Battle of Hogwarts

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Battle of Hogwarts is a fictional event involving two sides - Harry and Voldemort. Various combat models are discussed here, while limiting ourselves to the scope of the fictional world of Harry Potter, to determine the outcome of this battle with various initial conditions.

INTRODUCTION

Battle of Hogwarts is set in a fictional world. Two sides - Harry and Voldemort compete with each other. Each side is further divided into two more categories: Harry has the Order of the Phoenix and the students of Hogwarts, and Voldemort has the Death Eaters along with his followers. The wizards involved have specific powers for killing and these are decided based on setting of the fictional world. We try to model the problem through two techniques - applying the differential equations of 'Guerrilla Combat Model' of Lanchester type and Cellular Automata. The system is given initial conditions in the form of the number of fighters of each category and left to evolve.

There are four categories of fighters and they are represented as - **P**, **S**, **D** and **F** for Order of the Phoenix, students, Death Eaters and followers, respectively.

MODEL 1: DIFFERENTIAL EQUATIONS

This model describes combat between two homogeneous forces. Each force is able to kill all hostile operational units that have been detected with a killing rate k. The detection rate is a limiting factor here. This might be valid in a combat situation because the detection rate signifies the locality of a fighter, the region within which a fighter is able to operate. The killing rates will depend on the strength of the fighters. Thus, there is a dependence on detection rate as well as killing rate, giving a model which might be closer to practical combat.

Assumptions

This is a simple model which models the outcome of battle between two sides. The assumptions follow below.

- There are only four type of people in the battlefield - Order of the Phoenix, students, Death Eaters and followers
- The model assumes homogeneous forces: all of them fight only with magic wands.
- Any person ideally can come in contact with any other person at any given time.
- Interactions between any two people take place with a certain probability c. This acts as the detection rate for wizards.
- The model assumes that the only limiting factor to killing of either side is the detection probability and the killing rates.
- A single Order member detects a Death Eater with probability d and a single Death eater detects an arbitrary Phoenix member with a probability p' and killing rate p_k'. The coefficient thus becomes $p'*p_k' = p$.
- A single Death Eater detects and kills a student with probability s_1. While a student may be capable of detecting a Death Eater, he is incapable of killing a Death Eater.
- A single member of the Order detects and kills a follower with probability f_1. While a student may be capable of detecting an Order member, he is incapable of killing a Order member.
- A single follower detects and kills a student with probability s_2 and a single student detects and kills a follower with probability f_2.
- Death Eaters and Order members are better skilled at magic. Hence, it is easier for them to kill students and followers respectively; these kill rates are higher.
- No new fighters join the battle within the time frame that we are observing.
- The kill rates are constant in the time frame observed.

Model

The system of equations for our model are:

$$\frac{dP}{dt} = -pPD \tag{1}$$

$$\frac{dS}{dt} = -s_1SD - s_2SF \tag{2}$$

$$\frac{dD}{dt} = -dDP \tag{3}$$

$$\frac{dF}{dt} = -f_1FP - f_2FS \tag{4}$$

where, $\boldsymbol{p}$ is the rate at which a Death Eater kills a member of the Order. s_1 and s_2 are the rates at which a Death eater and a follower kills a student, respectively. $\boldsymbol{d}$ is the rate at which a member of the order kills a Death Eater. f_1 and f_2 are the rates at which a student and a member of the Order kill a follower, respectively.

Observations

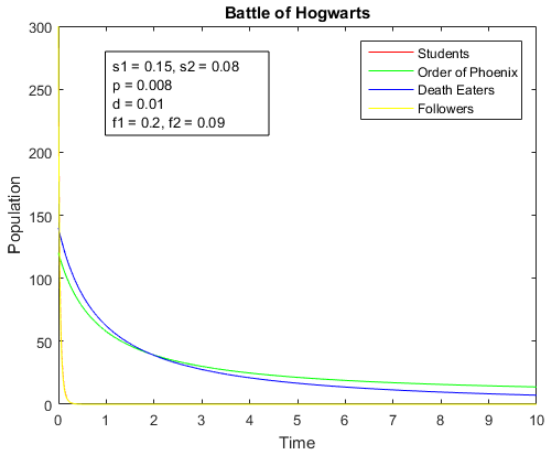

FIG. 1: Battle simulation with conditions where Harry Potter wins

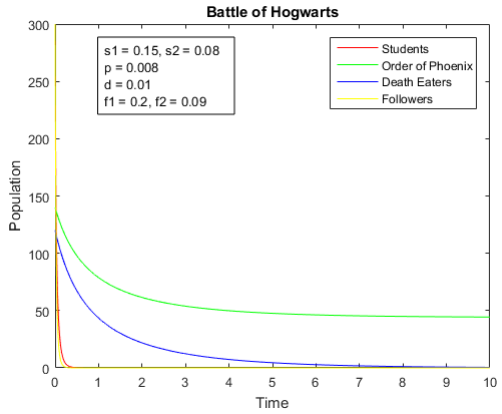

FIG. 2: Battle simulation with conditions where Voldemort wins

With this model the number of operational units cannot become negative. The outcome of the battle is decided based on the eventual number of wizards belonging to either side of the battle. Since Death Eaters and Order of Phoenix are much stronger than students and followers, the battle is won by the opposite side when either of these numbers goes to zero.

It can be seen from figure 1 that the outcome of the battle does not solely depend on the initial number of fighters but also on the killing and detection rates. Even though Death Eaters are more in number initially, Order of Phoenix win due to a higher Death Eater killing rate.

The following scenarios roughly describe the manner in which the winner of the battle is declared.

- If Pd > Dp → Harry Potter wins.
- If Pd < Dp → Voldemort wins.
- If Pd = Dp → mutual destruction.

Limitations

- With decreasing numbers of units involved in the battle, the logic of the model becomes weaker and collapses for number of units less than one.[2]
- These Differential Equations do not contain any information about the spatial distribution of armies or their movement.
- The killing rate is assumed to be constant. To be more realistic, this can be a decreasing function with time as the wizards would progressively lose power and strength.

MODEL 2: CELLULAR AUTOMATA

This model allows us to implement a protective spell, *Protego*, in addition to the killing spell *Avada Kedavra*. Since this is a spell which requires advanced magical skills, Hogwarts students and the followers would not be able to use them. Apart from that, Death Eaters do not feel the need to use this spell as they do not believe in protecting their own supporters. Only the Order members use this protective spell as they value the lives of their fellow fighters.

Another ability this model has implemented is the power to *apparate* i.e. to leave the current position and teleport to a new location. Since this is a difficult power

to master, only Death Eaters and Order members are capable of using it.

Assumptions

- There are only four type of people in the battlefield - Order of the Phoenix, students, Death Eaters and followers
- Any person can come in contact with only eight people neighbouring them.
- No new wizards join the battle in the time frame that we are observing.
- An empty cell is treated in the same way as a dead wizard/witch.
- If there is a greater number of Order of Phoenix in the neighborhood of a student, the student's life will be saved.
- Students cannot kill Death Eaters and Followers cannot kill Order members.
- The effectiveness of the killing spell is the same for all wizards.
- Each killing spell is effective only with a probability p.
- Given that the number of surrounding Order members and Death Eaters is the same, a student can be killed if the followers outnumber the neighboring students and vice versa.
- Order members and Death Eaters can apparate to new locations if all their neighboring enemies are dead.

Model

The life of a wizard who participated in the battle depends on the wizards/witches surrounding them. For each cell, the 8 neighbors are considered. If the number of enemies is higher than friends, the cell dies. Else, the cell survives.

If a cell has no more enemies surrounding it, Death Eaters and Order members relocate randomly to an empty location. They do not sit idle and apparate to a new location to continue their attack.

The model uses an absorbing boundary, with the boundary treated as empty cells.

The battle simulation begins with random locations of the wizards. This is done to minimize the effect of initial condition locations. Only the number of wizards fighting for each side affect the outcome.

Observations

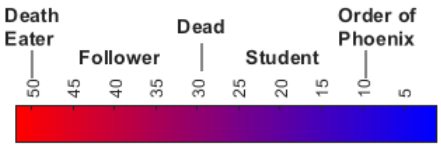

FIG. 3: Color legend for Cellular Automata simulations

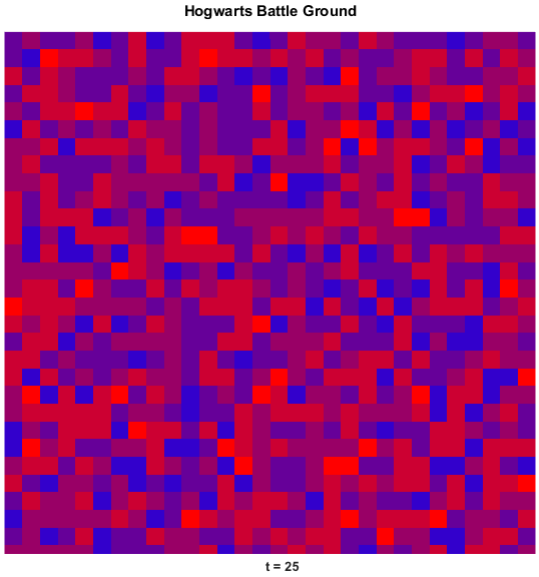

FIG. 4: Initial conditions where Harry Potter wins

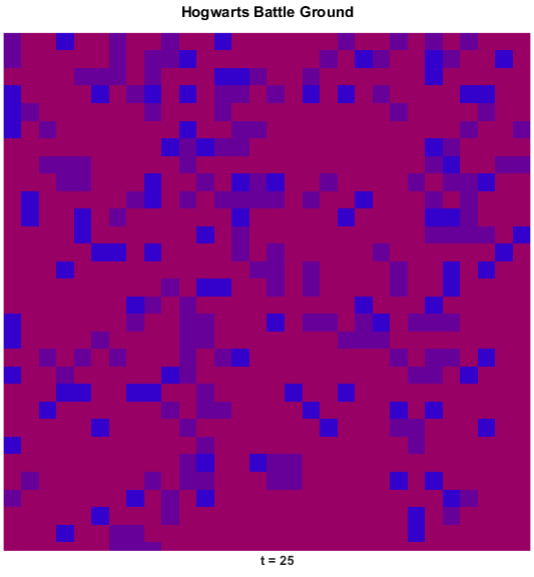

FIG. 5: Final conditions where Harry Potter wins

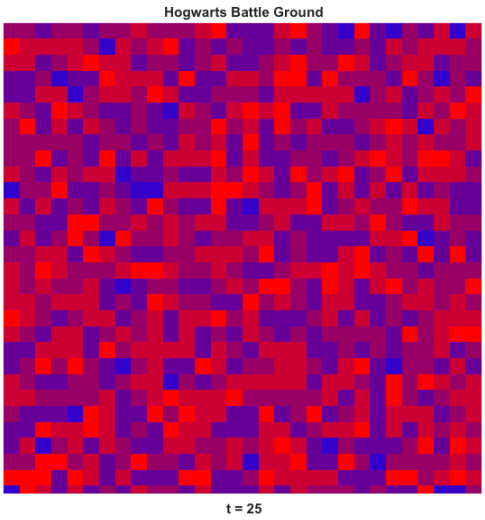

FIG. 6: Initial conditions where Voldemort wins

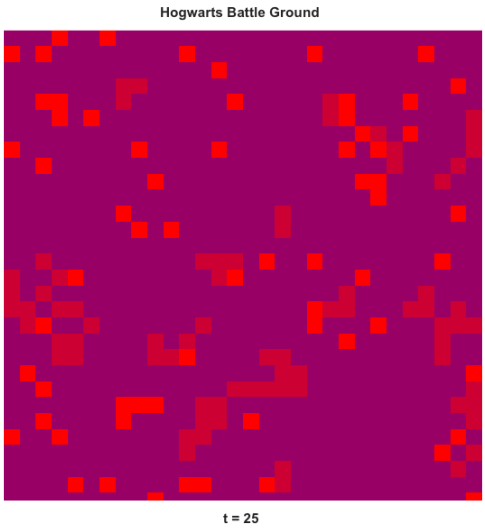

FIG. 7: Final conditions where Voldemort wins

The outcome of the battle is broadly decided on the number of wizards representing each category. Since Order members and Death Eaters are significantly more powerful than the students and followers, their initial numbers play a pivotal role.

Furthermore, inspite of having different protection rules for Order members and Death Eaters, this rule has minimal significance when it comes to the final result of the battle. Thus, the protection strategy of the Order of the Phoenix, though may seem righteous and an important strategy, does not affect the outcome. However, due to this strategy, the number of deaths sustained is lesser for the Harry Potter side when they win as compared to Voldemort and his Death Eaters. Thus, there is a demarcation in the margin of win for the two parties.

Limitations

- Each wizard is limited only to the eight wizards surrounding him. This might lead to discrepancies as the range of a wizard can extend further than the immediate neighbors.

- Each wizard has a killing probability which limits the performance of the wizard. To make a closer model, this probability can be different depending on the power of the wizard and the surroundings. This can also be modeled to decrease with time as fatigue sets in.

- Wizards apparate randomly, without any particular strategy. The reason behind this is that it is assumed that a wizard will not have an accurate idea about other regions of the battle ground and hence will move to a random location. Had the wizards moved to a location specifically where there were more enemies, there would be a very high probability of the wizard himself dying without inflicting much damage on the enemies. On the other hand, if the wizard apparated to a location with minimal enemies, the number of interactions would decrease and the battle would become stagnant. The strategy implemented here is inherently attacking in nature as the wizard apparates only when all neighboring enemies are dead. A more detailed strategy can be applied where a wizard apparates to a location where he is most useful.

CONCLUSION

Two models have been implemented to simulate the Battle of Hogwarts. Careful initial conditions and parameter values have been used to represent different scenarios which lead to the triumph of different sides of the battle.

Cellular Automata appears to form a closer model as the assumptions allow it to be more realistic in terms of the flexibility of the spells used and other powers. The scope of a fighter is limited to a close locality of the fighter, hence the neighborhood assumption of the cellular automata model is valid. However, most of these assumptions are also satisfied by the differential equation and hence this model is also accurate.

[1] J.K. Rowling, Harry Potter and the Deathly Hallows.
[2] Svend Clausen, "Warfare Can Be Calculated"
[3] Basic cellular automata code: http://press.princeton.edu/