Calculus Objectives Presentation

Intersections of vectors and vector-valued equations

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Subsection 1

Terminology

Definitions

- A **vector** is a quantity that has both a magnitude and a direction.
- The **intersection** of two vectors is the point where they meet.

Definitions

- A **vector** is a quantity that has both a magnitude and a direction.
- The **intersection** of two vectors is the point where they meet.

Subsection 2

Objective Defined

What is Objective 24?

Objective Definition

In this objective, you must be able to...
"Find points of intersection of intersecting vectors."

But what does that mean?

- Well, in baby terms...
  - We want to find where any number of vectors cross.

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Subsection 3

Modus Operandi

Formula

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Let $\vec{r_1}(t)$ and $\vec{r_2}(s)$ be two vector-valued functions in the form:

$$\vec{r_1}(t) = r_1 + t\vec{v_1}$$
$$\vec{r_2}(s) = r_2 + s\vec{v_2}$$

Then their point of intersection can be found by solving for t and s such that $\vec{r_1}(t) = \vec{r_2}(s)$.

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Formula

Formula (cont.)

Where:

- $\vec{r_1}(t)$ and $\vec{r_2}(s)$ are the vector-valued functions of the two lines
- t and s are the parameters used to determine the point of intersection
- r_1 and r_2 are the starting points of the lines
- $\vec{v1}$ and $\vec{v2}$ are the direction vectors of the lines

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Subsection 4

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- This works because the point of intersection must be on both lines simultaneously.

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Subsection 5

Example

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Problem

Two lines

$$\mathbf{v} = \begin{pmatrix} 7 \\ -3 \\ 1 \end{pmatrix} + \begin{pmatrix} -2 \\ 5 \\ 1 \end{pmatrix} t \text{ and } \mathbf{w} = \begin{pmatrix} 8 \\ -1 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ -4 \\ 0 \end{pmatrix} s$$

intersect at a point. Find the coordinates of the point of intersection.

Example

First, we need to find the vector equations for the lines. This can be done by adding each parameter to the specified coordinate:

$$v = \langle 7 - 2t, -3 + 5t, 1 + t \rangle$$
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It follows that if the two vectors are equal, their components must be equal:

$$\begin{cases} -2t + 7 = 8 + s \\ 5t - 3 = -1 - 4s \\ t + 1 = -1 \end{cases}$$

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Example

So, let's solve for a parameter (in this case, t):

$$1 + t = -1 \implies t = -2$$

Now that we know t, we can substitute it into the first vector equation to find the point of intersection.

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So, let's substitute $t = -2$ into v:

$$v = \langle 7 - 2(-2), -3 + 5(-2), 1 + (-2) \rangle = \begin{pmatrix} 11 \\ -13 \\ -1 \end{pmatrix}$$

Therefore, the point of intersection is $(11, -13, -1)$.

Alternatively, we could have substituted s into w. Always make sure you are plugging in the correct parameter into the correct vector equation.

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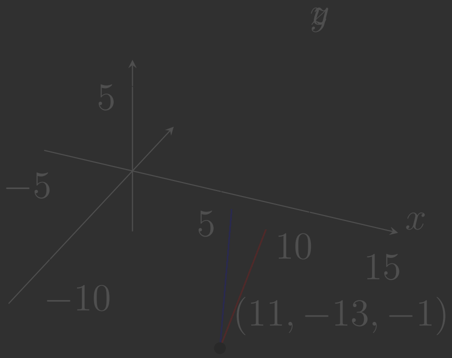

A substandard visual

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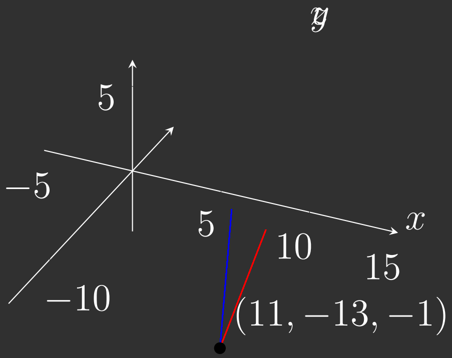

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Subsection 6

Gaussian Elimination

Gaussian Elimination

What is Gaussian Elimination?

- A method of solving a system of linear equations.
- A method of reducing a system of equations to a triangular matrix.
- A method of reducing a system of equations to a diagonal matrix.

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Gaussian Elimination

Why is Gaussian Elimination useful for this objective?

- Currently, we've only been dealing with the intersections of 3-dimensional vectors.
- But what if we wanted to find the intersection of an n-dimensional vector $\vec{v}$ such that $\vec{v}$ is spanned over all real numbers ($\vec{v} \in \mathbb{R}^n$)?
- Alternatively, what if we wanted to solve for the intersection of more than two vectors?
- This would make our system of equations quite complicated, and we wouldn't be able to solve it by hand... that is, unless we use Gaussian Elimination.

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Reduced row echelon form (RREF)

Definition: To be in RREF, a matrix A must satisfy the following 4 properties (if it satisfies just the first 3 properties, it is in REF):

1. If a row does not have only zeros, then its first nonzero number is a 1. We call this a *leading* 1.
2. The rows that contain only zeros (if there are any) are at the bottom of the matrix.
3. In any consecutive rows that do not have only zeros, the leading 1 in the lower row is farther right than the leading 1 in the row above.
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Write out the augmented matrix

A system of linear equation is generally of the form

$$A\,\boldsymbol{x} = \boldsymbol{b}\,, \tag{1}$$

where $A \in M(n \times m)$ and $\boldsymbol{b} \in \mathbb{R}^n$ are given, and $\boldsymbol{x} = (x_1, \dots, x_m)^T$ is the vector of unknowns. For example, the system

$$\begin{aligned} x_2 + 2\,x_3 - x_4 &= 1 \\ x_1 + x_3 + x_4 &= 4 \\ -x_1 + x_2 - x_4 &= 2 \\ 2\,x_2 + 3\,x_3 - x_4 &= 7 \end{aligned}$$

can be written in the form (1) with...

Write out the augmented matrix

$$A = \begin{pmatrix} 0 & 1 & 2 & -1 \\ 1 & 0 & 1 & 1 \\ -1 & 1 & 0 & -1 \\ 0 & 2 & 3 & -1 \end{pmatrix}, \qquad \boldsymbol{b} = \begin{pmatrix} 1 \\ 4 \\ 2 \\ 7 \end{pmatrix}.$$

Once you have the augmented matrix, you need to perform elementary row operations to get the matrix into RREF/REF. Then, you can solve for the unknowns. However, I will not be going any more in depth into Gaussian Elimination becaue it is out of the scope for this presentation and I don't want to kill myself writing LaTeX any further.

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Subsection 7

Objective Summary

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To find the point of intersection between two vectors:

- Find the vector equations for each vector.
- Set the two vectors equal to each other.
- Solve for a parameter.
- Substitute the parameter into one of the vector equations to find the point of intersection.

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Objective 26

Subsection 2

Terminology

Definitions

- A **vector-valued equation** is an equation that expresses a vector in terms of a parameter.
- A **line** is a straight path that extends infinitely in two directions.
- **Three-dimensional space** is the space in which three coordinates are needed to specify a point.

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Subsection 3

Objective Defined

What is Objective 26?

Objective Definition

In this objective, you must be able to...
"Write the vector-valued equation of a line in three dimensions."

But what does that mean?

- Well, in baby terms...
  - We want to write an equation that tells us where a line is in 3D space.

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Subsection 4

Modus Operandi

Formula

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A line in three dimensions can be defined by a vector-valued equation

$$\vec{r}(t) = r_0 + t\vec{v}$$

Formula

Formula (cont.)

Where:

- $\vec{r}(t)$ is the vector-valued equation of the line.
- r_0 is the starting point of the line.
- t is the parameter used to determine the position of any point on the line.
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Subsection 5

Explanation

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So, what was all of that?

- By using a vector-valued equation with a parameter t, we can represent any point on the line by substituting different values of t into the equation.
- The direction vector $\vec{v}$ determines the slope of the line, and the starting point a determines where the line begins.

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Subsection 6

Example

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Problem

What is the vector equation of a line that passes through the points (2, 4, -3) and (4, 1, 5)?

Example

First, we need to find the direction vector. Let point (2, 4, -3) equal A and point equal B:

$$\begin{aligned}\vec{AB} &= \langle 4-2, 1-4, 5-(-3)\rangle \\ &= \langle 2, -3, 8\rangle\end{aligned}$$

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Now, we need to get the starting point, or r_0.
Since we chose A to be the initial point of $\vec{AB}$, we can just use the coordinates of A as r_0.

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$$r = \langle 1 + 3t, 3 - 2t, -2 + 7t \rangle$$

We could have also done it in the opposite direction as long as we set r_0 to be B and set v to be $\vec{BA}$.

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$$r = \langle 1 + 3t, 3 - 2t, -2 + 7t \rangle$$

We could have also done it in the opposite direction as long as we set r_0 to be B and set v to be $\vec{BA}$.

Example

Now, we can put it all together:

$$
\begin{aligned}
r &= r_0 + t\vec{v} \\
r_0 &= (2, 4, -3) \\
\vec{v} = \vec{AB} &= \langle 2, -3, 8 \rangle \\
r &= \langle 1 + 3t, 3 - 2t, -2 + 7t \rangle
\end{aligned}
$$

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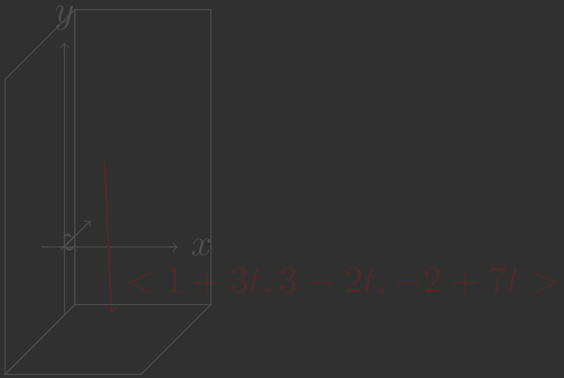
Example

A substandard visual



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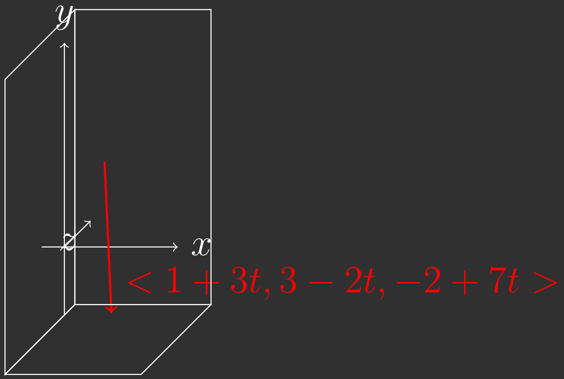

Subsection 7

Objective Summary

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To write the vector-valued equation of a line in three dimensions:

- Find the direction vector.
- Find the starting point.
- Write the vector-valued equation.

Remember: *The direction vector determines the slope of the line, and the starting point determines where the line begins.*

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Acknowledgments

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Any questions?