* smallest eigen value and corresponding eigen vector

For smallest eigen value and corresponding eigen vector we work on the inverse of given matrix and we find largest eigen value and corresponding eigen vector of inverse matrix. The smallest eigen value is the reciprocal of eigen value of inverse matrix and corresponding eigen vector is the eigen vector of inverse matrix.

Q Find smallest eigen value and corresponding eigen vector.

$$A = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & 4 \end{bmatrix}$$

Solution:

By calculator

$$B = A^{-1} = \begin{bmatrix} 0.0422 & -0.014 & -0.021 \\ -0.014 & 0.338 & 0.0070 \\ -0.021 & 0.0070 & 0.2605 \end{bmatrix}$$

$$\text{let } X_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$BX_0 = \begin{bmatrix} 0.0422 & -0.014 & -0.021 \\ -0.0140 & 0.338 & 0.0070 \\ -0.021 & 0.0070 & 0.2605 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Calculation look carefully while practising in exam.

$$= \begin{bmatrix} 0.0071 \\ 0.3310 \\ 0.2464 \end{bmatrix}$$

$$X_1 = \frac{1}{0.3310}\begin{bmatrix} 0.0071 \\ 0.3310 \\ 0.2464 \end{bmatrix}, \; k_1 = 0.3310$$

$$= \begin{bmatrix} 0.021 \\ 1 \\ 0.7444 \end{bmatrix}$$

$$BX_1 = \begin{bmatrix} 0.0422 & -0.014 & -0.021 \\ -0.0140 & 0.338 & 0.0070 \\ -0.0211 & 0.0070 & 0.2605 \end{bmatrix}\begin{bmatrix} 0.021 \\ 1 \\ 0.7444 \end{bmatrix}$$

$$= \begin{bmatrix} -0.0288 \\ 0.3429 \\ 0.2004 \end{bmatrix}$$

$$X_2 = \frac{1}{0.3429}\begin{bmatrix} -0.0288 \\ 0.3429 \\ 0.2004 \end{bmatrix} \quad k_2 = 0.3429$$

$$= \begin{bmatrix} -0.0839 \\ 1 \\ 0.5844 \end{bmatrix}$$

$$AX_2 = \begin{bmatrix} -0.0298 \\ 0.3432 \\ 0.161 \end{bmatrix}$$

$$X_3 = \frac{1}{0.3432}\begin{bmatrix} -0.0298 \\ 0.3432 \\ 0.161 \end{bmatrix}, \; X_3 = 0.3432$$

$$= \begin{bmatrix} -0.0868 \\ 1 \\ 0.4691 \end{bmatrix}$$

$\therefore$ smallest eigen value $\lambda = \frac{1}{0.3432}$

$\therefore \lambda = 2.9137$

Corresponding eigen vector $= \begin{bmatrix} -0.0868 \\ 1 \\ 0.4691 \end{bmatrix}$

Chapter-5

Solution of Differential Equation

Solution of Differential Equation by Euler's method:

Let $\frac{dy}{dx} = f(x,y),\ y(x_0) = y_0$ be a differential equation.

Expanding $y(x)$ in Taylor's series as;

$$y(x) = y(x_0) + (x-x_0)y'(x_0) + \frac{(x-x_0)^2}{2!}y''(x_0) + \frac{(x-x_0)^3}{3!}y'''(x_0) + \cdots \quad (1)$$

or, $y(x) = y(x_0) + (x - x_0)\, y'(x_0)$ Neglecting the terms containing higher power of $(x - x_0)$

or, $y(x_1) = y(x_0) + (x_1 - x_0)\, y'(x_0)$

or, $y_1 = y_0 + h\, y'(x_0)$ - (2)

From (1)

$\frac{dy}{dx} = f(x, y)$

or, $y' = f(x, y)$

or, $y'(x) = f\{x, y(x)\}$

or, $y'(x_0) = f\{x_0, y(x_0)\}$

or, $y'(x_0) = f(x_0, y_0)$

Now (2) is

$y_1 = y_0 + h f(x_0, y_0)$

$y_2 = y_1 + h f(x_1, y_1)$

$y_3 = y_2 + h f(x_2, y_2)$

$$\boxed{\therefore y_{n+1} = y_n + h f(x_n, y_n)}$$

(6) (a) Use Euler's method to solve the following equation for $y(1)$, using $h = 0.25$

$\frac{dy}{dx} = x + y + xy$, $y(0) = 1$

Solution:

$f(x, y) = x + y + xy$ - (1)

$y(0) = 1$ is given - (2)

Comparing (2) with

$y(x_0) = y_0$

$x_0 = 0$

$y_0 = 1$

we have,

$y_{n+1} = y_n + h f(x_n, y_n)$ Formula (3)

$y_1 = y_0 + h f(x_0, y_0)$ - (4)

Here, $f(x_0, y_0) = x_0 + y_0 + x_0 y_0$

$= 0 + 1 + 0 \times 1$

$= 1$

Now (4) is

$y_1 = 1 + 0.25 \times 1$

$y_1 = 1.25$

$y(x_1) = 1.25$

or, $y(x_0 + h) = 1.25$

or, $y(0 + 0.25) = 1.25$

$\therefore y(0.25) = 1.25$

from (3)

$y_2 = y_1 + h f(x_1, y_1)$ (5)

$x_1 = x_0 + h = 0 + 0.25 = 0.25$

$y_1 = 1.25$

$F(x_1, y_1) = x_1 + y_1 + x_1 \times y_1$

$= 0.25 + 1.25 + 0.3125$

$= 1.8125$

Now (5) is,

$y_2 = 1.25 + 0.25 \times 1.8125$

$y_2 = 1.703125$

$y(x_2) = 1.703125$

or, $y(x_1 + h) = 1.703125$

or, $y(0.25 + 0.25) = 1.703125$

$\therefore y(0.5) = 1.703125$

From (3)

$y_3 = y_2 + hF(x_2, y_2)$ — (6)

$F(x_2, y_2) = x_2 + y_2 + x_2 y_2$

$= 0.5 + 1.7031 + 0.5 \times 1.7031$

$= 3.0546$

Now (6) is

$y_3 = 1.7031 + 0.25 \times 3.0546$

$y_3 = 2.4667$

$y(x_3) = 2.4667$

or, $y(x_2 + h) = 2.4667$

or, $y(0.5 + 0.25) = 2.4667$

or, $y(0.75) = 2.4667$

From (3),

$y_4 = y_3 + hF(x_3, y_3)$ — (7)

$F(x_3, y_3) = x_3 + y_3 + x_3 y_3$

$= 0.75 + 2.4667 + 0.75 \times 2.4667$

$= 5.0667$

Now (7) is

$y_4 = 2.4667 + 0.25 \times 5.0667$

or, $y_4 = 3.7333$

or, $y(x_4) = 3.7333$

or, $y(x_3 + h) = 3.7333$

or, $y(0.75 + 0.25) = 3.7333$

$\therefore y(1) = 3.7333$ //

2074

(5)

Q. Solution of differential at $x = 0.5$ taking $h = 0.25$ by using Heun's method

From

$$\frac{dy}{dx} + 0.4y = 3e^{-x}, \quad y(0) = 5$$

Solution:

$$\frac{dy}{dx} = 3e^{-x} - 0.4y \quad -(1)$$

Or, $f(x,y) = 3e^{-x} - 0.4y$

$y(0) = 5$ given $-(2)$

Comparing (2) with

$y(x_0) = y_0$

$\therefore x_0 = 0$

$y_0 = 5$

We have,

$$y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_{n+1}, y_{n+1}^{e})\right]$$

formula (3)

$$y_1 = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{e})\right] \quad (4)$$

$f(x_0, y_0) = 3e^{-x_0} - 0.4y_0$

$= 3e^{-0} - 0.4 \times 5$

$= 3 - 2 = 1$

$x_1 = x_0 + h = 0 + 0.25 = 0.25$

$y_1^{e} = y_0 + hf(x_0, y_0)$

$= 5 + 0.25 \times 1$

$= 5.25$

$f(x_1, y_1^{e}) = 3e^{-x_1} - 0.4y_1^{e}$

$= 3e^{-0.25} - 0.4 \times 5.25$

$= 0.2364$

Now (4) is

$y_1 = 5 + \frac{0.25}{2}\left[1 + 0.2364\right]$

$\therefore y_1 = 5.1545$

$y(x_1) = 5.1545$

$\therefore y(0.25) = 5.1545$

From (3)

$$y_2 = y_1 + \frac{h}{2}\left[f(x_1, y_1) + f(x_2, y_2^{e})\right] \quad -(5)$$

$f(x_1, y_1) = 3e^{-x_1} - 0.4y_1$

$= 3e^{-0.25} - 0.4 \times 5.1545$

$= 0.2746$

$x_2 = x_1 + h = 0.25 + 0.25 = 0.5$

$y_2^{e} = y_1 + hf(x_1, y_1)$

$= 5.1545 + 0.25 \times 0.2746$

$= 5.2231$

$f(x_2, y_2^{e}) = 3e^{-x_2} - 0.4y_2^{e}$

$= 3e^{-0.5} - 0.4 \times 5.2231 = -0.2696$

Now (5)

$y_2 = 5.1545 + \frac{0.25}{2}\left[0.2746 - 0.2696\right]$

$y_2 = 5.1551$

$y(x_2) = 5.1551 \quad \therefore y(0.5) = 5.1551$

Fourth order or classical R-K method

2023
6 (a) Solve $\frac{dy}{dx} = \frac{2xy + e^x}{x^2 + xe^x}$ given $x_0 = 1, y_0 = 0$

For y at $x = 1.2, 1.4$

Solution:

$$F(x, y) = \frac{2xy + e^x}{x^2 + xe^x}$$

$x_0 = 1$

$y_0 = 0$

taking $h = 0.2$

$$y_{n+1} = y_n + \frac{h}{6}(m_1 + 2m_2 + 2m_3 + m_4) \quad \text{(i)}$$

$m_1 = f(x_n, y_n)$

$m_2 = F(x_n + h/2, y_n + m_1 h/2)$

$m_3 = F(x_n + h/2, y_n + m_2 h/2)$

$m_4 = F(x_n + h, y_n + m_3 h)$

$$\therefore y = y_0 + \frac{h}{6}(m_1 + 2m_2 + 2m_3 + m_4) \quad \text{(1)}$$

$m_1 = f(x_0, y_0)$

$= F(1, 0)$

$= \frac{2 \times 1 \times 0 + e^1}{1^2 + e^1}$ $\quad m_1 = 0.7310$

$m_2 = F(x_0 + h/2, y_0 + m_1 h/2)$

$= F\left(1 + \frac{0.2}{2}, 0 + \frac{0.7310 \times 0.2}{2}\right)$

$= F(1.1, 0.0731)$

$= \frac{2 \times 1.1 \times 0.0731 + e^{1.1}}{(1.1)^2 + 1.1 \times e^{1.1}}$

$m_2 = 0.7010$

$m_3 = F(x_0 + h/2, y_0 + m_2/2\, h)$

$= F\left(1.1, 0 + \frac{0.7010 \times 0.2}{2}\right)$

$= F(1.1, 0.0701)$

$= \frac{2 \times 1.1 \times 0.0701 + e^{1.1}}{(1.1)^2 + 1.1 \times e^{1.1}}$

$m_3 = 0.6995$

$m_4 = F(x_0 + h, y_0 + m_3 h)$

$= F(1 + 0.2, 0 + 0.6995 \times 0.2)$

$= F(1.2, 0.1399)$

$= \frac{2 \times 1.2 \times 0.1399 + e^{1.2}}{(1.2)^2 + 1.2 \times e^{1.2}}$

$m_4 = 0.6740$

Now (i) is

$y_1 = 0 + \frac{0.2}{6}(0.7310 + 2 \times 0.7010 + 2 \times 0.6995 + 0.6740)$

$\therefore y_1 = 0.1402$

$y(x_1) = 0.1402$

or, $y(x_0 + h) = 0.1402$

or, $y(1 + 0.2) = 0.1402$

$\therefore y(1.2) = 0.1402$

$y_2 = y_1 + \frac{h}{6}(m_1 + 2m_2 + 2m_3 + m_4)$.

$m_1 = f(x_1, y_1)$

$= f(1.2, 0.1402)$

$= \frac{2 \times 1.2 \times 0.1402 + e^{1.2}}{1.2 e^{1.2} + (1.2)^2}$

$= 0.6741$

$m_2 = f(x_1 + h/2, y_1 + m_1 h/2)$

$= f\left(1.2 + \frac{0.2}{2}, 0.1402 + 0.6741 \times \frac{0.2}{2}\right)$

$= f(1.3, 0.2076)$

$= \frac{2 \times 1.3 \times 0.2076 + e^{1.3}}{(1.3)^2 + 1.3 e^{1.3}}$

$= 0.6515$

$m_3 = 0.6506$

$m_4 = f(x_1 + h, y_1 + m_3 h)$

$= f(1.2 + 0.2, 0.1402 + 0.6506 \times 0.2)$

$= f(1.4, 0.2703)$

$= \frac{2 \times 1.4 \times 0.2703 + e^{1.4}}{(1.4)^2 + 1.4 e^{1.4}}$

$= 0.6300$

Now

$y_2 = 0.1402 + \frac{0.2}{6}[0.6741 + 2 \times 0.6515 + 2 \times 0.6506 + 0.6300]$

$y(x_2) = 0.2706$, $y(x_1 + h) = 0.2706$

or, $y(1.4) = 0.2706$ // $y(1.2 + 0.2) = 0.2706$

Q. Solve $y'' - y' - 2y = 3e^{2x}$, $y(0) = 0$, $y'(0) = -2$, for $y(0.2)$ and $y'(0.2)$ taking $h = 0.1$ by Euler's method.

Soln

$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 3e^{2x}$

or, $\frac{d}{dx}\left(\frac{dy}{dx}\right) - \frac{dy}{dx} - 2y = 3e^{2x}$ — (1)

putting $\frac{dy}{dx} = z$ then (1) is

$\frac{dz}{dx} - z - 2y = 3e^{2x}$

or, $\frac{dz}{dx} = 3e^{2x} + z + 2y$

$g(x, y, z) = 3e^{2x} + z + 2y$

$f(x, y, z) = z$

$$y_{n+1} = y_n + hf(x_n, y_n, z_n) \quad \text{(2)}$$
$$z_{n+1} = z_n + hg(x_n, y_n, z_n) \quad \text{(3)}$$

$$y_1 = y_0 + hf(x_0, y_0, z_0) \quad \text{(4)}$$
$$z_1 = z_0 + hg(x_0, y_0, z_0) \quad \text{(5)}$$

$y(0) = 0$
$y(x_0) = y_0$
$\therefore x_0 = 0$
$y_0 = 0$

$\therefore y'(0) = -2$ is given

We have, $\frac{dy}{dx} = z$
$y'(x) = z(x)$
$y'(0) = z(0)$
$\therefore z(0) = -2$

$f(x_0, y_0, z_0) = z_0 = -2$

$g(x_0, y_0, z_0) = 3e^{2x_0} + z_0 + 2y_0$
$= 3e^{2\times 0} + -2 + 2\times 0$
$= 3 - 2$
$= 1$

Now (4) and (5) are

$y_1 = 0 + 0.1 \times (-2)$
$\therefore y_1 = -0.2$
or, $y(x_1) = -0.2$
or, $y(x_0 + h) = -0.2$
or, $y(0 + 0.1) = -0.2$
$\therefore y(0.1) = -0.2$

$z_1 = -2 + 0.1 \times 1$
$= -2 + 0.1$
$\therefore z_1 = -1.9$
or, $z(x_1) = -1.9$
$\therefore z(0.1) = -1.9$

From (2) and (3)

$y_2 = y_1 + hf(x_1, y_1, z_1)$
$z_2 = z_1 + hg(x_1, y_1, z_1)$

$f(x_1, y_1, z_1) = z_1$
$= -1.9$

$g(x_1, y_1, z_1) = 3e^{2x_1} + z_1 + 2y_1$
$= 3e^{2\times 0.1} + (-1.9) + 2\times(-0.2)$
$= 1.3642$

$\therefore y_2 = -0.2 + 0.1 \times (-1.9)$
$y(x_2) = -0.39$
$y(x_1 + h) = -0.39$
$y(0.1 + 0.1) = -0.39$
$y(0.2) = -0.39$

$z_2 = -1.9 + 0.1 \times 1.364$
$z(x_2) = -1.7635$
$z(x_2) = -1.7635$
$z(0.2) = -1.7635$
$y'(0.2) = -1.7635$

$\therefore y(0.2) = -0.39$
$y'(0.2) = -1.7635$

Q(b) Solve for $y(0.4)$ using Heun's method.
$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = 6x$, $y(0) = 0$, $y'(0) = 1$

Solution:
$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = 6x \quad \text{--- (1)}$$
Putting
$$\frac{dy}{dx} = z$$
Now, (1) is
$$\frac{dz}{dx} + 2z - 3y = 6x$$
or, $$\frac{dz}{dx} = 6x + 3y - 2z$$

$f(x, y, z) = z$
$g(x, y, z) = 6x + 3y - 2z$

$\because y(0) = 0$, $x_0 = 0$, $y_0 = 0$

$y'(0) = 1$ is given
$z_0 = y'(0)$
$\therefore z_0 = 1$

We have,
$$y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n, z_n) + f(x_{n+1}, y_{n+1}^e, z_{n+1}^e)\right]$$
$$z_{n+1} = z_n + \frac{h}{2}\left[g(x_n, y_n, z_n) + g(x_{n+1}, y_{n+1}^e, z_{n+1}^e)\right]$$
$$\therefore y_1 = y_0 + \frac{h}{2}\left[f(x_0, y_0, z_0) + f(x_1, y_1^e, z_1^e)\right] \quad \text{--- (2)}$$

$$z_1 = z_0 + \frac{h}{2}\left[g(x_0, y_0, z_0) + g(x_1, y_1^e, z_1^e)\right] \quad \text{--- (3)}$$

$f(x_0, y_0, z_0) = 1$

$g(x_0, y_0, z_0) = 6x_0 + 3y_0 - 2z_0 = 6\times0 + 3\times0 - 2\times1$
$= -2$

$z_1^e = y_0 + hf(x_0, y_0, z_0)$
$= 0 + 0.2 \times (-2)$
$= 0.6$

$y_1^e = y_0 + hf(x_0, y_0, z_0)$
$= 0 + 0.2\times1$
$= 0.2$

$f(x_1, y_1^e, z_1^e) = z_1^e = 0.6$
$g(x_1, y_1^e, z_1^e) = 6x_1 + 3y_1^e - 2z_1^e$
$= 6\times0.2 + 3\times0.2 - 2\times0.6$
$= 0.6$

Now (2) and (3) are
$$y_1 = 0 + \frac{0.2}{2}[1 + 0.6]$$
$y(x_1) = 0.16$
$y(0.2) = 0.16$

$$z_1 = 0 + \frac{0.2}{2}[-2 + 0.6]$$
$z(x_1) = 0.86$
$z(0.2) = 0.86$

Also,
$$y_2 = y_1 + \frac{h}{2}\left[f(x_1, y_1, z_1) + f(x_2, y_2^e, z_2^e)\right] \quad (4)$$

$f(x_1, y_1, z_1) = z_1 = 0.86$

$g(x_1, y_1, z_1) = 6x_1 + 3y_1 - 2z_1$
$= 6\times0.2 + 3\times0.16 - 2\times0.86$
$= -0.04$

$x_2 = x_1 + h = 0.2 + 0.2 = 0.4$

$y_2^c = y_1 + hf(x_1, y_1, z_1)$
$= 0.16 + 0.2\times z_1$
$= 0.16 + 0.2\times0.86$
$= 0.3320$

$z_2^c = z_1 + hg(x_1, y_1, z_1)$
$= 0.86 + 0.2\times(-0.04)$
$= 0.852$

$f(x_2, y_2^c, z_2^c) = z_2^c = 0.852$

Now (4) is

$y_2 = 0.16 + \frac{0.2}{2}\times(0.86 + 0.852)$

$\therefore y(x_2) = 0.3312$

$\therefore y(0.4) = 0.3312$ //

2021
5 (b)

Solve $\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$, $y = 1$ and $\frac{dy}{dx} = 0$ when $x = 0$, apply RK-4th order for $y(0.2)$.

Solution:

$\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$ — (1)

$x_0 = 0$
$y_0 = 1$
$y'(0) = 0$

Taking $h = 0.2$

Putting $\frac{dy}{dx} = z$ then (1) is

$\frac{dz}{dx} + xz - y = 0$

$\frac{dz}{dx} = y - xz$

$f(x, y, z) = z$
$g(x, y, z) = y - xz$

$y_{n+1} = y_n + h/6\,[f_1 + 2f_2 + 2f_3 + f_4]$ — (2)

$z_{n+1} = z_n + h/6\,[g_1 + 2g_2 + 2g_3 + g_4]$ — (3)

$f_1 = f(x_n, y_n, z_n)$ $\qquad g_1 = g(x_n, y_n, z_n)$

$f_2 = f(x_n + h/2, y_n + f_1 h/2, z_n + g_1 h/2)$ $\qquad g_2 = g(\ldots$

$f_3 = f\left(x_n + \frac{h}{2}, y_n + f_2\frac{h}{2}, z_n + g_2\frac{h}{2}\right)$

$g_3 = g\left(x_n + \frac{h}{2}, y_n + f_2\frac{h}{2}, z_n + g_2\frac{h}{2}\right)$

$f_4 = f(x_n + h, y_n + f_3 h, z_n + g_3 h)$

$g_4 = g(x_n + h, y_n + f_3 h, z_n + g_3 h)$

Now, $f_1 = f(x_0, y_0, z_0) = z_0 = 0$

$g_1 = g(x_0, y_0, z_0) = y_0 - x_0 z_0 = 1 - 0 \times 0 = 1$

$f_2 = f\left(x_0 + \frac{h}{2}, y_0 + f_1\frac{h}{2}, z_0 + g_1\frac{h}{2}\right)$

$= f\left(0 + \frac{0.2}{2}, 1 + 0 \times \frac{0.2}{2}, 0 + 1 \times \frac{0.2}{2}\right)$

$= f(0.1, 1, 0.1)$

$= 0.1$

$g_2 = g(0.1, 1, 0.1) = 0.99$

$f_3 = f\left(x_0 + \frac{h}{2}, y_0 + f_2\frac{h}{2}, z_0 + g_2\frac{h}{2}\right)$

$= f\left(0.1, 1 + \frac{0.1 \times 0.2}{2}, 0 + \frac{0.99 \times 0.2}{2}\right)$

$= f(0.1, 1.01, 0.099)$

$= 0.099$

$g_3 = g(0.1, 1.01, 0.099) = 1.01 - 0.1 \times 0.099$

$= 1.0001$

$f_4 = f(x_0 + h, y_0 + f_3 h, z_0 + g_3 h)$

$= f(0 + 0.2, 1 + 0.099 \times 0.2, 0 + 1.0001 \times 0.2)$

$= f(0.2, 1.098, 0.2)$

$= 0.2$

Now (2) is

$y_1 = 1 + \frac{0.2}{6}[0 + 2 \times 0.1 + 2 \times 0.099 + 0.2]$

$\therefore y(x_1) = 1.0199$

$\therefore y(0.2) = 1.0199$

* Solve:

$\frac{d^2y}{dx^2} = 6x,\ y(1) = 2,\ y(2) = 9$

by shooting method

Solution

Putting $\frac{dy}{dx} = z$.

Now (1) is

$\frac{dz}{dx} = 6x$

$g(x, y, z) = 6x$

$f(x, y, z) = z$

We have,

$$y_{n+1} = y_n + h f(x_n, y_n, z_n)$$
$$z_{n+1} = z_n + h g(x_n, y_n, z_n)$$

$$\therefore y_1 = y_0 + h f(x_0, y_0, z_0)$$
$$z_1 = z_0 + h g(x_0, y_0, z_0)$$

Here, $x_0 = 1$

$y_0 = 2$

Let $z_0 = 2$

$$y_1 = y_0 + h z_0$$
$$y_1 = y_0 + z_0$$

$$y_1 = y_0 + z_0$$
or, $y(x_0 + h) = y_0 + z_0$

or, $y(1+1) = 2 + z_0$

$\therefore y(2) = 2 + 2$

$\therefore y(2) = 9$

$\therefore$ Required solution is

$y'(1) = 2$