3. Math :

1) Sigmoid function =

$$S(x) = \frac{1}{1 + e^{-x}}$$

Differentiating on both sides,

$$\frac{d}{dx}(S(x)) = \frac{(1+e^{-x})\frac{d}{dx}(1) - \frac{d}{dx}(1+e^{-x})}{(1+e^{-x})^2}$$

$$= \frac{e^{-x}}{(1+e^{-x})^2}$$

$$= \frac{(1+e^{-x})-1}{1+e^{-x}} \cdot \frac{1}{1+e^{-x}}$$

$$= \frac{1}{1+e^{-x}}\left(1 - \frac{1}{1+e^{-x}}\right)$$

$$= S(x)(1 - S(x))$$

2. Partial derivatives of f w.r.t each of its variables:

$$f(z, z, a, b) = y = \left(4ax^2 + a\right) + 3 + \sigma(z) + \left(\sigma(b)^2\right)$$

i) $\dfrac{\partial y}{\partial x} = 8ax \quad \left[\begin{array}{l} \because \text{ All the other terms except} \\ 4ax^2 = 0 \text{ and differentiation of} \\ 4ax^2 = 8ax \end{array}\right]$

ii) $\dfrac{\partial y}{\partial a} = 4x^2 + 1$

iii) $\dfrac{\partial y}{\partial z} = \sigma(z)(1 - \sigma(z)) \quad \left[\begin{array}{l} \text{The differentiation of} \\ \text{the sigma function is} \\ \text{proved in the prior} \\ \text{problem and taking} \\ \text{that gives you the} \\ \text{answer provided} \end{array}\right]$

iv) $\dfrac{\partial y}{\partial b} = 2(\sigma(b))\left(\dfrac{\partial}{\partial b}(\sigma(b))\right) \quad \left[\begin{array}{l} \text{Following the} \\ \text{chain rule of differen-} \\ \text{-tiation} \end{array}\right]$

$= 2(\sigma(b) \cdot \sigma(b)(1 - \sigma(b)))$

$= 2(\sigma(b)^2)(1 - \sigma(b))$