

HKN

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(Part 1)

3) Isothermal process

It is an Isothermal process in which temp of a system remains constant throughout the process. There is free exchange of heat between system and surrounding. So it is a closed system.

Consider an ideal gas is

Confined in the conducting cylinder as shown in figure. Let p , v and T are pressure, volume and Temp of a gas. In Isothermal process temp remain constant. Therefore.

Eqn of 1st law of thermodynamics

$$d\theta = du + dw$$

$$\text{or } \theta_{12} = w_{12} + u_2 - u_1 \quad \text{--- (1)}$$

We know that internal energy is a function of temp.

i.e. $u_2 - u_1 = du = C_V dT = 0$ since at const temp $T_2 - T_1 = 0$

∴ Eqn (1) will be

$$\theta_{12} = w_{12} + 0 \Rightarrow \theta_{12} = w_{12} \quad \text{--- (2)}$$

To find the work done during an Isothermal process, we consider an ideal gas filled in the conducting cylinder fitted with piston.

The amount of work done during Isothermal process

$$dw = pdv \Rightarrow dw = \int_{V_1}^{V_2} pdv$$

$$\text{or } w = \int_{V_1}^{V_2} pdv \quad \text{--- (3)} \text{ since in the process of work}$$

done, vol. change V_1 to V_2 by taking const Temp.

We know that $pv = nRT = \text{const. (R)}$

$$\therefore p = \frac{C}{V}$$

Then eqn (3) will be.

$$w = \int_{V_1}^{V_2} C \frac{dv}{V}$$

$$w = C \left[\ln \frac{V_2}{V_1} \right] = w = C \ln \left(\frac{V_2}{V_1} \right)$$

L.C.

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We know that for isothermal process

$$P_1 V_1 = P_2 V_2 = \text{const} (C.) \quad \text{thus, } \Delta Q_{12} = \Delta U_{12} \quad \text{in eqn (1)}$$

$$\therefore \Delta Q_{12} = \Delta U_{12} = P_1 V_1 \ln \left[\frac{V_2}{V_1} \right]$$

or

$$\Delta Q_{12} = \Delta U_{12} = P_2 V_2 \ln \left(\frac{V_2}{V_1} \right)$$

Hence here transferred to a control mass for an ideal gas during a constant temp. process is equal to work done by the gas.

(a) Adiabatic (polytropic) process

It is a thermodynamic process in which pressure, volume and temp. of the system change without exchange of heat between system and surrounding. Therefore, form 1st law of thermodynamics

$$dQ = dU + dW \quad \text{or} \quad \Delta Q_{12} = \Delta U_{12} + \Delta W_{12} \quad (1)$$

~~$\Rightarrow dU = dW$ where $dQ = 0$.~~ Suppose an n' meter of an ideal gas taken in a insulated cylinder as shown in figure

Also, we noted that

$$\Delta Q_{12} = \Delta U_{12} + \Delta W_{12}$$

\Rightarrow

~~$\therefore \Delta U_{12} = \Delta W_{12}$, no exchange of heat in between~~

System and surrounding. Suppose gas expand at (P_1, V_1, T_1) to (P_2, V_2, T_2) . If P is the pressure of the gas at an instant ~~at time~~ then the work done for small change in vol. dV is given

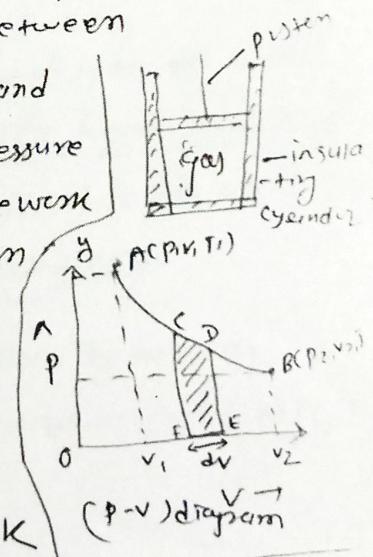
by $dW = PdV$. The total work done during whole process is

$$W = \int_{V_1}^{V_2} PdV \quad (2)$$

For an adiabatic process $PV^\gamma = \text{const.} = K$

$$\therefore P = \frac{K}{V^\gamma} \quad (3)$$

using (3) in eqn (2) we get



$$w = \int_{V_1}^{V_2} \frac{K dV}{V^\gamma}$$

$$\text{or } w = K \int_{V_1}^{V_2} V^{\gamma-1} dV$$

$$\text{or } w = K \left[\frac{V^{\gamma+1}}{\gamma+1} \right]_{V_1}^{V_2}$$

$$\therefore w = \frac{K}{\gamma-1} \left[V_2^{1-\gamma} - V_1^{1-\gamma} \right]$$

$$\text{or } w = \frac{1}{\gamma-1} (K V_2^{1-\gamma} - K V_1^{1-\gamma}) \quad \text{--- (4)}$$

We know that for adiabatic process

$$P_1 V_1^\gamma = P_2 V_2^\gamma = K \quad \text{so eqn (4) will be}$$

$$w = \frac{1}{\gamma-1} [P_2 V_2^{1-\gamma} - P_1 V_1^{1-\gamma}]$$

$$\text{or } w = \frac{1}{\gamma-1} (P_2 V_2 - P_1 V_1) \quad \text{--- (5)}$$

$$\therefore Q_{12} = n \left[\frac{R}{\gamma-1} + C_V \right] T_2 - T_1$$

$$\text{or } Q_{12} = n C_n (T_2 - T_1) \text{ where } C_n = \left(\frac{R}{\gamma-1} + C_V \right) \text{ is called}$$

Polytropic specific heat capacity of mole of gas.

(5) specific heat capacity?

⇒ Specific heat of a gas is defined in terms of standard mass system (kg), it is called principled specific heat capacity (C)

⇒ If specific heat of gas defined in terms of molar mass,

it is called molar specific heat capacity (C)

Specific heat capacity of a gas is explained by two ways

i.e. taking Volume constant (C_V) and pressure constant (C_P)

They are described as following:

(1) sp. heat capacity at constant volume:

(a) principical sp. heat capacity at gas at constant volume (C_V):

It is defined as the quantity of heat required to raise the temp. of 1 kg of a gas through 1 Kelvin temp. at const. volume

In SI unit $C_V \text{ is } J \text{ kg}^{-1} \text{ K}^{-1}$

PVT

→ For ideal eqn. of gas $PV = nRT$,
 $P_2 V_2 = nRT_2$

so, eqn (5) will be

$$w = \frac{1}{(1-\gamma)} (nRT_2 - nRT_1)$$

$$\therefore w = \frac{nR}{(1-\gamma)} (T_2 - T_1) \quad \text{--- (6)}$$

where $n = \text{mole of gas}$.

Using eqn (6) in eqn (5) we get

$$Q_{12} = \frac{nR}{(1-\gamma)} (T_2 - T_1) + nC_V(T_2 - T_1)$$

where $dU = nC_V(T_2 - T_1)$ or $dU =$

or $C_V = \text{molar sp. heat capacity of a gas at constant volume}$.

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(3) molar specific heat capacity at constant volume (C_V)

C_V is defined as the amount of heat required to raise the temp. of one mole of gas through one kelvin at const. volume. Its SI unit is $J \text{ mol}^{-1} \text{ K}^{-1}$. M be the molar mass of gas in kg. Then,

$$C_V = m C_V$$

specific heat of an ideal gas at constant volume can be defined in term of change in internal energy. So, internal energy of an ideal gas can be determined if its temp. and specific volume are given.

$$\therefore U = U(T, V)$$

$$dU = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV$$

For constant volume, $dV = 0$

$$\therefore dU = \left(\frac{\partial U}{\partial T}\right)_V dT$$

$$\text{or } dU = C_V dT \text{ where } C_V = \left(\frac{\partial U}{\partial T}\right)_V$$

For n mole of gas, $dU = n C_V dT$

$$\therefore m \text{ mass of gas } dU = m C_V dT$$

\therefore Specific heat at constant volume is defined as a change in specific internal energy per degree change in temp.

(2) Specific heat Capacity at constant pressures

(a) principal specific heat capacity at constant pressure (C_P). C_P is defined as the quantity of heat required to raise the temp. of 1 kg of gas through 1 K at constant pressure. In SI unit, C_P has $J \text{ kg}^{-1} \text{ K}^{-1}$.

(b) molar specific heat capacity at constant pressure (C_P)

C_P is defined as the amount of heat required to raise the temp. of 1 mole of gas through 1 K at constant pressure. If its units is $J \text{ mol}^{-1} \text{ K}^{-1}$. If m be the molar mass of a gas in kg. Then $C_P = M C_P$.

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The specific heat capacity of an ideal gas at constant pressure can be defined in terms of change in enthalpy. So we have, specific enthalpy

$$H = H(T, p)$$

Change in sp. enthalpy τ_p given as

$$dH = \left(\frac{\partial H}{\partial T}\right)_p dT + \left(\frac{\partial H}{\partial P}\right)_T dP$$

For constant pressure $dP = 0$

$$\therefore dH = \left(\frac{\partial H}{\partial T}\right)_p dT$$

or $dH = C_p dT$ where $C_p = \left(\frac{\partial H}{\partial T}\right)_p$ τ_p called sp. heat at constant pressure and τ_p defined as the change in sp. heat enthalpy per degree change in temp.

Qn 1 Show that $C_V > C_U$ [from +2 book]

② Derive the relation $[C_p - C_V = R]$ [from +2 book]

Second law of thermodynamics

Limitations of first law of thermodynamics

First law of thermodynamics is an important tool for understanding physical processes. Also no any process can violate it but has four limitations. They are given as following:

- ① When two bodies at different temp. are brought in thermal contact, heat flows always from hot body to cold body but first law of thermodynamics is silent to the question why heat doesn't flow from cold body to hot body. Therefore, first law of thermodynamics does not indicate the direction of heat flow.
- ② Heat can not be converted into useful work form a

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Single heat reservoir like sea. But why this is impossible,
 First law of thermodynamics has no answer to this question.
 That is first law of thermodynamics does not provide the

- Condition under which heat can be converted into work.
- ③ When a bullet strikes a target, K.E. of bullet converts into heat energy but target cannot provides energy to the bullet for fly back losing its heat why? First law of thermodynamics has no answer that whether the mechanical work and heat energy are interconvertible or not.
- ④ In a heat engine, allowed by first law of thermodynamics no 100% heat energy converts into mechanical work, why? First law of thermodynamics has no answer to this. That is first law of thermodynamics does not indicate that 100 to what extent, heat energy can be converted into mechanical work.

Second law of thermodynamics

Second law of thermodynamics deals with the nature or quality of heat energy and defines the direction in which the system can proceed. Hence, the main feature of 2nd law of thermodynamics is that it defines the directions of the processes.

2nd law of thermodynamics defines the process's direction in terms of a system property known as entropy. Thus, entropy is the randomness or disorder of a thermodynamics system. It is an extensive property of a system that is used to measure the amount of energy that is unavailable for work.

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It is denoted by S .

$$\Delta S = \frac{Q}{T} \quad \text{--- (1)}$$

where, Q = amount of heat energy transfers during a process
 T = absolute temp at which the process takes place
 we know that, Entropy change is equal to the sum of entropy change of system and surroundings. Therefore Total change of entropy

$$(\Delta S)_T = (\Delta S)_{\text{system}} + (\Delta S)_{\text{surrounding}} \quad \text{--- (2)}$$

If the system loses an amount of heat Q at a temp T_1 which is received by surroundings at temp T_2 .

So, $(\Delta S)_{\text{total}}$ can be calculated

$$(\Delta S)_{\text{system}} = -\frac{Q}{T_1} \quad \text{and} \quad (\Delta S)_{\text{surrounding}} = +\frac{Q}{T_2}$$

$$\therefore (\Delta S)_{\text{total}} = -\frac{Q}{T_1} + \frac{Q}{T_2} \quad \text{--- (3)}$$

If $(\Delta S)_{\text{total}} = +ve$, the process is spontaneous

" $(\Delta S)_{\text{total}} = -ve$, " " " non-spontaneous

" $(\Delta S)_{\text{total}} = 0$, the process is at equilibrium.

" $(\Delta S)_{\text{total}} = 0$, the process is at equilibrium.

(II) Statements of 2nd law of thermodynamics

(1) Kelvin Statement (About heat engine):

" It is impossible to get continuous work from a body by cooling it to a temp. lower than that of its surroundings."

This means that in heat engine, working substance absorbs heat from source and performs mechanical work then rejects certain amount of heat to the sink. But, no any heat engine converts all heat extracted from the source into work without rejecting to the sink.

In other words, presence of colder body is required for a reservoir continuous work. This means single heat transfer cannot transfer heat into mechanical work continuously.

② Clausius Statement (about refrigerators)

"It's impossible to make flow of heat from colder body to hotter body without doing external work on the working substance."

This means, in a refrigerator, working substance absorbs heat from the sink, certain external work is added on it so that finally large amount of heat is rejected to the source. But, no one refrigerator transfer heat from colder body to hotter body without adding external work on it.

③ Heat engine:

Heat engine is a device which converts heat into mechanical work continuously. For example, the gasoline engines in automobile, airplane, steam turbine of steam engine are heat engines. Heat engine consists of following components.

① Source: It is a body of infinite heat capacity maintained at heat temp. Source has greater temp. (T, K) than that of sink. Working substance can absorb any amount of heat from source keeping source temp. constant.

② Working substance: The material body of heat engine which burn to produce heat so that it perform mechanical work.

Ideal gas in Carnot's engine, 9% petrol vapour and 98% air in petrol engine are examples of working substance.

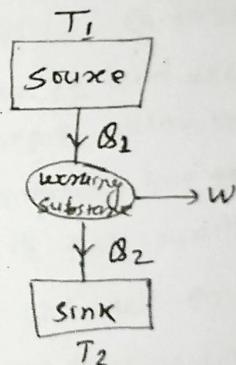
(ii) Sink: It is a body of infinite heat capacity maintained at low temp. Sink has lower temp. (T_2 K) than that of source. Any amount of heat can be rejected to sink keeping sink temp. constant.

(iii) Thermal efficiency efficiency of heat engine.

Heat engine operates on a cyclic process and receive heat from a high temp reservoir (source) convert part of this heat to work and then reject the remaining part of heat to a lower temp. reservoir (sink) during the cycle.

Suppose, working substance

extract \dot{Q}_1 amount of heat from the source at temp. T_1 K, perform certain mechanical work w and rejects remaining amount of heat \dot{Q}_2 to the sink at lower temp. T_2 K as shown in figure.



Therefore, $(\dot{Q}_1 - \dot{Q}_2)$ is the amount of heat converted into mechanical work. That is work done

$$w = \dot{Q}_1 - \dot{Q}_2 \quad \text{--- (1)}$$

Now, efficiency of heat engine is defined as the ratio of the external work obtained to the heat energy absorbed by the working substance from the source. It is denoted by η . So,

$$\eta = \frac{\text{External work done (w)}}{\text{Heat energy absorbed from the source} (\dot{Q}_1)}$$

$$\therefore \eta = \frac{\eta_1 - \eta_2}{\eta_1} \Rightarrow \eta = 1 - \frac{\eta_2}{\eta_1} \text{ since } \eta_1, \eta_2 = w.$$

In general efficiency is expressed in %. So,

$\eta = \left(1 - \frac{\eta_2}{\eta_1}\right) \times 100\%$. Since $\eta_1 > \eta_2$, efficiency is always less than 100%. ~~For cent percentage~~ Efficiency is possible only when $\eta_2 = 0$ but $\eta = 0$ is impossible because to drive engine, heat must be lost to the sink compulsorily.

In term of temp. efficiency of heat engine can be expressed as

$$\eta = \left(1 - \frac{T_2}{T_1}\right) \times 100\%$$

(ii) Refrigerator:

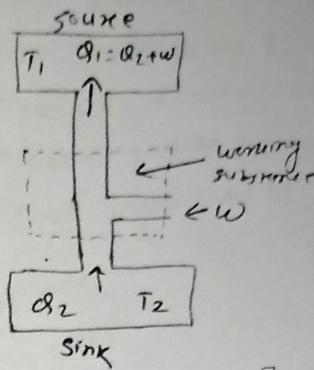
Refrigerator operates on a cyclic process which takes heat from a low-temp. reservoir at temp. T_2 and delivers it into the high-temp. reservoir at temp. T_1 with the help of external work. Refrigerator maintains the temp. of desired space lower than that of surroundings.

A refrigerator is essentially a heat engine working in the reverse direction. That means in refrigerator, working substance takes heat from cold body (interior of refrigerator), compressor supplies the necessary mechanical work input and sum of two energies is rejected to the hot reservoir (surrounding air). The schematic operation of a refrigerator is shown in figure below.

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SUPPOSE WORKING substance absorbs \dot{Q}_2 amount of heat from sink at temp T_2 on adding external work W unit by motor. Finally, large amount of heat \dot{Q}_1 is rejected to the source at higher temp. T_2 . Now, $\dot{Q}_1 = \dot{Q}_2 + W$. —①

Here, Input work (or energy) $W = \dot{Q}_1 - \dot{Q}_2$
Output work (or energy) = \dot{Q}_2



The efficiency of different refrigerators can be compared by their coefficient of performance (COP). Coefficient of performance (β) is defined as the ratio of heat extracted from the cold body (refrigerant) to work done by the motor.

$$\therefore \beta = \frac{\text{Output}}{\text{Input}} = \frac{\dot{Q}_2}{\dot{Q}_1 - \dot{Q}_2} \Rightarrow \frac{1}{\frac{\dot{Q}_1 - \dot{Q}_2}{\dot{Q}_2}} \Rightarrow \frac{1}{\left(\frac{\dot{Q}_1}{\dot{Q}_2} - 1\right)}$$

$$\therefore \beta = \frac{1}{\left(\frac{T_1}{T_2} - 1\right)} \Rightarrow \beta = \frac{T_2}{T_1 - T_2}$$

This shows that all reversible refrigerating machines working between the same two temp. have the same coefficient of performance. If β is clear from expression that if $T_1 - T_2$ is small, β will be higher. The best refrigerator removes maximum heat \dot{Q}_2 with least possible work input by motor. If no work were needed to operate a refrigerator, the coefficient of performance β would become infinite which is rather impossible. For practical refrigerator, β is always less than $\frac{T_2}{T_1 - T_2}$.

(II) Heat pump

Heat pump operates on a cyclic process which takes heat from a low temp. reservoir (surrounding) at T_2 and delivers into the high temp. reservoir (desired space) at T_1 with the help of external work done. Heat pump maintains the temp. of desired space higher than that of surroundings.

Consider a heat pump takes \dot{Q}_2 amount of heat from sink at temp. T_2 and supply \dot{Q}_1 to desired space at T_1 with the help of external work done (W). performance of heat pump is measured in terms of coefficient of performance (COP) β , which is defined as the ratio of desired effect and work supplied. i.e.

$$\beta = \frac{\text{Desire effect}}{\text{work input}}$$

where Desire effect = amount of heat supplied to the desired space (\dot{Q}_1).

$$\therefore (\beta)_{\text{heat pump}} = \frac{\dot{Q}_1}{W} = \frac{\dot{Q}_1}{\dot{Q}_1 - \dot{Q}_2} \Rightarrow \frac{1}{1 - \frac{\dot{Q}_2}{\dot{Q}_1}}$$

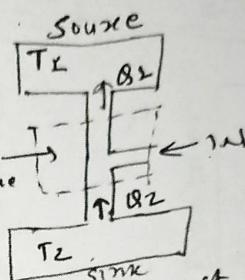
$$\text{In term of Temp, } (\beta)_{\text{heat pump}} = \frac{T_1}{T_1 - T_2} \Rightarrow \frac{1}{1 - \frac{T_2}{T_1}}$$

(II) Components of refrigerator and heat pump:

OR.

Construction and working of Refrigerator.

\Rightarrow A refrigerator and heat pump operates on a cyclic process called refrigeration cycle. The refrigeration cycle is the



[Block diagram of heat pump]

Throttling

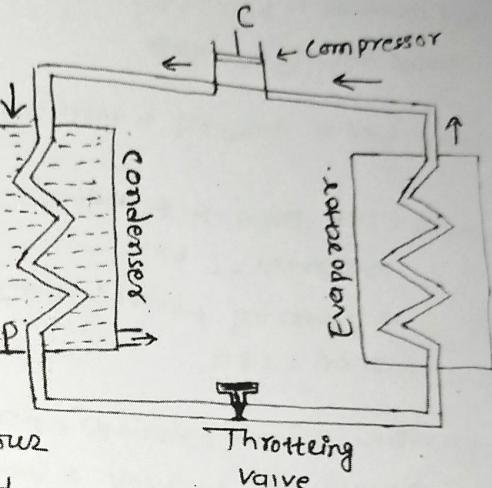
widely used for heating and cooling of desired space.

The working principle of common refrigerator has been explained with the support of schematic diagram as shown in figure below. Four components of refrigerator are

- (i) Compressor
- ii) Condenser
- iii) Throttling Valve, and
- iv) Evaporator.

The working procedure of refrigerator along with its components are described as following.

The working substance in a refrigerator is usually the vapour of some low boiling point liquid, such as chloro-fluoro carbon (CCl_2F_2 or freon), ammonia,



[Fig: Schematic diagram of refrigerator.]

Sulphur dioxide etc. The compressor (C) compresses the working substances adiabatically to a high pressure with the support of motor (external work is done here) so that gas becomes quite hot. This hot and high pressured gas is passed through condenser. Condenser is a spiral pipe (P) to which cold water is circulated. Now gas gets liquefied. This liquid is passed through the throttling valve (V) and it expands suddenly by the process called throttling process. This process causes the decrease in the temp. of gas. Then the gas is passed through the evaporator (spiral tube + cooling department containing objects to be cooled like food items) where

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liquid evaporators and gets cooled by taking heat from food items. Now, working substance leaves the cooling unit and then goes to the compressor (C) for fresh cycle.

Note:

Compressor \rightarrow high pressure - Temp. \rightarrow p, T and H increases but Entropy (S) remains constant.

Condenser \rightarrow high pressure - low temp. \rightarrow p = const, H and S decreases.

Throttling valve \rightarrow high press - low temp \rightarrow H = const, P & T = decreases

Evaporator \rightarrow low pressure - low temp \rightarrow p = const, and S = increases.

II Numerical problems:

① A gas is contained in a piston cylinder device initially at a pressure of 150 kPa and a volume of 0.04 m³. Calculate the work done by the gas when it undergoes the following process to a final volume of 0.1 m³.

② Constant pressure. ③ Constant temp. ④ $pV^{1.35} = \text{constant}$

Soln: we have given,

$$\text{initial pressure } (P_1) = 150 \text{ kPa}$$

$$\text{Volume } (V_1) = 0.04 \text{ m}^3$$

$$\text{Final } \text{Volume } (V_2) = 0.1 \text{ m}^3$$

work done by the gas = ?

④ At const. pressure, work done $W = PdV = P_1(V_2 - V_1)$

$$\therefore W = 150 \times 10^3 (0.1 - 0.04) \Rightarrow W = 9000 \text{ J}$$

⑤ At constant temp, work done $W = P_1V_1 \ln \left(\frac{V_2}{V_1} \right)$

$$= 150 \times 10^3 \times 0.04 \ln \left(\frac{0.1}{0.04} \right) = 5497 \text{ J}$$

⑥ For adiabatic process, $pV^\gamma = \text{const.}$ since $\gamma = 1.35$.

$$P_1V_1^\gamma = P_2V_2^\gamma \Rightarrow P_2 = P_1 \left(\frac{V_2}{V_1} \right)^\gamma = 1.35$$

$$P_2 = 150 \times 10^3 \left(\frac{0.04}{0.1} \right)^{1.35} \Rightarrow 43538.38 \text{ Pa.}$$

$$\text{Work done } W = \frac{1}{1-\gamma} (P_2V_2 - P_1V_1)$$

$$\frac{1}{1-1.35} \left(\text{Used the value at constant pressure} \right) \text{ find } W = 4703.3 \text{ J}$$

$$(W =)$$

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- (2) An ideal engine has an efficiency of 25%. If the source temp. is increased by 200°C , its efficiency gets doubled. Determine its source and sink temp.

Soln. Let Temp. of source = T_1 ,

" " Sink = T_2

efficiency

$$\eta = 1 - \frac{T_2}{T_1} \Rightarrow 0.25 = 1 - \frac{T_2}{T_1} \Rightarrow T_2 = 0.75 T_1$$

If double, i.e. $\eta_2 = 2\eta = 2 \times 0.25 \Rightarrow \eta_2 = 0.50$.

$$\therefore \eta_2 = 1 - \frac{T_2}{T_1}$$

$$0.50 = 1 - \frac{\cancel{T_2}}{T_1 + 200}$$

$$\frac{T_2}{T_1 + 200} = 0.5 \quad \text{--- (2)}$$

Use eq(1) in eq(2). we get

$$\frac{0.75 T_1}{T_1 + 200} = 0.5$$

$$\therefore T_1 = 2100\text{K}$$

$$\text{and } T_2 = 0.75 \times 2100 = 3150\text{K}$$

- (3) The source reservoir of a heat engine is at temp. of 400K and takes 400J of heat and rejects 20J of heat to the sink reservoir in each cycle. What are the efficiency of engine and temp. of sink?

Soln Hint: $T_1 = 400\text{K}$, $T_2 = ?$, $Q_1 = 400\text{J}$, $Q_2 = 20\text{J}$, $\eta = ?$

$$\eta \% = \frac{Q_1 - Q_2}{Q_1} \times 100\% = \frac{400 - 20}{400} \times 100\% = 95\%$$

Again, $\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \Rightarrow \frac{400}{20} = \frac{400}{T_2} \Rightarrow T_2 = 20\text{K}$.

- (4) An ideal heat engine operates between two reservoirs at two temp. In order to achieve 30% efficiency when the temp. of the sink is 50°C . what should be the temp. of the source?

Soln. Hint $\eta = 30\%$, $T_2 = 50^\circ\text{C} = (50 + 273)\text{K} = 323\text{K}$

$$T_1 = ? \text{ we have } \frac{T_1 - T_2}{T_1} \times 100\% = 30\% \Rightarrow T_1 = 461.43\text{K}$$

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Part 2

HKN

Dr. Hari Neupane (Physics)Chapter - 5

(Pg-1)

Capacitor and Dielectric
 X X X① Introduction:

A specially designed device to store charge and electric energy in a small space is called capacitor. Capacitor consists of two conducting surfaces (in the form of foil, thin film, electrolyte etc.) which are called plates. Plates are separated by insulating medium (air, plastic films, mica, glass, ceramics, papers, etc) called dielectrics. The plates connected to a battery is called collector plate and that earthed is called condenser plate. Effect of capacitor (Capacity to hold charges and energy) is called capacitance. Non-conducting medium (dielectric) are used to increase capacitance. There are various types of capacitors. The symbol of capacitor is $\text{---} \parallel \text{---}$.

② Capacitance of Capacitor:

Capacitance of a capacitor is an ability to store (hold) electric charge. Series of experiments show that charge given to a capacitor is directly proportional to electric potential developed across its plates. That is

$$q \propto V$$

or $q = CV$ where C = proportionality const.

$$\therefore C = \frac{q}{V} \quad \text{--- (1)}$$

Thus, 'Capacitance of capacitor can be defined as the ratio of amount of charge given to a capacitor to potential difference developed across its plates.'

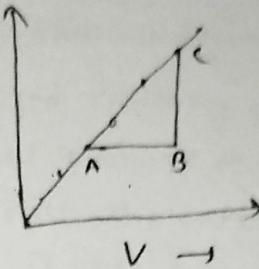
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Capacitance of a capacitor is also determined by the ratio of charge (q) vs potential difference (v) as shown in figure.

$$\text{In figure, slope} = \frac{BC}{AB} = \frac{\Delta q}{\Delta V}$$

$$\therefore \text{Slope} = \frac{q}{V} = C \text{ (capacitance).}$$



Unit of C:

$$C = \frac{q}{V} = \frac{\text{Coulomb}}{\text{Volts}} = \text{Farad (F)} \quad [\text{Graph. both } q \propto V]$$

\therefore Capacitance can be measured in Faraday (F)

Unit: ~~C~~ = C can also be measured in mF, μF, PF, nF etc.

~~1 mF = 10^3 F, 1 μF = 10^{-6} F, 1 PF = 10^{12} F, 1 nF = 10^{-9} F~~

Definition of 1 F:

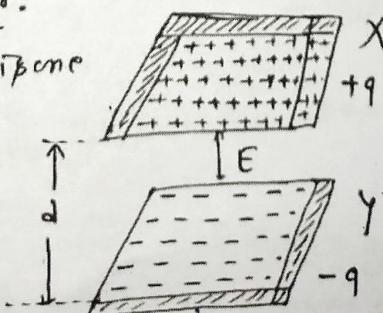
The capacitance of a capacitor is said to be 1 F if the application of 1 C charge causes the rise in electric potential between its plates through 1 V.

(3) Types of capacitors:

There are various types of capacitors. Among them here, we are discussed three types of capacitor and then calculated their capacitance respectively.

(1) parallel plate capacitor:

Parallel plate capacitor is one of the simplest forms of capacitor and consists of two (Collector plate and Condenser plate) parallel



PFY: parallel plate capacitor

Credits: Mahigya Dahal (Note), Nishan Paudel (Photo)

(3)

(Pys)

metal plates separated by dielectric medium. Suppose two parallel metal plates X and Y each of area A are separated by distance (d) as shown in figure.

Assume that plate X is charged positively with charge ($+q$) + q so that plate Y gets equal number of induced negative charge ($-q$) on its inner surface. The outer surface of Y is earthed. Let distance ' d ' is very small as compared to area (A) so that electric field (E) setup by charge + q and - q is uniform between plates. If σ be the surface charge density and ϵ is permittivity of medium between plates, then, Form Gauss's law

$$E = \frac{\sigma}{\epsilon} \quad \text{--- (1)} \quad (\text{since } E \cdot A = q / (4\pi\epsilon_0)$$

The potential difference between two plates is given by $V = Ed$ --- (2) Using eq (1) in eq (2) we get,

$$\text{or } V = \frac{\sigma}{\epsilon} d$$

$$\text{or } V = \frac{q}{A\epsilon} d \quad \text{where } \sigma = \frac{q}{A} = \text{surface charge density}$$

(3)

We know that capacitance of a capacitor $C = \frac{q}{V}$

so, eq (3) becomes

$$C = \frac{q}{V} \Rightarrow C = \frac{q'}{qd} \Rightarrow C = \frac{A\epsilon}{d} \quad \text{--- (4)}$$

We know that $K = \frac{\epsilon}{\epsilon_0} \Rightarrow \epsilon = K\epsilon_0$ where K = dielectric const.

So, eq (4) will be,

$$C = \frac{AK\epsilon_0}{d} \quad \text{--- (5)} \quad \text{This eq (5) gives the capacitance of a capacitor where dielectric medium is placed in both the plates.}$$

(4)

(13-4)

If the space between the plates is air or vacuum, Then eqn (5) will be

$$C_0 = \frac{\epsilon_0 A}{d} \quad \text{where } \epsilon_0 = \text{permittivity of vacuum.}$$

These eqn. (5) and (6) gives the capacitance of ~~cap~~ parallel plate capacitor.

(#) Factor affecting the capacitance of capacitor:

Capacitance of a capacitor is independent of nature of material of plate, amount of charges given to the plate and potential across the plates but depends on the following factors.

(i) The distance between the plates d (i.e. $C \propto \frac{1}{d}$); inversely proportional relation.

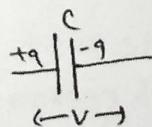
(ii) Area of each plate (A) (i.e. $C \propto A$); directly proportional relation.

(iii) Nature of dielectric (medium separating the plates) ϵ (i.e. $C \propto \epsilon$ or $C \propto \epsilon_0 k$); directly proportional relation.

(#) Energy stored in a charged capacitor:

When a capacitor is connected to a battery, Charging goes to some extent

and then further charging is opposed [Fy: energy stored in a capacitor] by electric field setup between plates of the capacitor. To charge the capacitor furthermore, the battery has to do work against developed electric field.



(5)

popgo - pg-5

This work done is stored as an electric potential energy and is obtained at the cost of chemical energy stored in the battery.

Suppose a capacitor of capacitance C has charge q and potential difference V at any time of its charging as shown in figure. If dW is small work done to store dq amount of charge to the capacitor then,

$$dW = (V + dV) dq$$

$$\therefore dW = V dq \quad (\text{since by storing small charge } dq, dV \approx 0 \text{ and } V + dV \approx V)$$

$$\therefore dW = \frac{q}{C} dq \quad \text{since } C = \frac{q}{V}$$

Now, total work done required to store q amount of charge to the capacitor,

$$W = \int dW = \int \frac{q}{C} dq$$

$$\text{or } W = \frac{1}{C} \left[\frac{q^2}{2} \right]_0^q$$

$$\text{or } W = \frac{1}{2} \frac{q^2}{C}$$

$$\therefore W = \frac{q^2}{2C} \quad \text{---(1)}$$

This work done is stored in the capacitor as its electric potential energy (U) i.e.

$$U = \frac{1}{2} \frac{q^2}{C} \Rightarrow U = \frac{1}{2} \frac{V^2 C^2}{C} \Rightarrow U = \frac{1}{2} C V^2 \quad \text{---(2)}$$

This gives the work done is stored in capacitor as its electric potential U .

Energy density:

Energy stored per unit volume of the space between the plates of a capacitor is called energy density. If A be the area of each plate of parallel plate capacitor with dielectric constant ϵ and d be the distance between plates. Then, energy density of capacitor is given by $U_d = \frac{\text{energy stored}}{\text{volume}}$

(6)

(P+3)(c)

$$U_d = \frac{1}{2} \frac{CV^2}{Ad} \quad (\text{since } V = A \times \text{distance})$$

$$\text{or } U_d = \frac{1}{2} \left(\frac{A\epsilon}{d} \right) \cdot V^2 \quad \text{since } C = \frac{A\epsilon}{d}$$

$$\text{or } U_d = \frac{1}{2} \epsilon \left(\frac{V}{d} \right)^2$$

$$\epsilon = \epsilon_0 K$$

$$\text{or } U_d = \frac{1}{2} \epsilon E^2 \quad \text{where } E = \frac{V}{d}$$

$$\therefore U_d = \frac{1}{2} \epsilon_0 K E^2 \quad \text{--- (3)}$$

Hence, eqn (3) is an expression of energy density.

(b) Cylindrical Capacitor:

Consider two cylinders of radii a and b are placed co-axially each of length l separated by small distance $(b-a)$ form a cylindrical capacitor.

As a Gaussian surface, we choose a cylinder of length l and radius r that enclosed just the charge q on the positive inner cylinder.

$$\text{From Gauss's law, } EA = \frac{q}{\epsilon_0} \Rightarrow E = \frac{q}{A\epsilon_0}$$

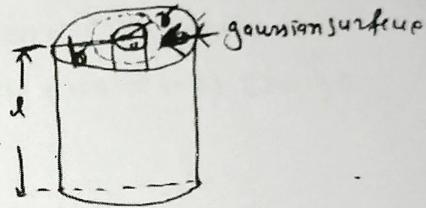
$$\text{or } E = \frac{q}{2\pi r \epsilon_0 l} \quad \text{where } A = 2\pi r l.$$

$$\text{Now } V = \int_a^b E \cdot dr$$

$$\text{or } V = \int_a^b \frac{q}{2\pi r \epsilon_0 l} \cdot dr \Rightarrow V = \int_a^b \left(\frac{q}{2\pi r \epsilon_0 l} \right) \frac{1}{r} dr$$

$$\text{or } V = \frac{q}{2\pi \epsilon_0 l} [\ln b - \ln a] \Rightarrow V = \frac{q}{2\pi \epsilon_0 l} \ln \left(\frac{b}{a} \right)$$

$$C = \frac{q}{V} \Rightarrow C = \frac{q}{\frac{q}{2\pi \epsilon_0 l} \ln \left(\frac{b}{a} \right)} \Rightarrow C = \frac{2\pi \epsilon_0 l}{\ln \left(\frac{b}{a} \right)}$$

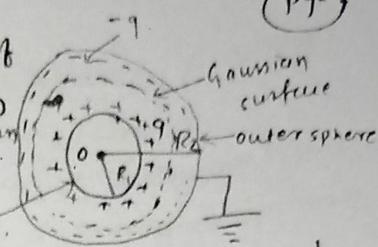


This gives the capacitance of a cylindrical capacitor.

(7)

(c) Spherical Capacitor:

A spherical capacitor consists of two concentric conducting spheres where outer one is earthed in figure.



Let R_1 and R_2 be the radii of inner and outer sphere respectively. When $+q$ charge is given to inner sphere then $-q$ charge is induced on inner surface of ~~outer~~ outer sphere. The outer surface is earthed so it does not store charge and electric potential due to it ($V_0 = 0$). If there is vacuum or air between two spheres, then the potential at inner sphere (V_i) = potential due to $(+q)$ charge + potential due to $(-q)$ charge.

$$V_i = \frac{1}{4\pi\epsilon_0} \frac{q}{R_1} + \frac{1}{4\pi\epsilon_0} \frac{(-q)}{R_2}$$

$$\text{or } V_i = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\text{or } V_i = \frac{q}{4\pi\epsilon_0} \frac{(R_2 - R_1)}{R_1 \cdot R_2}$$

Now potential difference between inner and outer sphere is $V = V_i - V_0$

$$\text{or } V = \frac{q}{4\pi\epsilon_0} \cdot \left(\frac{R_2 - R_1}{R_1 \cdot R_2} \right) - 0$$

$$\therefore V = \frac{q}{4\pi\epsilon_0} \left(\frac{R_2 - R_1}{R_1 \cdot R_2} \right) \quad \text{--- (1)}$$

Thus, capacitance of spherical capacitor is

$$(C = \frac{q}{V}) \Rightarrow \frac{q}{\frac{q}{4\pi\epsilon_0} \left(\frac{R_2 - R_1}{R_1 \cdot R_2} \right)} \Rightarrow C = 4\pi\epsilon_0 \left(\frac{R_1 \cdot R_2}{R_2 - R_1} \right) \quad \text{--- (2)}$$

This eqn (2) gives the capacitance of spherical capacitor

(8)

No 8

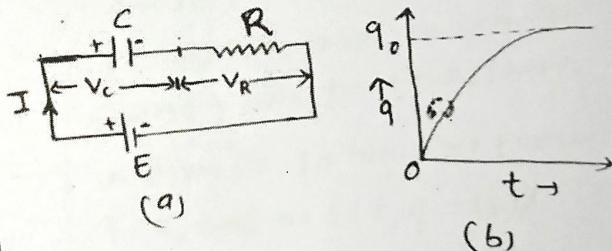
(ii) Applications of capacitors

Some applications of capacitors are given as following:

- ① Capacitors are used to store the energy (charge)
- ② Capacitors are used in high frequency microwave system
- ③ Capacitors are used in electrical measuring equipment such as in sensors.
- ④ Capacitors are used in electronics and telecommunications equipment
- ⑤ Capacitors are required in both residential and commercial appliances for example batteries, cameras, electronic chargers, LED lights, Audio equipment etc.
- ⑥ Capacitors are used in power system to control voltage and enhance the quality of the power supply

4 Charging and discharging of capacitor

Let us consider, charging of a capacitor of capacitance C through a resistor R in series as shown in figure (a). Suppose, E for an instant, q be the emf of the source having negligible internal resistance. Let a time $t=0$, charge in a capacitor $q=0$.



[Fig: (a) charging of a capacitor, (b) graph showing the charging nature of capacitor]

Suppose for an instant, q be the charge on capacitor and I be the current through R . Let, V_c and V_R are the potential difference across the capacitor C and resistor R respectively. Therefore, $E = V_c + V_R$.

$$\Rightarrow E = \frac{q}{C} + IR$$

$$\Rightarrow E = \frac{q}{C} + \frac{dq}{dt}R \quad \text{Since } I = \frac{dq}{dt}$$

(9)

(P9-2)

$$\text{or } \frac{dq}{dt} R = E - \frac{q}{C}$$

$$\text{or } \frac{dq}{dt} = \frac{EC - q}{RC}$$

$$\text{or } \frac{dq}{EC - q} = \frac{dt}{RC}$$

$$\text{or } \frac{dq}{q_0 - q} = \frac{dt}{RC} \quad \text{(1) where}$$

where $q_0 = EC$ is maximum charge that can be stored in the capacitor. Now integrating eqⁿ(1) we get

$$\int_{0}^q \frac{dq}{q_0 - q} = \int_{0}^t \frac{dt}{RC}$$

$$\text{or } -\log(q_0 - q) \Big|_0^q = \frac{1}{RC} [t]_0^t$$

$$\text{or } -\log(q_0 - q) + \log q_0 = \frac{t}{RC}$$

$$\text{or } \log\left(\frac{q_0}{q_0 - q}\right) = \frac{t}{RC}$$

$$\text{or } \log\left(\frac{q_0 - q}{q_0}\right) = -\frac{t}{RC}$$

$$\text{or } \frac{q_0 - q}{q_0} = e^{-t/RC}$$

$$\text{or } q_0 - q = q_0 e^{-t/RC}$$

$$\text{or } q = q_0 (1 - e^{-t/RC}) \quad \text{(2)}$$

$$\text{or } q = q_0 (1 - e^{-t/T}) \quad \text{(2)}$$

where $T = RC$ is called R-C time constant or charging time constant of the capacitor.

This eqⁿ(2) is called an eqⁿ of charging of a capacitor.

→ Charging time constant (relaxation time of circuit):

when $t = T$ then

$$q = q_0 (1 - e^{-T/RC})$$

$$q = q_0 (1 - e^{-1})$$

$$q = q_0 (1 - \frac{1}{e})$$

$$q = q_0 (1 - 0.37)$$

$$q = 0.63 q_0$$

Therefore, $q = 0.63 q_0$ when $t = RC - T$

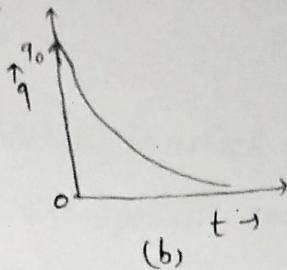
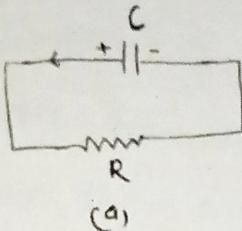
Hence, charging time constant (R-C time constant) of a capacitor is defined as the interval at which capacitor gets charge equal to 63% of maximum charge that it can ~~store~~ store.

If we plot a graph between charge q and time t as shown in figure (b). Initially at $t = 0$ sec $q = 0$, and at $t = T$, $q = q_0$ is maximum charge stored in a capacitor.

(10)

(11)

Discharging of a capacitor:



[Fig: (a) Discharging of a capacitor (b) graph showing the discharging nature of a capacitor]

Consider a capacitor of capacitance C is initially charged up to potential V_0 and charge q_0 . Then, $q_0 = CV_0$

Let us connect the two plates of the capacitor through a resistor R as shown in figure (a). So that capacitor starts discharging. Suppose after time 't' potential across plates becomes V and remaining charge becomes q . If I be the current through resistor R , then we have

$$V_C = VR$$

$$\frac{q}{C} = IR$$

$$\text{or } \frac{q}{C} = -\frac{dq}{dt} R$$

Where - sign indicates that q decreases with increase in time.
or, $\frac{dq}{q} = -\frac{dt}{RC}$ — (1)

Integrating eqn (1) we get

$$\int_{q_0}^q \frac{dq}{q} = \int_0^t -\frac{dt}{RC}$$

$$\text{or } [\log q]_0^q = -\frac{1}{RC} [t]_0^t$$

$$\text{or } \log\left(\frac{q}{q_0}\right) = -\frac{t}{RC}$$

$$\text{or } q = q_0 e^{-t/RC}$$

$$\text{or } q = q_0 e^{-t/T} \quad \text{--- (2)}$$

→ This eqn (2) gives the eqn of discharging of capacitor.

Discharging time constant:

The term $RC = T$ is called discharging time constant. If RC is high, it takes longer time for discharging. If RC is small, the capacitor takes shorter time for discharging.

If $t = T = RC$, then eqn (2) will be

$$q = q_0 e^{-t/T}$$

$$\text{or } q = q_0 e^{-1}$$

$$\text{or } q = 0.37 q_0 \quad \text{--- (3)}$$

Thus, the discharging time constant of a capacitor is defined as the time interval at which the charge on the capacitor is equal to 37% of the initial charge on the capacitor. If we plot a graph between charge q and time t as shown in figure (b), where discharging takes place with time.

Introduction

A dielectric is an insulator in which all the electrons are tightly bounded to the specific atoms or molecules so that there are no free electrons to carry current on the dielectric. A substance whose basic electric property is the ability to be polarized and in which an electrostatic field can exist. A dielectric material is used to prevent the leakage of electric charge in electrical engineering devices. There are two types of dielectric substances (molecules). They are given as following;

① Polar dielectric substance (molecule):

The molecule in which two charge centers (positive and negative) are displaced from each other and has permanent electric dipole moment is called polar dielectric substance or molecules and this process is called polarization. In polar dielectric substance, there is permanent displacement even in the absence of external field. The dipole will tend to align along the direction of applied field. In this case, centre of gravity (C.G.) of positive and negative ion separated by a certain distance. e.g. H_2O , NaCl , NO_2 etc.

Physical optics

Coherent sources of light

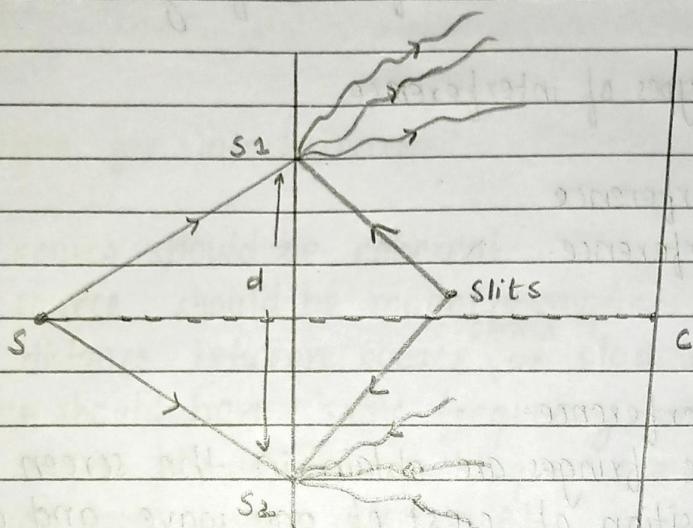


Fig: Coherent sources of light

Two sources of light are derived from single monochromatic source of light which emit continuous lightwaves having same wavelength, frequency and amplitude and are travelling in same medium (in phase or constant phase difference), then superimpose to give interference pattern. These sources of light are called coherent source of light. For e.g.: laser light.

Consider S_1 & S_2 are coherent source of light, derived from single monochromatic source of light emit continuous light wave having same wavelength, frequency and amplitude and travelling in same medium as shown in fig.

The ^{non-}~~non~~uniform distribution of light energy travelling in the medium, due to the superposition of light waves (crest+crest or trough+trough OR crest+trough or trough+crest) which are generated from the coherent sources of light. This phenomenon is called interference of light.

There are two types of interference.

- i) Constructive interference
- ii) Destructive interference.

I) Constructive interference.

The interference fringes are obtain in the screen due to the superposition of crest of one wave and crest of another wave or trough of one wave and trough of another wave. Maximum intensity (bright fringes) are seen, these types of interference is called constructive interference.

Path difference of constructive interference

$$(p.d) = n\lambda \quad (\text{where } n=0, 1, 2, 3, \dots)$$

II) Destructive interference.

The interference fringes are obtain on the screen due to the superposition of crest of one wave and trough of another wave or trough of one wave and crest of another wave. Minimum intensity (dark fringes) are seen, these types of interference is called destructive interference.

Path difference of destructive interference

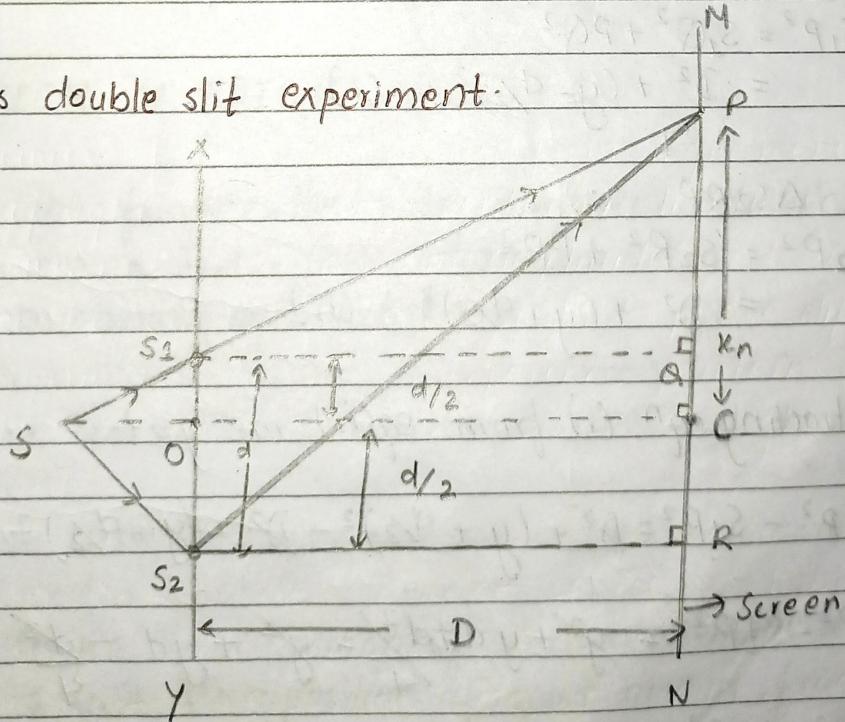
$$(p.d) = (2n+1)k/2 \quad (\text{where } n = 0, 1, 2)$$

$$OR, (2n-1)\alpha/2$$

- ## Condition for interference.

- i) The source should be coherent.
 - ii) The source should be monochromatic.
 - iii) The distance between source ^{should} be close to each other.
 - iv) Source should have same frequency, amplitude and wavelength.
 - v) The source should be travelling in same medium (in phase or constant phase difference)

- Young's double slit experiment



Consider a monochromatic source of light having wavelength λ . S_1 and S_2 are two slits separated by a small distance d . Due to the superposition of two waves both dark and bright fringes are formed on the screen. Let 'D' be the distance of screen from slit. Let 'y' be the position of point 'P' to the point 'C' i.e. center of the screen.

From the fig, we can say

$$PQ = PC - QC$$

$$= y - \frac{d}{2}$$

$$PR = PC - CR$$

$$= y + \frac{d}{2}$$

In $\triangle S_1QP$,

$$S_1P^2 = S_1Q^2 + PQ^2$$

$$= D^2 + (y - \frac{d}{2})^2 \dots (i)$$

In $\triangle S_2RP$

$$S_2P^2 = S_2R^2 + PR^2$$

$$= D^2 + (y + \frac{d}{2})^2 \dots (ii)$$

Subtracting eqn (i) from eqn (ii), we get

$$S_2P^2 - S_1P^2 = D^2 + (y + \frac{d}{2})^2 - D^2 - (y - \frac{d}{2})^2$$

$$\text{or, } S_2P^2 - S_1P^2 = y^2 + yd + \frac{d^2}{4} - y^2 + yd - \frac{d^2}{4}$$

$$\text{or, } S_2P^2 - S_1P^2 = 2yd$$

Hence, the path difference between two wave is

$$\alpha = S_2 P - S_1 P$$

$$\therefore S_2 P - S_1 P = \frac{2yd}{S_2 P + S_1 P}$$

Since, the fringes formed on the screen are closed with each other, we can write

$$S_1 P \approx S_2 P \approx D$$

Then,

$$\alpha = \frac{2yd}{D+D}$$

$$= \frac{yd}{D} \dots \text{(iii)}$$

POSITION OF BRIGHT FRINGES

For bright fringes, the path difference should be equal to nd , i.e. $\alpha = nd \dots \text{(iv)}$ (where $n = 0, 1, 2, \dots$)

Now, combining eqn (iii) & (iv)

$$\frac{yd}{D} = nd$$

$$\text{or, } y_n = \frac{ndD}{d}$$

For $n = 0$, $y_0 = 0$ ∴ position of central bright fringe.

For $n = 1$, $y_1 = \frac{D}{d}$ ∴ position of first bright fringe.

For $n = 2$, $y_2 = \frac{2D}{d}$ ∴ position of second bright fringe.

In general, for n^{th} bright fringe,

$$y_n = \frac{n \lambda D}{d}$$

$$\begin{aligned} \text{Fringe width (B)} &= y_2 - y_1 \\ &= \frac{2\lambda D}{d} - \frac{\lambda D}{d} \\ &= \frac{\lambda D}{d} \end{aligned}$$

POSITION OF DARK FRINGES

For dark fringes, the path difference should be equal to $(2n-1)\lambda/2$ e.g. $n = (2n-1)\lambda/2 \dots (v)$

where $n = 1, 2, 3, \dots$

Combining (iii) and (v), we get

$$\frac{yd}{D} = (2n-1)\lambda/2$$

$$\text{or, } y_n = (2n-1) \frac{\lambda D}{2d}$$

For $n = 1, y_1 = \frac{\lambda D}{2d} \therefore$ position of 1st dark fringe.

For $n = 2, y_2 = \frac{3\lambda D}{2d} \therefore$ position of 2nd dark fringe.

For $n = 3, y_3 = \frac{5\lambda D}{2d} \therefore$ position of 3rd dark fringe.

In general, for n^{th} dark fringes

$$y_n = \frac{(2n-1) \lambda D}{2d}$$

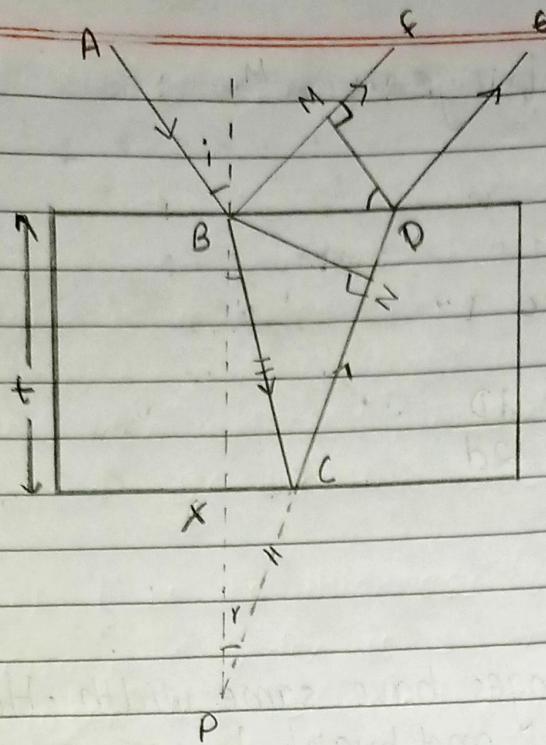
$$\text{Fringe width (B)} = y_2 - y_1$$

$$= \frac{3\lambda D}{2d} - \frac{\lambda D}{2d}$$

$$= \frac{\lambda D}{d}$$

All bright and dark fringes have same width. Hence, we can conclude both dark and bright fringes are formed in Young's Double Slit Experiment are of same width.

(i)



Interference by reflected light

$$p \cdot d = u (BC + CD) - BM$$

$$p \cdot d = u (BC + CD) - u ND$$

$$p \cdot d = u (BC + CD - ND)$$

$$p \cdot d = u (BC + CN)$$

$$p \cdot d = u (CP + CN)$$

$$p \cdot d = u PN \dots (1)$$

$$\text{In } \triangle BPN, \cos \gamma = \frac{PN}{BP}$$

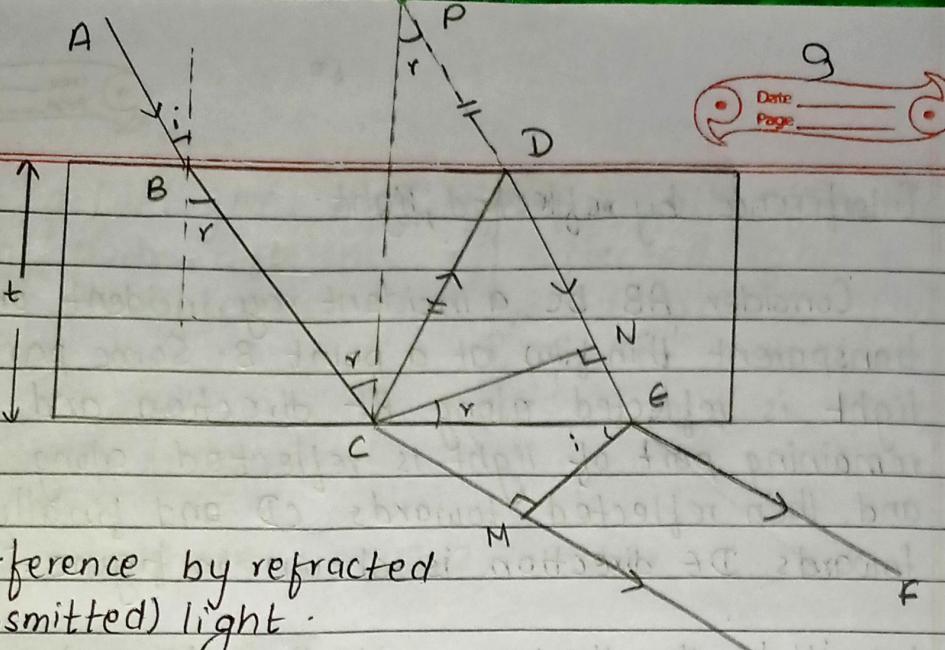
$$\therefore PN = BP \cos \gamma$$

$$\therefore PN = 2t \cos \gamma \text{ since,}$$

$$BP = BX + XP$$

$$= t + t \dots (2)$$

$$p \cdot d = 2ut \cos \gamma \dots (3)$$



Interference by refracted
(transmitted) light.

$$\text{path difference } P \cdot d = n (CD + DE) - CM$$

$$= n (CD + DE) - n NG$$

$$= n (CD + DE - NE)$$

$$= n (CD + DN)$$

$$= n (PD + DN)$$

$$\text{since } PD = CD$$

$$P \cdot d = n PN \dots (1)$$

$$\text{In } \triangle PCN, \cos r = \frac{PN}{PC}$$

$$PN = PC \cos r = \frac{PN}{PC}$$

$$PN = PC \cos r$$

$$PN = 2t \cos r \text{ (since } PC = t + t) \dots (2)$$

$$\therefore P \cdot d = 2nt \cos r \dots (3)$$

For bright fringes,

$$P \cdot d = n \lambda \text{ where } n = 0, 1, 2, \dots (4)$$

Equating eqn (3) and (4), we get

$$2nt \cos r = n \lambda \dots (5)$$

For dark fringes

$$P \cdot d = (2n+1) \lambda / 2 \text{ where } n = 0, 1, 2, \dots (6)$$

Equating eqn (3) and (6)

$$2nt \cos r = (2n+1) \frac{\lambda}{2} \dots (7)$$

(i) Interference by reflected light.

Consider AB be a incident ray, incident on a transparent thin film at a point B. Some part of light is reflected along BF direction and remaining part of light is reflected along BC and then reflected towards CD and finally refracted towards DE direction is shown in figure.

Let 't' be the thickness of thin firm having refractive index μ so that path difference between two reflected light (i.e DE and BF) is given by.

$$p.d = \mu (BC + CD) - BM$$

$$p.d = \mu (BC + CD) - \mu ND$$

$$p.d = \mu (BC + CD - ND)$$

$$p.d = \mu (BC + CN)$$

$$p.d = \mu (CP + CN)$$

$$p.d = \mu PN \dots (i)$$

In right angle $\triangle BPN$, $\cos r = \frac{PN}{BP} \Rightarrow PN = BP \cos r$

$[PN = 2t \cos r] \sin \dots (ii)$ since $BP = BX + XP = 2t$

Using eqn (i) in (ii), we get,

$$p.d = 2\mu t \cos r \dots (iii)$$

The eqn (iii) gives only apparent path difference. So, total path difference of reflected light is given by, path difference ($p.d$) = $2ut\cos r - \frac{\lambda}{2}$... (iv)

For bright fringe,

$$p.d = n\lambda, \text{ where } n=0, 1, 2, \dots \quad (\text{v})$$

Equating eqn (iv) and (v), we get

$$2ut\cos r - \frac{\lambda}{2} = n\lambda$$

$$\text{or, } 2ut\cos r = n\lambda + \frac{\lambda}{2}$$

$$\text{or, } 2ut\cos r = \frac{2n\lambda + \lambda}{2}$$

$$2ut\cos r = (2n+1)\frac{\lambda}{2} \dots (\text{vi})$$

For dark fringe,

$$p.d = (2n+1)\frac{\lambda}{2}, \text{ where } n=0, 1, 2, \dots \quad (\text{vii})$$

Equating equation (iv) and (vii), we get

$$2ut\cos r - \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$\text{or, } 2ut\cos r = (2n+1)\frac{\lambda}{2} + \frac{\lambda}{2}$$

$\therefore 2ut\cos r = n\lambda$,
where $n+1 \approx n$

$$\text{or, } 2ut\cos r = \frac{2n\lambda}{2} + \frac{\lambda}{2} + \frac{\lambda}{2}$$

(integral)

M

Newton's ring experiment

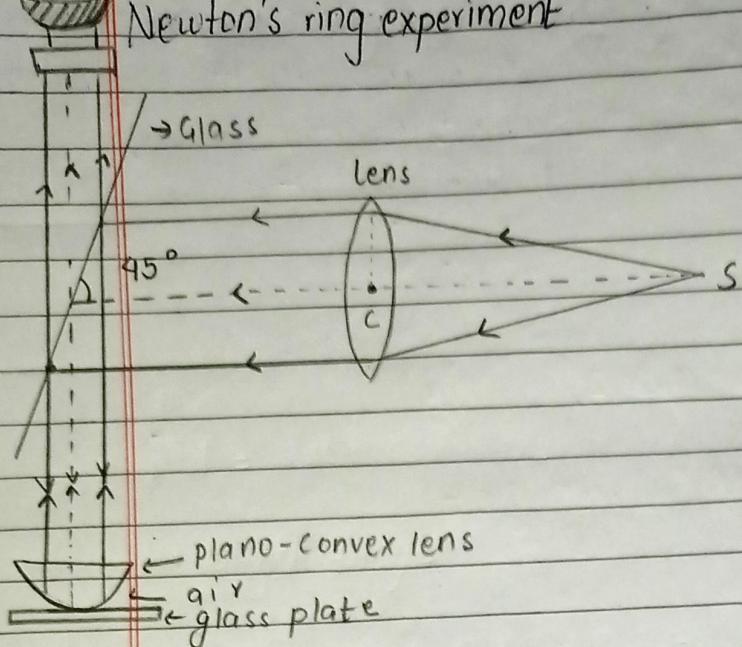
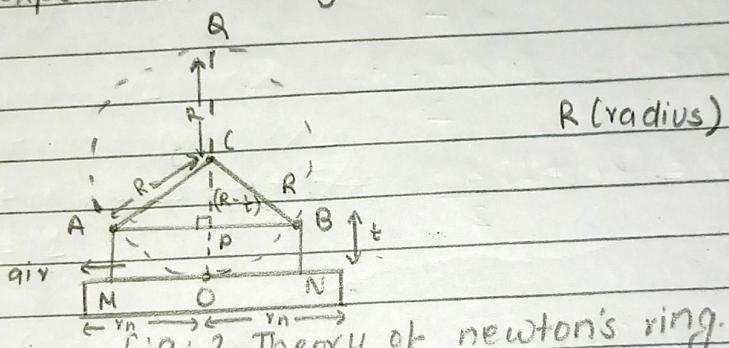


fig: 1: experiment arrangement of newton's ring



Theory :

$$\Delta CAP, CA^2 = AP^2 + CP^2$$

$$R^2 = r_n^2 + (R-t)^2$$

$$R^2 = r_n^2 + R^2 - 2Rt + t^2$$

$$R^2 - R^2 + 2Rt - t^2 = r_n^2 \quad t^2 = 0$$

$$\therefore r_n^2 = 2Rt$$

$$\therefore \frac{2t}{R} = r_n^2 \dots (1)$$

Define:

The circular interference fringes are formed in between plano convex lens and glass plate, which enclose the air film of varying thickness. These fringes were observed ^{first} by Newton is called Newton's ring experiment.

Experiment arrangement:

Theory:

Consider air film are enclosed in between plano convex lens and glass plate as shown in fig. 2, where circular fringes are obtained.

Let, t_n , r_n and R be the thickness of air film, radius of n^{th} circular rings and radius of curvature of glass (plano convex lens) respectively.

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a) Newton's ring by reflected rays

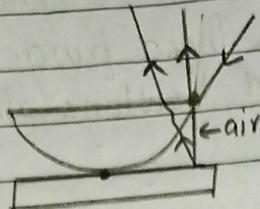


fig: 3: reflected ray

$$p \cdot d = 2ut \cos r - \lambda/2 \dots (ii)$$

For bright fringe:

$$pd = n\lambda, \text{ where } n = 0, 1, 2, 3 \dots (iii)$$

Equating eqn (ii) and (iii), we get

$$2ut \cos r - \lambda/2 = n\lambda$$

$$\text{or, } 2ut \cos r = n\lambda + \lambda/2$$

$$\text{or, } 2ut \cos r = (2n+1)\lambda/2$$

$$\text{If } u = 1$$

$$r = 0$$

$$2t = (2n+1)\lambda/2 \dots (iv)$$

Equating eqn (iv) and (i), we get,

$$\frac{r_n^2}{R} = (2n+1)\lambda/2$$

$$\therefore r_n = \sqrt{\frac{(2n+1)\lambda R}{2}} \dots (v)$$

equation (v) gives the radius of n^{th} circular ring.

$$\text{If } n=0, r_0 = \sqrt{\frac{\lambda R}{2}} \neq 0$$

$$n_1 = 1, r_1 = \sqrt{\frac{3\lambda R}{2}}$$

continue.



b) Newton's ring by transmitted rays.

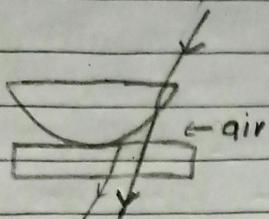


fig : 4: transmitted ray

$$p \cdot d = 2ut \cos r \dots (ii)$$

For bright fringe.

$$p \cdot d = n\lambda \dots (iii) \text{ where, } n = 0, 1, 2, 3$$

Equating eqn (ii) and (iii), we get

$$2ut \cos r = n\lambda$$

$$\text{If } u = 1$$

$$r = 0$$

$$\therefore 2t = n\lambda \dots (iv)$$

Equating eqn (iv) and (i), we get

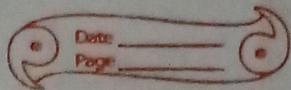
$$\frac{r_n^2}{R} = n\lambda$$

$$\text{or, } r_n = \sqrt{nR\lambda} \dots (v)$$

$$\text{If } n = 0, r_0 = 0$$

$$\text{If } n = 1, r_1 = \sqrt{\lambda R}$$

Central fringe of transmitted light is dark.



#. Continue

(a) Newton's ring formed by reflected rays.

for dark fringes:

$$p \cdot d = (2n+1) \frac{\lambda}{2} \dots (vi)$$

From eqn (ii) and (vi)

$$2ut \cos r - \frac{\lambda}{2} = (2n+1) \frac{\lambda}{2}$$

$$\text{or, } 2ut \cos r = 2n \frac{\lambda}{2} + \frac{\lambda}{2} + \frac{\lambda}{2}$$

$$\text{or, } 2ut \cos r = \lambda(n+1)$$

$$\therefore 2ut \cos r = n\lambda$$

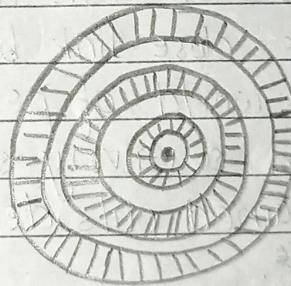
$$\text{if } u=1$$

$$r_0 = 0$$

$$\therefore 2t = n\lambda \dots (vii)$$

Equating eqn (vii) and (i), we get

$$\frac{r_n^2}{R} = n\lambda$$



$$r_n = \sqrt{n\lambda R} \dots (viii)$$

$$\text{If } n=0, r_0=0$$

$$\text{If } n=1, r_1 = \sqrt{\lambda R}$$

Central fringe of newton's ring is dark.

From above discussion, we conclude that centre fringe of newton's ring will be dark of reflected light.

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- Unit: 1 Mechanical oscillation - 6 hrs
Unit: 2 Wave motion - 4 hrs
Unit: 3 Acoustics - 4 hrs
Unit: 4 Photonic - 6 hrs
Unit: 5 Capacitor and Dielectric - 6 + 7 hrs
Unit: 6 Electromagnetism - 6 + 9 hrs
Unit: 7 Quantum Mechanics - 5 hrs
Unit: 8 Fundamental of thermodynamics and heat transfer - 8 hrs

Quantum Mechanics

Inadequacy of classical mechanics.

Classical physics deal with macroscopic phenomena. Most of the effect with which classical theory is concerned are directly observable or can be made observable with simple instruments. These are close link between classical physics and the world of perception. The phenomena which occur on a very small scale (atomic), can not be explained outside the frame of quantum mechanics.

Quantum mechanics, plays the remarkable unification of the fundamental physics by treating particles and radiation on the same footing. Some inadequacies of classic mechanics are:

- 1) Stability: The stability of atom atoms or molecules.
- 2) The enormous range of electronical conductivity of solid material.

- 3) The observed spectrum of black body radiation.
- 4) Specific heat of solid at low temperature.
- 5) The origin of discrete spectra of atoms.
- 6) The special phenomena like photoelectric effect, Compton effect, Raman effect, change of particle at high temperature, emission of α , β and γ , etc.

A. Stability.

It couldn't explain the ~~ext~~ stability.

According to classic mechanic when an atom or molecule move around nucleus, it loses energy in the form electromagnetic wave and its velocity decreases continuously. The ultimate result is that electrons come closer and closer to the nucleus until it collapse as shown in fig:

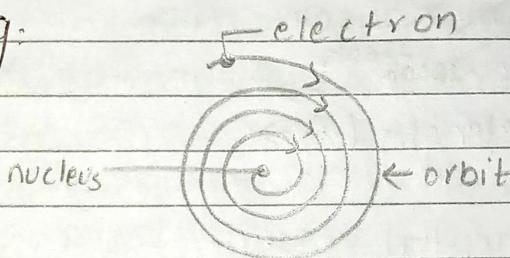


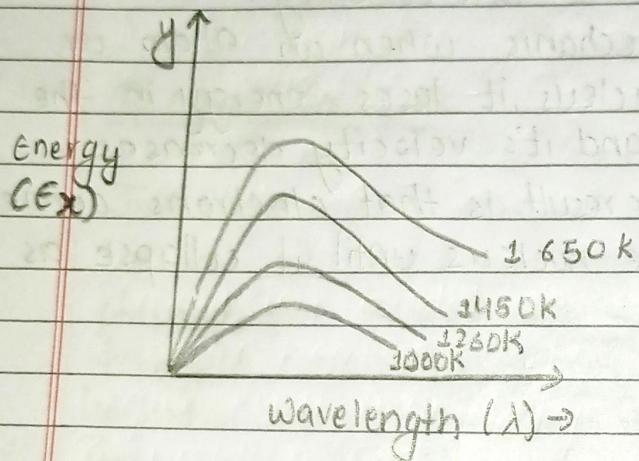
fig: collapse of electron

This shows the instability of atom, it is in contradiction to the observe fact of the ~~so~~ stability of atom. Thus, classical mechanics fail to explain the ~~stability~~ of atom.

of solid materials

- B. It couldn't explain enormous range of electrical conductivity
 The electrical conductivity has different values for different substances. These variation of electrical conductivity can't be explained classically and we need quantum mechanical approach.

- C. It couldn't explain the observed spectrum of black body radiation.



Experimental result for a black body at different temp.

Any one law of black body radiation account for the entire shape of black body radiation curve. Hence, anomalous situation has to be accepted that the fundamental assumption of classical theory i.e. energy can be absorbed or emitted by the oscillator continuously was at fault. Rayleigh and Wien's law explain the curve in black body radiation spectrum for longer and shorter wavelength respectively, but Planck's was

able to explain black body radiation spectrum assuming that matter can emit or absorb radiation only in discrete packets is called quanta. Each of energy ($E = h\nu$). It was found that distribution of energy in the spectrum of black body as found by Planck's radiation law, was in perfect agreement with the experiment for all wavelength. The experimental result for the perfect black body at different temp are shown in fig.

Inadequacy of classical mechanics:

- d) The specific heat capacity of solid was first studied by Dulong's and Petits on the basis of classical consideration and found that it is constant for all range of temperature in all the substances. They assume that C_v is equal to $3R$ i.e $C_v = 3R$ where, $R = Nk = 8.3 \times 10^3$ $J/kg\cdot K$. Hence, according to them, the specific heat capacity is constant for all ranges of temperature. But classical formula breaks down at low temperature. The specific heat of all solids drops sharply at low temperature and approaches to zero at temperature approaching $0K$. This discrepancy was removed by applying the Quantum mechanics.

de-Broglie's wave length of electron:

According to Quantum theory, heat is radiated from a body in the form of discrete particles called photons. The energy of photon is equal to $h\nu$

$$E = h\nu$$

So, Einstein assume some assumption and then found an expression.

$C_V = 3R$ for high temperature which is like classical theory but at low temperature

$$C_V = \frac{3Nk}{T} \left(\frac{\Theta E}{e^{\Theta E/kT}} - 1 \right)$$

where $\Theta E = \text{Einstein temperature} = \frac{h\nu}{k_B}$

So, Einstein model described the variation of specific heat capacity at low temperature region. Furthermore, Debye's also assume some assumption and then found specific heat capacity is same result like classical consideration at high temperature region. But at low temperature region $C_V \propto T^3$

This is very closed with experiment result. Hence, classical mechanics couldn't explain the observe variation of specific heat capacity of solid at low temperature. The variation of specific heat capacity with temperature as shown in figure.

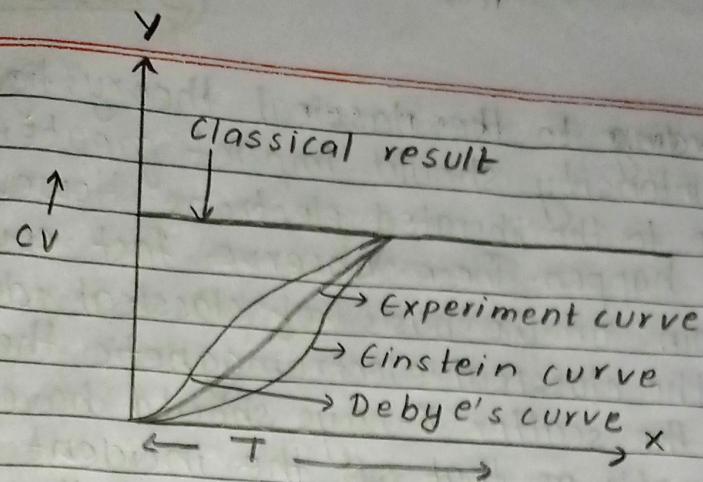


Fig: variation of specific heat capacity with temperature.

- 5) It couldn't explain the observed spectrum of the simplest atom like hydrogen atom (origin of discrete spectrum of atom).

According to the classical theory, the excited atoms of hydrogen emit electromagnetic radiation for all wave length continuously while it observe that they emit the radiation of certain wave length only. Hence, classical theory couldn't explain the observe spectrum of the simplest atom like hydrogen.

- 6) Classical mechanics couldn't explain the phenomena of photoelectric effect, compton effect.

Experimentally, we observe that in the photoelectric effect, the electron ejection take short time but classically, the photoelectric ejection should take long time.

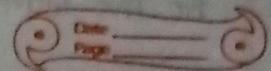


- ▶ Similarly, according to the classical theory, light of greater intensity should impact greater kinetic energy to the liberated electrons! However, this does not happen. These observe fact could not be explain on the basis of classical idea. According to the classical electromagnetic theory of radiation, the scatter X-rays should have the same wavelength as that of the incident X-ray on the target. But it is observed that there is shift in wavelength of X-ray incident. This effect couldn't be explain by classical theory. Hence, classical theory couldn't explain like photoelectric effect, compton effect, etc.

- De-Broglie's theory:

Dual nature of radiation:

The phenomena of interference, diffraction and polarization of light can only be explain on the basis of wave theory of light. This phenomena show that light possess wave nature. However, certain phenomena like photoelectric effect, compton effect and discrete emition and absorption of radiation can only be explain on the basis of Quantum theory. According to this theory light propagates in a small pocket or quanta and behave like particle. This shows that light possess particle nature. Thus, we say that light possess dual (wave and particle nature).



De-Broglie's theory:

The waves associated with a moving particle are called De-Broglie's wave or matter wave and the wavelength associated with matter wave is called de-Broglie's wavelength. De-Broglie's proposed the wavelength of material particle could be related to its momentum in the same way as for photon i.e. $\lambda = h/p$

(lambda)

This is for a particle of mass 'm' is moving with velocity (speed) v . The de-Broglie's wavelength is $\lambda = \frac{h}{p} = \frac{h}{mv}$ where $h = \text{Planck's constant}$ and $p = mv$ is the momentum of the particle.

* Derivation of de-Broglie's wavelength (λ)

According to Planck's theory of radiation,
 $E = hv \dots (i)$

According to Einstein theory,

$$E = mc^2 \dots (ii)$$

The energy of photon in two cases are same.

Therefore, equating eqn (i) and (ii), we get,

$$mc^2 = hv$$

$$\text{or, } mc^2 = h \frac{v}{\lambda}, \text{ where } v = \frac{\lambda}{\text{lambda}}$$

$$\text{or, } mc^2 = \frac{hc}{\lambda} \text{ where light has dual nature.}$$

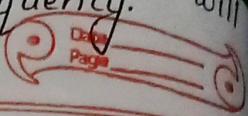
$$\frac{mc}{\lambda} = \frac{h}{mv} \quad \therefore d = h \quad \text{since } c = v$$

$$\text{or, } \lambda = \frac{h}{mc} \quad \therefore \lambda = \frac{h}{p} = \frac{h}{mv} \dots (iii)$$

This eqn (iii) is called de-Broglie's relation and is called de-Broglie's wavelength. #

It concludes greater be the momentum, shorter will be the wavelength or higher will be frequency.

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* de Broglie's wavelength of electron

Consider an electron 'e' of mass 'm' is moving with velocity 'v' by the potential 'V' volt, work done on the electron must be equal to the gain of kinetic energy of the electron.

Therefore,

$$\text{work done of an electron } E_k = eV \dots (i)$$

$$\text{K.E. of an electron} = \frac{1}{2} mv^2 \dots (ii)$$

Equating eqn (i) and (ii), we get,

$$eV = \frac{1}{2} mv^2$$

$$\text{or, } 2eV = \frac{m^2 v^2}{m}$$

$$\text{or, } 2eVm = m^2 v^2 = p^2$$

$$\therefore p = \sqrt{2eVm} \dots (iii)$$

We know that, de Broglie's wavelength of an electron

$$\lambda = \frac{h}{p} \dots (iv)$$

using equation (iii) and (iv), we get

$$\lambda = \frac{h}{\sqrt{2meV}} \dots (v)$$

The equation (v) gives the de-Broglie's wavelength of an electron in term of eV .



* de Broglie's wavelength in term of K.E:

⇒

Let E_K be the kinetic energy of moving particles,
then,

$$E_K = \frac{1}{2}mv^2$$

$$\text{or, } 2E_K = \frac{m^2v^2}{m}$$

$$\text{or, } m^2v^2 = 2mE_K$$

$$\therefore P = \sqrt{2mE_K} \dots \text{(i)}$$

We have, $\lambda = \frac{h}{P} \dots \text{(ii)}$

So, equating (i) and (ii), we get

$$\lambda = \frac{h}{\sqrt{2mE_K}}$$

c) De Broglie's wavelength in term of thermal equilibrium

If the particle is in thermal equilibrium at temperature T . Then, from kinetic theory of gases, average K.E of particle is given by

$$E_K = \frac{3}{2} k_B T \dots \text{(i)}$$

Where k_B = Boltzmann's constant

$$= 1.38 \times 10^{-23} \text{ J K}^{-1}$$

We know that,

$$\lambda = \frac{h}{\sqrt{2mE_K}} \quad \dots \text{(ii)}$$

Using eqn (i) and (ii), we get

$$\lambda = \frac{h}{\sqrt{\frac{2m^3 k_B T}{2}}} \quad \dots$$

$$\therefore \lambda = \frac{h}{\sqrt{3mk_B T}} \quad \dots \text{(iii)}$$

- Q.1) The most rapidly moving valence electron in metallic sodium at the absolute zero temp. has a K.E 3eV. Show its de-Broglie's wavelength is 7 Å.
(given $h = 6.63 \times 10^{-34} \text{ Js}$, $m = 9.1 \times 10^{-31} \text{ kg}$)

Given,

$$K.E = 3 \text{ eV}$$

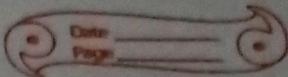
$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$m = 9.1 \times 10^{-31} \text{ kg}$$

$$\lambda = \frac{h}{\sqrt{2mE_K}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 3 \times 1.6 \times 10^{-19}}}$$

$$= 2.837 \times 10^{-10} \times 7.093 \times 10^{-12} \approx 7$$



Q.2) Find the energy of the neutron in units of eV whose de-Broglie's wavelength 1 \AA .

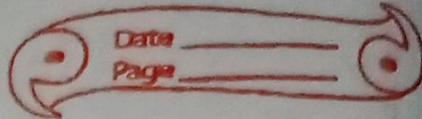
(given mass of neutron = $1.67 \times 10^{-27} \text{ kg}$, $h = 6.63 \times 10^{-34} \text{ Js}$)

Given,

$$\text{de-Broglie's wavelength } (\lambda) = 1\text{ \AA}$$

$$\text{mass of neutron} = 1.67 \times 10^{-27} \text{ kg}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$



Q.3) Calculate the energy of proton having de-Broglie's wavelength 0.5 fm . given, $m_p = 1.6 \times 10^{-27} \text{ kg}$
 $1 \text{ fm} = 10^{-15} \text{ m}$

Given,



Imp Wave function(4)

The probability that a particle will be found at a given place in a space at given instant of time is characterized by wave function

$$\psi(x, y, z, t) \text{ or } \psi(r, t)$$

It is called wave function. The function is either real or complex.

The only quantity having a physical meaning is the square of its magnitude. Therefore,

$$P |\psi|^2 = \psi^* \psi$$

where ψ^* is the complex conjugate of ψ and the quantity P is the probability density.

The probability of finding a particle in a volume $dx \cdot dy \cdot dz$ or d^3r or dv is $|\psi|^2 dx \cdot dy \cdot dz = |\psi|^2 d^3r$ or $|\psi|^2 dv$ since $\psi = \psi(r, t)$

$$\text{Now, } \psi(r, t) \psi^*(r, t) = |\psi(r, t)|^2$$

Maxwell gives the physical significance of ψ and according to him $\psi^* \psi$ gives the probability of finding particle at a point.

$$\iiint \psi^* \psi dx dy dz = 1$$

OR,

$$\int_{-\infty}^{\infty} \psi^* \psi dx dy dz = 1$$

The triple integral are extending over all possible value of x, y and z . A wave function Ψ satisfying this relation is called normal wave function.

Every acceptable wave function can be normalized by multiplying it with an appropriate constant is called normalizing constant.

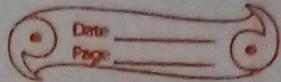
A wave function can be normalize by using relation

$$\int_{-\infty}^{\infty} \Psi^* \Psi dV = 1$$

Thus, it is called normalizing condition and Ψ is called normalize wave function. Thus, the wave function Ψ must have following properties:

- a) Ψ must be continuous.
- b) Ψ must be finite everywhere.
- c) Ψ must be single valued and
- d) The probability of finding the particle over all the space from minus infinity to plus infinity must be equal to 1. Therefore, the probability of finding a particle at a point in space is $|\Psi|^2$ also,

$$\int_{-\infty}^{\infty} |\Psi|^2 dx dy dz = 1$$



* Normalized wave function (Ψ)

The quantum mechanical particle represented by a wave packet described by the wave function $\Psi(r,t)$ which helps to measure the probability of finding the particle at a particular position w.r.t the origin of its region. The probability density is given by relation:

$$P = \Psi^*(r,t) \Psi(r,t) = |\Psi(r,t)|^2$$

If the particle is bounded by force to a limited region, the probability of finding the particle somewhere in the region at given time 't' must be:

$$\int |\Psi(r,t)|^2 dx dy dz = 1$$

Any wave function satisfying this equation is said to be normalized. If $\Psi(r,t)$ is not a normalized wave function, it will be normalized by multiplying any constant value 'A' then $A\Psi$ be a normalized wavefunction.

$$\int (A\Psi)^* (A\Psi) dx dy dz = 1$$

$$AA^* \int \Psi^* \Psi dx dy dz = 1$$

$$|A|^2 \int \Psi^* \Psi dx dy dz = 1$$

$$\therefore |A|^2 = \frac{1}{\int \Psi^* \Psi dx dy dz}$$

$$\left[|A| = \frac{1}{\sqrt{\int \Psi^* \Psi dx dy dz}} \right]$$

This gives the normalizing factor.

Hence, multiplying the wavefunction by constant 'A', a new ^{normalize}_n wavefunction $A\Psi$ is obtained:

Importance of normalize wavefunction.

- a) The solution of schrodinger wave equation (SWE) is a wavefunction. The amplitude of the wave function is arbitrary.

After normalization, the amplitude of wave function has a fix constant value, thus normalization is important to give the solution of schrodinger wave equation a constant fix amplitude.

The probability of finding the particle in space at a instant of time is only given by normalize wavefunction but non-normalizable wavefunction must be rejected the solution of schrodinger wave equation.

Therefore, any quantum phenomenon are not describe by non-normalize wave function.

[Q.1][II] Normalize the wave function!

$$\Psi(x) = A(\sin x + i \cos x) \text{ in a range } -l \leq x \leq l$$

$$\left[|A| = \frac{1}{\sqrt{2l}} \right] \text{Answer}$$



Schrodinger wave equation.

Schrodinger wave equation is a wave equation in the variable ψ and is the fundamental equation of quantum mechanics. In the sense that second law of motion is the fundamental equation of Newton's mechanics.

Schrodinger wave equations are of two types:

- Time dependent schrodinger wave equation.
- Time independent schrodinger wave equation

[I] Time dependent schrodinger wave equation.

Suppose wave function ψ of having periodic displacement $\psi(x, y, z, t)$ at any instant of time 't' moving with velocity 'u' is the wave velocity of the particles, then

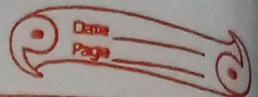
$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = \frac{1}{u^2} \frac{\partial^2 \psi}{\partial t^2} \quad \dots (i)$$

$$\text{or, } \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \psi = \frac{1}{u^2} \frac{\partial^2 \psi}{\partial t^2}$$

$$\text{or, } \nabla^2 \psi = \frac{1}{u^2} \frac{\partial^2 \psi}{\partial t^2} \quad \dots (ii), \text{ where}$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \quad \dots (iii) \text{ is called Laplacian operator or Nebla.}$$

The eqn(ii) is called wave equation of particle.



The solution of equation (ii) in the form of
 $\Psi(x, y, z, t) = \Psi_0(x, y, z) e^{-i\omega t} \dots (\text{iii})$

Since, $\Psi_0(x, y, z)$ is called amplitude of particle which is independent of time.

Let \vec{r} be the position vector which is expressed as $\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$

so, eqn (iii) becomes,

$$\Psi(r, t) = \Psi_0(r) e^{-i\omega t} \dots (\text{iv})$$

Differentiating eqn (iv) w.r.t 't' we get,

$$\frac{\partial \Psi(r, t)}{\partial t} = \Psi_0(r) e^{-i\omega t} (-i\omega)$$

$$\frac{\partial^2 \Psi(r, t)}{\partial t^2} = \Psi_0(r) e^{-i\omega t} (-i\omega) \cdot (i\omega)$$

$$\text{or, } \frac{\partial^2 \Psi(r, t)}{\partial t^2} = -\omega^2 \Psi_0(r) e^{-i\omega t}$$

$$\text{or, } \frac{\partial^2 \Psi(r, t)}{\partial t^2} = -\omega^2 \Psi(r, t) \dots (\text{iv})$$

From eqn (iv) in eqn (ii) we get,

$$\nabla^2 \Psi(r, t) = -\frac{\omega^2}{U^2} \Psi(r, t)$$

$$\text{or, } \nabla^2 \Psi + \frac{\omega^2}{U^2} \Psi = 0 \dots (\text{vi})$$

Since, $\Psi(r,t) = \Psi(shy)$ say,

We have,

$$\omega = 2\pi f$$

$$= 2\pi \frac{u}{\lambda}$$

$$\frac{\omega}{u} = \frac{2\pi}{\lambda}$$

$$\frac{\omega^2}{u^2} = \frac{4\pi^2}{\lambda^2}$$

so, eqn (vi) becomes,

$$\nabla^2 \Psi + \frac{4\pi^2}{\lambda^2} \Psi = 0 \dots (vii)$$

We have from de-Broglie's relation

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

(Lemma)

$$\text{or, } \lambda^2 = \frac{h^2}{m^2 v^2}$$

So eqn (vii) becomes,

$$\nabla^2 \Psi + \frac{4\pi^2 m^2 v^2}{h^2} \Psi = 0$$

$$\nabla^2 \Psi + \frac{mv^2}{\left(\frac{h}{2\pi}\right)^2} \Psi = 0$$

$$\nabla^2 \psi + \frac{1}{2} mv^2 \cdot \frac{2m}{\hbar^2} \psi = 0 \quad \dots \text{(viii)}$$

where, $\frac{\hbar}{2\pi} = \hbar$

We know that, total energy is equal to .

$$K.E + P.E$$

$$E = \frac{1}{2} mv^2 + V \quad (\text{where, } P.E = V)$$

$$\frac{1}{2} mv^2 = E - V$$

So, eqn(viii) becomes,

$$\nabla^2 \psi + (E - V) \frac{2m}{\hbar^2} \psi = 0$$

$$\therefore \nabla^2 \psi + \frac{2m}{\hbar^2} (E - V) \psi = 0 \dots \text{(ix)}$$

To find time dependent schrodinger wave equation by eliminating ' E ' from eqn(ix) from this, we have eqn (iii)

$$\Psi(r,t) = \psi_0(r) e^{-i\omega t}$$

Differentiate w.r.t 't' we get

$$\frac{\partial \Psi(r,t)}{\partial t} = \psi_0(r) e^{-i\omega t} (-i\omega)$$

$$\text{or, } \frac{\partial \Psi}{\partial t} = -i\omega \Psi$$

$$= -i2\pi f \Psi$$

Energy of photon $E = hF$
 $\Rightarrow F = \frac{E}{h}$

$$\frac{\partial \Psi}{\partial t} = -i2\pi \frac{E}{h} \Psi$$

$\therefore E\Psi = i\hbar \frac{\partial \Psi}{\partial t}$ where, $E = i\hbar \frac{\partial}{\partial t}$ is called energy operator.

Q) What is energy operator?

Using the value of 'E' in the eqn (ix), we get

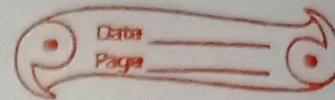
$$\nabla^2 \Psi + \frac{2m}{\hbar^2} \left(i\hbar \frac{\partial}{\partial t} - v \right) \Psi = 0$$

$$\nabla^2 \Psi + \frac{2m}{\hbar^2} i\hbar \frac{\partial \Psi}{\partial t} - \frac{2mv}{\hbar^2} \Psi = 0$$

$$\text{or, } \frac{2m}{\hbar^2} i\hbar \frac{\partial \Psi}{\partial t} = -\nabla^2 \Psi + \frac{2mv}{\hbar^2} \Psi$$

$$\text{or, } i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Psi + \frac{2mv\hbar^2}{\hbar^2 2m} \Psi$$

$$\text{or, } i\hbar \frac{\partial \Psi}{\partial t} = \left[-\frac{\hbar^2}{2m} \nabla^2 + v \right] \Psi$$



where, $H = \left(\frac{-\hbar^2}{2m} \nabla^2 + v \right)$ is called hamiltonian operator of a system.

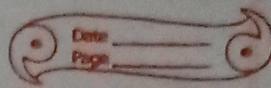
$$\text{so, } i\hbar \frac{\partial \Psi}{\partial t} = H\Psi$$

$$\therefore E\Psi = H\Psi \text{ where } E = i\hbar \frac{\partial}{\partial t}$$

$$E\Psi(x, t) = H\Psi(x, t) \dots (x)$$

This is called the time dependent schrodinger wave equation. In term of operator,

$$\overset{\text{1}}{T}\Psi = \overset{\text{1}}{H}\Psi \dots (xi)$$



Time independent schrodinger wave equation

We have time dependent schrodinger wave equation i.e. $E \Psi(r, t) = H \Psi(r, t)$

OR,
 $i\hbar \frac{\partial \Psi(r, t)}{\partial t} = H \Psi(r, t)$

} ... (i)

where $\Psi(r, t)$ is the function of space and time.
 If the hamiltonian (H) is not an explicitly function of time, the system is conservative system. So $H = H(r)$
 The solution of equation (i) can be written as:

$$\Psi(r, t) = \Psi(r) f(t) \dots (ii)$$

Using eqn (ii) in eqn (i)

$$-i\hbar \frac{\partial}{\partial t} \Psi(r) f(t) = H \Psi(r) f(t)$$

$$\text{or, } i\hbar \frac{\partial f(t)}{\partial t} = H \Psi(r) \dots (iii)$$

eqn (iii)

These can be solved by the method of separation of variable.

let E_n be the separation variable which is independent to the both position and time (that is constant). So, eqn (iii) becomes

$$\frac{i\hbar \partial f(t)}{f(t)} = \frac{H \Psi(r)}{\Psi(r)} = E_n = \text{constant}$$

then either

$$\left. \begin{aligned} i\hbar \frac{\partial f(t)}{\partial t} &= E_n \\ f(t) &\quad \end{aligned} \right\} \dots (iv)$$

$$\text{OR, } \frac{H \Psi(r)}{\Psi(r)} = E_n$$

From eqⁿ (iv) we get $H\psi(r) = E_n \psi(r)$

$$\text{or, } \left(-\frac{\hbar^2 \nabla^2}{2m} + V \right) \psi(r) = i\hbar \frac{\partial}{\partial t} \psi(r) \quad \} \dots (v)$$

$$\text{or, } \nabla^2 \psi(r) + \frac{2m}{\hbar^2} (E - V) \psi(r) = 0$$

This eqⁿ(v) is called time independent schrodinger wave equation, and is also called eigen value equation.

The set of Eigen value of Hamiltonian operator is known as energy spectrum. This energy spectrum is discrete or continuous or both.

► Energy and momentum operators: To find the solution of time dependent schrodinger wave equation can be written $\psi(r, t) = \psi_0(r) e^{-i\omega t} \dots (i)$

Let Diff. eqⁿ(i) w.r.t t

$$\frac{\partial \psi(r, t)}{\partial t} = \psi_0(r) e^{-i\omega t} (-i\omega)$$

$$\text{or, } \frac{\partial \psi(r, t)}{\partial t} = (-i\omega) \psi(r, t)$$

$$\therefore \frac{\partial \psi(r, t)}{\partial t} = -i2\pi F \psi(r, t) \dots (ii)$$

We have, energy of photon

$$E = hF$$

$$\text{or, } F = \frac{E}{h} \dots (iii)$$

Using eqⁿ(iii) in eqⁿ(ii), we get

$$\frac{\partial \Psi(r,t)}{\partial t} = -i\frac{2\pi}{h}(\epsilon)\Psi(r,t)$$

$$\epsilon\Psi(r,t) = \frac{h}{-i2\pi} \frac{\partial \Psi(r,t)}{\partial t}$$

$$\epsilon\Psi(r,t) = -\frac{\hbar}{i} \frac{\partial \Psi(r,t)}{\partial t}$$

since $\frac{h}{2\pi} = \hbar$

$$\therefore \epsilon\Psi(r,t) = -\frac{\hbar}{i} \frac{\partial \Psi(r,t)}{\partial t} \times \frac{i}{i}$$

$$\text{or, } \epsilon\Psi(r,t) = i\hbar \frac{\partial \Psi(r,t)}{\partial t}$$

$\therefore \epsilon = i\hbar \frac{\partial}{\partial t}$ is called energy operator.

We know that,

$$E = K \cdot E + P \cdot E$$

$$E = \frac{1}{2}mv^2 + V \quad \text{Since } V = P \cdot E$$

$$\text{or, } E = \frac{1}{2m}m^2v^2 + V$$

$$\text{or, } E = \frac{P^2}{2m} + V \dots (i)$$

We know that,

$$H = -\frac{\hbar^2}{2m}\nabla^2 + V \dots (ii)$$

$$\text{Or, } E = -\frac{\hbar^2}{2m} \nabla^2 + V \dots (\text{since } H\Psi = E\Psi) \quad (\text{iii})$$

Comparing eqn (i) and (iii), we get

$$\frac{P^2}{2m} = -\frac{\hbar^2}{2m} \nabla^2$$

$$\text{Or, } P^2 = -\hbar^2 \nabla^2$$

$$\text{Or, } P^2 = -\frac{\hbar^2}{i^2} \nabla^2$$

$$\therefore P = \frac{\hbar}{i} \nabla \times i$$

$\therefore P = i\hbar \nabla \dots (\text{iv})$ This is called momentum operator in 3-dimⁿ term.

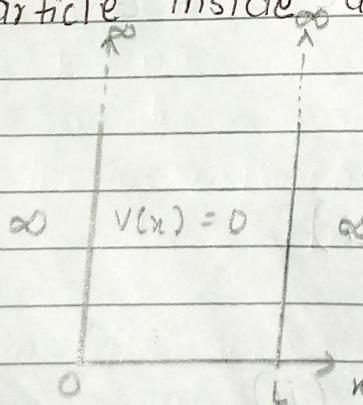
$$\text{In one dim, } P_x = -i\hbar \frac{\partial}{\partial x}$$

$$P_y = -i\hbar \frac{\partial}{\partial y}$$

$$P_z = -i\hbar \frac{\partial}{\partial z}$$

Application of Schrodinger Wave Equation.

- 1) Particle inside an infinite potential well:



[Fig: infinite potential well]

free particle:

$$\nabla^2 \psi + \frac{2m}{\hbar^2} (E - V) \psi = 0$$

$$\left(-\frac{\hbar^2 \nabla^2}{2m} + V \right) \psi = i\hbar \frac{\partial \psi}{\partial t}$$

$$H\psi = E\psi$$

Consider an electron of mass 'm' confined to remain within a one dimensional line in a crystal of length 'L' bounded by infinite potential well at the ends of length as shown in figure. Then, the potential for free particle (electron) in one dimension. Potential is given as

$$V(x) = 0 \} \text{ for } 0 < x < L \\ = \infty \} \text{ For } x \leq 0 \text{ and } x \geq L$$

Now, the wave function $\psi_n(x)$ of the electron in the solution of schrodinger equation. Therefore, schrodinger wave equation of particle in one dimension is given by:

$$\frac{\partial^2 \psi_n(x)}{\partial x^2} + \frac{2m(E-V)}{\hbar^2} \psi(x) = 0$$

For free particle $V(x) = 0$ in $0 < x < L$

$$\text{So, } \frac{\partial^2 \psi_n(x)}{\partial x^2} + \frac{2m}{\hbar^2} E_n \psi_n(x) = 0$$

$$\text{or, } \frac{\partial^2 \psi_n(x)}{\partial x^2} + k^2 \psi_n(x) = 0 \dots (i)$$

where, $k^2 = \frac{2m}{\hbar^2} E$

$$\begin{aligned} V(x) &= 0 \text{ for } 0 < x < L \\ &= \infty \text{ for } x \leq 0, x \geq L \end{aligned}$$

$$\frac{\partial^2 \psi}{\partial t^2} = \frac{1}{L^2} \frac{\partial^2 \psi}{\partial x^2}$$

The solution of equation (i) can be written as

$$\Psi_n(x) = A \sin kx + B \cos kx \quad \dots \text{(ii)}$$

Using boundary condition in eqn (ii), we get

$$\Psi_n(x) = 0 \text{ at } x = 0 \text{ & } x = L$$

Equation (ii) becomes,

$$0 = A \sin k \cdot 0 + B \cos k \cdot 0$$

$$\therefore B = 0$$

so eqn (ii) will be,

$$\Psi_n(x) = A \sin kx + 0 \cos kx$$

$$\therefore \Psi_n(x) = A \sin kx$$

At $x = L$,

$$\Psi_n(x) = 0 = A \sin kL$$

$$\therefore \sin kL = 0$$

or, $\sin kL = \sin n\pi$ where $n = 0, 1, 2, \dots, n$

$$\text{or, } kL = n\pi$$

$$k = \frac{n\pi}{L}$$

1. The wave function

$$\Psi_n(x) = A \sin \left(\frac{n\pi}{L} x \right) \dots \text{(iii)}$$

Using the normalization condition for wave function in $0 < x < L$, we get,

$$\int_0^L |\psi_n(x)|^2 dx = 1$$

$$\text{Or, } |A|^2 \int_0^L \sin^2\left(\frac{n\pi}{L}x\right) dx = 1$$

$$\text{Or, } \frac{|A|^2}{2} \int_0^L 2 \sin^2\left(\frac{n\pi}{L}x\right) dx = 1$$

Since $2 \sin^2 x = 1 - \cos 2x$

$$\text{Or, } \frac{|A|^2}{2} \left[\int_0^L 1 dx - \int_0^L \cos\left(\frac{2n\pi}{L}x\right) dx \right] = 1$$

$$\text{Or, } \frac{|A|^2}{2} [x]_0^L - 0 = 1$$

$$\text{Or, } \frac{|A|^2}{2} [L - 0] = 1$$

$$\text{Or, } |A|^2 = \frac{2}{L}$$

$$\text{Or, } |A| = \sqrt{\frac{2}{L}}$$

The normalized function:

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L}x\right) \dots (\text{iv})$$

This eqn (iv) gives the normalized wave function of a particle confined inside the infinite potential well.

For energy:

We know that,

$$k^2 = \frac{2m}{\hbar^2} E_n$$

$$\text{or, } E_n = \frac{\hbar^2 k^2}{2m}$$

$$\text{or, } E_n = \frac{\hbar^2}{2m} \left(\frac{n\pi}{L} \right)^2, \text{ where } k = \frac{n\pi}{L}$$

$$\therefore E_n = \frac{\hbar^2 n^2 \pi^2}{2m L^2} \dots (\text{v})$$

This equation gives the energy of n th particle inside the infinite potential well.

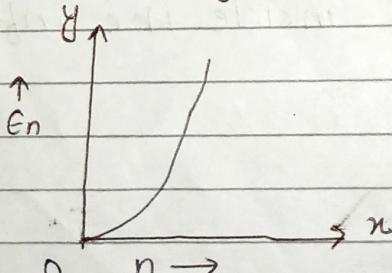
For eqn (v), we get,

$$E_n \propto n^2 \dots (\text{vi})$$

$$\text{where } \frac{\hbar^2 \pi^2}{2m L^2} = \text{constant}$$

This relation shows that energy of electron is directly proportional to square of state of electron.

If we plot a graph in between E_n and n as shown in fig (i)



[Fig. 1: Graph between E_n and n]

Again, we have normalized wave function

$$\Psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

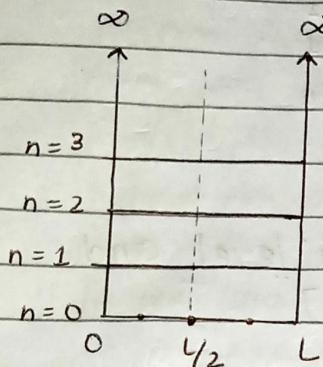


Fig. 2:

If $n=0$ at $x=0$

$$\Psi_0(0) = \sqrt{\frac{2}{L}} \sin \frac{n\pi 0}{L} = 0$$

If $n=0$ at $x=L$

$$\Psi_0(L) = \sqrt{\frac{2}{L}} \sin \frac{n\pi L}{L} = 0$$

If $n=1$, at $x=0$

$$\Psi_1(0) = \sqrt{\frac{2}{L}} \sin \frac{n\pi 0}{L} = 0$$

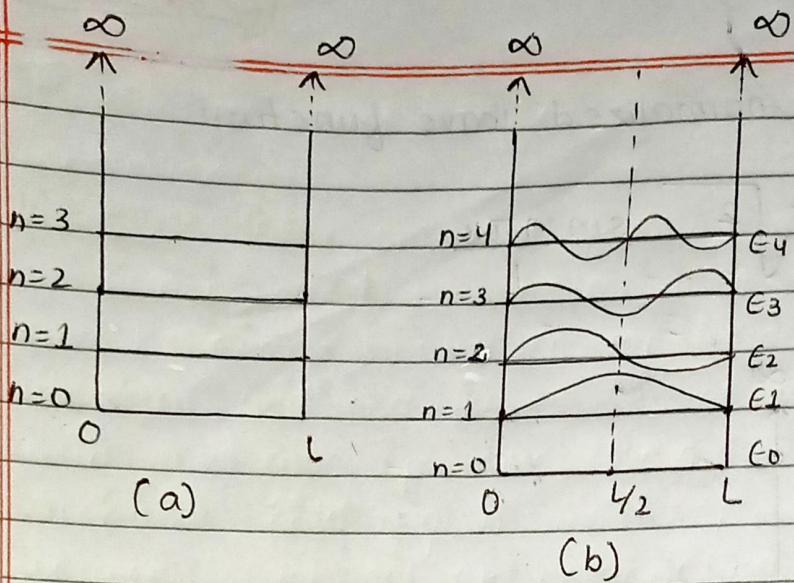
If $n=1$, at $x=L$

$$\Psi_1(L) = \sqrt{\frac{2}{L}} \sin \frac{n\pi L}{L} = 0$$

If $n=1$ at $x=L/2$

$$\Psi_1(L/2) = \sqrt{\frac{2}{L}} \sin \frac{n\pi}{L} \cdot \frac{L}{2}$$

$$\Psi_1(L/2) = \sqrt{\frac{2}{L}} = \text{maximum}$$



[Fig. 3 (a) Energy level (b) Energy level and corresponding eigen function]

Thus, wave function is maximum at $x = \frac{L}{2}$ for $n=1$ as shown in fig. 3(b)

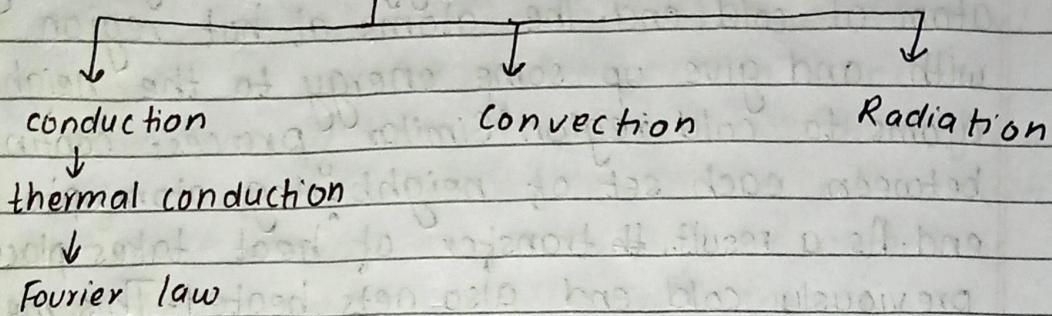
Similarly for $n=2, 3, \dots$ get such type of nature of wave as shown in fig 3(b) & hence eqn(v) gives that energy level are discrete which implies the allowed energy level.

The energy eigen value and wave function of free particles inside the one dimensional infinite potential well as shown in fig 3 (a) & (b)

Chapter: 8

Transfer of heat:

Modes of heat transfer



Heat transfer: It is the common form of energy which is continuously being transferred from one body to another body. In earlier time, during the development of concept of heat and temperature. People have the concept of **inequality** for flow of heat. According to this concept, heat always flows from a body of high temperature to a body of low temperature. Actually, heat always flows until there is difference in temperature.

There are 3 different modes of transfer of heat. They are:

- a) **Conduction:** It is the process of transfer of heat from one point to another point of a body carried out by means of **direct collision** between rapidly vibrating atoms at hot region and slowly vibrating atoms at cold region.

There is no actual transfer of particle during conduction.

When one end of solid is heated, atoms at heat hot end is vibrate with greater amplitude and have more kinetic energy than neighbouring atom at cold end. The atoms at hot region collide with and give up some energy to the neighbouring atoms to cold region. Similar process continue between each set of neighbouring atom upto next end. As a result, transfer of heat takes place and previously cold end also gets heated. This method transfer of heat is called conduction.

- b) Convection: It is the process of transfer of heat in fluid (liquid & gas) medium by means of actual motion of heated particle from higher temperature region to lower temperature region.

Heated particle carry heat and move from hot region to cold region but cold particles move in opposite direction. The current setup in the process is called convection current. This method is not possible in solid and vacuum.

Heating of water, land breezes, sea breezes, wind etc are the example of convection process.

c) Radiation: It is the process of transmission of heat from one point to another point without need of any material medium. Radiation doesn't heat the medium through which heat energy passes. In radiation, heat is transferred in the form of electromagnetic wave (radiation) and travel with speed of light (3×10^8 m/sec) in vacuum.

Heat energy coming to the earth is from the surface of sun is an example of radiation process. Heat energy radiated by an object is called radiant energy or thermal energy.

Statement and assumption of Fourier law of thermal conductivity. [Imp.]

The ability of a body to conduct heat is measured in terms of thermal conductivity. Therefore, an ability of material which conduct heat through its surface is called its thermal conductivity. Thermal conductivity generally occurs in solid medium.

Statement :

Fourier law states that the rate of heat flow in solid is directly proportional to the cross section area perpendicular to the flow axis and negative of temperature gradient over the length of path of conduction.

According to Fourier law, the rate of heat flow (Q) through the homogenous solid

is directly proportional to the area (A) of the section at the right angle to the direction of heat flow and to the temperature difference (dT) ^{along} ~~to the~~ path of heat flow i.e. $\frac{Q}{t} = -k A \frac{dT}{dx}$

where, $\frac{Q}{t}$ = rate of flow of heat ($J/sec = \text{watt}$)

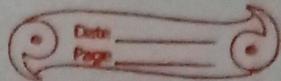
K = thermal conductivity ($\text{W/m}\cdot\text{K}$)

A = area of cross section

dT = change in temperature along the direction of heat flow

dx = thickness of the object (m)

In the Fourier heat conduction equation, the negative sign implies that heat is flowing from higher temperature to lower temperature region. Therefore, it is provided to compensate for the negative nature of temperature gradient.



Assumption in fourier law of heat conduction.

Following are assumptions of fourier law of heat conduction:

1. The thermal conductivity of a material is constant throughout the material.
2. There is no internal heat generation that occurs in the body.
3. The temp-gradient ($\frac{dT}{dx}$) is considered at constant.
4. The heat flow is unidirectional and takes place under steady-state conditions.
5. The surfaces are isothermal.

Temperature gradient:

The rate of fall in temperature with distance along the direction of heat flow is called temperature gradient. It is expressed as - $\frac{dT}{dx}$ and its unit is

Kevin per meter (K/m). Here, dT is the small change in temp over a small distance dx , and -ve sign indicates that temp falls with distance.

One dimension steady state heat conduction through plane wall.

Consider parallel sided plane wall of thickness 'l' and uniform cross-section area

A. As shown in figure let k be the thermal conductivity of wall material through which heat is flowing only in x -direction. [fig: plane wall]

let T_1 and T_2 are the temperature of higher temperature face-1 and lower temperature face-2. The small change of temp dT when the particles conduct a very small distance dx .

According to Fourier law, the rate of heat transfer is given by,

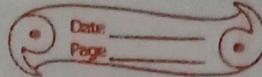
$$\frac{dQ}{dt} = -kA \frac{dT}{dx}$$

$$\text{or, } \frac{dQ}{dt} dx = -kA dT \dots (1)$$

To obtain the rate of flow of heat through this whole wall is obtained by integrating eqn (1), we get,

$$\int_0^L \frac{dQ}{dt} dx = \int_{T_1}^{T_2} -kA dT$$

$$\text{or } \frac{dQ}{dT} \int_0^L dx = -kA \int_{T_1}^{T_2} dT$$



$$\text{or, } \frac{dQ}{dt} [L - 0] = -KA[T_2 - T_1]$$

$$\text{or, } \frac{dQ}{dt} = \frac{KA(T_1 - T_2)}{L}$$

$$\text{or, } \frac{dQ}{dt} = \frac{T_1 + T_2}{L/KA} \dots (2)$$

where L/KA = thermal resistance (R_{TH})

$$\frac{dQ}{dT} = \frac{T_1 - T_2}{R_{TH}} \dots (3)$$

This eqn (3) gives the rate of flow of heat through the wall.

If R_{TH} increases, $\frac{dQ}{dt}$ decreases and vice versa.

Black body and black body radiation.

A perfect black body is one which absorbs heat radiation of all wavelengths falling on it and emits radiation of all possible wavelengths when heated.

For a perfect black body absorptance (a) = 1 and emissivity (e) = 1. When a body absorbs all radiant energy, it neither reflects nor it transmits. As a result body appears black and known as black body. Sun emits radiations of all wavelengths. So, it can be

considered as black body. A good absorber is a good emitter and can emit the radiation of all possible wavelengths.

A perfect black body cannot be realized in actual practice. The nearest approach to a perfectly black body is surface coated with lamp black or platinum black. Such a surface absorbs 96% to 98% of the incident radiation. Ferry's body is an example of perfectly black body realized in practice.

Ferry's black body:

Ferry's black body is a close approximation of perfectly black body, which can be realized in practice. Ferry's body consists of a double walled copper sphere with a small fine hole O and a conical projection P opposite to O as shown in figure.

A heating filament is kept in between two walls of the sphere and inner surface of sphere is coated with lamp black or platinum black.

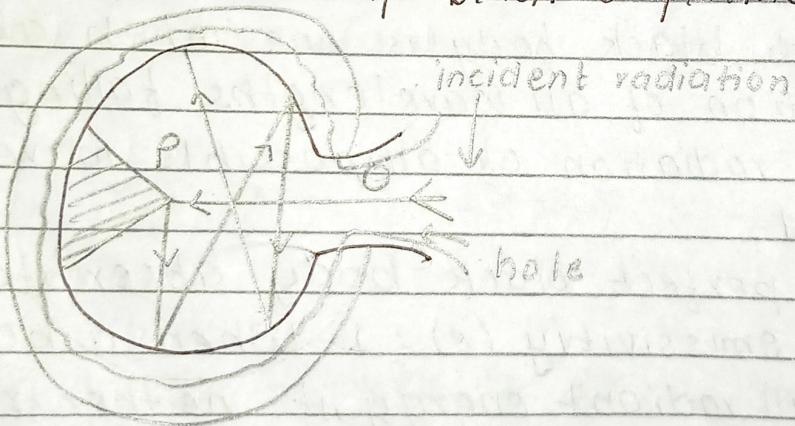
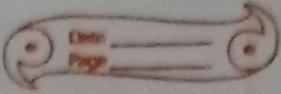


Fig: Ferry's black body.



When heat radiation falls on the conical projection 'P' through hole 'O', projection 'P' prevents the direct reflection of incident radiation rather it causes multiple reflections of radiations inside the sphere. Most of the incident radiation is absorbed by lamp black or platinum black and remaining part is also lost by multiple reflection. When the sphere is heated by passing electric current to the heating filament, radiation of all possible wavelengths come out from the hole 'O'. Hence, the hole 'O' behaves as a black body.

Emissive power:

It is the amount of heat radiations radiated by a unit area of a black body per unit time at a particular temperature. It is denoted by ϵ . Mathematically,

$$\text{Emissive power } (\epsilon) = \frac{\text{Energy radiated } (Q)}{\text{Area } (A) \times \text{time } (t)}$$

$$= \frac{Q}{A \times t}$$

$$\epsilon = \frac{Q/t}{A} = \frac{P}{A}$$

$$\text{Unit of } \epsilon = \frac{\text{J/sec}}{\text{m}^2} = \text{W/m}^2$$

According to Stefan's - Boltzmann law,

$$\epsilon = \sigma T^4 \text{ where } \sigma = \text{Stefan-Boltzmann constant}$$

$$= 5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$$

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Emissivity (ϵ): is defined

Emissivity of a body, as the ratio of emissive power of a given body to the emissive power of a perfectly black body of same size at same temperature. It is denoted by ϵ .

$$\therefore \epsilon = \frac{\text{emissive power of body}}{\text{emissive power of perfect black body}}$$

$$= \frac{E_b}{E_b}$$

Emissivity has no unit and dimension. Value of ϵ depends upon the nature of the surface of the emitting body. Its value ranges from 0 to 1.

For a perfectly black body, $\epsilon = 1$

For a black body $\epsilon < 1$.

For a perfectly reflecting surface

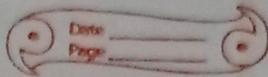
$$\epsilon = 0$$

Stefan's Boltzmann's law of black body radiation:

Stefan's declared experimentally that energy radiated by body depends only on its temperature and he put forward a law known as Stefan's law.

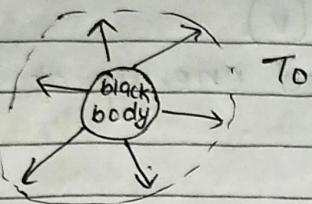
Stefan's - Boltzmann law states that "Emissive power of a black body is directly proportional to the fourth power of absolute temperature of surface of the body." let E be the emissive power of a black body at absolute temperature (T). Then, according to Stefan's - Boltzmann law.

$$E \propto T^4$$



or, $E = \sigma T^4 \dots (i)$ where σ is proportionality constant is called Stefan's constant. The value of σ in SI system is $5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$

If a body is not perfectly black and has its emissivity ϵ . Then, $E = \epsilon \sigma T^4 \dots (ii)$



[big: Stefan's law of black body radiation]

$$\text{or, } \frac{Q}{A \cdot t} = \epsilon \sigma T^4 \text{ or } \frac{Q/t}{A} = \epsilon \sigma T^4$$

$$\Rightarrow \text{Power} = \epsilon \sigma T^4$$

$$P = \epsilon A \sigma T^4 \dots (iii)$$

This relations given above are valid only if the temperature of the surrounding is zero Kelvin. But, if there is a surrounding T_o (more than 0 K) then surrounding also emits heat radiation which is absorbed by black body. Hence, relation is modified by Boltzmann for the net emissive power as

$$E' = \sigma (T^4 - T_o^4) \dots (iv)$$

This is called Stefan's Boltzmann law.

If a body is not perfectly black body, the net emissive power $\epsilon' = e \sigma (T^4 - T_0^4)$

$$\frac{Q'}{A} = e \sigma (T^4 - T_0^4)$$

$$\text{or, } \frac{Axt}{A} = e \sigma (T^4 - T_0^4)$$

$$\therefore P = e A \sigma (T^4 - T_0^4) \dots (v)$$

This gives power of radiated energy of a black body.

Some numerical problems:

- 1) An insulating material having a thermal conductivity of $0.08 \text{ W/m}\cdot\text{K}$ is used to limit the heat transfer of 80 W/m^2 for a temp of 15°C across the opposite faces. Find the thickness of the material?

⇒ Sol'n:-

$$k = 0.08 \text{ W/m}\cdot\text{K}$$

$$\frac{Q}{A} = 80 \text{ W/m}^2$$

$$\Delta T = 15^\circ\text{C} = 15 \text{ K}$$

$$dx = ?$$

Now,

$$Q = k A \frac{dT}{dx}$$

$$dx = \frac{Q}{AkdT}$$

$$dx = 0.15 \text{ cm}$$

2) A brick wall 12cm thick and $5m^2$ surface area exposed to 50°C at one face and 20°C to another face. If the thermal conductivity of the material is $1.5 \text{ W/m}\cdot\text{K}$. Find the heat transfer rate.

$$\Rightarrow \Delta x = 12 \text{ cm}$$

$$A = 5 \text{ m}^2$$

$$\Delta T = 50^\circ\text{C} - 20^\circ\text{C}$$

$$= 30^\circ\text{C}$$

$$= 30 \text{ K}$$

$$k = 1.5 \text{ W/m}\cdot\text{K}$$

$$\frac{Q}{t} = ?$$

$$\frac{Q}{t} = k \frac{A \Delta T}{dx}$$

$$= 1875 \text{ watt}$$

3) Find the rate of heat lost from a brick wall ($k = 0.7 \text{ W/m}\cdot\text{K}$) of length 5m, height 4m and 0.25m thick. The temp of the inner surface is 50°C and that of outer surface is 30°C . Also, calculate the distance from the inner surface at which the temp. is 40°C

\Rightarrow SOLN:-

$$k \text{ of brick wall} = 0.7 \text{ W/m}\cdot\text{K}$$

$$\text{Area}(A) = 5 \text{ m} \times 4 \text{ m} \\ = 20 \text{ m}^2$$

$$\Delta x = 0.25 \text{ m}$$

$$\Delta T = 50^\circ - 30^\circ \text{C}$$

$$= 20^\circ \text{C}$$

$$= 20 \text{ K}$$

$$\frac{dQ}{dt} = k A \frac{\Delta T}{\Delta x}$$

$$= 1120 \text{ watt}$$

Again, $\Delta x' = ?$

$$\text{where } \frac{dQ}{dt} = k A \frac{(T_1 - T_2)}{\Delta x}$$

$$\text{or, } 1120 = 10.7 \times 20 \frac{(50 - 40)}{\Delta x'}$$

$$\therefore \Delta x' = 0.125 \text{ m}$$

- Q4) A 1.2m long tube with outer diameter of 4cm outside temp of 120°C is exposed to an ambient air at 20°C . If the heat transfer coefficient between the tube surface and the air is $20 \text{ W/m}^2 \text{K}$. Find the rate of heat transfer from the tube to the air.

\Rightarrow Soln:-

$$\Delta x = 1.2 \text{ m}$$

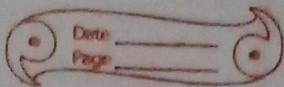
$$\Delta T = 120^\circ \text{C} - 20^\circ \text{C}$$

$$= 100^\circ \text{C}$$

$$= 100 \text{ K}$$

$$\text{heat transfer rate (h)} = \frac{k}{\Delta x}$$

$$= 20 \text{ W/m}^2 \text{K}$$



$$\begin{aligned}
 A &= 2\pi R dx \\
 &= 2\pi \times 2 \times 10^{-2} \times 12 \\
 &= 0.1508 \text{ m}^2 \\
 \frac{dQ}{dt} &= k \frac{dT}{dx} A \\
 &= 301.593 \text{ watt.}
 \end{aligned}$$

(5) A furnace inside temp of 2250 K has a glass circular viewing of 6cm diameter. If the transmissivity of glass is 0.08 makes calculation to the heat loss from the glass window due to radiation.

\Rightarrow Sol:-

$$\begin{aligned}
 A &= \frac{\pi}{4} d^2 \\
 &= \frac{\pi}{4} (6 \times 10^{-2})^2
 \end{aligned}$$

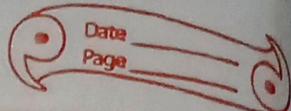
$$e = 0.08, T = 2250 \text{ K}$$

Now,

$$\epsilon = \sigma T^4$$

$$Q = \sigma T^4 \times e \text{ for not perfect body.}$$

$$\begin{aligned}
 \frac{Q}{t} &= \sigma T^4 \cdot A e \\
 &= 328.53 \text{ watt}
 \end{aligned}$$



Chapter 8.

8.1 Fundamentals of thermodynamics:

Introduction:

When two stones strike together spark may be produced. When we rub our hands each other, we feel warm. In both cases, heat is produced in the expense of mechanical energy. Also, when certain mass of ice falls from a height, it may melt. This is due to conversion of gravitational potential energy into heat energy. We have other examples such as cars, bikes, airplanes, etc. fuel is burnt to produce heat which causes the expansion of gases inside the cylinder and hence mechanical work is obtained. In all cases, we mentioned the principle of conservation of energy is not violated. Transformation of heat into mechanical work and vice-versa has provides us the foundation for the construction of heat engines (like bike, scooter, cars, etc) and refrigerators. Thermodynamics is the branch of physics which deals with the interrelationship between heat and mechanical work. Nowadays thermodynamics has been more appreciable in other field like electrical, chemical and magnetic fields.

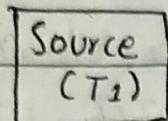
Note:

Thermodynamics \Rightarrow Thermo + dynamics = heat + study
of motion

It is the branch of physics which deals with
the transformation of heat energy into mechanical
work and vice versa. Thermodynamics deals
macroscopically i.e it deals with bulk system
but not with the individual molecule.

Efficiency of heat engine (η)

$$\eta = \frac{\text{output}}{\text{input}}$$



$$\eta = \frac{w}{Q_1}$$

$$\eta = \frac{Q_1 - Q_2}{Q_1}$$

$$\eta = \frac{Q_1 - Q_2}{Q_1} \quad \text{fig: Black diagram of heat engine}$$

$$\eta = \left(1 - \frac{Q_2}{Q_1} \right)$$

$$\eta \cdot \% = \left(1 - \frac{Q_2}{Q_1} \right) \times 100\%$$

In $^{\circ}\text{C}$ temp:

$$\eta \cdot \% = \left(1 - \frac{T_2}{T_1} \right) \times 100\%$$

Coefficient of performance of heat pump (β)

$$\beta = \frac{\text{output}}{\text{input}}$$

$$\beta = \frac{Q_1}{w}$$

$$\beta = \frac{Q_1}{Q_1 - Q_2}$$

$$\beta = \frac{Q_1 - Q_2}{Q_2}$$

$$\beta = 1$$

$$\left(1 - \frac{Q_2}{Q_1} \right)$$

$$\beta \cdot \% = 1$$

$$\left(1 - \frac{Q_2}{Q_1} \right) \times 100\%$$

in term of temp.

$$\beta \cdot \% = 1$$

$$\left(1 - \frac{T_2}{T_1} \right) \times 100\%$$

Coefficient of performance of refrigerator (β)

$$\beta = \frac{\text{output}}{\text{input}}$$

$$\beta = \frac{Q_2}{w}$$

$$\beta = \frac{Q_2}{Q_1 - Q_2}$$

$$\beta = 1$$

$$\frac{Q_1 - Q_2}{Q_2} \quad \text{fig: Black diagram of refrigerator}$$

$$\beta = 1$$

$$\left(\frac{Q_1 - 1}{Q_2} \right)$$

$$\beta \cdot \% = 1$$

$$\left(\frac{Q_1 - 1}{Q_2} \right) \times 100\%$$

In term of temp.

$$\beta \cdot \% = 1$$

$$\left(\frac{T_1 - 1}{T_2} \right) \times 100\%$$



. Thermodynamic system (Types of thermodynamic system) and its types.

An assembly of large number of particles bounded by a closed structure (boundary surface) having certain value of macroscopic parameters (pressure, volume and temp) is called thermodynamic system. Anything outside the boundary surface of system is called its surrounding. Thermodynamic system interacts with its surrounding either by exchanging energy and matter in different ways or by performing work.

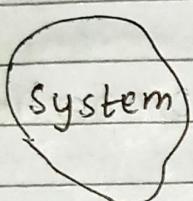


fig: Thermodynamic system.

On the basis of exchange of energy (heat) and matter, systems are classified into three types. They are:

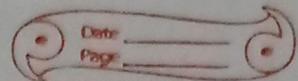
- 1) Open system: Thermodynamic system which can exchange both matter and energy with its surrounding is known as open system. Eg: gas contained in an open cylinder.

- 2) Closed system: Thermodynamic system which can exchange only energy but not matter with its surroundings is called closed system. eg: gas contained in a closed and conducting cylinder fitted with piston.
- 3) Isolated system: Thermodynamic system which can exchange neither energy nor matter with its surrounding is called isolated system. eg: gas contained in a perfectly insulated cylinder fitted with piston.

- Thermodynamical variables and properties.

An equilibrium state of a thermodynamic system is specified by the value of its parameters like pressure (P), volume (V) and temp (T) etc. These parameters are called thermodynamic variables.

On the other hand, thermodynamic property is any characteristics of thermodynamic variable and other additional terms like mass, internal energy, total energy, enthalpy, entropy, viscosity, thermal conductivity, etc.



Thermodynamic properties are generally classified into two groups. They are:

- 1) Intensive properties: The properties which are independent of the mass of the system are called intensive properties. Eg: Temperature, pressure.
- 2) Extensive properties: The properties which depend on the size or extent of the system are called extensive properties. Eg: Total mass, total volume and total momentum.

- Thermodynamic equilibrium:

Thermodynamic deals with equilibrium states. Equilibrium means a state of balance. An equilibrium state, there is no existed unbalanced potentials within a system. A system will be in a state of thermal equilibrium, if the condition for the following three types of equilibrium are satisfied:

- 1) mechanical equilibrium: A system is in mechanical equilibrium, if there is no change in pressure at any point of the system with time.
- 2) Thermal equilibrium: A system is said to be in thermal equilibrium if all parts of it exhibit the same value of temperature.

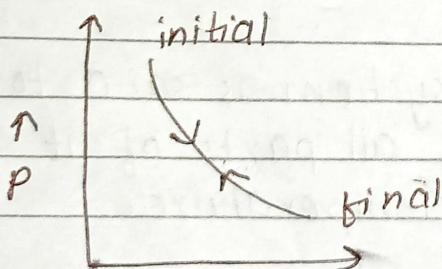
3) chemical equilibrium: A system is in chemical equilibrium if its chemical composition doesn't change with time, that is no chemical reaction occurs.

- Reversible and irreversible process.

a) Reversible process:

If the effects produced by a process on the system as well as on the surrounding can be completely restored to their initial states and no changes are left in any of the system taking part in the process or in the surrounding then the process is said to be a reversible process.

Any process, taking place slowly and controlled manner. All the mechanical processes taking place under the action of conservative force are reversible process. In reversible process, a system can retrace in the opposite direction. So that it passes through exactly the same states in all respect as in the direct process.

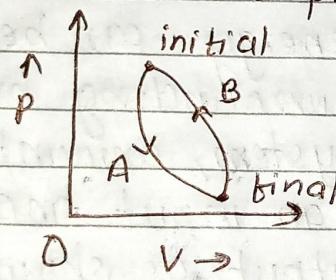


Conditions for the reversible process:

- 1) The process should be infinitely slow.
- 2) There should be no loss of energy during the process.
- 3) System and surrounding should be in thermodynamic equilibrium during the process.
- 4) No dissipative force of friction, viscosity, electrical resistance etc should be presented.

b) Irreversible process:

The process which produce a permanent change in the thermodynamic state of the system and can't be retraced in the opposite order are called irreversible process. Almost all process are in the nature of irreversible process.



Eg: rusting of iron, decay of matter, heat produced by friction.

- Laws of thermodynamics:

First law of thermodynamics is based on the principle of conservation. It states that "energy can neither be created nor can be destroyed but can change one form to another form." This law can be expressed in closed and open systems.

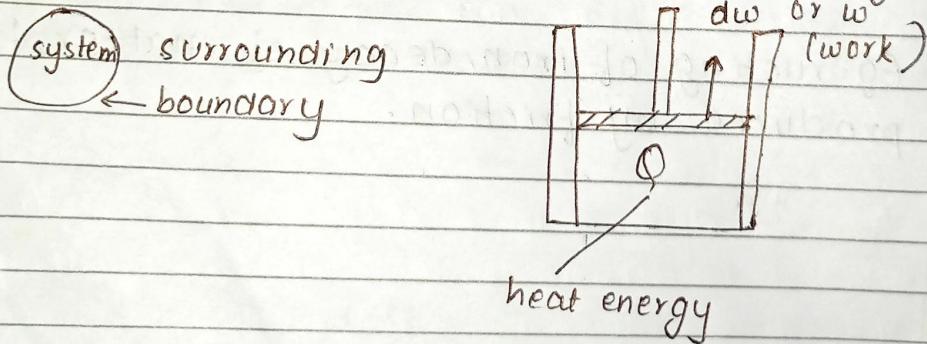
For a closed system 1st law of thermodynamics can be expressed ^{based} on the energy conservation.

In open system, it can be explained based on the main mass conservation and energy conservation. They are described as following:

- First law of thermodynamics for closed system (For control mass):

In the closed system, energy can be transferred between system and surrounding but not matter.

Consider a closed system, amount of heat energy Q is given as an input and external work done (w) as an output as shown in fig.





Using the law of conservation of energy,

Energy entering the system - Energy leaving the system = change in total energy of the system.
i.e. $E_{in} - E_{out} = \Delta E$

$$Q - W = \Delta E \dots (i)$$

The total energy of the system (i.e. change of energy) is the sum of K.E, P.E and initial internal energy)

$$\Delta E = \Delta K.E + \Delta P.E + \Delta U \dots (ii) \text{ where } \Delta U = \text{change in internal energy of system.}$$

In the case of closed system, change in K.E and P.E are negligible as compared to the change in internal energy. Therefore, from eqn (ii) we get,

$$\Delta E = \Delta U \dots (iii)$$

Using eqn (iii) in eqn (i), we get,

$$Q - W = \Delta U$$

$$dQ - dW = dU$$

$$\text{or, } dQ = dU + dW \dots (iv)$$

This is an equation of 1st law of thermodynamics.

Case I: If the closed system undergoes in cyclic process. The conservation of energy equation can be written as,

$$\oint (dQ - dw) = \oint du$$

For cyclic process $\oint du = 0$

$$\therefore \oint (dQ - dw) = 0$$

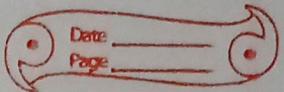
$$\text{or, } \oint dQ = \oint dw$$

Therefore, net amount of applied heat energy (Q) is equal to net amount of external work done (w).

Case II: For any thermodynamical process from one to another of a closed systems.

$$Q_{12} - w_{12} = u_2 - u_1$$

$$Q_{12} = w_{12} + u_2 - u_1$$



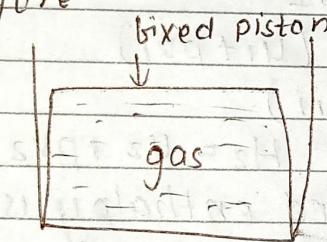
- Application of 1st law of thermodynamic of a closed system:

Some application of 1st law of thermodynamics are described as following.

- Constant volume (Isochoric process):

It is a thermodynamic process in which volume of gas remains constant as the gas is kept in a non-expanding chamber.

Consider the working substance (gas) is contained in a rigid vessel in a constant volume process, making the system's boundaries immovable and preventing any work from being done on or by the system. The rigid vessel as shown in figure.



When the heat energy is added in the system, there is no change of volume. So, $dV = V_2 - V_1 = 0$

Total work done $dW = pdV = 0$

We know that, from 1st law of thermodynamics

$$Q_{12} = W_{12} + U_2 - U_1$$

$$Q_{12} = U_2 - U_1 \text{ since, } dW = W_{12} = 0$$

$$\text{Thus, } Q_{12} = (\Delta U)_{12}$$

Hence, heat transferred to control mass during a constant volume process is equal to the increase in internal energy of the system.

2) Isobaric process:

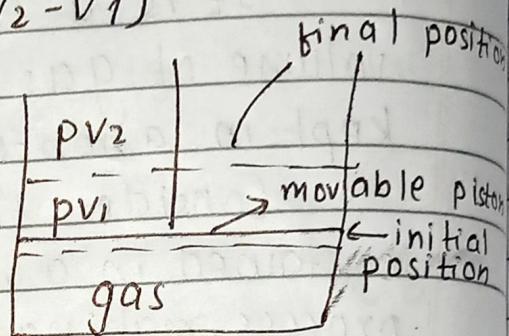
It is thermodynamic process in which the working substance is taken in an expanding chamber and heat is supplied to it so that volume and temp. of system change without any change in pressure.

In the isobaric process, work done.

$$\int dW = W_{1,2} = \int_{V_1}^{V_2} PdV = P(V_2 - V_1)$$

$$\text{where } P_1 = P_2 = P$$

$$\therefore W_{1,2} = PV_2 - PV_1 \dots (i)$$



From 1st law of thermodynamics,

$$Q_{1,2} = W_{1,2} + U_2 - U_1$$

$$Q_{1,2} = PV_2 - PV_1 + U_2 - U_1$$

$$Q_{1,2} = (U_2 + PV_2) - (U_1 + PV_1)$$

$$Q_{1,2} = H_2 - H_1 \dots (ii)$$

where $H_1 = U_1 + PV_1$ and $H_2 = U_2 + PV_2$

since $H = U + PV$ is called enthalpy is equal to the sum of internal energy and product of pressure and volume of the gas.

From eqn (ii), we conclude that heat transferred to a control mass during a constant pressure process is equal to the increase in the enthalpy of the system.