatrix}4 & 5 & -1\end{bmatrix}\begin{bmatrix}8\\0\\2\end{bmatrix}\\
&= [4\cdot8+5\cdot0+(-1)\cdot2]\\
&= [30]
\end{aligned}$$

1.2 Transpose of a Matrix

The transpose of an (m x n) matrix is the (n x m) matrix where the rows and columns are swapped.

If $B = \begin{bmatrix}4 & 2\\-1 & 0\\3 & 5\end{bmatrix}$, $B^T = \begin{bmatrix}4 & -1 & 3\\2 & 0 & 5\end{bmatrix}$

$$
\begin{aligned}
B \cdot B^T &= \begin{bmatrix} 4 & 2 \\ -1 & 0 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} 4 & -1 & 3 \\ 2 & 0 & 5 \end{bmatrix} \\
&= \begin{bmatrix} 4 \cdot 4 + 2 \cdot 2 & 4 \cdot (-1) + 2 \cdot 0 & 4 \cdot 3 + 2 \cdot 5 \\ (-1) \cdot 4 + 0 \cdot 2 & (-1) \cdot (-1) + 0 \cdot 0 & (-1) \cdot 3 + 0 \cdot 5 \\ 3 \cdot 4 + 5 \cdot 2 & 3 \cdot (-1) + 5 \cdot 0 & 3 \cdot 3 + 5 \cdot 5 \end{bmatrix} \\
&= \begin{bmatrix} 20 & -4 & 22 \\ -4 & 1 & -3 \\ 22 & -3 & 34 \end{bmatrix}
\end{aligned}
$$

- The transpose of a matrix is **always** multiplicative with the original.
- There is also a **main diagonal** that is the diagonal from the top left to the bottom right.
- The **trace** of a square matrix A is equal to the sum of all the elements on the main diagonal: tr(A)

1.2.1 Transpose Matrix Properties

- $(A^T)^T = A$
- $(A + B)^T = A^T + B^T$
- $(A - B)^T = A^T - B^T$
- $(kA)^T = kA^T$
- $(AB)^T = B^T A^T$