IT461 Assignment 1: Generating random variables

(1) Use simulation to approximate the following integrals. Compare with exact answers whenever answers are known

a)
$$\int_0^1 (1-x^2)^{3/2} dx$$

b) $\int_0^\infty x(1+x^2)^{-2} dx$

(2) For the uniform random variables $U_1, U_2, ...$ define

$$N = \min\{n : \sum_{i=1}^{n} U_i > 1\}$$

Estimate $\mathbb{E}(N)$ by generating 100,1000 and 10000 values of N.

- (3) Give two algorithms to generate the random variable X such that P(X=1)=0.3, P(X=2)=0.2, P(X=3)=0.35, P(X=4)=0.15. Write code to implement these algorithms.
- (4) Write a program that generates a random variable having density

$$f(x) = 30(x^2 - 2x^3 + x^4) \quad 0 \le x \le 1$$

(5) Write a program to simulate the first 10 time units of a non homogeneous Poisson process having intensity

$$\Lambda(t) = \begin{cases} \frac{t}{5} & 0 \le t < 5\\ 1 + 5(t - 5) & 5 \le t \le 10 \end{cases}$$

(6) Write a program to generate a normal random variable with mean 0 and variance 1 using the polar method.