$$\begin{aligned} x - y + 2z - 3w &= 0 \\ y - 2z + 2w &= 0 \\ -4z + 15w &= 0 \end{aligned}$$

Choosing $w = k$, an arbitrary constant, we have $z = \frac{15}{4}k$, $y = \frac{11}{2}k$ and $x = k$. Giving various values to k, we get an infinite number of solutions.

Exercise 1.8

1. Discuss the consistency and inconsistency of the system of linear non-homogeneous and homogeneous equations with the help of rank of matrices and augmentation of linear equations.

2. Test the consistency of the following system of equations and solve, if found consistent.
   i. $5x + 3y + 7z = 4,\ 3x + 26y + 2z = 9,\ 7x + 2y + 10z = 5$
   ii. $2x + 3y + 4z = 11,\ x + 5y + 7z = 15,\ 3x + 11y + 13z = 25$
   iii. $3x + 3y + 2z = 1,\ x + 2y = 4,\ 10y + 3z = -2,\ 2x - 3y - z = 5$
   iv. $x + y + z = -1,\ 3x + y - 2z = -2,\ 2x + 4y + 7z = 7$
   v. $x - y + z = 1,\ 2x - 2y + 3z = 2,\ x + 2y - z = 3$
   vi. $x + y + z = 3,\ x + 2y + 3z = 4,\ 2x + 3y + 4z = 7$
   vii. $x - 3y - 8z = -10,\ 3x + y - 4z = 0,\ 2x + 5y + 6z = 13$

3. Determine for what values of λ and μ the equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ have
   (i) no solution
   (ii) a unique solution
   (iii) an infinite number of solutions.

4. Find for what value of k the set of equations $2x - 3y + 6z - 5t = 3$, $y - 4z + t = 1$, $4x - 5y + 8z - 9t = k$ has (i) no solution (ii) an infinite number of solutions?

5. Find the value of λ for which the system of equations $x + y + 4z = 1$, $x + 2y - 2z = 1$, $\lambda x + y + z = 1$ will have a unique solution ?

6. Show that the equations $3x + 4y + 5z = a$, $4x + 5y + 6z = b$, $5x + 6y + 7z = c$ do not have a solution unless $a + c = 2b$. Also, solve the equations when $a = b = c = 1$.

7. Solve completely the following system of homogeneous equations by matrix method :
   (i) $2x - y + z = 0,\ 3x + 2y + z = 0,\ x - 3y + 5z = 0$
   (ii) $x + 2y + 3z = 0,\ 3x + 4y + 4z = 0,\ 7x + 10y + 12z = 0$
   (iii) $x - y + 2z - 3w = 0,\ 3x + 2y - 4z + w = 0,\ 5x - 3y + 2z + 6w = 0$
   (iv) $2x - 2y + 5z + 3w = 0,\ 4x - y + z + w = 0,$
   $3x - 2y + 3z + 4w = 0,\ x - 3y + 7z + 6w = 0$

8. Find the value of k such that the system of equations $2x + 3y - 2z = 0$, $3x - y + 3z = 0$, $7x + ky + 5z = 0$ has non trivial solution ? Also find the solutions.

9. Find the value of k such that the system of equations $4x + 9y + z = 0$, $kx + 3y + kz = 0$, $x + 4y + 2z = 0$ has non trivial solution ? Also find the solutions.

□□□

Answers:

2. i. *consistent with infinite solutions viz.* $x = \dfrac{-16k+7}{11}$, $y = \dfrac{k+3}{11}$, $z = k$

   ii. *consistent,* $x = 2, y = -3, z = 4$

   iii. *consistent,* $x = 2, y = 1, z = -4$

   iv. *inconsistent*

   v. *consistent,* $x = \dfrac{5}{3}, y = \dfrac{2}{3}, z = 0$

   vi. *consistent with infinite solutions viz.* $x = k + 2$, $y = 1 - 2k, z = k$

   vii. *consistent with infinite solutions viz.* $x = -1 + 2k$, $y = 3 - 2k, z = k$

3. i. $\lambda = 3$ *and* $\mu \neq 10$ ii. $\lambda \neq 3$ *and* μ *may have any value.*

   iii. $\lambda = 3$ *and* $\mu = 10$

4. i. $k \neq 7$ ii. $k = 7$

5. $\lambda \neq \dfrac{7}{10}$ 6. $x = k + 1,\ y = -1 - 2k,\ z = k$

7. i. *The trivial solution* $x = y = z = 0$ *is the only solution.*

   ii. *The trivial solution* $x = y = z = 0$ *is the only solution.*

   iii. $k, \dfrac{13}{2}k, \dfrac{17}{4}k, k$.

   iv. $\dfrac{5}{9}k, 4k, \dfrac{7}{9}k, k$

8. $k = 5, x = \dfrac{-7a}{11}, y = \dfrac{12a}{11}, z = a$ *where a is an arbitrary number.*

9. $k = 1, x = 2\lambda, y = -\lambda, z = \lambda$