

NCIT

NEPAL COLLEGE OF INFORMATION TECHNOLOGY

Assessment Fall-2023

Level: Bachelor

Year : 2024

Programme: BE. For(CE_D/M) SE D/M)/Civil

Full Marks: 100

Course: Calculus _II

Pass Marks: 45

Time : 3 hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. Evaluate $\int_0^1 \int_{4y}^4 e^{x^2} dx dy$ [5]

b. Evaluate $\iiint_V x^2 dx dy dz$ over the region bounded by the plane $x=0$, $y=0$, $z=0$ and $x+y+z=a$ [5]

c. find the volume of the solid whose base in the region in the xy -plane. That is bounded by the parabola $y = 4 - x^2$ and the line $y = 3x$ while top of the solid is bounded by $z = x + 4$ [5]

2. (a) solve by power series method : $y'' + y = 0$ [8]

(b) Express $f(x) = x^3 - 5x^2 + x + 2$ in terms of legendre polynomials [7]

Or

Solve the Bessel equation $x^2 y'' + xy' + (x^2 - v^2)y = 0$

3. (a) (i) Find the laplace transform of $f(t) = t^2 e^{-t} \sin t$ [4+4]

(ii) find the inverse laplace transform of $F(s) = \log\left(\frac{s+a}{s+b}\right)$

(b) By using laplace transform solve the initial value problem. [7]

$y'' + 2y' + 17y = 0$ $y(0) = 0$ $y'(0) = 12$

4. (a) A particle moves along the curve $x = t^3 + 1$, $y = t^2$, $z = 2t + 5$ find the component of its velocity and acceleration at $t=1$ in the direction $\vec{i} + \vec{j} + 3\vec{k}$ [5]

(b) If $\phi = x^3 + y^3 + z^3 - 3xy$ find $\text{div}(\text{grad}\phi)$. 5

(c) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = yz\vec{i} + (xz + 1)\vec{j} + xy\vec{k}$ and C is any path from $(1, 0, 0)$ to $(2, 1, 4)$.

5. (a) Find $\iint_S (\vec{F} \cdot \hat{n}) dS$ for $\vec{F} = 3x\vec{i} + xz\vec{j} + z^2\vec{k}$, S bounded by

$$z = 4 - x^2 - y^2 \text{ and } XY \text{ plane.}$$

(b) Evaluate $\iint_S (\vec{F} \cdot \hat{n}) dS$ by using Gauss divergence theorem if $\vec{F} = (4xz, -y^2, yz)$ S is the Cube bounded by $0 \leq x \leq 1, 0 \leq y \leq 1$ and $0 \leq z \leq 1$ [8]

OR

Evaluate $\iint_C \vec{F} \cdot d\vec{r}$ by using Stoke's theorem where $\vec{F} = (y^3, 0, x^3)$ and C is the boundary of the triangle with vertices $(1, 0, 0), (0, 1, 0), (0, 0, 1)$.

6. (a) find the fourier series for the function $f(x) = x^2$ in the interval $0 \leq x \leq 2\pi$ [7]

(b) Find the fourier cosine as well as fourier sine series of function $f(x) = e^x$ in the interval $0 \leq x \leq L$ [8]

7. Attempt any two. (5*2=10)

(a) Find the solution of $u_x + u_y - u = 0$ given that $u(x, 0) = 2$

(b) find the general solution of linear first order constant coefficient partial differential equation.

(c) Find the convolution of the function $e^t * e^{-t}$