Date:

Experiment No. 5

DETERMINATION OF THE CAPACITANCE OF A GIVEN CAPACITOR BY CHARGING AND DISCHARGING THROUGH RESISITOR.

APPARATUS REQUIRED:

a) Capacitor b) Microammeter $(0 - 100\ \mu A)$ c) Stop watch

d) Connecting wires e) Variable high resistance box f) Battery

THEORY:

Charge is the intrinsic property. Modern technology revolves around the use of charge & current.

Capacitor is a device used to store charge or electric potential energy. Capacitance is its ability to store charge.

For Charging:

Let a capacitor having capacitance 'C' is connected in series with a resistor of resistance 'R', ammeter 'A', a battery with emf E and a two way key as shown in figure. When the key X is on the capacitor, it is in charging mode. The positive and negative charges appear on the plates and oppose the flow of electrons. As the charges accumulate, the potential difference between the plates of capacitor increases and the charging current falls asymptotically to zero.

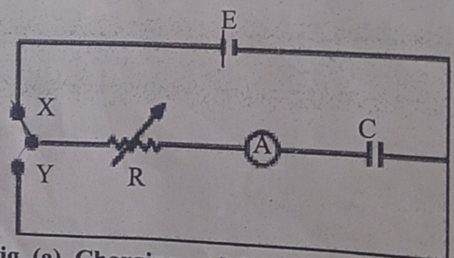

Fig. (a). Charging and discharging of capacitor

Applying Kirchhoff 's loop rule,

$$E = V_C + V_R = \frac{q}{C} + IR$$

$$= \frac{q}{C} + \frac{dq}{dt} R$$

$$\Rightarrow \quad \frac{dq}{dt}R = E - \frac{q}{C} = \frac{EC-q}{C}$$

$$\Rightarrow \quad \frac{dq}{dt} = \frac{1}{RC}(EC - q)$$

$$\Rightarrow \quad \frac{dq}{(EC-q)} = \frac{1}{RC};dt$$

For a capacitor, q= CV and maximum change, $q_0 = CE$

$$\therefore \quad \frac{dq}{q_0-q} = \frac{dt}{RC}$$

Integrating, we get

$$-\ln(q_0 - q) = \frac{t}{RC} + A \text{ .... (i)}$$

Where A is the integrating constant.

To find A :-

At t = 0, q=0

So, equation(i) becomes

$$-\ln(q_0) = A \text{ .... (ii)}$$

Using equation (i) & (ii)

$$\Rightarrow \quad -\ln(q_0 - q) = \frac{t}{RC} - \ln q_0$$

$$\Rightarrow \quad -[\ln(q_0 - q) - \ln q_0] = \frac{t}{RC}$$

$$\Rightarrow \quad \ln\left(\frac{q_0-q}{q}\right) = -\frac{t}{RC}$$

$$\Rightarrow \quad q_0 - q = q_0 e^{-\frac{t}{RC}}$$

$$\therefore \quad q = q_0\left(1 - e^{-\frac{t}{RC}}\right) \text{ ... .(iii)}$$

where, $q_0 = EC$ is the maximum charge stored in the capacitor. Differentiating equation (iii) with respect to time,

$$\frac{dq}{dt} = -q_0\left(-\frac{1}{RC}\right)e^{-\frac{t}{RC}}$$

$$\therefore \quad I = \frac{q_0}{RC}e^{-\frac{t}{RC}} = \frac{EC}{RC}e^{-\frac{t}{RC}} = \frac{E}{R}e^{-\frac{t}{RC}} = I_0 e^{-\frac{t}{RC}}$$

$$\therefore \quad I = I_0 \ e^{-\frac{t}{RC}} \text{ .... (iv)}$$

Where, $I_0 = \frac{E}{R}$ is the maximum current

Again from (iv),

$$\frac{I_0}{I} = e^{\frac{t}{RC}}$$

or, $$\ln\left(\frac{I_0}{I}\right) = \frac{t}{RC}$$

$\ln\left(\frac{I_0}{I}\right) = \frac{t}{RC}$ ....(v) , which is the required equation for determination of C in this experiment.

Equation (iii) and (iv) are called the charging equation in terms of charge and current respectively.

Equation (v) shows that 'C' can be determined from the slope of straight line obtained from the plot between $\ln\left(\frac{I_0}{I}\right)$ and 't' as shown in figure.

The half-life of the circuit, $T_{1/2}$ is the time for the current to decrease to half of its initial value.

i.e. when $t = T_{1/2}, I = \frac{I_0}{2}$

From equation (v),

$$\ln(2) = \frac{T_{1/2}}{RC}$$

or, $$C = \frac{T_{1/2}}{R\ln 2}$$

or, $$C = \frac{T_{1/2}}{0.693R} \quad \text{........(vi)}$$

Time constant (or relaxation time) :-

The term RC in equation (iii) and (iv) is called time constant, where t = RC, from equation (iii),

$$q = q_0(1 - e^{-1})$$

$$= q_0(1 - 0.37) = 0.63q_0$$

or, $$q = 0.63q_0 = 63\% \text{ of } q_0$$

Hence, the time constant of charging circuit is defined as the time in which the capacitor charges by about 63% of its maximum charge.

For discharging of capacitor:-

When the capacitor is fully charged and switch Y is on, discharging occurs in the capacitor through resistor.

Using Kirchhoff 's voltage or second law,

$$0 = V_C + V_R = \frac{q}{C} + IR \qquad [\text{since e.m.f. (E)} = 0]$$

or, $$0 = \frac{q}{C} + \frac{dq}{dt}.R$$

or, $$R\frac{dq}{dt} = -\frac{q}{C}$$

or, $$\frac{dq}{dt} = -\frac{q}{RC}$$

or, $$\frac{dq}{q} = -\frac{1}{RC}dt$$

Integrating,

$$\int_{q_0}^{q} \frac{dq}{q} = -\frac{1}{RC}\int_0^t dt$$

or, $$[\ln q]_{q_0}^{q} = -\frac{t}{RC}$$

or, $$\ln q - \ln q_0 = -\frac{t}{RC}$$

or, $$\ln\frac{q}{q_0} = -\frac{t}{RC}$$

or, $$\frac{q}{q_0} = e^{-\frac{t}{RC}}$$

$\therefore$ $$q = q_0 e^{-t/RC} \;\ldots\ldots\ldots\ldots\ldots(v)$$

Differentiating with respect to time,

$$\frac{dq}{dt} = -\frac{q_0}{RC}.e^{-\frac{t}{RC}} = -\frac{EC}{RC}e^{-\frac{t}{RC}}$$

$$= -\frac{E}{R}e^{-\frac{t}{RC}}[\because q_0 = EC]$$

$$\therefore I = -I_0 e^{-\frac{t}{RC}} \ldots\ldots\ldots(vi)$$

The negative sign is due to the opposite direction of the flow of charge (or current) during discharging as compared to that during charging.

Where, $I_0 = \frac{E}{R}$ is the maximum current

$\therefore$ $$I = I_0 e^{-\frac{t}{RC}} \ldots\ldots\ldots (vii)\text{ (in magnitude)}$$

Equation (v) and (vi) are called the discharging equations in terms of charge and current respectively. Here negative sign in the exponential term indicates decrease in charge with time.

Time constant: The factors RC in the discharging equation is called capacitive time constant of discharging circuit. It has dimensions of time.

When $t = RC$, equation (vii) becomes

$$q = q_0 e^{-1} = \frac{q_0}{e} = 0.37 q_0.$$

$$q = 0.37 q_0 = 37\% \text{of } q_0.$$

$\therefore$

Hence time constant of a discharging circuit is the time at which the charge stored in capacitor fall to 37% of its initial value.

Equation (iv) and (viii) show that magnitude of charging current is equal to magnitude of discharging current.

PROCEDURE:

1. Connect the given capacitor ($C\mu F$), resistor($R\,K\Omega$), two way key (X, Y), ammeter $(0 - 200\ \mu A)$ and battery as shown in figure.
2. Close the switch X, charging starts and ammeter gives the maximum reading. This is I_0 for time $t = 0$. The current then starts decreasing.
3. The maximum current I_0 is noted and then the current I for fixed interval of time (say 5 or 10 sec) is noted.
4. Changing the polarity of ammeter disconnect X and connect Y, note the data as in steps (2) and (3) .
5. Find half life $T_{1/2}$ from the plot of I and t.
6. Plot a graph between In $\left(\frac{I_0}{I}\right)$ and t for all cases and find the capacitance of capacitor from slope of the plot.

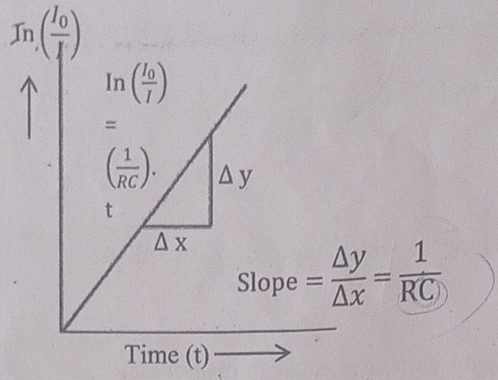

Fig.(b) Plot between ln (I_0/I) and 't'

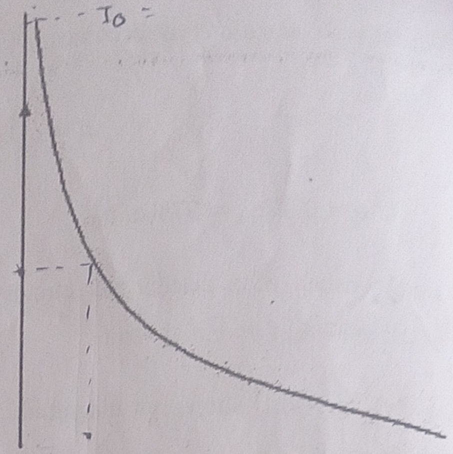

Fig.(c) Plot of I~t showing exponential nature

OBSERVATIONS :

Least count of clock =

S.N.	Time (t) sec.	Current (I)		Resistance (R) ohm
		Charging	Discharging	
1.	0			
2.	5			
3.				
4.				
5.				
6.				
7.				
8.				
9.				
10.				
11.				
12.				
13.				
14.				
15.				

CALCULATIONS:

For charging

	S.N.	I_0/I	$\ln I_0/I$	C	$\bar{C}$	$C_i - \bar{C}$	$(C_i - \bar{C})^2$	σ_C
I_0	1.							
I	2.							
"	3.							
"	4.							
"	5.							
"	6.							
"	7.							
"	8.							
"	9.							
"	10.							
"	11.							
"	12.							
"	13.							
"	14.							
	For Discharging							
"	15.							
"	16.							
"	17.							
"	18.							
"	19.							
"	20.							
"	21.							
"	22.							
"	23.							
"	24.							
"	25.							
"	26.							
"	27.							
"	28.							

RESULTS:

The value of capacitance =

Standard value of capacitance =

Percentage error =

The value of 'C' from graph (Charging) =

The value of 'C' from graph (discharging) =

The value of capacitance from half life $(C) = \frac{T_{1/2}}{0.693R}$

CONCLUSIONS;

PRECAUTIONS: