

Dynamic risk spillovers from oil to stock markets: Fresh evidence from GARCH copula quantile regression based CoVaR model*

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ABSTRACT

This study proposes a GARCH copula quantile regression model to capture the downside and upside tail dependence between oil price change and stock market returns at different risk levels. In the model, ten copulas are provided to measure the nonlinearity of the tail dependence with the marginal distribution built on the GARCH family models. Using daily price data of stock markets in ten important economies and Brent oil market, we estimate the downward and upward risk spillovers from oil to stock markets. The empirical results suggest strong evidence of risk spillover effects from oil to stock markets. Furthermore, oil has the largest downside and upside risk spillover effects on the Brazilian and Mexican stock markets, respectively. And the US stock market displays the smallest downside and upside risk spillovers from the oil market. We also find evidence that the downside risk spillovers are larger than upside risk spillovers, a finding which is consistent with the flight-to-quality phenomenon. Finally, the dynamic risk spillover effects show heterogeneity over time and are comparatively different for each country. Our results provide significant implications for portfolio managers and international regulators who want to optimize their investment portfolios and maintain stock market stability.

Keywords: Risk spillover; Oil market; Stock market; CoVaR; GARCH copula quantile regression

JEL Classifications: C58; G14; G15; Q43

1. Introduction

Crude oil market is one of the most strategic markets with distinct characteristics. The strategic importance of oil continuously draws finance researchers' attention to study its dynamics (Awartani et al., 2016; Benlagha and Omari, 2021; Husain et al., 2019; Lin et al., 2019). A plethora of literature has emerged studying the relationship between oil and stock markets, for example, most studies have supported the hypothesis of a significant link between oil prices and stock prices (Jones and Kaul, 1996; Park and Ratti, 2008; Sadorsky, 1999; Hammoudeh and Li, 2005), in the same vein, the literature acknowledges the spillover effect from oil to stock markets (Anand and Paul, 2021; Wang and Wu, 2018; Xu et al., 2019). However, most of these studies examine the average relationship of the two markets. As is well known, the stock market and the oil market frequently exhibit very sharp price fluctuations, so the relationship in extreme market conditions is more important than the average relationship. Thus, monitoring and controlling extreme market risks and their spillovers between the two markets are very important for managing investment risk, portfolio allocation, and establishing policy concerning the stability of an economy and its financial markets.

The oil shock in October 1973 caused international oil prices to nearly quadruple in just a few months. In the case of WTI, after rising from \$3.07 per barrel at the end of September 1973 to \$11.65 per barrel at the end of January 1974, the oil price has repeatedly surged and plunged over the past 45 years, showing high volatility. When the second oil shock occurred, the price of WTI crude oil soared and rose to \$39.50 in April 1980. Large fluctuation dynamics are similar in case of Brent oil price. It was \$11.93 in September 1988 but fell to \$9.91 in November 1998 when the Asian financial crisis occurred. In June 2008, when the global financial crisis occurred, it rose to \$138.40, and in March 2020, when the COVID-19 pandemic occurred, it fell to \$14.85. Although the pandemic is still underway, the price has risen again to \$87.74 a barrel in January 2022.

Changes in crude oil prices affect stock prices through several economic channels. First, since crude oil is one of the most basic and important raw materials for industrial production, an increase in crude oil price can directly affect production by increasing the scarcity of raw materials and raising the marginal cost of production (Rasche and Tatom, 1977). In other words, an increase in oil prices can raise inflation, lower the utilization rate of facilities, and increase unemployment, which can darken the outlook for economic growth. This is called the supply-side shock effect or fundamental effect, and the resulting recession can have a negative impact

on the stock market. Second, an increase in the goods price caused by an increase in oil prices reduces the profits of companies that are highly dependent on oil and energy, and as a result, the stock price of those companies decreases. This path can be called the inflation effect. In addition, since some of the effect of the oil price increase is passed on to consumers, consumers' cost of living increases, and the demand for money increases in the process. At this time, if there is no change in the money supply, the short-term interest rate rises, which increases the company's financing cost and increases the discount rate for future earnings, causing the stock price to fall (Bernanke et al., 1997). This pathway can be called the real balance effect. Third, rising oil prices can affect economic activity and stock prices by transferring purchasing power from oil-importing countries to oil-exporting countries, which can be called income transfers and aggregate demand effect (Fried and Schulze, 1975; Dohner, 1981). The transfer of purchasing power due to an increase in oil prices reduces consumer demand in oil importing countries and increases consumer demand in oil-exporting countries. As a result, stock prices of oil-importing countries fall and those of oil-exporting countries rise. Fourth, uncertainty due to the increase in oil price volatility can affect real economic activity and stock prices. This path can be called the uncertainty effect. Bernanke (1983) argued that when uncertainty in the economy increases due to factors such as oil price fluctuations, consumption and investment are postponed to the future, thereby negatively affecting firms in the economy. In particular, Ferderer (1996) presented the reason to keep an eye on oil price volatility from the result that the real economy is greatly affected by the uncertainty caused by oil price volatility rather than the oil price level itself.

Regarding the uncertainty path, many studies focus on the impact of volatility of oil prices to stock market price dynamics. However, only a few studies analyzed extreme risk transmission from the oil market to the stock market when the market condition is bad. In this study, we focus on the fourth channel.

We aim to uncover the nature of extreme risk spillover from the oil market to the developed and large emerging countries. For this purpose, we estimate the downside and upside risk spillovers from the Brent oil market to the stock markets of ten large developed and emerging economies using daily data spanning from January 2, 2001 to December 31, 2021, which include several important episodes, including the global financial crisis and the ongoing COVID-19 pandemic periods.

Our study contributes to the literature in the following directions: First, we have revisited the oil-stock price relationship to discover fresh evidence employing a novel CoVaR model

based on GARCH copula quantile regression (CQR) recently developed by Tian and Ji (2022). This approach enables us to capture the upside tail dependence between oil and stock markets at different risk levels. In the new model, ten copulas are provided to measure the nonlinearity of the downside and upside tail dependence with the marginal distribution built on the GARCH family models. This method can further enhance our understanding of risk transmission between oil and stock markets. The findings obtained will bring more information to policymakers, investors, portfolio managers and traders for hedging and risk diversification. Second, compared to previous studies, we have employed a long daily data from January 2, 2001 to December 31, 2021 with 5,261 observations to provide a fresh and accurate structure between the markets, which highlight the special experience of the global financial crisis and ongoing COVID-19 pandemic episodes. Third, we analyze a large dataset of ten developed and emerging countries which is not incorporated in other studies.

The study finds (1) oil price shock exhibits the largest downside and upside risk spillovers on the Brazilian and Mexican stock markets, respectively. And the US stock market displays the smallest risk spillovers from the oil market. (2) The downside and upside risk spillovers are asymmetric, with upside risk spillovers less than downside risk spillovers, which is consistent with the phenomenon of flight-to-quality. (3) The dynamic risk spillover effects show heterogeneity over time and are comparatively different for each country.

The remainder of this paper is organized as follow. Section 2 briefly discusses the related literature. Section 3 outlines the methodology. Section 4 explains the sample data. Section 5 reports and discusses the empirical results and highlights their policy implications. Finally, section 6 concludes.

2. Literature review

Following the pioneering study of Bernanke (1983), many studies explored the uncertainty path that uncertainty due to the increase in oil price volatility affects the real economy. Among others, Ferderer (1996) revealed that uncertainty due to oil price volatility has an influence on the total cost and marginal cost of firm's investment activity, which in turn affects the real economy more in the short-run rather than in the long-run. Elder and Serletis (2010) also reported that uncertainty caused by oil price volatility has a significantly negative influence on the real economy, and in particular, has a negative impact on durable goods

consumption and fixed investment. Serletis and Xu (2019) confirmed that uncertainty about oil prices had a negative influence on the real economic growth rate and found that the larger the magnitude of oil price fluctuations during an oil price rise, the longer the shock duration and the greater the impact. They also reported that when oil prices fall, large fluctuations positively affected the real economic growth rate. These studies are extended to uncover the effect of oil price shocks on stock price fluctuation.

2.1. Studies on the impact of oil price on stock price

Since Hamilton (1983) revealed that the oil price shock caused a recession in the US economy, empirical studies on the impact of oil price changes on the real economy and the stock market have continued. However, not all empirical findings are consistent. Jones and Kaul (1996), Sadorsky (1999) and Basher et al. (2012) reported empirical analysis results that an unexpected rise in oil prices negatively affects stock prices. In particular, Park and Ratti (2008) analyzed the influence of oil price shocks on the stock prices in oil-exporting countries and oil-importing countries, respectively. They revealed that in Norway, an oil-exporting country, when oil prices rise, the stock market also rises, but, in other European countries, the rise in oil prices negatively affect the stock market. Cunado and de Gracia (2014) explored the effects of oil price changes on the stock returns employing vector error correction model. They discovered evidence of a significantly negative impact of an oil price shock on stock market returns in most oil-importing European countries. Alsalman and Herrera (2015) explored the oil price innovations on the US stock markets and disclosed that positive oil price shocks decrease stock returns of the US. Huang et al. (2017) revealed that oil price increase and decrease both significantly influence stock returns.

However, few studies revealed that oil price changes did not significantly affect stock returns (Chen et al., 1986; Huang et al., 1996; Wei, 2003; Apergis and Miller, 2009; Sukcharoen et al., 2014) or that oil price effects are ambiguous (Miller and Ratti, 2009; Mollick and Assefa, 2013). These studies pointed out that not all oil price innovations are identical and stock market fluctuations do not respond identically to oil market shocks.

Whereas, few studies provide opposing evidence that both prices move in the same direction. Mohanty et al. (2011) investigated the linkage between changes in oil prices and stock price returns in Gulf Cooperation Council (GCC) countries, which are crude oil exporters. They presented evidence that, unlike crude oil-importing countries, oil price changes have a positive relationship with stock price returns. Silvapulle et al. (2017) also revealed that oil

prices have had a significantly positive influence on stock prices in large net oil-importing countries, with the coefficient being significantly higher before the global financial crisis than after the crisis. Jiang and Yoon (2020) found that the stock prices are more affected by oil prices in oil-exporting countries than in oil-importing countries. They explained that the finding indicates that the economic structure of oil-exporting countries are strongly linked to crude oil production. Alamgir and Amin (2021) found that oil prices and stock prices fluctuate in the same direction in South Asian countries such as India.

2.2. Studies on the volatility spillovers between oil and stock markets

Some studies examined volatility transmission between oil and stock markets using the multivariate GARCH-class models. Among others, Hammoudeh et al. (2004) investigated the interaction of volatility among US oil prices and oil industry stock indices employing VECM and GARCH models. They discovered that the oil futures market has a echoing volatility transmission to the stock prices of some oil sectors and a volatility-dampening effect on the stock prices of other sectors. Hammoudeh et al. (2010) uncovered that oil market volatility has differing impacts on the US stock sector. Arouri et al. (2011) explored the linkage between oil and stock markets in the GCC countries and discovered evidence for the existence of substantial volatility transmission between oil and GCC stock markets. Filis et al. (2011) explored the time-varying volatility correlations between oil and stock markets on both oil-importing and oil-exporting countries. They discovered that the time-varying correlation increases positively in respond to demand-side oil market shocks. Chkili et al. (2014) investigated the connectedness between oil and US stock markets employing a DCC (dynamic conditional correlations)–FIAPARCH (fractional integrated asymmetric power ARCH) model. They discovered that the DCC between the oil and US stock prices are influenced by economic and geopolitical events. Guesmi and Fattoum (2014) examined co-movements and dynamic volatility spillovers between oil and stock price returns of oil-exporting and oil-importing countries by employing an asymmetric DCC-GARCH model. They found volatility correlation increases positively in response to shocks from global business cycle fluctuations or world turmoil. Joo and Park (2017) explored the linkage between oil price returns and stock price returns employing a DCC-GARCH-in-mean model and found that there are times when uncertainty in the oil market negatively affects the stock market. Xu et al. (2019) discovered strong evidence of asymmetries in bad volatility shocks by measuring directional spillover index. Yun and Yoon (2019) analyzed the influence crude oil price change on the stock price

and volatility of airlines companies employing a VAR-BEKK-GARCH model. They revealed evidence of volatility linkage from oil price and stock prices of airlines. Ahmed and Huo (2021) investigated volatility dependence across the oil, commodity futures and Chinese stock markets employing a trivariate VAR-BEKK-GARCH model. They discovered evidence supporting for volatility transmission from the oil prices to the Chinese stock prices.

2.3. Studies in the extreme risk spillovers between oil and stock markets

As stock markets are highly linked to oil market and changes in oil prices affect stock markets, extreme fluctuation in a market can substantially influence the other, this is mostly apparent during financial crises (Zhao et al., 2021). Previous studies have shown that the interdependence between oil and stock markets are almost stable and have exception in financial crises were cross-market linkages increase (Bampinas and Panagiotidis, 2017; Martín-Barragán et al., 2015), although a large body of empirical research exist on the risk transmission between oil and stock markets, the studies couldn't identify the interdependence during extreme (bullish and bearish) markets conditions. To understand the dynamics, researchers have focused on conditional variance transmission which cannot reveal the whole story of risk relations, later copula has been utilized for the same purpose (Aloui et al., 2013; Wen et al., 2012). However, copula alone don't cover lagged effects at the risk level, contemporaneous dependence is hard to identify the causality of risk transmissions, and the downside (upside) risk dependence stand on the default assumption of positive correlation between market returns and the literature confirms that oil and stock markets might be negatively correlated (Du and He, 2015), as a result, some recent research tried to uncover the linkage in extreme market condition employing advanced approaches such as multivariate copula and quantile regression (Li and Wei, 2018; Mensi et al., 2014, 2015, 2017a; Nguyen and Bhatti, 2012; Sukcharoen et al., 2014; Yu et al., 2020).

2.4. Studies related to empirical methodology

Currently, the most popular approach for calculating the risk spillover effect is the conditional value at risk (CoVaR) model (Girardi and Ergün, 2013; Adrian and Brunnermeier, 2016). Essentially, CoVaR is a type of conditional quantile, many researchers employed the quantile regression model (Koenker and Bassett, 1978) to investigate the risk spillovers due to the greater flexibility in estimating the correlation among financial variables (see Acharya et al. 2012; Bernal et al., 2014; Adrian and Brunnermeier, 2016; Morelli and Vioto, 2020).

However, financial risks in different financial markets are nonlinearly correlated at high-risk levels (Chao et al., 2015). A CoVaR model based on linear quantile regression ignores the nonlinear tail dependence among different financial variables, which may lead to distorted estimates of the risk spillover effects. To solve this question, many new models have been developed.¹ Among these models, the GARCH copula CoVaR model can describe the nonlinearity of tail dependence and capture the features of fat tails, skewness, serial correlation and volatility clustering of the marginal distributions.

A further extension of the GARCH copula model is to allow the tail dependence to be time-varying.² However, this approach tends to cause unstable VaRs of asset portfolios. To mitigate this problem, a new Garch CQR model is developed by Tian and Ji (2022) to calculate downside CoVaR and risk spillovers. This new model allows the downside tail dependence to vary with different risk levels. Moreover, it also has advantages in accurately describing the nonlinearity of downside tail dependence at different risk levels and the characteristics of volatility clustering and serial correlation of the marginal distribution.

3. Methodology

Following Tian and Ji (2022), we propose a GARCH CQR model accurately capturing the upside tail dependence. This is in contrast to Tian and Ji (2022) in which the model is used to capture the downside risk spillover using the downside CoVaR (DCoVaR) method. Consequently, the downside and upside CoVaR and risk spillovers can be calculated by the GARCH CQR-based DCoVaR and UCoVaR model, respectively. Finally, based on the bootstrap Kolmogorov-Smirnov (KS) tests, we introduce the significance and asymmetry tests.

¹ Such as the partial quantile regression CoVaR model (Chao et al., 2015), the asymmetric CoVaR model (López-Espinosa et al., 2015), TENET (tail-event driven network) (Härdle et al., 2016), the wavelet CoVaR model (Teply and Kvapilíková, 2017), the multiple-CoVaR model (Bernardi et al., 2017), the GARCH copula CoVaR model (e.g., Huang et al., 2009; Reboredo and Ugolini, 2015; Reboredo et al., 2016; Karimalis and Nomikos, 2018; Ji et al., 2018; Sun et al., 2020).

² See Patton (2006), Giacomini et al. (2009), Ausin and Lopes (2010), Hafner and Reznikova (2010), Patton (2012), Creal et al. (2013), Reboredo et al. (2016), and Wu et al. (2021).

3.1. CoVaR model

We apply the risk measure ΔCoVaR to estimate downside and upside risk spillovers from crude oil market to stock market. Firstly, we review the risk measure VaR. For a stock market i , the downside $\text{VaR}_{\alpha,t}^i$ and upside $\text{VaR}_{1-\alpha,t}^i$ at a confidence level $(1-\alpha)$ are defined as:

$$\Pr(r_{it} \leq \text{VaR}_{\alpha,t}^i) = \Pr(r_{it} \geq \text{VaR}_{1-\alpha,t}^i) = \alpha.$$

The given confidence level $(1-\alpha)$ implies that the probability of the maximum possible loss greater than the VaR is less than or equal to α . It is obvious that for a portfolio manager with a long position (a short position), the risk measure VaR is related to downside risk (upside risk).

According to the VaR measure, the CoVaR measure (Adrian and Brunnermeier, 2016) is defined as follows. Given confidence level $(1-\tau)$, the downside $\text{CoVaR}_{\tau|\beta,t}^{S|i}$ and upside $\text{CoVaR}_{1-\tau|1-\beta,t}^{S|i}$ for the stock market S , conditional on the downside $\text{VaR}_{\beta,t}^i$ and upside $\text{VaR}_{1-\beta,t}^i$ for the returns of the oil market i at the confidence level $(1-\beta)$ satisfy

$$\Pr(r_{st} \leq \text{CoVaR}_{\tau|\beta,t}^{S|i} | r_{it} = \text{VaR}_{\beta,t}^i) = \Pr(r_{st} \geq \text{CoVaR}_{1-\tau|1-\beta,t}^{S|i} | r_{it} = \text{VaR}_{1-\beta,t}^i) = \tau.$$

Here, r_{it} and r_{st} are the returns of oil markets i and stock market S , respectively. Therefore, the risk spillover effect $\Delta\text{CoVaR}_{\tau|\beta,t}^{S|i}$ of one oil market i on the stock market S at confidence level $(1-\tau)$ can be defined as follows,

$$\Delta\text{CoVaR}_{\tau|\beta,t}^{S|i} = \text{CoVaR}_{\tau|\beta,t}^{S|i} - \text{CoVaR}_{\tau|0.5,t}^{S|i}, \quad (1)$$

where $\text{CoVaR}_{\tau|\beta,t}^{S|i}$ and $\text{CoVaR}_{\tau|0.5,t}^{S|i}$ are the VaR of the stock market S conditional on the oil market i being in a distress state and a benchmark state, respectively.

Similarly, the upside risk spillover effect can be calculated by the following equation:

$$\Delta\text{CoVaR}_{1-\tau|1-\beta,t}^{S|i} = \text{CoVaR}_{1-\tau|1-\beta,t}^{S|i} - \text{CoVaR}_{1-\tau|0.5,t}^{S|i}. \quad (2)$$

The estimation of the downside risk spillover by the GARCH CQR-based DCoVaR model has been proposed by Tian and Ji (2022). In the following subsections, we will derive the GARCH CQR-based UCoVaR model.

3.2. Marginal distribution model

In this subsection, we introduce the ARMA-GARCH model, the most widely used approach to describe the properties of serial correlations, volatility clustering and conditional heteroskedasticity of financial returns. In general, the ARMA(p,q)-GARCH(m,s) model is constructed as follows:

$$r_t = \mu_t + a_t = \varphi_0 + \sum_{i=1}^p \varphi_i r_{t-i} + \sum_{j=1}^q \psi_j a_{t-j} + a_t, \quad (3)$$

where p and q are nonnegative integers and φ_i and ψ_j are the autoregressive and moving average parameters, respectively. $a_t = \sigma_t \varepsilon_t$, σ_t^2 is the conditional variance that has dynamics as given by the GARCH model:

$$\sigma_t^2 = \omega + \sum_{i=1}^m \alpha_i a_{t-i}^2 + \sum_{j=1}^s \beta_j \sigma_{t-j}^2, \quad (4)$$

where $\{\varepsilon_t\}$ is a sequence of i.i.d. random variables with mean 0 and variance 1, $\omega > 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$ and $\sum_{j=1}^{max(m,s)} (\alpha_j + \beta_j) < 1$.

To allow for asymmetric effects between positive and negative asset returns, the EGARCH model (Nelson, 1991) is proposed as follows:

$$\ln \sigma_t^2 = \omega + \sum_{i=1}^m g_i(\varepsilon_{t-i}) + \sum_{j=1}^s \beta_j \ln \sigma_{t-j}^2, \quad (5)$$

where again $\{\varepsilon_t\}$ is a sequence of i.i.d. random variables with mean 0 and variance 1, and $g_i(\varepsilon_t) = (\alpha_i \varepsilon + \gamma_i (|\varepsilon_t| - E|\varepsilon_t|))$. It is obvious that

$$g_i(\varepsilon_t) = \begin{cases} (\alpha_i - \gamma_i) \varepsilon_t + \gamma_i E|\varepsilon_t|, & \varepsilon_t < 0 \\ (\alpha_i + \gamma_i) \varepsilon_t + \gamma_i E|\varepsilon_t|, & \varepsilon_t \geq 0 \end{cases}$$

where $E|\varepsilon_t|$ is the expected value of absolute standardized innovation ε_t . Thus, the parameter α_i captures the sign effect and γ_i the magnitude effect, which denotes the asymmetry of the volatility for positive and negative returns which is commonly attributed to the leverage effect of equity returns.

The standardized residuals generally exhibit the characteristics of both kurtosis and skewness, which follow a standardized skew Student's t distribution (SSST) (Tsay, 2013). Let $X \sim SSST(\xi, \nu)$ be the SSST distribution, and its PDF (probability density function) is

$$d(\varepsilon_t|\xi,v) = \begin{cases} \frac{2}{\xi + \xi^{-1}} \varsigma f(\xi(\varsigma \varepsilon_t + w)|v), & \varepsilon_t < \frac{w}{\varsigma} \\ \frac{2}{\xi + \xi^{-1}} \varsigma f((\varsigma \varepsilon_t + w)/\xi|v), & \varepsilon_t \geq \frac{w}{\varsigma} \end{cases}$$

where $f(\cdot|v)$ is the PDF of the SST distribution:

$$f(\varepsilon_t|v) = \frac{\Gamma((v+1)/2)}{\Gamma(v/2)\sqrt{\pi(v-2)}} (1 + \varepsilon_t^2/(v-2))^{-(v+1)/2}$$

where $\Gamma(\cdot)$ is the gamma function and $v > 2$ is the degree of freedom. ξ^2 is equal to the ratio of probability masses above and below the mode of the distribution; hence, ξ is the skewness parameter, $w = \frac{\Gamma((v+1)/2)\sqrt{v-2}(\xi - \xi^{-1})}{\Gamma(v/2)\sqrt{\pi}}$, and $\varsigma^2 = (\xi + \xi^{-2} - 1) - w^2$.

3.3. CQR model

Let the cumulative distribution functions (CDFs) be $u = F_X(x)$ and $v = F_Y(y)$, respectively. They can be connected by the copula function $C(u,v;\delta)$ with parameter δ and we can get the joint distribution function $H(x,y) = C(F_X(x), F_Y(y); \delta)$ (Sklar, 1959). The one-parameter copula family used in this study includes Clayton copula, Joe copula, Gumbel copula, Galambos copula, Hüsler-Reiss copula and their 180-degree rotated forms (Joe, 1997; Nelsen, 2006), which are shown in Table 1.

Table 1. Copula models

Copula models	Copula function	Parameter
Clayton	$C^C(u,v;\delta) = (u^{-\delta} + v^{-\delta} - 1)^{-1/\delta}$	$\delta \in [0, +\infty)$
Rotated Clayton	$C^{RC}(u,v;\delta) = u + v - 1 + C^C(1-u, 1-v; \delta)$	$\delta \in [0, +\infty)$
Joe	$C^J(u,v;\delta) = 1 - ((1-u)^\delta + (1-v)^\delta - (1-u)^\delta(1-v)^\delta)^{1/\delta}$	$\delta \in [1, +\infty)$
Rotated Joe	$C^{RJ}(u,v;\delta) = u + v - 1 + C^J(1-u, 1-v; \delta)$	$\delta \in [1, +\infty)$
Gumbel	$C^G(u,v;\delta) = \exp(-((-\log u)^\delta + (-\log v)^\delta)^{1/\delta})$	$\delta \in [1, +\infty)$
Rotated Gumbel	$C^{RG}(u,v;\delta) = u + v - 1 + C^G(1-u, 1-v; \delta)$	$\delta \in [1, +\infty)$
Galambos	$C^{Ga}(u,v;\delta) = uv \exp(((-\log u)^{-\delta} + (-\log v)^{-\delta})^{-1/\delta})$	$\delta \in [0, +\infty)$
Rotated Galambos	$C^{RGa}(u,v;\delta) = u + v - 1 + C^{Ga}(1-u, 1-v; \delta)$	$\delta \in [0, +\infty)$
Hüsler-Reiss	$C^{HR}(u,v;\delta) = \exp(\Phi(\delta^{-1} + 0.5\delta \log(\log u / \log v)) \log u + \Phi(\delta^{-1} + 0.5\delta \log(\log v / \log u)) \log v)$	$\delta \in [0, +\infty)$
Rotated Hüsler-Reiss	$C^{RHR}(u,v;\delta) = u + v - 1 + C^{HR}(1-u, 1-v; \delta)$	$\delta \in [0, +\infty)$

Note: Φ is the CDF of standard normal distribution.

Based on the definition of conditional CDF $H(y|x)$, we have

$$\tau = H(y|x) = Pr(Y \leq y|X = x) = \lim_{\Delta x \rightarrow 0} Pr(Y \leq y|x \leq X \leq x + \Delta x) = \frac{\partial C(u,v;\delta)}{\partial u}, \quad (6)$$

where $C_1(v|u;\delta) = \frac{\partial C(u,v;\delta)}{\partial u}$ is conditional copula. The conditional copula functions of the copula families shown in Table 1 are presented in Table 2. Fixing the conditional probability of Y given $X = x$ at quantile τ ; we can get

$$v = C_1^{-1}(\tau|u;\delta), \quad (7)$$

by solving $\tau = C_1(v|u;\delta)$ for v . Equation (7) presents the τ^{th} copula quantile curve for (u,v) .

Table 2. Conditional distributions of copula models

Copula models	Conditional distribution functions
Clayton	$C_1^C(v u;\delta) = (1 + u^\delta(v^{-\delta} - 1))^{-(1+\delta)/\delta}$
Rotated Clayton	$C_1^{RC}(v u;\delta) = 1 - (1 + (1-u)^\delta((1-v)^{-\delta} - 1))^{-(1+\delta)/\delta}$
Joe	$C_1^J(v u;\delta) = (1 + (1-u)^{-\delta}(1-v)^\delta - (1-v)^\delta)^{(1-\delta)/\delta}(1 - (1-v)^\delta)$
Rotated Joe	$C_1^{RJ}(v u;\delta) = 1 - (1 + u^{-\delta}v^\delta - v^\delta)^{(1-\delta)/\delta}(1 - v^\delta)$
Gumbel	$C_1^G(v u;\delta) = u^{-1}C^G(u,v;\delta)(1 + (\log v/\log u)^\delta)^{(1-\delta)/\delta}$
Rotated Gumbel	$C_1^{RG}(v u;\delta) = 1 - (1-u)^{-1}C^G(1-u,1-v;\delta)(1 + (\log(1-v)/\log(1-u))^\delta)^{(1-\delta)/\delta}$
Galambos	$C_1^{Ga}(v u;\delta) = u^{-1}C^{Ga}(u,v;\delta)(1 - (1 + (\log u/\log v)^\delta)^{-(1+\delta)/\delta})$
Rotated Galambos	$C_1^{RGa}(v u;\delta) = 1 - (1-u)^{-1}C^{Ga}(1-u,1-v;\delta)(1 - (1 + (\log(1-u)/\log(1-v))^\delta)^{-(1+\delta)/\delta})$
Hüsler-Reiss	$C_1^{HR}(v u;\delta) = C^{HR}(u,v;\delta)u^{-1}\Phi(\delta^{-1} + 0.5\delta\log(\log u/\log v))$
Rotated Hüsler-Reiss	$C_1^{RHR}(v u;\delta) = 1 - C^{HR}(1-u,1-v;\delta)(1-u)^{-1}\Phi(\delta^{-1} + 0.5\delta\log(\log(1-u)/\log(1-v)))$

Note: See the notes for Table 1.

Considering $u = F_X(x)$ and $v = F_Y(y)$, equation (7) can be rewritten as

$$F_Y(y) = C_1^{-1}(\tau|F_X(x);\delta). \quad (8)$$

Therefore, we can get the CQR function for (x,y) at quantile τ as follows

$$y = F_Y^{-1}(C_1^{-1}(\tau|F_X(x);\delta)), \quad (9)$$

where $F_Y^{-1}(\cdot)$ is the quantile function.

Similarly, based on the definition of upside CoVaR, for the $(1-\tau)^{th}$ conditional quantile of Y given $X = x$, we have

$$\tau = Pr(Y \geq y|X = x) = 1 - Pr(Y \leq y|X = x) = 1 - C_1(v|u;\delta), \quad (10)$$

or

$$1 - \tau = C_1(v|u;\delta).$$

Solving $1 - \tau = C_1(v|u;\delta)$ for v yields the $(1 - \tau)^{th}$ CQR curve for (u,v) as the following equation:

$$v = C_1^{-1}(1 - \tau|u;\delta). \quad (11)$$

Therefore, the CQR function for (x,y) at quantile $(1 - \tau)$ is

$$y = F_Y^{-1}(C_1^{-1}(1 - \tau|F_X(x);\delta)). \quad (12)$$

Among all the above-mentioned copulas presented in Tables 1 and 2, Clayton copula, rotated copulas of Joe, Gumbel, Galambos and Hüsler-Reiss can describe downside tail dependence and upside tail independence. In contrast, the rotated Clayton copula, Joe copula, Gumbel copula, Galambos copula and Hüsler-Reiss copula can capture upside tail dependence and downside tail independence. Thus, the corresponding CQR function could properly describe the lower or upper tail dependence between random variables (X,Y) or (U,V) . To illustrate this desirable property, we generate 2000 random values of (U,V) for different copula with different parameters δ , and the marginal distributions of X and Y follow the SSST distribution with different parameters. We plot the CQR curves for ten copula families in Appendix A displaying different tail dependence behaviour.

3.4. GARCH CQR-based UCoVaR model

$F_{i,t}$ and $F_{s,t}$ denote the CDFs of r_{it} and r_{st} , returns of oil market i and stock market S , respectively. Thus, according to the definition of upside CoVaR and equation (10), we have

$$\begin{aligned} \tau &= Pr(r_{st} \geq CoVaR_{1-\tau|1-\beta,t}^S | r_{it} = VaR_{1-\beta,t}^i) \\ &= 1 - Pr(r_{st} \leq CoVaR_{1-\tau|1-\beta,t}^S | r_{it} = VaR_{1-\beta,t}^i) \\ &= 1 - H(CoVaR_{1-\tau|1-\beta,t}^S | VaR_{1-\beta,t}^i) = 1 - C_1(F_{s,t}(CoVaR_{1-\tau|1-\beta,t}^S | F_{i,t}(VaR_{1-\beta,t}^i); \delta)) \\ &= 1 - C_1\left(D_s\left(\frac{CoVaR_{1-\tau|1-\beta,t}^S - \mu_{st}}{\sigma_{st}}\right) \middle| D_i\left(\frac{VaR_{1-\beta,t}^i - \mu_{it}}{\sigma_{it}}\right); \delta\right) \end{aligned} \quad (13)$$

or

$$1 - \tau = C_1\left(D_s\left(\frac{CoVaR_{1-\tau|1-\beta,t}^S - \mu_{st}}{\sigma_{st}}\right) \middle| D_i\left(\frac{VaR_{1-\beta,t}^i - \mu_{it}}{\sigma_{it}}\right); \delta\right), \quad (14)$$

where D_i and D_s denote the CDFs of ε_{it} and ε_{st} , the standardized residuals of r_{it} and r_{st} ; μ_{it} , μ_{st} and σ_{it} , σ_{st} are the conditional mean and standard deviation of the returns of oil market i and stock market S , estimated by equations (3), (4) or (5).

According to equations (11) and (12), equation (14) is equivalent to

$$D_s\left(\frac{CoVaR_{1-\tau|1-\beta,t}^{S|i}-\mu_{st}}{\sigma_{st}}\right) = C_1^{-1}\left(1-\tau\left|D_i\left(\frac{VaR_{1-\beta,t}^i-\mu_{it}}{\sigma_{it}}\right);\delta\right.\right) = C_1^{-1}(1-\tau|D_i(VaR_{1-\beta,t}^{\varepsilon_i});\delta) \quad (15)$$

Therefore, the upside CoVaR can be estimated by

$$\frac{CoVaR_{1-\tau|1-\beta,t}^{S|i}-\mu_{st}}{\sigma_{st}} = D_s^{-1}(C_1^{-1}(1-\tau|D_i(VaR_{1-\beta,t}^{\varepsilon_i});\delta)), \quad (16)$$

or

$$CoVaR_{1-\tau|1-\beta,t}^{S|i} = \mu_{st} + \sigma_{st}D_s^{-1}(C_1^{-1}(1-\tau|D_i(VaR_{1-\beta,t}^{\varepsilon_i});\delta)), \quad (17)$$

where D_s^{-1} is the quantile function of ε_{st} . Following Tian and Ji (2022), we can estimate the parameter δ in model (17) by interior point algorithm for nonlinear quantile regression model (Koenker and Park, 1994) at the $(1-\tau)^{th}$ quantile:

$$Q_{1-\tau}(\varepsilon_{st}|\varepsilon_{it}) = \theta_{1-\tau} + \eta_{1-\tau}D_s^{-1}(C_1^{-1}(1-\tau|D_i(\varepsilon_{it});\delta_{1-\tau})), \quad (18)$$

based on $(\varepsilon_{it}, \varepsilon_{st})$, $t = 1, 2, \dots, T$, where $Q_{1-\tau}(\varepsilon_{st}|\varepsilon_{it})$ is the $(1-\tau)^{th}$ conditional quantile of ε_{st} given ε_{it} , $\eta_{1-\tau}$ is the zooming parameter and $\theta_{1-\tau}$ is the panning parameter.

Therefore, given confidence levels $(1-\tau)$ and $(1-\beta)$, the upside CoVaR of the stock market S conditional on the upside value at risk of the oil market i being $VaR_{1-\beta,t}^i$ can be obtained as follows:

$$\begin{aligned} CoVaR_{1-\tau|1-\beta,t}^{S|i} &= \mu_{st} + \sigma_{st}(\theta_{1-\tau} + \eta_{1-\tau}D_s^{-1}(C_1^{-1}(1-\tau|D_i(\varepsilon_{it});\delta_{1-\tau}))) \\ &= (\mu_{st} + \theta_{1-\tau}\sigma_{st}) + \sigma_{st}\eta_{1-\tau}D_s^{-1}(C_1^{-1}(1-\tau|1-\beta;\delta_{1-\tau})). \end{aligned} \quad (19)$$

Equation (19) is the GARCH CQR-based UCoVaR model. Meanwhile, the following equation is the GARCH CQR-based DCoVaR model (Tian and Ji, 2022):

$$CoVaR_{\tau|\beta,t}^{S|i} = (\mu_{st} + \theta_{\tau}\sigma_{st}) + \sigma_{st}\eta_{\tau}D_s^{-1}(C_1^{-1}(\tau|\beta;\delta_{\tau})). \quad (20)$$

In particular, the upside and downside CoVaRs of the stock market S conditional on oil market i being in its benchmark state ($\beta = 0.5$) can also be calculated by equations (19) and

(20), respectively. Therefore, the downward and upward risk spillover effects are determined by

$$\Delta CoVaR_{\tau|\beta,t}^{S|i} = \sigma_{st}\eta_{\tau}(D_s^{-1}(C_1^{-1}(\tau|\beta;\delta_{\tau}) - D_s^{-1}(C_1^{-1}(\tau|0.5;\delta_{\tau}))), \quad (21)$$

and

$$\Delta CoVaR_{1-\tau|1-\beta,t}^{S|i} = \sigma_{st}\eta_{1-\tau}(D_s^{-1}(C_1^{-1}(1-\tau|1-\beta;\delta_{1-\tau}) - D_s^{-1}(C_1^{-1}(1-\tau|0.5;\delta_{1-\tau}))). \quad (22)$$

It is worth noting that when applying equation (19) to calculate the upside risk spillover effect, the copula function should be selected from rotated Clayton copula, Gumbel copula, Joe copula, Hüsler-Reiss copula and Galambos copula, which can describe the lower tail independence and upper tail dependence between financial returns. Meanwhile, regarding the downside risk spillovers, the copula function in equation (20) is the optimal one of Clayton copula, rotated copulas of Gumbel, Joe, Hüsler-Reiss and Galambos, which can capture the upper tail independence and lower tail dependence.

3.5. Two-sample bootstrap Kolmogorov-Smirnov (KS) test

To determine whether oil market contributes significantly to the stock market, we adopt the two-sample bootstrap KS test (Abadie, 2002) to compare the CDFs of the dynamics of benchmark CoVaR and downside (or upside) CoVaR. The associated statistic of the significance test is defined as follows:

$$D_{mn} = (mn/(m+n))^{0.5} \sup |F_m(x) - G_n(x)|, \quad (23)$$

where $F_m(x)$ and $G_n(x)$ are the CDFs of the dynamics of benchmark CoVaR and downside (or upside) CoVaR, respectively, and m and n represent the size of the two samples. The null hypotheses of the significance test for downside and upside risk spillovers are defined as follows:

$$H_0: \Delta CoVaR_{\tau|\beta,t}^{S|i} = CoVaR_{\tau|\beta,t}^{S|i} - CoVaR_{\tau|0.5,t}^{S|i} = 0,$$

and

$$H_0: \Delta CoVaR_{1-\tau|1-\beta,t}^{S|i} = CoVaR_{1-\tau|1-\beta,t}^{S|i} - CoVaR_{1-\tau|0.5,t}^{S|i} = 0.$$

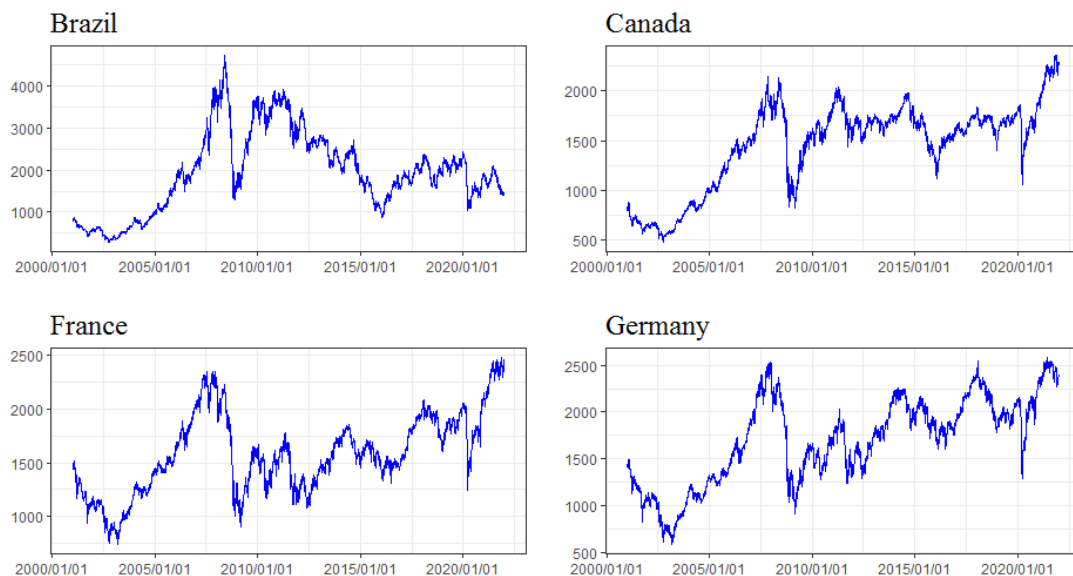
Finally, the asymmetry tests can assess whether the upside risk spillover of oil market i contributes more or equal to the stock market than the downside risk spillovers. The asymmetry test can also be implemented by the two-sample bootstrap KS test. The null hypothesis of the

asymmetry test is defined as follows:

$$H_0: CoVaR_{1-\tau|1-\beta,t}^{S|i} \geq |CoVaR_{\tau|\beta,t}^{S|i}|.$$

4. Data

In the empirical study, we investigate the risk spillovers from the Brent oil market to the stock markets in ten economies, including Brazil, Canada, France, Germany, Italy, Mexico, Russia, South Africa, the UK and the US. From the Wind database, we selected the daily data of the MSCI indices from 1 January 2001 to 31 December 2021 (5,262 observations) to represent the stock market indices. Fig. 1 presents the long-term trends of the MSCI price indices and the Brent oil price over the period analyzed. Note that the huge fluctuations of eleven price indices are quite similar. Specifically, the extreme risk events, such as the global financial crisis, the European debt crisis and the COVID-19 pandemic, usually resulted in an extreme downward trend of the eleven price indices.



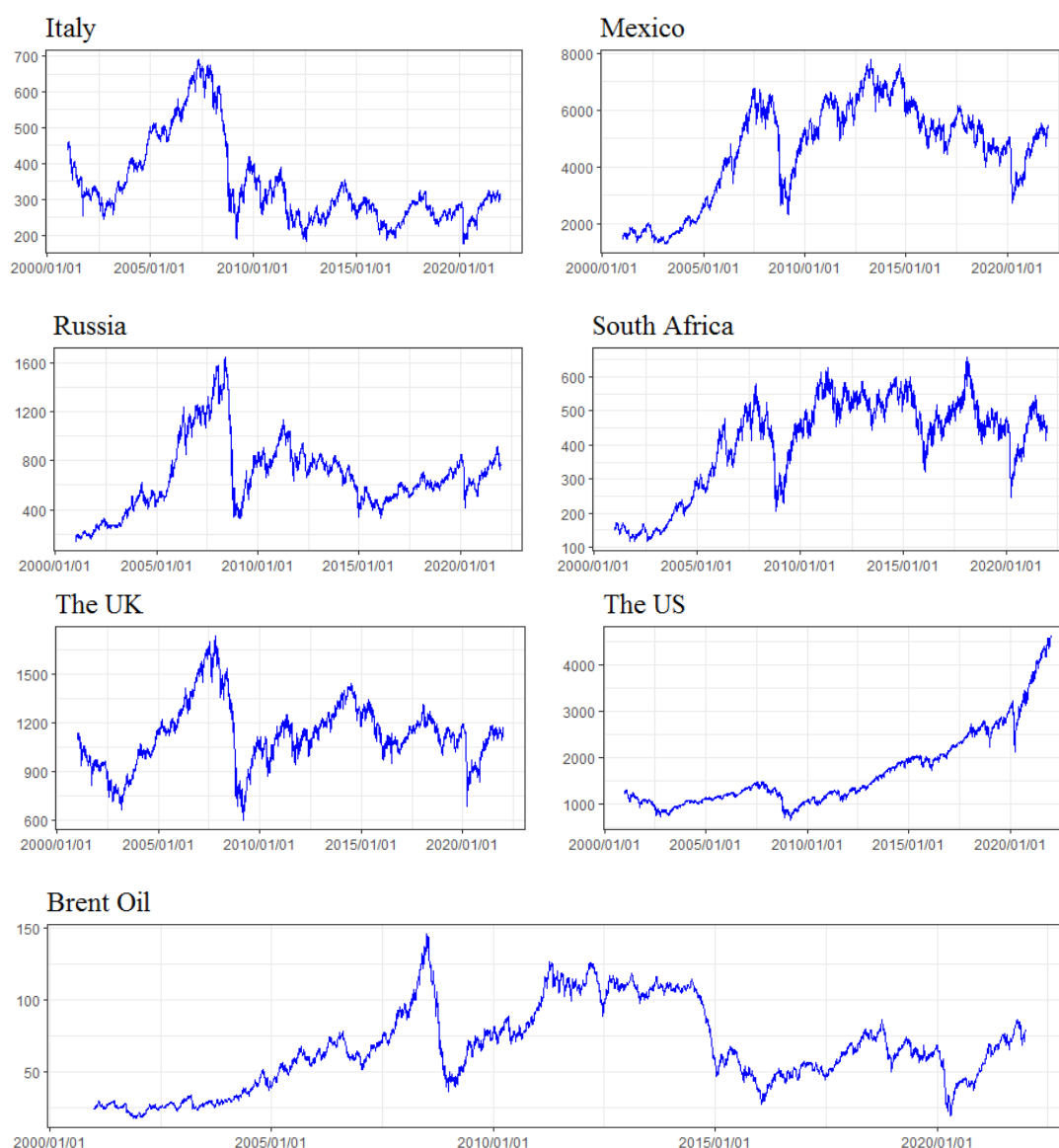
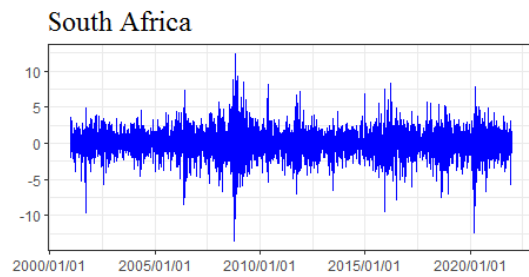
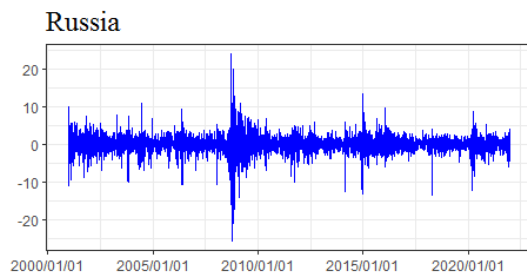
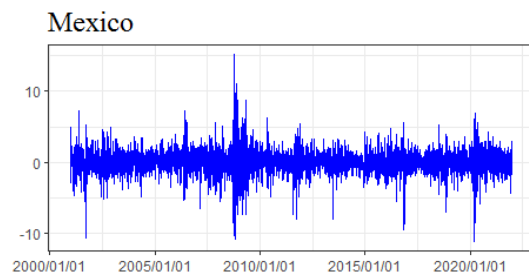
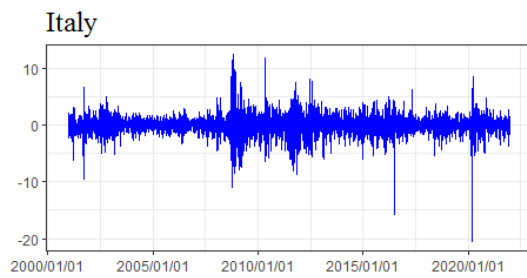
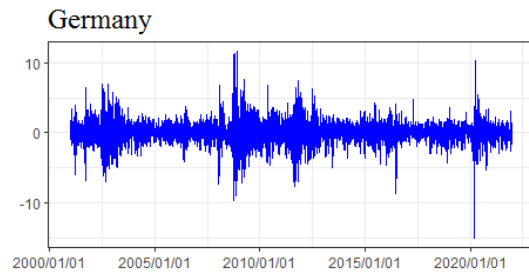
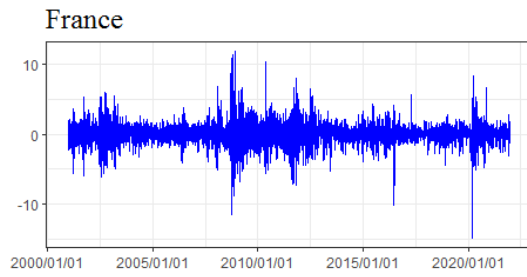
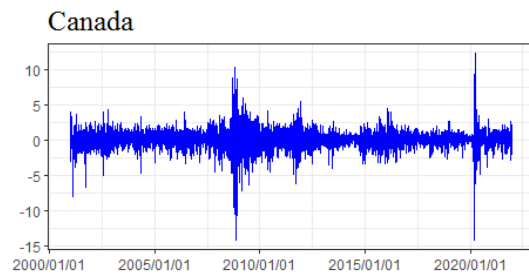
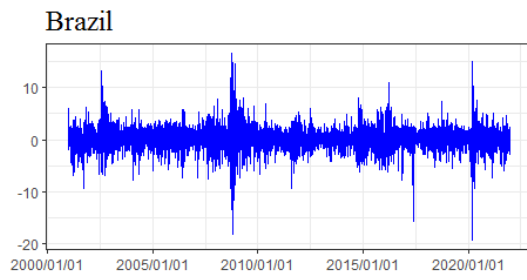


Fig. 1. MSCI indices of different financial markets

Fig. 2 plots the returns of these price indices which are given as $r_t = 100 \times (\ln P_t - \ln P_{t-1})$. Obviously, there is also a similar volatility clustering along with the occurrence of the global financial crisis, the European debt crisis and the COVID-19 pandemic. However, the reactions of different financial markets to extreme shocks have been heterogeneous over time. These characteristics provide an opportunity to explore risk spillovers from oil market to the stock markets.



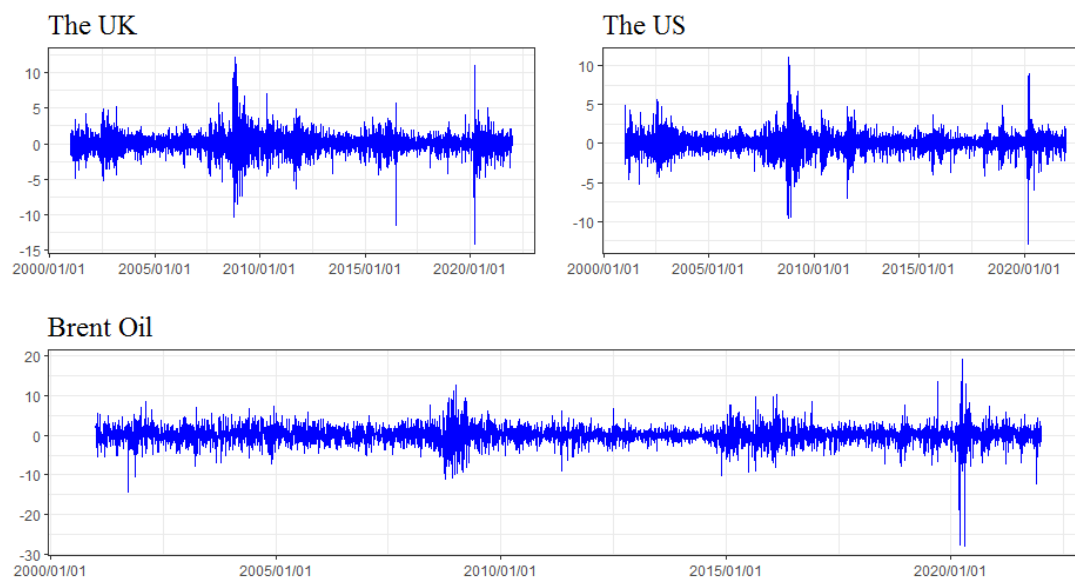


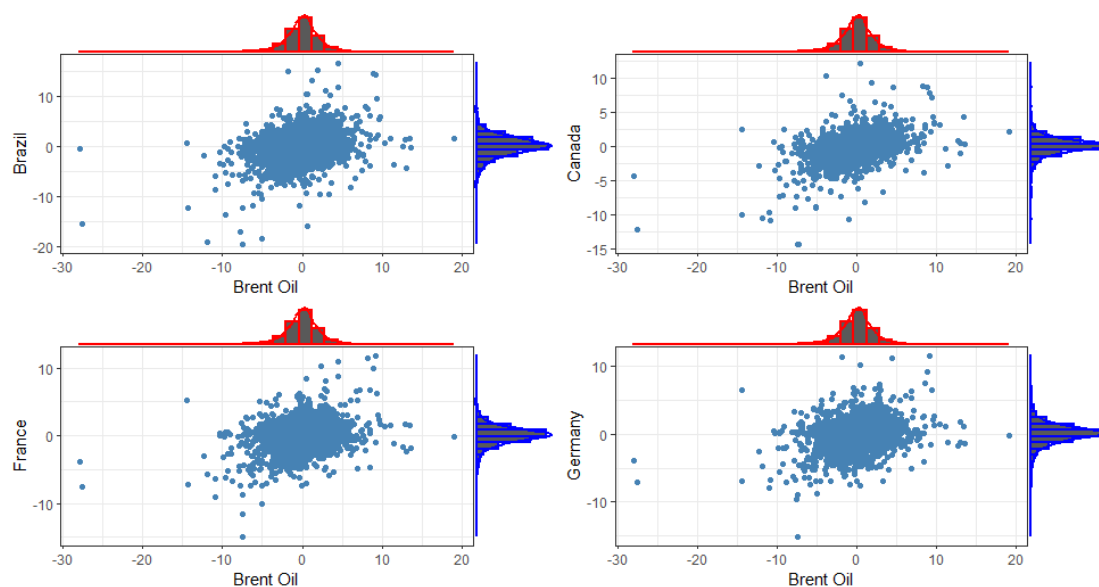
Fig. 2. Returns of the of different financial markets

Table 3. Descriptive statistics of MSCI index returns of stock markets and oil market

Statistics	Brazil	Canada	France	Germany	Italy	Mexico	Russia	South Africa	The UK	The US	Brent oil
Mean	0.016	0.022	0.018	0.029	0.021	0.028	0.019	0.002	0.023	0.017	0.019
Maximum	16.618	14.042	20.311	19.486	15.633	24.987	15.159	14.234	23.976	12.353	10.073
Minimum	-19.433	-17.173	-20.349	-17.791	-19.946	-20.672	-11.183	-20.401	-25.593	-13.566	-11.412
Median	0.074	0.014	0.062	0.056	0.011	0.028	0.062	0.023	0.065	0.106	0.086
S.D.	2.249	1.735	2.098	1.675	1.926	1.936	1.671	1.926	2.343	1.858	1.213
Skewness	-0.459	-0.152	-0.244	-0.659	-0.458	-0.279	-0.269	-0.508	-0.494	-0.447	-0.727
Kurtosis	8.110	6.966	9.120	12.318	8.521	12.104	6.212	7.328	11.645	4.456	9.118
Q0.05	-3.485	-2.765	-3.155	-2.539	-2.989	-2.944	-2.532	-2.962	-3.506	-3.020	-1.872
Q0.95	3.222	2.595	3.115	2.339	2.819	2.743	2.430	2.961	3.352	2.790	1.690
J-B	15024***	10966***	18810***	34608***	16564***	33109***	8769***	12342***	30798***	4659***	19223***
ρ	0.316***	0.432***	0.291***	0.260***	0.301***	0.284***	0.323***	0.293***	0.330***	0.267***	1.00***
Lj	91.598	61.262	126.890	95.767	94.720	51.688	116.780	32.663	105.020	74.617	262.130
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.026]	[0.000]	[0.000]	[0.000]
ARCH	1460.400	1051.300	1092.500	398.160	468.580	1082.400	1205.000	568.820	1214.800	1046.400	1287.300
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]

Notes: The data source is the Wind database. J-B denotes the Jarque-Bera statistics for normality. ρ is the pearson correlation coefficient between Brent crude oil and stock market returns. Lj denotes the Ljung-Box statistics for serial correlation in financial returns calculated with lag 20. ARCH is Engle's LM test for the ARCH effect in financial returns up to 20th order. p values are in square brackets and *** means rejection of the null hypothesis at 1% significance level.

Table 3 reports the descriptive statistics of these financial market returns. The means and medians of returns are close to zero, and high standard deviations of the returns imply large dispersion in volatility. All financial returns have negative skewness values and high values for the kurtosis statistic, consistent with the properties of sharp peaks, fat tails and being skewed for the return distributions. At the same time, the normality of stock returns is rejected by Jarque-Bera statistics. Furthermore, the results of Ljung-Box test reject the null hypothesis of autocorrelations at lag 20 at the 5% significance level; and Engle's Lagrange multiplier (LM) test reveals strong evidence of ARCH effects in all the financial return series at the 5% significance level. Finally, the correlation coefficients between returns of Brent crude oil market and stock markets are positive and significantly different from zero, which is in line with the scatter plots presented in Fig. 3. The scatter plots showing a nonlinear relationship in the upper and lower tails indicates that we should use the nonlinear model to study the risk spillover effect from oil market to stock markets.



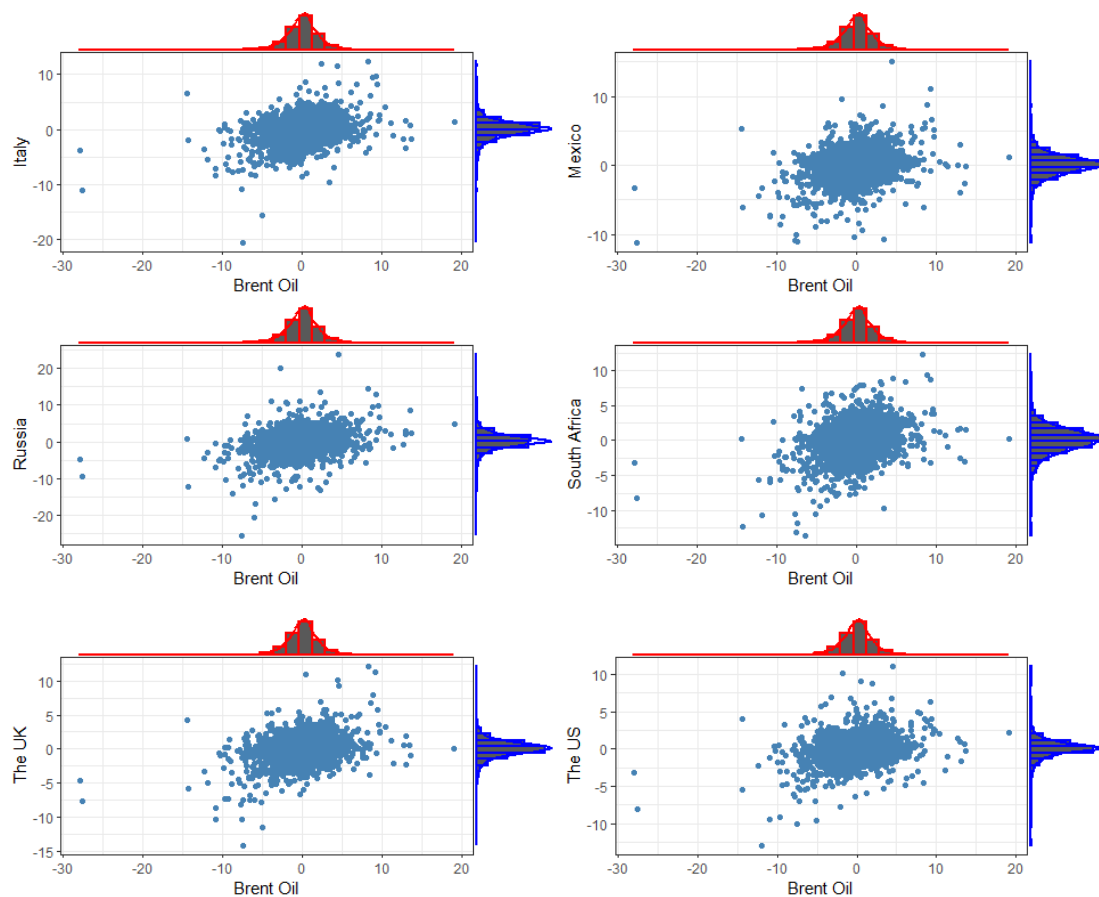


Fig. 3. Scatter plots of returns between the oil market and the stock markets

5. Empirical results

5.1. Estimates of the marginal distribution

As indicated in section 3, to capture the distribution properties of heavy tails, skewness, autocorrelation and volatility clustering, marginal distribution for oil and stock market returns are built on the ARMA-GARCH family models with the standard normal distribution, standardized Student's t distribution (SST) and SSST distribution, respectively. Table 4 shows the LLF (loglikelihood) and AIC (Akaike information criterion) criteria for model selection. Based on the values of AIC and LLF, the ARMA(1,1)-EGARCH(1,1) model with SSST innovation fit the financial returns best.

Table 5 presents the estimated parameters, the Ljung-Box and ARCH tests for model adequacy. Ljung-Box test applied to the standardized residuals (and the square of the standardized residuals) of the ARMA(1,1)-EGARCH(1,1) model with SSST innovation does

not reject the null hypothesis of autocorrelations at lag 20 at the 5% significance level. The Engle's LM test suggests the absence of ARCH effects in all the return series at the 5% significance level. The estimates of parameters and the standard deviations show that the ARMA(1,1)-EGARCH(1,1) model is adequate. Furthermore, the parameter estimates of the SSST distribution confirm that the standardized residuals do not follow the normal distribution, which is consistent with the negative values for skewness and high values for the kurtosis statistic reported in Table 3.

Table 4. Selections of marginal distribution

Countries	Distribution	ARMA(1,1)-GARCH(1,1)		ARMA(1,1)-EGARCH(1,1)	
		LLF	AIC	LLF	AIC
Brazil	Norm	-11029.800	4.195	-10994.634	4.182
	SST	-10917.382	4.153	-10890.717	4.143
	SSST	-10908.534	4.150	-10880.509	4.140
Canada	Norm	-7809.080	2.971	-7745.608	2.947
	SST	-7723.887	2.939	-7686.053	2.925
	SSST	-7693.777	2.928	-7650.474	2.912
France	Norm	-8700.268	3.310	-8620.990	3.280
	SST	-8597.730	3.271	-8529.335	3.246
	SSST	-8574.133	3.263	-8506.518	3.237
Germany	Norm	-8885.630	3.380	-8803.253	3.349
	SST	-8790.516	3.344	-8726.097	3.320
	SSST	-8769.788	3.337	-8708.576	3.314
Italy	Norm	-9193.586	3.497	-9125.864	3.472
	SST	-9082.730	3.456	-9032.564	3.437
	SSST	-9058.546	3.447	-9009.435	3.428
Mexico	Norm	-9269.542	3.526	-9204.993	3.502
	SST	-9167.409	3.488	-9122.598	3.471
	SSST	-9159.190	3.485	-9110.810	3.467
Russia	Norm	-10703.825	4.071	-10686.584	4.065
	SST	-10500.733	3.995	-10494.396	3.993
	SSST	-10494.503	3.993	-10486.081	3.990
South Africa	Norm	-10189.843	3.876	-10125.837	3.852
	SST	-10115.540	3.848	-10074.176	3.833
	SSST	-10098.796	3.842	-10051.949	3.825
The UK	Norm	-7975.335	3.034	-7905.093	3.008
	SST	-7880.987	2.999	-7826.575	2.978
	SSST	-7860.406	2.991	-7805.899	2.971
The US	Norm	-7129.271	2.713	-7037.372	2.678
	SST	-6996.184	2.662	-6903.931	2.628
	SSST	-6956.026	2.647	-6861.118	2.612
Oil	Norm	-11136.266	4.236	-11085.504	4.217
	SST	-10976.083	4.175	-10947.553	4.165
	SSST	-10966.274	4.172	-10933.979	4.160

Notes: Norm, SST and SSST denote the standard normal distribution, standardized Student's t distribution and standardized skew Student's t distribution, respectively. LLF and AIC are the value of the log likelihood function and Akaike information criterion.

Table 5. Parameter estimates of ARMA(1,1)-EGARCH(1,1) model with SSST innovation

Parameters	Brazil	Canada	France	Germany	Italy	Mexico	Russia	South Africa	The UK	The US	Oil
φ_0	0.008 (0.024)	0.021*** (0.007)	0.016* (0.009)	0.016 (0.011)	0.011 (0.015)	0.012 (0.019)	0.039* (0.022)	0.005 (0.024)	0.005 (0.008)	0.032*** (0.009)	0.012 (0.021)
φ_1	-0.002 (0.027)	-0.167 (0.560)	0.637*** (0.031)	-0.698*** (0.037)	-0.615*** (0.034)	-0.079 (0.083)	-0.074** (0.036)	-0.040 (0.055)	0.607*** (0.022)	0.363*** (0.038)	-0.074** (0.032)
ψ_1	0.062** (0.025)	0.213 (0.554)	-0.660*** (0.030)	0.685*** (0.038)	0.585*** (0.035)	0.163** (0.081)	0.108*** (0.036)	0.073 (0.055)	-0.638*** (0.022)	-0.431*** (0.038)	0.019 (0.034)
ω	0.026*** (0.002)	0.001 (0.002)	0.006*** (0.002)	0.007*** (0.002)	0.010*** (0.002)	0.014*** (0.002)	0.018*** (0.002)	0.023*** (0.002)	0.003 (0.002)	-0.005* (0.003)	0.013*** (0.001)
α_1	-0.071*** (0.008)	-0.078*** (0.007)	-0.105*** (0.009)	-0.092*** (0.007)	-0.086*** (0.008)	-0.091*** (0.009)	-0.051*** (0.008)	-0.089*** (0.008)	-0.097*** (0.008)	-0.148*** (0.009)	-0.052*** (0.007)
β_1	0.981*** (0.001)	0.987*** (0.001)	0.984*** (0.001)	0.985*** (0.001)	0.983*** (0.001)	0.980*** (0.001)	0.985*** (0.001)	0.978*** (0.001)	0.982*** (0.001)	0.978*** (0.001)	0.991*** (0.001)
γ_1	0.126*** (0.013)	0.142*** (0.012)	0.136*** (0.013)	0.130*** (0.014)	0.137*** (0.014)	0.123*** (0.013)	0.164*** (0.019)	0.115*** (0.013)	0.158*** (0.015)	0.150*** (0.013)	0.113*** (0.008)
ξ	0.916*** (0.018)	0.845*** (0.017)	0.875*** (0.017)	0.893*** (0.017)	0.875*** (0.017)	0.910*** (0.018)	0.933*** (0.018)	0.876*** (0.018)	0.882*** (0.017)	0.837*** (0.016)	0.905*** (0.017)
v	8.035*** (0.813)	11.110*** (1.499)	7.988*** (0.803)	8.261*** (0.895)	8.288*** (0.870)	8.320*** (0.881)	5.870*** (0.454)	10.045*** (1.256)	8.456*** (0.901)	7.115*** (0.682)	6.886*** (0.614)
Lj	20.604 [0.421]	27.613 [0.119]	21.614 [0.362]	18.848 [0.532]	18.180 [0.576]	22.304 [0.324]	21.994 [0.341]	20.603 [0.421]	16.955 [0.656]	21.558 [0.365]	17.088 [0.647]
Lj2	15.831 [0.727]	23.568 [0.262]	14.417 [0.809]	13.861 [0.838]	30.798 [0.058]	23.246 [0.277]	13.867 [0.837]	15.494 [0.748]	23.575 [0.261]	13.625 [0.849]	17.670 [0.609]
ARCH	15.240 [0.763]	24.984 [0.202]	13.999 [0.831]	13.571 [0.852]	30.904 [0.056]	22.745 [0.302]	14.564 [0.801]	15.452 [0.750]	23.667 [0.257]	13.122 [0.872]	17.175 [0.642]

Notes: Lj2 denotes the Ljung-Box statistics of squared residual with lag 20. See also note of Table 3.

5.2. Estimates and selections of the copulas

In this subsection, we will select the optimal copula functions for the oil market paired with each stock market based on the standardized residuals $(\varepsilon_{it}, \varepsilon_{st})$, by employing the inference function for margins (IFM) method (Nelsen, 2006). According to the LLF values presented in Table 6, the downside and upside tail dependence structure between ten stock markets and the oil market can be captured best by the rotated Gumbel copula and Gumbel copula, respectively. Therefore, we can estimate the downside and upside CoVaRs of the stock markets by the rotated Gumbel CQR model and the Gumbel CQR model, respectively.

Table 6. Parameter estimates for copulas and model selection statistics

Tail dependence	Copulas	Brazil		Canada		France		Germany		Italy	
		$\hat{\delta}$	LLF	$\hat{\delta}$	LLF	$\hat{\delta}$	LLF	$\hat{\delta}$	LLF	$\hat{\delta}$	LLF
Downside	Clayton	0.325	177.21	0.519	379.88	0.310	168.23	0.273	136.84	0.305	165.48
	Rotated Joe	1.245	153.03	1.407	339.78	1.234	151.83	1.206	126.53	1.227	148.28
	Rotated Gumbel	1.192	196.09	1.318	435.52	1.178	180.82	1.155	146.93	1.175	178.78
	Rotated Galambos	0.432	187.22	0.576	426.87	0.416	175.18	0.387	139.78	0.413	172.89
	Rotated Hüsler-Reiss	0.756	174.71	0.926	402.16	0.742	167.90	0.705	132.57	0.737	164.51
Upside	Rotated Clayton	0.284	127.85	0.478	311.81	0.243	98.99	0.205	71.41	0.248	99.15
	Joe	1.204	102.5	1.375	272.70	1.170	78.39	1.141	57.4	1.176	80.66
	Gumbel	1.181	163.20	1.312	394.20	1.158	131.90	1.135	100.20	1.161	134.00
	Galambos	0.420	155.21	0.566	379.47	0.392	124.17	0.363	90.26	0.395	124.29
	Hüsler-Reiss	0.747	146.80	0.916	359.51	0.708	114.56	0.669	81.37	0.713	114.80
Tail dependence	Copulas	Mexico		Russia		South Africa		The UK		The US	
		$\hat{\delta}$	LLF	$\hat{\delta}$	LLF	$\hat{\delta}$	LLF	$\hat{\delta}$	LLF	$\hat{\delta}$	LLF
Downside	Clayton	0.283	142.99	0.382	228.92	0.310	168.00	0.348	205.59	0.258	127.65
	Rotated Joe	1.211	130.51	1.296	201.33	1.231	150.62	1.263	186.89	1.186	110.19
	Rotated Gumbel	1.160	155.26	1.232	259.37	1.175	179.02	1.202	226.07	1.147	135.64
	Rotated Galambos	0.395	147.71	0.476	245.66	0.416	176.23	0.446	222.12	0.378	131.22
	Rotated Hüsler-Reiss	0.712	138.37	0.801	224.49	0.744	169.74	0.783	215.33	0.694	124.89
Upside	Rotated Clayton	0.219	81.78	0.348	179.68	0.238	96.35	0.288	135.42	0.218	77.76
	Joe	1.149	65.71	1.261	156.40	1.166	75.07	1.210	111.10	1.155	63.96
	Gumbel	1.140	110.40	1.224	234.00	1.155	127.70	1.187	175.30	1.141	105.6
	Galambos	0.372	103.72	0.467	217.74	0.389	121.47	0.426	167.92	0.370	95.92
	Hüsler-Reiss	0.687	97.30	0.796	201.82	0.708	114.80	0.754	159.28	0.681	88.06

Notes: LLF is the value of loglikelihood function.

5.3. Estimates of the CQR models

Based on $(\varepsilon_{it}, \varepsilon_{st})$, $t = 1, 2, \dots, 5,261$, the rotated Gumbel CQR model and the Gumbel CQR model are estimated at the 5% and 95% quantiles, respectively. The results of the coefficients estimates are presented in Table 7.

Table 7. Coefficient estimates of the CQR model

Quantiles	Estimates	Brazil	Canada	France	Germany	Italy
$\tau = 5\%$	$\hat{\delta}_\tau$	1.160*** (0.106)	1.578*** (0.564)	1.205*** (0.102)	1.217*** (0.149)	1.311*** (0.106)
	$\hat{\theta}_\tau$	-0.204 (0.566)	-0.804 (0.503)	-0.342 (0.366)	-0.405 (0.538)	-0.394 (0.390)
	$\hat{\eta}_\tau$	0.873*** (0.319)	0.578*** (0.225)	0.789*** (0.194)	0.769*** (0.287)	0.781*** (0.190)
$1 - \tau = 95\%$	$\hat{\delta}_{1-\tau}$	4.750* (2.840)	2.641** (1.240)	1.910* (1.147)	1.322*** (0.303)	1.977* (1.161)
	$\hat{\theta}_{1-\tau}$	1.412*** (0.116)	1.022*** (0.220)	1.028** (0.491)	0.621*** (0.551)	1.105*** (0.357)
	$\hat{\eta}_{1-\tau}$	0.303*** (0.062)	0.466*** (0.103)	0.432* (0.252)	0.701** (0.315)	0.351* (0.182)
Quantiles	Estimates	Mexico	Russia	South Africa	The UK	The US
$\tau = 5\%$	$\hat{\delta}_\tau$	1.201*** (0.149)	1.234*** (0.158)	1.314*** (0.089)	1.199*** (0.134)	1.216*** (0.129)
	$\hat{\theta}_\tau$	-0.396 (0.536)	-0.349 (0.320)	-0.365 (0.329)	-0.344 (0.500)	-0.393 (0.461)
	$\hat{\eta}_\tau$	0.768*** (0.295)	0.741*** (0.182)	0.790*** (0.174)	0.805*** (0.268)	0.750*** (0.239)
$1 - \tau = 95\%$	$\hat{\delta}_{1-\tau}$	1.954* (1.104)	2.868** (1.353)	1.500*** (0.439)	1.851** (0.899)	1.889* (1.012)
	$\hat{\theta}_{1-\tau}$	1.170*** (0.282)	1.189*** (0.191)	0.913*** (0.316)	1.043*** (0.311)	1.167*** (0.247)
	$\hat{\eta}_{1-\tau}$	0.364*** (0.124)	0.355*** (0.077)	0.463*** (0.162)	0.427*** (0.146)	0.301** (0.117)

Notes: Standard errors are in parentheses. ***, ** and * indicate significance at 0.01, 0.05 and 0.1 levels, respectively.

The estimates of δ in Table 7 show that the Canadian and Brazilian stock markets have the strongest downside and upside tail dependence with the oil market, respectively. Moreover, the panning parameters θ are estimated at negative values for 5% quantile and at positive values for 95% quantile, while the zooming parameters η are smaller than one.

5.4. Dynamic risk spillovers from oil to ten stock markets

Given the 95% confidence level ($\tau = \beta = 5\%$), the downside $CoVaR_{\tau|\beta,t}^{S|Oil}$ and upside $CoVaR_{1-\tau|1-\beta,t}^{S|Oil}$ of the stock markets can be calculated by equations (19) and (20), where the parameters have been estimated in subsections 5.1 and 5.3. To determine whether oil market has contribution to the stock markets, we apply the significance test to compare the dynamic CoVaRs of the stock markets. The results of significance test in the second columns of Table 8 indicate the rejection of the null hypothesis at the 1% significance level. Therefore, the oil market significantly contributes to the stock markets in ten countries.

Table 8. Results of significance test and asymmetric test

Countries	Statistics		
	Downside	Upside	Asymmetric
	$H_0: \Delta CoVaR_{\tau \beta,t}^{S Oil} = 0$	$H_0: \Delta CoVaR_{1-\tau 1-\beta,t}^{S Oil} = 0$	$\Delta CoVaR_{1-\tau 1-\beta,t}^{S Oil} \geq \Delta CoVaR_{\tau \beta,t}^{S Oil} $
Brazil	0.484*** [0.000]	0.331*** [0.000]	0.613*** [0.000]
Canada	0.423*** [0.000]	0.357*** [0.000]	0.344*** [1.000]
France	0.379*** [0.000]	0.288*** [0.000]	0.378*** [0.000]
Germany	0.396*** [0.000]	0.370*** [0.000]	0.148*** [0.000]
Italy	0.394*** [0.000]	0.262*** [0.000]	0.563*** [0.000]
Mexico	0.515*** [0.000]	0.397*** [0.000]	0.603*** [0.000]
Russia	0.453*** [0.000]	0.348*** [0.000]	0.410*** [0.000]
South Africa	0.545*** [0.000]	0.422*** [0.000]	0.615*** [0.000]
The UK	0.370*** [0.000]	0.271*** [0.000]	0.395*** [0.000]
The US	0.329*** [0.000]	0.168*** [0.000]	0.615*** [0.006]

Notes: The null hypothesis $\Delta CoVaR_{\tau|\beta,t}^{S|Oil} = 0$ means that there is no difference between the dynamic CoVaR of the stock market conditional on the oil market in the distress state and benchmark state. The null hypothesis $\Delta CoVaR_{1-\tau|1-\beta,t}^{S|Oil} \geq |\Delta CoVaR_{\tau|\beta,t}^{S|Oil}|$ means that the upside risk spillovers of stock market is greater or equal to the upside risk spillovers. p values for the KS test statistic are in square brackets. See also note of Table 7.

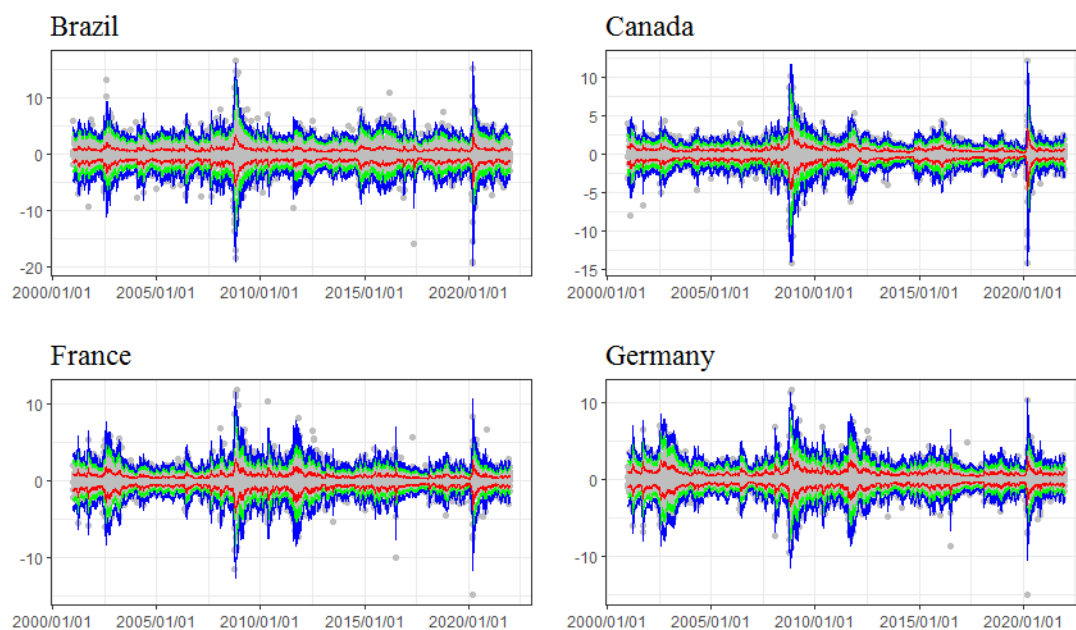
Panels A and B in Table 9 present the descriptive statistics of downside and upside risk spillovers. We see that the mean absolute value of the downside and upside risk spillovers from oil market to the Brazilian and Mexican stock markets are the largest, while risk spillovers to the US' stock market is the smallest. The standard deviation of the risk spillovers for the Mexican stock market is the highest.

Moreover, the comparison of risk spillovers in Panels A and B indicates that the mean absolute values of the downside risk spillovers are larger than that of the upside risk spillovers. That is the risk spillover effects exhibit significant asymmetric characteristics, which is corroborated by the asymmetry test reported in Table 8. The significant asymmetry between downside and upside risk spillovers suggests that the upside risk of oil market has lower impacts on the stock markets than its downside risk has, which is consistent with the flight-to-safety phenomenon. Furthermore, the asymmetric spillovers also reflect the asymmetric sensitivity of international capital holders, with portfolio managers and global investors responding less to bullish markets than to bearish ones. If oil price decrease dramatically, the stock price tends to fall sharply, and stockholders have tendencies to sell stocks to avoid risks. This type of responses causes capital outflows from the stock market and results in larger downside risks. Similarly, rising oil prices will lead to capital inflows into the stock market. But the capital outflows following bearish oil market can affect the stock to a larger extent than capital inflows following bullish oil market.

Table 9. Summary statistics for risk spillovers to ten stock markets

Quantiles	Ranking	Mean	S.D.	Max	Min	Median
Panel A: Downside risk spillovers						
Brazil	1	-1.398	0.534	-0.637	-5.955	-1.285
Canada	8	-0.864	0.485	-0.229	-4.707	-0.750
France	6	-0.988	0.489	-0.319	-4.218	-0.861
Germany	7	-0.969	0.460	-0.353	-3.622	-0.852
Italy	4	-1.055	0.486	-0.374	-4.507	-0.949
Mexico	2	-1.371	0.648	-0.555	-6.612	-1.202
Russia	3	-1.194	0.398	-0.572	-4.379	-1.109
South Africa	9	-0.862	0.439	-0.281	-4.268	-0.760
The UK	5	-0.990	0.392	-0.438	-4.212	-0.899
The US	10	-0.759	0.450	-0.213	-4.820	-0.635
Panel B: Upside risk spillovers						
Brazil	3	0.860	0.329	3.664	0.392	0.790
Canada	7	0.628	0.352	3.417	0.166	0.545
France	5	0.665	0.329	2.838	0.215	0.579
Germany	2	0.861	0.409	3.218	0.314	0.757
Italy	8	0.591	0.272	2.524	0.210	0.531
Mexico	1	0.962	0.455	4.638	0.389	0.843
Russia	4	0.769	0.257	2.821	0.368	0.714
South Africa	9	0.567	0.289	2.806	0.185	0.500
The UK	6	0.633	0.251	2.695	0.280	0.575
The US	10	0.337	0.200	2.140	0.094	0.282

Fig. 4 presents the dynamics of CoVaR and Δ CoVaR for the ten stock markets during the period under analysis at the 0.95 confidence level. First, the dynamic CoVaR and Δ CoVaR for each stock market are comparatively different indicating that the impact of extreme risk in the oil market on stock markets' extreme risk tends to vary by country. While the CoVaR and Δ CoVaR also have the similar shape, showing regional characteristics. For example, the risk spillovers to the American, Brazilian, Canadian and Mexican stock market follow the same trend. Moreover, the abrupt changes of the CoVaR and Δ CoVaR clearly reflect the impacts of some important risk events, such as the global financial crisis in 2008, the European debt crisis in 2010 and the COVID-19 crisis in 2020. For example, the CoVaR and Δ CoVaR for the US stock market were extremely large during the global financial crisis and the COVID-19 crisis. The European debt crisis and Brexit also had a great impact on risk spillover effects for the UK, Germany, France and Italy. Specifically, due to the Russia-Ukraine conflict, the Russian stock market was severely affected by the volatile oil market. Therefore, the sharp down and up movements in the stock markets are not only from the macroeconomic environment but also the external geopolitical shocks which affect the oil price. Finally, this graphical evidence also suggests that the downside risk spillovers are shown to be noticeably greater than upside risk spillovers, which is consistent with the results of asymmetric tests in Table 8.



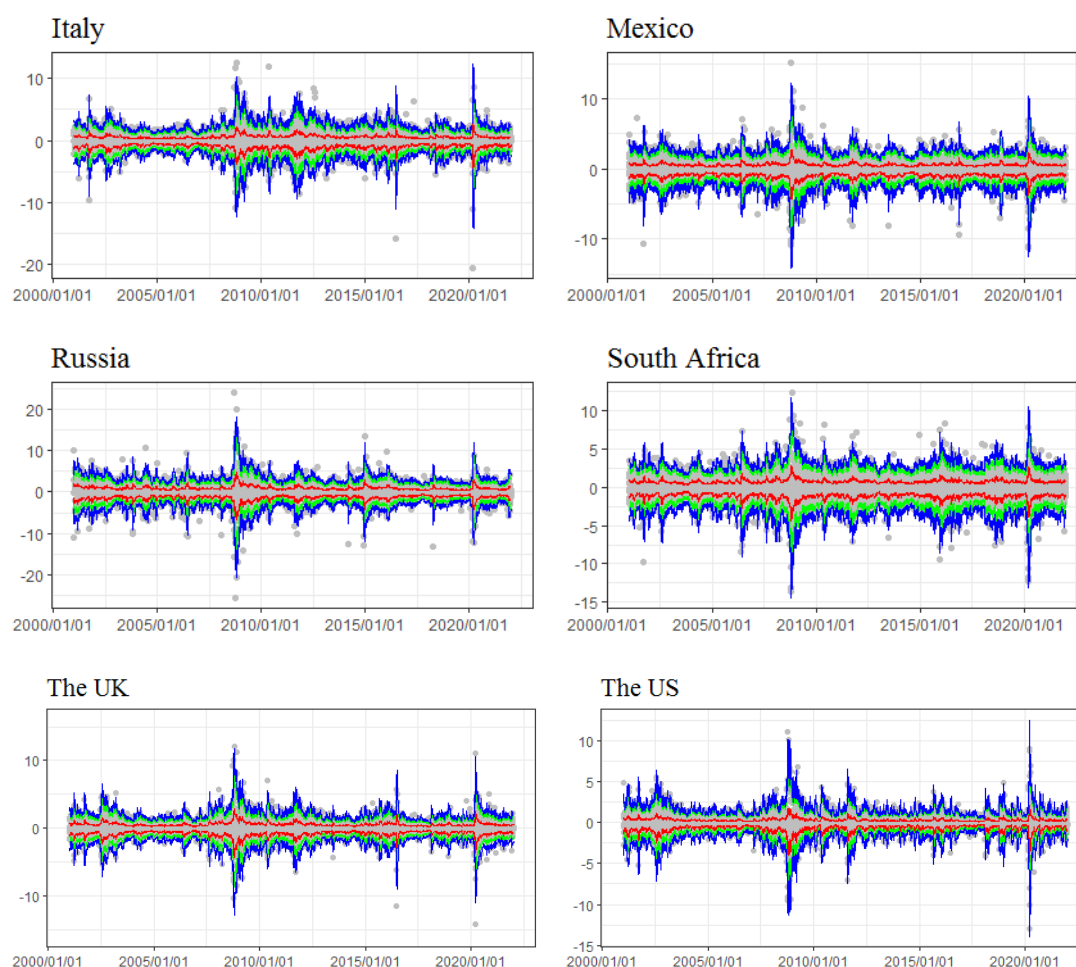


Fig. 4. Dynamics CoVaR and risk spillovers

Notes: In each subfigure, the gray points are the stock market returns for ten economies. The green lines and blue lines stand for the CoVaRs of the benchmark state and the distressed state, respectively. The red lines stand for the downside or the upside risk spillovers.

5.5. Implications for global investors and international supervisory authorities

The findings in this paper have important policy implications for and global investors. First, the asymmetric risk spillovers, with downside risk spillovers larger than upside risk spillovers suggests that the fund managers and global investors with long positions on the stock markets will face larger risk over the times of bearish oil market than those with short positions during the periods of bullish oil market. Consequently, to reduce asset losses due to the oil market's risk spillovers, fund managers and global investors should evaluate comprehensively the risk measurement of risk contagions and accordingly adjust their positions to optimize portfolio strategy.

Moreover, the greater downside risk spillover from oil market to the Brazilian stock

market indicates that portfolio managers with long positions of the Brazilian stock market could suffer larger risk during the period of bearish oil market than of other stock markets. Thus, portfolio managers and global investors should close their long positions or allocate proper instruments to hedge the downside risk spillovers timely during the period of the oil market crisis, especially for global investors with long positions of the Brazilian stock market. For greater upside risk spillovers from oil market to the Mexican stock market, the policy implications are similar, but considering the opposite positions, upside risk and bullish oil market.

Considering the fact that the sharp changes of the oil price could trigger the extreme risk of stock markets, the financial regulators should closely monitor and effectively contain the impacts of the oil market's extreme risk. Specifically, identifying the ranking of the risk spillovers to the stock markets conditional on the oil market returns decrease or increase dramatically, can help regulatory authorities precisely locate the riskiest stock markets. Therefore, it is necessary for the supervisory authorities to regulate the Brazilian and Mexican stock markets rather than simply focusing on the supervision of the stock markets with the higher market capitalization, such as the US stock market.

6. Conclusions

Estimating the downside and upside risk spillovers from the oil market to the stock markets and accordingly identifying the riskiest stock markets are essential for international capital holders and supervisory authorities. To accurately evaluate the downside risk spillovers, Tian and Ji (2022) propose the GARCH CQR model that can describe the nonlinearity of the downside tail dependence structure between financial variables. As is known, measuring the upside risk and its spillovers is also critical especially for global investors with short positions of the stock markets. In response to the problem that the model in Tian and Ji (2022) cannot capture the nonlinearity of the upside tail dependence between financial variables, this study constructs a GARCH CQR-based UCoVaR model to calculate upside CoVaR and risk spillovers.

In the empirical study, based on the MSCI daily data from January 2000 to June 2021, we assess the risk contribution of Brent crude oil to stock markets in ten important economies, using the GARCH CQR-based DCoVaR and UCoVaR models. The empirical results reveal that oil displays the largest downside and upside risk spillover effects on the Brazilian and

Mexican stock markets, respectively. And the US stock market displays the smallest risk spillovers from the oil market. We also find that the downside and upside risk spillovers show the asymmetric feature, with upside risk spillovers less than downside risk spillovers, which is consistent with the phenomenon of flight-to-quality. Moreover, the dynamic risk spillover effects show heterogeneity over time and are comparatively different for each country. Finally, based on these findings, we provide important implications for international capital holders and supervisory authorities optimizing the investment portfolios and formulating supervision policy.

Statement of Conflict of Interest:

The authors declare no conflict of interest.

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Appendix A. Simulations of different CQR models

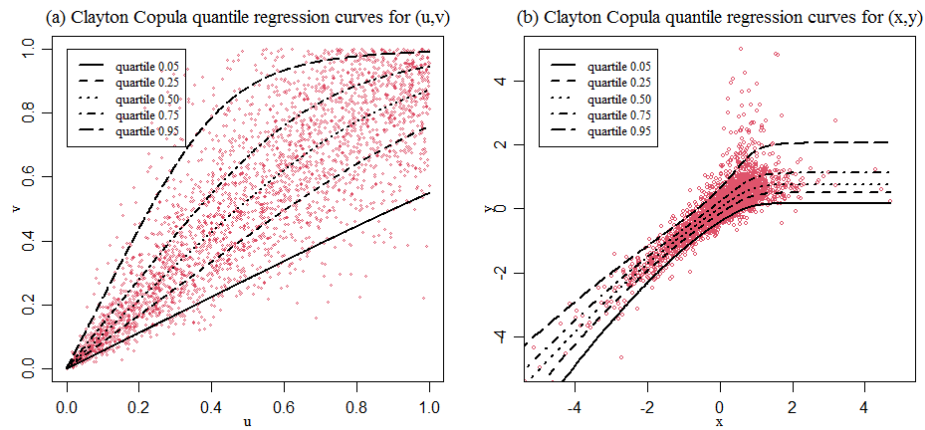


Fig. A1. The Clayton CQR curves

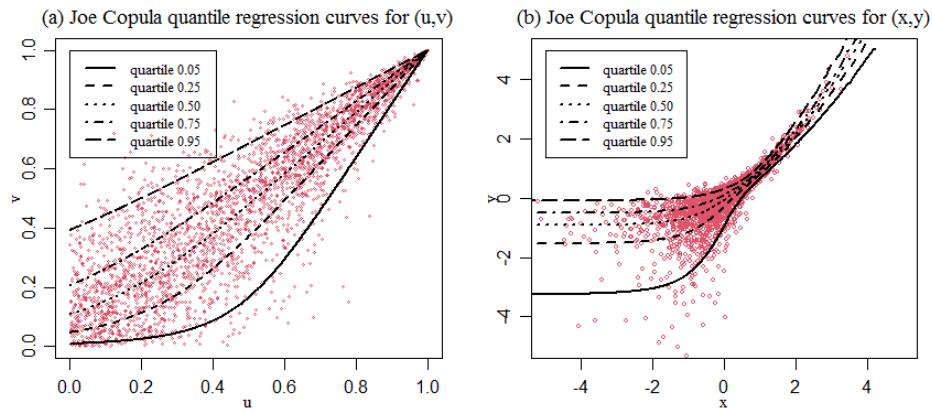


Fig. A2. The Joe CQR curves

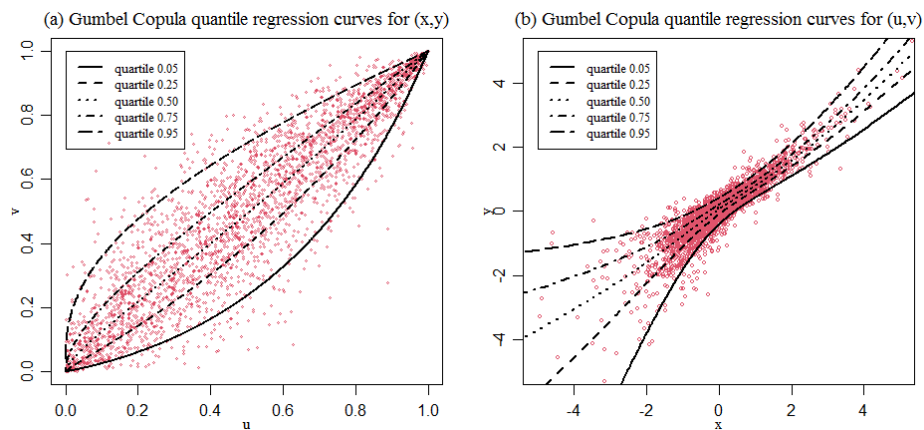


Fig A3. The Gumbel CQR curves

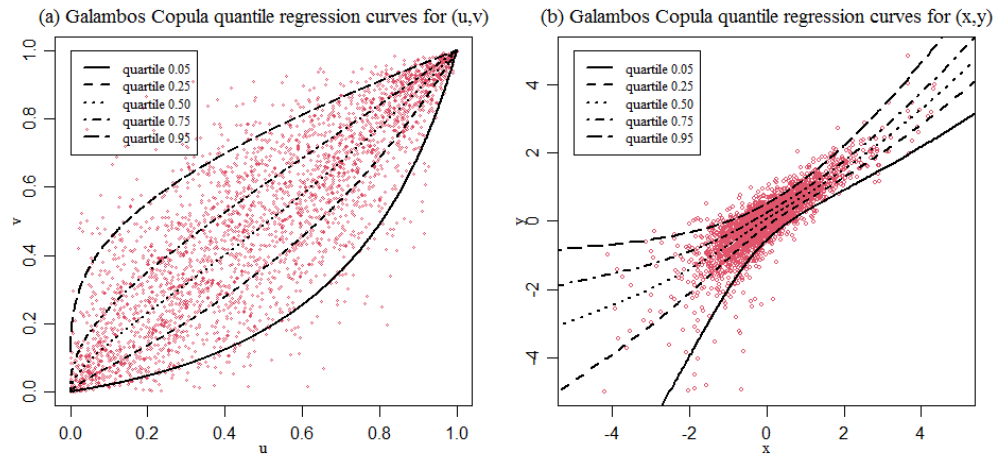


Fig. A4. The Galambos CQR curves

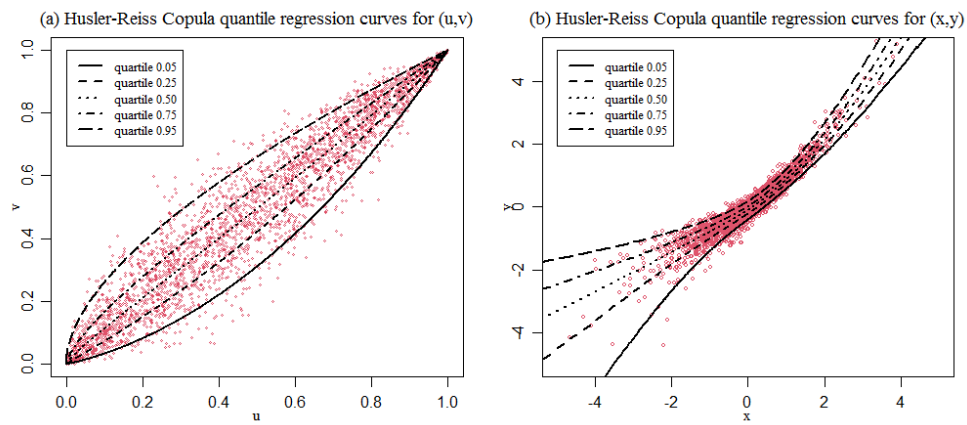


Fig. A5. The Hüsler-Reiss CQR curves

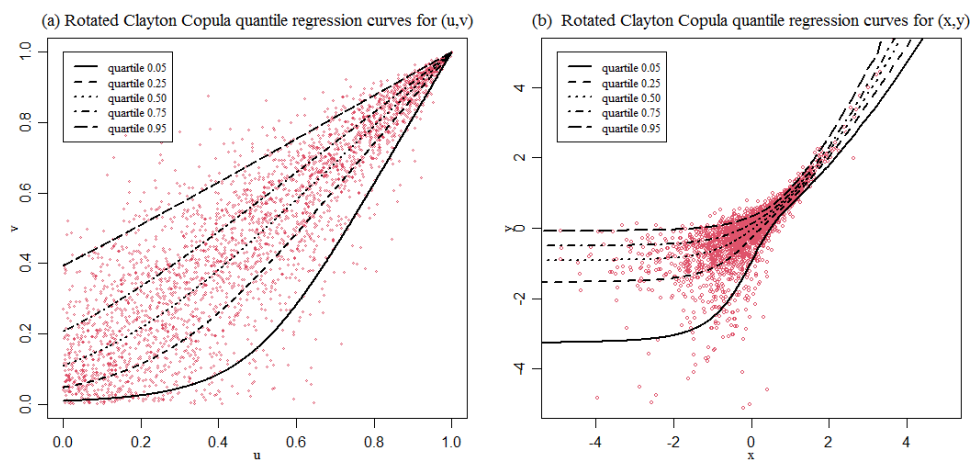


Fig. A6. The rotated Clayton CQR curves

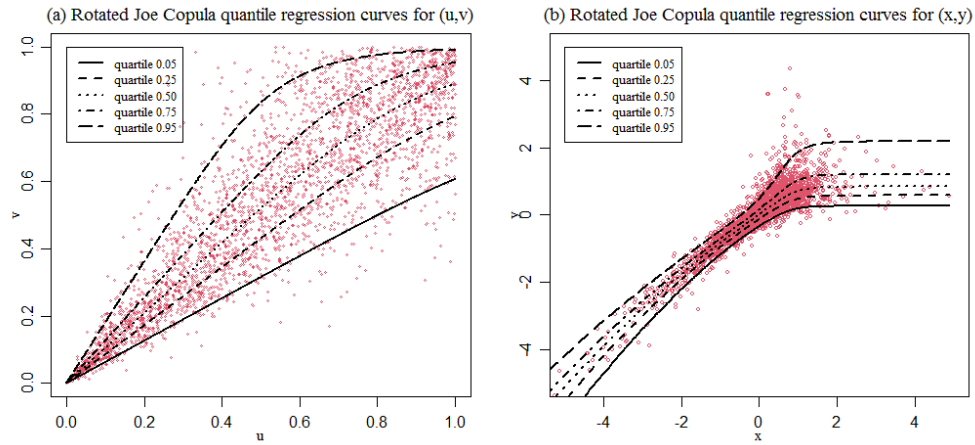


Fig. A7. The rotated Joe CQR curves

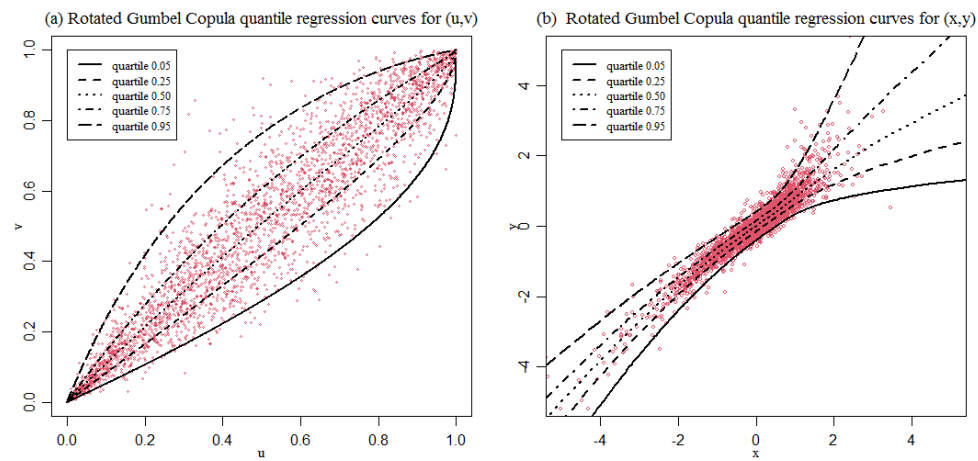


Fig. A8. The rotated Gumbel CQR curves

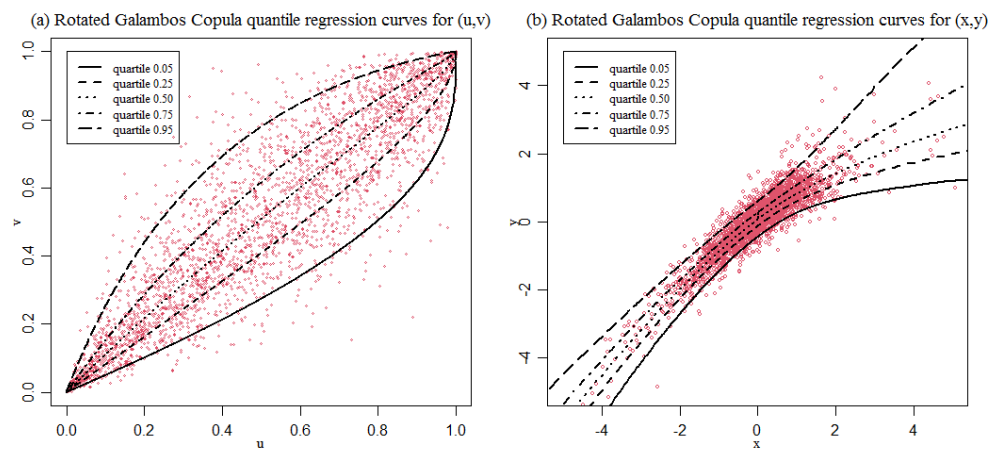


Fig. A9. The rotated Galambos CQR curves

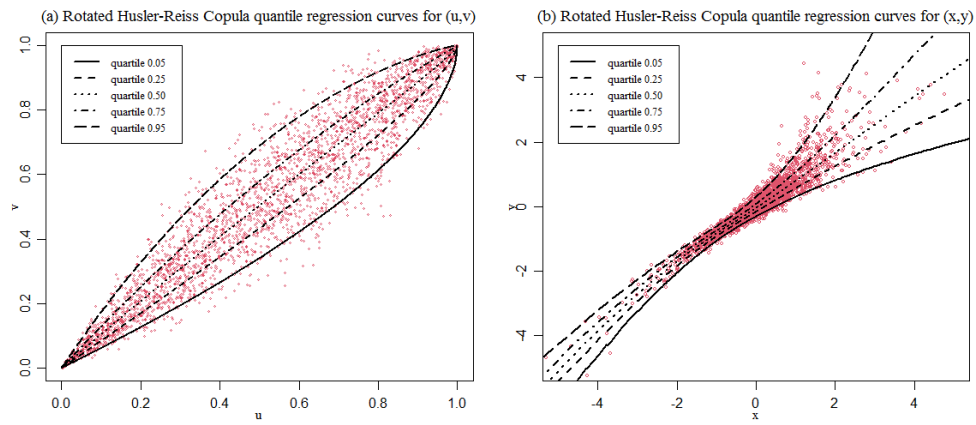


Fig. A10. The rotated Hüsler-Reiss CQR curves