Mechanical Oscillation.

Simple Harmonic Motion (SHM): The motion in which the restoring force is directly proportional to the displacement from the mean position and is opposite to it is called simple harmonic motion.

i.e $F \propto x$

or, $F = -kx$ ... (i)

where, the negative sign is due to the opposite direction of f and x and k is constant called force constant

Define:

From eq^n (i)

$k = \frac{f}{x}$ (in magnitude)

Hence, the force constant is defined as the restoring force per unit displacement.

Unit :

$k \longrightarrow$ N/m (SI unit)

dyne/cm

$1N = 10^5$ dyne

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Differential equation of SHM

$\because v = \frac{dx}{dt}$

From eq^n (i), $F = -kx$

or, $ma = -kx$

$a = \frac{dv}{dt}$

or, $m\frac{d^2x}{dt^2} = -kx$

$= \frac{d}{dt}\left(\frac{dx}{dt}\right)$

or, $m\frac{d^2x}{dt^2} + kx = 0$

let $\omega^2 = k/m$

$\omega = \sqrt{k/m}$

or, $\frac{d^2x}{dt} + \left(\frac{k}{m}\right) x = 0$

or, $\frac{d^2x}{dt} + \omega^2 x = 0$ ... (ii)

ω = angular frequency

Eq^n (ii) is called differential equation of SHM.

Formula:

a) $F = -kx$

b) $\frac{d^2x}{dt} + \omega^2 x = 0$

The solution of eq^n (ii) can be written as

$x = x_m \cos(\omega t + \phi)$ ... (iii)

OR, $x = x_m \sin(\omega t + \phi)$ ... (iv)

Eq^n (iii) or (iv) is the required solution of SHM.

Here, x_m = maximum displacement from mean position

ω = angular frequency

ϕ = phase constant ; $\omega t + \phi$ = phase

Note:

In SHM

variable: x, t, K.E, P.E, $\omega t + \phi$ (phase), v, a

constant: x_m, T, f, T.E, ϕ, ω, k

Q) Show that $x = x_m \sin(\omega t + \phi)$ and $x = x_m \cos(\omega t + \phi)$ satisfied SHM.

~~LHS =~~

$$\frac{d^2x}{dt} + \omega^2 x$$

Putting $x = x_m \sin(\omega t + \phi)$

$$L.H.S = \frac{d^2 x_m \sin(\omega t + \phi)}{dt^2} + \omega^2 x_m \sin(\omega t + \phi)$$

$$= x_m \frac{d}{dt}\left[\frac{d\sin(\omega t + \phi)}{d(\omega t + \phi)} \cdot \frac{d(\omega t + \phi)}{dt}\right] + \omega^2 x_m \sin(\omega t + \phi)$$

$$= x_m \frac{d}{dt}\left[\cos(\omega t + \phi)\, \omega\right] + \omega^2 x \sin(\omega t + \phi)$$

$$= x_m \omega \frac{d \cos(\omega t + \phi)}{d(\omega t + \phi)} \frac{d(\omega t + \phi)}{dt} + \omega^2 x_m \sin(\omega t + \phi)$$

$$= -\omega^2 x_m \sin(\omega t + \phi) + \omega^2 x_m \sin(\omega t + \phi)$$

$$= 0$$

$$= R.H.S$$

$$\frac{d^2x}{dt^2} + \omega^2 x$$

Putting $x = x_m \cos(\omega t + \phi)$

$$L.H.S = \frac{d^2 x_m \cos(\omega t + \phi)}{dt^2} + \omega^2 x_m \cos(\omega t + \phi)$$

$$= x_m \frac{d}{dt}\left[\frac{d\cos(\omega t + \phi)}{d(\omega t + \phi)} \cdot \frac{d(\omega t + \phi)}{dt}\right] + \omega^2 x_m \cos(\omega t + \phi)$$

$$= x_m \frac{d}{dt}\left[-\sin(\omega t + \phi)\omega\right] + \omega^2 x_m \cos(\omega t + \phi)$$

$$= -x_m \omega \frac{d \sin(\omega t + \phi)}{d(\omega t + \phi)} \frac{d(\omega t + \phi)}{dt} + \omega^2 x_m \cos(\omega t + \phi)$$

$$= -x_m \omega^2 \cos(\omega t + \phi) + \omega^2 x_m \cos(\omega t + \phi)$$

$= 0$

$= R.H.S_{//}$

The range of x lies between $-x_m$ to $+x_m$

Characteristics of SHM:

1) Displacement (x): It is given by $x = x_m \cos(\omega t + \phi)$ ... (i)

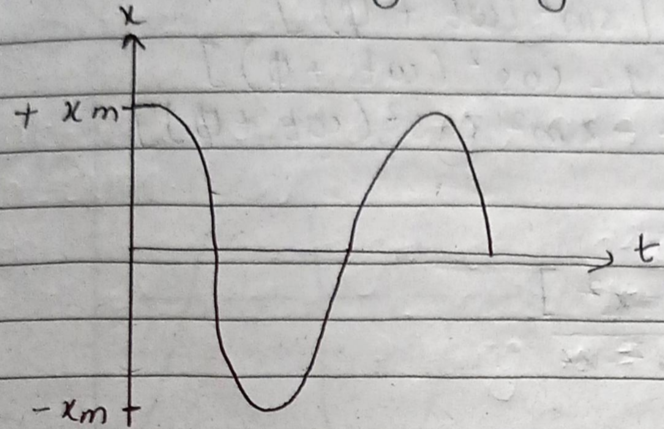


Fig: variation of x vs t

2) Velocity (v): It is the rate of change of displacement w.r.t time. $v = \frac{dx}{dt} = \frac{d}{dt}[x_m \cos(\omega t + \phi)]$ [using eq^n (i)]

or, $v = x_m[-\sin(\omega t + \phi)]\omega$

$= -\omega x_m \sin(\omega t + \phi)$ ... (ii)

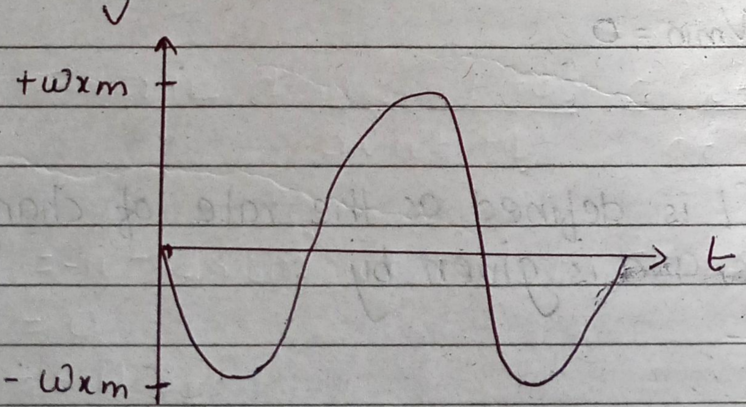


Fig: Variation of V with t

Variation of v with x :

Squaring eqⁿ (ii),

$$v^2 = \omega^2 x_m^2 [\sin^2(\omega t + \phi)]$$
$$= \omega^2 x_m^2 [1 - \cos^2(\omega t + \phi)]$$
$$= \omega^2 [x_m^2 - x_m^2 \cos^2(\omega t + \phi)]$$

Using eqⁿ (i)

$$v^2 = \omega^2 [x_m^2 - x^2]$$
$$\therefore v = \pm \omega \sqrt{x_m^2 - x^2}$$

Cases:

[I] At mean position i.e at $x = 0$

$$\therefore v = \pm \omega x_m$$
$$\therefore v_{max} = \omega x_m$$

[II] At the extreme positions,

$$x = \pm x_m$$
$$\therefore v = v_{min} = 0$$

[III] Acceleration (a) : It is defined as the rate of change of velocity w.r.t time, and is given by :

$$a = \frac{dv}{dt}$$

Using eqⁿ (ii),

$$a = \frac{d}{dt} [-\omega x_m \sin(\omega t + \phi)]$$

or, $a = -\omega x_m \cos(\omega t + \phi)\omega$ ......(iii)

$\therefore a = -\omega^2 x$ [using eq^n (i)] ... (iv) [Using (i) in (iii)]

* Graph of a vs t

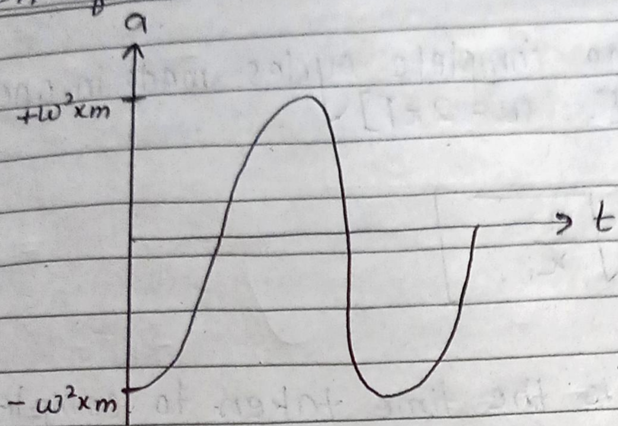


Fig: variation of a with t

Cases:

(i) $x = 0$

$a = 0$ [using eq^n (iv)]

i.e a_{min} at $x = 0$

) $x = \pm x_m$

$\therefore a = \pm \omega^2 x_m$

i.e $a_{max} = \omega^2 x_m$ at $x = \pm x_m$

) Angular frequency (ω)

From eq^n (iv),

$\omega^2 = \frac{a}{x}$, in magnitude

$\therefore \omega = \sqrt{\frac{a}{x}}$

$\Rightarrow \omega = \sqrt{\frac{\text{acceleration}}{\text{displacement}}}$ $\therefore \omega = \sqrt{\frac{K}{m}}$

5) Frequency (f): It is the complete cycles made in one second, is given by: $f = \frac{\omega}{2\pi}$ $[\because \omega = 2\pi f]$

$$\left[\therefore f = \frac{1}{2\pi}\sqrt{\frac{a}{x}}\right]$$

6) Time period (T): It is the time taken to complete one oscillation and is given by: $T = \frac{1}{f}$ or $T = \frac{2\pi}{\omega}$

$\therefore T = 2\pi\sqrt{\frac{x}{a}}$ $\left[\because \omega = 2\pi f = 2\pi/T\right]$

7) Phase: It is given by $(\omega t + \phi)$.

8) phase constant: It is given by ϕ

* Energy in SHM:

The particle or the body in SHM has both kinetic and potential energy, which are given by:

$K.E = \frac{1}{2}mv^2$

Using eq^n (ii)

$v = -\omega x_m \sin(\omega t + \phi)$

$\therefore K.E = \frac{1}{2} m\omega^2 x_m^2 \sin^2(\omega t + \phi)$

And,

$P.E = \frac{1}{2} kx^2$ [$P.E = mgh$]

$= \frac{1}{2} m\omega^2 x_m^2 \cos^2(\omega t + \phi)$ [$\because \omega^2 = k/m$]

HW Q.1) Proof: $P.E = \frac{1}{2} kx^2$

Now, Total Energy is given by;

$T.E\,(E) = K.E + P.E$

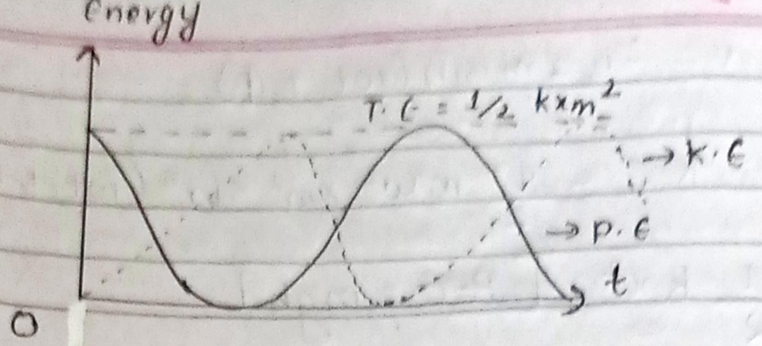
$= \frac{1}{2} m\omega^2 x_m^2 \sin^2(\omega t + \phi) + \frac{1}{2} m\omega^2 x_m^2 \cos^2(\omega t + \phi)$

$= \frac{1}{2} m\omega^2 x_m^2 [\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)]$

$$\left[\therefore E = \frac{1}{2} m\omega^2 x_m^2\right]$$

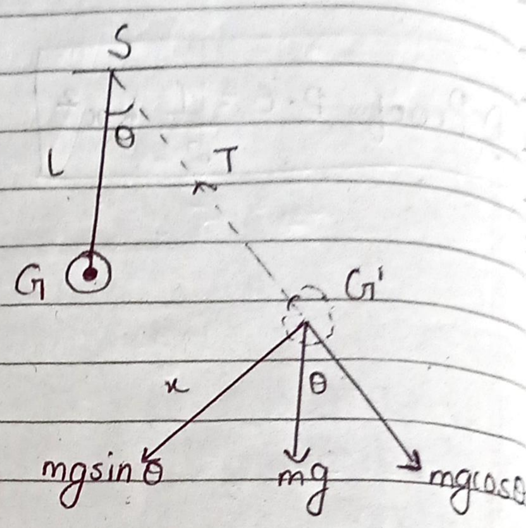
Also, $E = \frac{1}{2} k x_m^2$ $\left[\text{since } \omega^2 = \frac{k}{m}\right]$

This shows that the total energy remains constant although K.E and P.E vary with time.



Examples of SHM

1) Simple pendulum:



$T = mg\cos\theta$

Restoring force,

$F = -mg\sin\theta$

or, $m\dfrac{d^2x}{dt^2} = -mg\sin\theta$

or, $\dfrac{d^2x}{dt^2} + g\theta = 0$

Here, $\theta = \dfrac{x}{l}$ [where $a = \dfrac{d^2x}{dt^2}$ and $\sin\theta \approx \theta$ for small angle θ]

$\therefore \dfrac{d^2x}{dt^2} + g\left(\dfrac{x}{l}\right) = 0$

$\dfrac{d^2x}{dt^2} + \left(\dfrac{g}{l}\right)x = 0$

Let $\omega^2 = \dfrac{g}{l}$

$\omega = \sqrt{\dfrac{g}{l}}$, angular frequency

$$\therefore \frac{d^2x}{dt^2} + \omega^2 x = 0 \cdots (i)$$

Eqⁿ (i) is the differential equation of SHM. Hence, the motion of SHM.

- To find T:

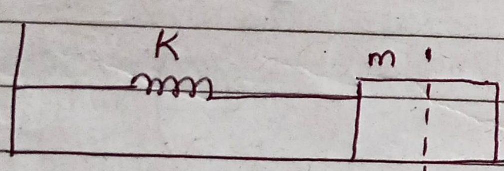
Since, $T = \frac{2\pi}{\omega}$ $\left[\because \omega = 2\pi F = \frac{2\pi}{T}\right]$

$$T = \frac{2\pi}{\sqrt{\frac{g}{l}}}$$

$$\therefore T = 2\pi\sqrt{\frac{l}{g}}$$

2) Mass-spring system

a) Horizontal:



$F \propto x$

or, $F = -kx$ [where k = spring constant]
[Unit = Nm^{-1}]

or, $ma = -kx$

or, $m\frac{d^2x}{dt^2} = -kx$

or, $\frac{md^2x}{dt^2} + kx = 0$

let $\omega^2 = \frac{k}{m}$

$\therefore \omega = \sqrt{\frac{k}{m}}$, angular frequency

$\therefore \frac{d^2x}{dt^2} + \omega^2 x = 0 \cdots (i)$

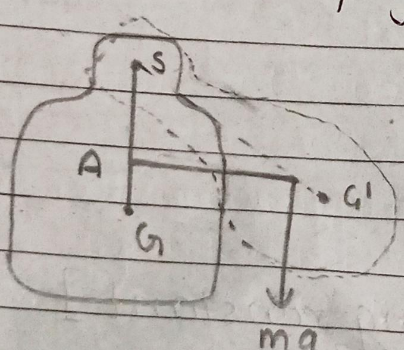
Eqⁿ(i) is the differential equation of SHM. Hence, the motion of the horizontal mass spring system is SHM.

3) To find T:

Since, $T = \frac{2\pi}{\omega}$

or, $T = 2\pi\sqrt{\frac{m}{k}}$

3) Compound or Real or physical pendulum



A compound pendulum is a rigid body capable of being rotated in a vertical plane about horizontal axis.

Consider a compound pendulum suspended at

I = momentum Interia

the point (S) having centre of gravity at G. let the pendulum be displace through a angle θ then the pendulum comes back to the mean position due to the restoring torque which is given by $\tau = I\alpha$ ...(i)

where τ = moment of Torque

α = angular acceleration

$= \frac{d^2\theta}{dt^2}$ ... (ii)

Also, $\tau = mgG'A$ [$\because \tau$ = force × ⊥r distance]

or, $\tau = -mgl\sin\theta$ ... (iii)

where in $\Delta SAG'$

$\sin\theta = \frac{G'A}{SG'} = \frac{G'A}{l}$

$\therefore G'A = l\sin\theta$

From (i), (ii) and (iii)

$$I\frac{d^2\theta}{dt^2} = -mgl\sin\theta$$

[~~Fro~~ For small angle θ, $\sin\theta \simeq \theta$]

or, $I\frac{d^2\theta}{dt^2} + mgl\theta = 0$

or, $\frac{d^2\theta}{dt^2} + \left(\frac{mgl}{I}\right)\theta = 0$

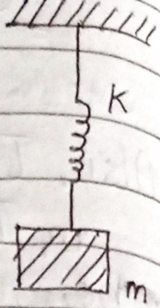
let $\omega^2 = \frac{mgl}{I}$

$\therefore \omega = \sqrt{\frac{mgl}{I}}$, angular frequency

$\therefore \frac{d^2\theta}{dt^2} + \omega^2\theta = 0$ ... (iv)

Eq^n(iv) is the required differential equation of SHM. Hence, the motion of the compound pendulum is simple harmonic. Eq^n (iv) is also the equation of angular harmonic motion.

ii) Vertical mass spring system



In case, even if the spring is vertical, then gravity has no effect on force constant or the time period of oscillation. The weight 'mg' of the body produces on initial elongation 'x_0' such that $mg - kx_0 = 0$. If the displacement is 'x' from the equilibrium position, then the total elastic force will be $-k(x_0+x)$. Hence, the effective restoring force will be:

$$mg - k(x_0 + x) = -kx$$

or, $F = -kx$

or, $m\frac{d^2x}{dt^2} = -kx$

or, $\frac{d^2x}{dt^2} + \left(\frac{k}{m}\right)x = 0$

or, $\frac{d^2x}{dt^2} + \omega^2 x = 0$ ...(i) Let $\left[\omega^2 = \frac{k}{m}, \omega = \sqrt{\frac{k}{m}}\right]$

This means that gravity has no effect on the oscillation. Eq^n (i) is the differential equation of S.H.M. Hence the motion is simple harmonic.

To find T:

$$T = \frac{2\pi}{\omega}$$

$\therefore T = 2\pi\sqrt{\frac{m}{k}}$

Compound Pendulum

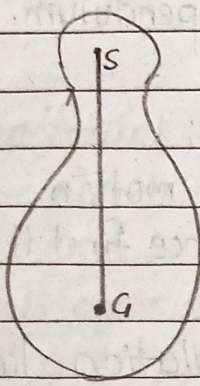
To find Time period in term of Moment of Inertia:

Since $T = \frac{2\pi}{\omega}$ $\left[\because \omega = 2\pi f = \frac{2\pi}{T}\right]$

$$\therefore T = 2\pi\sqrt{\frac{I}{mgl}} \quad \ldots (v)$$

This is the expression of time period in terms of moment of inertia.

Now, I = moment of inertia of the pendulum about an axis passing through the point S and pendicular to the axis of the pendulum.



Then, $I = I_{CG} + ml^2 \quad \ldots \cancel{(vi)}$
[From theorem of parallel axes of M.I.]

or, $I = mk^2 + ml^2 \quad \ldots (vi)$

where, $I_{CG} = mk^2$, k is the radius of gyration. (Unit: m)

Using equation (vi) in (v),

$$T = 2\pi\sqrt{\frac{\cancel{m}(k^2+l^2)}{\cancel{m}gl}}$$

$$= 2\pi\sqrt{\frac{k^2/l + l^2/l}{g}}$$

$$\therefore T = 2\pi\sqrt{\frac{\frac{k^2}{l} + l}{g}} \quad \ldots (vii)$$

This is the time period in terms of radius of gyration.

Recall :

$$T_{simple\ pendulum} = 2\pi\sqrt{\frac{l}{g}}$$

let $L = l + \frac{k^2}{l}$

$$T = 2\pi\sqrt{\frac{L}{g}} \quad \cdots (viii)$$

Then, $L = l + \frac{k^2}{l}$, is called the equivalent length of ~~equivalent~~ simple pendulum.

OR, the ~~equivalent~~ reduced length of the ~~simple~~ compound pendulum.

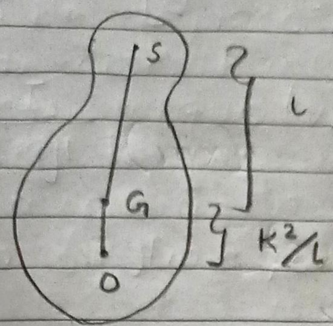
Then eq^n (viii) gives the expression of time period in terms of equivalent length of simple pendulum.

Q) What is SHM Show that the motion of compound pendulum is simple harmonic, hence find its time period.

Q) Show that the point of oscillation lies ~~belo~~ beyond CG.

Q) Show that point of suspension and the point of oscillation are interchangeable.

Point of oscillation.



The point which lies at a distance of K^2/l from CG and below it on the axis of the pendulum is called point of oscillation.

Q) Show that the point of suspension and the point of oscillation are interchangeable.

⇒ Solⁿ:-

For the point of suspension 'S', the time period is given by $= 2\pi\sqrt{\frac{l+k^2/l}{g}}$ ..... (~~viii~~ix)

let $k^2/l = l'$ ..... (x)

Then eqⁿ (ix) becomes,

$$T = 2\pi\sqrt{\frac{l+l'}{g}} \quad \text{.....(xi)}$$

for the point of oscillation 'O'.

$$T' = 2\pi\sqrt{\frac{l'+k^2/l'}{g}} \quad \text{.....(xii)}$$

From eq^n (x) $\frac{K^2}{l'} = l$

eq (xii) becomes,

$$T' = 2\pi\sqrt{\frac{l' + l}{g}} \quad \ldots (xiii)$$

From eq^n (xi) and (xiii),

$T = T'$

i.e the period is same for the point of suspension and the point of oscillation. Hence, they are interchange-able.

Q) What are the conditions for the maximum and minimum time period?

⇒ Maximum time period:

Since the time period is given by

$$T = 2\pi\sqrt{\frac{l + K^2/l}{g}},$$

$T = \infty$ (maximum) when

(i) $g = 0$

(ii) $l = \infty$

(iii) $l = 0$ i.e CG is itself the point of oscillation.

Minimum time period:

Since, the time period is given by:

$$T = 2\pi\sqrt{\frac{l + k^2/l}{g}}$$

Squaring on both sides,

$$T^2 = 4\pi^2\left(\frac{l + k^2/l}{g}\right)$$

Differenting with respect to l.

$$\frac{dT^2}{dl} = \frac{4\pi^2}{g}\,d\left(l + k^2/l\right)$$

or, $\dfrac{dT^2}{dl} = \dfrac{4\pi^2}{g}\left(1 + k^2(-1)l^{-2}\right)$

or, $\cancel{\dfrac{dT^2}{dl}}\ 2T\dfrac{dT}{dl} = \dfrac{4\pi^2}{g}\left(1 - \dfrac{k^2}{l^2}\right) \quad \cdots (xiv)$

Since $T \neq 0$,

$\dfrac{dT}{dl} = 0$ (for T to be minimum)

or, $\cancel{\ni} 0 = \dfrac{4\pi^2}{g}\left(1 - \dfrac{k^2}{l^2}\right)$

Since $\dfrac{4\pi^2}{g} \neq 0$

$$\frac{1-k^2}{l^2}=0$$

or, $\frac{\cancel{\theta} k^2}{l^2}=1$

or, $l^2=k^2$ or, $l=\pm k$

$\therefore l=k$ [using +ve value]

From eqn (xiv)

$$T\frac{dT}{dl}=\frac{2\pi^2}{g}\left(1-\frac{k^2}{l^2}\right)$$

Differentiating w.r.t 'l'.

$$T\frac{d^2T}{dl^2}+\cancel{\frac{dT}{dl}}^{0}\cancel{\frac{dT}{dl}}^{0}=\frac{2\pi^2}{g}\left(0-k^2(-2)l^{-3}\right)$$

or, $$\frac{Td^2T}{dl^2}=\frac{4\pi^2}{g}\frac{k^2}{l^3}$$

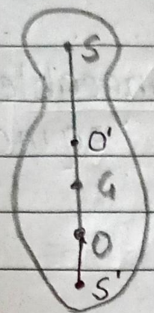
Since $T>0$ and R.H.S >0

$$\frac{d^2T}{dl^2}>0$$

$\therefore$ T is minimum when $l=k$

continue...

Q) Show that there are 4 points that are collinear on the axis for which the period is same.



First, show that S and O are interchangeable.
Now, taking $SG = l_0 = S'G$
$GO = \frac{k^2}{l} = GO'$, we find

The period, $T = 2\pi\sqrt{\frac{l + k^2/l}{g}}$ still remains same.

Thus, the period is same for S, O, S' and O'. Thus, there are 4 collinear points for which T is same.

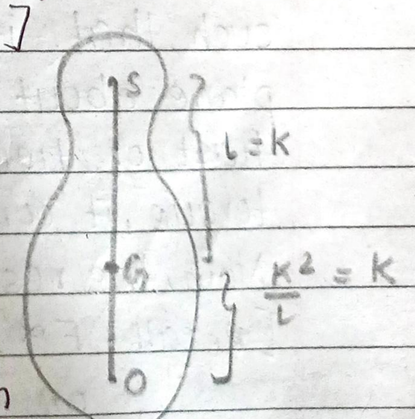
continue....

T is minimum when $l = k$ (proved)

Then, $T_{min} = 2\pi\sqrt{\frac{k + k^2/k}{g}}$ $[\because l = k]$

or, $T_{min} = 2\pi\sqrt{\frac{k+k}{g}}$

$\therefore T_{min} = 2\pi\sqrt{\frac{2k}{g}}$

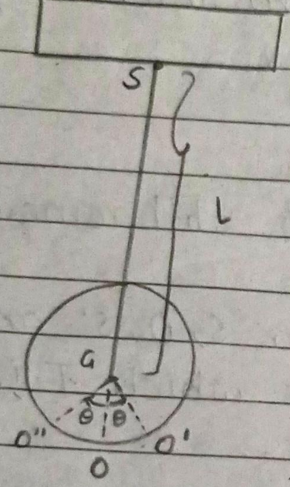


Hence, time period is minimum when $l = k$ alternatively the time period is minimum point of suspension and point of oscillation are equidistant from ~~CV~~ CG.

Note:

1) T is same for S and O interchangeable.
2) T is minimum for S and O are equidistant from C.G. (For $l = k$)

Torsion Pendulum:



A torsion pendulum is a circular disc suspended by a wire of length 'l' and modulus of rigidity η such that it is capable of rotate in a horizontal plane about a vertical axis. Let θ be the angle of twist or twisting angle then due to the restoring torque, it comes back to the mean position.

Now, the restoring torque is such that $\tau \propto \theta$

[Recall: $F \propto x$ or, $F = -kx$]

or, $\tau = -C\theta$ ..... (i)

PO

The negative sign is due to the opposite direction of τ and θ, where C is constant of proportion called torsion constant of the wire. Also, $C = \frac{\pi \eta r^4}{2l}$ ... (ii), η = Modulus of rigidity of wire

r = radius of the wire

l = length of the wire

Also, $\tau = I\alpha$ [Recall $F = ma$ → $\frac{d^2x}{dt^2}$]

$$= I\frac{d^2\theta}{dt^2} \quad \cdots (iii)$$

where I = M.I of the disc

α = Angular acceleration

$$= \frac{d^2\theta}{dt^2}$$

From (i) and (iii),

$$I\frac{d^2\theta}{dt^2} = -C\theta$$

$$\text{or, } \frac{d^2\theta}{dt^2} + \left(\frac{C}{I}\right)\theta = 0 \quad \cdots (iv)$$

Let $\frac{C}{I} = \omega^2$

or, $\omega = \sqrt{\frac{C}{I}}$, angular frequency

$\therefore$ Eq^n (iv) becomes,

$$\frac{d^2\theta}{dt^2} + \omega^2\theta = 0 \quad \cdots (v)$$

Eq^n(v) is the required differential equation of SHM. Hence the motion of torsion pendulum is simple harmonic.

To find time period.

Since $T = \frac{2\pi}{\omega}$

$$\therefore T = \frac{2\pi}{\sqrt{\frac{C}{I}}}$$

$$\therefore T = 2\pi\sqrt{\frac{I}{C}} \quad \cdots (vi)$$

Recall :-

$$T_{(mass \cdot spring\ system)} = 2\pi\sqrt{\frac{m}{k}}$$

$$T_{(compound \cdot pendulum)} = 2\pi\sqrt{\frac{I}{mgl}} = \sqrt{\frac{l + k^2/l}{g}}$$

(also done in lab)

To find η:

Using (ii) in (vi)

$$T = 2\pi\sqrt{\frac{I}{\frac{\pi\eta r^4}{2l}}}$$

Squaring,

$$T^2 = 4\pi^{\cancel{2}} \cdot \frac{I}{\frac{\cancel{\pi}\eta r^4}{2l}}$$

$$= \frac{8\pi I L}{\eta r^4}$$

$$\therefore \eta = \frac{8\pi I L}{T^2 r^4} \quad \text{(vii)}$$

~~Wave:~~

CHAPTER: 2 WAVE (motion)

It is the periodic disturbance of the material medium such that energy is transfered from one point to another point of the medium without the actual transport of the medium.

The particles vibrate in SHM.

Types of wave:

1) Mechanical wave: The wave which needs the material medium for its propagation is called mechanical wave. For e.g: sound wave, water wave, scismic wave, etc.

They are of two types:

a) Longitudinal wave: The wave in which the particles of the medium vibrate in a direction parallel to the direction of propagation of the wave is called longitudinal wave. Eg: sound wave

b) Transverse wave: The wave in which the particles of the medium vibrate in a direction perpendicular to the direction of propagation of the wave is called transverse wave. Eg:

2) Electromagnetic wave (chapter 6):
The wave which travels with the speed of light in vacuum and doesn't need the material medium for its propagation is called electromagnetic wave. It has the electric field components and magnetic field component oscillating perpendicular to each other and also perpendicular to the direction of propagation of the wave. It is transverse in nature. For example: light wave.
Modern communication technology is based on the electromagnetic wave.

3) De-Broglie wave or matter wave (chapter 7):
The wave associated with the particle or matter in motion is called de-Broglie or matter wave.

4) Gravitational wave:
The wave associated with gravitational force is called gravitational wave. It is produced due to the distortion in the space time due to the very massive astronomical objects like black holes, neutron stars, etc. In other words, gravitational wave is the ripple in the space time.

Gravitational wave has opened the new window to the properties working and beginning of the cosmos or the universe.

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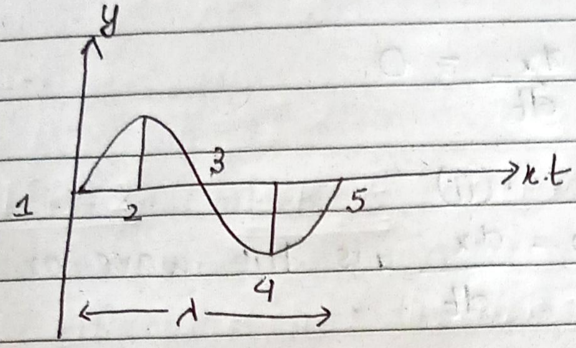
* Equation of plane progressive wave / travelling wave.

The equation of plane progressive wave travelling along +ve x-direction is given by $y = A\sin(\omega t - kx) \cdots (i)$



where y = particle displacement
A = maximum ~~displacement~~ distance from the mean position i.e amplitude.
ω = angular frequency
$= 2\pi f$, f = frequency
k = wave number or progagation constant
$= \frac{2\pi}{\lambda}$, λ = wavelength.

Equation (i) is also the particle displacement equation.

Q) Equation: $y = 0.01\,m \sin\pi(0.01x - 2t)$
$A = ?$ $k = ?$ $f = ?$ maximum particle
$\omega = ?$ $\lambda = ?$ v = wave velocity = ? speed = ?

Here $\phi = \omega t - kx$ is phase

For a given wave phase,

$$\text{phase} = \omega t - kx = \text{constant}$$

$$\therefore \frac{d}{dt}(\omega t - kx) = 0$$

$$\frac{\omega\, dt}{dt} - k\frac{dx}{dt} = 0$$

$$\omega - k\vartheta = 0 \quad \cdots (ii)$$

where $\vartheta = V_p = \frac{dx}{dt}$, is the wave or

phase velocity

$$V = V_p = \frac{\omega}{k} \quad \cdots [\text{From eq}^n\ (ii)]$$

Since, $\omega = 2\pi f$

$$k = \frac{2\pi}{\lambda}$$

$$\therefore V = V_p = \frac{2\pi f}{\frac{2\pi}{\cancel{f}\lambda}} = \lambda f$$

$$\therefore V = V_p = \frac{\omega}{k} = \lambda f$$

$$\left[\text{Recall} : v = \frac{d}{t} \qquad v = \lambda f = \lambda\frac{1}{T} = \frac{\lambda}{T} = \frac{d}{t} \right]$$

Now, the particle velocity is given by $V_u = \frac{dy}{dt}$

$$= \frac{d[A\sin(\omega t - kx)]}{dt}$$

$$= A \cancel{si} \cos(\omega t - kx).\omega$$

or, $u = \omega A \cos(\omega t - kx)$

The maximum value of u is $u_{max} = \omega A$

[Recall :- $v = \pm\omega\sqrt{x_m^2 - x^2}$

$v_{max} = \omega x_m$ (magnitude)]

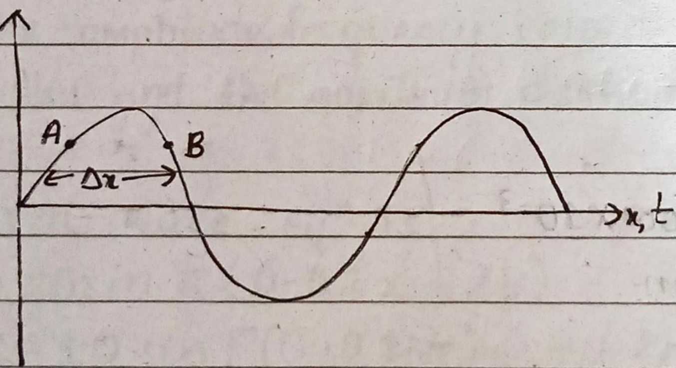
Board exam:

Q) A wave has phase velocity of 350 m/s and has a frequency 500 Hz.

(a) How far apart are the two points 60° ~~outer~~ out of phase.

(b) What is the phase difference between the two points at a certain displacement at 10^{-3} sec apart?

⇒ Solⁿ:-



Here, given, wave of phase velocity (V or V_p) = 350 m/s

frequency (f) = 500 Hz

(a) Since the phase is given by $\phi = \omega t - kx$

$\therefore \Delta\phi = \omega \Delta t - k\Delta x$

At the same time, $\Delta t = 0$

$\therefore \Delta\phi = -k\Delta x$

then, $\Delta\phi = 60° = \frac{\pi}{3}$ rad

$\therefore \frac{\cancel{\pi}}{3} = -\frac{2\cancel{\pi}}{\lambda}\Delta x$

$\therefore$ since $v = \lambda f$

$\frac{1}{3} = \frac{2}{v/f}\Delta x$

or, $\frac{1}{3} = \frac{2}{350/500}\Delta x$

$\therefore \Delta x = 0.116$ m (in magnitude)

≈ 0.12 m

(b) At a certain displacement (point)

$\Delta x = 0$

By question, $\Delta t = 10^{-3}$ sec

$\therefore \Delta\phi = \omega\Delta t$

$= 2\pi f \Delta t$

$= 2\pi \times 500 \times 10^{-3}$

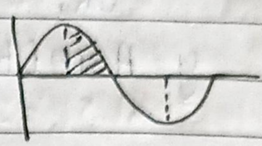
$= \pi$ radian

Board

Q) The time taken to move a particular point from the maximum displacement to zero is 0.17 sec. Find the time period, frequency. If the wavelength of the wave is 1.94 m, find the wave velocity.

⇒ Solⁿ:-

Note: To draw figure



Given,

Wavelength $(\lambda) = 1.94$ m

Time period = ?

Frequency (f) = ?

$t = 0.17$ sec

(i) $T = 4t$

$= 0.68$ s

~~(iii)~~ $F = \frac{1}{T} = 1.47$ Hz

(ii) $v = \lambda f$

$= 2.85$ m/s

Q) The equation of transverse wave travelling in a rope is given by $y = 10 \sin \pi (0.01x - 2t)$. Find the amplitude, frequency, velocity and wavelength of the wave. Also find the maximum particle speed of the medium.

The given wave eqⁿ is

$y = 10 \sin \pi (0.01x - 2t)$

$= 10 \sin [(0.01\pi)x - (2\pi)t] \dots (i)$

Comparing eqⁿ(i) with

$y = A \sin (kx - \omega t)$

Note :

1) $y = A\sin(\omega t - kx)$ or $y = A\sin(kx - \omega t)$ } wave along +ve x-direction

2) $y = A\sin(\omega t + kx)$ OR, $y = A\sin(kx + \omega t)$ } wave along -ve x-direction

(A) $a = 10\,cm$

(B) $k = 0.01\pi \Rightarrow \frac{2\cancel{\pi}}{\lambda} = 0.01\cancel{\pi}$

$\therefore \lambda = 200\,cm$

(C) $\omega = 2\pi$

$2\pi f = 2\pi$

$\therefore f = 1\,Hz$

(D) V or $V_p = \lambda f = \cancel{10}\ 200\,cm/s$

(E) $u_{max} = \omega A = 2\pi \times 10$

$= 62.8\,cm/s$

Assignment : Example and exercises book को

Note: $y = A\sin(\omega t + kx)$
$y = A\sin(kx + \omega t)$ } along -ve x-axis

• Relation between wave velocity (v) and particle velocity (u)

since, $y = A\sin(\omega t - kx)$ ... (i)

$$u = \frac{dy}{dt} = A\omega\cos(\omega t - kx) \quad \ldots (ii)$$

$$\frac{dy}{dx} = A\cos(\omega t - kx)(-k)$$

$$= -Ak\cos(\omega t - kx) \quad \ldots (iii)$$

Also (ii) can be written as

$$u = Avk\cos(\omega t - kx) \quad [\because v = \omega/k \text{ or, } \omega = kv]$$

$$\text{or, } u = (-v)[-Ak\cos(\omega t - kx)] \quad \ldots (iv)$$

Using (iii) in (iv)

$$u = (-v)\frac{dy}{dx}$$

$$\frac{dy}{dt} = (-v)\frac{dy}{dx}$$

This is the relation between wave velocity (v) and particle velocity (u)

where $\frac{dy}{dt}$ = particle velocity at a point.

v = wave velocity

$\frac{dy}{dx}$ = slope of displacement curve at the poin

- Differential Equation of wave [2nd order differential equation]

[Recall: SHM = Diff eqn (2nd order diff. eqn)

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

Soln :- $y = A\sin(\omega t - kx) \cdots (i)$

$$\frac{dy}{dt} = A\cos(\omega t - kx)\,\omega \cdots (ii)$$

$$= AVK\cos(\omega t - kx) \quad [\because V = \omega/k]$$

$$\frac{d^2y}{dt^2} = A\omega^2[-\sin(\omega t - kx)]$$

$$= V^2[-Ak^2\sin(\omega t - kx)] \cdots (iii)$$

Similarly,

Differentiating both sides of (i) w.r.t 'x'

$$\frac{dy}{dx} = A\cos(\omega t - kx)(-k)$$

$$= -Ak\cos(\omega t - kx)$$

Again,

$$\frac{d^2y}{dx^2} = -Ak[-\sin(\omega t - kx)](-k)$$

$$\therefore \frac{d^2y}{dx^2} = -Ak^2\sin(\omega t - kx) \cdots (iv)$$

Now,

using (iv) in (iii)

$$\frac{d^2y}{dt^2} = v^2\frac{d^2y}{dx^2}$$

This is the required differential equation of wave which is second order.

* Energy Intensity and Power of the plane progressive wave.

The energy carried by a wave is due to the energy carried by a vibrating particles of the medium in SHM. The energy of a single particle vibrating in SHM is given by:

$$E = K.E + P.E$$
$$= \frac{1}{2} mu^2 + \frac{1}{2} ky^2 \quad \text{where } k \text{ is force constant.}$$

where, m = mass of single particle

y = particle displacement
$= A\sin(\omega t - kx)$

u = particle velocity
$= \frac{dy}{dt}$
$= A\omega\cos(\omega t - kx)$

Also, $k = m\omega^2$ $\left[\because \text{In SHM} \Rightarrow \omega^2 = \frac{k}{m}\right]$, force constant

$$\therefore E = \frac{1}{2} m\omega^2 A^2 \cos^2(\omega t - kx) + \frac{1}{2} m\omega^2 A^2 \sin^2(\omega t - kx)$$

$$= \frac{1}{2} m\omega^2 A^2 \quad \left[\because \text{Recall: In SHM, } E = \tfrac{1}{2} m\omega^2 x_m^2\right]$$

let n be the no. of particles per unit volume of the medium. Then 'nE' gives the energy of particles per unit volume of the medium called energy density (V)

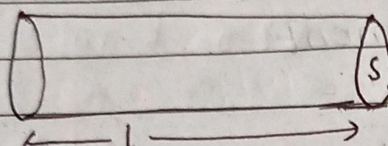
$$\therefore V = nE$$
$$= n\frac{1}{2} m\omega^2 A^2$$

$= \frac{1}{2}(mn)\omega^2 A^2$

$\therefore U = \frac{1}{2}\rho\omega^2 A^2$ unit of $U = Jm^{-3}$

where $\rho = mn$, density of the medium.

Now,
consider a cylinder of cross sectional area 'S' of length 'L'



Then, volume of the cylinder $(V) = SL$
The total energy of vibrating particle is given by,

$W = UV$

$= \frac{1}{2}\rho\omega^2 A^2 SL$

Now, the intensity of wave is defined as energy carried by the wave per unit area of the cross sectional per unit time.

i.e. Intensity $(I) = \frac{\text{Energy}(W)}{\text{Area}(S) \times \text{time}(t)}$ Unit of $I = Jm^{-2}s^{-1}$ $= Wm^{-2}$

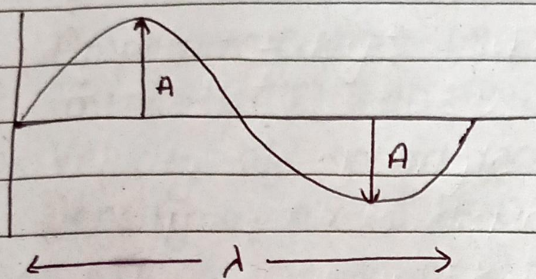
$= \dfrac{\frac{1}{2}\rho\omega^2 A^2 SL}{S \times t}$

Vimp. $\boxed{I = \frac{1}{2}\rho v A^2\omega^2}$ where, $v = \frac{L}{t}$ is the wave/phase velocity.

Recall: $U = \frac{1}{2} \rho \omega^2 A^2$

$$I = \frac{1}{2} \rho v \omega^2 A^2$$

For a given plane progressive wave, amplitude 'A' is constant; therefore I is constant.



$A = \text{constant}$

$\therefore I = \text{constant}$.

Q) Derive an expression for the intensity of a plane progressive wave/travelling wave. OR show that intensity is constant.

OR, Intensity varies with the square of frequency.

Note: $I \propto \rho$

$I \propto v$

$I \propto \omega^2$ or $I \propto f^2$

$I \propto A^2$

For a spherical wave:

$$I = \frac{P}{4\pi r^2} \quad \ldots (i) \quad \left[\because I = \frac{\text{Energy}}{A \times t} = \frac{\text{power } (P)}{4\pi r^2}\right)$$

where, P = power of the source

r = distance

$\therefore I \propto \frac{1}{r^2}$; for a spherical wave

Also, using $I = \frac{1}{2} \rho v \omega^2 A^2$ in (i)

$$\frac{1}{2} \rho v \omega^2 A^2 = \frac{P}{4\pi r^2}$$

$$\therefore A = \frac{1}{\omega r} \sqrt{\frac{P}{2\pi \rho v}}$$

$$A \propto \frac{1}{r}$$

* Summary:

1) Plane progressive wave:

$$I = \frac{1}{2} \rho v \omega^2 A^2$$

A = constant

I = constant

2) Spherical wave:

$$I = \frac{P}{4\pi r^2}$$

$$A \propto \frac{1}{r}$$

$$I \propto \frac{1}{r^2}$$

cc = cm3

Q) Calculate the frequency of vibration of air particles in plane progressive wave of amplitude 2.18×10^{-10} m and intensity 10^{-10} W/m², velocity of sound wave in air (v) = 340 m/s, density of air is 0.00129 gm/cc.

⇒ Solⁿ:-

Given,

Amplitude (A) = 2.18×10^{-10} m

Intensity (I) = 10^{-10} W/m²

Velocity of sound wave in air (v) = 340 m/s

Density of air (ρ) = 0.00129 gm/cc

$= 0.00129 \times 1000$ kg/m³

$= 1.29$ kg/m³

Frequency of vibration of air particles (f) = ?

Now,

We know that,

$$I = \frac{1}{2}\rho v \omega^2 A^2$$

or, $10^{-10} = \frac{1}{2} \times 1.29 \times 340 \times (2\pi f)^2 \times (2.18 \times 10^{-10})^2$

or, $2 \times 10^{-10} = 1.7297 \times 10^4 \times 4.7524 \times 10^{-20} \times f^2$

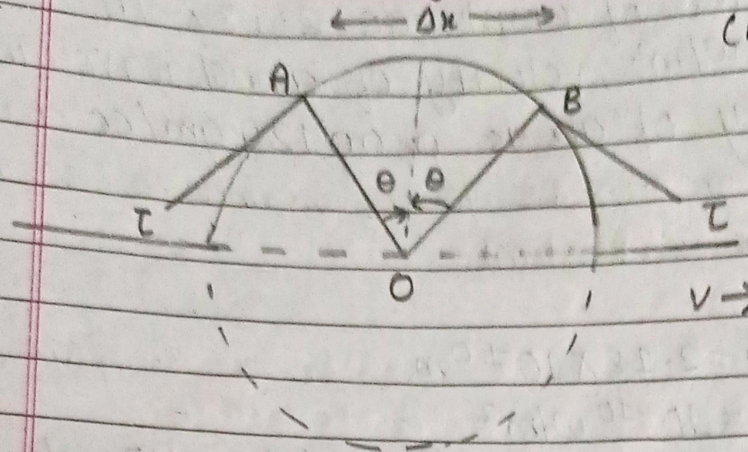
or, $f = \sqrt{\frac{2 \times 10^{-10}}{8.220 \times 10^4 \times 10^{-20}}}$

or, $f = \sqrt{\frac{2 \times 10^{-10+16}}{8.220}}$

$f = \sqrt{0.24330 \times 10^6}$

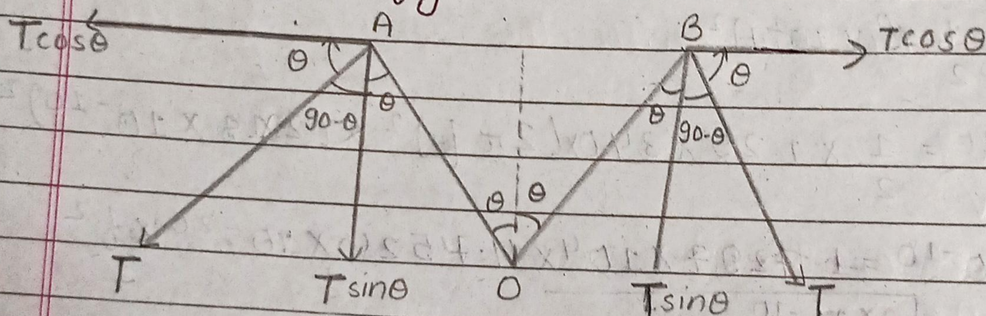
$= 493.2544$ Hz

* Speed or Velocity along a stretched string (one-dimensional medium)



fig(1)

Consider a stretched string of linear mass density μ (i.e $\frac{\text{mass}}{\text{length}}$). The string element AB is of length Δx has the tangential tensions at A ~~aan~~ and B as shown in fig (1) above.



fig(2)

As shown in the figure (2), the vertical component of T adds up to provide the necessary centripetal force which sets the string element AB into circular motion.

Then,

$$T\sin\theta + T\sin\theta = \frac{\Delta m v^2}{R} \quad ; \Delta m = \mu \Delta x$$

where Δm is the mass of the string element AB.

μ = linear mass density.

$$\therefore 2T\sin\theta = \frac{\mu \Delta x \times v^2}{R}$$

$$\text{or, } T(2\sin\theta) = \frac{\mu \Delta x \times v^2}{R}$$

For small angle θ

$\sin\theta \approx \theta$

$$\therefore T(2\theta) = \frac{\mu \Delta x\, v^2}{R}$$

$$\text{or, } T\frac{\Delta x}{R} = \frac{\mu \Delta x}{R} v^2$$

$$\text{or, } T = \mu v^2$$

$$v = \sqrt{\frac{T}{\mu}}, \quad v \propto \sqrt{T} \text{ and } v \propto \frac{1}{\sqrt{\mu}}$$

* Rate of transfer of energy along the stretched string.

The kinetic energy associated with the string element of length 'dx' is given by:

$$dK = \frac{1}{2} dm\, u^2$$

where, dm = mass of string element

$= \mu dx$

u = particle velocity

$= \frac{dy}{dt}$; $y = A\sin(\omega t - kx)$ is the particle displacement.

$= A\omega\cos(\omega t - kx)$

$\therefore dK = \frac{1}{2} \mu\, dx\, \omega^2 A^2 \cos^2(\omega t - kx)$

The rate of transfer of K.E is given by,

$$\frac{dK}{dt} = \frac{1}{2} \mu v \omega^2 A^2 \cos^2(\omega t - kx)$$

where, $v = \frac{dx}{dt}$ is the wave or phase velocity.

Now, the average rate of transfer of K.E is given by,

$$\left[\frac{dK}{dt}\right]_{avg} = \frac{1}{2} \mu v \omega^2 A^2 [\cos^2(\omega t - kx)]_{avg}$$

$$\therefore \left[\frac{dK}{dt}\right]_{avg} = \frac{1}{4} \mu v \omega^2 A^2$$

Similarly, the average rate of transfer of potential energy is given by,

$$\left[\frac{dU}{dt}\right]_{avg} = \frac{1}{4} \mu v \omega^2 A^2$$

Therefore, the average rate of transfer of total energy is $\left[\frac{dE}{dt}\right]_{avg} = \left[\frac{dK}{dt}\right]_{avg} + \left[\frac{dU}{dt}\right]_{avg}$

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Vimp $P_{avg} = \frac{1}{2} \mu \omega^2 v A^2$

Recall: $I = \frac{1}{2} \rho v \omega^2 A^2$

I : $\frac{\text{energy}}{\text{Area} \times \text{time}}$

ρ : $\frac{\text{mass}}{\text{area} \times \text{length}}$

Board

Q.no.1) A stretched string has linear density 525 gm/m and is under tension of 45 N. A sinusoidal wave with frequency of 120 Hz and amplitude at 8.5 mm is sent along the string from one end. At what average rate does the wave transport energy?

⇒ Solⁿ:-

Given, linear density (μ) = 525 gm/m $= \frac{525}{1000}$

$= 0.525$ kg/m.

Tension (T) = 45 N

frequency (f) = 120 Hz

Amplitude (A) = 8.5 mm

$= 8.5 \times 10^{-3}$ m

$P_{avg} = ?$

Now, we know that,

$$P_{avg} = \frac{1}{2} \mu v \omega^2 A^2$$

$$= \frac{1}{2} \mu \sqrt{\frac{T}{\mu}} (2\pi f)^2 \times A^2$$

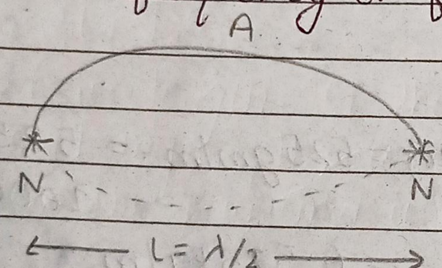
$$= \frac{1}{2} \, 0.525 \sqrt{\frac{45}{0.525}} \times 4\pi^2 (120)^2 \times (8.5\times10^{-3})$$

$= 99.7184$ watt

Standing wave and resonance:

When a string stretched between two clamps is made to oscillate at certain frequency, the interference produces standing wave pattern with nodes and antinodes. This condition is called resonance and the string is said to resonate at certain frequencies called resonant frequencies.

(i) Fundamental frequency or frequency of 1st harmonic:

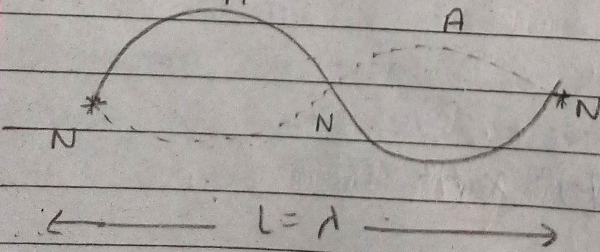


$\Rightarrow L = \lambda/2 \Rightarrow \lambda = 2L$

$$V = \sqrt{\frac{T}{\mu}}$$

$$f_0 = \frac{V}{\lambda} = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

(ii) Frequency of second harmonic or 1st overtone.

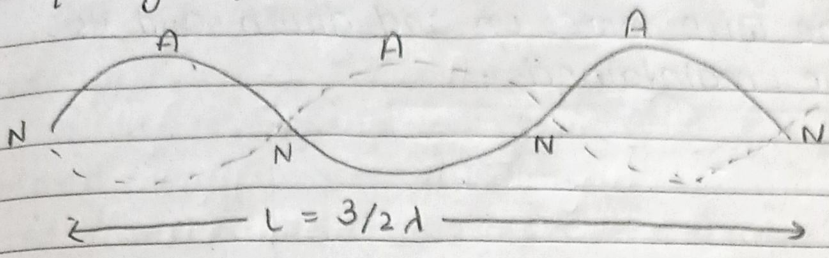


$\lambda = L$

$$f_1 = \frac{V}{\lambda} = \frac{V}{L} = \frac{2V}{2L} = 2f_0$$

$\therefore f_1 = 2f_0$

(iii) Frequency of third harmonic or 2^{nd} overtone:



$\Rightarrow \lambda = \frac{2L}{3}$ $\qquad f_2 = V/\lambda$

$= 3V/2L$

$f_2 = 3f_0$ and so on

$f_n = (n+1)f_0$

Thus, both odd and even harmonics are present.

* **Sonometer**: A sonometer consists of a hollow sounding box one end of the wire is fixed at one end while the second end passes over a pulley fixed at the other end of box, and carries a hanger on which slotted weights can be slipped. The vibrating length of the wire can be adjusted by means of two sharp knife edges, over which the wire passes. The horse-shape magnet is placed at the middle of the wire. An alternating current of low voltage is passed through the wire, when a current carrying is placed in an uniform magnetic field, it will experience magnetic force and is deflected. According to fleming's left hand rule, if the current flows from left to right and magnetic field is directed in the direction opposite to our face, the wire experience upward force and is deflected upward. After next half cycle of a.c, the current flows from right to left and

the wire is deflected downward and so on. In this way, the wire moves up and down and its vibration are maintained.

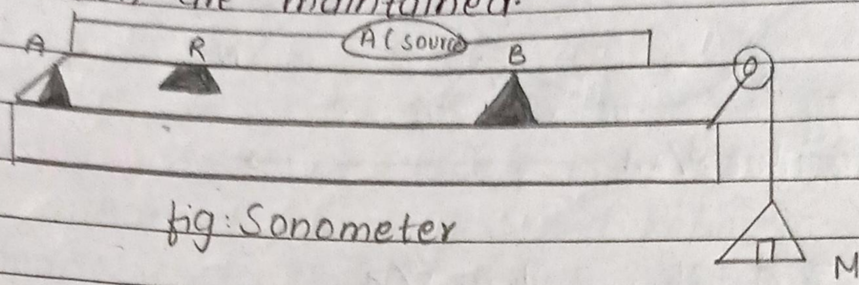


fig: Sonometer

When the wire resonates the frequency of a.c main is equal to the frequency of vibration of the string. According to laws of transverse vibration of string, the frequency of fundamental mode is,

$$f = \frac{1}{2l}\sqrt{\frac{T}{\mu}} \quad \ldots (i)$$

where, l is length of the vibrating pigment of the wire. $T = mg$, is the tension in the wire, m is mass placed in the pan and μ is mass per unit length of wire.

$$T = 4\mu f^2 \cdot l^2$$

If d be the diameter of the wire, then area of cross section of wire $= \frac{\pi d^2}{4}$

volume of wire = Area of cross-section x length

mass of wire = volume x density

$$= \frac{\pi d^2}{4} \times l \times \rho$$

mass per unit length of wire $(\mu) = \frac{\pi d^2}{4} \times \rho$

Here, ρ = density of material of wire.

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Unit 6: Electromagnetic

Electromagnetic oscillations:

Three types:

a) Free or LC oscillation.

b) Damped or LCR oscillation.

c) Forced / driven or L·CR with a·c source oscillation.

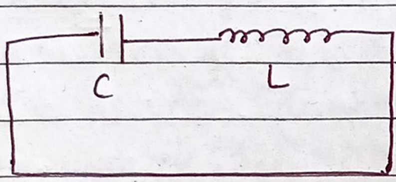
Note:

L → inductor

C → capacitor

R → resistor

1) Free or undamped or L·C oscillation.



Consider a fully charged capacitor connected in series within inductor as shown in figure. The capacitor stores the electric energy and the inductor stores the magnetic energy which are given by $U_E = \frac{1}{2}\frac{q^2}{C}$

and $U_B = \frac{1}{2} LI^2$ [Recall: In SHM: $KE = \frac{1}{2}mv^2$, $P.E = \frac{1}{2}kx^2$]

Now, the total energy is given by:

$$U = \cancel{E}\ U_E + U_B$$

$$= \frac{1}{2}\frac{q^2}{C} + \frac{1}{2}LI^2$$

Then, $\frac{dU}{dt} = 0$ [Note: $H = I^2Rt$] $\therefore H/t = I^2R$

Q 8

or, $\frac{d}{dt}\left[\frac{1}{2C}q^2 + \frac{1}{2}LI^2\right] = 0$

or, $\frac{1}{2C}\frac{dq^2}{dq}\cdot\frac{dq}{dt} + \frac{1}{2}L\frac{dI^2}{dI}\cdot\frac{dI}{dt} = 0$

or, $\frac{1}{\cancel{2}C}\cancel{2}q\, I + \frac{1}{\cancel{2}}\cancel{2}IL\frac{d^2q}{dt^2} = 0$

[Using $I = \frac{dq}{dt}$ and $\frac{dI}{dt} = \frac{d^2q}{dt^2}$]

or, $\frac{q}{C}I + LI\frac{d^2q}{dt^2} = 0$

Since $I \neq 0$

$\frac{q}{C} + L\frac{d^2q}{dt^2} = 0$

or, $L\frac{d^2q}{dt^2} + \frac{1}{C}q = 0 \quad \cdots (i)$

or, $\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0 \quad \cdots (ii)$

[Recall: SHM : $F \propto x$, $F = -kx$

$ma = -kx$

$m\frac{d^2x}{dt} + kx = 0$

$\frac{d^2x}{dt^2} + \omega^2 x = 0$

$\omega = \sqrt{k/m}$

Date
Page

Eqⁿ (i) is the form of $m\frac{d^2x}{dt^2} + kx = 0$, so

Comparing, $m = L$
$k = 1/C$
$x = q$

Also, eqⁿ (ii) is of the form of the differential eqⁿ of SHM.

$$\frac{d^2x}{dt^2} + \omega^2 x = 0, \ \omega = \sqrt{k/m}, \text{ angular frequency} \quad \ldots (iii)$$

Comparing, $\omega^2 = \frac{1}{LC}$

or, $\omega = \sqrt{\frac{1}{LC}}$, undamped angular frequency

$f = f_r = \frac{\omega}{2\pi} \quad [\because \omega = 2\pi f]$

$$\boxed{\therefore f = f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \frac{1}{2\pi\sqrt{LC}}}$$

Here, $f = f_r$ is called undamped frequency.
Here, the charge oscillates simple harmonically in L.C circuit
This is called L.C oscillation.
The solution of eqⁿ (iii) can be written as:
$x = x_m \cos(\omega t + \phi)$
Similarly, the solution of eqⁿ (ii) can be written as:
$q = q_m \cos(\omega t + \phi) \quad \ldots (iv)$
$I = \frac{dq}{dt} = -\omega q_m \sin(\omega t + \phi)$

Again,

$$\frac{d^2q}{dt^2} = -\omega^2 q_m \cos(\omega t + \phi) \cdots (v)$$

Using (iv) and (v) in (ii),

$$-\omega^2 q_m \cos(\omega t + \phi) + \frac{1}{LC} q_m \cos(\omega t + \phi) = 0$$

Here, $q = q_m \cos(\omega t + \phi) \neq 0$,

$$-\omega^2 + \frac{1}{LC} = 0$$

$$\therefore \omega^2 = \frac{1}{LC}$$

$$\Rightarrow \omega = \sqrt{\frac{1}{LC}}, \text{same as before}$$

Energy:

$$U_E = \frac{1}{2}\frac{q^2}{C}$$

$$= \frac{1}{2C} q_m^2 \cos^2(\omega t + \phi) \text{ [Using (iv)]}$$

and $U_B = \frac{1}{2} LI^2$

$$= \frac{1}{2} L\omega^2 q_m^2 \sin^2(\omega t + \phi)$$

since $\omega^2 = \frac{1}{LC} \Rightarrow \omega^2 L = \frac{1}{C}$

$\therefore U_B = \frac{1}{2C} q_m^2 \sin^2(\omega t + \phi)$

Then, $T.E(U) = U_E + U_B$

$= \frac{1}{2C} q_m^2$ which is constant or conserved.

Hence, the total energy is constant although, the kinetic and potential vary with time as in S.H.M.

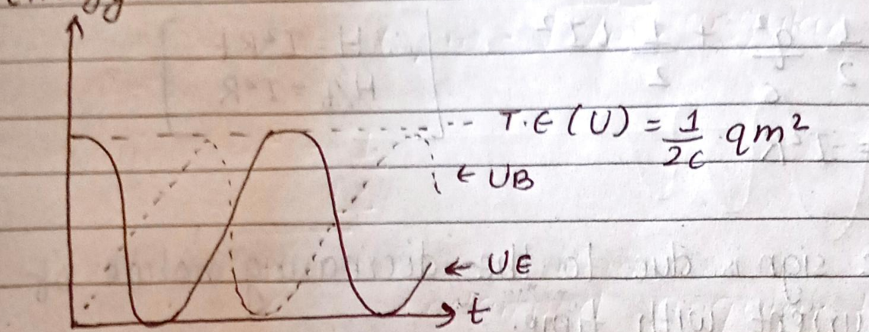


fig: Energy Vs t graph

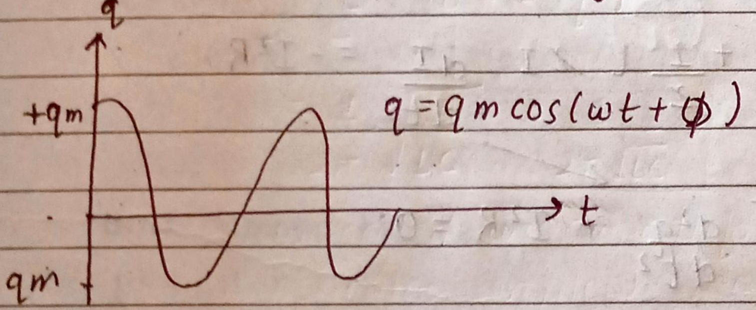
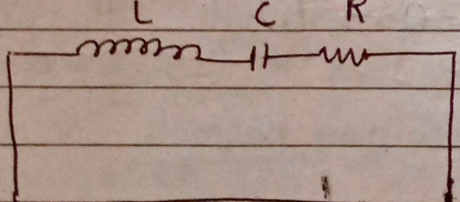


fig: q vs t graph

2) L.C.R or damped oscillation:



Consider a fully charged capacito connected in series within inductor and ~~top~~ resistor as shown in fig.

The capacitor stores the electrically energy and resistor the inductor stores the magnetic energy, which are given by:

$$U_E = \frac{1}{2}\frac{q^2}{C}$$

$$U_B = \frac{1}{2} L I^2$$

Then $U = U_E + U_B$

$$= \frac{1}{2}\frac{q^2}{C} + \frac{1}{2} L I^2$$

$$\left[\because H = I^2 R t \quad H/t = I^2 R\right]$$

Now, $\frac{dU}{dt} = -I^2 R$

The negative sign is due to the decreasing nature of charge or current with time.

or, $\frac{1}{\cancel{2}C}\cancel{2}q\frac{dq}{dt} + \frac{1}{\cancel{2}} L \cancel{2} I \frac{dI}{dt} = -I^2 R$

or, $\frac{q}{C} I + L I \frac{d^2q}{dt^2} + I^2 R = 0$

Since $I \neq 0$,

$$\frac{q}{C} + L\frac{d^2q}{dt^2} + IR = 0$$

or, $L \cdot \frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C} q = 0 \cdots (i)$

Comparing eq^n (i) with the damped harmonic motion.

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0 \cdots (ii)$$

$m = L$, inductance

$b = R$, resistance

(b = damping factor)

$k = \frac{1}{c}$, c = capacitance

$x = q$

Solution of eq^n (ii) can be written as:

$$x = x_m e^{-\left(\frac{b}{2m}\right)t} \cos(\omega' t + \phi)$$

Recall:
Th/Lab
RC cht:
$I = I_0 e^{-\frac{t}{RC}}$

where, ω' = damped angular frequency

$$= \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$

and damped frequency is

$$f' = \frac{\omega'}{2\pi}$$

$$\therefore f' = \frac{1}{2\pi}\sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$

Similarly, the solution of eq^n (i) can be written as:

$$q = q_m e^{-(R/2L)t} \cos(\omega' t + \phi),$$

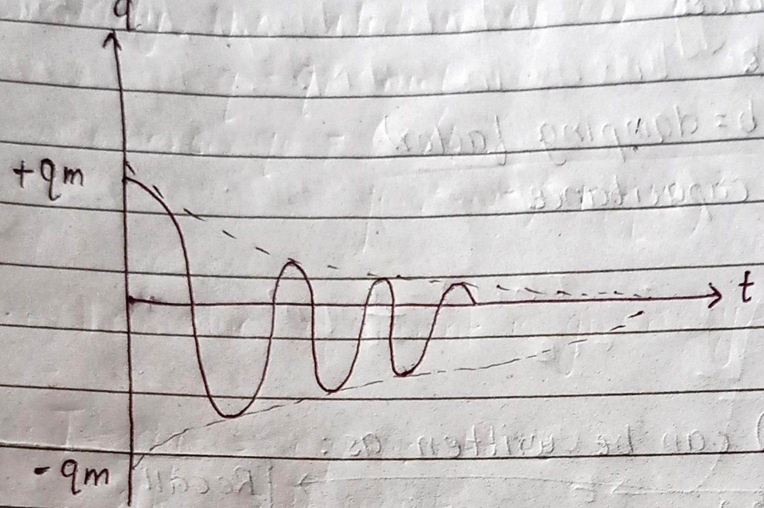
where ω' = damped angular frequency

$$= \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

damped frequency is given by:

$$f' = \frac{\omega'}{2\pi}$$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$



$$\omega' = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

Cases:

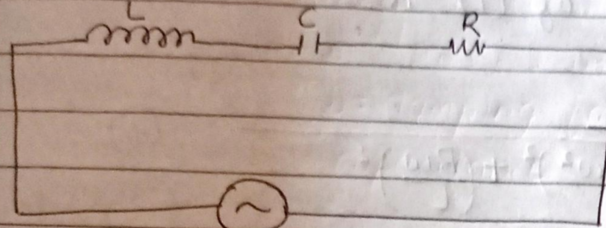
1) If $\frac{1}{LC} > \left(\frac{R}{2L}\right)^2$

or, if $\frac{1}{LC} > \frac{R^2}{4L^2}$, ω' is positive and the discharge of capacitor is ~~at~~ oscillative

2) If $\frac{1}{LC} = \frac{R^2}{4L^2}$, ω' is 0 and the discharge is critically damped.

3) If $\frac{1}{LC} < \frac{R^2}{4L^2}$, ω' is $-ve$ and the discharge is non oscillating.

3) Forced or driven or LCR with a.c source oscillation.



a c source

Consider a circuit containing an inductor, capacitor and a resistor connected in series with an a.c source of Emf $E = E_0 \sin \omega t$

Then, KVL gives

$$V_R + V_L + V_C = E$$

or, $$IR + L\frac{dI}{dt} + \frac{q}{C} = E_0 \sin \omega t$$

or, $$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = E_0 \sin \omega t \quad \ldots (i)$$

Eqn (i) is the differential equation of forced or driven LCR oscillation. Comparing eqn (i) with the differential equation of force mechanical oscillation.

$$M\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0 \sin \omega t \quad \ldots (ii)$$

we get,

$M = L$ $\qquad F_0 = E_0$

$b = R$ $\qquad x = q$

$K = 1/C$

The solution of eqⁿ (ii) can be written as:

$$x = A\sin\omega t$$

where $A = \dfrac{F_0/m}{\sqrt{(\omega_0^2-\omega^2)^2+\left(\frac{b\omega}{m}\right)^2}}$

Similarly, the solution of (i) is:

$$q = q_0\sin\omega t$$

where $q_0 = \dfrac{\epsilon_0/L}{\sqrt{(\omega_0^2-\omega^2)^2+\left(\frac{R\omega}{L}\right)^2}}$ ....(iii)

Now, the value of current is given by

$$I = \frac{dq}{dt} = \omega q_0\cos\omega t$$

$$= I_0\cos\omega t$$

where $I_0 = \omega q_0$ ...(iv)

Now, using (iii) in eqⁿ (iv)

$$I_0 = \frac{\omega(\epsilon_0/L)}{\sqrt{(\omega_0^2-\omega^2)^2+\left(\frac{R\omega}{L}\right)^2}}$$

or, $I_0 = \dfrac{\epsilon_0(\omega/L)}{\frac{\omega}{L}\sqrt{\left(\omega_0^2\frac{L}{\omega}-\omega^2\frac{L}{\omega}\right)^2+R^2}}$

or, $I_0 = \dfrac{\epsilon_0}{\sqrt{\left(\frac{1}{LC}\frac{L}{\omega}-\omega L\right)^2+R^2}}$

where $\omega_0^2 = \frac{1}{LC}$ $\qquad$ $z = \sqrt{(X_C - X_L)^2 + R^2}$

$X_L = \omega L$

$X_C = \frac{1}{\omega C}$

$$\therefore I_0 = \frac{E_0}{\sqrt{\left(\frac{1}{\omega C} - \omega L\right)^2 + R^2}}$$

$$= \frac{E_0}{\sqrt{(X_C - X_L)^2 + R^2}} = \frac{E_0}{Z}$$

where $Z = \sqrt{R^2 + (X_C - X_L)^2}$ is called impedance

$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$, X_C = capacitance reactance.

$X_L = \omega L = 2\pi f L$, X_L = inductive reactance.

Here, Z will be minimum when $X_C = X_L$

$$\text{or, } \frac{1}{\omega C} = \omega L$$

$$\text{or, } \omega^2 = \frac{1}{LC}$$

$$\therefore \omega = \frac{1}{\sqrt{LC}} = \omega_r$$

$$\therefore f = \frac{\omega}{2\pi}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = f_r$$

At resonance, the driving frequency is equal to the natural frequency and the current in the circuit becomes maximum, this occurs when z is minimum. For this condition

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = f_r$$

* Quality factor (Q):

$$Q = \frac{V_L}{V_R} \text{ or } \frac{V_C}{V_R}$$

$$= \frac{\cancel{I}X_L}{\cancel{I}R} = \frac{\omega L}{R} = \frac{2\pi f L}{R}$$

or,

$$Q = \frac{\cancel{I}X_C}{\cancel{I}R} = \frac{X_C}{R} = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

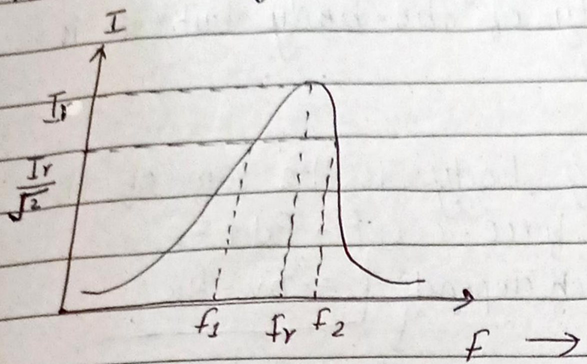
Alternatively,

Q = Energy stored across L / Energy ~~stored~~ dissipiated across R

$$= \frac{I^2 X_L t}{I^2 R t}$$

$$\frac{X_L}{R} = \frac{\omega L}{R} \text{ same as before.}$$

If we plot a graph of current vs frequency for a given resistance, we get



The ~~quat~~ quality factor is given by:

$$Q = \frac{2\pi f_r}{f_2 - f_1}$$

Here, $f_2 - f_1$ = band width

The quality factor ~~cap~~ characterisizes the sharpness of resonance. The quality factor is also the figure of merit that enable us to compare the different coils. If 'Q' is large, the curve is sharp.

Types of oscillation (in SHM)

1) Free oscillation.
2) Damped oscillation
3) Force or driven oscillation.

Damped oscillation: The free oscillation is an ideal concept, there always exists a resistive force which reduces the amplitude and energy of vibration of the body, and finally becomes zero, such a vibration is called the damped vibration ~~of~~ or oscillation.

For low velocity, the damping force is directly proportional to the velocity of the body but it is oppositively directed to it.

The net force on oscillating body is the sum of the restoring force and resistive force i.e $F = F_d + F_r$

$= -bv - kx$

b = damping constant which depend upon the nature of damping material.

v = velocity

k = force constant

Using $F = ma$

$ma = -bv - kx$

or, $m\frac{d^2x}{dt^2} + bv + kx = 0$ ..(i)

since $V = \frac{dx}{dt}$

or, $m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$ ...(i)

Also, $\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$ ... (ii)

or, $\frac{d^2x}{dt^2} + 2\delta\frac{dx}{dt} + \omega^2 x = 0$ ... (iii)

where $2\delta = \frac{b}{m} = \frac{1}{\tau}$

$\omega^2 = \frac{k}{m}$, τ = relaxation time

or, $\omega = \sqrt{\frac{k}{m}}$, natural frequency or, $x = x_m e^{-\frac{b}{2m}t} \cos(\omega t + \phi)$

or,

$x = x_m e^{-\frac{b}{2m}t} \sin(\omega' t + \phi)$

Eq^n (i) is the differential equation of damped oscillation. The angular frequency of damped oscillation is given by:

$$\omega' = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$

or, $$\boxed{\omega' = \sqrt{\omega^2 - \delta^2}}$$

The damped frequency is given by:

$$f' = \frac{\omega'}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

Time period $T' = \frac{1}{f'}$

Cases:

1) if $\frac{k}{m} > \frac{b^2}{4m^2}$, ω' is +ve and the system oscillates with decreasing amplitude, such oscillation is called damped oscillation.

2) if $k/m = \frac{b^2}{4m^2}$, ω' is zero, in this case there will be no oscillation and the system returns to the equilibrium condition in the shortest time, such oscillation is called critically damped.

3) if $K/m < \frac{b^2}{4m^2}$, ω' will be ~~-ve and~~ imaginary, then there will be no oscillation and the system is called overdamped.

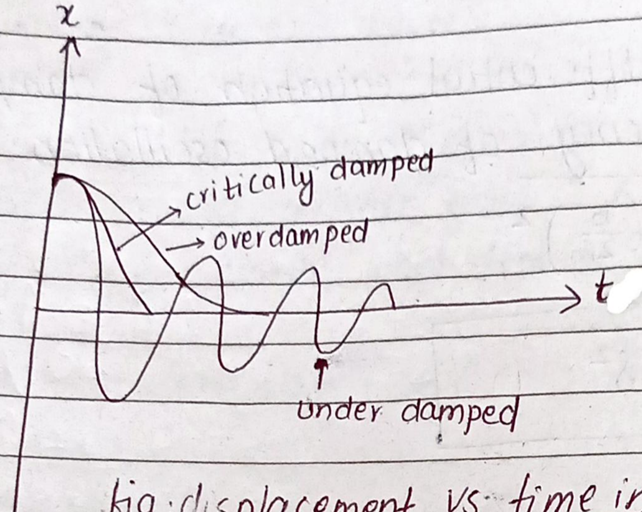


fig: displacement vs time in damped oscillation.

* FORCED OR DRIVEN OSCILLATION:

If an external force is applied on a damped harmonic oscillator, the oscillation is sustained then the total force acting on the body is given by:

$$F = F_r + F_d + F_{external}$$

$$\text{or, } ma = -kx - bv + F_0 \sin \omega t$$

$$\text{or, } m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0 \sin \omega t \quad \cdots (i)$$

Equation (i) is the differential equation of the force or driven oscillation whose solution can be written as:

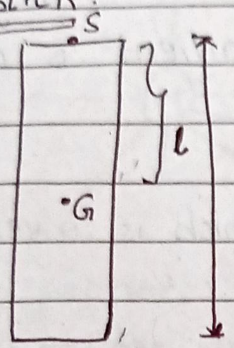
$$[\; x = x_m \sin \omega t \quad \text{or, } x_m \sin(\omega t + \phi) \;]$$

where, $x_m = \dfrac{F_0/m}{\sqrt{(\omega_0^2 - \omega^2)^2 + \left(\frac{b\omega}{m}\right)^2}}$

$$\phi = \tan^{-1}\left[\frac{\omega_0^2 - \omega^2}{\omega \frac{b}{m}}\right]_{//}$$

Here, ω_0 is natural frequency and ω is the frequency of forced oscillation.

* Meter stick:



$L = 1m$

$$l = \frac{1}{2}m = 0.5m$$

$$T = 2\pi\sqrt{\frac{l + k^2/l}{g}}$$

$$g = 9.8 m/s^2$$

$$k = \frac{\text{Total length}}{\sqrt{12}} = \frac{1m}{\sqrt{12}}$$

$$= 0.289\ m$$

$$f = \frac{1}{T} = \quad Hz \ (\text{or } s^{-1})$$

$$T = \quad sec$$

$$\omega = 2\pi f = \quad rad s^{-1}$$

Unit 6: Electromagnetism.

* Maxwell's equations: EM wave

Scalars: q, I, ρ, ϕ
Vectors: $\vec{E}, \vec{B}, \vec{J}$
Recipe:
Electrostatics
Electricity
Magnetism

In 3-d:

a) Gradient of a scalar.

Let ϕ be a scalar, then it's gradient is denoted by

$$\vec{\nabla}\phi = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\phi$$

$$= \hat{i}\frac{\partial \phi}{\partial x} + \hat{j}\frac{\partial \phi}{\partial y} + \hat{k}\frac{\partial \phi}{\partial z}$$, which is a vector.

Example: $E = -\frac{dV}{dr}$

$$\boxed{\vec{E} = -\vec{\nabla}V}$$

electric field is negative of potential gradient.

b) Divergence of a vector:

Let $\vec{A}$ be a vector. Then its divergence is denoted by $\vec{\nabla}\cdot\vec{A}$ and is given by

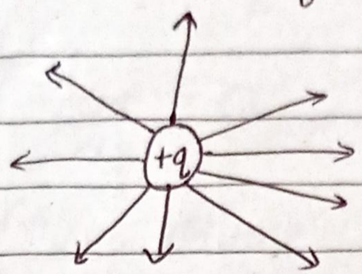
$$\vec{\nabla}\cdot\vec{A} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot\left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right)$$

$$= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$, which is a scalar.

∴ Hence, divergence of a vector is scalar.

Physical significance of divergence:

Physically, the divergence gives the information of the source or sink of the quantity



c) Curl of a vector: (cross product)

Let $\vec{A}$ be a vector, then the curl is denoted by $\vec{\nabla} \times \vec{A}$ and is given by

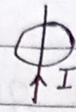
$\vec{\nabla} \times \vec{A}$ & is given by

$$\vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{vmatrix}$$ which is a vector

Hence curl of a vector is a vector.

Physical significance of curl.

Physically curl gives the rotation of a quantity.



* **Two theorems:**

a) Gauss's divergence theorem.
b) Stoke's theorem.

a) Gauss's divergence theorem:

It states that the surface integral of a vector over a closed surface is equal to volume intregal of divergence of that vector over the volume enclosed by that surface i.e

$$\oint_s \vec{A} \cdot \vec{ds} = \oint_v (\vec{\nabla} \cdot \vec{A}) dv$$

b) Stoke's theorem:

It states that the line integral of a vector over a closed loop is equalto surface integral of curl of that vector over the surface enclosed by that loop.

i.e $\oint_l \vec{A} \cdot \vec{dl} = \oint_s (\vec{\nabla} \times \vec{A}) \cdot \vec{ds}$

Note:

ine integral and surface integral are always taken of vector.

b) Volume intregal is always taken of a scalar.

v. imp

* Continuity equation or equation of continuity:

Q) Derive the continuity equation.

$$\vec{\nabla}\cdot\vec{J} = \frac{ds}{dt} = 0$$

Note: $\rho = q/v$, volume charge density

$J = I/A$, current density

Consider a volume distribution of charge such that volume charge density is given by

$$\left.\begin{aligned} \rho &= \frac{q}{v} \\ \Rightarrow q &= \rho v \end{aligned}\right\}\text{ you can skip}$$

$dq = \rho \cdot dv$

$\therefore q = \oint_v dq = \oint_v \rho dv \quad \cdots (i)$

Now, the current is given by rate i.e $I = -\frac{dq}{dt} \quad \cdots (2)$

The negative sign is due to the decreasing nature of charge or current with time.

Now, the current density is given by $J = \frac{I}{A}$

or, $I = JA$

or, $I = \oint \vec{J} \cdot \vec{ds}$ ... (3)

Using (1) and (3) in (2),

$$\oint_S \vec{J} \cdot \vec{ds} = -\frac{d}{dt} \oint_V \rho dv$$

$$= \oint_V \left(-\frac{d\rho}{dt} \right) dv$$

Using gauss divergence theorem:

$$\oint_V (\vec{\nabla} \cdot \vec{J}) dv = \oint_V \left(-\frac{d\rho}{dt} \right) dv$$

On equating

$$\vec{\nabla} \cdot \vec{J} = -\frac{d\rho}{dt}$$

or, $\vec{\nabla} \cdot \vec{J} + \frac{d\rho}{dt} = 0$ which is the continuity equation.

Case: For a steady state charge

ρ = constant

$\therefore \frac{d\rho}{dt} = 0$

Hence, the continuity eq^n becomes $\vec{\nabla} \cdot \vec{J} = 0$

VV imp

Q) Convert the integral form of maxwell's equation to the differential form, or derive the maxwell's equation and write their physical meaning or significance.

⇒ Solⁿ:-

There are four sets of maxwell equation in the intregal form and differential Form, which are given below:

S.N	Laws	Intregal Form	Differential Form
1.	Gauss's law in electrostatics	$\Phi_E = \frac{1}{\epsilon_0} q$ i.e. $\oint_S \vec{E} \cdot \vec{ds} = q/\epsilon_0$	$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$
2.	Gauss's law in magneto statics	$\Phi_B = 0$ or, $\oint_S \vec{B} \cdot \vec{ds} = 0$	$\vec{\nabla} \cdot \vec{B} = 0$
3.	Faraday's law of electromagnetic induction.	$\mathcal{E} = -\frac{d\Phi_B}{dt}$ or, $\oint_l \vec{E} \cdot \vec{dl} = -\frac{d\Phi_B}{dt}$	$\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$
4.	Modified Ampere's law or Ampere Maxwell law.	$\oint_l \vec{B} \cdot \vec{dl} = \mu_0 \left(I + \epsilon_0 \frac{d\Phi_E}{dt}\right)$	$\vec{\nabla} \times \vec{B} = \mu_0 \left(\vec{J} + \epsilon_0 \frac{d\vec{E}}{dt}\right)$

Solⁿ:-

[A] Maxwell's first equation is Gauss's law in electrostat which states that the total electric flux passing through a closed surface $\phi_E = \frac{1}{\epsilon_0} q$, where q is charge enclosed by that circuit.

$$\phi_E = \frac{1}{\epsilon_0} q$$

$$\text{or, } \oint_S \vec{E} \cdot \vec{ds} = \frac{q}{\epsilon_0} \qquad \therefore q = \oint_V \rho \, dv$$

Using Gauss divergence theorem in L.H.S

~~It states that the~~

$$\oint_V (\vec{\nabla} \cdot \vec{E}) \, dV = \frac{1}{\epsilon_0} \oint_V \rho \, dv$$

$$= \oint_V \frac{\rho}{\epsilon_0} \, dv$$

On equating

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

This is first maxwell equation, which is maxwell first equation in differential form.

Physical significance:

The first equation is $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$ which tells that divergence of the electric field is non-zero.

[B]

A single charge can act as a source of electric field, or electric monopole can exist in nature or charge can be isolated.

[B] Maxwell's second equation is Gauss's law in magnetostatics which tells that the total magnetic flux passing through a closed surface is equal to zero i.e $\phi_B = 0$.

or, $\oint_S \vec{B} \cdot \vec{ds} = 0$

Using G.d.t in L.H.S, we get,

$\oint_V (\vec{\nabla} \cdot \vec{B}) dv = 0$

On equating, we get,

$\vec{\nabla} \cdot \vec{B} = 0$ which is Maxwell's second equation in the differential form.

Physical significance: The second equation is $\vec{\nabla} \cdot \vec{B} = 0$ which tells that the divergence of the magnetic field is zero i.e there is no source or sink of the magnetic field and magnetic line of force always formed closed loop.

Magnetic poles always occur in pairs i.e magnetic monopoles doesn't exit.

[C] Maxwell's third equation is Faraday's law of electromagnetic induction which states that an induced e.m.f is produced due to the change of magnetic flux

i.e $\epsilon = -\frac{d\phi_B}{dt}$

or, $\oint_l \vec{E} \cdot \vec{dl} = -\frac{d\phi_B}{dt}$, which is Maxwell's third equation in intregal form.

[Recall: KVL (2nd) = $\Sigma emf = \Sigma p.d$

$\epsilon \approx V$

$\underset{\uparrow \text{ electric field}}{E} = -\frac{dv}{dr} = \frac{V}{L}$

or, $V = EL$

or, $dv = \vec{E} \cdot \vec{dl}$

$V = \oint dv$

$\epsilon = \oint_l \vec{E} \cdot \vec{dl}$]

Using Stoke's theorem in L.H.S, $\therefore \phi_B = \oint_S \vec{B} \cdot \vec{ds}$

$$\oint_S \vec{\nabla} \times \vec{E} \, \vec{ds} = -\frac{d}{dt} \oint_S \vec{B} \cdot \vec{ds}$$

$$= \oint_S \left(-\frac{\vec{dB}}{dt} \right) \cdot \vec{ds}$$

On equating, $\vec{\nabla} \times \vec{E} = -\frac{\vec{dB}}{dt}$ which is Maxwell's third equation in the differential form.

Physical significance:
The third equation is $\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$, which tell that the change in magnetic field produces electric field.

[D] Maxwell's forth equation is the modified Ampere's law.
The ampere's law states that the line integral of magnetic field around a closed loop is equal to μ_0 times the current enclosed by that loop i.e

$$\oint_l \vec{B} \cdot d\vec{l} = \mu_0 I$$

From third equation,

$$\epsilon = \oint_l \vec{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt}$$

We see that, the change in magnetic flux produces the electric field. By symmetry, Maxwell assumed that the change in electric flux should also produce magnetic field. Hence, the modified Ampere law becomes

$$\oint_l \vec{B} \cdot d\vec{l} = \mu_0 \left(I + \epsilon_0 \frac{d\phi_E}{dt} \right) \cdots (1)$$

Here, $I_d = \epsilon_0 \frac{d\phi_E}{dt}$, is the displacement current.

This current is produced due to the change in the electric flux. Note: I = conduction current.

$$I = \oint_s \vec{J} \cdot \vec{ds} \quad \& \quad \phi_E = \oint_s \vec{E} \cdot \vec{ds}$$

Using Stroke's theorem in L.H.S, we get,

$$\oint_s (\vec{\nabla} \times \vec{B}) \vec{ds} = \mu_0 \left[\oint_s \vec{J} \cdot \vec{ds} + \epsilon_0 \frac{d}{dt} \oint_s \vec{E} \cdot \vec{ds} \right.$$

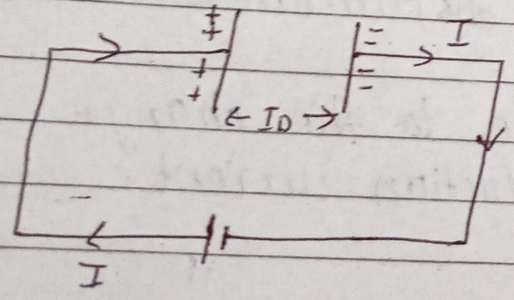
On equating,

$$\vec{\nabla} \times \vec{B} = \mu_0 \left[\vec{J} + \epsilon_0 \frac{d\vec{E}}{dt} \right]$$, which is Maxwell's forth equation in the differential form.

Physical significance:

The forth equation is $\vec{\nabla} \times \vec{B} = \mu_0 [\vec{J} + \epsilon_0 \frac{d\vec{E}}{dt}]$ which tells that magnetic field is produced due to the conduction current density $\vec{J}$ and displacement current density, $\vec{I_d} = \epsilon_0 \frac{d\vec{E}}{dt}$, which is produced due to change in electric field.

Q.1) Show that current between the plates of a capacitor is the displacement current.



The displace current is given by;

$$I_d = \epsilon_0 \frac{d\phi_E}{dt} \quad \cdots (i)$$

The charge on the plates of the capacitor is given by,

$q = CV$

where, C = capacitance of the capacitor

V = voltage across the capacitor

Now,

By Gauss's theorem,

$E = \frac{\sigma}{\epsilon_0}$, where σ = surface charge density

E = electric field.

$E = \frac{q}{\epsilon_0 A}$ [$\because \sigma = q/A$ where A is area of surface over which it flows]

or, $q = \epsilon_0 E A$ ... (ii)

Now, Diff eq^n (ii) w.r.t time, we get,

$\frac{dq}{dt} = \frac{d}{dt}(\epsilon_0 E A)$

$= \epsilon_0 \frac{d\phi_E}{dt}$ [Recall: $\phi_E = \oint_S \vec{E} \cdot \vec{ds} = E \cdot A$]

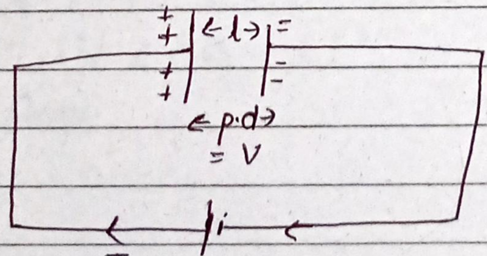
$= I_d$ [$\because$ From eq^n (i)]

which prove the current between the plates of a capacitor is the displacement current.

Q.no.3) Show that the current between the plates of the capacitor is given by,

$I_d = C \frac{dv}{dt}$

Solution:



Short cut way,

For a capacitor, $q = CV$, where C = capacitance and V = p.d or voltage

q = charge,

$$I = \frac{dq}{dt} = \frac{d}{dt}(CV) = C\frac{dV}{dt}$$

Alternatively,

$$I_d = \epsilon_0 \frac{d\phi_E}{dt}$$

$$= \epsilon_0 \frac{d}{dt}(EA)$$

$$= \epsilon_0 A \frac{dE}{dt} \quad \left[\because E = -\frac{dV}{dr} = \frac{V}{l}\right]$$

$$= \epsilon_0 A \frac{d}{dt}\left(\frac{V}{l}\right)$$

$$= \frac{\epsilon_0 A}{l}\frac{dV}{dt}$$

$$= C\frac{dV}{dt}$$

where $C = \frac{\epsilon_0 A}{l}$ is the capacitance of a parallel plate capacitor.

Date ______
Page ______

Q) Given a 1 μF capacitor, how can you produce a displace-ment current of 1A?

Solution,

Given: $I_d = 1A$

$C = 1\,\mu F = 10^{-6} F$

since, $I_d = \frac{C\,dv}{dt}$

or, $\frac{dv}{dt} = \frac{I_d}{C}$

or, $\frac{dv}{dt} = \frac{1}{10^{-6}}$

or, $\frac{dv}{dt} = 10^{-6}\ v/s$

Q) Given a parallel plate capacitor with circular plates. Find Id.
[Given: radius(r) = 8 cm = 0.08 m]

$\frac{dE}{dt} = 10^{12}\ Vm^{-1}s^{-1}$ $\left[\because E = \frac{V}{l}\right.$

Soln:-

$I_d = \epsilon_0 A \frac{dE}{dt}$

$= 8.854 \times 10^{-12} \times \pi r^2 \times 10^{12}$

$I_d = 8.854 \times 3.14 \times (0.08)^2$

$= 0.1779$ Amp.

Summary on Id:

Id is produced due to changed in,

① q : $I_d = \frac{dq}{dt}$ $q = CV$

② V : $I_d = C\frac{dV}{dt}$ $I_d = \frac{dq}{dt}$

③ E : $I_d = \epsilon_0 A \frac{dE}{dt}$

④ ϕ_E : $I_d = \epsilon_0 \frac{d\phi_E}{dt}$

v.v. imp

Qno.1 Write Maxwell's equations in free space and device wave equations for the electric and magnetic fields
OR, Show that light is an electromagnetic wave.
OR,
Show that electric and magnetic field propagate in Vaccum, with the speed of light.

⇒ Solution:

In free space or vaccum, there is no matter. Hence, no charge, therefore volume charged density $(\rho) = 0$

Hence, no current density (i.e $\vec{J} = 0$) Then Maxwell's equation in free space becomes.

i) $\vec{\nabla} \cdot \vec{E} = 0$ [$\because \rho = 0$]

ii) $\vec{\nabla} \cdot \vec{B} = 0$

iii) $\vec{\nabla} \cdot \vec{E} = -\frac{d\vec{B}}{dt}$

iv) $\vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{d\vec{E}}{dt}$ $[\because \vec{J} = 0]$

Now, taking the curl of the 3rd equation,

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\vec{\nabla} \times \frac{d\vec{B}}{dt}$$

$$= \frac{-d}{dt}(\vec{\nabla} \times \vec{B})$$

Recall: $\vec{A} \times (\vec{B} \times \vec{C}) = bac - abc$

or

bac - cab

$$= \vec{B}(\vec{A} \cdot \vec{C}) - (\vec{A} \cdot \vec{B})\vec{C}$$

or, $\vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - (\vec{\nabla} \cdot \vec{\nabla})\vec{E} = \frac{-d}{dt}(\vec{\nabla} \times \vec{B})$

Using Maxwell's 1st and 4th equations,

$$0 - \nabla^2 \vec{E} = \frac{-d}{dt}\left(\mu_0 \cdot \epsilon_0 \frac{d\vec{E}}{dt}\right)$$

or, $-\nabla^2 \vec{E} = -\mu_0 \epsilon_0 \frac{d^2\vec{E}}{dt^2}$

or, $\frac{d^2\vec{E}}{dt^2} = \frac{1}{\mu_0 \epsilon_0} \nabla^2 \vec{E}$ ---- (i)

Comparing (i) with diff eq^n of wave

$$\frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2}$$

In 3-d

$\frac{d^2y}{dt^2} = v^2 \nabla^2 y$ Here, v = wave ~~lengt~~ velocity

we get $Y = C$

$$v^2 = \frac{1}{\mu_0 \epsilon_0}$$

or, $V = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$, V = wave or phase velocity.

$$= \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8.854 \times 10^{-12}}}$$

$$= 3 \times 10^{8}\ m/s$$

= C, speed of light

Similarly,

$$\frac{d^2\vec{B}}{dt} = \frac{1}{\mu_0 \epsilon_0} \nabla^2 \vec{B} \quad \cdots (2)$$

Eq^n (1) and (2) are the wave equations for the electric and magnetic fields.

Date
Page

* Maxwell's equations and wave equations in dielectric (non-conducting):

$$\vec{\nabla} \cdot \vec{E} = 0$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$$

$$\vec{\nabla} \times \vec{B} = \mu \epsilon \frac{d\vec{E}}{dt}$$

where μ = permeability of dielectric
ϵ = permittivity of dielectric.

Taking curl of 3rd eqn,

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\vec{\nabla} \times \frac{d\vec{B}}{dt}$$

or, $\vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - (\vec{\nabla} \cdot \vec{\nabla})\vec{E} = -\frac{d}{dt}(\vec{\nabla} \times \vec{B})$

Using Maxwell's 1st and 4th equations;

$$-\nabla^2 \vec{E} = -\mu\epsilon \frac{d^2\vec{E}}{dt^2}$$

or, $\frac{d^2\vec{E}}{dt^2} = \frac{1}{\mu\epsilon} \nabla^2 \vec{E}$

Comparing with the differential equation of wave:

$$v^2 = \frac{1}{\mu\epsilon}$$

or, $V = \frac{1}{\sqrt{\mu \epsilon}}$

$= \frac{1}{\sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}}$

$= \frac{C}{\sqrt{\mu_r \epsilon_r}}$ $\left[\because C = \frac{1}{\sqrt{\mu_0 \epsilon_0}}\right]$

since $\mu_r > 1$ and $\epsilon_r > 1$,

$V < C$

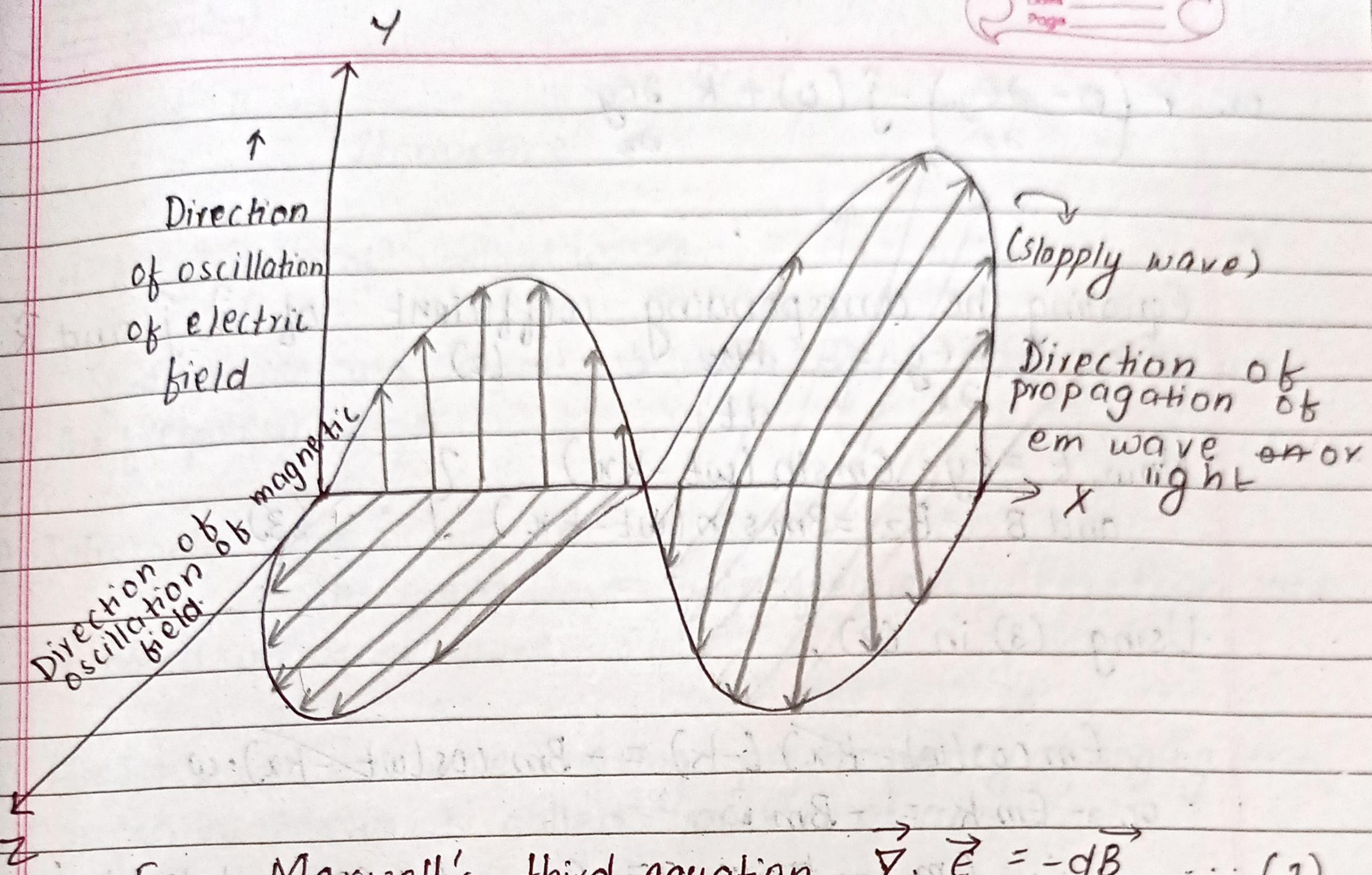
[Here, μ_r = relative permeability of medium > 1
ϵ_r = relative permittivity of medium > 1]

Hence, the electromagnetic wave propagating in the dia electric medium has velocity less than that in the free space or vaccum.

Q) Derive the relation between the ~~three~~ electric and magnetic fields in relation to the speed of EM wave. OR Show that $\frac{E}{B} = C$ or $\frac{E_m}{B_m} = C$

⇒ Solⁿ:-

Since, light is an electromagnetic wave, it has the electric and magnetic field components oscillating perpendicular to each other and also perpendicular to the direction of propagation of the electromagnetic wave or light.



From Maxwell's third equation $\vec{\nabla}.\vec{E} = -\frac{d\vec{B}}{dt}$ ... (1)

let the E and B be the electric field and magnetic field be oscillating along Y and Z axis respectively such that $\vec{E} = 0\hat{i} + E_y\hat{j} + 0\hat{k}$ and $\vec{B} = 0\cdot\hat{i} + 0\hat{j} + B_z\hat{k}$

Using these values in (1)

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & E_y & 0 \end{vmatrix} = -\frac{d}{dt}(B_z\hat{k})$$

or, $\hat{i}\left(0-\frac{\partial E_y}{\partial z}\right)-\hat{j}(0)+\hat{k}\,\frac{\partial E_y}{\partial x}$

Equating the corresponding coefficient of $\hat{i}$, $\hat{j}$ and $\hat{k}$

$$\frac{\partial E_y}{\partial x} = -\frac{dB_z}{dt} \quad \dots (2)$$

Now, $E = E_y = E_m \sin(\omega t - kx)$
and $B = B_z = B_m \sin(\omega t - kx)$ $\Big\}$ $\dots (3)$

Using (3) in (2),

$$E_m \cancel{\cos(\omega t - kx)}(-k) = -B_m \cancel{\cos(\omega t - kx)}\cdot\omega$$

or, $-E_m k = -B_m \omega$

$\therefore \frac{E_m}{B_m} = \frac{\omega}{k} = c$, $c =$ speed of light / (electromagnetic wave or radiation).

$\therefore \frac{E_m}{B_m} = c$

Also, $\frac{E}{B} = c$

Also, $\frac{E_m}{B_m} = \frac{E_m/\sqrt{2}}{B_m/\sqrt{2}}$

$= \frac{E_{rms}}{B_{rms}} = c$

pg 52 to Numerical (57)

Unit : 3

Acoustics

Classification of sound waves:

Sound waves are classified into 3 types depending upon the frequency range.

a) Infrasonic wave.
The sound waves with frequency less than 20Hz is called infrasonic wave.

b) Audible wave: The wave having ~~pre~~ frequency ranging from 20 Hz to 20KHz is called audible wave.

c) Ultrasonic wave: The sound wave having frequency greater than 20KHz are called ultrasonic wave. These are inaudible to our ears.

* Acoustics of building

Factors affecting the acoustics of building:

(a) Extraneous noice
(b) Loudness
(c) Echolon effect.
(d) Focusing
e) Echo
f) Resonance

(g) Reverberation

* ~~Reverberation~~ The persistance of sound for some time even though the source of ~~source~~ sound has ceased is called reverberation. OR, The multiple reflection of sound from the reflecting surfaces is called reverberation.

The condition for reverberation is that the distance should be less than 17 m.

Causes of reverberation:

a) The fall in intensity of sound in the room is reduced exponential and so it will take longer time to become zero.

b) Due to the multiple reflections from walls, ceiling, floor or other material, the sound reverberates inside the hall for longer time.

Time of reverberation:

The duration for which the sound can be heard after the source has ceased to produce the sound is called reverberation time.

In fact, it is also defined as the time taken by sound to fall its intensity to 1 million of its original value is called reverberation time.

i.e at $t = T$,

$I = 10^{-6} I_0$, I_0 = initial intensity

or at $t = T$

$$\frac{I}{I_0} = 10^{-6} = \frac{1}{10^6} \quad \#$$

* Factors affecting the acoustics of building:

1) Extraneous noice.
- There may be penetration of sound between rooms.
- For this the wall must be covered with sound absorbing material and doors must have heavy curtains. (There should be no penetration of sound between rooms).

2) Loudness: The loudness of sound may vary with position. To make proper loudness everywhere, wooden reflecting surface can be kept above the speaker; low ceiling is also helpful in reflecting sound towards audience.

3) Echelon effect: If there is a series of steps between floors or levels (or a set of railing) the sound produced in front of such a structure may produce a musical note due to regular successive echoes of sound, such effect is called echelon effect. If this note is in audible range, the listener will hear it prominently.
- To avoid Echelon effect steps are covered with carpet. (There should be no Echelon effect.)

4) Focusing:

- There may be concentration of sound or a ~~zero~~ zone of silence in any part of the hall.
- For uniform distribution curved surface and projection should be designed one method is to make the wall in front of audience parabolic with speaker at its focus. (~~Theire~~ There must be proper focusing of sound every where in the hall.)

5) Echo:

- Echo is a reflection of sound that arrives at the listener with a delay after the direct sound, as the distortion of sound takes place because of echo.
- The average interval between two syllable spoken by a person is about 0.2 sec. These walls and ceilings should be made in such a way that the reflection of sound takes place in more than 0.2 sec. Otherwise, it creates a confusion due to overlapping of direct and reflected sound.

(There should no echo)

6) Resonance.

- The resonance of any audio frequency note causes the sound of different intensity than that of direct one. For the hall of large size, the resonance Frequency is much below the audible limit and harmful effects due to resonance will be not ~~per~~ present.

7) Reverberation:

- The persistance of sound for some time even when the source of sound has ceased is called reverberation. This is due to the multiple reflection of sound from various part of the hall.
- It can be controlled by proper maintenance of absorbing material i.e providing window, door, carpets on the floor, heavy curtains, using full capacity of audience, etc.

Numericals.

Q) The time of reverberation of an empty hall and with 500 audience in hall is 1.5 sec and 1.4 sec respectively. Find the reverberation time with 800 audience in the hall.

⇒ Soln:

According to question,

$$1.5 = \frac{0.158V}{\alpha s} \quad \cdots (i)$$

$$1.4 = \frac{0.158V}{\alpha s + 500} \quad \cdots (ii)$$

Dividing eqn (i) by (ii)

$$\frac{1.5}{1.4} = \frac{\alpha s + 500}{\alpha s}$$

or, $1.5\alpha s = 1.4\alpha s + 700$

$\therefore \alpha s = 700$

Then eqn (i)

$$V = \frac{1.5 \times \alpha s}{0.158}$$

$$= \frac{1.5 \times 7000}{0.158}$$

$$= 66455.7$$

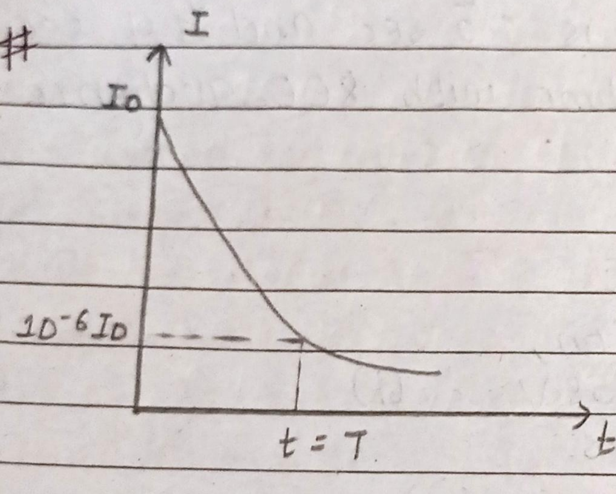
Therefore, time of reverberation with 800 audience,

$$T_3 = \frac{0.158 V}{\alpha s + 800}$$

$$= \frac{0.158 \times 66455.7}{7000 + 800}$$

$$= \underline{1.346 \text{ sec}}$$

Continue #



Absorption of sound:

The coefficient of absorption or the absorbtio coefficient is defined as the ratio of sound energy absorbed by the surface to the sound energy absorbed by an equal area of a perfect absorbed such as an open window.

The unit of absorption coefficient is sabine (s).
The sound energy absorbed by one ~~f~~ square feet of perfect absorber is called one sabine.

If A is the effective surface area for a surface having total surface area (S), the absorption coefficient 'α' is given by the relation,

$$A = \alpha S$$

If $\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_n$ be the absorbtion coefficient for each reflecting surface and $S_1, S_2, S_3, \ldots, S_n$ be the corresponding area then the average value of absorbtion coefficient is given by,

$$\alpha = \frac{\alpha_1 S_1 + \alpha_2 S_2 + \alpha_3 S_3 + \cdots + \alpha_n S_n}{S_1 + S_2 + S_3 + \cdots + S_n}$$

$$= \frac{\Sigma \alpha_i S_i}{\Sigma S_i}$$

$$\alpha = \frac{\Sigma \alpha_i S_i}{S}$$

→ Goal $T = \frac{0.158\,V}{\alpha S}$

Sabine's reverberation formula.

Let I be the average intensity of sound at any instant of time 't' then the fall in intensity in the interval δt is given by,

$$\delta I = -\alpha n I \delta t \quad \cdots (i)$$

where, the -ve sign is due the decreasing nature of intensity with time.

α = coefficient of absorption

n = number of reflection per second.

To find n;

It is found that the average distance travelled by sound between the successive reflection is given by Jaeger's formula.

$d = \frac{4V}{S}$

where, V = volume of the hall / auditorium / room

S = area

If 'v' be the velocity of sound. Then, the time between two successive reflections is given by:

$$t = \frac{d}{v} \quad \left[\because \text{velocity} = \frac{\text{distance}}{\text{time}}\right]$$

$$= \frac{4V}{Sv}$$

Then, $n = \frac{1}{t} = \frac{Sv}{4V} \quad \cdots (ii)$

v - velocity

Using (ii) in (i)

$$\delta I = -\alpha \frac{SV}{4V} I \delta t$$

In the limiting case,

$\delta t > 0, \delta I \to 0$ so,

$$dI = -\alpha \frac{SV}{4V} I \, dt$$

$$\text{or, } \frac{dI}{I} = -\alpha \frac{SV}{4V} dt$$

Integrating,

$$\int_{I=I_0}^{I} \frac{dI}{I} = -\alpha \frac{SV}{4V} \int_{t=0}^{t} dt$$

$$\text{or } \left| \ln I \right|_{I_0}^{I} = -\alpha \frac{SV}{4V} t$$

$$\text{or, } \ln I - \ln I_0 = -\alpha \frac{SV}{4V} t$$

$$\text{or, } \ln\left(\frac{I}{I_0}\right) = -\alpha \frac{SV}{4V} T$$

By the defination of reverberation time at $t = T$,

$I = 10^{-6} I_0$

$$\Rightarrow \frac{I}{I_0} = 10^{-6}$$

$$\therefore \ln(10^{-6}) = -\alpha \frac{SV}{4V} T$$

$$\text{or, } -13.816 = -\alpha \frac{SV}{4V} T$$

or, $T = \frac{55.262\,V}{\alpha S v}$

STP or NTP, $v = 332\ m/s$

$T = \frac{0.166\,V}{\alpha S}$

At $v = 340\ m/s$; $T = \frac{0.162\,V}{\alpha S}$

At $v = 350\ m/s$, $T = \frac{0.158\,V}{\alpha S}$ } S-I unit

V = cubic feet
S = sq. feet

Imp

Hence, the reverberation time is directly proportional to the volume of the hall or auditorium and inversely proportional to the absorbtion coefficient and total surface area.

For good acoustics of the hall, the reverberation time should have the appropriate value. If it is too large, there may be multiple reflections and overlapping of sound causes confusion to listner.

If it is too small, the sound vanishes intantaneously gives the dead effect. The suitable value of reverberation time is 1.03 second for a hall of 10,000 cubic feet capacity.

Ultrasonic wave or Ultrasound.

(Indroduction, production and applications of ultrasonic wave), ultrasonic method in non-destructive testing.

The sound wave having (longitudinal mechanical wave) frequency greater than 20KHz are called ultrasonic wave. These aren't audible.

ImP Methods of production:

UNIT: 5

Capacitor and Dielectric

Dielectric: It is a non-conducting or insulating material which doesn't conduct electricity. Example: glass, plastic.
Types of dielectric are:

a) non-polar dielectric.
b) polar dielectric.

[A] Non-polar dielectric: The dielectric in which positive and negative charges have the centre of gravity concided is called non polar dielectric.
In this case, the electric dipole moment is zero.
When the external electrical field is applied, dipoles are formed with the dielectric. These dipoles are called the induced dipole and tend to allign along the direction of the field. This phenomenon is called polarization.
Example: H_2, N_2, Carbon dioxde CO_2, etc.

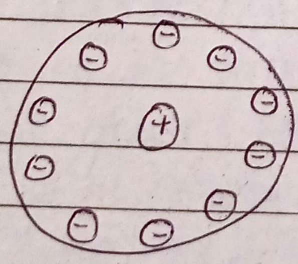


fig: non polar dielectric.

B] Polar dielectric: The dielectric in which the positive and negative charges have asymmetrical charge distribution i.e the positive and negative charges are separated by a finite distance.

Due to the finite separation between the positive and negative charges, such dielectric posses some electric dipole moment.

When the external ~~externo~~ electric field is applied, the randomly oriented dipole tends to allign along the applied electric field, but the alignment is not complete due to thermal agitation.

Example: N_2O, ~~HNO_3~~, HCL, NH_3, H_2O

* Dielectric constant (K): The dielectric constant is defined as the ratio of capacitance of capacitor with dielectric (C) to its capacitance without dielectric (C₀) i.e

$$K = \frac{C}{C_0} \quad \text{or} \quad \frac{C_{med}}{C_0}$$

Recall: Columb's law

$$K = \epsilon_r = \frac{F_{air}}{F_{med}} = \frac{E_{air}}{E_{med}}$$

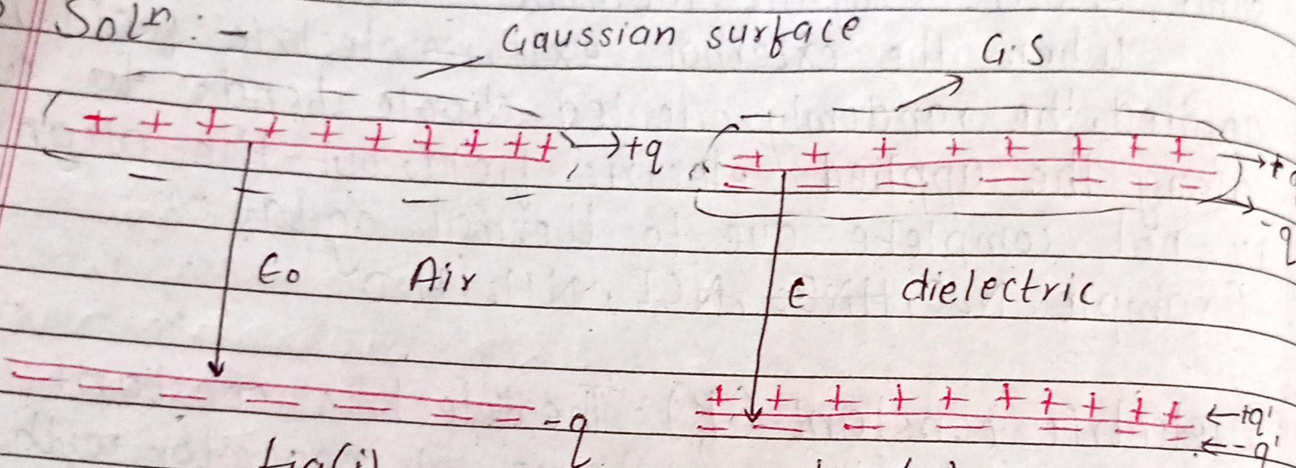
$$= \frac{\epsilon_m}{\epsilon_0} = \frac{C_{med}}{C_0} > 1$$

Also, $K = \frac{V_0}{V}$ [Recall: Electric field $(E) = \frac{V}{d}$]

* Dielectric and gauss's law:

Discuss Gauss's law in dielectric.
OR, derive the relation between the three electric vectors.
OR, Derive $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$

⇒ Solⁿ:-



fig(i) fig(ii)

Consider a parallel plate capacitor having plate area 'A' and plate separation 'd'. Let E_0 be the electric field between the plates in the presence of air. The upper plate has +q charge and the lower plate has -q charge. These charges are called free charge.

By Gauss's law,

$$\phi_E = \frac{1}{\epsilon_0} q_{enclosed}$$

$$\text{or, } \oint_S \vec{E} \cdot \vec{dA} = \frac{1}{\epsilon_0} q$$

$$\text{or, } E_0 A = \frac{q}{\epsilon_0}$$

or, $E_0 = \frac{q}{\epsilon_0 A}$ ... (i)

vvimp Fig (ii) let a dielectric of dielectric constant K be inserted between the plates such that $-q'$ and $+q'$ charges are induced on the upper and lower surfaces of dielectric.

These charges are called induced or polarized charges.

Then by Gauss's law.

$$\phi_E = \frac{1}{\epsilon_0} q_{enclosed}$$

$$\text{or } \oint_S \vec{E} \cdot d\vec{A} = \frac{1}{\epsilon_0}(q - q')$$

$$\text{or, } EA = \frac{q}{\epsilon_0} - \frac{q'}{\epsilon_0}$$

$$\text{or, } E = \frac{q}{\epsilon_0 A} - \frac{q'}{\epsilon_0 A} \quad \text{... (ii)}$$

Since $k = \frac{E_0}{E}$

$$E = \frac{E_0}{k} = \frac{q}{\epsilon_0 A k} \quad \text{... (iii)}$$

[Using (i)]

From (ii) and (iii)

$$\frac{q}{\epsilon_0 A k} = \frac{q}{\epsilon_0 A} - \frac{q'}{\epsilon_0 A} \quad \text{... (iv)}$$

Two tasks:

1) To prove $q' < q$:

From (iv)

$$\frac{q'}{\cancel{\epsilon_0 A}} = \frac{q}{\cancel{\epsilon_0 A}} - \frac{q}{\cancel{\epsilon_0 A}K}$$

or, $q' = q(1 - 1/k)$

V.Vimp

For a dielectric $k > 1$

$\therefore q' < q$

Case: In air, $k = 1$

$\therefore q' = 0$

2) To prove: $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$

From eq^n (iv),

$$\frac{q}{\epsilon_0 A} = \frac{q}{\epsilon_0 AK} + \frac{q'}{\epsilon_0 A}$$

or, $\frac{q}{A} = \epsilon_0 \left(\frac{q}{\epsilon_0 AK}\right) + \frac{q'}{A}$ .. (v)

Here,

$D = \frac{q}{A} = \frac{\text{Free charge}}{\text{Area}}$ is the magnitude of the displacement vector.

$E = \frac{q}{\epsilon_0 A K}$, electric field in the presence of dielectric

$P = \frac{q'}{A} = \frac{\text{induced or polarized charge}}{\text{Area}}$ is the magnitude of the polarization vector.

Hence, eq^n(v) becomes,

$$D = \epsilon_0 E + P$$

In the vector rotation,

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

This is the required relation between the three electric vector D, E and P. This is gauss law in dielectric.

Note: $P = \frac{q'}{A} = \frac{q' \cdot d}{A \cdot d} = \frac{\text{displacement}}{\text{volume}}$

Polarization.

When a dielectric is introduced between the plates of capacitor, some charges are induced on the dielectric, then polarization (P) is defined as the induced charge with in the electric per unit area

i.e $P = \frac{q'}{A}$

Note: dipole moment (P)
= magnitude of either charge (q')
x separation between the charges (d)

$\Rightarrow P = q'd$

Unit $P \rightarrow$ cm

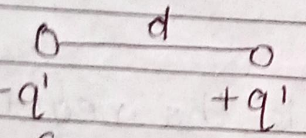


Fig: dipole

Then,

$$P = \frac{q'}{A} = \frac{q'd}{A \cdot d} = \frac{\text{dipole moment (P)}}{\text{volume}}$$

Also, $P = \frac{q'}{A}$ = surface polarization & charge density.

When a dielectric is placed in an external electric field, the atoms or ions or molecules of the dielectric get allign such that the dielectric acquires a certain dipole moment. This phenomenon is called polarization.

The polarization 'P' is directly proportional to the applied field i.e $P \propto E$

$P = \alpha E$ 'α' is called polarizability.

Define polarizability:

Polarizability is defined as the dipole moment per unit electric field applied.

Note: Also dipole moment $(P) \propto E$

$\therefore P = \alpha E$

Unit of α is Faradm^2 / Fm^2

Now,

$D = \epsilon_0 E + P$

$P = D - \epsilon_0 E$

$= \epsilon E - \epsilon_0 E$

ϵ = permitivity of medium/dielectric

$= \epsilon_0 \epsilon_r$; ϵ_r is relative permitivity

ϵ_0 = permitivity of vaccum

or, $P = (\epsilon_0 \cancel{\epsilon_0}) E$

$= (\epsilon_0 \epsilon_r - \epsilon_0) E$

$= \epsilon_0 (\epsilon_r - 1) E$

Imp [$P = \epsilon_0 \chi E$] where χ (Chi) $= \epsilon_r - 1$ is called susceptibility.

In vaccum or air,

$\epsilon_r = 1$

$\chi = 0$

$P = 0$

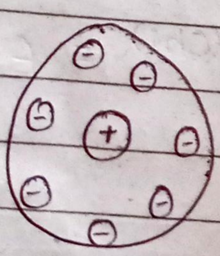
$\therefore$ Hence, there is no polarization in free space.

* Types of polarization.

(1) Electric polarization

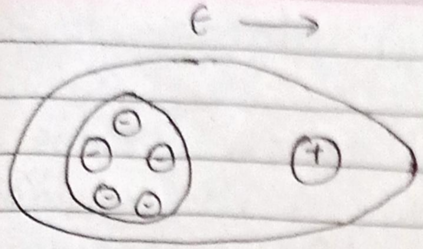
(2) Ionic polarization.

[1] Electric polarization: In the absence of external electric field, there is no dipole moment in the atom of dielectric. When the electric field is applied, the negative charges (electrons) will be shifted in the direction opposite to the field and the positive charges (nucleus) will be shifted in the direction of the field. This phenomenon is called polarization.



i) $p = 0$,
$E = 0$

ii)



$p \neq 0$
$E \neq 0$

To find electronic polarizability.

The induced dipole moment is given by,

$P = \alpha E$ ... (i) when α (alpha) is the electronic polarizability.

Atomic polarizability OR coefficient of electronic polarization.

We know,

$$F = ZeE$$

Also, $F = \frac{mv^2}{r} = m\omega^2 r \quad [\because v = \omega r]$

Equating, $ZeE = m\omega^2 r$

or, $r = \frac{ZeE}{m\omega^2}$

Also, $E = \frac{m\omega^2 r}{Ze}$ ... (ii)

From eq^n (i),

$$P = \alpha \frac{m\omega^2 r}{Ze}$$

or, $\alpha = \frac{PZe}{m\omega^2 r}$

Since,

$p = er$, dipole moment

$$\alpha = \frac{erZe}{m\omega^2 r}$$

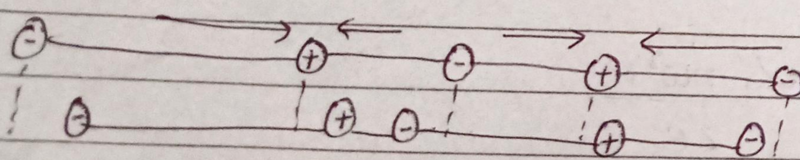
$$\therefore \alpha = \frac{Ze^2}{m\omega^2} = \alpha_e$$

[2] Ionic polarization:

The ionic crystals show or exhibit ionic polarization. KCL, NaCL are ionic crystals. Each pairs of +ve and -ve negative ions forms a dipole. Since these dipoles are lined one after another, there is no net dipole moment.

When an external electric field is applied, the negative ions move in the direction opposite to the electric field and the positive ions move in the direction of electric field. Hence, ionic crystals are polarized.

Note: Dipole moment is a vector.
(direction is from -ve to +ve charge.)



$E_0 = 0$

$P_{net} = P_+ - P_- = 0$

fig: Ionic polarization.

Clausius - Mosotti Equation

This equation relates the macroscopic quantity (parameter) ϵ_r and the microscopic quantity (parameter) 'α'

Derivation:

The actual field experienced by a molecule of the dielectric is called the local field and is given by the sum of applied field and the field due to polarization.

i.e $E_{local} = E + \frac{P}{3\epsilon_0}$ ← polarization.

The induced dipole moment is given by $\alpha \times E$,

i.e $p = \alpha E_{loc}$

For 'N' molecules or dipoles, then total polarization is given by $P = Np$

$= N\alpha E_{loc}$

Using (1),

$$P = N\alpha\left(E + \frac{P}{3\epsilon_0}\right) \quad \ldots (ii)$$

Also, $P = \epsilon_0 \chi E \quad \ldots (iii)$

From (ii),

$$P = N\alpha E + \frac{N\alpha P}{3\epsilon_0}$$

or, $P\left(1 - \frac{N\alpha}{3\epsilon_0}\right) = N\alpha E$

$$\therefore P = \frac{N\alpha E}{1 - \frac{N\alpha}{3\epsilon_0}} \quad \cdots (iv)$$

Equating (iii) and (iv),

$$\frac{N\alpha E}{1 - \frac{N\alpha}{3\epsilon_0}} = \epsilon_0 \chi E$$

or, $\dfrac{N\alpha E \cdot 3\epsilon_0}{3\epsilon_0 - N\alpha} = \epsilon_0 \chi E$

or, $\dfrac{3N\alpha}{3\epsilon_0 - N\alpha} = \chi$

or, $3N\alpha = 3\epsilon_0 \chi - N\alpha\chi$

or, $N\alpha(\chi + 3) = 3\epsilon_0 \chi$

or, $\dfrac{N\alpha}{3\epsilon_0} = \dfrac{\chi}{\chi + 3} \quad \cdots (v)$

Using $\chi = \epsilon_r - 1$,

$$\therefore \frac{N\alpha}{3\epsilon_0} = \frac{\epsilon_r - 1}{\epsilon_r + 2} \quad \cdots (vi)$$

Eq^n (v) and (vi) is the required Clausiuss-Mossotti equation.

The clausiuss mosotti eqⁿ for the electronic and ionic polarization becomes,

$$\frac{N\alpha_e}{3\epsilon_0} = \frac{\chi}{\chi+3} = \frac{\epsilon_r - 1}{\epsilon_r + 2}$$

$$\text{or, } \frac{N\alpha_i}{3\epsilon_0} = \frac{\chi}{\chi+3} = \frac{\epsilon_r - 1}{\epsilon_r + 2}$$

For electronic polarization, the clausiss mosotti eqⁿ written in terms of optical frequency is

$$\frac{N\alpha_e}{3\epsilon_0} = \frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{n^2 - 1}{n^2 + 2}$$ [where, $\epsilon_r = n^2$, n = optical frequency]

Note: $\alpha \approx 10\alpha_e$
So, ionic solid have large dielectric constant.

Unit 4: Photonics

4.1. laser
4.2: Fiber optics or optical fiber

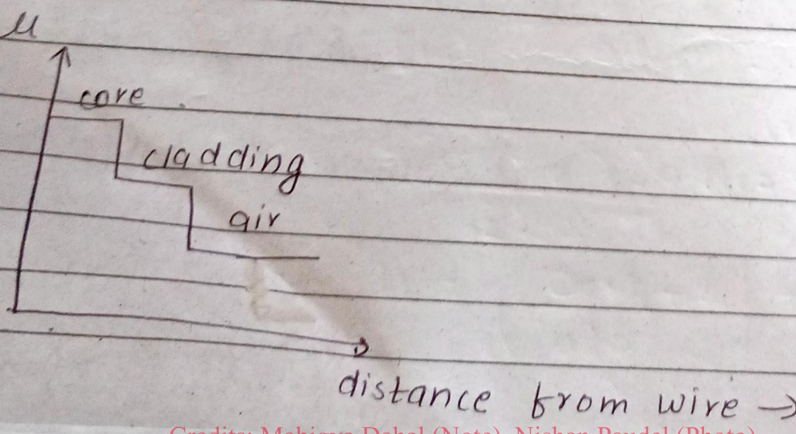
Optical Fiber:

principle: Total internal reflection

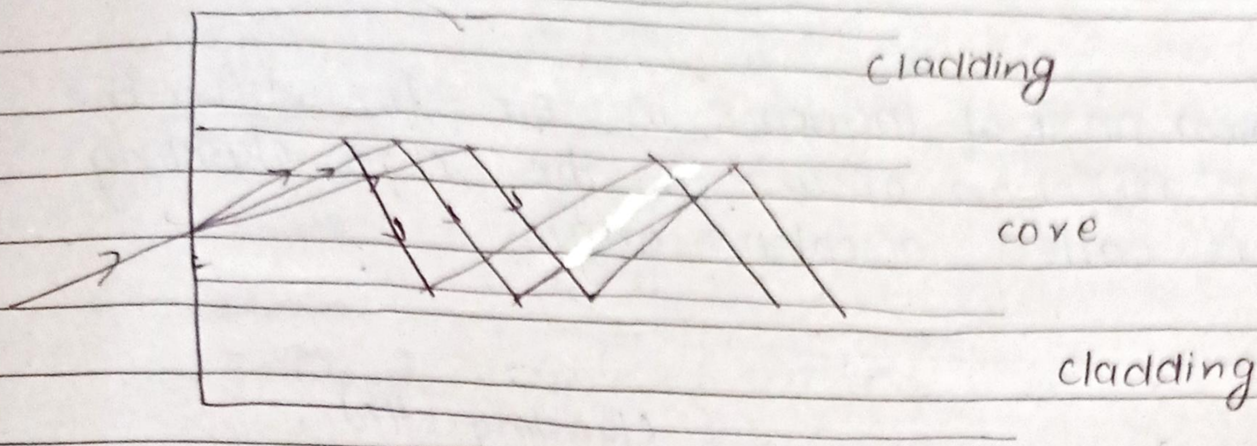
- denser → core → pure silica dopped with impurities → $\mu_{core} > \mu_{cladding}$
- rarer medium → cladding → pure silica

~~Typica~~ Types of optical fiber:

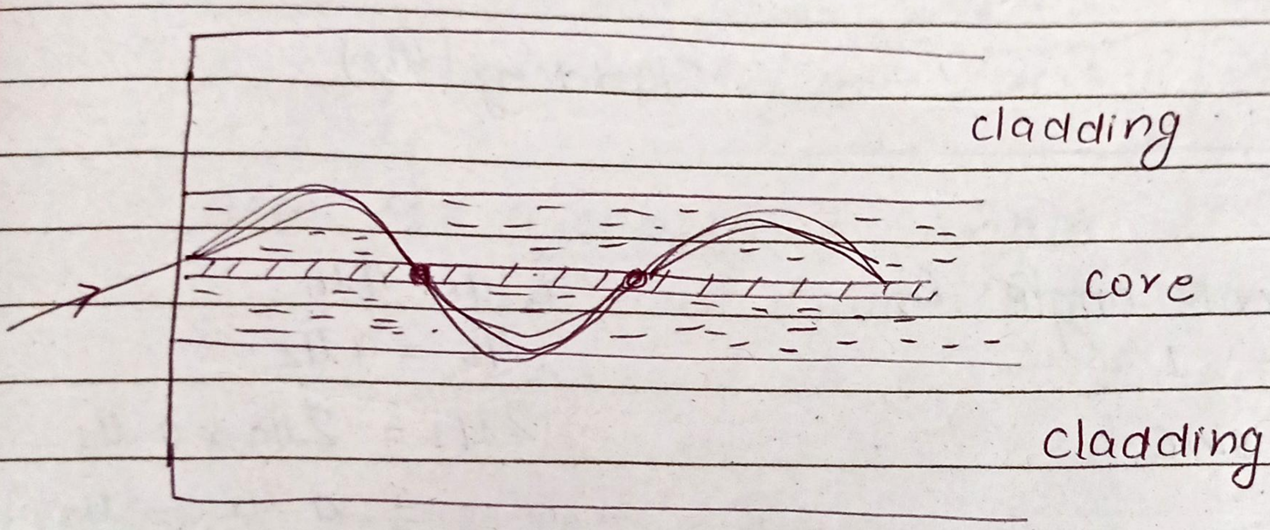
1) Single mode o.f
Multi-mode o.f
Step mode o.f
Graded mode o.f



* Step index of



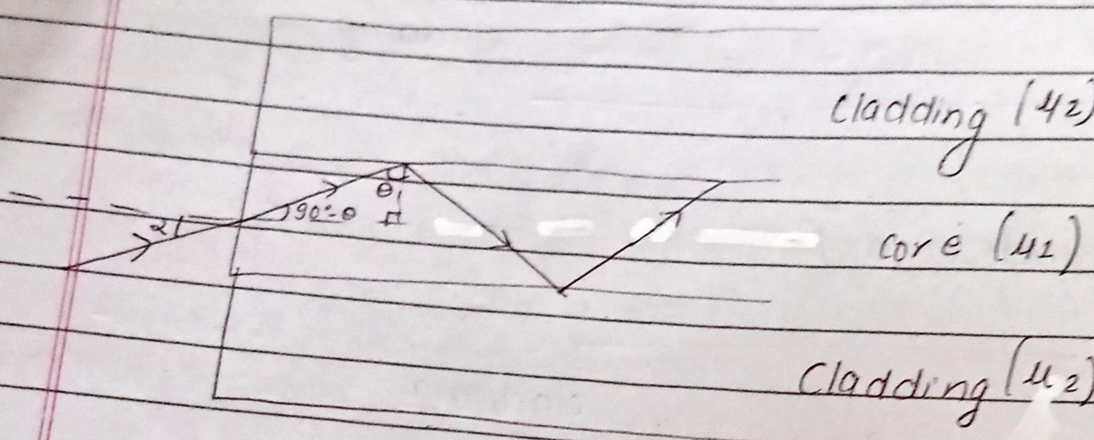
* Graded index o.F.



Core, cladding, coating

* Acceptance and Numerical Aperature.

The maximum angle of incidence in air for which the total internal reflection occurs at the core cladding interface is called acceptance angle.



For the critical angle C,

$${}_2\mu_1 = \frac{1}{\sin C}$$

$$\mu_1 = {}_a\mu_1$$
$$\mu_2 = {}_a\mu_2$$
$${}_2\mu_1 = {}_2\mu_a \times {}_a\mu_1 = \frac{{}_a\mu_1}{{}_a\mu_2} = \frac{\mu_1}{\mu_2}$$

$$\& \quad {}_2\mu_1 = \frac{\mu_1}{\mu_2}$$

$$\therefore \frac{1}{\sin C} = \frac{\mu_1}{\mu_2}$$

$$\text{or, } \sin C = \frac{\mu_2}{\mu_1} \quad \cdots (i)$$

Again, $${}_a\mu_1 = \mu_1 = \frac{\sin\alpha}{\sin(90°-\theta)}$$

Credits: Mahigya Dahal (Note), Nishan Paudel (Photo)

Here, $\theta \approx C$

$$\therefore \mu_1 = \frac{\sin\alpha}{\sin(90^\circ - C)}$$

$$= \frac{\sin\alpha}{\cos C}$$

or, $\cos C = \frac{\sin\alpha}{\mu_1}$

or, $1 = \frac{\mu_2^2}{\mu_1^2} + \frac{\sin^2\alpha}{\mu_1^2}$

or, $\mu_1^2 = \mu_2^2 + \sin^2\alpha$

$\therefore \sin^2\alpha = \mu_1^2 - \mu_2^2$

$\therefore \sin\alpha = \sqrt{\mu_1^2 - \mu_2^2} = \mu_1\sqrt{2\Delta}$

$$\left[\therefore \alpha = \sin^{-1}\sqrt{\mu_1^2 - \mu_2^2}\right]$$

Here, α = acceptance angle

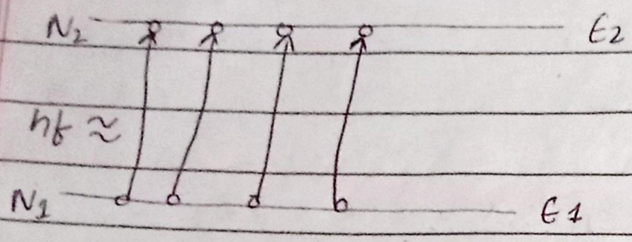
& $\sqrt{\mu_1^2 - \mu_2^2}$ = numerical aperture.

* Fractional refraction index

$$\Delta = \frac{\mu_1 - \mu_2}{\mu_1}$$

Full form:

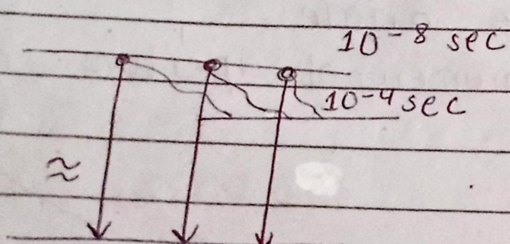
LASER: Light Amplification by stimulated emission of radiation.



$$E_2 - E_1 = hf$$

optical pumping

$$\frac{dN_1}{dt} = e \times E \cdot C \times N_1$$



* Induced absorption, optical pumping, Einstein coefficient, spontaneous emission, population inversion, meta stable state, helium neon laser and its construction & working, semi conductor laser, stimulated emission.

Q) A glass clad fiber is made with core glass of refractive index 15 and cladding is doped to give a fractional index difference of 5×10^{-4}. Determine
1) The cladding index 2) The critical internal reflection angle. 3) The external critical acceptance angle.
4) Numerical aperture.

$n_2 = \mu_2$

$n_1 = \mu_1$

⇒ Given,

refractive index of core, $\mu_1 = 1.5$
fractional refractive index difference or change, $\Delta = \frac{\mu_1 - \mu_2}{\mu_1} = 5 \times 10^{-4}$

i) Then, refractive index of cladding, $n_2 = ?$

$$\Delta n_1 = n_1 - n_2$$
$$n_2 = n_1 - \Delta n_1$$
$$= n_1(1 - \Delta)$$
$$= 1.4925$$

ii) $\sin C = \frac{n_2}{n_1}$

$$\text{or, } C = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$
$$= \cancel{88.2^\circ}\ 87^\circ$$

iii) $\alpha = \sin^{-1}\sqrt{\mu_1^2 - \mu_2^2}$ (Acceptance angle)

$$= \sin^{-1}\sqrt{(1.5)^2 - (1.4925)^2}$$
$$= 2.71^\circ$$

iv) Numerical aperture $= \sqrt{\mu_1^2 - \mu_2^2}$

$$\text{Or, } NA = n_1\sqrt{2\Delta} = 1.5\sqrt{2 \times 5 \times 10^{-4}}$$
$$= 0.0474$$

Credits: Mahigya Dahal (Note), Nishan Paudel (Photo)

Q) An optical fiber has fractional index difference of 0.2 and cladding refractive index 1.59. Determine the acceptance angle for the fiber in water in which refractive index is 1.33.

→ Given,

fractional index difference $\Delta = 0.2$

$n_2 = 1.59$

$n_1 = ?$

Now, $\Delta = \frac{n_1 - n_2}{n_1}$

or, $0.2 = \frac{n_1 - 1.59}{n_1}$

or, $0.2 n_1 = n_1 - 1.59$

or, $n_1 = 1.9875$

Acceptance angle in water,

$(\theta_i)_{max} = \sin^{-1} \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$

α

$= 63.72°$ ← n_0 ← refractive index of medium

Thus, the light wave index at an angle less than 63.72° will propagate along the optical fiber and the light ray incident at the angle greater than 63.72° escape out.

Q) Calulate the refractive ~~index~~ indices of core and cladding materials of the fiber for following data:

Numerical aperture = 0.22

Fractional refractive index change, $\nabla = 0.012$

Given,

Fractional refractive index change, $\nabla = \frac{n_1 - n_2}{n_1}$

$= 0.012$

Numerical aperture $= 0.22$

$NA = n_1 \sqrt{2\Delta}$

$0.22 = n_1 \sqrt{2 \times 0.012}$

$n_1 = 1.420$

Now, $\nabla = \frac{n_1 - n_2}{n_1}$

or, $0.012 = \frac{1.420 - n_2}{1.420}$

$\therefore n_2 = 1.402$ //

LASER

- He-Ne laser

- Acoustics:

- Ultrasonic method in non destructive testing.
- Application of ultrasonic wave in medical field.
- Application of ultrasonic wave in NDT
- Limitation of ultrasonic wave in NDT

SABINE Numerical

Board Q) A lecture hall of volume 12×10^4 m^3 has a total absorption of $13200 m^2$ of open window unit. Entery of students into the hall raises the absorption by another $13200 m^2$ of open window unit. Find the change in reverberation time.

⇒ Given, Case I:

Before the entry of the study:

The reverberation time is given by

$$T = \frac{0.158 V}{\alpha S}$$

$= 1.44 sec$

Case II:

After the entry of student, the reverberation time is given by:

$$T' = \frac{0.158V}{\alpha S + (\alpha S)_1}$$

where, $(\alpha S)_1 = 13200\ m^2$

$$\therefore T' = \frac{0.158}{\alpha S + (\alpha S)_1}$$

$$= 0.72\ sec$$

change in reverberation time

$$= T - T'$$

$$= 0.72\ sec$$

Board

Q) The size of an empty assembly hall has a dimension $20 \times 15 \times 5\ cm^3$ and the reverberation time is 3.5 sec. What area of the wall should be covered by curtain clothe to reduce the reverberation time by 2.5; if the absorption coefficient of cotton clothe is 0.5. Also calculate the average absorption coefficient of the hall.

⇒ Given,

$$V = 20 \times 15 \times 5\ cm^3$$

$$= 20 \times 15 \times 5 \times 10^{-6}\ m^3 = 0.0015\ m^3$$

$$T = \frac{0.158V}{\alpha s}$$

$\alpha s = 6.77 \times 10^{-6}$

Total surface area of bell / hall $= 2(lb + bh + hl)$

$= 2(20 \times 15 + 15 \times 5 + 20 \times$

$= 950\ cm^2$

$= 0.095\ m^2$

Absorption coefficient of curtain cloth $(\alpha_1) = 0.5$

The new reverberation time is $(T_1) = 3.5 - 2.5$

$= 1$

Then,

$T_1 = \dfrac{0.158\,V}{\alpha s + \alpha_1 S_1}$

or, $1 = \dfrac{0.158 \times 0.0015}{6.77 \times 10^{-5} + 0.5 \times S_1}$

or, $6.77 \times 10^{-5} + 0.5 S_1 = 2.37 \times 10^{-4}$

or, $S_1 = 4.60 \times 10^{-4}\ m^2$

Average absorption coefficient

$\bar{\alpha}$ or $\alpha_{avg} = \dfrac{\Sigma \alpha s}{\Sigma s} = \dfrac{6.77 \times 10^{-5}}{\cancel{0.0015}\ 0.095}$

$= 7.12 \times 10^{-4}$

SHM

Q) What is the amplitude of the oscillation of a system acted by a sinusoidally varying force of amplitude 0.01 N and frequency 100 Hz acting on a system with the mass of 50 gram, restoring capacity 64 N/m and damping constant 20gms^{-1}. [Ans: 5.087×10^{-7} m]

⇒ Given,

$f_0 = 0.01$ N

$k = 64$ Nm^{-1}

$b = 20$ gms^{-1} $= 0.02$ kgs^{-1}

$f = \cancel{5}100$ Hz

$m = 50$ gm $= 0.05$ kg

$\omega = 2\pi f$

$= 2\pi \times 100$

$= 628$ rad/s

Amplitude of oscillation,

$$x_m = \frac{f_0/m}{\sqrt{(\omega_0^2 - \omega^2)^2 + (\frac{b\omega}{m})^2}}$$

where, $\omega_0 = \sqrt{\frac{k}{m}} = 35.78$ rad s^{-1}

$x_m = \frac{0.01/0.05}{\quad}$

Q) The optical index of refraction and the dielectric constant for glass are 1.45 and 6.5 respectively. Calculate the percentage of ionic polarizability?

Given,

$n = 1.45$

$\epsilon_r = 6.5$

The clausius mossoti equation is,

$$\underbrace{\frac{N(\alpha_i + \alpha_e)}{3\epsilon_0}}_{\text{for ionic}} = \underbrace{\frac{\epsilon_r - 1}{\epsilon_r + 2}}_{\text{for electronic}} \quad \ldots (i)$$

For optical frequencies,

$\epsilon_r \rightarrow n^2$ for electronic polarization.

$$\frac{N\alpha_e}{3\epsilon_0} = \frac{n^2 - 1}{n^2 + 2} \quad \ldots (ii)$$

Dividing (ii) by (i)

$$\frac{n^2 - 1}{n^2 + 2} \times \frac{\epsilon_r + 2}{\epsilon_r - 1} = \frac{\alpha_e}{\alpha_i + \alpha_e}$$

Then, % of ionic polarizability is,

$$\frac{\alpha_i}{\alpha_i + \alpha_e} \times 100\%$$

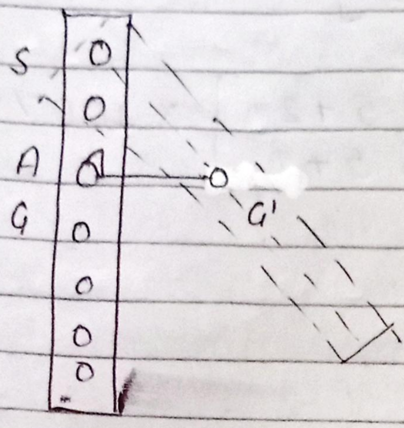
$$= \left[1 - \frac{\alpha_e}{\alpha_i + \alpha_e}\right] \times 100\%$$

$$= \left[1 - \frac{n^2 - 1}{n^2 + 2} \times \frac{\epsilon_r + 2}{\epsilon_r - 1}\right] \times 100\%$$

$$= \left[1 - \frac{1.45^2 - 1}{1.45^2 + 2} \times \frac{6.5 + 2}{6.5 + 1}\right] \times 100\%$$

$$= 58.46\%$$

Bar pendulum:



$SG = SG' = l$

$T = I\alpha = -mgl\sin\theta$

or, $\frac{Id^2\theta}{dt^2} + mgl\theta = 0$

[For small angle θ, $\sin\theta \approx \theta$]

or, $\frac{d^2\theta}{dt^2} + \left(\frac{mgl}{I}\right)\theta = 0$

Let $\omega^2 = \frac{mgl}{I}$

or, $\omega = \sqrt{\frac{mgl}{I}}$, angular frequency

$\therefore \frac{d^2\theta}{dt^2} + \omega^2\theta = 0$ which is the differential equation of SHM. Hence the motion of bar pendulum is S.H.

To find T:

since $T = \frac{2\pi}{\omega}$ $\left[\because \omega = 2\pi f = \frac{2\pi}{T} \text{ or } T = \frac{2\pi}{\omega}\right]$

$T = 2\pi\sqrt{\frac{I}{mgl}}$

Since $I = I_{CG} + ml^2$

$= mk^2 + ml^2$

$= m(k^2 + l^2)$

$$T = 2\pi\sqrt{\frac{k^2/l + l}{g}} \quad \ldots (i)$$

To find g and k:

Squaring (i)

$$T^2 = \frac{4\pi^2}{g}\left(\frac{k^2}{l} + l\right)$$

$$T^2 = \frac{4\pi^2}{g}\left(\frac{k^2 + l^2}{l}\right)$$

or, $$T^2 l = \frac{4\pi^2}{g} l^2 + \frac{4\pi^2 k^2}{g} \quad \ldots (ii)$$

1st method:

From (ii),

$$\left(\frac{4\pi^2}{g}\right) l^2 + (-T^2) l + \frac{4\pi^2}{g} k^2 = 0 \quad \ldots (iii)$$ which is quadratic in l.

Recall: quadr. eqn : $ax^2 + bx + c = 0$

$x = \alpha, \beta$

$$\left[\text{Then } \alpha + \beta = \frac{-b}{a} \ \& \ \alpha\beta = \frac{c}{a}\right]$$

let the roots of (iii) be l_1 and l_2

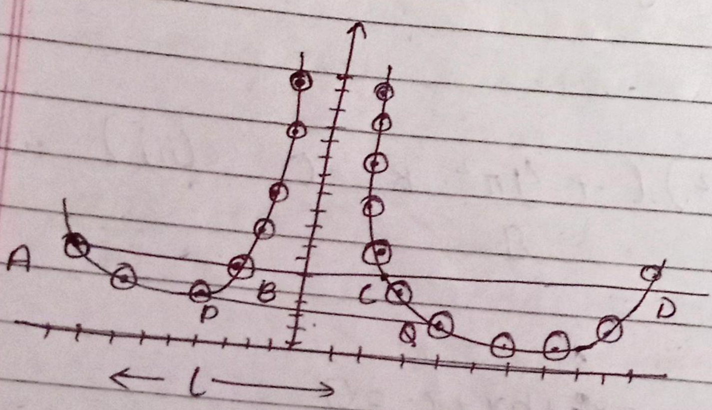
Then, $l_1 + l_2 = \dfrac{-(-T^2)}{\dfrac{4\pi^2}{g}} = \dfrac{gT^2}{4\pi^2}$

$l = \dfrac{gT^2}{4\pi^2}$, where $l = l_1 + l_2$

$g = \dfrac{4\pi l}{T^2}$

$l_1 l_2 = \dfrac{\dfrac{4\pi^2 k^2}{g}}{\dfrac{4\pi^2}{g}} = k^2$

$k = \sqrt{l_1 l_2}$ (Taking +ve sign)



$OA = l = l_1$

$OC = \dfrac{k^2}{l} = l_2$

From the graph,

$OA = l_1$

$OC = l_2$

Also, $OE = T$

Thus, g & k can be determined

2^{nd} method

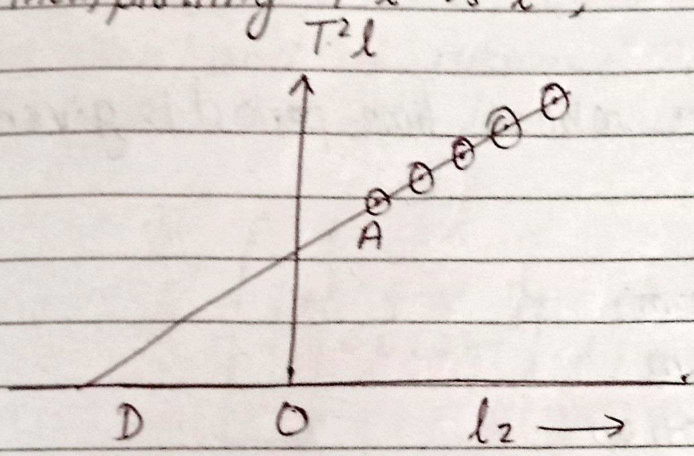
From eq^n(ii);

$$T^2 l = \left(\frac{4\pi^2}{g}\right) l^2 + \frac{4\pi^2}{g} k^2$$

let $l^2 = x$

$T^2 l = y$

Then, plotting $T^2 l$ vs l^2,



Then, $m = \text{slope} = \frac{4\pi^2}{g}$

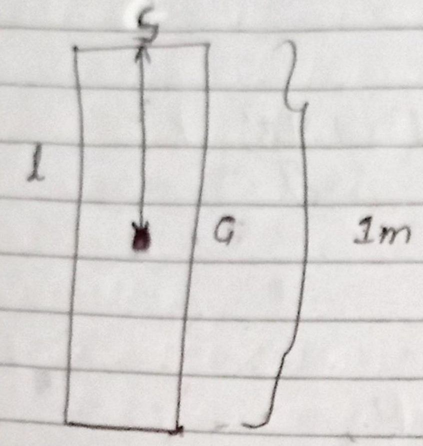
and y- intercept $(c) = \frac{4\pi^2}{g} k^2$

Here, $m = \frac{OA}{OD}$ & $C = OA$

Thus, g & k can be determined

Q) A meter end is suspended at one end. Find its time period, frequency, angular frequency.

⇒ Sol^n:-



It is a bar pendulum whose time period is given by

$$T = 2\pi\sqrt{\frac{l + \frac{k^2}{l}}{g}}$$

where $l = SG = 1/2\ m$
$= 0.5\ m$

k = radius of gyration
$= \frac{\text{total length}}{\sqrt{12}}$
$= \frac{1}{\sqrt{12}}\ m$
$= 0.289\ m$

$g = 9.8\ m/s^2$

$$T = 2\pi\sqrt{\frac{k^2/l + l}{g}}$$

$$= 2\pi\sqrt{\frac{\frac{(0.289)^2}{0.5} + 0.5}{9.8}} = 1.63\ sec$$

$f = \frac{1}{T}$

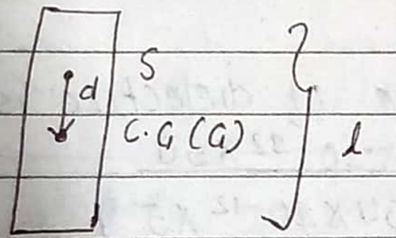
$= \quad Hz$

$\omega = 2\pi f$

$= 2\pi \times f$

$= \quad rad/sec$

Q) Show that if a uniform stick of length l is mounted. Show its to rotated about horizontal axis perpendicular to stick and at distance from the centre of gravity. The period has minimum value, when $l = 0.289\,l$



l mounted,

when $d = 0.289\,l$

$T = 2\pi\sqrt{\frac{d + k^2/d}{g}}$

T will be minimum

$d = k = \frac{\text{total length}(l)}{\sqrt{12}}$

$d = \frac{1}{\sqrt{12}}\,l$

$= 0.289\,l$

pg 45 (pink book)
Qno. 15, 24, 23

Capacitor and dielectric pg 105

8) A parallel plate has no capacitance of 100×10^{-12} F and plate are of $100\,cm^2$ and mica is used as dielectr at 50 p.d. Calculate electric field intensity and magnitude of induced charge.

⇒ Given,

$C = 100 \times 10^{-12}$ F

$A = 100\,cm^2 = 100 \times (1cm)^2$

$= 100 \times (10^{-2} m)^2$

$= 10^{-2}\,m^2$

mica, $k = 5.4$

$V = 50V$

Now,

1) Electric field in presence of dielectric is given by

$$E = \frac{q}{\epsilon_0 A k} = \frac{CV}{\epsilon_0 A k} = \frac{100 \times 10^{-12} \times 50}{8.854 \times 10^{-12} \times 5.4}$$

$$= 1.05 \times 10^{4}\ V/m$$

2) $$q' = q\left(1 - \frac{1}{k}\right)$$

$$= 4.07 \times 10^{-9}\ C$$

Date ______
Page ______

Capacitor

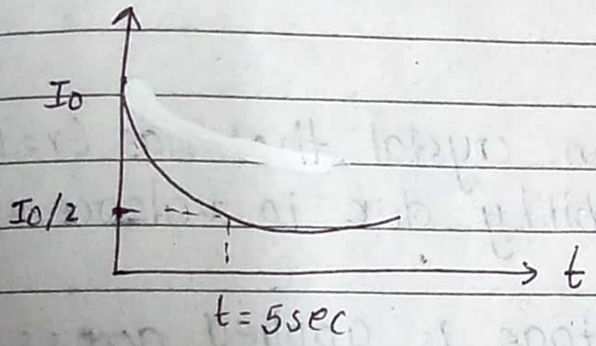
RC circuit (pg 107)

Q) Obtain the charging time constant of capacitor in a RC circuit such that current through the resistor decrease by 50 % of its peak value in 5 sec.

⇒ Solⁿ:-

For the charging,
Current is given by:

$I = I_0 e^{-t/RC}$



By the question,

$I = 50\%$ of I_0

$= \frac{I_0}{2}$

$t = 5 sec$

$\therefore \frac{I_0}{2} = I_0 e^{-t/RC}$

or, $\frac{1}{2} = e^{-t/\tau}$ $\tau = RC$, time constant

Taking ln, $\ln\left(\frac{1}{2}\right) = \frac{-t}{\tau}$

or, $+0.693 = -\frac{t}{\tau}$

or, $\tau = \frac{t}{\tau}$

or, $\tau = \frac{t}{0.693}$

$= \frac{5}{0.693}$

$= 7.21$ sec

Gaussian.

Q) Consider a pure silicon crystal that has $\epsilon_r = 11.9$
a) What the polarizibility due to valence electrons per Si atom.
b) Suppose that voltage is applied across Si crystal sample. By how much is the local field greater than applied field?

⇒ Given,

$\epsilon_0 = 8.85 \times 10^{-12}$ F/m

$N = 5 \times 10^{28}$ atoms per m^3

$\epsilon_r = 11.9$

Now,

~~A~~ Clausius - Mossoti equation,

i) $\frac{N\alpha E}{3\epsilon_0} = \frac{\epsilon_r - 1}{\epsilon_r + 2}$

$\alpha_e = 4.17 \times 10^{-40}\ Fm^2$

b) $E_{loc} = E + \frac{P}{3\epsilon_0} \quad \ldots (i)$

$P = \epsilon_0 \chi E$

$= \epsilon_0 (\epsilon_r - 1) E$

Substitute P

$\frac{E_{loc}}{E} = 4.63$

Credits: Mahigya Dahal (Note), Nishan Paudel (Photo)

$m \rightarrow$ milli $\rightarrow 10^{-3}$
$M \rightarrow$ Mega $\rightarrow 10^{6}$

Electromagnetism

EM oscillation:

1) You're given an inductor of 1 mH. ~~If~~ How can you make it oscillate it at MHz? (or how can you make it oscillatory circuit)?

⇒ Soln:-

$L = 1mH = 10^{-3}$ H
$f = \cancel{mH}$ 1 MHz $= 10^{6}$ Hz
$C = ?$

For LC oscillatory,

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

$$\text{or, } f^2 = \frac{1}{4\pi^2}\frac{1}{LC}$$

$$\text{or, } C = \frac{1}{4\pi^2 f^2 L}$$

$$= 4 \times 10^{-3}\ F_{//}$$

Hence, using capacitor of capacitance 4×10^{-3} Farad. We can make oscillating circuit.

Q. If 10 milli henry (mH) inductor and two capacitor of 5 microfarad and two microfarad (μF) are given. Find the two resonate frequencies that can be obtained by connecting these elements ~~in~~ in different ways.

⇒ Soln:-

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$L = 10\,mH = 10 \times 10^{-3}\,H$
$C_1 = 5\,\mu F = 5 \times 10^{-6}\,F$
$C_2 = 2\,\mu F = 2 \times 10^{-6}\,F$

(i) For C_1 and C_2 in series,

$$= \frac{1}{C_1} + \frac{1}{C_2}$$

$$= \frac{C_1 + C_2}{C_1 \cdot C_2}$$

$$= \frac{C_1 \cdot C_2}{C_1 + C_2}$$

$$= \frac{5 \cdot 2}{5+2}$$

$$= \frac{10}{7}\,\mu F$$

$$= \frac{10}{7} \times 10^{-6}\,F$$

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

=

=

(ii) For C_1 and C_2 in parallel

$C = C_1 + C_2$
$= 5 + 2$
$= 7\,\mu F$
$= 7 \times 10^{-6}\,F$

$$\therefore F = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

$=$

$= 60\ Hz$

Q) An LC circuit is converted into a LCR circuit inserting a resistance of $10\ \Omega$. Calculate the percentage in frequency in this conversion.

$\Rightarrow$ Given,

$R = 10\ \Omega$

$L = 10\ mH = 10 \times 10^{-3}\ H$

$C = 10\ \mu F = 10 \times 10^{-6}\ F$

i) For LC oscillation,

$$\text{Frequency} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

$$= 503.29\ Hz$$

ii) For LCR oscillation

$$F' = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

$$= 496.96\ Hz$$

Date ____
Page ____

$\therefore$ % change in frequency $= \frac{f - f'}{f} \times 100\%$

$= 1.25\%$

Board

Q) A circuit has L = mH and C = 10 μF. How much resistance should be added to circuit so that frequency of oscillation will be 1% less than that of LC oscillation.

⇒ Solution:

$L = 10\ mH = 10 \times 10^{-3}\ H$

$C = 10\ \mu F = 10 \times 10^{-6}\ F$

$R = ?$

By question,

$f' = f - 1\%$ of f

$= 99\%$ of f

$= 0.99 f$

where $f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = 503.29\ Hz$

Also, $f' = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$

$R = 8.706\ \Omega$

$= 0.99 \times 503.54$

$= 498.504\ Hz$

or, $\frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R}{4L^2}} = 498.50$

or,

Q) At some distance from transmitter of radio station the magnitude of em wave emitted by radio station is found to be 1.6×10^{-4} T. If frequency of broadcast is 1020 KHz then find ~~ex~~ speed, wavelength and maximum eletric field of em wave.

⇒ Sol^n:-

$f = 1020$ KHz $= 1020 \times 10^{3}$ Hz

$B_m = 1.6 \times 10^{-4}$ T

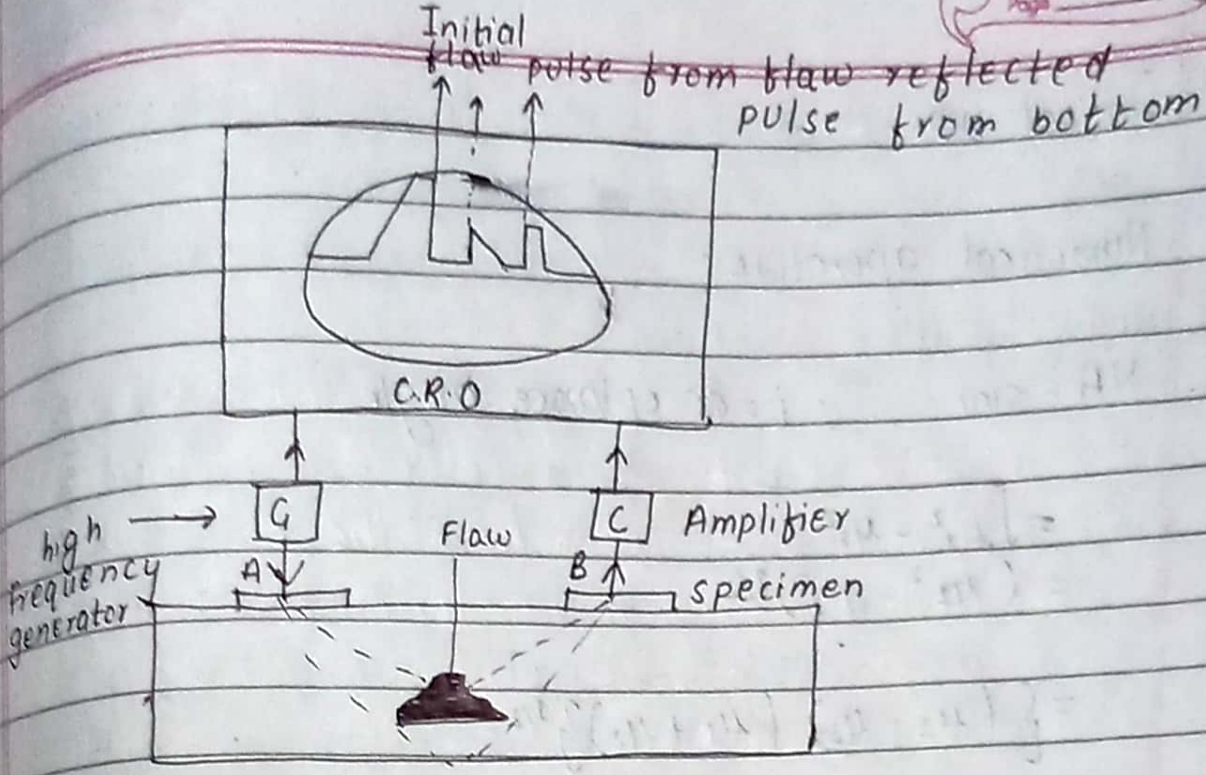
i) $c = \dfrac{1}{\sqrt{\mu_0 \epsilon_0}} = \dfrac{1}{\sqrt{4\pi \times 10^{-7} \times 8.85 \times 10^{-12}}} = 3 \times 10^{8}$ m/s

ii) $\lambda = \dfrac{C}{F} = \dfrac{3 \times 10^{8}}{1020 \times 10^{3}} = 294.12$ m

iii) $E_m = CB_m = 3 \times 10^{8} \times 1.6 \times 10^{-4}$
$= 4.8 \times 10^{4}$ V/m //

~~Q)~~ Ultrasonic method in non-destructive testing (NDT)

Ultrasonic waves in NDT uses high frequency to conduct examinations and make measurements. This can be used for flaw detection levaluation, dimensional measurements, material characterization and more.



A typical NDT consists of an ultrasonic frequency generator and a cathode ray oscilloscope (CRO), transmitting transducer (A), ~~redu~~ receiving transducer (B) and an amplifier. When there is discontinuity (such as a crack) in the wave path, part of energy will be reflected back from the flaw surface. The echoes of the reflected beam are captured by using a cathode ray oscilloscope. By examining echoes on CRO, flaws can be detected and their sizes can be estimated.

Optical fiber

Numerical aperature:

$NA = \sin i$; i = acceptance angle

$= \sqrt{\mu_1^2 - \mu_2^2}$

$= (\mu_1^2 - \mu_2^2)^{1/2}$

$= \{(\mu_1 - \mu_2)(\mu_1 + \mu_2)\}^{1/2}$

$= \left\{ \frac{\mu_1 - \mu_2}{\mu_1} \cdot \frac{\mu_1 + \mu_2}{2} \cdot 2\mu_1 \right\}^{1/2}$

Here, Δ = fractional refractive index change.

$= \frac{\mu_1 - \mu_2}{\mu_1}$

or, $NA = \left\{ \Delta \frac{\mu_1 + \mu_2}{2} \, 2\mu_1 \right\}^{1/2}$

since $\mu_1 \approx \mu_2$, $\frac{\mu_1 + \mu_2}{2} \approx \mu_1$

$NA = \{ \Delta \mu_1 \; 2\mu_1 \}^{1/2}$

$NA = \mu_1 \sqrt{2\Delta}$

Normalized frequency (V-number)

It denotes the no. of modes that can propagate through a fiber.

It characterisizes the optical fiber. It is given by:

$$V = \frac{2\pi a}{\lambda}\sqrt{\mu_1^2 - \mu_2^2}$$

Here, a = radius of wire

λ = free space wavelength

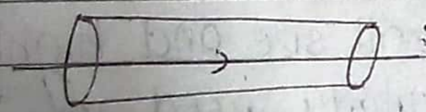
μ_1 = refractive index of wire

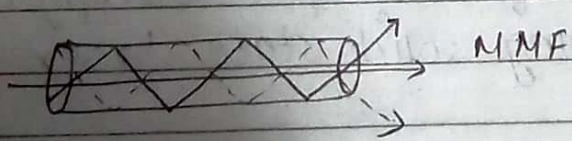
μ_2 = refractive index of cladding.

Note:

i) If $V < 2.405$, it is called single mode fiber (SMF) which support only one single mode.

ii) If $V > 2.405$, it is called MMF which support multi modes.





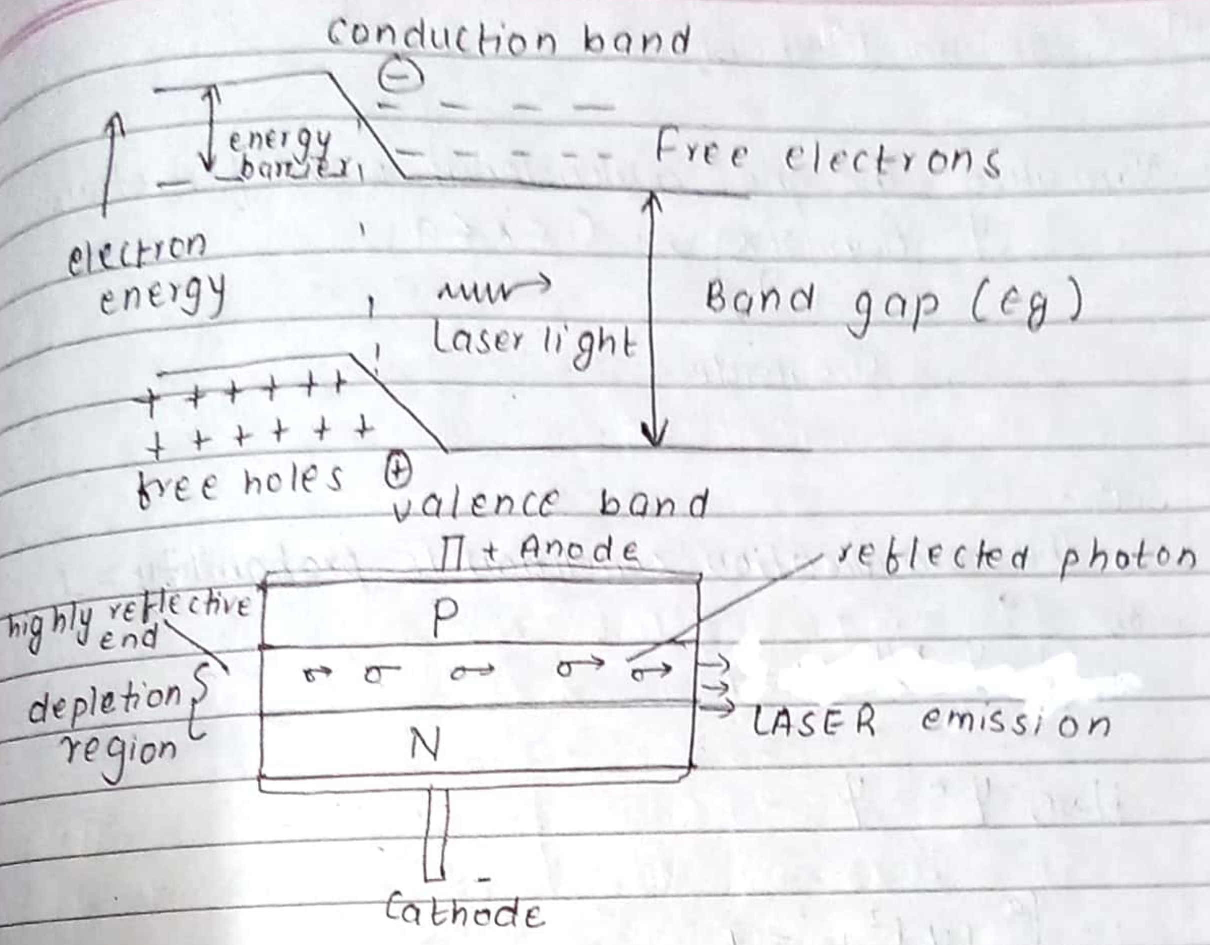
LASER

- Semiconductor laser:

It is basically the P-N junction diode When an external supply voltage is applied to the P-N junction, electrons from the N region and holes from the P-region are forced into the junction when they collide, they neutralize each other and emit recombination radiation.

For a semiconductor laser, the energy of photon emitted as recombination radiation is equal to N. The active medium of semiconductor laser is PN junction. If the active medium or junction is made up of single type of semiconductor material, the semiconductor laser is known as homojunction laser. While if the junction is made of different types of semiconductor material, then semiconductor laser is known as heterojunction laser.

Because of their small size and highly efficient pumping, they are particularly used in ~~obti~~ optical data transformer and storage, spectorscopy, medicine and pumping solid state laser.


conduction band
energy barrier
Free electrons
electron energy
laser light
Band gap (eg)
free holes
valence band
+ Anode
reflected photon
highly reflective end
P
depletion region
N
LASER emission
-
cathode

Board Quantum Mechanics:

Q) Normalize the one dimensional wave function.

$$\psi = A\sin\left(\frac{\pi x}{a}\right), \quad 0 < x < a$$

$$= 0, \text{outside}$$

⇒ Solⁿ:-

The normalization condition is probability = 1

or, $\int_0^a \psi^* \psi \, dx = 1$

Here, $\psi^* = \psi$

$\therefore \int_0^a |\psi|^2 dx = 1$

or, $\int_0^a A^2 \sin^2 \frac{\pi x}{a} \, dx = 1$

or, $\frac{A^2}{2} \int_0^a 2\sin^2 \frac{\pi x}{a} \, dx = 1$

or, $\frac{A^2}{2} \int_0^a \left[1 - \cos \frac{2\pi x}{a}\right] dx = 1$

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Page ______

or, $\frac{A^2}{2}\left[|x|_0^a - \left| \frac{\sin\frac{2\pi x}{a}}{\frac{2\pi x}{a}} \right|_0^a \right] = 1$

or, $\frac{A^2}{2}\left[a - a/2\pi \left\{ \sin\left(\frac{2\pi}{a}\cdot a\right) - \sin 0 \right\} \right] = 1$

or, $\frac{A^2}{2}\left[a - (0-0) \right] = 1$

$\therefore \frac{A^2}{2}\cdot a = 1$

$\therefore A = \sqrt{\frac{2}{a}}$

Hence, the normalized wave function is ψ or $\psi(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{\pi x}{a}\right)$, $0 < x < a$

$= 0$, outside$_{//}$

Board

Q) An electron moving is a wave has wave function $\psi(x) = 2\sin 2\pi x$. Find the probability of finding the electron in the region $x = 0.25$ to 0.5 m

Soln:-

$\psi(x) = 2\sin 2\pi x$

probability, $P = \int_{0.25}^{0.5} \psi^* \psi \, dx$

Here, $\psi^* = \psi$

$$\therefore P = \int_{0.25}^{0.5} 2^2 \sin^2 2\pi x \, dx$$

$$= 2\int_{0.25}^{0.5} 2\sin^2 2\pi x \, dx$$

$$= 2\int_{0.25}^{0.5} \left[1 - \cos 4\pi x\right] dx$$

$$= 2\left[\left|x\right|_{0.25}^{0.5} - \left|\frac{\sin 4\pi x}{4\pi}\right|_{0.25}^{0.5}\right]$$

$$= 2\left[0.25 - \frac{1}{4\pi}\left\{\sin 4\pi(0.5) - \sin 4\pi(0.25)\right\}\right]$$

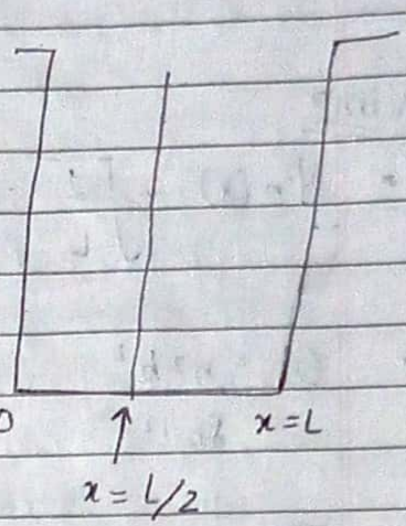
$$= 0.5$$

Board

A particle is moving in 1-D box of infinite potential. Evaluate the probability of finding the particle within range 1A° at the center of box when it is in lowest energy state.

⇒ Solⁿ:-

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L}\right)x$$



For the lowest energy state,

$n=1$

$$\therefore \psi_1(x) = \sqrt{\frac{2}{L}} \sin\frac{\pi}{L}x$$

At the centre, $x = L/2$

$$\therefore \psi_1(x)\Big|_{x=L/2} = \sqrt{\frac{2}{L}} \sin\frac{\pi}{L}\cdot\frac{L}{2}$$

$$= \sqrt{2/L}\,\sin\frac{\pi}{2} = \sqrt{\frac{2}{L}}$$

The probability of finding the particle at the centre is $P' = \psi_1^*\psi_1 = |\psi_1|^2 = \frac{2}{L}$

The probability of finding the particle within range 1A° at the center of box when it is in lowest energy state is $P = P' \times \Delta x$

$$= \frac{2}{L}\,1A°$$

Here, length of the box is not given,
let $L = 10A^\circ$
Then, $P = \frac{2}{10A^\circ} \cdot 1A^\circ$
$= 0.2$

V.Vimp

- $\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L} x\right), n = 1, 2, 3 \cdots$

- $E_n = \frac{n^2 h^2}{8mL^2}, n = 1, 2, 3, \cdots$

Thermodynamics:

Q) A gas contained in a piston cylinder device initia at pressure of 150 KPa and volume of $0.04 m^3$. Calculate the working done by the gas when it undergoes the following process to a final volume of $0.1 m^3$.

a) Constant pressure
b) constant temperature
c) $PV^{1.35}$ = constant

Here, initial pressure $P_1 = 150$ kpa
$V_1 = 0.04 m^3$
final state volume $V_2 = 0.1 m^3$

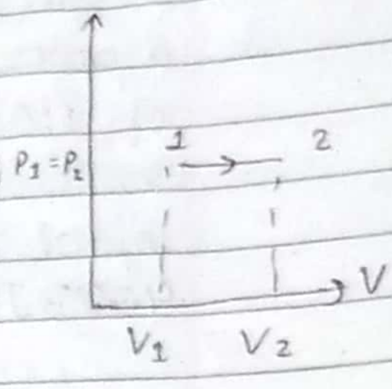
a) Constant pressure,

Work done

$W = P_1 (V_2 - V_1)$

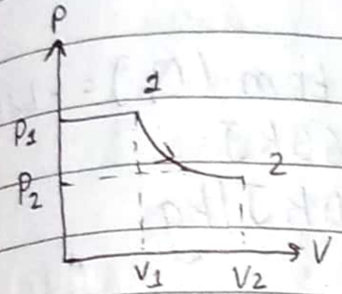
$= 150 \times 10^3 \; (0.1 - 0.04)$

$= 9000 \; J$



b) For constant temperature

Work done, $W = P_1 V_1 \ln\left(\frac{V_2}{V_1}\right)$



$= 150 \times 10^3 \times 0.04 \ln\left(\frac{0.1}{0.04}\right)$

$= 5497.7 \; J$

c) For polytropic process ($PV^{1.35}$ = constant)

Now,

$P_1 V_1^{1.35} = P_2 V_2^{1.35}$

$P_2 = P_1 \left(\frac{V_1}{V_2}\right)^{1.35}$

$= 150 \times 10^3 \left(\frac{0.04}{0.1}\right)^{1.35}$

$= 43538.38 \; Pa$

Then, work done during the process

$$W = \frac{P_1 V_1 - P_2 V_2}{(1 - n)}$$

$$= \frac{150 \times 10^3 \times 0.04 - 43538.38 \times 0.1}{1 - 1.35}$$

$$= \frac{-1646.16}{-0.35}$$

$= 4703.3 \; J$

Q) A control mass containing 0.5 kg of a gas undergoes a process in which there is a heat transfer of 120 kJ from the system to the surroundings. Work done on the system is 60 kJ. If the initial specific internal energy of the system in 400 kJ/kg, determine its final specific energy.

⇒ Soln:-

Mass of gas $(m) = 0.5\,kg$
Total heat transfer from the system $(Q) = -120\,kJ$
Work done on the system $(W) = -60\,kJ$
Initial specific energy $(u_1) = 400\,kJ/kg$
Final specific energy $(u_2) = ?$

Total heat transfer is given by

$$Q = \Delta U + W$$
$$= (U_2 - U_1) + W$$
$$= m(u_2 - u_1) + W$$
$$-120 = 0.5(u_2 - u_1) - 60$$
$$\text{or, } u_2 - u_1 = -\frac{120+60}{0.5}$$
$$\text{or, } u_2 = -120 + 400 = 280\,kJ/kg$$

Q) A 1.2m long tube with outer diameter of 4cm having outside temperature of 120° is exposed to the ambient air at 20°C. If the heat transfer coefficient between the tube surface and the air is 20 W/m²K. Find the rate of heat transfer from the tube to the air.

⇒ Solⁿ:-

length of the tube (L) = 1.2m

Outer radius (R) = 2cm = 2×10^{-2} m

Outside temp. of tube (T_A) = 120°C

Temp of Air (T_B) = 20°C

Heat transfer coeff (h) = 20 W/m²K

Area of tube (A) = $2\pi RL$

$= 2\pi \times 2 \times 10^{-2} \times 1.2$

$= 0.1508\ m^2$

Then, Rate of heat transfer $\frac{dQ}{dt} = hA(T_A - T_B)$

$= 20 \times 0.1508 \times (120 - 20)$

$= 301.593$ watt.

Q.13) Find out the energy radiated per minute from the filament of a lamp at 1000k if the surface area of the lamp is $10 \times 10^{-5}\,m^2$ and relative emittance is 0.95.

⇒ Given,

Temperature $(T) = 100K$

surface area $(A) = 10 \times 10^{-5}\,m^2$

Relative emittance $(e) = 0.95$

Time (t) = 1min = 60 sec

Rate of energy radiation $(E) = ?$

From Stefan Boltzmann's law, we have,

$E = \sigma e A T^4 t = 5.67 \times 10^{-8} \times 0.95 \times 10 \times 10^{-5} \times (1000)^4 \times 60$

$= 323 \times 10^2\,J$

Hence, the value of energy radiated per minute from the filament of the given lamp is $325 \times 10^2\,J$.

Board A simple harmonic transverse wave is propagating along a string towards the left direction as shown in figure. The figure shows a plot of the displacement as a function of position at time $t = 0$. The string tension is 36 N and its linear density is 25 gm/m. Calculate i) The amplitude

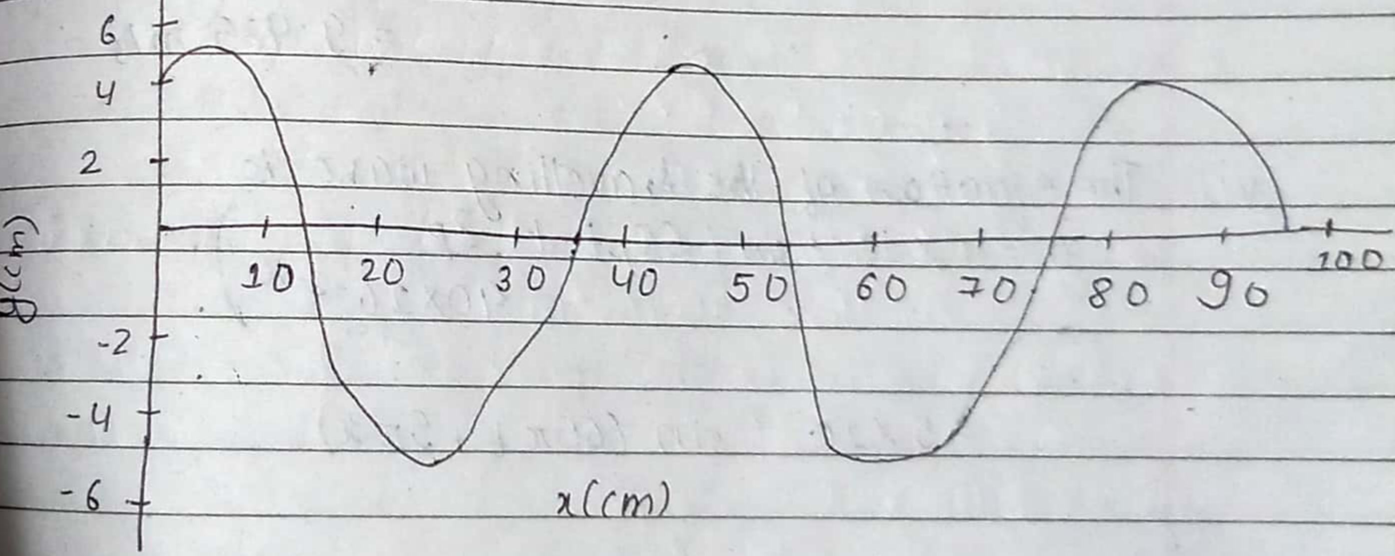
ii) The wave length

iii) Wave speed

iv) The period

v) The maximum particle speed in the string.

vi) Write an equation describing the travelling wave.



Solution:

$T = 3.6\ N$,

$\mu = 25\ gm/m = 25 \times 10^{-3}\ kg/m$

i) Amplitude $(A) = 5cm = 5 \times 10^{-2}\ m$

ii) Wavelength $(\lambda) = 40cm = 40 \times 10^{-2}\ m$

iii) wave speed $v = \sqrt{\frac{T}{m}} = \sqrt{\frac{3.6}{25 \times 10^{-3}}} = 12\ m/s$

iv) Frequency $(f) = \frac{v}{\lambda} = \frac{12}{40 \times 10^{-2}} = 30\ Hz$

Time period $(T) = \frac{1}{f} = \frac{1}{30} = 0.033\ sec$

v) Maximum particle speed $(V_{max}) = A\omega$

$= A(2\pi f)$

$= 5 \times 10^{-2} \times 2\pi \times 30$

$= 9.425\ m/s$

vi) The equation of the travelling wave is

$$y = 5 \times 10^{-2} \sin\left(60pt + \frac{2p}{40 \times 10^{-2}} x\right)$$

$$= 5 \times 10^{-2} \sin(60\pi t + 5\pi x)$$

Q) Obtain the time constant of a capacitor in RC circuit such that the current through the resistor decreases to one third of its peak value in 5 seconds!

⇒ Solⁿ:-

$I = I_0 e^{-t/RC}$

$\frac{I_0}{3} = I_0 e^{-t/\tau}$

or, $\frac{1}{3} = e^{-t/\tau}$ $\because \tau = RC$

Taking ln

$\ln\left(\frac{1}{3}\right) = -t/\tau$

$\cancel{-} 1.098 = \cancel{-} t/\tau$

$1.098 = \frac{5}{\tau}$

$\therefore \tau = \frac{5}{1.098} = 4.55$ sec.

Q) A wave is propagating on a long stretched string along its length taken as +ve x-axis. The wave equation is given by $y = y_0 e^{-\left(\frac{t}{T} - \frac{x}{\lambda}\right)^2}$ where $y_0 = 4mm$, $T = 1sec$. and $\lambda = 4m$.

i) Find the velocity of the wave
ii) Find the function $f(t)$ giving the displacement of the particle at $x=0$
iii) Find the function $g(x)$ giving the shape of the string at $t=0$
iv) Plot the shape $g(x)$ of the string at $t=0$
v) Plot the shape of the string at $t = 5sec$

⇒ Solutions:

$$y = y_0 e^{-\left(\frac{t}{T} - \frac{x}{\lambda}\right)^2}$$

$y_0 = 4mm = 4\times10^{-3} m$

$T = 1$ sec. and $f = 1 Hz$

$\lambda = 4cm = 4\times10^{-2} m$

i) Velocity of the wave $(v) = f\lambda = 1\times4\times10^{-2}$
$= 4\times10^{-2} m/s$

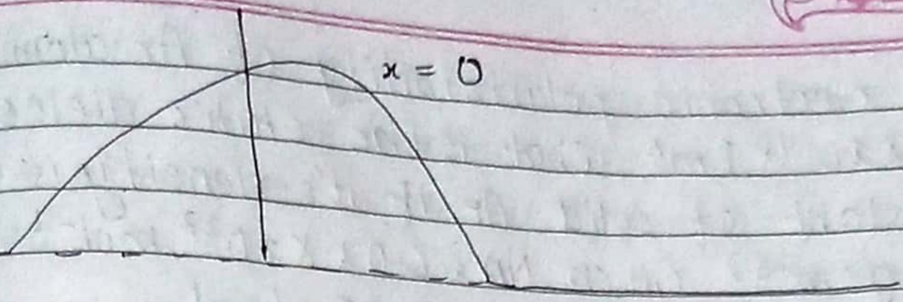
ii) At $x=0$

$$f(t) = y_0 e^{-\left(\frac{t}{T}\right)^2}$$

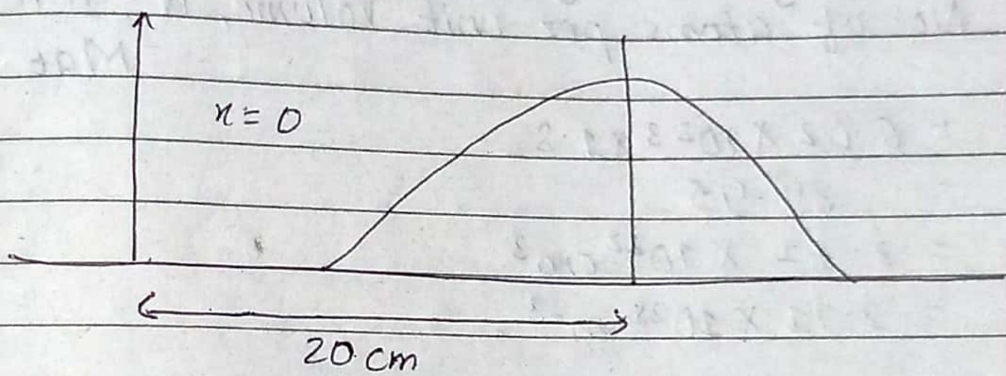
iii) At $t=0$

$$g(x) = y_0 e^{-\left(\frac{-x}{\lambda}\right)^2} = y_0 e^{-\left(\frac{x}{\lambda}\right)^2}$$

iv) $t=0$



v) for $t = 5$ sec, distance $(d) = v \times t = 4 \times 10^{-2} \times 5$
$= 0.2\,m$



Q) The electronic polarizibility of Ar atom is $1.7 \times 10^{-40}\, Fm^2$. What is the static dielectric constant of solid Ar if it's density is $1.8\, gcm^{-3}$? Given $N_A = 6.02 \times 10^{23}\, mol^{-1}$
Atomic mass of Ar = 39.95 g/mol

⇒ Soln:-
Here, $\alpha_e = 1.7 \times 10^{-40}\, Fm^2$
density $d = 1.8\, gcm^{-3}$
No. of atoms per unit volume, $N = \dfrac{dN_A}{M_{at}}$

$$= \frac{6.02 \times 10^{23} \times 1.8}{39.95}$$
$$= 2.71 \times 10^{22}\, cm^3$$
$$= 2.71 \times 10^{28}\, m^{-3}$$

From Calusius - Massotti equation,

$$\frac{N\alpha_e}{3\epsilon_0} = \frac{\epsilon_r - 1}{\epsilon_r + 2}$$

$$\frac{N\alpha_e \epsilon_r}{3\epsilon_0} + \frac{2N\alpha_e}{3\epsilon_0} = \epsilon_r - 1$$

$$\epsilon_r \left(\frac{N\alpha_e}{3\epsilon_0} - 1\right) = -\left[1 + \frac{2N\alpha_e}{3\epsilon_0}\right]$$

$$\epsilon_r = \frac{1 + \frac{2N\alpha_e}{3\epsilon_0}}{1 - \frac{N\alpha_e}{3\epsilon_0}} = 1.63$$

Q) A paralle plate capacitor has circular plates of 8 cm radius and 1mm separation. What charge will appear on the plates if a potential difference of 100V is applied?