IT461 Assignment 3: Markov Chains

(1) Simulate the Land of Oz example given in class for 300 days.

(2) Simulate the stepping stone model of Example 11.2 of Laurie Snell book with $n = 10$ for two colors. Observe what happen's at after a long time. Can you explain why this is happening?

(3) Simulate the drunkard's walk with lattice points $\{0, 1, 2, 3, 4\}$ with 0 and 4 being the absorbing states. Start the random walk in state 1. Estimate i)The expected number of times that the drunkard is in state 3 ii)The expected time to absorbtion iii)Probability that the chain will be absorbed in state 4 . Compare with the theoretical formula's in section 11.2

(4) Simulate the rat's maze example (Example 11.22) and keep track of the number of times that the rat is in state j for $j = 1, 2, .., 9$. Compare the empirical stationary distribution of this Markov chain to the theoretical one.

(5) A discrete time queueing system of capacity $n = 50$ consists of the person being served and those waiting to be served. The queue length x is observed each second. If $0 < x < n$, then with probability $p = 0.4$, the queue size is increased by one by an arrival and, inependently, with probability $r = 0.5$, it is decreased by one because the person being served finishes service. If $x = 0$, only an arrival (with probability p) is possible. If $x = 50$, an arrival will depart without waiting for service, and so only the departure (with probability r)of the person being served is possible. This is a Markov chain with states given by the number of customers in the queue. Write a computer program to simulate the queue. Have your program keep track of the proportion of the time that the queue length is j for $j = 0, 1, ..., n$ and the average queue length. Can you estimate the stationary vector ? Test what happens when $p = 0.5$ and $r = 0.4$?