Experiment No. 2 Date:

DETERMINATION OF THE VALUE OF MODULUS OF REGIDITY OF THE WIRE GIVEN AND MOMENT OF INERTIA OF A CIRCULAR DISC USING TORSION PENDULUM.

APPARATUS REQUIRED:

a) Torsion pendulum set, b) Thin wire, c) Stop watch, d) Screw gauge,

e) Meter scale, f) Spirit level, g) Balance.

THEORY:

A disc suspended at its mid-point by a long and thin wire to a rigid support constitutes a torsion pendulum. It is called so because when it is twisted and then released, it executes torsional vibrations about the wire as axis.

If the pendulum is turned through an angle $'\theta'$, the wire exerts a restoring torque proportional to angular displacement $'\theta'$,

i.e. $\tau \propto \theta$

or, $$\tau = -C\theta \text{ ....(i)}$$

Where $'C'$ is called torsion constant of the wire which depends upon its property. It is defined as the restoring torque per unit twist in the wire and given by

$$C = \frac{\pi\eta r^4}{2l}$$

Where, η = modulus of rigidity of wire

r = radius of wire

l = length of wire

The torque is also given by

$$\tau = I\alpha \text{.......(ii)}$$

Equations (i) and (ii) give $I\alpha = -C\theta$

or, $$\alpha = -\frac{C\theta}{I}$$

or, $$\frac{d^2\theta}{dt^2} + \frac{C\theta}{I} = 0$$

or, $$\frac{d^2\theta}{dt^2} + \omega^2\theta = 0$$

This shows that the motion of torsion pendulum is simple harmonic.

Here, $\omega^2 = \frac{C}{I}$

or, $$\omega = \sqrt{\frac{C}{I}}$$

or, $$\frac{2\pi}{T} = \sqrt{\frac{C}{I}}$$

or, $$T = 2\pi\sqrt{\frac{I}{C}} \quad \text{..........(iii)}$$

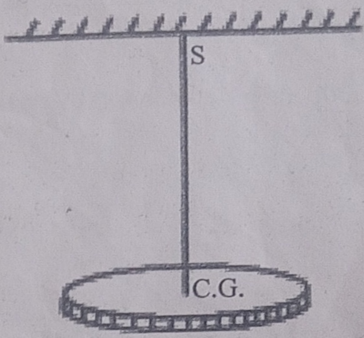


Fig. Torsion pendulum

This is the time period of torsion pendulum.

Let I_1 be the moment of inertia of circular disc, then its time period is given by,

$$T_1 = 2\pi\sqrt{\frac{I_1}{C}}$$

or, $$T_1^{\,2} = 4\pi^2\frac{I_1}{C} \quad \text{...........(iv)}$$

Now a circular ring is placed on the circular disc co-axially with the wire, then the period of oscillation for this combination is given by

$$T_2 = 2\pi\sqrt{\frac{I_1 + I_2}{C}}$$

Where, I_2 is the moment of inertia of circular ring, or, $$T_2^{\,2} = 4\pi^2\left(\frac{I_1+I_2}{C}\right) \quad \text{..................(v)}$$

Subtracting equation (iv) from (v),

$$T_2^{\,2} - T_1^{\,2} = \frac{4\pi^2 I_2}{C} \quad \text{..................(vi)}$$

$$= \frac{4\pi^2 I_2}{\pi\eta r^4} \times 2l$$

$\therefore$ $$\eta = \frac{8\pi I_2 l}{(T_2^{\,2} - T_1^{\,2})\, r^4} \quad \text{..................(vii)}$$

Again, dividing equation (iv) by (vi)

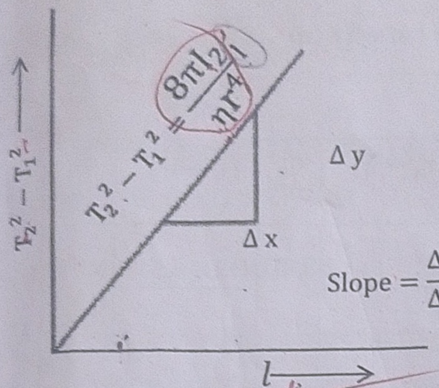
$$\frac{T_1^{\,2}}{T_2^{\,2} - T_1^{\,2}} = \frac{I_1}{I_2}$$

$\therefore$ $$I_1 = \left(\frac{T_1^{\,2}}{T_2^{\,2} - T_1^{\,2}}\right) I_2 \quad \text{..............(viii)}$$

PROCEDURES:

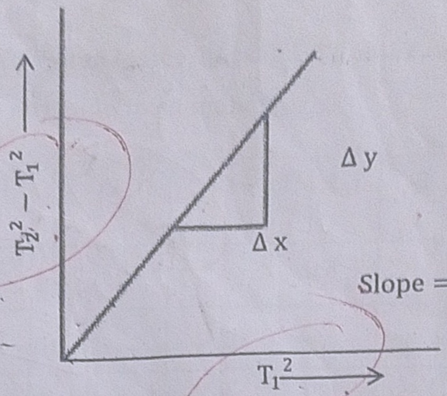
1. Suspend the circular disc with a ring from a rigid support.
2. Take the circular ring out of the circular disc and place it over the rigid support.
3. Twist slightly the disc in horizontal plane. Then, note the time taken for 10 oscillation and calculate time period T_1 for circular disc.
4. Now place the circular ring on the circular disc in such a way that the axis of the wire passes through centre of gravity of the ring.
5. Again, note the time period T_2 for disc and ring for the same length as in case of circular disc.
6. Repeat the process for different lengths of wire.
7. Plot a graph between $(T_2^2 - T_1^2) \sim l$ and $(T_2^2 - T_1^2) \sim T_1^2$ and find η and I_1

$8\pi I_2 l$



Slope $= \frac{\Delta y}{\Delta x} = \frac{8\pi I_2}{\eta r^4}$

Fig . Graph between $T_2^2 - T_1^2$ and l



Slope $= \frac{\Delta y}{\Delta x} = \frac{I_2}{I_1}$

Fig. Graph between $T_2^2 - T_1^2$ and T_1^2

OBSERVATIONS:

1. Mass of circular ring (M) = 320g
2. Mass of circular disc (M') = 850g
3. Radius of suspension wire (r) = 0.023 cm
4. Radius of circular disc (R) =
5. Internal radius of circular ring (R_1) =
6. External radius of circular ring (R_2) =
7. Moment of inertia of circular ring $I_2 = \frac{M(R_1^2 + R_2^2)}{2} =$

Table No.1. Determination of time period

S. N.	Length (l)	Determination of T_1 (Disc)				Determination of T_2 (Disc+Ring)			
		Time for ~~10~~ 05 oscillation			Time period (T_1)	Time for ~~10~~ 05 oscillation			Time period (T_2)
		1	2	Mean	$T_1 = T_1/10$	1	2	Mean	
1.	6								
2.	15								
3.									
4.									
5.									

Table No.2. Measurement of modulus of rigidity of wire (η) and moment of inertia of disc (I_1)

S.N.	ℓ	T_1^2	T_2^2	$T_2^2 - T_1^2$	η	$\bar{\eta}$	I_1	$\bar{I}_1$	$(\eta_i - \bar{\eta})^2$	$\sigma_{\bar{\eta}}$	$(I_i - \bar{I}_1)^2$	σ_{I_1}
1.												
2.												
3.												
4.												
5.												

RESULTS:

1. The value of modulus of rigidity of wire (η) =
2. Standard value of η =
3. Percentage error =
4. The value of moment of inertia of circular disc (I_1) =
5. Standard value of $I_1 = \frac{M'R^2}{2} =$
6. Percentage error in I_1 =
7. The value of η from graph =
8. The value of I_1 from graph =

CONCLUSION:

745

PRECAUTIONS: