Question 6

$$m(x) = \frac{1}{N} \sum_{i=1}^{N} x_i$$

$$Cov(X,Y) = \frac{1}{N} \sum (x_i - m(x))(y_i - m(Y)))$$

$$s^2 = \frac{1}{N} \sum (x_i - m(x))^2$$

1. $m(a+bx) = \frac{1}{N} \sum_{i=1}^{N} (a + bx_i) = a + b \frac{1}{N} \sum_{i=1}^{N} x_i = a + bm(x)$

2. $Cov(X, X) = \frac{1}{N} \sum (x_i - m(x))^2 = s^2$

3. $Cov(X, a+bY) = \frac{1}{N} \sum (x - m(x))(a + by_i - m(a+bY)) = b \, Cov(X, Y)$

4. $Cov(a+bx, a+bY) = b^2 \, Cov(X, Y)$

5. $med(a+bx) = a + b \, med(x) \Rightarrow$ True when $b > 0$

   $IQR(a+bx) = b \, IQR(x) \Rightarrow$ Not true, it would not be $a + b \, IQR(x)$

6. Say $X = \{1, 4\}$

   $m(x) = \frac{1+4}{2} = 2.5$

   $x^2 = \{1^2, 4^2\} = \{1, 16\}$

   $m(x^2) = \frac{1+16}{2} = 8.5$

   $(m(x))^2 = (2.5)^2 = 6.25$

   $(m(x))^2 \neq m(x^2)$

   $\sqrt{x} = \{1, 2\}$

   $m(\sqrt{x}) = \frac{1+2}{2} = 1.5$

   $\sqrt{m(x)} = \sqrt{2.5} \approx 1.58$

   $m(\sqrt{x}) \neq \sqrt{m(x)}$