Pokhara University

Level: Bachelor	Semester: xxxx	Year: xxxx
Program: BE		Full Marks: 100
Course: Calculus II	Time: 3 hrs.	Pass Marks: 45

MODEL QUESTION

Candidates are required to give their answer in their own words as far as practicable. The figures in the margin indicate full marks.

Attempt all questions.

1\.

a) Evaluate the integral: $\int_0^2 \int_{y^2}^4 y\cos(x^2)dxdy$. **5**

b) Evaluate the integral: $\int_0^1 \int_0^{(1-x)} \int_0^{(x+y)} e^z dzdydx$. **5**

c) Find the volume in the first octant bounded by the co-ordinate planes, the cylinder $x^2 + y^2 = 1$ and the plane z+ y = 3. 5

2\.

a) Solve by using power series: $y'' - 4xy' + (4x^2 - 2)y = 0$. **7**

b) (i) Express: $2x^2 - 4x + 2$ as Legendre polynomial. **4+4**

(ii) Show that: $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\sin x$.

OR

Find the solution of Bessel's Equation.

$$x^2 y'' + xy' + (x^2 - n^2)y = 0$$

3\.

a) (i) State first shifting theorem of Laplace Transform and find the Laplace transform of $t\,cosat$. 4+4

(ii) Find the inverse Laplace transform of the function $\frac{s+1}{s^2(s+3)}$.

b) Apply Laplace transform to solve the initial value problem 7

$y'' + 4y' + 3y = e^{-t}, \quad y(0) = y'(0) = 1$

4. a) A particle moves along the curve (t^3+1, t^2, $2t+5$). Find the component of the velocity and acceleration at t=1 along $\vec{i}+\vec{j}+3\vec{k}$. **5**

b) If $\phi = \ln(x^2+y^2+z^2)$, then find $grad\phi$ and $div(grad\phi)$. **5**

c) Evaluate $\oint_C (x^2-3y)dx + (x+siny)dy$ where C is the boundary of the triangle with vertices $(0,0), (1,0)$ and $(0,2)$. **5**

5. a) Find $\iint_S (\vec{F}.\vec{n})ds$, for $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$, **7**

$\vec{r} = (u\cos v, \quad u\sin v,\ 3v); 0 \le u \le 1, 0 \le v \le 2\pi.$

b) Evaluate $\oint_C (\overrightarrow{F}.\overrightarrow{dr})$ by using Stoke's theorem, where $\vec{F} = (y, xz^3, -zy^3)$ and $C: x^2+y^2 = 4, z = 3.$ **8**

OR

State Gauss divergence theorem and use it to evaluate the surface integral $\iint_S \overrightarrow{F}.\hat{n}\,ds$ for $\vec{F} = 4xz\hat{\imath} - y^2\hat{\jmath} + yz\,\hat{k}$ and S is a cube $0 \le x \le 1, 0 \le y \le 1$ and $0 \le z \le 1.$

6. a) Find the Fourier series of $f(x) = \frac{x^2}{2}$ for $-\pi \le x \le \pi$ and deduce that **7**

$$1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\ldots\ldots\ldots\ldots\ldots\ldots = \frac{\pi^2}{6}.$$

b) Find Fourier half range cosine and sine sires of $f(x) = e^x$ in $(0, L)$. 8

7. Attempt any two questions: $[\mathbf{2 \times 5 = 10}]$

a) Derive one dimensional traffic flow model using conservation law.

b) Find the breaking time for $u_t + u\,u_x = 0$, $u(x,0) = e^{-x^2}$.

c) Evaluate $\int_{(1,0,2)}^{(-2,1,3)} [(6xy^3+2z^2)dx + 9x^2y^2dy + (4xz+1)dz].$