———Anish Goyal and Zachary Cobb———

Vector-Valued Equations

1. $\langle 10, 0, 3 \rangle$ is parallel to a line. (0, 7, 7) is a point on the line. What is the vector-valued equation of the line?

2. Two points $A(-7, 20, 5)$ and $B(21, 16, -8)$ lie in a line. What is the vector-valued equation?

Intersections of Vectors

3. Two vectors $\vec{r_1}$ and $\vec{r_2}$ equal $\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} t$ and $\begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix} + \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix} s$, respectively. At what point do they intersect?

4. Two lines share a point $P(5, 6, 1)$. $\vec{r_1}$ has point (-2, 7, 1) and line $\vec{r_2}$ has the point (7, 20, -16). What are the vector-valued equations of both lines and the respective scalars (t, s) that make the lines intersect?

ANSWER KEY

1. $\langle 10, 0, 3 \rangle$ is parallel to a line. $(0, 7, 7)$ is a point on the line. What is the vector-valued equation of the line?

$$r_0 = (0, 7, 7)$$

$$\vec{n} = \langle 10, 0, 3 \rangle \quad \vec{r} = r_0 + \vec{n}$$

$$\vec{r} = \begin{pmatrix} 0 \\ 7 \\ 7 \end{pmatrix} + \begin{pmatrix} 10 \\ 0 \\ 3 \end{pmatrix} t$$

2. Two points $A(-7, 20, 5)$ and $B(21, 16, -8)$ lie in a line. What is the vector-valued equation?

$$\vec{AB} = \langle 28, -4, -13 \rangle$$

$$r_0 = (-7, 20, 5)$$

$$\vec{r} = \begin{pmatrix} -7 \\ 20 \\ 5 \end{pmatrix} + \begin{pmatrix} 28 \\ -4 \\ -13 \end{pmatrix} t$$

ANSWER KEY

3.　Two vectors $\vec{r_1}$ and $\vec{r_2}$ equal $\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} t$ and $\begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix} + \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix} s$, respectively. At

what point do they intersect?

$$\vec{r_1} = (1, 2, 0) + \langle 2, -1, 1 \rangle t$$

$$\vec{r_2} = (-2, 2, 1) + \langle -3, 2, -1 \rangle s$$

$$2 - t = 2 + 2s$$

$$t = 1 - s$$

$$2 - (1 - s) = 2 + 2s$$

$$1 + s = 2 + 2s$$

$$-1 = s \therefore t = 2$$

$$\vec{r_1} = (1 + 2t, 2 - t, t)$$

$$= (1 + 2(2), 2 - 2, 2)$$

$$= (5, 0, 2)$$

ANSWER KEY

4. Two lines share a point $P(5, 6, 1)$. $\vec{r_1}$ has point (-2, 7, 1) and line $\vec{r_2}$ has the point (7, 20, -16). What are the vector-valued equations of both lines and the respective scalars (t, s) that make the lines intersect?

$$\vec{v_1} = \langle 7, -1, 0 \rangle$$

$$\vec{v_2} = \langle -2, -14, 17 \rangle$$

$$\vec{r_1} = (-2, 7, 1) + \langle 7, -1, 0 \rangle$$

$$\vec{r_2} = (7, 20, -16) + \langle -2, -14, 17 \rangle$$

$$-2 + 7t = 7 - 2s$$

$$7 - t = 20 - 18s$$

$$1 = -16 + 17s$$

$$s = 1, t = 1$$