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1

Chapter

MECHANICAL OSCILLATION

CHAPTER OUTLINE

After completion of this chapter, you will be able to know:

- Free oscillation, Damped oscillation and Forced oscillation (Physical meaning and equations).
- Compound pendulum, Minimum and maximum time period in compound pendulum. Interchangeability of point of suspension and point of oscillation in compound pendulum. Torsion pendulum, Determination of modulus of elasticity of material using torsion pendulum.

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INTRODUCTION

Oscillation is a very general term, it indicates any kind of to and fro motion. Oscillation can repeat at fixed intervals or not.

Periodic motion is more specific type of oscillation, where the body returns to each state after a fixed period i.e. it repeats at fixed intervals.

Simple harmonic motion is a very exact term with a very specific meaning. It is a kind of periodic motion in which the body's acceleration at each point is directly proportional to its displacement from mean point of its motion and the acceleration is always directed towards the mean position.

For SHM to take place.

1. The motion should be periodic.
2. Acceleration is directly proportional to the displacement of the particle and directed towards mean position.
3. The restoring force should be directly proportional to displacement of the body from its mean position.

In SHM, since the motion is periodic, so this is expressed in terms of harmonic (periodic) functions sine, cosine, or combination of both.

If there are frictional or damping forces present, as there almost always are, these will generally cause the amplitude of the motion gradually decrease, which is not exactly simple harmonic. So, simple harmonic motions are rare in nature.

FREE OSCILLATION

The oscillation of a body or system with its natural frequency. These oscillation possesses constant amplitude and period without any external force. Ideally, free oscillation does not undergo damping. But in all natural systems damping is observed unless and until any external force is supplied to overcome damping. In such a system, the amplitude frequency and energy all remain constant.

Consider a particle of mass is oscillating freely. Let the displacement made by the particle be x.

Then the restoring force is given by

$$F = -kx \qquad \ldots (i)$$

And the PE of the particle can be written as

$$P.E. = \int -F dx$$

$$= -\int -kx dx$$

$$= \frac{1}{2}kx^2 \qquad \ldots (ii)$$

Also, the K.E. of the particle is

$$K.E. = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 \qquad \ldots (iii)$$

Hence the total energy of the system at any Instant will be

$$E = \frac{1}{2}kx^2 + \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 \qquad \ldots (iv)$$

The total energy remains constant for free oscillation

$$\therefore \quad \frac{1}{2}kx^2 + \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 = \text{constant}$$

Diff. the equation with respect to t on b.s. we get,

$$\frac{d}{dt}\left\{\frac{1}{2}kx^2 + \frac{1}{2}m\left(\frac{dx}{dt}\right)^2\right\} = 0$$

$$\text{or,} \quad \frac{1}{2}k\,2x\frac{dx}{dt} + \frac{1}{2}m\,2\left(\frac{dx}{dt}\right)\frac{d^2x}{dt^2} = 0$$

$$\text{or,} \quad \frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

$$\text{or,} \quad \frac{d^2x}{dt^2} + \omega^2 x = 0 \qquad \ldots (v)$$

Where $\omega = \sqrt{\frac{k}{m}}$

Equation (v) is similar to the differential equation of SHM. So, the solution of free oscillation can be written as.

$$x = A\sin(\omega t + \phi)$$

Which given the frequency $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

DAMPED OSCILLATION

The free vibration of a body is an ideal concept. There always exist a resistive force which reduce the amplitude and energy of vibration of the body and finally become zero. Such a vibration of a body is called damped vibration or oscillation.

The amplitude of the oscillation gradually decreases to zero as a result of damping force. For low velocity, the damping force is directly proportional to the velocity of the body but directed opposite to it. The net force on the oscillating body is the sum of restoring force and damping force i.e.

$$F = F_d + F_r \qquad \ldots (i)$$

Here $F_d = -bv$ is damping force.

'b' is called damping constant depends upon the nature of damping material and v is the velocity.

and $F_r = -kx$ is the restoring force.

$$\therefore \quad F = -bv - kx$$

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or, $ma = -bv - kx$

$$\Rightarrow \quad m\frac{d^2}{dt^2} + b\frac{dx}{dt} + kx = 0$$

$$\Rightarrow \quad \frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$$

or, $$\frac{d^2x}{dt^2} + 2\delta\frac{dx}{dt} + \omega_0^2 x = 0 \qquad \ldots \text{(ii)}$$

This is differential equation of damped harmonic isolation. Where, $2\delta = \frac{b}{m} = \frac{1}{\tau}$, τ is called relaxation time

$\omega_0 = \sqrt{\frac{k}{m}}$ is the natural angular frequency.

The solution of equation (ii) is, $x = x_m e^{\frac{-bt}{2m}} \sin(tf + \phi)$ ... (iii)

The angular frequency of damped harmonic oscillation is given by,

$$\omega = \sqrt{\omega_0^2 - \delta^2} = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$

or, Frequency $(f) = \frac{1}{2\pi}\sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$

$\therefore$ Time period of damped oscillator, $T = \frac{1}{f} = \frac{2\pi}{\sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}}$

If these is no damping i.e. $b = 0$, $T = 2\pi\sqrt{\frac{m}{k}}$, which is the time period of free oscillation.

Equation (iii) is, $x = x_m e^{\frac{-bt}{2m}} \sin(\omega t + \phi) = A \sin(\omega t + \phi)$

Where, $A = x_m e^{\frac{-bt}{2m}}$ ... (iv) is called amplitude of damped oscillation.

Mean life time: The time duration in which the amplitude drops to $\frac{1}{e}$ of its initial value is called mean life time.

Therefore from (iv)

$$\frac{1}{e}x_m = x_m e^{\frac{-bt}{2m}} \qquad \Rightarrow \frac{bt}{2m} = 1$$

$$t = \tau = \frac{b}{2m}$$

Here, $\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$

Case I: If $\frac{b^2}{4m^2} > \frac{k}{m}$ or, $b^2 > 4mk$ the ω is an imaginary. In such case there is no oscillation and the system moves slowly back to equilibrium position such an oscillation is called over damped.

Case II: If $\frac{b^2}{4m^2} = \frac{k}{m}$ or, $b^2 = 4mk$ then $\omega = 0$. In this case there will be no oscillation and the system returns to equilibrium condition in shortest time. Such an oscillation is called critically damped.

Case III: If $\frac{k}{m} > \frac{b^2}{4m^2}$ or, $b^2 < 4mk$. In this case the system oscillates with decreasing amplitude, such oscillation is under damped oscillation.

The total energy of damped harmonic oscillation is given by,

$$E = \frac{1}{2}m\omega^2A^2 = \frac{1}{2}m\omega^2 x_m^2 e^{\frac{-bt}{2m}} = E_0 e^{\frac{-bt}{2m}}$$

Therefore, the total mechanical energy of damped harmonic oscillator decrease exponentially with time.

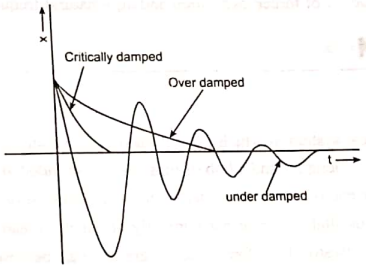

Fig: The displacement function x with time 't' for damped oscillator.

FORCED OSCILLATION

If an external force is applied on a damped harmonic oscillation such that it recovers the energy lost in damping, the oscillation of the system remains continuous. However the body vibration with the frequency of the external source. This kind of oscillation is called forced oscillation [Note: the external force should be periodic so that it can maintain periodic motion].

Now the total force acting on the body is, restoring, force, damping force and external periodic force.

$$\therefore \quad F = f_r + F_d + F_{ext}$$

$$= -kx + (-bv) + F_0 \sin\omega t.$$

or, $ma = -kx - bv + F_0 \sin\omega t.$

$$m\frac{d^2x}{dt^2} + b\frac{bx}{dt} + kx = F_0 \sin\omega t$$

$$\Rightarrow \quad \frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = \frac{F_0}{m}\sin\omega t$$

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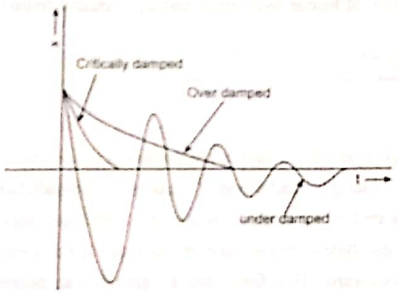

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$$m\frac{d^2x}{dt^2} + b\frac{bx}{dt} + kx = F_0 \sin\omega t$$

$$\Rightarrow \quad \frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = \frac{F_0}{m}\sin\omega t$$

$$\Rightarrow \quad \frac{d^2x}{dt^2} + 2\delta\frac{dx}{dt} + \omega_0^2 = \frac{F_0}{m}\sin\omega t \qquad \ldots (i)$$

This is the differential equation for forced harmonic motion.

The solution of this equation is.

$$x = x_m \sin(\omega t + \phi) \qquad \ldots (ii)$$

Solving equation (i) and (ii) we get,

$$\text{Amplitude, } x_m = \frac{F_0/m}{\sqrt{(\omega_0^2 - \omega^2) + \left(\frac{b\omega}{m}\right)^2}}$$

$$\text{and } \phi = \tan^{-1}\left[\frac{\omega_0^2 - \omega^2}{\omega\left(\frac{b}{m}\right)}\right]$$

Here ω = frequency of forced oscillation and ω_0 = natural frequency.

EXAMPLES OF SHM

Simple Pendulum

A simple pendulum is an ideal system in which a point mass is suspended at one end of mass-less string. Consider a simple pendulum of length l and a bob of mass m is suspended at the free end of the pendulum as shown in figure. The other end of the string is suspended at the rigid support. If the bob is displaced by an angle θ, and released, it oscillates simple harmonically about the mean position. The weight of the mass, mg acts vertically downward. This force due to gravity can be resolved into two components; mgcosθ is the force along the string and mgsinθ perpendicular to the string. The tension of the string (T) is balanced by mgcosθ. The component mg sinθ provides the restoring force which tends to bring the pendulum back to its mean position. That is

Restoring force (F) = – mg sinθ

Where the negative sign shows that the restoring force acts opposite to the displacement (x) of the bob.

If a is the acceleration of the bob, then

$$F = ma = -mg\sin\theta$$

$$ma = -mg\sin\theta$$

$$\frac{d^2x}{dt^2} = -g\sin\theta$$

$$\frac{d^2x}{dt^2} + g\sin\theta = 0$$

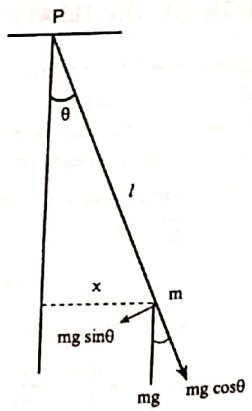


Fig: Simple Pendulum

For small θ, sinθ ≈ θ

$$\frac{d^2x}{dt^2} + g\theta = 0 \qquad \ldots (i)$$

If θ is small, x can be considered to be nearly equal to the length of arc, therefore,

$$x = l\theta$$

Substituting for x in equation (i)

$$l\frac{d^2\theta}{dt^2} + g\theta = 0$$

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\theta = 0 \qquad \ldots (2)$$

This equation is similar to equation of simple harmonic motion.

$$\frac{d^2\theta}{dt^2} + \omega^2\theta = 0$$

Thus, for small θ, simple pendulum executes simple harmonic motion with an angular frequency,

$$\omega = \sqrt{\frac{g}{l}}$$

$$\text{Time period, } T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{g}}$$

Limitations

1. No heavy point mass can be realized in practice.
2. The string cannot be weightless and perfectly inextensible as we assume.
3. The motion will not be simple harmonic if the angular displacement (θ) is large.

Physical or Compound Pendulum

A compound pendulum is just a rigid body of what ever shape, capable of oscillating about a horizontal axis passing through it. All real pendulums are physical pendulum.

Figure (1) shows a compound pendulum, free to rotate about a horizontal axis passing through the point of suspension S. In its normal position of rest, it's c.g. G, lies vertically below S. The distance between S and G is the length 'l' of the pendulum.

Let the pendulum be given small angular displacement 'θ', so that it's c.g. takes new position G'. Due to the weight mg acting vertically downward at G, it constitutes a restoring torque whose action is to tend to bring the pendulum back into its original position.

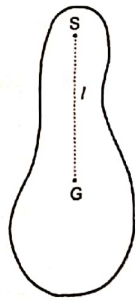


Fig. Compound Pendulum

The restoring torque is

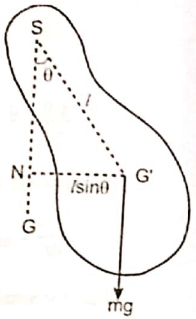
$$\tau = -mg(G'N)$$

$$= -mgl\sin\theta \quad \ldots (i)$$

-ve sign indicates that torque is oppositely directed to the displacement θ.

If I is the moment of inertia of the pendulum about the axis of suspension and α, its angular acceleration then,

$$\tau = I\alpha = I\frac{d^2\theta}{dt^2} \quad \ldots (ii)$$



From (1) and (2), $I\frac{d^2\theta}{dt^2} = -mgl\sin\theta$

Since, $\sin\theta = \theta - \theta^3/3! + \theta^5/5! + \ldots\ldots$ if θ is small, $\sin\theta \approx \theta$.

Therefore, $I\frac{d^2\theta}{dt^2} = -mgl\theta$ or $\frac{d^2\theta}{dt^2} = -\frac{mgl}{I}\theta$

The pendulum thus executes SHM

[Since, $\alpha = \frac{d^2\theta}{dt^2} \propto \theta$ i.e acceleration is directly proportional to displacement]. Therefore for large θ the motion will not be SHM.

or, $$\frac{d^2\theta}{dt^2} + \frac{mgl}{I}\theta = 0.$$

or $$\frac{d^2\theta}{dt^2} + \omega^2\theta = 0$$

Here,

$$\omega^2 = \frac{mgl}{I} \Rightarrow \omega = \sqrt{\frac{mgl}{I}}$$

$$\Rightarrow \quad f = \frac{1}{2\pi}\sqrt{\frac{mgl}{I}}$$

Time period, $T = \frac{1}{f} = 2\pi\sqrt{\frac{I}{mgl}}$

If k is the radius of gyration of the pendulum. Then from theorem of parallel axis, the total moment of inertia of the pendulum about the axis through point of suspension is,

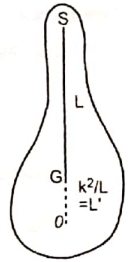
$$I = I_c + ml^2 = mk^2 + ml^2$$

Therefore, $T = 2\pi\sqrt{\frac{m(k^2 + l^2)}{mgl}} = 2\pi\sqrt{\frac{k^2/l + l}{g}}$

Thus the time period of compound pendulum is same as that of a simple pendulum of length $L = (k^2/l + l)$. This length 'L' is therefore called the length of an equivalent simple pendulum. Since $k^2 > 0$ i.e. $k^2/l > 0$ (positive), therefore, $L > l$, i.e. the length of equivalent simple pendulum (L) is always greater than l, the length of compound pendulum.

Centre of Suspension and Centre of Oscillation

Centre of suspension



The point at which the vertical plane passing through c.g. of the pendulum meets the axis of rotation is called its point or centre of suspension.

Centre of oscillation

A point 'O' on the other side of the pendulum, in the vertical plane passing through c.g. of the pendulum at a distance k^2/l from the c.g. is called centre or point of oscillation.

Inter changeability of centre of suspension and oscillation

The time period of compound pendulum is

$$T = 2\pi\sqrt{\frac{k^2/l + l}{g}}$$

Here, l = length of pendulum = distance between point of suspension and centre of gravity

And k^2/l = distance between point of oscillation and c.g.

Let, $\frac{k^2}{l} = l' \Rightarrow k^2 = ll'$

Then, $T = 2\pi\sqrt{\frac{l' + l}{g}}$

If we now invert the pendulum, so that it oscillate about the axis of oscillation, then.

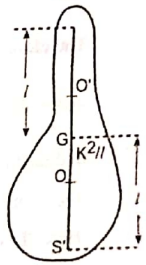
l' = length of pendulum = distance between point of suspension and c.g. and l = distance between point of oscillation and c.g.

The new time period, $T' = 2\pi\sqrt{\frac{k^2/l' + l'}{g}} = \sqrt{\frac{\frac{ll'}{l'} + l'}{g}}$ $[\because \frac{k^2}{l} = l']$

Therefore, $T' = 2\pi\sqrt{\frac{l + l'}{g}} = T$

Thus the centre of suspension and oscillation are reciprocal to each other. i.e. the time period of the pendulum is the same about either.

Existence of four co - linear points



Let the pendulum is in position as shown in figure. Here S and O are point of suspension and oscillation at a distance l and k^2/l from the c.g. Now take radius equal to SG = l with G as centre so as to cut the vertical axis at S' such that S' G = l.

Again take radius equal to GO = k^2/l and cut the vertical axis on the other side at O' such that GO' = k^2/l

Therefore the time period of pendulum about S' and O' be same as that of axis through S and O. Thus there are four points S, O, S' and O' collinear with the c.g. of the pendulum about which its time period is same.

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Maximum and minimum time periods of compound pendulum

The time period of compound pendulum is

$$T = 2\pi\sqrt{\frac{k^2/l + l}{g}} \quad \ldots (1)$$

Squaring both side $T^2 = \frac{4\pi^2}{g}(k^2/l + l)$

Differentiating w.r.t 'l'.

$$2T\frac{dT}{dl} = \frac{4\pi^2}{g}\left(-\frac{k^2}{l^2} + 1\right) \quad \ldots (2)$$

T will be minimum or maximum when $dT/dl = 0$

or, $4\pi^2/g\left(\frac{-k^2}{l^2} + 1\right) = 0 \Rightarrow (-k^2/l^2) + 1 = 0$

$\Rightarrow$ $k^2/l^2 = 1$

or, $\frac{k^2}{l} = l$

or, $k = \pm l$

or, $k = l$ [Since, k can't be negative]

Again differentiating equation (2) w.r.t. l

$$2T\frac{d^2T}{dl^2} + 2.\frac{dT}{dl}.\frac{dT}{dl} = \frac{4\pi^2}{g}\left(\frac{2k^2}{l^3}\right)$$

$$2T\frac{d^2T}{dl^2} + 2\left(\frac{dT}{dl}\right)^2 = \frac{4\pi^2}{g}\frac{2k^2}{l^3}$$

$\Rightarrow$ $\frac{d^2T}{dl^2} = \frac{4\pi^2}{gkT} > 0$ for $l = k$ and $dT/dl = 0$

This shows that $\frac{d^2T}{dl^2}$ is positive.

Therefore, T is minimum when $l = k$

and minimum time period, $T_{min} = 2\pi\sqrt{\frac{k^2/k + k}{g}} = 2\pi\sqrt{\frac{2k}{g}}$

and the time period is maximum when $l = 0$

For $l = 0$, $T = \infty$ or a maximum.

Here, the distance of point of suspension from c.g = l

and The distance of point of oscillation from c.g = $k^2/l = l^2/l = l$

Therefore, time period of compound pendulum is minimum when the distance of point of suspension from c.g = distance of point of oscillation from c.g

Inertia

The inability of a body to change its state of rest or of uniform linear motion by itself is called inertia. In linear motion mass is the measures of its inertia.

Moment of Inertia

The quantity which plays the same role in rotational motion as mass does in linear motion is called moment of inertia.

The moment of inertia of a rigid body about a given axis of rotation is the sum of products of the masses of various particles of the body and the square of their respective distances from the axis, that is,

$$I = \Sigma mr^2 = Mk^2$$

Where $M = m_1 + m_2 + \ldots\ldots$ is the total mass of the body.

k is the effective distance of its particle from axis called it's radius of gyration.

Radius of Gyration

It is defined as the distance from axis of rotation at which if all the masses of the body were supposed to be concentrated the moment of inertia of the body would same as the moment of inertia of the body with actual distribution of the particle.

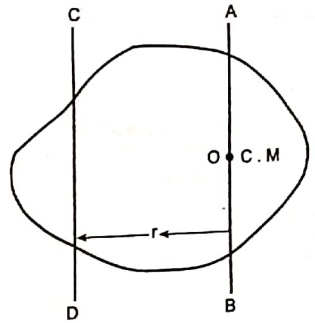
$$I = Mk^2 = \Sigma mr^2 = M\left(\frac{r_1^2 + r^2 + r_3^2 + \ldots\ldots + r_n^2}{n}\right)$$

$$K = \sqrt{\frac{r_1^2 + r_2^2 + r_3^2 + \ldots\ldots + r_n^2}{n}}$$

Thus it is also defined as the root mean square of the distances of the particles of the body from the axis of rotation.

Theorem of Parallel Axis

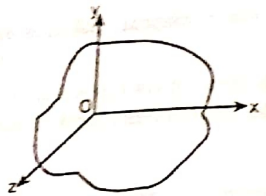
It states that the moment of inertia of a body about any axis is equal to its moment of inertia about a parallel axis passing through it's centre of mass, plus the product of the mass of the body and the square of the distance between the two axis.



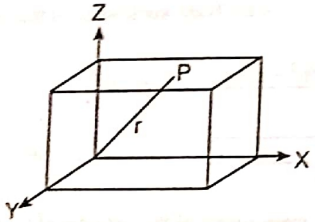
$$I = I_{cm} + Mr^2 = Mk^2 + Mr^2$$

Theorem of Perpendicular Axis

i. **For plane laminar body:** The moment of inertia of a plane lamina about an axis perpendicular to the plane is equal to the sum of the moments of the inertia about two mutually perpendicular axis in the plane of lamina which intersect that axis. i.e. $I_z = I_x + I_y$.



ii. **For three dimensional body:** It states that the sum of moments of inertia of a three dimensional body about its three mutually perpendicular axis is equal to twice the total moment of inertia of the body about the origin.



$$r^2 = x^2 + y^2 + z^2$$

i.e. $I_x + I_y + I_z = 2\sum mr^2$

Bar Pendulum

A bar pendulum is the simplest form of a compound pendulum, consist of uniform metal bar having equally spaced holes drilled along its length on either side of c.g.

The time period of bar pendulum is

$$T = 2\pi\sqrt{\frac{L}{g}} \qquad .....(i)$$

Where $L = k^2/l + l$, is the length of equivalent simple pendulum.

Therefore, $T = 2\pi\sqrt{\frac{k^2/l + l}{g}}$

Squaring both sides, $T^2 = 4\pi^2\left(\frac{k^2 + l^2}{lg}\right)$

$$\text{or,} \quad lT^2 = \frac{4\pi^2}{g}l^2 + \frac{4\pi^2}{g}k^2 \qquad ...(ii)$$

$$\text{or,} \quad \frac{4\pi^2}{g}l^2 - T^2 . l + \frac{4\pi^2}{g} . k^2 = 0$$

Which is the quadratic equation in l so it possesses two roots.

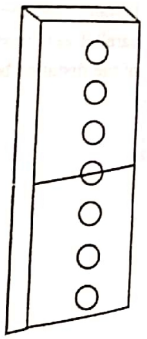


Fig: Bar Pendulum

If l_1 and l_2 are two roots

The sum of roots

$$l_1 + l_2 = -\frac{(-T^2)}{(4\pi^2/g)} = \frac{gT^2}{4\pi^2}$$

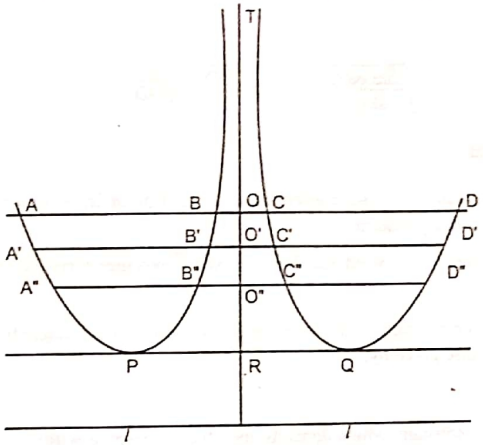
[Since, for quadratic equation $ax^2 + bx + c = 0$, the sum of roots = $-b/a$ and the product of roots = c/a]

$$\text{or,} \quad g = \frac{4\pi^2}{T^2}(l_1 + l_2) = \frac{4\pi^2}{T^2}L \qquad ...(iii)$$

And the product of roots $l_1 . l_2 = \left(\frac{4\pi^2}{g}k^2\right) / \frac{4\pi^2}{g} = k^2$

$$k^2 = l_1 . l_2 \Rightarrow k = \pm\sqrt{l_1 l_2} \qquad .....(4)$$

Here l_1 and l_2 are two values of l for one side for which the time period will be same. [or $l_1 = k^2/l$, $l_2 = l$ such that $l_1 + l_2 = L$, total length of pendulum (equivalent length of simple pendulum)



First the time periods of bar pendulum at different value of it's length are determined by setting it into small oscillation. A graph is then plotted between the lengths of pendulum along the x-axis and the time periods (T) along y - axis.

The experiment is repeated with the holes on the other side of the c.g. of the pendulum and a similar graph is drawn along side the first on the same graph paper.

A horizontal line ABOCD be drawn parallel to the x - axis so as to cut the curve in points A,B,C and D. These four points are collinear points having same time period.

Then AC = BD = L represents the length of equivalent simple pendulum. Now, the acceleration due to gravity can be determined by using equation (3)

[Here, $AC = AO + OC = l + \frac{k^2}{l} = L$ and $BD = BO + OD = \frac{k^2}{l} + l = L$]

Also in figure (2), $AO = l_1$ & $OC = l_2$ or $BO = l_2$ and $OD = l_1$

Therefore the radius of gyration can be calculated using equation (4).

Ferguson Method: (to find g and k)

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Again plotting a graph between lT^2 and l^2 in accordance with equation (2), it will be a straight line as shown in figure (3)

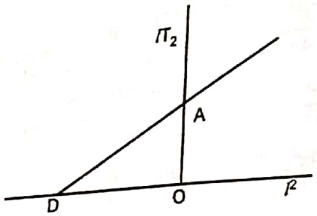


Figure (3)

Here, slope $= \frac{4\pi^2}{g} \Rightarrow g = \frac{4\pi^2}{\text{slope}}$

and intercept $= \frac{4\pi^2}{g} k^2$

or intercept = slope. $k^2 \Rightarrow k = \sqrt{\frac{\text{intercept}}{\text{slope}}} = \sqrt{OA.\frac{OD}{OA}} = \sqrt{OD}$

Torsion Pendulum

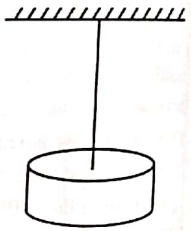
A rigid body, like a cylinder or a disc, suspended at its mid point by a long and thin wire to a rigid support, constitutes a torsoinal pendulum.

It is so called because when it is twisted and then released it executes torsional vibrations or oscillations about the wire as axis.

If the disc is turned through an angle θ, the wire too gets twisted through angle θ. This twisted wire exert a restoring torque on the disc given by.

$$\tau = -C\theta \qquad \ldots(i)$$

Where, C is called torsoin constant, which depends upon the property of wire.



The value of C is, $C = \frac{\pi\eta r^4}{2l}$; where, r = radius of wire, l = length of wire, η = modulus of rigidity of wire.

But $\tau = I\alpha$ $\qquad \ldots(ii)$

From (1) and (2) $I\alpha = -C\theta$ or $I.\frac{d^2\theta}{dt^2} = -C\theta \Rightarrow \frac{d^2\theta}{dt^2} + \frac{C}{I}\theta = 0$

or, $$\frac{d^2\theta}{dt^2} + \omega^2\theta = 0 \qquad \ldots(iii)$$

Here $\omega^2 = \frac{C}{I}$

$$\Rightarrow \quad \omega = \sqrt{\frac{C}{I}} \Rightarrow f = \frac{1}{2\pi}\cdot\sqrt{\frac{C}{I}}$$

Therefore, time period, $T = \frac{1}{f} = 2\pi\sqrt{\frac{I}{C}}$ $\qquad \ldots(iv)$

This is the time period of torsoinal pendulum.

The solution of equation (3) is $\theta = \theta_m \sin(\omega t + \phi)$.

The time period is

$$T = 2\pi\sqrt{\frac{I}{C}} \Rightarrow T^2 = 4\pi^2\frac{I}{C}$$

or, $$T^2 = 4\pi^2 I \times \frac{2l}{\pi\eta r^4} = \frac{8\pi I l}{\eta r^4}$$

$$\Rightarrow \quad \text{Modulus of rigidity, } \eta = \frac{8\pi I\, l}{T^2 r^4}$$

Here I is the moment of inertia of disc about the wire as axis.

[For disc, $I = \frac{1}{2}MR^2$]

Let I_1 be the moment of inertia of circular disc, then its time period is given by,

$$T_1 = 2\pi\sqrt{\frac{I_1}{C}}$$

or, $$T_1^2 = 4\pi^2\frac{I_1}{C} \qquad \ldots(v)$$

Now a circular ring is placed on the circular disc co-axially with the wire, then the period of oscillation for this combination is given by

$$T_2 = 2\pi\sqrt{\frac{I_1 + I_2}{C}}$$

Where, I_2 is the moment of inertia of circular ring,

or, $$T_2^2 = 4\pi^2\left(\frac{I_1 + I_2}{C}\right) \qquad \ldots(vi)$$

Subtracting equation (iv) from (v),

$$T_2^2 - T_1^2 = \frac{4\pi^2 I_2}{C} \qquad \ldots(vii)$$

$$= \frac{4\pi^2 I_2}{\pi\eta r^4} \times 2l$$

$\therefore$ $$\eta = \frac{8\pi I_2 l}{(T_2^2 - T_1^2)r^4} \qquad \ldots(viii)$$

Again, dividing equation (iv) by (vi)

$$\frac{T_1^2}{T_2^2 - T_1^2} = \frac{I_1}{I_2}$$

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$$\therefore \quad I_1 = \left(\frac{T_1^2}{T_2^2 - T_1^2}\right) I_2 \quad \ldots \text{(ix)}$$

And, moment of inertia of circular ring $I_2 = \frac{M(R_1^2 + R_2^2)}{2}$

R_1 and R_2 are the internal and external radius of the circular ring.

Note: Here to derive the relation for T no approximation is used like in the case of compound or simple pendulum. [Where we used $\sin\theta \sim \theta$ for small amplitude] The time period of a torsional pendulum therefore remains un effected even if the amplitude be large].

Difference between Compound and Torsion Pendulum

Compound Pendulum	Torsion Pendulum
1. Definition: A compound pendulum is a rigid body of whatever shape capable of oscillating about a horizontal axis passing through it.	1. Definition: A torsion pendulum is a rigid body capable of executing torsional vibration about an axis passing through it's c.g.
2. It oscillates about any axis passing through it other than c.g.	2. It oscillates about an axis along the wire passing through c.g.
3. It executes angular vibration.	3. It executes torsional vibration.
4. c.g. moves in an arc.	4. c.g remains fixed.
5. Time period $T = 2\pi\sqrt{\frac{k^2/l + l}{g}}$	5. Time period, $T = 2\pi\sqrt{I/C}$ where, $C = \frac{\pi\eta r^4}{2l}$
6. Time period is dependent of 'g'.	6. Time period is independent of g.
7. The approximation, for small θ, $\sin\theta \approx \theta$ is used to derive the time period. If θ is large the time period of compound pendulum is affected and the motion remain no longer simple harmonic. Therefore θ should be small.	7. No approximation is used to derive the time period. Therefore the time period of torsion pendulum remains unaffected even if the amplitude be large, provided the elastic limit of the suspension wire is not exceeded.

Solved Example

Example 1: If the relaxation time of a damped harmonic oscillator is 50 sec. Find the time period for which the amplitude and energy of oscillator falls to $\frac{1}{e}$ times the initial value.

Solution:

Here relaxation time $\tau = 50$ sec

We know,

$$x = x_m e^{\frac{-bt}{2m}} = x_m \cdot e^{\frac{-t}{2\tau}} \qquad \left[\tau = \frac{m}{b}\right]$$

Here, $x = \frac{x_m}{e}$ and $t = T$

$$\frac{x_m}{e} = x_m e^{\frac{-T}{2\tau}} \Rightarrow e^{-1} = e^{\frac{-T}{2\tau}}$$

$$\Rightarrow \quad \frac{T}{2\tau} = 1 \Rightarrow T = 2\tau = 2 \times 50 = 100 \text{ sec}$$

We know $E = E_0 e^{\frac{-bt}{m}} = E_0 e^{\frac{-t}{\tau}}$

When $E = \frac{E_0}{e}$, $t = T$

$$\therefore \quad \frac{E_0}{e} = E_0 e^{\frac{-t}{\tau}} \Rightarrow e^{-1} = e^{-\frac{t}{\tau}}$$

$$\Rightarrow \quad \frac{-t}{\tau} = 1 \Rightarrow T = \tau = 50 \text{ sec}$$

Example 2: The amplitude of lightly damped oscillator decreases by 3% during each cycle. What fraction of the energy of the oscillator is lost in each cycle.

Solution:

Given, Amplitude, $x_m = A - 3\% \text{ of } A = A - \frac{3A}{100} = \frac{97}{100}A = 0.97\,A$

$\therefore$ Energy lost in each full oscillation, $= \frac{1}{2} m\omega^2 A^2 - \frac{1}{2} m\omega^2 x_m^2$

$$= \frac{1}{2} m\omega^2 A^2 - \frac{1}{2} m w^2 (0.97\,A)^2$$

$$= \frac{1}{2} m\,\omega^2 A^2 [1 - 0.97^2] = E_0 \times 0.591 = 0.0571\,E_0.$$

Fraction of energy lost in each cycle $= \frac{\Delta E}{E_0} = 0.0591$ Ans.

Example 3: What is amplitude of the oscillation of a system acted by a sinusoidally varying force of amplitude 0.01 N and frequency 100 Hz acting on a system with the mass of 50 gm, restoring capacity 64 Nm^{-1} and damping constant 20gms^{-1}?

Solution

Amplitude of varying force (F_0) = 0.01 N

Frequency (f) = 100 HZ

$\therefore$ $\omega = 2\pi \times 100 = 628$ rad/sec

Mass (m) = 50 gm = 0.05 kg

Restoring constant (k) = 64 Nm^{-1}

Damping constant (b) = 20 gms^{-1} = 0.02 kg s^{-1}

$\therefore$ The amplitude of oscillation is given by

$$x_m = \frac{F_0/m}{\sqrt{(\omega o^2 - \omega^2)^2 + \left(\frac{b\omega}{m}\right)^2}}$$

$$\Rightarrow \quad \text{Here, } \omega_0 \sqrt{\frac{k}{m}} = \sqrt{\frac{64}{0.05}} = 35.78$$

$$= \frac{0.01/0.05}{\sqrt{(628^2 - 35.78^2)^2 + \left(\frac{0.02 \times 628}{0.05}\right)^2}} = \frac{0.2}{393103.87} = 5.087 \times 10^{-7} \text{ m}$$

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Example 4: A Damped oscillator has mas 250 gm, spring constant 85N/m and damping constant 70 g/s (i) How long does it take for the amplitude of the damped oscillator to drop to half of its initial value (ii) How long does it take for the mechanical energy to drop to half of its initial value?

Solution:

Mass of block m = 250 gm = 0.25 kg, spring constant k = 85 N/m

Damping constant b = 70 g/s = 0.07 kg/s

(i) The amplitude is $A = x_m e^{\frac{-bt}{2m}}$

According to questions, $A = \frac{x_m}{2} = x_m e^{\frac{-bt}{2m}}$

or, $t = \frac{2 \times 0.25}{0.07} \ell n\, 2 = 5$ second

(ii) The energy is, $E = E_0 e^{\frac{-bt}{m}}$

Here, $E = E_0/2 = E_0 e^{-bt/m}$

or, $e^{bt/m} = 2$

$\frac{bt}{m} = \ell n\, 2 \quad \Rightarrow t = \frac{m}{b} \ell n\, 2 = \frac{0.025}{0.07} \times \ell n 2$

$\therefore$ T = 2.5 second

Example 5: A vibrating system consists of mass of 400 gm and spring of spring constant 250 N/m. The frictional force acting on it is directly propositional to velocity with the coefficient 4 g/sec. What is the amplitude of oscillation if a periodic force of amplitude 1N and angular frequency of 20 Hz is applied to the system.

Solution:

Mass of body m = 400 gm = 0.4 kg, spring constant k = 250 N/m

$\therefore$ Natural frequency, $(\omega_0) = \sqrt{\frac{k}{m}} = 25$ rad/sec

Damping constant (b) = 4 gm/sec = 0.004 kg/sec.

Static force (F_0) = 1N

Frequency (f)= 20 Hz, $\omega = 2\pi f = 2\pi \times 20 = 125.6$

$$X_m = \frac{F_0/m}{\sqrt{(\omega_0^2 - \omega^2)^2 + \left(\frac{b\omega}{m}\right)^2}}$$

$$= \frac{1/0.4}{\sqrt{(25^2 - 125.6^2)^2 + \left(\frac{0.004 \times 125.6}{0.4}\right)^2}} = 1.65 \times 10^{-4}\ \text{m}$$

Example 6: A 0.4 kg mass is moving on the end of a spring with force constant k = 300 N/m and is acted on by a damping force F = – bv (i) calculate the frequency of oscillation if b = g kg /s (ii) for what value of 'b' will motion will be critically damped.

Solution:

(i) Damping constant b= 9 kg/s

$$\therefore \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2} = \sqrt{\frac{300}{0.4} - \left(\frac{9}{2 \times 0.4}\right)^2}$$

$\omega = \sqrt{750 - 126.6} = 24.97$

$\therefore$ Frequency $(f) = \frac{\omega}{2\pi} = \frac{24 - 97}{2 \times 3.24} = 3.98$Hz.

(2) For critical damping we have

$\frac{b}{2m} = \omega_0$

$\Rightarrow$ $b = 2m\,\omega_0$

$= 2 \times 0.4 \times \sqrt{\frac{k}{m}}$

$= 0.8 \times \sqrt{\frac{300}{0.4}} = 0.8 \times \sqrt{750} = 21.9$

$\therefore$ damping constant 'b' = 21.9 kg/s

Example 7: If the relaxation time of a damped harmonic oscillator is 50 sec, find the time in which the amplitude falls to $\frac{1}{e^3}$ of its initial value and energy of the system falls to $\frac{1}{e^4}$ of its initial value.

Solution:

Here, relaxation time $\tau = 50$ sec

The amplitude is $x = x_m e^{\frac{-bt}{2m}}$ $x_m e^{\frac{-t}{2\tau}}$ $\left[\because 2\delta = \frac{b}{m} = \frac{1}{\tau}\right]$

Here, $x = \frac{x_m}{e^3}$, t = T

$\therefore$ $\frac{x_m}{e^3} = x_m e^{\frac{-bt}{2m}} \Rightarrow e^{-3} = e^{\frac{-bt}{2m}}$

$\frac{-bt}{2m} = 3 \Rightarrow T = \frac{5m}{b} = b\tau = 6 \times 50 = 300$ sec

Again, Energy $E = E_0 = e^{\frac{-bt}{m}} = E_0 e^{\frac{-t}{\tau}}$

Here, $E = \frac{E_0}{e^4}$ t = T

$\therefore$ $\frac{E_0}{e^4} = E_0 e^{\frac{-T}{\tau}} \Rightarrow e^{-4} = e^{\frac{-T}{\tau}}$

$\therefore$ $T = 4\tau = 4 \times 50 = 200$ sec.

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Example 8: What value of b should be in the system having mass m = 0.1 kg and force constant 10N/m for the maximum amplitude to decay to 90% of its initial value in 5 cycles.

Solution:

m = 0.1 kg,

k = 10N/m

Time to decay the amplitude (t) $= 5T$

$$= 5 \times 2\pi\sqrt{\frac{m}{k}}$$

$$= 5 \times 2\pi\sqrt{\frac{0.1}{10}}$$

$$= 3.14 \text{ sec}$$

We have,

Amplitude $A = x_m e^{\frac{-bt}{2m}}$

or, $\frac{x_m}{2} = x_m e^{\frac{-bt}{2m}}$

or, $\frac{1}{2} = e^{\frac{-bt}{2m}}$

Taking ℓn on both side,

or, $\frac{-bt}{2m} = -0.693$

or, $b = \frac{0.693 \times 2 \times 0.1}{3.14}$

$= 0.044$ kg/sec.

Example 9: An oscillatory motion of a body is represented by $y = ae^{i\omega t}$ where y is displacement in time t, a is its amplitude and ω is angular frequency. Show that the motion is simple harmonic.

Solution:

Here, $y = ae^{i\omega t}$

$$\frac{dy}{dt} = (i\omega)\, ae^{i\omega t}$$

$$\frac{d^2y}{dt^2} = (i\omega)^2\, ae^{i\omega t} = -\omega^2 y$$

$$\frac{d^2y}{dt^2} + \omega^2 y = 0$$

This is the equation of simple harmonic motion. Therefore, the motion is simple harmonic.

Example 10: A linear spring whose force constant is 0.2 N/m hangs vertically supporting a 1kg mass at rest. The mass is pulled down a distance 0.2m and then released. What will be its maximum velocity? Also find the frequency of vibration.

Solution:

Spring constant (k) = 0.2 N/m

Mass (m) = 1kg

Amplitude (A) = 0.2m

1. $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{0.2}{1}} = 0.447$

   Maximum velocity $V_{max} = A\omega = 0.2 \times 0.447 = 0.089$ m/sec.

2. Frequency $(f) = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 0.071$Hz.

Example 11: A particle of mass 10 gm moves under a potential $V_x = 8\times10^5 x^2$ ergs/gm, where x is in cm. Deduce the time displacement relation when the total energy is 8×10^5 ergs.

Solution:

Here potential $V_x = 8\times10^5\, x^2$ ergs/gm

Potential energy acquired by particle,

$U = m\, V_x = 10\times8\times10^5 x^2$

$= 8\times10^6\, x^2$ ergs

Since P.E. is maximum at extreme position, that is when x = A (maximum displacement or amplitude). Where total energy is only P.E.

$\therefore \quad 8\times10^6 A^2 = 8\times10^5$

$$A^2 = \frac{1}{10}$$

$$A = \frac{1}{\sqrt{10}} \text{ cm}$$

Now, force acting on particle, $F = -\frac{dU}{dx} = -16\times10^6 x$

$$\Rightarrow \quad m\frac{d^2x}{dt^2} = -16\times10^6 x$$

$$m\frac{d^2x}{dt^2} + (16\times10^6)x = 0$$

$$\frac{d^2x}{dt^2} + \left(\frac{16\times10^6}{m}\right)x = 0$$

Therefore, $\omega^2 = \frac{16\times10^6}{m} = \frac{16\times10^6}{10} = 16\times10^5$

$$\omega = \sqrt{16\times10^5} = 400\sqrt{10}$$

The time displacement relation is given by

$$x = \frac{1}{\sqrt{10}}\sin(400\sqrt{10}\, t + \phi)$$

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Example 12: A sinusoidal wave travels along a string. The time for a particular point to move from maximum displacement to zero is 0.17 S. What are the

(i) Period and frequency?

(ii) The wavelength is 1.40 m; what is the wave speed?

Solution:

Let T be the time period of oscillation.

The time interval between two extreme (maximum) position $= \frac{T}{2}$

Then, the time interval between mean position and one maximum position $= \frac{T}{4}$

(i) Time period (T) = 4 × (Time for maximum displacement to zero) = 4 × 0.17 S = 0.68 sec.

and Frequency $(f) = \frac{1}{T} = \frac{1}{0.68} = 1.47$ Hz

(ii) $\lambda = 1.40$ m, wave speed $(v) = f\lambda = 1.47 \times 1.40 = 2.05$ m/S

Example 13: A small body of mass 0.1 kg is undergoing a SHM of amplitude 0.1m and period 2sec. (1) What is the maximum force on the body? (2) If the oscillations are produced in the spring, what should be the force constant?

Solution:

Mass of the body, m = 0.1kg

Amplitude, A = 0.1m

Time period, T = 2sec

i. $F_{max} = ?$

ii. k = ?

We have,

$$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow T^2 = \frac{4\pi^2 m}{k}$$

$$k = \frac{4\pi^2 m}{T^2} = 0.986 \text{ N/m}$$

$$F_{max} = kA = 0.0986 \text{ N.}$$

Example 14: If a particle moves in a potential energy field $U = U_0 - ax + bx^2$, where a and b are positive constants, a) obtain an expression for the force acting on it as a function of position b) At what point does the force vanish? c) Is this a point of stable equilibrium d) calculate the force constant, time period and frequency.

Solution:

a) The force acting on the particle is given by

$$F = -\frac{dU}{dx} = -\frac{d}{dx}(U_0 - ax + bx^2) = a - 2bx$$

b) The force vanishes at the point where $\frac{dU}{dx} = 0$

i.e. $a - 2bx = 0$

$$x = \frac{a}{2b}$$

c) Here, $\frac{d^2U}{dx^2} = \frac{d}{dx}\left(\frac{dU}{dx}\right) = \frac{d}{dx}(-a + 2bx) = 2b$, which is positive. The point $x = \frac{a}{2b}$ represents the point of minimum potential energy. It is therefore, a point of stable equilibrium.

d) We have $F = a - 2bx$, from this relation if is clear that F is a linear restoring force with force constant k = 2b.

Therefore, time period, $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{2b}}$

and frequency, $f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{2b}{m}}$

Example 15: Show that the time period of oscillation of loaded spring is $T = 2\pi\sqrt{\frac{x}{g}}$.

Solution:

If k is the force constant, we have

$mg = kx$

$$\frac{m}{k} = \frac{x}{g}$$

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{x}{g}}$$

Example 16: Show that if a uniform stick of length 'l' is mounted so as to rotate about a horizontal axis perpendicular to the stick and at a distance 'd' from the centre of gravity the period has minimum value when $d = 0.289l$.

Solution:

Here, Total length of stick = l

We have for a rod, $I = mk^2 = \frac{ml^2}{12} \Rightarrow k = \frac{l}{\sqrt{12}} = 0.289l$.

The time period of compound pendulum is minimum when length of pendulum (d) = radius of gyration (k)

Therefore, $d = k = 0.289l$.

Example 17: A uniform circular disc of radius 'R' oscillates in a vertical plane about horizontal axis. Find the distance of the axis of rotation from the centre for which the time period is minimum. Find the value of time period also.

Solution:

For a circular disc, $I = mk^2 = \frac{mR^2}{2}$

$$\Rightarrow \quad k = \frac{R}{\sqrt{2}}$$

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Time period is minimum when, $l = k = \frac{R}{\sqrt{2}}$

Since, $T = 2\pi\sqrt{\frac{\frac{k^2}{l}+l}{g}}$

Therefore,

$$T_{min} = 2\pi\sqrt{\frac{k+k}{g}}$$

$$= 2\pi\sqrt{\frac{2k}{g}}$$

$$= 2\pi\sqrt{\frac{2}{g}\cdot\frac{R}{\sqrt{2}}}$$

$$= 2\pi\sqrt{\frac{1.41R}{g}}$$

Example 18: A metal disc of radius 0.5m oscillates in its own plane about an axis passing through a point on its edge. Find the length of equivalent simple pendulum.

Solution:

Radius of disc (R) = 0.5m

Therefore, length of pendulum l = R = 0.5m

For disc we have,

$$I = mk^2 = \frac{mR^2}{2} \Rightarrow k = \frac{R}{\sqrt{2}}$$

Therefore, Length of equivalent simple pendulum $L = \frac{k^2}{l} + l$

$$L = \frac{R^2}{2}\cdot\frac{1}{R} + R = \frac{R}{2} + R = \frac{3R}{2}$$

L = 0.75m

Example 19: A thin straight uniform rod of length l = 1m and mass m = 160 gm hangs from a pivot at one end. (a) What is it's time period for small amplitude oscillation? (b) What is the length of a simple pendulum that will have the same time period?

Solution:

Here, length of rod, l' = 1m

Therefore, length of pendulum $l = \frac{l'}{2}$ = 0.5m

Mass, m = 160 gm = 0.16kg

Since, the moment of inerfra rod about an axis passing through one end is

$$I = \frac{1}{3}ml'^2 = \frac{1}{3}\times 0.16\times 1 = 0.053 \text{ kgm}^2$$

a. $T = 2\pi\sqrt{\frac{I}{mgl}} = 2\pi\sqrt{\frac{0.053}{0.16\times 9.8\times 0.5}} = 1.63$ sec.

b. $T = 2\pi\sqrt{\frac{k^2/l + l}{g}} = 2\pi\sqrt{\frac{L}{g}}$

$$\Rightarrow L = \frac{gT^2}{4\pi^2}$$

$$= \frac{9.8\times 1.63^2}{4\times\pi^2}$$

$$= 0.66\text{m}$$

Example 20: A wire has torsional constant of 2 Nm/rad. A disc of radius 5cm and mass 100 gm is suspended at it's centre. What is the frequency?

Solution:

Here,

C = 2Nm/rad, R = 5cm = 0.05m, m = 100 gm = 0.1kg

We have, $I = \frac{1}{2}mR^2 = \frac{1}{2}\times 0.1\times 0.05^2 = 1.25\times 10^{-4}$ kgm^2

and, $T = 2\pi\sqrt{\frac{I}{C}}$

$$= 2\pi\sqrt{\frac{1.25\times 10^{-4}}{2}}$$

= 0.04965 Sec.

Therefore,

Frequency $(f) = \frac{1}{T}$ = 20.14 Hz.

Example 21: A flat uniform circular disc has mass of 3kg and radius of 70cm. It is suspended in a horizontal plane by a wire attached to it's centre. If the disc is rotated through 2.5 radian about the wire, a torque of 0.6Nm is required to maintain that orientation. Calculate i) The rotational inertia of the disc about the wire ii) The torsion constant iii) The angular frequency of the torsion pendulum when it is set oscillating.

Solution:

Here, m = 3kg, R = 70cm = 0.7m, θ = 2.5 radian and τ = 0.6 Nm.

i. $I = \frac{1}{2}mR^2 = \frac{1}{2}\times 3\times 0.7^2 = 0.735$ kgm^2

ii. $\tau = C\theta \Rightarrow C = \frac{\tau}{\theta} = \frac{0.6}{2.5} = 0.24$ Nm/rad

iii. $\omega = \sqrt{\frac{C}{I}} = \sqrt{\frac{0.24}{0.735}} = 0.571$ rad/sec.

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Example 22: A solid sphere of radius 0.3m executes torsional oscillation of time period $2\pi\sqrt{12}$ sec at the end of suspension wire where upper end is fixed in a rigid support. If the torque constant of the wire be 6×10^{-3} Nm/rad, calculate the mass of the sphere.

Solution:

Here, $R = 0.3m$, $T = 2\pi\sqrt{12}$ sec, $C = 6\times10^{-3}$ Nm/rad, $m = ?$

We have, $T = 2\pi\sqrt{\frac{I}{C}} \Rightarrow I = \frac{CT^2}{4\pi^2} = \frac{6\times10^{-3}\times4\pi^2\times12}{4\pi^2}$

$I = 7.2\times10^{-2}$ kg m^2

Since, for sphere, $I = \frac{2}{5}mR^2$

$m = \frac{5I}{2R^2} = 2kg.$

THEORETICAL ANSWER QUESTIONS

1. Derive the differential equation of damped oscillatory motion and show that mechanical energy decrease with increase in time.
2. Explain clearly the terms free, damped and forced vibration. Develop the differential equation of a particle executing damped vibration in a medium. Explain the physical meaning of each term and each constant in the equations.
3. What is forced oscillation? Derive differential equation for forced oscillation and show that amplitude at resonance is inversely proportional to the damping constant of medium.
4. Explain how the amplitude of oscillation vary with time.
5. Explain the sharpness of resonance.
6. Give the necessary theory of forced vibration and deduce the condition for amplitude and velocity resonance. Relate sharpness of resonance with quality factor.
7. How does the amplitude of forced oscillation vary with the frequency of external period force. Derive the relation for sinusoidally varying periodic external force.
8. Derive an expression for the time period of a physical pendulum and establish the interchangeability of centre of oscillation and suspension.
9. Distinguish between simple and physical pendulum. Deduce the time period of a compound pendulum and show that it is minimum when the length of the pendulum is equal to radius of gyration.
10. Derive an expression for the time period and radius of gyration of compound pendulum.
11. What are the limitation of a simple pendulum?
12. Why compound pendulum is preferred than simple pendulum?
13. Derive time period of torsion pendulum and find an expression for modulus of rigidity of the suspension wire.
14. In which condition time period of compound pendulum is maximum and minimum.

15. Describe with necessary theory how you will determine the value of the modulus of rigidity of a metal in laboratory using torsional pendulum.
16. For a compound pendulum prove that the minimum time period is obtained if the point of suspension and point of oscillation are equidistance from C.G.
17. Point out similarities and dissimilarities between the oscillations of bar pendulum and torsional pendulum.
18. Show that motion of a compound pendulum is angular simple harmonic and derive its time period.
19. Differentiate between damped oscillation and forced oscillation. Show that the center of oscillation of compound pendulum lies beyond the center of gravity.
20. Derive an expression for the period and radius of gyration of a compound pendulum and show that centre of oscillation and centre of suspension are interchangeable.
21. Discuss different types of oscillations and hence determine the time period of compound pendulum and compare it with time period of simple pendulum.
22. Derive a relation to determine the radius of gyration of a compound pendulum. Why determination of the acceleration due to gravity is more accurate from a compound pendulum than a simple pendulum.
23. What is compound pendulum? Deduce the expression for the time period of a compound pendulum and compare this period with torsional pendulum.
24. Differentiate between linear and angular harmonic motion. Show that the motion of torsion pendulum is angular harmonic motion. Also find it's time period.
25. Obtain an expression for the time period of a compound pendulum and show that it's time period is unaffected by the fixing of a small additional mass to it's centre of suspension.
26. Show that the period of torsion pendulum remains unaffected even if the amplitude be large, provided that the elastic limit of the wire is not exceeded.
27. Show that there are four collinear points within compound pendulum having same time period. Give their physical significance.
28. Deduce the expression for the time period of a compound pendulum and formulate the equivalent length of the simple pendulum.

NUMERICAL PROBLEMS

1. A 2 kg mass hangs from a spring. A 200 gm body hung below the mass stretches the spring 2 cm further. If the 200 gm body is removed and the mass in set in oscillation, find the period of the motion. **[Ans: 0.897 Sec.]**
2. A 500 gm block oscillates on the end of a spring whose force constant in k = 50 N/m. The mass moves in a fluid which offers a resistive force F = – bv where b = 0.15 kg/m. find the period of the motion. **[Ans: 0.628 Sec.]**
3. A mass of 1kg is suspended from a spring of spring constant 25 N/m. If the undamped frequency is $\frac{2}{\sqrt{3}}$ times the damped frequency, what will be the damping factor? **[Ans: 5 kg/sec]**

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4. A 0.5 kg block is attached to a spring (k = 12.5 N/m). The damped frequency is 0.2% lower than the natural frequency (a) what is damping constant? (b) How does the amplitude vary in time? (c) What is critical damping constant? [Ans: 0.316 kg/sec, 3.16 kg/sec]
5. A meter stick swings about pivot at one end, at distance 'l' from the stick's centre of mass. Calculate the period of oscillation using parallel axis theorem. What would be the length of the simple pendulum that would have the same period? [Ans: 1.63 Sec, 0.66 m]
6. A physical pendulum consists of a meter stick that is pivoted at a small hole drilled through the stick at a distance x from C.G. The period of the oscillation is observed to be 2.5 Sec. Find the distance x. [Ans: 5.57 cm]
7. A meter stick swings about a pivot point at its one end. What is the time period of oscillation? What is the distance of the pivot from the centre of oscillation of the stick? [Ans: 1.63 Sec, 0.66 m]
8. A thin rod of mass 0.1kg and length 0.1m is suspended by a wire, which passes through center and is perpendicular to its length. The wire is twisted and the rod is set into oscillation. The period is found to be 2 sec. When a flat body in the shape of an equilateral triangle is suspended similarly through its center of mass, the period is found to be 6 sec. Find the moment of inertia of triangle about this axis. [Ans: 7.5×10^{-4} kgm²]
9. A uniform circular disc of a diameter 20cm vibrates about a horizontal axis perpendicular to its plane and at a distance of 5cm from the center. Calculate, the period of oscillation and the equivalent length of simple pendulum. [Ans: 0.779 sec, 0.151 m]
10. A metallic disc of mass 0.5kg and radius of 0.1m is suspended by wire of length 40cm and radius 1mm. The torsion oscillation of the disc is found to have a period of 2.5sec. Find the modulus of rigidity of wire. [Ans: 4.02×10^{9} N/m²]
11. A uniform circular disc whose radius R is 12.6cm is suspended as a physical pendulum from a pivot point on it's rim a) What is its period? b) At what radial distance r < R, is there a pivot point that gives the same period. [Ans: 0.873 Sec, 6.3 cm]
12. A solid cylinder of mass 500 gm is suspended by a thin wire in such a way that its axis remains vertical. If the period of torsion oscillation be 2.5 seconds what will be the period if the cylinder is replaced by a solid sphere of same radius and mass 400 gm? [Ans: 2 Sec]

□□□

Chapter 2

WAVE MOTION

CHAPTER OUTLINE

After completion of this chapter, you will be able to know:

- Introduction of wave, wave velocity and particle velocity, types of waves and their applications, Speed of transverse wave in stretched string, energy, power and intensity of plane progressive wave, standing wave and resonance, sonometer,.

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INTRODUCTION

Wave motion is a form of disturbance which travels through a medium due to the repeated periodic motion of the particles of the medium about their mean positions. There is transfer of energy from one point to another through successive vibration of particles.

Characteristics of Wave Motion

1. The wave motion is the disturbance produced in the medium due to repeated periodic motion of the particles of the medium.
2. Wave travels in forward direction while the particles of the medium vibrate about their mean position.
3. In each vibration, a particle hands over some of its energy to the next particles which it receives from previous particle.
4. The velocity of wave (called wave velocity) is different from the velocity of particle (called particle velocity).

   wave velocity $v = f\lambda$

   particle velocity $u = \frac{dy}{dt} = \frac{d}{dt}(A\sin\omega t) = A\omega\cos\omega t$

6. Each particle in the medium starts vibrating a little later than the preceding one.
7. Wave velocity is uniform while particle velocity is different at different position. It is maximum at mean position and minimum at extreme position.

TYPES OF WAVES

a. On the basis of modes of vibration of the particle of medium there are two types of waves.

1. **Transverse wave:** When the particles of the medium vibrate about their mean position in a direction perpendicular to the direction of propagation of disturbance, the wave is called transverse wave. In a transverse wave motion, disturbance travels in the form of crest and trough.

   Example: transverse wave produced in water when a stone is thrown in a pond, transverse wave produced in a stretched string when one end is fixed at wall and other end is moved up and down.

2. **Longitudinal wave**

   If the particles of the medium propagating the wave motion vibrate in the direction of propagation, the wave is called longitudinal wave. In longitudinal wave motion, disturbance travels in the form of compression and rarefactions.

   Example: Sound wave in air, longitudinal wave produced in spring when it is suddenly compressed and released.

b. on the basis of necessity of medium for propagation there are two types of waves.

1. **Mechanical wave:** The wave which require medium to propagate are called mechanical ware, Eg. Sound waves, water waves, seismic wave. It can be longitudinal or transverse in nature.
2. **Electromagnetic wave:** The wave which does not require medium to propagate are called non-mechanical or electro magnetic wave.

   Eg: light waves, radio waves, x - ray, γ - ray etc.

   It is transverse in nature.

c. There are two types of wave on the basis of state of motion of particle of the medium.

1. **Progressive wave:** A wave that travels from one region to another region of medium through the successive vibration of the particle of the medium is called progressive wave.

   In progressive wave motion there is transfer of energy from one point to another and no particle of the medium is permanently at rest. All those waves discussed above falls on this category.

2. **Stationary waves:** When two progressive waves of the same amplitude and frequency travelling through a medium with the same speed but in opposite direction superimpose on each other, stationary or standing wave is formed. The wave is so called stationary because it does not travel in either direction and all the particles of the medium are permanently at rest. There is not flow of energy along the wave.

EQUATION OF PLANE PROGRESSIVE WAVE

Consider a wave travelling along +ve x-axis as shown in figure. Let a, ω and T be the amplitude, angular velocity and time period of the wave. This means all the particles vibrating in the medium have same amplitude a, same angular velocity ω and same time period T.

Let y be the displacement of the particle vibrating at 'O'. Then it's displacement at any instant 't' is given by.

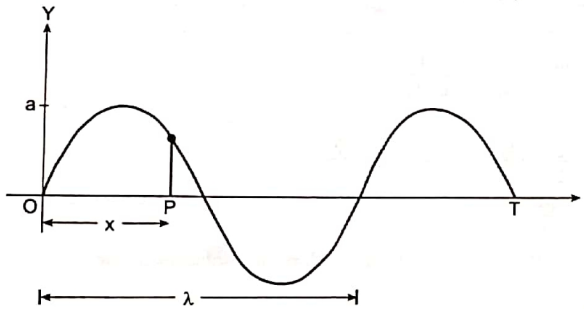
$$y = a\sin\omega t \quad \ldots (i)$$

Consider another particle at point 'P' at a distance x from 'O'.

This means path difference between the particle at 'P' and 'O' is x and let corresponding phase difference be ϕ.

Then the displacement of the particle at 'P' at same time 't' is,

$$y = a\sin(\omega t - \phi) \quad \ldots (ii)$$



We know, path difference, λ ⇒ phase difference, 2π

path difference, 1 ⇒ phase difference, 2π/λ

path difference, x ⇒ phase difference, $\frac{2\pi}{\lambda}.x$

$$\text{Therefore, } \phi = \frac{2\pi}{\lambda}.x \quad \ldots (iii)$$

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Substituting ϕ for equation (2) $y = a \sin(\omega t - \frac{2\pi}{\lambda} . x)$ ... (iv)

$\Rightarrow \quad y = a\sin(\omega t - kx)$

where $k = \frac{2\pi}{\lambda}$ is called the wave number or propagation constant.

or, $\quad y = a \sin(2\pi ft - \frac{2\pi}{\lambda} . x)$

$= a \sin \frac{2\pi}{\lambda} (f\lambda . t - x)$

$= a \sin \frac{2\pi}{\lambda} (vt - x) = a \sin k (vt - x)$ ... (v)

Equation (4) or (5) represents plane progressive wave. If the wave travels along -ve x direction or from right to left, it will arrive at 'P' before 'O'. So the vibration at P will lead 'O' and wave equation becomes.

$$y = a \sin(\omega t + kx) = a \sin \frac{2\pi}{\lambda} (vt + x)$$

RELATION BETWEEN WAVE VELOCITY AND PARTICLE VELOCITY

The equation of plane progressive wave is

$y = a \sin(\omega t - kx)$ ... (i)

Differentiating this equation w.r.t 't' and x.

$\frac{dy}{dt} = a\omega \cos(\omega t - kx)$ ... (ii)

and $\frac{dy}{dx} = a(-k) . \cos(\omega t - kx)$ ... (iii)

Dividing equation (2) by equation (3)

$$\frac{\frac{dy}{dt}}{\frac{dy}{dx}} = -\frac{\omega}{k} \Rightarrow \frac{dy}{dt} = -\frac{\omega}{k}\frac{dy}{dx}$$

or, $\frac{dy}{dt} = -\frac{2\pi f}{2\pi/\lambda}\frac{dy}{dx} = -f\lambda\frac{dy}{dx} = -v\frac{dy}{dx}$

$$\frac{dy}{dt} = -v\frac{dy}{dx}$$

Therefore, particle velocity = - (wave velocity) × slope of displacement curve.

Particle acceleration

The equation of plane progressive wave is, $y = a \sin(\omega t - kx)$

Particle velocity, $v_p = \frac{dy}{dt} = a\omega \cos(\omega t - kx)$

Particle acceleration, $a_p = \frac{d^2y}{dt^2} = -a\omega^2 \sin(\omega t - kx)$

$a_p = -\omega^2 y.$

Therefore, maximum velocity, $(v_p)_{max} = a\omega$

and maximum acceleration, $(a_p)_{max} = -\omega^2 a$

Differential Equation of Wave Motion

The equation of plane progressive wave is,

$y = a \sin(\omega t - kx)$ ... (i)

Differentiating w.r.t. time 't'

$\frac{dy}{dt} = a\omega \cos(\omega t - kx)$

Again differentiating w.r.t. to time 't'

$\frac{d^2y}{dt^2} = -a\omega^2 \sin(\omega t - kx) = -\omega^2 y$ ... (ii)

Differentiating equation (1) w.r.t. x.

$\frac{dy}{dx} = a(-k) \cos(\omega t - kx)$

Again differentiating with 'x'

$\frac{d^2y}{dx^2} = -a(-k)^2 \sin(\omega t - kx) = -k^2 y$ ... (iii)

Dividing equation (2) by (3)

$$\frac{d^2y/dt^2}{d^2y/dx^2} = \frac{\omega^2}{k^2} = \left(\frac{2\pi f}{2\pi/\lambda}\right)^2 = (f\lambda)^2 = v^2$$

$\frac{d^2y}{dt^2} = v^2\frac{d^2y}{dx^2}$ ... (iv)

This is the general (differential) equation of wave motion. It's solution is, $y = a \sin(\omega t - kx)$.

ENERGY, POWER, INTENSITY OF PLANE PROGRESSIVE WAVE

In progressive wave there is no transfer of the medium but there is always transfer of energy in the direction of propagation of the wave. Energy transfer in wave is done by vibrating particle of the medium.

The potential energy is given by

$$P.E. = -\int_o^y F.dy = -\int_o^y -kydy = \frac{ky^2}{2} = \frac{1}{2}ky^2$$

Here k is force constant and given by $k = m\omega^2$

Therefore, $P.E = \frac{1}{2}m\omega^2 y^2 = \frac{1}{2}m\omega^2 a^2 \sin^2(\omega t - kx)$ ... (i)

Potential energy per unit volume $= \frac{1}{2}\rho\omega^2 a^2 \sin^2(\omega t - kx)$

The kinetic energy is given by

$$K.E. = \frac{1}{2}mv^2 = \frac{1}{2}m\left\{\frac{d}{dt}[a \sin(\omega t - kx)]\right\}^2$$

$$K.E. = \frac{1}{2}m a^2 \omega^2 \cos^2(\omega t - kx)$$

K.E. per unit volume $= \frac{1}{2}\rho a^2 \omega^2 \cos^2(\omega t - kx)$

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Therefore, Total energy per unit volume $= \frac{1}{2}\rho a^2\omega^2$

Average K.E. over a complete wave length

$$= \frac{1}{\lambda}\int_0^{\lambda} \frac{1}{2}\rho a^2\omega^2 \cos^2(\omega t - kx)\,dx$$

$$= \frac{1}{\lambda}\left(\frac{1}{2}\rho a^2\omega^2\right)\int_0^{\lambda}\left[\frac{1}{2}(1+\cos 2(\omega t - kx))\right]dx$$

$$= \frac{1}{4}\frac{\rho a^2\omega^2}{\lambda}\left[\int_0^{\lambda} dx + \int_0^{\lambda}\cos\{2(\omega t - kx)\}\,dx\right]$$

$$= \frac{1}{4}\frac{\rho a^2\omega^2}{\lambda}\times\lambda = \frac{1}{4}\rho a^2\omega^2 = \frac{1}{2}\text{ total energy,}$$

$=$ constant

Average P.E. over a complete wave length.

$$= \frac{1}{\lambda}\int_0^{\lambda}\frac{1}{2}\rho a^2\omega^2\sin^2(\omega t - kx)\,dx$$

$$= \frac{1}{2}\frac{\rho a^2\omega^2}{\lambda}\int_0^{\lambda}\left[\frac{1}{2}(1-\cos 2(\omega t - kx))\right]dx$$

$$= \frac{1}{4}\frac{\rho a^2\omega^2}{\lambda}.\lambda = \frac{1}{4}\rho a^2\omega^2 = \frac{1}{2}\text{ Total energy}$$

$=$ constant.

This shows that the average K.E per unit volume and P.E per unit volume are equal and each equal to half the total energy per unit volume.

Here, it is seen that both K.E and P.E depend upon the values of x and t, but K.E per unit volume, P.E. per unit volume and Total energy density are independent of either i.e. constants.

Power of progressive wave: The total energy transported by wave per unit time is called power.

Here, total energy per unit volume.

$$E/V = \frac{1}{2}\rho a^2\omega^2$$

$$\Rightarrow \quad E = \frac{1}{2}V\rho a^2\omega^2,$$

where V is the volume of medium it is given by, $V = A \times l$

A - cross sectional area through which wave travels.

l - is the distance travelled by wave.

Also, $l = vt$, v is the speed of wave and t is the time

Therefore, $V = A \times vt$

Substituting V for E,

$$E = \frac{1}{2}A\,v\,t\,\rho\,a^2\omega^2$$

Rate of energy transfer or power $P = E/t = \frac{1}{2}A\,v\,\rho\,a^2\omega^2$

Intensity of progressive ware:

The rate of flow of energy per unit area of cross-section through which wave travels is called energy current or energy flux or intensity of progressive wave.

$$I = \frac{P}{A} = \frac{1}{A}\left(\frac{1}{2}A\,v\,\rho\,a^2\omega^2\right)$$

$$I = \frac{1}{2}v\,\rho a^2\omega^2$$

Hence, both power and intensity of travelling wave is directly proportional to square of amplitude.

VELOCITY OF TRANSVERSE WAVE ALONG A STRETCHED STRING

Consider a portion PABQ of a stretched string in which a transverse wave is travelling from left to right with velocity v. consider a small string element AB of length ΔL forming on arc of a circle of radius 'R' and subtending an angle AOB = θ at the centre of that circle.

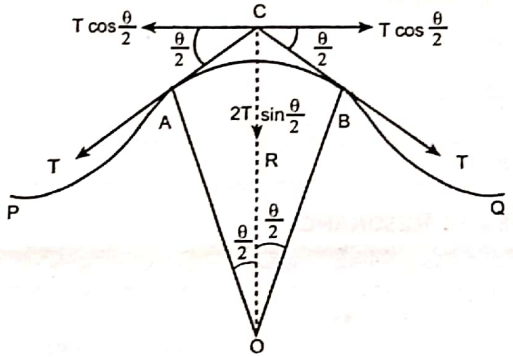

Figure 1: Snapshot of Transfer Wave Along a Stretched String

Let T be the tension at A and B. The directions of these tensions are tangential to the element at A and B.

Resolving the tension at A into two rectangular components T sin θ/2 as vertical and T cos θ/ 2 as horizontal component.

Similarly the tension at B can be resolved into two components T sin θ/2 as vertical and T cos θ/2 as horizontal component.

The horizontal components are equal and opposite so they cancelled each other and the vertical component gets added since they are along same direction along CO to give resultant tension,

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$T \sin \theta/2 + T \sin \theta/2 = 2T \sin \theta/2$

As θ is small, $\sin \theta/2 \approx \theta/2$

Therefore, resultant tension $= 2\,T.\,\theta/2 = T\theta$ . . . (i)

Since the string segment AB is moving in an arc of circle. This resultant tension is balanced by the centripetal force towards the centre of the circle.

i.e. $T\,\theta$ = centripetal force $= \frac{mv^2}{R}$ . . . (ii)

Where m = mass of string element, If μ is mass per unit length then $m = \mu\,\Delta L$

Also from figure $\Delta L = R\theta \Rightarrow \theta = \frac{\Delta L}{R}$

Substituting m and ΔL for equation (2),

$$T.\frac{\Delta L}{R} = \frac{\mu.\,\Delta L}{R}.\,v^2$$

$$\Rightarrow \quad T = \mu.\,v^2$$

Therefore, $v = \sqrt{\frac{T}{\mu}}$ . . . (iii)

Which is the expression for velocity of transverse wave along stretched string. This shows that velocity of wave depends upon the tension and linear density of the string, not on the frequency of the wave.

Frequency of wave is given by, $f = \frac{v}{\lambda} = \frac{1}{\lambda}\sqrt{\frac{T}{\mu}}$.

The average rage of energy transport,

$$\text{Power (P)} = \frac{1}{2}\,\rho A v \omega^2 a^2$$

$$= \frac{1}{2}\,\frac{m}{A \times l}\,A v \omega^2 a^2$$

$$= \frac{1}{2}\,\mu v \omega^2 a^2$$

STANDING WAVE AND RESONANCE

When a string stretched between two clamps is made to oscillate at certain frequency, the interference produces standing wave pattern with nodes and large antinodes. This condition is called resonance and the string is said to resonate at these certain frequencies called resonant frequencies.

Let a string be stretched between two clamps by a fixed distance 'l'. A node must exist at each of its ends because each end is fixed and cannot oscillate. A simplest pattern that meets this requirement is shown in fig (1).

For this pattern, $\lambda/2 = l$

$\lambda = 2l$

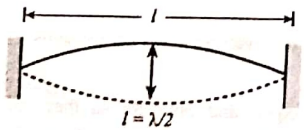

Figure (1)

A second simple pattern meeting the requirement of nodes at fixed string is shown in figure (2). For this pattern.

$\lambda = l = 2\frac{l}{2}$

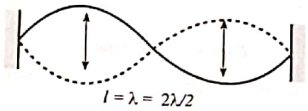

Figure (2)

A third pattern is as shown in figure (3)

Here $l = 3\lambda/2$

$\lambda = 2\frac{l}{3}$

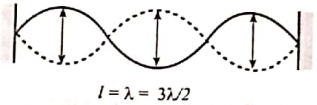

Figure (3)

Continuing this process we observe that if the left going and right going traveling waves are to set up resonance by their interference, they must have the wave length

$\lambda = \frac{2l}{n}$, n = 1, 2, 3 .....

The resonant frequencies are given by the relation,

$f = \frac{v}{\lambda} = n\,.\,\frac{v}{2l}$ for n = 1, 2, 3, ...(1)

The oscillation made with lowest resonant frequency $f_1 = \frac{v}{2l}$ for n = 1 is called fundamental mode or first harmonic. The second harmonic is the oscillation mode with n = 2 and so on.

SONOMETER

A sonometer consist of a hollow sounding box. One end of the wire is fixed at one end while the second end passes over a pulley fixed at the other end of box, and carries a hanger on which slotted weights can be slipped. The vibrating length of the wire can be adjusted by means of two sharp knife edges, over which the wire passes. The horse-shoe magnet is placed at the middle of the wire. An alternating current of low voltage is passed through the wire, when a current carrying conductor is placed in an uniform magnetic field, it will experience a magnetic force and is deflected. According to fleming's left hand rule, if the current flows from left to right and magnetic field is directed in the direction opposite to our face, the wire experience upward force and is deflected upward. After next half cycle of a.c., the current flows from right to left and the wire is deflected downward and so on. In this way, the wire moves up and down and its vibrations are maintained.

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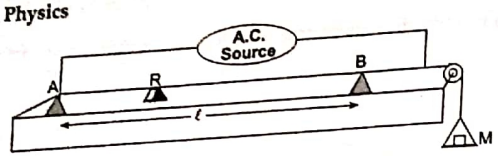

Figure 4 : Sonometer

When the wire resonates the frequency of a.c. mains is equal to the frequency of vibration of the string.

According to laws of transverse vibration of string, the frequency of fundamental mode is,

$$f = \frac{1}{2l}\sqrt{\frac{T}{\mu}} \quad \ldots (i)$$

Where, l is length of the vibrating pigment of the wire

$T = mg$, is the tension in the wire, m is mass placed in the pan and μ is mass per unit length of wire.

$$T = 4\mu f^2 . l^2 \quad \ldots (ii)$$

If d be the diameter of the wire, then area of cross-section of wire $= \frac{\pi d^2}{4}$

Volume of wire = Area of cross-section × Length

∴ Mass of wire = Volume × Density

$$= \frac{\pi d^2}{4} \times l \times \rho$$

∴ Mass per unit length of wire $(\mu) = \frac{\pi d^2}{4} \times \rho$

Here, ρ = density of material of wire.

Solved Example

Example 1: The equation of transverse wave travelling in a rope is given by $y = 10 \sin \pi (0.01 x - 2t)$ centimeters. Find the amplitude, frequency, velocity and wave length of the wave.

Solution:

The given equation is $y = 10 \sin \pi (0.01x - 2t)$

Comparing this equation with the general displacement equation for a transverse wave,

$y = A \sin (kx - \omega t)$

a. $A = 10$ cm

b. $k = 0.01\pi \Rightarrow \frac{2\pi}{\lambda} = 0.01\pi \Rightarrow \lambda = \frac{2}{0.01} = 200$ cm $= 2$m

c. $\omega = 2\pi \Rightarrow 2\pi f = 2\pi \Rightarrow f = 1$ Hz

d. The wave speed is $v = f\lambda = 2$m/s

Example 2: Calculate the wavelength, frequency, speed of the wave and maximum particle velocity in the wave represented by $y = 20 \sin\pi (2t - 0.05x)$, the values of x and y are in centimeters.

Solution:

We have,

$$y = 20 \sin\pi (2t - 0.05x) \quad \ldots (i)$$

Comparing equation (1) with the equation of progressive wave $y = A \sin(\omega t - kx)$

$A = 20$ cm $= 20 \times 10^{-2}$ m

$\omega = 2\pi \Rightarrow 2\pi f = 2\pi \Rightarrow f = 1$ Hz

$k = \pi\, 0.05\ (cm^{-1}) = \pi\, 0.05 \times 100\ m^{-1} = 5\pi\ m^{-1}$

i. Wave length, $\lambda = \frac{2\pi}{k} = \frac{2\pi}{5\pi} = 0.4$ m

ii. Speed of wave, $v = f\lambda = 1 \times 0.4 = 0.4$ m/sec

iii. Maximum particle velocity $(V_{max}) = A\omega = 20 \times 10^{-2} \times 2\pi = 1.25$ m/sec

Example 3: What is amplitude, the wave length and the velocity of the wave represented by $y = 5\sin(6\pi t + 4x)$. Where distance and time are measured in SI units?

Solution:

$$y = 5 \sin (6\pi t + 4x) \quad \ldots (i)$$

Comparing this equation with

$$y = A \sin (\omega t + kx) \quad \ldots (ii)$$

we get,

$A = 5$m, $\omega = 6\pi$, $k = 4$

we have,

$\omega = 2\pi f$

$6\pi = 2\pi f$

∴ $f = 3$ Hz

Also, $k = \frac{2\pi}{\lambda}$; $\lambda = \frac{2p}{4} = 1.57$ m

Velocity of wave (v) $= f\lambda$

$= 3 \times 1.57$

$= 4.712$ m/s.

Example 4: A boy claps his hands once every second and hears the echo from a distance building. He hears the echo of each clap mid-way between it and the next clap. If the velocity of sound is 340 m/s what is the distance of the building from the body?

Solution:

V = 340 m/s, Let d be the distance between building and the boy.

Total distance travel by the sound (D) = 2d

Time interval (t) $= \frac{1}{2}$ sec. = 0.5 sec.

We have,

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$$v = \frac{D}{t}$$

$$340 = \frac{2d}{0.5}$$

$$d = \frac{340 \times 0.5}{2} = 85 \text{ m}$$

Example 5: **Calculate the wave length, frequency, speed of the wave and the maximum particle velocity in the wave represented by, $y = 10 \sin(8\pi t - 0.08\,\pi x)$. The value of x and y are in CGS system.**

Solution:

$$y = 10 \sin(8\pi t - 0.08\,\pi x) \quad \ldots (i)$$

Comparing this equation with

$$y = A \sin(\omega t - kx) \quad \ldots (ii)$$

We get,

$$A = 10 \text{ cm} = 10 \times 10^{-2} \text{ m}, \ \omega = 8\pi, \ k = 0.08\,\pi$$

(i) Frequency $(f) = \frac{\omega}{2\pi} = \frac{8\pi}{2\pi} = 4$ Hz

(ii) $k = \frac{2\pi}{\lambda}$

or, Wave length $(\lambda) = \frac{2\pi}{k} = \frac{2\pi}{0.08\,\pi} = 25 \text{ cm} = 25 \times 10^{-2}$ m

(iii) Speed of the wave $(v) = f\lambda = 4 \times 25 \times 10^{-2} = 1$ m/sec

(iv) Maximum particle velocity $(v_{max}) = A\,\omega = 10 \times 10^{-2} \times 8\,\pi = 2.512$ m/sec

Example 6: Here are the equation of three waves

(i) $y(x, t) = 2 \sin(4x - 2t)$

(ii) $y(x,t) = 2 \sin(3x - 4t)$ and 3) $y(x, t) = 2 \sin(3x - 3t)$. Write down the equation in accordance with (i) wave speed (ii) maximum particle velocity (maximum transverse speed) in descending order.

Solution:

For first wave, $y(x, t) = 2 \sin(4x - 2t)$

Comparing this equation with $y = A \sin(kx - \omega t)$

$$A = 2, \ \omega = 2, \ k = 4$$

Wave speed $= \frac{\omega}{k} = \frac{2}{4} = 0.5$

Maximum particle velocity $= A\omega = 2 \times 2 = 4$

For second wave,

$$A = 1, \ k = 3, \ \omega = 4$$

Wave speed $= \omega/k = 4/3 = 1.33$

Maximum particle velocity $= A\omega = 2 \times 4 = 8$

For third wave,

$$A = 2, \ K = 3, \ \omega = 3$$

Wave speed $= \omega/k = 3/3 = 1$

Maximum particle velocity $= A\omega = 2 \times 3 = 6$

The descending order is, 2nd, 3rd and 1st wave.

Example 7: Calculate the ratio of intensity of following two waves $y_1 = 6 \sin(0.4t - 25x)$ cm and $y_2 = 2.5 \sin(3.2t - 200x)$ cm.

Solution:

For first wave, $y_1 = 6 \sin(0.4t - 25x)$

$$a = 6\text{cm}, \ \omega = 0.4, \ k = 25 \Rightarrow v = \frac{\omega}{k} = \frac{0.4}{25} = 0.016 \text{ m/s}$$

For second wave, $y_2 = 2.5 \sin(3.2t - 200x)$

$$a = 2.5 \text{ cm}, \ \omega = 3.2, \ k = 200 \Rightarrow v = \frac{3.2}{200} = 0.016 \text{ m/s}$$

$$\therefore \frac{I_1}{I_2} = \frac{1/2\, v_1 \rho \omega_1^2 a_1^2}{1/2\, v_2 \rho \omega_2^2 a_2^2} = \frac{0.4^2 \times 6^2}{2.5^2 \times 3.2^2} = 0.09$$

Example 8: If the intensity of wave is 1.0×10^6 W/m² at 50km from a source, what is the intensity at 10 km from the source.

Solution:

Since Intensity, $I \propto \frac{1}{(\text{distance})^2}$ i.e. $I \propto \frac{1}{r^2}$

Therefore, $\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2} \Rightarrow \frac{1 \times 10^6}{I_2} = \frac{(10 \times 10^3)^2}{(50 \times 10^3)^2} = \frac{1}{25}$

$$I_2 = 25 \times 10^6 \text{ Watt/m}^2 = 2.5 \times 10^7 \text{ Watt/m}^2$$

Example 9: Calculate frequency of vibration of air particles in plane progressive wave of amplitude 2.18×10^{-10} m and intensity 10^{-10} W/m², the velocity of sound in air is 340 m/S and density of air is 0.00129 gm/cc.

Solution:

$$A = 2.18 \times 10^{-10} \text{ m}, \ I = 10^{-10} \text{ W/m}^2, \ v = 340 \text{ m/sec},$$

$$\rho = 0.00129 \text{ gm/cc} = \frac{0.00129 \times 10^{-3}}{10^{-6}} = 1.29 \text{ kg/m}^3$$

We have,

$$I = \frac{1}{2} v \rho \omega^2 a^2$$

$$\omega^2 = \frac{2I}{v \rho a^2}$$

$$\omega = \sqrt{\frac{2I}{v \rho a^2}}$$

$$2\pi f = \sqrt{\frac{2I}{v \rho a^2}}$$

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$$f = \frac{1}{2\pi}\sqrt{\frac{2I}{v\rho a^2}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{2 \times 10^{-10}}{340 \times 1.29 \times (2.18 \times 10^{-10})^2}}$$

$f = 493$ Hz.

Example 10: A progressive and stationary, simple harmonic wave having frequency 250 Hz and e having same velocity 30 m/S

(i) Determine the phase difference between two vibrating point in a progressive wa at a distance of 10 cm.

(ii) Wave equation of progressive wave if amplitude is 0.03 m.

(iii) Distance between nodes in stationary wave.

Solution:

Here, $f = 250$ Hz, $v = 30$ m/S

We have, $v = f\lambda \Rightarrow \lambda = \frac{v}{f} = \frac{30}{250}$

$\lambda = 0.12$ m

i. Since, path difference, $\lambda \Rightarrow$ Phase difference, 2π

Path difference, $1 \Rightarrow$ Phase difference, $2\pi/\lambda$

Path difference 10 cm (= 0.1m) $\Rightarrow$ Phase difference, $\frac{2\pi}{\lambda} \times 0.1$

Therefore, phase difference $= \frac{2\pi}{\lambda} \times 0.1 = \frac{2\pi}{0.12} \times 0.1 = 5.24°$

ii. Here, amplitude, $A = 0.03$ m

The wave equation is given by,

$y = A \sin(\omega t - kx)$

$$y = 0.03 \sin\left(2\pi f t - \frac{2\pi}{\lambda}x\right)$$

$$y = 0.03 \sin\left(2\pi \times 250\, t - \frac{2\pi}{0.12}x\right)$$

$y = 0.03 \sin(500\,\pi t - 52.36x)$

$y = 0.03 \sin(1570.8\, t - 52.36\, x)$

iii. The distance between two adjacent nodes in a stationary wave is $\frac{\lambda}{2} = \frac{0.12}{2} = 0.06$ m.

Example 11: A wave of frequency 500 cycles /sec has a phase velocity of 350 ms^{-1} (i) How far apart are two points 60° out phase (ii) what is the phase difference between two displacements at certain points at times 10^{-3} sec apart.

Solution:

Here, $f = 500$ cycles/sec, $v = 350\ ms^{-1}$

Since, $v = f\lambda \Rightarrow \lambda = \frac{v}{f}$

$$\lambda = \frac{350}{500} = 0.7 \text{ m}$$

i) We have, phase difference, $2\pi \Rightarrow$ path difference, λ.

Phase difference, $1 \Rightarrow$ path difference, $\frac{\lambda}{2\pi}$

Phase difference, $60° \Rightarrow$ path difference, $\frac{\lambda}{2\pi} \times 60$

Therefore, path difference $= \frac{0.7}{2 \times 180} \times 60 = 0.116$ m

ii) Here the time period, $T = \frac{1}{f} = \frac{1}{500} = 2 \times 10^{-3}$ sec

For time period 'T' seconds the phase difference $= 2\pi$

For time period 1 sec the phase difference $= \frac{2\pi}{T}$

For time 10^{-3} sec the phase difference $= \frac{2\pi}{T} \times 10^{-3} = \frac{2\pi}{2 \times 10^{-3}} \times 10^{-3} = \pi$ rad

Example 12: A piano wire with mass 5 gm and length 90 cm is stretched with tension of 25 N. A wave with frequency 100 Hz and amplitude 1.6 mm travels along the wire (a) calculate the average power carried by the wave (b) What happens to the power if the wave amplitude is halved.

Solution:

Mass (m) = 5 gm = 5×10^{-3} kg, length (l) = 90 cm = 0.9 m

Then, $\mu = \frac{m}{l} = \frac{5 \times 10^{-3}}{0.9} = 5.55 \times 10^{-3}$ kg/m

Tension (T) = 25 N

Frequency (f) = 100 Hz

amplitude (a) = 1.6 mm = 1.6×10^{-3} m

Speed of the wave (v) $= \sqrt{\frac{T}{\mu}} = \sqrt{\frac{25}{5.55 \times 10^{-3}}} = 67.082$ m/s

a. Average power (P) $= \frac{1}{2}\mu v \omega^2 a^2$

$$= \frac{1}{2} \times 5.55 \times 10^{-3} \times 67.082 \times (2\pi \times 100)^2 \times (1.6 \times 10^{-3})^2 = 0.188 \text{ watt}$$

If amplitude is halved then, $a' = \frac{a}{2}$

Average power (P') $= \frac{1}{2}\mu v \omega^2 \left(\frac{a}{2}\right)^2 = \frac{1}{4}\left(\frac{1}{2}\mu v \omega^2 a^2\right) = \frac{1}{4} \times 0.188 = 0.0470$ watt.

Example 13: A source of sound has a frequency of 512 Hz and amplitude of 0.25 cm. What is the flow of energy across a square cm per second. If the velocity of sound in air is 340m/sec and density of air is 0.00129 gm/cm^3.

Solution:

Frequency, f = 512 Hz,

Amplitude, a = 0.25cm = 0.25×10^{-2}m,

velocity of sound in air,

v = 340m/sec, density of air,

$\rho = 0.00129\ gm/cm^3$

$= \frac{0.00129 \times 10^{-3}}{10^{-6}}\ kg/m^3$

$= 1.29\ kg/m^3$

The flow of energy per unit area per unit time, i.e. intensity of wave,

$I = \frac{1}{2} v\rho\omega^2 a^2 = \frac{1}{2} \times 340 \times 1.29 \times (2\pi \times 512)^2 \times (0.25 \times 10^{-2})^2 = 14170.26\ Watt/m^2$

$= 1.417\ Watt/cm^2$

Example 14: Two progressive wave of equal amplitude and frequency travelling opposite direction superimpose to each other so as to form a standing wave of equation $y = A \cos kx \sin\omega t$ where, $A = 1$ mm, $k = 1.57\ cm^{-1}$ and $\omega = 78.5\ sec^{-1}$. Find,

i. Velocity of progressive wave
ii. Node closest to the origin $x > 0$
iii. Antinode closest to the origin $x > 0$
iv. Amplitude of resultant wave when $x = 2.33$ cm

Solution:

The equation of standing wave is

$y = A \cos kx \sin\omega t$ ... (i)

with $A = 1\ mm = 1 \times 10^{-3}$ m, $k = 1.57 cm^{-1} = 157\ m^{-1}$ $\omega = 78.5\ sec^{-1}$

As, progressive waves of equal amplitudes equal frequencies are moving in opposite directions, their equations can be written as

$y_1 = \frac{A}{2} \sin(\omega t - kx)$ ... (ii)

$y_2 = \frac{A}{2} \sin(\omega t + kx)$ ... (iii)

$\therefore\ y = y_1 + y_2$

$= \frac{A}{2} [\sin(\omega t - kx) + \sin(\omega t + kx)$

$= A \cos kx \sin \omega t$

i. Wave velocity of progressive wave,

$v = \frac{\omega}{k} = \frac{78.5}{157} = 0.5\ m/s$

ii. For a node amplitude is zero

$\therefore\ \cos kx = 0 = \cos\frac{p}{2}$

$x = \frac{\pi/2}{157} = 0.01$ m

iii. For an antinode amplitude is maximum

$\therefore\ |\cos kx| = 1 = \cos n\pi$

$x = \frac{p}{k} = \frac{p}{157} = 0.02$ m.

iv. The amplitude of vibration $= |A \cos k x|$

$= 10^{-3} \cos(157 \times 2.33 \times 10^{-2})$

$= 10^{-3} \cos 3\ (3.658\ rad)$

$= 10^{-3} \cos\left(3.658 \times \frac{180}{\pi}\right)^{o}$

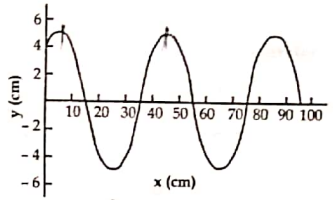
$= 10^{-3} \cos(209.59^o)$

$= -10^{-3}(0.869)$

$= -8.69 \times 10^{-4}$ m

Example 15: A simple harmonic transverse wave is propagating along a string towards the left direction as shown in figure. The figure shows a plot of the displacement as a function of position at time $t = 0$. The string tension is 3.6 N and its linear density is 25 gm/m. Calculate

i. The amplitude
ii. The wave-length
iii. Wave speed
iv. The period
v. The maximum particle speed in the string
vi. Write an equation describing the travelling wave.



Solution:

$T = 3.6$ N, $\mu = 25\ gm/m = 25 \times 10^{-3}\ kg/m$

i. Amplitude $(A) = 5\ cm = 5 \times 10^{-2}$ m

ii. Wavelength $(\lambda) = 40\ cm = 40 \times 10^{-2}$ m

iii. wave speed $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{3.6}{25 \times 10^{-3}}} = 12\ m/s$

iv. Frequency $(f) = \frac{v}{\lambda} = \frac{12}{40 \times 10^{-2}} = 30$ Hz

and Time period $(T) = \frac{1}{f} = \frac{1}{30} = 0.033$ sec.

v. Maximum particle speed $(v_{max}) = A\omega = A(2\pi f) = 5 \times 10^{-2} \times 2\pi \times 30 = 9.425\ m/s$

vi. The equation of the travelling wave is

$y = 5 \times 10^{-2} \sin\left(60\pi t + \frac{2p}{40 \times 10^{-2}} x\right)$

$= 5 \times 10^{-2} \sin(60\pi t + 5\pi x)$

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Example 16: A wave of frequency 500 cycles/sec. has a phase velocity of 350 m/sec. How for apart are two points 60° out of phase.

Solution:

Frequency $f = 500$ cycles/sec
Phase velocity $v = 350$ m/sec
Phase difference $= 60°$
Path difference = ?

Since $v = f\lambda \Rightarrow \lambda = \frac{v}{f} = \frac{350}{500}$

Since phase difference $2\pi \Rightarrow$ path difference λ

Phase difference 1 $\Rightarrow$ path difference $\frac{\lambda}{2\pi}$

Phase difference 60° $\Rightarrow$ path difference $\frac{\lambda}{2\pi} \times 60°$

Therefore, path difference $= \frac{\lambda}{2\pi} \times 60° = \frac{\lambda}{6} = \frac{350}{500 \times 6} = 0.117$m

Example 17: The equation of transverse wave on a string is $y = (2.0\text{mm}) \sin [(20\text{m}^{-1})\, x - (600\ \text{S}^{-1})\, t]$. The tension on the string is 15 N. (i) What is the wave speed (ii) Find the linear density of the string in grams per meter.

Solution:

The given equation is,

$$y = (2.0\ \text{mm}) \sin [(20\ \text{m}^{-1})\, x - (600\ \text{S}^{-1})\, t]$$

Comparing this equation with general displacement equation

$$y = A \sin (kx - \omega t)$$

$$k = 20\ \text{m}^{-1} \text{ and } \omega = 600\ \text{S}^{-1}$$

Therefore, velocity, $v = \frac{\omega}{k} = \frac{600}{20} = 30$ m/S

Here, tension on the string, $T = 15$ N

Since, $v = \sqrt{\frac{T}{\mu}} \Rightarrow v^2 = \frac{T}{\mu} \Rightarrow \mu = \frac{T}{v^2}$

$$\mu = \frac{15}{30^2} = \frac{1}{60}\ \text{kg/m}$$

$$= \frac{1000}{60}\ \text{gm/m} = 16.67\ \text{gm/m}$$

Example 18: A stretched string has linear density 525 gm/m and is under tension of 45N. A sinusoidal wave with frequency 120 Hz and amplitude 8.5 mm is sent along the string from one end. At what average rate does the wave transport energy.

Solution:

Here, $\mu = 525$ gm/m $= 0.525$ kg/m, $T = 45$ N

Velocity of wave $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{45}{0.525}} = 9.26$ m/sec

$f = 120$ Hz, amplitude, $a = 8.5$ mm $= 8.5 \times 10^{-3}$m

The average rate at which wave transport energy

$$P = \frac{1}{2} v\, \mu \omega^2 a^2 = \frac{1}{2} \times 9.26 \times 0.525 \times (2\pi \times 120)^2 \times (8.5 \times 10^{-3})^2$$

$$= 99.74 \text{ J/sec} = 99.74 \text{ Watt.}$$

Example 19: Show that for a wave on string, the kinetic energy per unit length of string is K.E. $= \frac{T}{2}$, where T is the tension on the string.

Solution:

We know, K.E $= \frac{1}{2} m v^2$

K.E. per unit length $= \frac{1}{2} \frac{m}{l} v^2 = \frac{1}{2} \mu v^2$

$$= \frac{1}{2} \mu . \frac{T}{\mu} \qquad \left[\text{Since, } v = \sqrt{\frac{T}{\mu}}\right]$$

$$= \frac{T}{2}$$

Example 20: One end of 14 m long rubber tube with total mass 0.8 kg is fastened to a fixed support. A cord Attached to the other end passes over a pulley and supports an object with a mass of 7.5 kg. The tube struck a transverse blow at one end. Find the time required for the pulse to reach the other end.

Solution:

Mass of rubber tube, $m = 0.8$ kg, length $l = 14$ m

Mass per unit length, $\mu = \frac{m}{l} = 0.057$ kg/m

Mass of object, $M = 7.5$ kg

Tension on rubber tube, $T = Mg = 7.5 \times 9.8 = 73.5$ N

Therefore, $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{73.5}{0.057}} = 35.9$ m/sec

Thus, time required for the pulse to reach the other end,

$$T = \frac{l}{v} = \frac{14}{35.9} = 0.39 \text{ sec}$$

Example 21: Two strings have been tied together with a knot and then stretched between two rigid supports. The string have linear density $\mu_1 = 1.4 \times 10^{-4}$ kg/m and $\mu_2 = 2.8 \times 10^{-4}$ kg/m. Their lengths are $L_1 = 3.0$ m and $L_2 = 2.0$ m, and first string is under a tension of 400 N. Simultaneously, on each string a pulse is sent from the rigid support end towards the knot, which pulse reaches the knot first?

Solution:

For first string, $\mu_1 = 1.4 \times 10^{-4}$ kg/m, length $L_1 = 3$m, $T = 400$ N

Therefore, $v_1 = \sqrt{\frac{T}{\mu_1}} = \sqrt{\frac{400}{1.4 \times 10^{-4}}} = 1690.3$ m/sec

The time required to pulse to reach the knot, $t_1 = \frac{L_1}{v_1} = \frac{3}{1690.3}$

$\Rightarrow \quad t_1 = 1.77 \times 10^{-3}$ sec

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For second string, $\mu_2 = 2.8 \times 10^{-4}$ kg/m, $L_2 = 2$m, T = 400 N

Therefore, $v_2 = \sqrt{\frac{T}{\mu_2}} = \sqrt{\frac{400}{2.8 \times 10^{-4}}} = 1195.228$ m/.sec

The time required to pulse on 2nd string to reach the knot,

$$t_2 = \frac{L_2}{v_2} = \frac{2}{1195.228} = 1.673 \times 10^{-3} \text{ sec}$$

Since $t_2 < t_1$ thus the pulse on second string reaches the knot first.

Example 22: A wave is propagating on a long stretched string along its length taken as the positive x-axis. The wave equation is given by $y = y_0 e^{-\left(\frac{t}{T} - \frac{x}{\lambda}\right)^2}$. Where, $y_0 = 4$mm, T = 1Sec. and $\lambda = 4$ m.

i. Find the velocity of the wave.
ii. Find the function f(t) giving the displacement of the particle at x = 0
iii. Find the function g(x) giving the shape of the string at t = 0
iv. Plot the shape g(x) of the string at t = 0
v. Plot the shape of the string at t = 5 sec.

Solution:

$$y = y_0 e^{-\left(\frac{t}{T} - \frac{x}{\lambda}\right)^2} \quad \ldots(i)$$

$y_0 = 4$ mm $= 4 \times 10^{-3}$ m
T = 1 Sec. and f = 1 Hz
$\lambda = 4$ cm $= 4 \times 10^{-2}$ m

i. Velocity of the wave $(v) = f\lambda = 1 \times 4 \times 10^{-2} = 4 \times 10^{-2}$ m/s

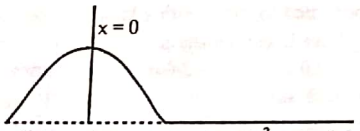
ii. At x = 0

$$f(t) = y_0 e^{-\left(\frac{t}{T}\right)^2}$$

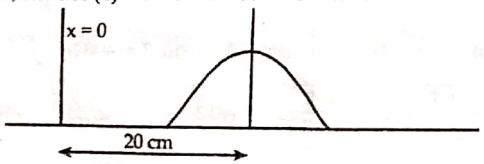
iii. At t = 0

$$g(x) = y_0 e^{-\left(\frac{-x}{\lambda}\right)^2} = y_0 e^{-\left(\frac{x}{\lambda}\right)^2}$$

iv. for t = 0



v. For, t = 5 sec, distance (d) = v × t = $4 \times 10^{-2} \times 5 = 0.2$ m



Example 23: A transverse sinusoidal wave is generated at one end of a long horizontal string by a bar which moves up and down through a distance of 0.5 m. The motion is continuous and repeated regularly twice each second. If the string has linear mass density of 0.005 kg/m and is kept under a tension of 2 N. Find the speed, amplitude, time period and wave length of the wave motion.

Solution:

f = 2 Hz, T = 2N, x = 0.5 m, $\mu = 0.005$ kg/m

We have, $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{2\text{ N}}{0.005\text{ kg/m}}}$

Speed (v) = 20 m/s

Also, $v = \omega\sqrt{A^2 - x^2}$

or, $20 = 2\pi f\sqrt{A^2 - (0.5)^2}$

or, $400 = 4\pi^2 f^2 [A^2 - (0.5)^2]$

or, $\frac{400}{4\pi^2 (2^2)} = A^2 - 0.25 \Rightarrow A^2 = 2.785$

$\Rightarrow$ Amplitude (A) = 1.67 m

Time period $(T) = \frac{1}{f} = \frac{1}{2} = 0.5$ sec

Wave length $(\lambda) = \frac{v}{f} = \frac{20}{2} = 10$ m

Example 24: A stretched string has a linear mass density of 5.0 gm/cm and a tension of 10 N. A wave on this string has an amplitude of 0.12 mm and a frequency of 100 Hz and is travelling in –ve x direction. At what average rate does the wave transport energy? Write the wave equation with appropriate units.

Solution:

Here, $\mu = 5\text{gm/cm} = \frac{5 \times 10^{-3}}{10^{-2}} = 0.5$ kg/m

Tension, T = 10 N

Velocity, $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{10}{0.5}} = 4.47$ m/sec

Here, amplitude, $a = 0.12$ mm $= 0.12 \times 10^{-3}$ m Frequency, f = 100 Hz

Since, $v = f\lambda \Rightarrow \lambda = \frac{v}{f} = \frac{4.47}{100} = 0.0447$m Therefore, $k = \frac{2\pi}{\lambda} = 141\text{ m}^{-1}$

Now, $P = \frac{1}{2}\mu v\omega^2 a^2 = \frac{1}{2} \times 0.5 \times 4.47 \times (2\pi \times 100)^2 \times (0.12 \times 10^{-3})^2 = 6.35 \times 10^{-3}$ watt.

The equation of wave travelling in –ve x direction is

$y = a \sin(\omega t + kx) = (0.12\text{mm}) \sin[(2\pi f)t + (141\text{m}^{-1})x] = 0.12 \text{ mm} \sin[(628\text{s}^{-1})t + (141\text{ m}^{-1})x]$

THEORETICAL ANSWER QUESTIONS

1. How standing waves are produced on a string? Write down the difference between travelling wave and standing wave.
2. Show that for a plane progressive wave, on the average, half the energy is kinetic and half-potential.
3. What is wave motion? Write the equation of sine wave travelling on the string; apply the theory of supper position to find the maxima and minima of interference pattern on the string.
4. Calculate amount of energy transmitted along a stretched string when a wave passes through it.
5. Differentiate between plane progressive wave and stationary wave. Show that intensity of a particular wave for a medium remains constant.
6. Deduce an expression for velocity of transverse waves in a stretched string.
7. What is super position of wave? Describe interference of wave.
8. In the progressive wave, show that the potential energy and kinetic energy of every particle will change with time but the average K.E per unit volume and P.E. per unit volume remains constant.
9. Distinguish between wave velocity and particle velocity. Obtain the relation for acceleration of the particle in wave motion.

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10. The six strings of a guitar are of the same length and are under same tension, but have different thickness. On which string the wave travels the fastest? Answer with mathematical expression.
11. Define wave velocity and particle velocity, obtain the relation between them.
12. Derive a relation for speed of transverse wave in a stretched string and show that the average rate of energy transfer is $\frac{1}{2}\mu v\omega^2 A^2$, where symbol carry their usual meaning.
13. Derive the differential equation of transverse wave of a stretched string with applied tension T and mass per unit length μ. Also find the velocity of the wave propagating through the string.
14. Starting from the progressive wave equation $y = a \sin \frac{2\pi}{\lambda}(vt - x)$. Prove that the differential equation. $\frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2}$
15. What is energy density of wave? Derive its expression.
16. What is meant by wave motion? What are its characteristics? Explain how it transports energy and derive a relation for the average rate at which the wave transports the energy along the string.
17. Differentiate between travelling wave and mechanical wave. Find an expression for the energy and power of a traveling wave.

NUMERICAL PROBLEMS

1. Calculate the frequency and maximum particle velocity due to wave represented by $y(x, t) = 0.03 \sin(60\pi t - 0.03\pi x)$. The values of x and y are in centimeters. [Ans: 30 Hz, 5.65 m/s]
2. The displacement equation of a transverse plane wave at any instant is given by $y = 0.03 \text{ Sin } (3\pi t - 0.03\pi x)$, where x and t are in meter and second. Calculate wave-length, frequency and velocity of the wave. Also find the phase difference between two points 0.05 m apart. [Ans: 66.67 m, 1.5 Hz, 100 m/s, 4.71×10^{-3} rad.]
3. What power is transmitted by a transverse wave of amplitude 1 cm and frequency 2 Hz propagating through a string of linear mass density 1 gm/m and under tension 20 N. [Ans: 1.16×10^{-3} W]
4. A string 2.72m long has a mass of 263 gm. The tension in the string is 36.1 N. What must be the frequency of traveling waves of amplitude 2.5 mm in order that average transmitted power is 85.5 W? [Ans: 608.4 Hz]
5. Calculate the minimum intensity of audibility in watts per square cm from a note of 1000 Hz if the amplitude of vibration is 10^{-9}cm. Given density of air is 0.0013 gm/cc and velocity of sound in air is 340 m/s. [Ans: 8.725×10^{-13} W/cm²]
6. A piano wire with mass 3 gm and length 80 cm is stretched with tension of 25 N. A wave with frequency 120 Hz and amplitude 1.6 mm travels along the wire a) calculate the average power carried by the wave b) what happens to the power if the wave amplitude is halved. [Ans: 0.22 W, 0.055 W]
7. The speed of a transverse wave on a string is 170 m/s, when the string tension is 120 N. To what value must the tension be changed to raise the wave speed to 180 m/sec? [Ans: 134.53 N]
8. A rod vibrating at 12 Hz generates harmonic waves with amplitude of 1.5 mm in a s string of linear mass density 2 gm/m. If the tension in the string is 15 N, what is the average power supplied by the source. [Ans: 1.108×10^{-3} W]
9. A sound wave of frequency 300 Hz has an intensity of 1 μ W/m². If density of air is 1.29 kg/m³ and speed of sound in air is 330 m/sec. What is the amplitude of air oscillation caused by this wave? [Ans: 3.636×10^{-4} m]
10. In a sonometer experiment, the resonance length is found to be 0.5 m with mass 5 kg and vibrates at 100 Hz. Find the linear density of the wire. [Ans: 0.0049 kg/m]
11. A wire whose linear density is 5×10^{-3} kg/m is stretched between two knife edges with a tension of 450 N. The wire resonates at a frequency of 420 Hz. Find the resonance length of the wire. [Ans: 0.357 m]

□□□

3 Chapter

ACOUSTICS

CHAPTER OUTLINE

After completion of this chapter, you will be able to know:

- Classification of sound waves, Acoustics of building, Reverberation of sound, absorption coefficient, Noise pollution and its control, Sound insulation, Sabine equation.
- Introduction, production and applications of ultrasonic wave. Ultrasonic method in non-destructive testing.

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INTRODUCTION

Sound is a vibration that travels in the form of longitudinal waves through the medium. This means that sound waves are waves in which the particles of the medium vibrate in the same direction as the wave. Because sound waves require a medium to propagate, they are referred to as mechanical waves. The most fundamental properties of sound waves are wavelength, frequency, amplitude, speed, intensity etc. There are different types of sound including, audible, inaudible, unpleasant, pleasant, soft, loud, noise and music. Sound waves are classified into three groups based on their frequency range.

1. **Infrasonic waves:** These are sound waves with frequencies less than 20 hertz. These frequencies are inaudible to humans. Infrasound is used by scientists to detect earthquakes and volcanic eruptions, to map underground rock and petroleum formations, and to study heart activity. Despite our inability to hear infrared waves, many animals use them to communicate in nature.
2. **Audible waves:** These are sound waves with frequencies ranging from 20 hertz to 20,000 hertz (20 kHz). These frequencies are audible to humans.
3. **Ultrasonic waves:** Ultrasonic waves are sound waves with frequencies greater than 20 kilohertz. These frequencies are inaudible to humans.

ACOUSTICS OF BUILDING

The branch of physics which deals with the planning of a building or a hall with a view to provide best audible sound to the audience is called acoustics of building.

Factors Affecting the Acoustics of Building

1. **Extraneous Noise**
   - There may be penetration of sound between rooms.
   - For this the wall must be covered with sound absorbing material and doors must have heavy curtains. (There should be no penetration of sound between rooms)
2. **Loudness**
   - The loudness of sound may vary with position.
   - To make proper loudness every where, wooden reflecting surface can be kept above the speaker. Low ceiling is also help full in reflecting sound towards audience.
   (The sound must sufficiently be loud every where).
3. **Echelon effect**
   - If there is a series of steps between floors or levels (or a set of railing), the sound produced in front of such a structure may produce a musical note due to regular successive echoes of sound. Such an effect is called Echelon effect. If this note is in audible range, the listener will hear it prominently.
   - To avoid Echelon effect steps are covered with carpet.
   (There should be no Echelon effect)
4. **Focusing**
   - There may be concentration of sound or a zone of silence in any part of the hall.
   - For uniform distribution curved surface and projection should be designed. One method is to make the wall in front of audience parabolic with speaker at its focus.
   (Their must be proper focusing of sound every where in the hall.

5. **Echo**
   - Echo is a reflection of sound that arrives at the listener with a delay after the direct sound. The distortion of sound takes place because of echo.
   - The average interval between two syllables spoken by a person is about 0.2 sec. These walls and ceilings should be made in such a way that the reflection of sound takes place in more than 0.2 sec. Otherwise it creates a confusion due to overlapping of direct and reflected sound.
   (There should not be echo).
6. **Resonance**
   - The resonance of any audio frequency note causes the sound of different intensity than that of direct one.
   - For the hall of large size, the resonance frequency is much below the audible limit and harmful effects due to resonance will not be present.
7. **Reverberation**
   - The persistence of sound for some time even when the source of sound has ceased is called Reverberation. This is due to the multiple reflection of sound from various part of the hall.
   - It can be controlled by proper maintenance of absorbing material i.e. providing window, doors, carpets on the floor, heavy curtains, using full capacity of audience etc.

REVERBERATION

The persistence of sound for some time even when the source of sound has ceased is called reverberation.

Causes:

i. The fall in intensity of the sound produced in room is exponential and so it will take longer time to become zero.

ii. Due to multiple reflection from walls, ceilings and floor or other material, the sound reverberates inside the hall for longer time.

Time of Reverberation

The duration for which the sound can be heard after the source has ceased to produce the sound is called reverberation time.

OR: The time taken by sound intensity to fall it's intensity equal to one millionth of its original value (i.e. by a factor of 10^{-6}) is called time of reverberation. (i.e. time to fall loudness by 60dB)

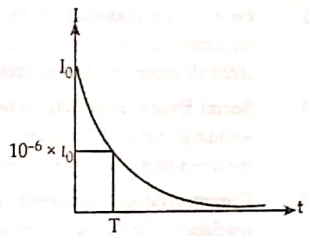

Figure (1): Variation of intensity of sound

Absorption of Sound

Coefficient of absorption of sound for the surface of material is defined as the ratio of sound energy absorbed by the surface to the sound energy absorbed by an equal area of a perfect absorber such as an open window.

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The unit of absorption coefficient is Sabine. The sound energy absorbed by 1. sq.ft. of perfect absorber is called one Sabine.

If A is effective surface area for surface having total surface area S. The absorption coefficient 'α' is given by the relation

$$A = \alpha S$$

If $\alpha_1, \alpha_2, \ldots \alpha_n$ are absorption coefficient for each reflecting surface and $S_1, S_2, S_3 \ldots\ldots$ are corresponding area, then the average value of absorption coefficient is.

$$\alpha = \frac{\alpha_1 S_1 + \alpha_2 S_2 + \ldots\ldots\ldots + \alpha_n S_n}{S_1 + S_2 + S_3 + \ldots\ldots + S_n} = \Sigma \frac{\alpha_i S_i}{S_i}$$

$$\Rightarrow \quad \alpha = \frac{\Sigma \alpha_i S_i}{S}$$

NOISE POLLUTION

Any undesired or irritating sound that has an impact on the health and well being of people and other living things is referred to as noise pollution. It is not harmful to air, soil and water but affects the animals including humans. Sound is typically described in terms of the loudness (amplitude) and the pitch (frequency) of the wave. Loudness is measured in logarithmic units called decibels (dB).

Humans have a hearing range called as audible range. Audible range depends upon frequency and loudness of sound. For a person with normal hearing, frequency ranges from 20 to 20,000 Hz and loudness ranges from 0 to 120 dB. Sound is measured in decibels (dB). A decibel value above 80 is considered to be noise pollution.

Causes of Noise Pollution

1. **Domestic Sources:** People utilize devices heavily in their daily lives and are constantly surrounded by them. The quantity of noise created by appliances like TVs, mobile phones, mixer grinders, pressure cookers, vacuum cleaners, washing machines, dryers, coolers, and air conditioners is also a factor, but it frequently lowers the quality of life.
2. **Poor urban planning:** In the majority of developing nations, poor urban planning is also quite important. Large families living in tiny spaces, parking lots, street noise, honking, and business areas all contribute to noise pollution, which disturbs the social environment.
3. **Social Events and Celebrations:** Most social events have a high level of noise. Whether it's a wedding, party, pub, disco, fire work, place of worship with loud speakers, election rallies, meetings of political parties etc, typically break the restrictions and contribute to noise pollution.
4. **Commercial and Industrial activities:** The majority of industries employ large, noise-producing machinery. Additional equipment that contributes to noise production includes Tractors, Thrashers, Harvesters, Tube Wells, Powered Tillers, Compressors, Generators, Exhaust Fans, and Grinding Mills.
5. **Construction Activities:** Construction activities like mining, construction of bridges, dams, buildings, stations, roads, flyovers take place in almost every part of the world. To fulfill the demands of a population that is always growing, these construction projects should be continued. Additionally, it generates noise pollution.

6. **Transportation:** Heavy noise is produced by a large number of automobiles on the road, planes, and trains. A regular individual would lose their capacity to hear well due to the extreme noise level.

Effects of Noise Pollution

1. **Auditory effect:** Long-term exposure to noise causes the internal ear to slowly deteriorate, which can result in hearing loss or deafness. Continuous exposure to noise levels greater than 90 dB may cause it. It might be either temporary or permanent. Explosions and other loud noises can also instantly make a person deaf by rupturing the ear drums or harming the cochlea. Hearing loss is frequently connected to one's profession.
2. **Non Auditory effect:** Noise can occasionally cause emotional disruptions and people to lose their cool. It has been found to decrease productivity at work, emotional disruption, may obstruct getting enough rest and sleep. It includes the pale complexion of the skin, tensing of the voluntary muscles, reduction of gastric secretions, an increase in blood pressure, and the abrupt release of hormone, which magnifies neuro muscular tension, nervousness, irritability, and anxiety. Loud noise can lead to problems and make it difficult for people to freely communicate.
3. **Effects on Animals:** Noise pollution makes it difficult for animals to use sound for navigation, finding food, mating, and avoiding predators, affecting many animals ability to survive. It may cause animal migration on an ecosystem level. Their movement may have an impact on crop output. Because many animals such as bats pollinate bananas, peaches and other cash crops.
4. **Effect on non-living things:** The noise booms cause cracks in walls of buildings as well as in hills. Sonic boom can break window panes and buildings.

Preventing Noise Pollution

1. **Noise reduction at source:** It can be done using by designing, fabricating and using quieter machines to replace the noisy ones. Also Covering noise-producing machine with sound-absorbing materials to check noise production.
2. **Control at receiver's end:** To minimize occupational exposure, ear protection equipment such as earplugs, earmuffs, noise helmets, headphones, etc. must be made available to workers in noisy environments.
3. **Acoustic Zoning:** There should be quiet zones near residential areas, educational institutions, and hospitals. Distancing noisy industrial areas, bus terminals and railway stations, aerodromes, and so on from residential areas, i.e. increasing the distance between source and receiver.
4. **Sound Insulation at Construction Stages:** The construction techniques of walls, windows, doors, and floors, as well as the selection of appropriate building materials, all contribute to better acoustic control in buildings.
5. **Planting of Trees:** Green muffler scheme involves planting green trees and shrubs along roads, hospitals, educational institutions etc. to reduce noise to a considerable extent.
6. **Legislative Measures:** To address the threat of noise pollution, strict legislative measures must be implemented. Noise standards must be strictly followed, banning pressure horns in automobiles, minimum use of loudspeakers and amplifiers especially near silence zones.

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SOUND INSULATION

Sound insulation is a type of barrier that prevents sound waves from permeating. It is demonstrated by the sound transmission loss, which is expressed as the decibel difference between incident and permeated sound. The sound that people want to insulate can be divided into two types:

i. Air - borne sound (due to air vibration), and

ii. Solid-borne sound (due to the impact on solids or solid vibration).

Sound permeation follows the "mass law" of acoustics. The mass area ratio of a wall or plate determines its sound insulation property. The greater the mass, the more difficult it is to vibrate this material, and thus the better the insulating property. As a result, dense and heavy materials (clay brick, reinforced concrete, steel plate, etc.) are preferable as sound insulators The unconnected structure is the best way to insulate solid-borne sound. That means installing elastic liner between the wall and the spandrel beam (an edge beam that runs along the exterior wall of each floor of the building) as well as between the building frame and the wallboard. The elastic liner can be made of felt, cork, rubber, or elastic carpet . Do not mix up sound absorbing and sound insulating materials. It is important to note that good sound absorbing material is light, loose, and porous.

SABINE'S FORMULA FOR REVERBERATION TIME

Let 'I' be the average intensity of sound at any instant of time and δI be the fall in intensity due to absorption in a small instant of time δt, then,

$$\delta I = -\alpha n I \delta t \qquad \ldots (i)$$

where,

n = number of reflection of sound waves per second

α = absorption coefficient of all absorbing surfaces.

It has been shown by Jaeger that sound travels an average distance 4V/S between successive reflections.

where V = volume of the hall

S = Total surface area of the reflecting surface.

If v is the velocity of sound, then time between successive reflection is, $\frac{\text{Distance}}{\text{Velocity}} = \frac{4V}{Sv}$.

$$\text{Therefore, } n = \frac{Sv}{4V} \qquad \ldots (ii)$$

putting value of 'n' in equation (i)

$$\delta I = -\alpha . \frac{Sv}{4V} I \delta t$$

$$\text{or, } \frac{\delta I}{I} = -\alpha \frac{Sv}{4V} \delta t$$

can be written in limit as

$$\frac{dI}{I} = -\frac{\alpha S v}{4V} dt$$

Let I_o be the value of intensity at t = 0 and I_t be it's value after time 't'. then integrating above equation.

$$\int_{I_o}^{I_t} \frac{dI}{I} = -\frac{\alpha S v}{4V} \int_{o}^{t} dt$$

$$\text{or, } [\ell n I]_{I_o}^{I_t} = \frac{-\alpha S v}{4V} . t$$

$$\Rightarrow I_t = I_o \exp\left(-\frac{\alpha S v}{4V} . t\right)$$

For reverberation time, t = T, $I_t = I_o / 10^6 = I_o \times 10^{-6}$

Therefore,

$$10^{-6} = \exp\left(\frac{-\alpha S v}{4V} . T\right) \Rightarrow \exp\left(\frac{\alpha S v}{4V} . T\right) = 10^6$$

$$\text{or, } \frac{\alpha S v}{4V} . T = ln(10^6) = 6\, ln(10)$$

$$\therefore T = \frac{6\, ln(10) \times 4 \times V}{\alpha S v} = \frac{55.262V}{\alpha S v}$$

Since v = 350 m/sec at room temp.

$$\text{Therefore, } T = \frac{0.158V}{\alpha S} \text{ in S.I unit} \qquad \ldots (iii)$$

The is called Sabine formula.

If we put v = 1120 ft/sec, the expression for reverberation time will be,

$$T = \frac{0.05V}{\alpha S} \text{ in F.P.S.}$$

This shows that Reverberation time is directly proportional to the volume of auditorium and inversely proportional to absorption coefficient and total surface area.

For good acoustics of a hall, the reverberation time should have appropriate value. If it is too large, there may be multiple reflections and over lapping of sound cousing confusion to listner. If it is too small, the sound vanishes instanteneously and gives the dead effect. The suitable value of reverberation time is 1.03 sec for a hall of 10,000 cu ft capacity.

Equation (3) is called Sabine's formula.

ULTRASONICS

The longitudinal mechanical wave of frequency greater than 20 KHz are called ultrasonic waves. These are not audible.

Audible waves (Ordinary sound): The longitudinal mechanical waves whose frequency lies in the range 20 Hz to 20KHz and can produce sensation of hearing in our ears are called audible waves and those having frequency less than 20 Hz are called infrasonic.

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Properties of Ultrasonic Waves

i. Ultrasonic waves vibrate at a frequency greater than the audible range for humans (20 kHz).

ii. Their wavelengths are shorter. As a result, their penetrating power is high.

iii. They are unable to pass through a vacuum.

iv. Ultrasonic waves travel at the speed of sound in the medium. In a homogeneous medium, th travel at a constant velocity but travel faster in denser medium.

v. They undergo reflection, refraction and absorption.

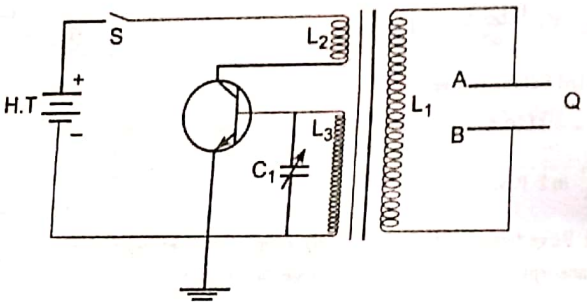
vi. They have high energy content. They can be transmitted over a large distance without m loss of energy.

vii. They produce intense heat when they are passed through objects.

Production of Ultrasonic Waves

1. Piezo - Electric Generation

If mechanical pressure is applied to the opposite faces of certain crystal slices (such as quar Rochelle salt) cut in a way that their faces are perpendicular to optic axis, the other pair of oppo faces develop opposite electric charges resulting a potential difference. This effect is called piez electric effect.



Conversely, if a periodic potential is applied to the opposite faces of the crystal, the periodic chang in dimension of crystal would take place. This periodic change in length of crystal produc ultrasonic waves. For the generation of ultrasound one can supply the periodic voltage of frequen equal to that of ultrasound which is to be produced.

The experimental circuit diagram is as shown above 'Q' is a thin slice of crystal cut with its oppos faces are perpendicular to optic axis. The crystal is placed between two metal plates A and B. Th plates A and B are connected to the primary of transformer which is coupled inductively to th oscillator circuit. The variable condenser C_1 is adjusted so that frequency of the oscillatory circuit i equal to the natural frequency of one of the modes of vibration of crystal. This will ensu mechanical vibration in the crystal due to linear expansion and contraction.

The frequency of vibration is.

$$f = \frac{n}{2l}\sqrt{\frac{Y}{\rho}}, \text{ where, } n = 1, 2, 3.... \text{ (modes of vibration)}$$

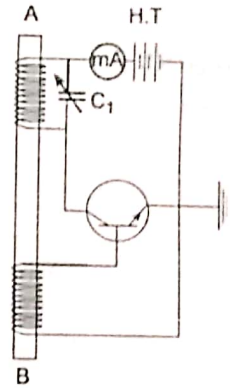
Y = Young's modulus of elasticity.

ρ = density of the crystal

l = length of crystal.

2. Magnetostriction generator

When a ferromagnetic material in the form of bar, is subjected to alternating magnetic field, it expands and contracts alternately with twice the frequency of the applied magnetic field. These longitudinal expansions and contraction produce ultrasonic waves.



The experimental arrangement is as shown in figure. AB is a bar of ferromagnetic material. Initially bar is magnetized by passing current. The condenser C_1, is adjusted so that the frequency of oscillatory circuit is the same as the natural frequency of bar.

Since the rod is open at both end, for fundamental mode of vibration, frequency is given by

$$f = \frac{1}{2l}\sqrt{\frac{Y}{\rho}}$$

Where

l = length of rod

ρ = density of rod

Y = Young's modulus of elasticity.

Applications of Ultrasonic Waves in Science and Engineering

i. It is used to detect flaws or cracks in metals.

ii. SONAR is a technique which stands for Sound Navigation and Ranging. SONAR uses ultrasonics for the detection and identification of underwater objects like ships, submarines, iceberg, etc., in the ocean.

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iii. It is used for soldering aluminium coil capacitors, aluminium wires and plates.

iv. It is used to weld some metals which cannot be welded by electric or gas welding.

v. It is used for cutting and drilling holes in metals.

vi. It is used to form stable emulsions of even immiscible liquids like water and oil or water and mercury which finds application in the preparation of photographic films, face creams, etc.

vii. It acts like a catalytic agent and accelerates the chemical reactions.

Applications of Ultrasonic Waves In Medical Field

Imp.

i. It is used as a diagnostic technique to find tumors, breast cancer, and the development of the fetus.

ii. It is used to remove kidney stones and brain tumors without shedding any blood.

iii. It is used to remove the broken teeth.

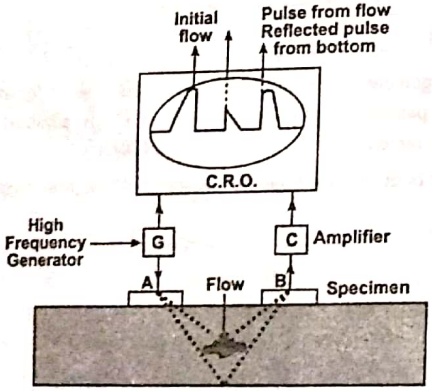
iv. It is used for sterilizing milk and for killing bacteria.

Imp.

v. It is used to study the blood flow velocities in blood vessels.

ULTRASONIC METHOD IN NON-DESTRUCTIVE TESTING

Ultrasonic waves in NDT uses high frequency sound energy to conduct examinations and make measurements. This can be used for flaw detection/evaluation, dimensional measurements, material characterization, and more.



A typical NDT consists of an ultrasonic frequency generator and a cathode ray oscilloscope (CRO), transmitting transducer (A), receiving transducer (B) and an amplifier. When there is a discontinuity (such as a crack) in the wave path, part of the energy will be reflected back from the flaw surface. The echoes of the reflected beam are captured by using a cathode ray oscilloscope. By examining echoes on CRO, flaws can be detected and their sizes can be estimated.

Advantages of Ultrasonic Waves in NDT

i. High penetrating power enables the identification of defects present deep within the component.

ii. Extremely high sensitivity, allowing for the detection of minute faults.

iii. More precise than other nondestructive methods in determining the thickness of components with parallel surfaces and the depth of internal faults.

iv. The capacity to estimate flaws' size, orientation, shape, and nature.

v. Not harmful to surrounding workers or processes, and has no impact on nearby materials and equipment.

vi. Able to operate in a portable or highly automated manner.

Limitations of Ultrasonic Waves in NDT

i. For ultrasound to be transmitted, the surface must be reachable.

ii. Skill and training is more extensive than with some other methods.

iii. It normally requires a coupling medium to promote the transfer of sound energy into the test specimen.

iv. It is challenging to evaluate materials that are rough, have irregular shapes, are extremely small, extremely thin, or are not uniform.

v. Cast iron and other coarse grained materials are difficult to inspect due to low sound transmission and high signal noise.

vi. Linear defects oriented parallel to the sound beam may go undetected.

vii. Reference standards are required for both equipment calibration and the characterization of flaws.

Solved Example

Example 1: The time of reverberation of an empty hall and with 500 audience in the hall is 1.5 sec and 1.4 sec respectively. Find the reverberation time with 800 audience in the hall.

Solution:

According to question, $1.5 = \frac{0.158\ V}{\alpha.S}$ ... (i)

and $1.4 = \frac{0.158\ V}{\alpha.S + 500}$ ... (ii)

Dividing equation (1) by (2)

$$\frac{1.5}{1.4} = \frac{\alpha S + 500}{\alpha S} \Rightarrow 1.5\ \alpha S = 1.4\alpha S + 700$$

$$\Rightarrow \quad \alpha S = 7000$$

From equation (1), $V = \frac{1.5 \times \alpha S}{0.158} = \frac{1.5 \times 7000}{0.158} = 66455.7$

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Therefore, time of reverberation with 800 audience,

$$T_3 = \frac{0.158\ V}{\alpha S + 800} = \frac{0.158 \times 66455.7}{7000 + 800} = 1.346 \text{ sec}$$

Example 2: The time of reverberation of an empty hall and with 600 audiences is 1.8 sec. and 1.6 sec respectively. Find the reverberation time with 1000 audiences in the hall.

Solution:

According to question,

$$T_1 = 1.8 = \frac{0.158V}{aS} \quad \ldots (i)$$

$$\text{and } T_2 = 1.6 = \frac{0.158V}{aS + 600} \quad \ldots (ii)$$

Dividing equation (i) and (ii)

or, $1.125\ \alpha S = \alpha S + 600 \Rightarrow \alpha S = 4800$

From equation (1)

$$V = \frac{1.8 \times 4800}{0.158} = 54683.54$$

Therefore time of reverberation with 1000 audience

$$T_3 = \frac{0.158V}{\alpha S + 1000} = \frac{0.158 \times 54683.54}{4800 + 1000} = 1.4896 \text{ sec.}$$

Example 3: The time of reverberation of an empty hall is 1.5 sec with 500 audiences present in the hall the reverberation time falls to 1.4 sec. Find the number of persons present in the hall if the reverberation time falls down to 1.312 sec.

Solution:

$$\text{For first case, } 1.5 = \frac{0.158\ V}{\alpha s} \quad \ldots (i)$$

$$\text{For second case, } 1.4 = \frac{0.158\ V}{\alpha s + 500\ \alpha_m} \quad \ldots (ii) \quad \alpha_m = \text{absorption coefficient for a person}$$

$$\text{For third case, } 1.312 = \frac{0.158\ V}{\alpha S + n\alpha_m} \quad \ldots (iii) \quad n = \text{number of person}$$

Dividing (1) by (2)

$$\frac{1.5}{1.4} = \frac{\alpha s + 500\ \alpha_m}{\alpha s} = 1 + \frac{500\ \alpha_m}{\alpha s}$$

$$\Rightarrow \quad 500\ \alpha_m = 0.07143\ \alpha s \quad \ldots (iv)$$

Again dividing (1) by (3),

$$\frac{1.5}{1.312} = \frac{\alpha s + n\alpha_m}{\alpha s} = 1 + \frac{n\alpha_m}{\alpha s}$$

$$\Rightarrow \quad n\alpha_m = 0.1433\ \alpha s \quad \ldots (v)$$

Again dividing (5) by (4)

$$\frac{n}{500} = \frac{0.1433}{0.07143}$$

$$n = 1003$$

Example 4: What is the reverberation time for a hall with length 12m, breadth 11m, and height 9m. If the coefficients of absorption of walls, ceiling and floor are 0.02, 0.04 and 0.08 respectively.

Solution:

Volume of the hall, $V = 12 \times 11 \times 9 = 1188\ m^3$

Area of floor = Area of ceiling = $l \times b = 132\ m^2$

Area of walls = $l \times h + l \times h + b \times h + b \times h = 2\ (l+b).\ h = 2\ (12 + 11) \times 9 = 414\ m^2$

Total absorption, $\alpha s = 414 \times 0.02 + 132 \times 0.04 + 132 \times 0.08 = 24.12$

$$\text{Reverberation time, } T = \frac{0.158\ V}{\alpha s} = \frac{0.158 \times 1188}{24.12} = 7.78 \text{ sec}$$

Example 5: A lecture hall with a volume of 4500 m^3 is found to have a reverberation time of 1.5 sec. What is the total absorbing power of all the surfaces in the hall? If the area of the sound absorbing surface is 1600 m^2, calculate average absorption coefficient.

Solution:

Volume of hall, $V = 4500\ m^3$

Reverberation time, $T = 1.5$ sec

$$\text{Since } T = \frac{0.158\ V}{\alpha S} \Rightarrow \alpha S = \frac{0.158V}{T} = \frac{0.158 \times 4500}{1.5} = 474$$

Absorbing power of all surfaces, $\alpha s = 474$ Sabine.

Area of absorbing surface, $S = 1600\ m^2$

$$\text{Average absorption coefficient, } \alpha = \frac{\alpha S}{S} = \frac{474}{1600} = 0.3$$

Example 6: A class room has dimensions $20 \times 15 \times 5\ m^3$. The reverberation time is 3.5 sec. Calculate the total absorption of its surfaces and the average absorption coefficient.

Solution:

Volume of the room (V) = $20 \times 15 \times 5 = 1500\ m^3$

Total surface area (ΣS) $= 2\ (lb + lh + bh)$

$= 2\ (20 \times 15 + 20 \times 5 + 15 \times 5) = 950\ m^2$

$$\text{Then, Reverberation time } T = \frac{0.158V}{aS}\ 3.5 = \frac{0.158 \times 1500}{\alpha S} \Rightarrow \alpha S = 67.71\ m^2$$

$$\text{Also, Average absorption coefficient } (\alpha_{avg}) = \frac{\Sigma \alpha S}{\Sigma S} = \frac{67.71}{950} = 0.0713$$

Example 7: The volume of a room is 600 m^3, wall area of room is 220m^2, the floor and ceiling area each is 120 m^2. If average absorption coefficient for walls is 0.03, for ceiling is 0.80 and for floor is 0.06 calculate average absorption coefficient and reverberation time.

Solution:

Here, $V = 600\ m^3$

Absorption due to wall = $\alpha_1 S_1 = 220 \times 0.03 = 6.6$

Absorption due to floor = $\alpha_2 S_2 = 0.06 \times 120 = 7.2$

Absorption due to ceiling = $\alpha_3 S_3 = 0.80 \times 120 = 96$

Total absorption $\alpha S = \alpha_1 s_1 + \alpha_2 s_2 + \alpha_3 s_3 = 109.8$ Sabines

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$$T = \frac{0.158\ V}{\alpha s} = \frac{0.158 \times 600}{109.8} = 0.86\ sec$$

Average absorption coefficient, $\alpha = \frac{\Sigma \alpha S}{\Sigma S} = \frac{109.8}{220 + 120 + 120}$

Therefore, $\alpha = 0.239$

Example 8: Calculate the reverberation time of small hall of 1500 m^3 having seating capacity of persons when i) The hall is empty and ii) with full capacity of the audience for the follow data.

Surfaceacto	Areas	Coefficient of absorption
plastered wall	112 m^2	0.03
wooden floor	130 m^2	0.06
plastered ceiling	170 m^2	0.04
wooden doors	20 m^2	0.06
cushioned chairs	120	0.5
Audience	120	0.44

Solution:

The absorption due to plastered wall = $\alpha_1 s_1 = 112 \times 0.03 = 3.36$

The absorption due to wooden floor = $\alpha_2 s_2 = 130 \times 0.06 = 7.8$

The absorption due to plastered ceiling = $\alpha_3 s_3 = 170 \times 0.04 = 6.8$

The absorption due to wooden doors = $\alpha_4 s_4 = 20 \times 0.06 = 1.2$

The absorption due to cushioned chairs = $\alpha_5 s_5 = 120 \times 0.5 = 60$

The absorption due to audience = $\alpha_6 s_6 = 120 \times 0.44 = 52.8$

1. Total absorption with out audience = 79.16

Therefore, $T = \frac{0.158\ V}{\alpha s} = \frac{0.158 \times 1500}{79.16} = 2.99\ sec$

2. Total absorption with full capacity of audience = 79.16 + 52.8 = 131.96

$$T = \frac{0.158\ V}{\alpha s} = \frac{0.158 \times 1500}{131.96} = 1.8\ sec$$

Example 9: Entry of people in an auditorium of volume 1,60,000 cu. ft and total absorption of 10 sq. ft of open window raises the absorption by 600 sq. ft. Find the change reverberation time.

Solution:

Volume of auditorium, V = 160, 000 cu. ft

Total absorption, αs = 1000 sq.ft

$$T = \frac{0.05\ V}{\alpha s} = \frac{0.05 \times 160000}{1000} = 8\ sec$$

Total absorption due to entry of people = 1000 + 600 = 1600 sq.ft

Hence, Reverberation time, $T = \frac{0.05 \times 160000}{1600} = 5\ sec$

Change in reverberation time = 8 – 5 = 3 sec

Example 10: A hall with floor is 15 × 30 m^2 along with height of 6m in which 500 people occupy upholsters seat and the remainder sit on wooden chair, optimum reverberation time for orchestral music is 1.34 sec and absorption coefficient per person is 0.44

a. Calculate the total absorption to be provided by the walls, floor and ceiling when the hall is fully occupied.

b. Calculate the reverberation time if only half upholstered seats are occupied. (α = 0.02 for each wooden chair)

Solution:

Volume of hall, V = 15 × 30 × 6 = 2700 m^3

Area of walls = 2 (l + b) h = 2 (15 + 30) × 6 = 540 m^2

Area of floor = 15 × 30 = 450 m^2

Area of ceiling = 15 × 30 = 450 m^2

We have, $T = \frac{0.158\ V}{\alpha s} \Rightarrow \alpha s = \frac{0.158\ V}{T}$

$$\alpha s = \frac{0.158 \times 2700}{1.34} = 318.36$$

Total absorption, αs = 318.36

Absorption due to audience = 500 × 0.44 = 220

a. Absorption provided by walls, floor and ceiling = 318.36 – 220 = 98.36

b. If only half seats are occupied, then absorption due to 250 people = 250 × 0.44 = 110 and absorption due to 250 wooden seats = 250 × 0.02 = 5

Total absorption for this case = 318.36- 220 + 110 + 5 = 213.36

$$T = \frac{0.158\ V}{\alpha S} = \frac{0.158 \times 2700}{213.36} = 2.0\ sec$$

Example 11: A quartz crystal of thickness 0.001 m is vibrating at resonance. Calculate the fundamental frequency. Given, young's modulus for quartz (γ) = 7.9 × 10^{10} N/m^2 and, density of quartz (ρ) = 2.650 × 10^3 kg/m^3.

Solution:

We have the relation for fundamental frequency.

$$f = \frac{1}{2l}\sqrt{\frac{\gamma}{\rho}}$$

In this case, $\gamma = 7.9 \times 10^{10}$ N/m^2

$\rho = 2.650 \times 10^3$ kg/ m^3

$l = 0.001$ m

Therefore, $f = \frac{1}{2 \times 0.001}\sqrt{\frac{7.9 \times 10^{10}}{2.65 \times 10^3}} = 2.73 \times 10^6$ Hz

Example 12: A room has dimension of 6m × 4m × 5m find,

1. mean free path of sound wave in the room.
2. The number of reflections made per second by the sound wave with the walls of the room (Take velocity of sound in air = 350 ms^{-1})

Solution:

i. Mean free path $(\lambda) = \frac{4\,(\text{volume of room})}{\text{Total surface area}} = \frac{4V}{S}$

Here volume of the room $(V) = 6 \times 4 \times 5 = 120\ m^3$

and total surface area $(S) = 2\,(lb + bh + hl)$

$= 2\,(6 \times 4 + 4 \times 5 + 6 \times 5) = 148\ m^2$

Mean free path $(\lambda) = \frac{4 \times 120}{148} = 3.243$ m

ii. The no of reflection made per second, $N = \frac{\text{velocity of sound}}{\text{mean free path}} = \frac{350}{3.243} = 107.9 = 108$

Example 13: A lecture hall of volume $12 \times 10^4\ m^3$ has a total absorption of 13200 m^2 of open window unit. Entry of students into the hall raises the absorption by another 13200 m^2 of open window unit. Find the change in reverberation time.

Solution:

Volume of the hall, $V = 12 \times 10^4 m^3$

Total absorption, $\alpha s = 13200\ m^2$ of open window unit.

Reverberation time $(T) = \frac{0.158\,V}{\alpha s} = \frac{0.158 \times 12 \times 10^4}{13200} = 1.44$ sec

Due to the entry of students the raise in absorption = 13200 m^2

New total absorption, $\alpha s = 13200 + 13200 = 26400\ m^2$

New, reverberation time,

$$T_1 = \frac{0.158\,V}{\alpha s} = \frac{0.158 \times 12 \times 10^4}{26400} = 0.72 \text{ sec}$$

Therefore, change in reverberation time = 1.44 - 0.72 = 0.72 sec

Example 14: The size of an empty assembly of bell has dimension 20 × 15 × 5 cm^3 and the reverberation time is 3.5 sec. What area of the wall should be covered by curtain cloth to reduce the reverberation time by 2.5 if the absorption coefficient of curtain cloth is 0.5. Also calculate the average absorption coefficient of the bell.

Solution:

Volume of the bell, $V = 20 \times 15 \times 5 \times 10^{-6}\ m^3 = 0.0015\ m^3$

We have,

$$T = \frac{0.158\,V}{\alpha S}$$

or, $$3.5 = \frac{0.158 \times 0.0015}{\alpha S}$$

$$\alpha S = \frac{0.158 \times 0.0015}{3.5} = 6.77 \times 10^{-6}$$

Total surface area of bell = 2 (lb + lh + bh)

$= 2\,(20 \times 15 + 20 \times 5 + 15 \times 5) = 950\ cm^2 = 0.095\ m^2$

Absorption coefficient of curtain cloth $(\alpha_1) = 0.5$

Now, New reverberation time $T_1 = 3.5 - 2.5 = 1$ sec

Then, $$T_1 = \frac{0.518\,V}{\alpha S + \alpha_1 S_1}$$

$$1 = \frac{0.158 \times 0.0015}{6.77 \times 10^{-6} + 0.5\,S_1}$$

or, $$0.5\,S_1 = 2.37 \times 10^{-4} - 6.77 \times 10^{-6}$$

or, $$S_1 = \frac{2.30 \times 10^{-4}}{0.5}$$

$\therefore$ $$S_1 = 4.60 \times 10^{-4}\ m^2$$

Also, average absorption coefficient of bell, $\alpha = \frac{\Sigma \alpha S}{\Sigma S} = \frac{6.77 \times 10^{-6}}{0.095} = 7.12 \times 10^{-5}$

Example 15: A hall has a volume 2265 m^3. Its total absorption is equivalent to 92.9 m^2 of open window. What will be the effects of reverberation time if audiences fills the hall and thereby increase the absorption by another 92.9m^2.

Solution:

Volume of hall $(V) = 2265 m^3$

Total absorption $(\alpha S) = 92.9 m^2$ of open window unit

Reverberation time $(T) = \frac{0.158V}{\alpha S} = \frac{0.158 \times 2265}{92.9} = 3.852$ sec.

Due to the audiences the raise is absorption = 92.9 m^2

New total absorption, $\Sigma \alpha S = 92.9 + 92.9 = 185.8\ m^2$

New Reverberation time $(T_1) = \frac{0.158 \times 2265}{185.8} = 1.926$ sec.

Therefore change in Reverberation time = 3.852 – 1.926 = 1.926 sec.

THEORETICAL ANSWER QUESTIONS

1. What is reverberation? Derive Sabine's reverberation formula and explain its significance.
2. What is ultrasonic waves? Discuss a method for the production of Ultrasonic wave.
3. How are ultrasonic wave produced? How these wave are used to detect cracks in metals. Mention some of the properties of ultrasonic waves.
4. Distinguish between ultrasonic and infrasonic. What are the application of ultrasonic waves?
5. What do you understand by noise pollution. Discuss briefly the causes of noise pollution.
6. What is noise pollution? How can you prevent the noise pollution?
7. How do you differentiate sound insulation and sound absorption. Write the effects of noise pollution.
8. Derive the necessary equation for reverberation time and mention the factors affecting the acousting of building.
9. Explain the magnetostriction method of production of ultrasonics.

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10. What are ultrasonic waves? Discuss briefly the application of ultrasonic wave in non-destructive testing.
11. Explain ultrasonic method in non-destructive testing write its advantages and limitation.
12. Discuss a method of production of ultrasonic wave. Write the applications of ultrasonic wave in medical field and in science and engineering.
13. What is reverberation? Discuss the factors that determines reverberation time in an auditorium.
14. Define absorption coefficient of sound. Derive relation between reverberation time and absorption coefficient for acoustically good hall.
15. Give an account of bad acoustic properties of a hall. Derive the expression for reverberation time in a good acoustics of a hall.
16. Discuss the Sabine's formula to obtain the expression for growth and decay of sound energy in room.
17. Discuss the factor reverberation for quality control of sound in public buildings. How it is different from echo?
18. Give the physical principle of the generation of Ultrasound by using piezoelectric material.
19. What is reverberation? Explain the causes of reverberation in theatre and how can it be reduced.
20. What are the factors that affect the build up and decay of sound level in an auditorium. Define reverberation time.

NUMERICAL PROBLEMS

1. Calculate the reverberation time in a hall measuring 40×100×20ft. With the following parameters (i) 7500 sq. ft of plaster, $\alpha_1 = 0.03$ (ii) 600 sq. ft. of wood and floor etc $\alpha_2 = 0.06$ (iii) 400 sq. ft. of glass $\alpha = 0.025$ (iv) 600 seats, $\alpha_4 = 0.3$ (v) Audience of 500 persons $\alpha_5 = 4.0$ per person. [Ans: 1.632s]
2. For an empty assembly hall of size 20 × 15 × 10m^3 the reverberation time is 3.5 sec. Calculate the absorption coefficient of the hall. What area of the wall should be covered by the curtain so as to reduce the reverberation time to 2.5 sec? Give the absorption coefficient of curtain cloth is 0.5. [Ans: 0.104, 108.342]
3. A lecture hall with a volume of 45,000 cu. ft. is found to have a reverberation time of 1.5 sec. What is total absorbing power of all the Surface in the hall? If the area of the sound absorbing surface is 8000sq. ft. Calculate average absorption coefficient. [Ans: 1500 sq.ft., 0.187]
4. A hall has a volume of 80000 sq. ft. Its absorption coefficient is equivalent to 1000 sq. ft. of an open window. Calculate the reverberation time. [Ans: 4s]
5. The volume of room is 980 m^3. The wall area of the room is 150 m^2, ceiling area 95 m^2 and floor area 90m^2. The average sound absorption coefficient (i) for wall is 0.03 (ii) for ceiling 0.80 and (iii) for the floor is 0.06. Calculate the average sound absorption coefficient and reverberation time. [Ans: 0.256, 1.80s]
6. Calculate the fundamental frequency of vibration when a quartz crystal of thickness 0.15 cm vibrating at resonance. Give that Y = 7.9 × 10^{10} N/m^2 and ρ = 2650 kg/m^3. [Ans: 1.819 MHz]
7. Calculate the frequency of 40 mm length of pure iron rod. Given the density of pure iron 7.25 ×10^3 kg/m^3 and its Young's modulus is 115 × 10^9 N/m^2. Can you use it in magnetostriction oscillator to produce ultrasonic waves? [Ans: 49.784 kHz]

□□□

4 Chapter

PHOTONICS

CHAPTER OUTLINE

After completion of this chapter, you will be able to know:

- Laser: Introduction of laser, Principles of generation of laser light (induced absorption, spontaneous emission, stimulated emission, population inversion, pumping, metastable state), He–Ne laser, Semiconductor laser, Applications of laser.
- Fiber optics: Introduction, Types of optical fiber, Principle of propagation of light wave through optical fiber (Acceptance angle), Numerical aperture, Applications of optical fiber in communications, Optical fiber sensors.

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(A) LASER

INTRODUCTION

In an ordinary source of light the billions and billions of atoms or molecules produce light of all possible wave length in all possible direction. So due to superposition of different types of wave (different in phase, plane of polarization etc), the intensity decreases with distance. The light produced by ordinary source of light is not unidirectional, not monochromatic and in out of phase.

The light from a source come as the sum of total radiations produced by all individual atoms or molecules in the source. In laser, the radiations given out by all the emitters in the source are in mutual agreement not only in phase but also in the direction of emission and plane of polarization. Such monochromatic and highly coherent sources are the laser sources. The word LASER stands for Light Amplification by Stimulated Emission of Radiations.

PRINCIPLES OF GENERATION OF LASER LIGHT

Induced Absorption

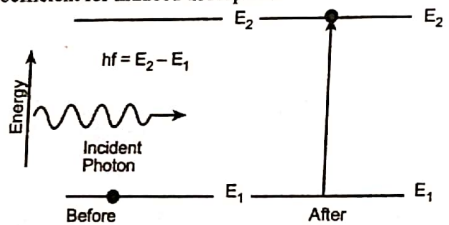
If the atom is initially in the lower state E_1, it can be raised to higher energy state E_2 by imparting it with a photon of energy $E_2 - E_1 = hf$, f = frequency of incident photon. The photon is absorbed by the atom and reached to higher energy state E_2. This phenomenon of absorption of photon by atom is called induced absorption. The rate of induced absorption is given by,

$$R_{abs} = B_{12}\,\rho\,N_1$$

Where, N_1 = Population of atoms at E_1

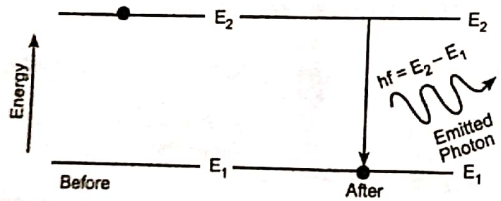
ρ = Energy density of incident beam

B_{12} = Einstein's coefficient for induced absorption.



Spontaneous Emission

An atom cannot stay in the excited state for a longer time. In a time of about 10^{-8} second the atom returns to the lower energy state by releasing a photon of energy $E_2 - E_1 = hf$. The emission of a photon by an atom without any external impetus is called spontaneous emission.



The rate of spontaneous emission is given by, $R_{sp} = A_{21} N_2$

Where, N_2 = Number of atoms at excited state.

A_{21} = Einstein's coefficient for spontaneous emission.

Characteristic of Spontaneous Emission

1. The process of spontaneous emission is probabilistic.
2. The process of spontaneous emission is not controllable from out side.
3. The instant of emission, direction of propagation, the phase and the plane of polarization of each photon are all random.
4. The light resulting from this process is not monochromatic.
5. The intensity goes on decreasing rapidly with distance from source.
6. Light emitting from this process is incoherent.
7. The net intensity is proportional to the number of radiating atoms.
   i.e. $I_{total} = NI$, where N = Number of atoms and
   I = Intensity of light emitted by one atom.

Stimulated Emission

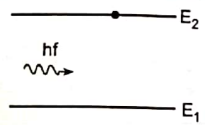
The atoms in the excited state can be returned to original state before spontaneous emission by the incident of an external photon with the emission of both incident and stimulated (induced) photon. This type of emission of radiation is called the stimulated emission or induced emission. The rate of stimulated emission of photon is given by

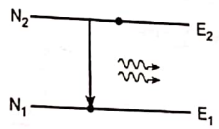
$$R_{st} = B_{21}\,\rho N_2$$

Where B_{21} = Einstein's coefficient for stimulated emission.

N_2 = Number of atoms at excited state.

ρ = Energy density of incident photon





Characteristic of Stimulated Emission

1. The process of stimulated emission is controllable from outside.
2. The photon induced propagates in the same direction as that of incident photon and has same frequency, phase and plane of polarization as that of incident photon.
3. In this process, two photon emerges with the incidence of one photon. These two photons traveling in the same direction interact with two more excited atom and generate two more photon and so on.
4. All the light waves generated in the medium are due to one initial wave and all of the waves are in phase. So the waves are coherent and interfere constructively to produce larger intensity.
5. The net intensity is proportional to the square of number of radiating atoms.
   $I_{total} = N^2 I$, I = intensity of light radiation emitted by one photon.
6. The resulting light is of high intensity and said to be amplified.

Pumping and Population Inversion

To achieve a high percentage of stimulated emission, a majority of atoms should be at the higher energy level than at the lower level. The establishment of situation in which the number of atoms in the higher energy level is greater than that of lower energy level is called population inversion.

The process carried out to achieve population inversion by supplying energy to laser medium is called pumping. When the pumping is done by light energy it is called optical pumping.

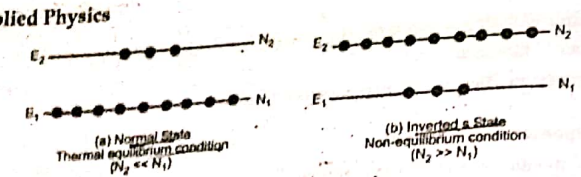


Figure: Population inversion

Active Medium

Atoms are generally characterized by a large number of energy levels. How ever all types of atom are not suitable for laser operation. Even in a medium consisting of different species of atoms, only a small fraction of atoms of a particular type have energy level suitable for achieving population inversion such atoms can produce more stimulated emission than spontaneous emission and causes amplification of light. These atoms are called active centers. The rest of the medium acts as host and supports active center. The medium hosting the active center is called active medium.

Metastable State

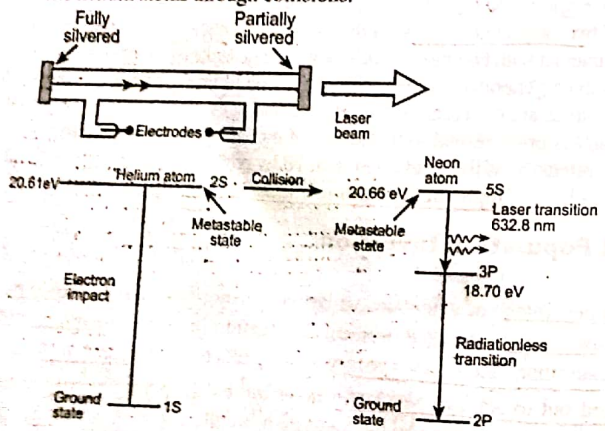
Normally the life time of an atom in excited state is very short (10^{-8} sec). How ever the atoms of special elements can stay for longer time in the higher energy state where the life time is greater than 10^{-4} sec. Such higher state is called meta stable state. There is no radiative transitions when the electron jumps to the metastable state from the excited state. The emission of radiation occurs when it goes from metastable state to the ground state.

Characteristics of Laser Light

1. Laser light is highly mono chromatic.
2. Laser light is highly coherent (Two independent source can act as coherent sources)
3. Laser light is highly directional.
4. It is highly intense.

HELIUM-NEON LASER

The schematic diagram for He-Ne laser is as shown in figure. It consist of a long discharge tube filled with a mixture of Helium and Neon gas in the ratio 10:1. Neon atoms are the active center and have energy level suitable for laser transition while helium atoms help in exciting Neon atom. Electrodes are provided in the discharge tube to produce discharge in the gas. When the power is switched on, a high voltage of about 10 KV is applied across the gas. It is sufficient to ionize the gas. The electrons and ions produced in the process of discharge are accelerated towards the anode and cathode respectively. The energetic electrons excite helium atoms through collisions.



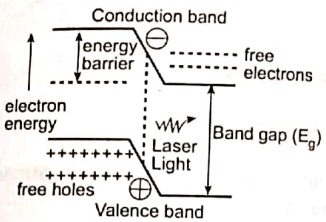
One of the excited level of He is 2S at 20.61 ev above the ground level. It is metastable state and excited helium cannot return to ground level through spontaneous emission. How ever it can return to ground level by transferring its excess energy to a Neon atom through collision. Such an energy transfer can take place when two colliding atom have identical energy levels. One of the excited level 5S of Neon is at 20.66eV, which is nearly at the same level as 2S of Helium atom. The kinetic energy of Helium atoms provides the additional 0.05 eV required for excitation of the Neon atoms. Helium atoms drop to the ground state after exciting the Neon atoms. This is pumping mechanism in He - Ne laser.

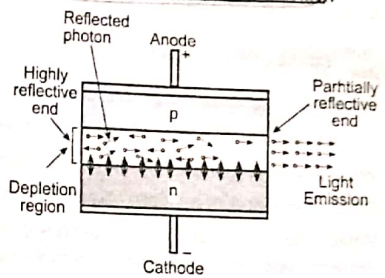
The upper state (5S) of Neon atoms is a meta stable state. Therefore Neon atoms accumulate in this upper state. A state of population inversion is established between 5S and 3P levels. Random photons emitted causes stimulated emission. The transition from 5S to 3P of Neon generate a laser beam of wave length 6328Å°. fter the intensity has reached optimum value, a highly monochromatic and intense beam of laser light emerges through half silvered mirror.

SEMICONDUCTOR LASER

Semiconductor lasers are basically PN junction diodes. When a p-type semiconductor comes into immediate contact to an n-type semiconductor, a PN junction forms at the interface.

When an external supply voltage is applied to the PN junction, electrons from the n-region and holes from the p-region are forced into the junction. These electrons and holes are attracted to one another.





When they collide, they neutralize each other and emit recombination radiation. For a semiconductor laser, the energy of the photon emitted as recombination radiation is equal to the band gap of the material. The active medium of the semiconductor laser is a PN junction. In this laser, mirror is not used as in other resonator or cavity for optical feedback to sustain laser oscillation.

The reflectivity due to the refractive indices of two layers of a semiconductor laser is used for optical feedback. The end faces of two types of semiconductors (i.e. P-type and n-type) are cleaved and are perfectly parallel to each other for achieving optical feedback.

If the active medium or junction is made of a single type of semiconductor material, then the semiconductor laser is also known as homojunction laser.

On the other hand, if the junction is made of different types of semiconductor materials, then the semiconductor laser is known as a heterojunction laser.

Because of their small size and highly efficient pumping, they are particularly useful in optical data transmission and storage, spectroscopy, medicine, and pumping solid-state lasers.

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APPLICATIONS OF LASER

1. Since the intensity of laser light is high, they have been used in drilling holes in hard metals and diamond. Laser light can also be used to cut, hard material such as glass. (cutting and drilling).
2. Laser light is highly coherent, so it is used in radios and television. These can be modulated to send large number of message simultaneously (modulation.)
3. They are used for 'blood less surgery' in hospitals. Many medical equipment are made in the principle of laser. (Medical)
4. They are used in scientific research. Such as in the study of chemical and crystalline structure of various molecule, in Raman spectroscopy, in astronomy. (to find distance between the stars) (Scientific Research).
5. They are even used in wars to target missiles. (war)
6. Many modern electronic devices such as CD players, laser printers, memory cards etc are made in the principle of laser (Modern equipments)
7. Holography and fiber optics are also the result of laser (holography, fiber)
8. High power lasers are used to carry thermo-nuclear reactions.
9. Lasers are used for separating isotopes of an element (to separate isotopes)

THEORETICAL ANSWER QUESTIONS

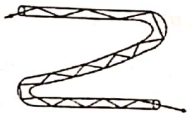
1. What are the principles of generation of Laser light?
2. Explain the basic principles of production of laser light and its application.
3. Write down the characteristics of a laser beam.
4. Discuss briefly working principles of Laser. How laser beam can be obtained by using Ruby Laser.
5. What is meant by LASER? Explain the terms induced absorption, Spontaneous emission, Stimulate emission, population inversion, metastable state and pumping.
6. Explain why laser action cannot occur without population inversion.
7. How laser is Superior to the normal light? Explain, what are requirement for producing laser action? How are they achieved?
8. Differentiate between active center and active medium.
9. Explain the construction and working of He-Ne laser with a suitable energy level diagram.
10. Differentiate between LASER and ordinary light.
11. What is the role of He and Ne atoms in He-Ne laser.
12. What is the importance of laser?
13. Write down the principles of laser action and explain the construction and working principle of He-Ne laser.
14. What are the characteristics of laser light? Explain briefly about semiconductor laser.
15. How the recombination radiation is emitted in semiconductor laser?

□□□

(B) FIBER OPTICS

INTRODUCTION

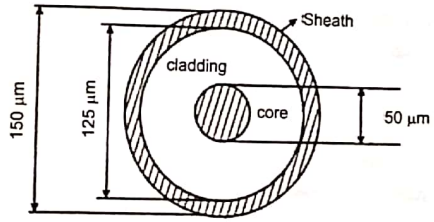
An optical fiber is a transparent conduit, as thin as human hair, made up of glass or clear plastic, designed to guide light waves along its length.



An optical fiber works on principle of total internal reflection. The movement of light is in a zig-zag path and some fraction of light may escape through side walls but major fraction comes out from the other end of the fiber.

An optical fiber consists of following three regions

1. ***Core:*** It is innermost light guiding region.
2. ***Cladding:*** Core is surrounded by a co-axial middle region known as the cladding. The refractive index of cladding is always lower than that of the core.
3. ***Sheath:*** The outermost region is called sheath. The sheath protects the core and cladding from moisture, abrasion contamination and give mechanical strength to fiber.



In optical fiber light travels through core and basic purpose of cladding is to confine the light to the core. The function of sheath is to protect core and cladding from external factor.

The core is made of silica doped with some impurities to increases its refractive index. The cladding is made by pure silica.

MODES OF PROPAGATION

In optical fiber, only certain directions (or path) are allowed for the propagation of light wave. These allowed paths are called modes of propagation.

The light rays traveling through a fiber may be classified as 1) zig–zag rays 2) axial rays.

1) **Zig–zag rays:** The rays traveling inside the fiber get repeatedly reflected at the interface of core and cladding. Some of them may be in phase and get intensified while some other be in out of phase and fade out due to destructive interference. The light ray paths along which the waves are in phase are the allowed modes of propagation.

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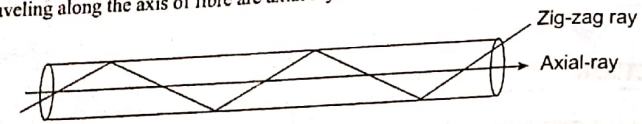
2) **Axial rays:** The light ray path along the axis of fiber is also a allowed modes of propagation. The rays traveling along the axis of fibre are axial rays.

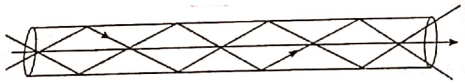


TYPES OF OPTICAL FIBER

A. On the basis of modes of propagation there are two types of optical fibers

1. **Single mode fiber:** In single mode fiber there is only one mode of propagation (only axial mode). It has smaller core diameter. It is step index fibre.

2. **Multimode fibre:** In multimode fibre there are two or more than two modes of propagation, so it has large core diameter.



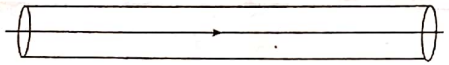
B. on the basis of index profile optical fibres are further classified into two groups, (a) Step index fibre and (b) graded index fibre.

The single mode fibre is usually of step index type but the multimode optical fibre is of both step index and graded index type.

An index profile is the plot of refractive index along y-axis and distance from core along x-axis. It gives the variation of refractive index with distance as we move outward from core.

1. Single Mode Step Index Fibre

i. In single mode fibre, there is only one mode of propagation i.e. along the axis only, called zero order mode. So it is named single mode fibre (SMF).



ii. In this type of fibre, core has higher constant refractive index (n_1 = 1.52) and cladding has lower constant refractive index (n_2 = 1.48). So refractive index changes abruptly at core cladding interface. Therefore it is called step index fibre.

iii. It has smaller core diameter of 4μm which corresponds to some of the wave lengths of light waves.

iv. Normalized frequency (V-number) is less than 2.405.

v. It has low value of fractional refractive index change (Δ).

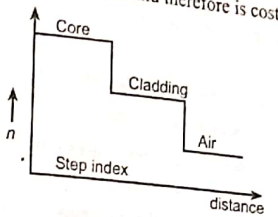
vi. Numerical aperture (N.A.) is small.

vii. The value of acceptance angle (θ_o) is small.

viii. Since there is only one mode of propagation, so there is no (or less) intermodal dispersion and hence the data rate is high.

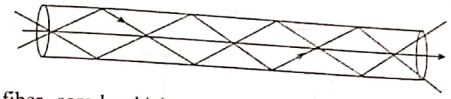
ix. It has high transmission loss (because of abrupt change in refraction index)

x. It is difficult to manufacture and handle and therefore is costly.



2. Multimode Step Index Fibre

i. In this type of fibre there are multiple modes of propagation.



ii. In this type of fiber, core has higher constant refractive index (n_1 = 1.52) and cladding has lower constant refractive index (n_2 = 1.42). So refractive index changes abruptly at core - cladding interface. Therefore it is called step index fiber.

iii. It has larger core diameter of 100 μm, which is very large as compared to the wave length of light.

iv. Normalized frequency (v-number) is greater than 2.405.

v. It has higher value of fractional refractive index change (Δ)

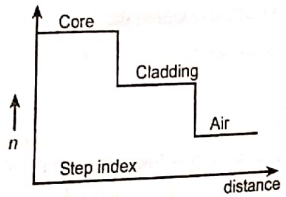
vi. Numerical aperture is high (0.3)

vii. The value of acceptance angle is larger.

viii. Since there is multiple mode of propagation, (and NA is more) so there is high intermodal dispersion and hence the data rate is low.

ix. It has high transmission loss (because of abrupt change in refractive index).

x. It is easy to manufacture and is less costly.



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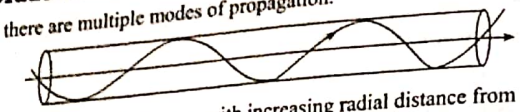
3. Multimode Graded Index Fibre

i. In this fibre, there are multiple modes of propagation.



ii. The refractive index of core decreases with increasing radial distance from the axis.

iii. It has larger core diameter of 100 μm, which is very large as compared to the wave length of light.

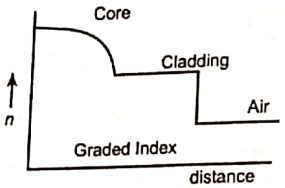
iv. Numerical aperture decreases with increasing radial distance from the axis.

v. The acceptance angle also deceases with radial distance from the axis.

vi. The number of modes in graded index fibre is about half of that in multimode step index fibre. So it has lower dispersion than in multimode step index fibre.

vii. It has low transmission loss because the continuous decrease in the refractive index causes bending of light towards the center of the core.

viii. It's manufacture is more complex than step index Multimode Fibre (MMF).



FRACTIONAL REFRACTIVE INDEX CHANGE

The ratio of difference in refractive indices of the core and the cladding to the refractive index of core is called fractional refractive index change. Fractional refractive index change, $\Delta = \frac{n_1 - n_2}{n_1}$. Since, $n_1 > n_2$ so Δ is always +ve.

Acceptance Angle

Consider an optical fibre through which light is launched at one end. Let n_1, n_2 and n_0 be the refractive indices of core, cladding, and the medium from which light is launched into the fibre.

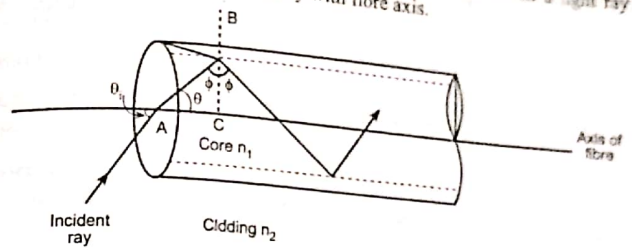
Assume a light ray enters the fibre at an angle θ_i to the axis of the fibre. The ray refracts at an angle θ_r and strikes the core-cladding interface at an angle ϕ. The angle ϕ is greater than critical angle ϕ_c, and total internal reflection occurs. So the light will stay within the fibre.

Applying Snell's law at the incident edge, we get,

$$\frac{\sin\theta_i}{\sin\theta_r} = \frac{n_1}{n_0} \quad \ldots (i)$$

If θ_i is increased beyond a limit, ϕ will drop below the critical value ϕ_c and the ray escapes from the dewall of the fibre. The largest value of θ_i occurs when $\phi = \phi_c$.

Acceptance angle is the maximum value of angle of incidence θ_i for which a light ray can propagate through fibre. It is the angle made by incident ray with fibre axis.



From figure in Δ ABC, $\theta_r = 180 - 90 - \phi = 90 - \phi$

$$\sin\theta_r = \sin(90-\phi) = \cos\phi \quad \ldots (ii)$$

From equation (1) and equation (2)

$$\frac{\sin\theta_i}{\sin\theta_r} = \frac{n_1}{n_0} \Rightarrow \frac{\sin\theta_i}{\cos\phi} = \frac{n_1}{n_0}$$

$$\sin\theta_i = \frac{n_1}{n_0}\cos\phi$$

For $\phi = \phi_c$, $\theta_i = \theta_{i\,max}$ [Here, θ_{imax} is the acceptance angle]

Therefore, $\sin\theta_{i\,max} = \frac{n_1}{n_0}\cos\phi_c \quad \ldots (iii)$

But at critical angle, total internal reflection just occurs

Applying Snell's law at core to cladding interface.

$$\frac{\sin\phi_c}{\sin 90} = \frac{n_2}{n_1}$$

$$\sin\phi_c = \frac{n_2}{n_1} \text{ and } \cos\phi_c = \sqrt{1-\sin^2\phi_c} = \sqrt{1-\frac{n_2^2}{n_1^2}} = \sqrt{\frac{n_1^2-n_2^2}{n_1^2}}$$

Substituting $\cos\phi_c$ for equation (3),

$$\sin\theta_{i\,max} = \frac{n_1}{n_0}\sqrt{\frac{n_1^2-n_2^2}{n_1^2}} = \frac{\sqrt{n_1^2-n_2^2}}{n_0} \quad \ldots (iv)$$

$$\boxed{\theta_{i\,max} = \sin^{-1}\frac{\sqrt{n_1^2-n_2^2}}{n_0}} \quad \ldots (v)$$

$\propto$ max (medium)

which is the expression for acceptance angle.

If the incident ray is launched from air medium,

$$\boxed{\theta_0 = \theta_{i\,max} = \sin^{-1}\sqrt{n_1^2-n_2^2}.}$$

air

The light ray incident at an angle less than $\theta_{i\,max}$ will propagate along the fibre and the light ray incident at an angle larger than $\theta_{i\,max}$ will escape out.

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NUMERICAL APERTURE (N.A.)

Numerical aperture is the measure of the amount of light that can be accepted by a fibre. Mathematically it is defined as the sine of the acceptance angle i.e.

$$N.A. = \sin\theta_0$$

$$= \sqrt{n_1^2 - n_2^2}$$

$$= \{(n_1-n_2)(n_1+n_2)\}^{1/2} = \left(\frac{n_1-n_2}{n_1} \cdot \frac{n_1+n_2}{2} \cdot 2n_1\right)^{1/2}$$

Since fractional refractive index change $\Delta = \frac{n_1 - n_2}{n_1}$

$$N.A. = \left(\Delta \cdot \frac{n_1+n_2}{2} \cdot 2n_1\right)^{1/2}$$

Approximating $\frac{n_1+n_2}{2} \approx n_1$ as $n_1 > n_2$

$$N.A. = (\Delta . n_1 . 2n_1)^{1/2} = \sqrt{2n_1^2\Delta}$$

$$N.A. = n_1\sqrt{2\Delta}$$

It is seen that N.A. only dependent on the refractive indices of core and cladding. The value of N.A. ranges from 0.13 to 0.50. Larger value of N.A. means the fibre can accept more light from the source.

NORMALIZED FREQUENCY: (V–NUMBER)

The V-number determines the number of modes that can propagate through a fibre.
It is an important parameter that characterize an optical fibre. Mathematically it is give by.

$$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2}$$

Where a = radius of the core, λ = free space wave length, n_1 and n_2 are refractive indices for core and cladding.

For $V < 2.405$, fibre can support only one mode and is called single mode fibre (SMF)

For $V > 2.405$, fibre can support number of modes simultaneously and is called multimode fibre (MMF)

pplication

ptical fibre has a wide variety of applications. Some of them are discussed below.

Medical application

- The optical fibre technique is employed for endoscopy, which helps in the study of inaccessible part of human body.
- This can be employed to join a detached retina or rectify other eye defects using laser.

- In cardiology, optical energy transmitted through a optical fibre is used to evaporate built-up plaque that is blocking an artery.
- The optical fibre technology is used in the treatment of cancer.

2. **In Communications**

An optical fibre communication system is similar to traditional communication system, with the exception that this system is highly efficient.

A traditional communication system consist of a transmitter, a receiver and an information path way. Generally the information to be communicated is non electrical message which is first converted into electrical message by transducer. These are modulated with a wave of very high frequency called carrier wave [In fibre communication, laser light plays the role of carrier wave and can carry large number of messages].

The message travels along transmission channel and is received at the receiver. The receiver demodulates the modulated wave and message is fed to transducer to transform it into original form.

Higher the band width of carrier signal larger will be the number of message that it can transmit. The band width of light wave used in optical fibre communication is appreciably high than that of micro waves used in traditional communication system. So the optical fibre communication system is highly efficient than traditional one.

Advantages of Optical Fiber Communication

1. Because the light waves plays the role of carrier waves, the rate of data transmission is very high.
2. Band width of light wave is greater than that of electrical wave so greater amount of information carried over in optical fiber.
3. Optical fiber are light in weight and flexible.
4. There is less interference of waves so the cross talk is negligible in fiber communication.
5. Optical fiber have longer life time.
6. There is no corrosion and tolerability over high temperature.
7. Easy to maintain and more reliable.
8. Optical fiber transmit light wave not electrical so they are safer than copper cables.

3. **Optical fibre sensors**

The property of variation of refractive index under the action of external force in optical fibre is of crucial importance. In optical fibre sensor, LED act as a source of light, photo detector measures the output intensity.

a) **Thermometer (Heat sensor):** If the optical fibre is subjected to heating, it causes a change in refractive index. As temperature increses, the refractive index of cladding increases. Thus the difference in refractive indices of core and cladding reduces and light gets leak away from the cladding. Due to this effect the output intensity decreases. The reverse effect occurs when the temperature is decreased. By observing the variation in emergent intensity, the temperature at different place can be notified.

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b) **Smoke and pollution detector:** If foreign particle (due to smoke and pollution) are present in a region between two optical fibre the variation in emergent intensity determines the extent of foreign particle between the two fibre.

c) A loop of optical fibre can used to determine the level of liquid inside an unreachable place. A part of cladding at which optical fibre is going to touch liquid level is scraped. A bare (naked) core losses more light when it is immerged in liquid than when it is in air. Therefore sudden change of out coming intensity indicates liquid level.

d) The other sensors such as pressure sensor, current sensor, voltage sensor, magnetic field sensor are made on the basis of optical properties of optical fibre.

4. **Military Application**

Optical fibres have a lot of application in various warfare's and military operations. For control mechanism or communication, a lot of copper cables are to be transported by an aircraft, a ship or tank. However with fibre optics, the weight is enormously reduced and communication network is greatly improved. With fibre communication secrecy is also maintained.

Fibre guided missiles are used in recent wars. Sensors mounted on the missile transmit video information through the optical fibre to a ground control van and receives commands from the van again. The control van continuously monitors the missile, so that it can hit the target.

Solved Example

Example 1: A glass clad fibre is made with core glass of refractive index 1.5 and cladding is doped to give a fractional index difference of 5×10^{-4}. Determine i) the cladding index ii) The critical internal reflection angle iii) The external critical acceptance angle iv) numerical aperture.

Solution:

Here, refractive index of core, $n_1 = 1.5$

fractional refractive index change, $\Delta = 5 \times 10^{-4}$

i. We have, $\Delta = \frac{n_1 - n_2}{n_1} \Rightarrow n_1 - n_2 = n_1\Delta \Rightarrow n_2 = n_1 - n_1\Delta$

$\Rightarrow \quad n_2 = 1.5 - (1.5 \times 0.005) = 1.4925$

$\therefore \quad n_2 = 1.4925$

ii. Let ϕ_c be the critical internal reflection angle then,

$$\sin\phi_c = \frac{n_2}{n_1} \Rightarrow \phi_c = \sin^{-1}\left(\frac{1.4925}{1.5}\right) = 88.2°$$

iii. The external critical acceptance angle is given by

$$\theta_0 = \sin^{-1}\sqrt{n_1^2 - n_2^2}$$

$$= \sin^{-1}\left(\sqrt{1.5^2 - 1.4925^2}\right)$$

$$= 112°$$

iv. N.A. $= n_1\sqrt{2\Delta}$

$$= 1.5\sqrt{2 \times 0.005} = 0.0474$$

Example 2: An optical fibre has fractional index difference of 0.2 and a cladding refractive index 1.59. Determine acceptance angle for the fibre in water in which has refractive index of 1.33.

Solution:

Here, $\Delta = 0.2$, $n_2 = 1.59$, $n_o = 1.33$

Since, $\Delta = \frac{n_1 - n_2}{n_1} \Rightarrow n_1 - n_2 = n_1\Delta \Rightarrow n_1 - n_1\Delta = n_2$

$$n_1(1 - \Delta) = n_2 \Rightarrow n_1 = \frac{n_2}{1 - \Delta} = \frac{1.59}{1 - 0.2} = 1.9875$$

Again, Acceptance angle,

$$\theta_{i\,max} = \sin^{-1}\frac{\sqrt{n_1^2 - n_2^2}}{n_o} = \sin^{-1}\frac{\sqrt{1.9875^2 - 1.59^2}}{1.33}$$

$$\theta_{i\,max} = 63.72°$$

Example 3: Calculate the refractive indices of the core and cladding materials of a fiber from following data. Numerical aperture (NA) = 0.22 and fractional refractive index change $\Delta = 0.012$.

Solution:

Here, N.A = 0.22

$$\Rightarrow \quad n_1\sqrt{2\Delta} = 0.22$$

$$\Rightarrow \quad n_1 = \frac{0.22}{\sqrt{2\Delta}}$$

$$= \frac{0.22}{\sqrt{2 \times 0.012}}$$

$$= 1.42$$

Given, $\Delta = 0.012$

$$\Rightarrow \quad \frac{n_1 - n_2}{n_1} = 0.012$$

$$n_1 - n_2 = 0.012\, n_1$$

$$\Rightarrow \quad n_2 = n_1 - 0.012\, n_1$$

$$\Rightarrow \quad n_2 = 1.402$$

THEORETICAL ANSWER QUESTIONS

1. Write the principle of optical Fiber.
2. Write short notes on propagation of light wave through optical Fiber.
3. What is an optical fiber? How is it made? Write down main differences between step index and graded index multimode optical fibers with well diagrams.

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4. Show that numerical aperture (N.A.) = $\mu_{core}\sqrt{2\Delta}$, where symbols carry their usual meaning.
5. Define acceptance angle and numerical aperture of an optical fiber, develop a relation betwe them. Also write down their significance.
6. Explain the application of optical fiber in communication system.
7. Compare a monomode step index fiber with a multimode step index fiber.
8. Explain the physics behind the function of optical fiber.
9. What are the application of optical fiber in communication system as well as medical science.
10. Give the basic principle and structure of optical fiber. How the various refractive indices have to b related to get better working of an optical fiber.
11. What are the monomode and multimode optical fiber.
12. What are the advantages of optical fiber communication over co-axial copper cab communication?
13. What is fiber optics? What are their importances? Also explain the types of optical fiber.
14. Discuss the types of optical fiber and explain the working principle of optical fiber?
15. Describe about the principle of working of a optical fibre and its application.

NUMERICAL PROBLEMS

1. In an optical fiber, the core material has refractive index 1.6 and refractive index of clad material 1.3. Calculate the value of critical angle and the angle of acceptance. [Ans: 54.3°, 60.3
2. Calculate the numerical aperture and hence the acceptance angle for an optical fiber. Given tha the refractive indices of core and cladding are 1.45 and 1.40 respectively. [Ans: 0.2626, 22.1
3. A light ray enters from air to a fiber. The fiber has refractive index of core is equal to 1.5 and that o cladding is 1.48. Find the critical angle, fractional refractive index, the acceptance angle an numerical aperture. [Ans: 80.63°, 0.0133, 14.13°, 0.24
4. The refractive indices for core and cladding for a step index fiber are 1.52 and 1.41 respectively Calculate i) critical angle ii) numerical aperture iii) the maximum incidence angle. [Ans: 68.07° 0.578, 34.59

□□□

5 Chapter

CAPACITOR AND DIELECTRIC

CHAPTER OUTLINE

After completion of this chapter, you will be able to know:

- Capacitor: Introduction, Types of capacitor, Charging and discharging of capacitor.
- Dielectric: Introduction, Dielectric constant, electric flux density, Polarization, Polarization in free space, Gauss law in dielectric, Electronic and Ionic polarization (Clausius-Mossotti equation).

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INTRODUCTION

A capacitor is a device that stores electric potential energy or electric charge. Two conducting plates enclosing an insulating (dielectric) material form a capacitor. To store charge it is connected in series with power supply. To transfer charge from one conductor to the other some work must be done. This work done is stored as electric potential energy.

CAPACITANCE

For a capacitor, the ratio of the charge on each conducting plate to the potential difference is a constant quantity, called the capacitance.

$$\frac{\text{Charge}}{\text{potential difference}} = \frac{q}{V} = C \text{ (capacitance)}$$

$$\text{unit} = \frac{\text{coulomb}}{\text{volt}} = \frac{C}{V} = \text{Farad}$$

It is also defined as the 'ability of a capacitor to store electric potential energy'.

The capacitance depends on the size and shapes of the conductors and on the insulating material between them. Due to the presence of dielectric capacitance increase. This happen because redistribution of charge called polarisation takes place with in the insulating material.

TYPES OF CAPACITORS

Before studying about types of capacitor we need to know the method for calculation of capacitance.

Calculating the capacitance

i. Assume a charge q on the plate under consideration.

ii. Calculate the electric field $\vec{E}$ between the plates in terms of this charge using Gauss's law.
If q is the charge on each plate of capacitor then from Gauss's law.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_o} \quad .....(1)$$

Here q is charge enclosed by Gaussian surface and $\oint \vec{E}\, dA$ is the net electric flux through the surface. The Gaussian surface is constructed so that it completely encloses the charge on the positive plate. The Gaussian surface will be such that $\vec{E}$ will have a uniform magnitude E and the vector $\vec{E}$ and $d\vec{A}$ will be parallel.
Therefore, equation (1) reduces to

$$E A = \frac{q}{\epsilon_o} \Rightarrow q = \epsilon_o E A \; [\cos\theta = \cos 0 = 1]$$

iii. Knowing $\vec{E}$, calculate the potential difference between the plates.
Now, the potential difference between the plates of capacitor is given by

$$V = \int_i^f \vec{E} \cdot d\vec{r}$$

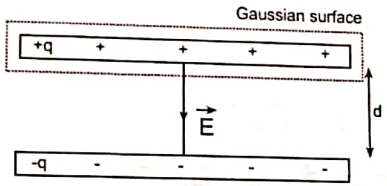
iv. Now capacitance, $C = \frac{q}{V}$

TYPES OF CAPACITOR

1. **Parallel plate capacitor**

The arrangement of two parallel conducting plates each with area A separated by small distance 'd' form a parallel plate capacitor.

The plates are considered so large and so close together that, we can neglect the fringing of the electric field that occurs near the edges of the plates and assume that the electric field has same magnitude and direction everywhere.



We draw a Gaussian surface that encloses just the charge 'q' on the positive plate.

From Gauss law, $\int \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_o}$

$$\Rightarrow \quad E A = \frac{q}{\epsilon_o}$$

$$q = \epsilon_o E A \Rightarrow E = \frac{q}{\epsilon_o A}$$

$$\text{Now, } V = \int_o^d E.dr = E \int_o^d dr = E\,[d]_o^d = Ed = \frac{qd}{\epsilon_o A}$$

$$\text{Capacitance of capacitor is, } C = \frac{q}{V} = \frac{q \times \epsilon_o A}{q\,d} = \frac{\epsilon_o A}{d}$$

$$C = \frac{\epsilon_o A}{d}$$

Therefore, capacitance depends only on the geometry of the capacitor i.e. area of the plates and distance between them.

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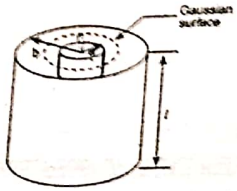
2. **Cylindrical capacitor**

Two cylinders of radii a and b placed co - axially each of length 'l'. Separated by small distance (b - a) form a cylindrical capacitor.

We assume $l >> b$ so that fringing of electric field that occurs at the end of the cylinder can be neglected.



As a Gaussian surface, we choose a cylinder of length l and radius r that enclosed just the charge q on the positive inner cylinder.

From gauss law, $\int \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_o} \Rightarrow EA = \frac{q}{\epsilon_o} \Rightarrow E = \frac{q}{\epsilon_0 A}$

$$E = \frac{q}{2\pi \epsilon_o lr}$$

Now, $V = \int_a^b E.dr$

$$\Rightarrow \quad V = \int_a^b \frac{q}{2\pi\epsilon_o lr} dr = \int_a^b \left(\frac{q}{2\pi\epsilon_o l}\right)\frac{1}{r} dr$$

$$V = \frac{q}{2\pi\epsilon_o l}[ln\, r]_a^b = \frac{q}{2\pi\epsilon_o l}[ln\, b - ln\, a]$$

$$V = \frac{q}{2\pi\epsilon_o l}[ln\, b - ln\, a] = \frac{q}{2\pi\epsilon_o l} ln\, b/a$$

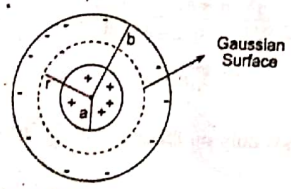
Now, $C = \frac{q}{V} = \frac{1}{q\, ln\left(\frac{b}{a}\right)} . 2\pi\epsilon_0 l$

Capacitance, $C = \frac{2\pi\epsilon_o l}{ln\, b/a}$

Therefore, Capacitance depends only upon geometrical factor l, b and a.

- **Spherical Capacitor**

Two concentric spheres of radii a and b such that b > a insulated from each other form a spherical capacitor.



The inner conductor is solid sphere of radius a and the outer conductor is hollow sphere of radius b. We draw a sphere as a Gaussian surface of radius 'r' concentric with two spheres such that a < r < b.

From gauss law, $\int \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_o}$

$$\Rightarrow \quad E . A = \frac{q}{\epsilon_o}$$

or, $E = \frac{q}{\epsilon_o A} = \frac{q}{4\pi\epsilon_o r^2}$

Now, potential, $V = \int_a^b \vec{E} . d\vec{r}$

$$= \frac{q}{4\pi\epsilon_o}\int_a^b \frac{dr}{r^2} = \frac{q}{4\pi\epsilon_o}\left[-\frac{1}{r}\right]_a^b$$

$$= \frac{q}{4\pi\epsilon_o}\left[-\frac{1}{b}+\frac{1}{a}\right] = \frac{q}{4\pi\epsilon_o}\left[\frac{1}{a}-\frac{1}{b}\right] = \frac{q}{4\pi\epsilon_o}\left[\frac{b-a}{ab}\right]$$

$$V = \frac{q}{4\pi\epsilon_o}\left[\frac{b-a}{ab}\right]$$

Therefore, Capacitance of spherical capacitor is:

$$C = \frac{q}{V} = q \times \frac{4\pi\epsilon_0 ab}{q(b-a)} = \frac{4\pi\epsilon_0 ab}{(b-a)}$$

Hence the capacitance of spherical capacitor depends only on geometrical factor i.e. 'a' and 'b'.

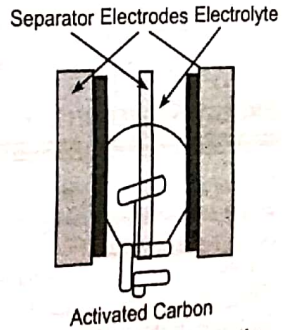
If $b >> a$ or $b \to \infty$ $C = 4\pi\epsilon_0 a$

Applications of Capacitors

i. It is used to store the energy.
ii. The capacitor is required in both residential and commercial appliances such as batteries, fans, cameras, coolers, electronic chargers, LED lights, audio equipment, etc.
iii. It is used in the power system to control voltage and enhance the quality of the power supply.
iv. It is used in electronics and telecommunications equipment.
v. It is used in electrical measuring equipment (for example- sensors)
vi. It is used in high frequency microwave system.

SUPERCAPACITOR

A supercapacitor, also known as an ultra-capacitor, isa capacitor with a much higher capacitance value than a conventional capacitor. The basic principle of the operation of a supercapacitor is the same as that of a conventional capacitor. The supercapacitor uses electrodes of much larger surface area. A supercapacitor's electrodes are often covered with activated carbon as an electrode material. Instead of a dielectric substance, the supercapacitor has a separator between its electrodes.



SuperCapacitor Construction

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Types of Supercapacitors

The supercapacitors are classified into three types, electrostatic double layer capacitors, electrochemic pseudo-capacitors and hybrid capacitors.

1. **Electrostatic Double Layer Capacitors:** These capacitors are made up of two electrodes, separator, and an electrolyte. The electrolyte is a dissolved combination of positive and negati ions in water. A separator separates the two electrodes. Carbon electrodes or derivatives wi substantially greater electrostatic double-layer capacitance are used in these supercapacitors. Th charge separation of electrostatic double-layer capacitors is smaller than that of a typical capacit ranging from 0.3 to 0.8 nm.
2. **Electrochemical Pseudo-Capacitors:** These capacitors utilize conducting polymer electrodes metal oxide electrodes with large electrochemical pseudocapacitance. These types of compone store electrical energy by electron charge transfer between electrode and electrolyte.
3. **Hybrid Capacitors:** Hybrid capacitors are created by combining double-layer capacitor a pseudo-capacitor techniques. Electrodes of different properties are employed in these compone One electrode can show electrostatic capacitance, while the other can show electrochemi capacitance.

Properties of Supercapacitor

i. Supercapacitors have relatively high capacitance (up to 2 kF) and lower voltage ratings (2.5 to 2.7V).
ii. Supercapacitors have comparatively very high energy density (1 Watt hr /kg to 5 Watt hr /kg).
iii. The charging and discharging times of a supercapacitor vary from milliseconds to seconds.
iv. The operating temperature for supercapacitor is about – 40 °C to +85 °C.
v. The cost of supercapacitor is high.

Applications of Supercapacitors

i. It is used in electric vehicles, wind turbines and industrial electrical motors etc.
ii. It is used in ICs, RAM, CMOS, Clocks and micro computers etc.
iii. It is used in cordless electric screwdrivers that charge quickly.
iv. It is used to generate LED flashlights in digital cameras.
v. It replaces electrolytic capacitor banks in uninterrupted power supply (UPS) systems.

Energy Stored in Electric Field

The energy stored in the electric field between the plates of a capacitor can be visualized as the to amount of work done to charge a capacitor. This work done is stored in the capacitor in the form electric potential energy. This stored energy can be used again by discharging of a capacitor. The amou of work done to charge a capacitor is same as the amount of energy required to transfer charges from o plate to another plate of capacitor.

Suppose at an instant a charge dq is transferred from one plate to another. The applied potential differen at that instant be 'V'. Then the amount of work done required is, $dW = Vdq$

The total work done to store charge 'q' is

$$W = \int_0^q Vdq = \int_0^q \frac{q}{C}dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^q = \frac{q^2}{2C}$$

or, $$W = \frac{(CV)^2}{2C} = \frac{C^2V^2}{2C} = \frac{1}{2}CV^2 \quad \text{[Since, } q = CV\text{]}$$

Therefore, Electric potential energy $U = W = \frac{1}{2}CV^2$

This relation holds for all types of capacitor.

Energy Density of Electric Field

In a parallel plate capacitor, the electric field has nearly the same value at all the points between the plates. Thus the energy density i.e. the potential energy per unit volume between the plates should be uniform.

Energy density, $\mu = \frac{U}{\text{Volume}} = \frac{U}{A \times d} = \frac{CV^2}{2Ad}$

For parallel plate capacitor, $C = \frac{\epsilon_0 A}{d}$

Therefore, $\mu = \frac{\epsilon_0 A}{d} \cdot \frac{V^2}{2Ad} = \frac{1}{2}\epsilon_0 \left(\frac{V}{d}\right)^2$

$$\mu = \frac{1}{2}\epsilon_0 E^2$$

Although we derived this result for the special case of a parallel plate capacitor. It holds good for every type of capacitor, whatever may be the sources of electric field.

Charging and Discharging of Capacitor

Let a capacitor having capacitance 'C' is connected in an electric circuit with a resistance 'R', ammeter A, a battery with emf ε and a key. When the key x is on, The positive and -ve charges appear on the plates and oppose the flow of electrons. As the charges accumulate, the potential difference between the plates increases and the charging current falls to zero.

Applying kirchhoff's loop rule to the circuit after the switch 'X' is on.

$$\varepsilon = V_C + V_R = \frac{q}{C} + IR$$

or, $$\frac{q}{C} + IR = \varepsilon$$

$$\frac{q}{C} + \frac{dq}{dt}R = \varepsilon$$

$$R.\frac{dq}{dt} = \varepsilon - \frac{q}{C} = \frac{\varepsilon C - q}{C} = \frac{q_0 - q}{C}$$

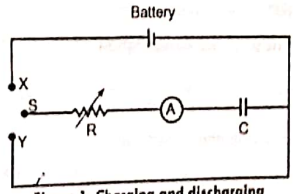

Figure 1: Charging and discharging of a capacitor

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where $q_o = \varepsilon C$ is the maximum charge stored in the capacitor.

$$\Rightarrow \quad \frac{dq}{q_o - q} = \frac{dt}{RC}$$

Integrating both sides, we obtain

$$\int_o^q \frac{dq}{q_o - q} = \frac{1}{RC}\int_o^t dt$$

$$\Rightarrow \quad -\int_o^q \frac{-dq}{q_o - q} = \frac{1}{RC}\cdot t$$

$$\Rightarrow \quad -\left[\ell n(q_o - q)\right]_o^q = \frac{t}{RC} \Rightarrow \left[\ell n(q_o - q)\right]_o^q = -\frac{t}{RC}$$

$$\Rightarrow \quad ln\,(q_o - q) - ln\,q_o = -\frac{t}{RC}$$

$$\text{or,} \quad ln\frac{q_o - q}{q_o} = -\frac{t}{RC} \Rightarrow \frac{q_o - q}{q_o} = e^{-t/RC}$$

$$\text{or,} \quad q_o - q = q_o e^{-t/RC}$$

$$q = q_o(1 - e^{-t/RC}) \qquad \ldots (i)$$

Which is charging equation for capacitor.

Differentiating this equation w.r.t. time, the current during charging is.

$$I = \frac{dq}{dt} = -q_o\left(\frac{-1}{RC}\right)e^{-t/RC} = \frac{q_o}{RC}e^{-t/RC}$$

$$= I = \frac{\varepsilon C}{RC}e^{-t/RC} \text{ [Since, } q_o = EC] \Rightarrow I = \frac{E}{R}e^{-t/RC}$$

Therefore, $I = I_o e^{-t/RC}$ ... (ii) (where $I_o = \varepsilon/R$ is the maximum current)

This is the charging equation in terms of current.

The term RC in equation (i) and (ii) is called capacitive time constant of circuit. It has dimension of time.

When, $t = RC$, from equation (1)

$$q = q_o(1 - e^{-1}) = q_o(1 - 0.37) = 0.63\,q_o = 63\% \text{ of } q_o$$

Hence the time constant of a charging circuit is defined as the time in which capacitor charges by about 63% of its maximum charge.

The potential across capacitor is

$$V_C = \frac{q}{C} = \frac{q_o}{C}(1 - e^{-t/RC}) = \varepsilon(1 - e^{-t/RC}) \quad \ldots (iii)$$

The potential across resister is

$$V_R = IR = I_o R e^{-t/RC} = \varepsilon e^{-t/RC} \qquad \ldots (iv)$$

The plot of charge (q) with time is as shown figure (2) and the plot of current (I) with time (t) in accordance with equation (ii) is as shown in figure (3).

Equation (iii) show that, the plot of V_C w.r.t. time is same as that of q and t. And equation (iv) shows that the plot of V_R and t is same as that of I and t.

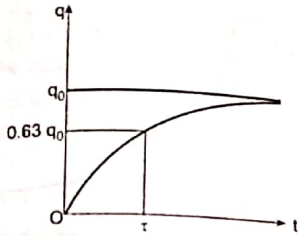

Figure 2: Variation of charge in capacitor while charging

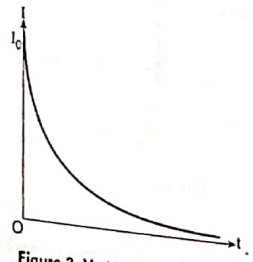

Figure 3: Variation of current while charging

Discharging of Capacitor

When capacitor is fully charged and switch Y (in figure (1)) is on. discharging occurs in the capacitor through resistor.

Using Kirchhoff's rule, $0 = V_C + V_R = \frac{q}{C} + IR = \frac{q}{C} + \frac{dq}{dt}\cdot R$

$$\Rightarrow R\frac{dq}{dt} = -\frac{q}{c} \Rightarrow \frac{dq}{dt} = -\frac{q}{RC} \Rightarrow \frac{dq}{q} = -\frac{1}{RC}dt$$

$$\text{Integrating,} \int_{q_o}^{q}\frac{dq}{q} = -\frac{1}{RC}\int_o^t dt = [ln\,q]_{q_o}^{q} = -\frac{t}{RC}$$

$$\Rightarrow ln\,q - ln\,q_o = -\frac{t}{RC} \Rightarrow ln\frac{q}{q_o} = -\frac{t}{RC} \Rightarrow \frac{q}{q_o} = e^{-t/RC}$$

$$q = q_o e^{-t/RC} \qquad \ldots (v)$$

Differentiating w.r.t. time.

$$\frac{dq}{dt} = -\frac{q_o}{RC}e^{-t/RC} = -\frac{\varepsilon C}{RC}e^{-t/RC} = -\frac{\varepsilon}{R}e^{-t/RC}$$

$$I = -I_o e^{-t/RC} \qquad \ldots (vi)$$

Equations (v) and (vi) are called discharging equation in terms of charge and current. Here -ve sign indicates decrease in charge with time. Equations (ii) and (vi) show that magnitude of charging current is equal to discharging current.

The factor RC in discharging equation is called capacitive time constant of discharging circuit. It has dimensions of time.

When,

$$t = RC, \text{ from equation (5) } q = q_o e^{-1} = \frac{q_o}{e} = 0.37\,q_o$$

$$q = 37\% \text{ of } q_o$$

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Hence, time constant of a discharging circuit is the time at which the charge stored in capacitor fall to 37% of its initial value.

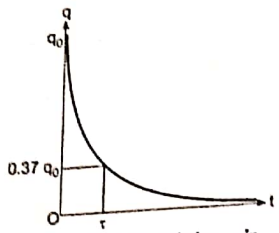


Figure 4: Variation of charge in capacitor while discharging

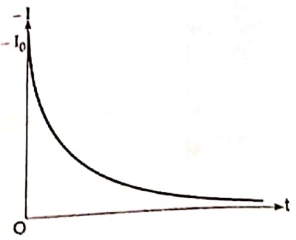


Figure 5: Variation of current while discharging

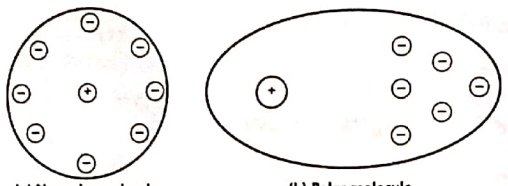
DIELECTRICS

Introduction

A dielectric is a non conducting or insulating material which does not conduct electricity such as rubber, glass, plastic etc.

1. **Nonpolar dielectrics:** The dielectric in which positive and negative charges have symmetrical charge distribution about their centre is called nonpolar dielectric. In such cases the centre of gravity of positive and negative charges coincide with each other, hence the electric dipole moment is zero.

   When external field is applied, dipoles are formed with in the dielectric. These dipoles are called induced dipole and tend to align with the external field. This phenomenon is called polarisation Ex H_2, N_2, CO_2 etc.



(a) Nonpolar molecule (b) Polar molecule

2. **Polar dielectrics:** The dielectric in which positive and negative charges have asymmetrical charge distribution about their centre is called polar dielectric.

   In Such cases the centre of gravity of positive and negative charges does not coincide with each other. Due to the finite separation between +ve and -ve charges such dielectric possess some electric dipole moment. So in polar dielectric –ve and +ve charges form dipoles, with orientation in different direction.

   When External field is applied, the randomly oriented dipoles tend to align along the applied electric field but the alignment is not complete due to thermal agitation.

   Eg: N_2O, NH_3, HCl, H_2O

Dielectric Constant (K)

Dielectrics are kept in between the two conducting plates of a capacitor. By placing the dielectrics between the plates of capacitor it solves the mechanical problem of maintaining two large metal plates at a very small separation with out actual contact. By placing the dielectrics between the plates of capacitor the capacitance of the capacitor increases.

The dielectric constant is defined as the ratio of capacitance of capacitor with dielectric (C) to its capacitance with out dielectric (C_o).

$$K = \frac{C}{C_o}$$

$K = \frac{\epsilon}{\epsilon_0}$

Consider a capacitor with magnitude of charge 'q' on each plate and potential difference V_0. When an uncharged sheet of dielectric is inserted the potential decreases to smaller value V.

$$\text{Now, } K = \frac{C}{C_0} = \frac{q/V}{q/V_o} = \frac{V_o}{V} \Rightarrow V = \frac{V_o}{K}$$

In the similar manner the electric field is decreased by the factor K i.e..

$$K = \frac{E_0}{E} \quad \begin{matrix}\text{(Electric field befor dielectric is introduced)} \\ \text{(Electric field after dielectric is introduced)}\end{matrix} \quad [\text{Since } E = Vd]$$

$$\Rightarrow E = \frac{E_o}{K}$$

Where, $E = E_o - E_p$, E_p is Electric field due to polarisation charges.

Note: The value of K is always greater or equal to 1.

From above discussion, following points can be concluded. By introducing a dielectric in between the plates of a capacitor:

1. The capacitance of capacitor increases by K times.
2. The potential difference between the plates of a capacitor is decreases by a factor $\frac{1}{K}$.
3. The electric field across the plates of capacitor decreases by a factor 1/K.
4. The energy stored in a capacitor is

$$U_o = \frac{1}{2} C_o V_o^2$$

$$\text{With dielectric, } U = \frac{1}{2} CV^2 = \frac{1}{2} C_o K \left(\frac{V_o}{K}\right)^2 = \frac{1}{2} C_o V_o^2 \cdot \frac{1}{K}$$

$$\Rightarrow U = U_o \cdot \frac{1}{K} = \frac{U_0}{K}$$

Therefore, energy stored is reduced by a factor of $\frac{1}{K}$

Note

The dielectric constant is also known as relative permittivity.

$$\left[\text{Since, } K = \frac{C}{C_o} = \frac{(\epsilon A)/d}{(\epsilon_0 A)/d} = \frac{\epsilon}{\epsilon_0} = \epsilon_r\right]$$

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POLARIZATION

When a dielectric is introduced between the plates of capacitors, some charges induce on dielectric. The polarization (P) can be defined as the induced charge with in the dielectric per unit area i.e. polarization $= \frac{q'}{A}$ (q' = induced charge)

The polarization of the dielectric is defined as the dipole moment per unit volume and represented by polarization vector P

i.e. $P = \frac{\sum p_i}{V}$, p_i = dipole moment of ith molecule

V = volume

If +q and −q are polarization charges and 'd' be the distance between them, $P = \frac{q.d}{V} = \frac{q}{Area} = \sigma$

Therefore, polarization vector (P) is equivalent to surface polarization charge density.

When dielectric is placed in an external electric field, the electrically charged particles (atoms, ions, molecules) get's arranged in such order that dielectric acquires a certain electric moment. Such phenomenon is called polarization.

The polarization is directly proportional to applied field i.e. $P \propto E$

$$P = \alpha E$$

Where α is constant of proportionality called polarizibility.

When E = 1 unit, $P = \alpha$

Hence, polarizaibility (α) is dipole moment produced by field of unit strength.

If there are N dipoles, $P = N\alpha E$

Unit of α is Farad m^2

In vacuum, The electric flux density is given by $D_o = \epsilon_o E$, when the dielectric of permittivity ϵ is inserted between the plates of capacitor, $D = \epsilon E$. The increase in flux density when dielectric is inserted is called polarization i.e.

$$\begin{aligned} P &= D - D_o \\ &= \epsilon E - \epsilon_o E \\ &= \epsilon_o \epsilon_r E - \epsilon_o E \\ &= \epsilon_o (\epsilon_r - 1) E \\ &= \epsilon_o \chi E \end{aligned}$$

$$P = \epsilon_o \chi E.$$

where $\chi = \epsilon_r - 1$ is called dielectric susceptibility.

In vacuum (free space), $\epsilon_r = \frac{\epsilon_o}{\epsilon_o} = 1 \Rightarrow \chi = 0, \Rightarrow P = \epsilon_o \chi E = 0$

Therefore, there is no polarization in free space.

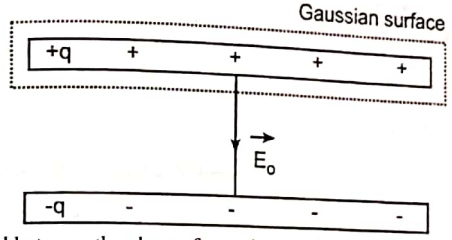
DIELECTRICS AND GAUSS'S LAW

Consider a parallel plate capacitor of plate area A having charge q on each plate.

Now draw a Gaussian surface that just encloses the charge q on the positive plate, let E_o be the magnitude of the field and ϵ_o be the permittivity of air between two plates then from Gauss' law.

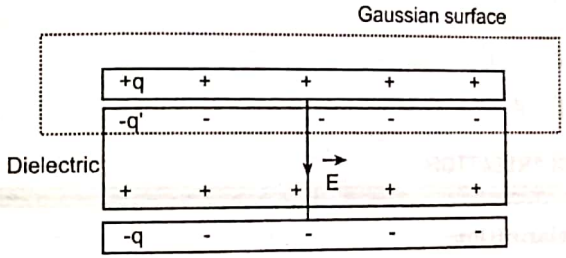
$$\oint \vec{E_o}.d\vec{A} = \frac{q}{\epsilon_o} \Rightarrow E_o A = \frac{q}{\epsilon_o}$$

$$E_o = \frac{q}{\epsilon_o A} \quad \ldots (i)$$



Now a dielectric is placed between the plates of capacitor as shown in figure (2). Now the induced charge due to the charge q on the top plate is q' which is distributed on the top face of the dielectric.

Take the same Gaussian surface, the net charge enclosed by it is, q - q'.



Now using Gauss' law

$$\oint \vec{E}.d\vec{A} = \frac{q - q'}{\epsilon_o}$$

$$E.A = \frac{q - q'}{\epsilon_o}$$

$$\Rightarrow E = \frac{q - q'}{\epsilon_o A} \quad \ldots (ii)$$

Where E is the electric field across capacitor with dielectric.

Since, the effect of dielectric in capacitor is to decrease electric field by a factor of 1/K so,

$$E = \frac{E_o}{K}$$

$$\frac{q - q'}{\epsilon_o A} = \frac{q}{\epsilon_o A} \cdot \frac{1}{K} \Rightarrow q - q' = \frac{q}{K}$$

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Now we can write the Gauss' law as:

$$\oint \vec{E}.d\vec{A} = \frac{q-q'}{\epsilon_o} = \frac{1}{\epsilon_o}.\frac{q}{K}$$

or, $$K\oint \vec{E}d\vec{A} = \frac{q}{\epsilon_o} \quad \ldots \text{(iii)}$$

This is the Gauss' law with dielectric although derived for parallel plate capacitor, holds good every where.

Equation (iii) shows that Gauss law for a dielectric of dielectric constant K can be obtained just by introducing K on the left side in the general form.

• To derive relation between $\vec{D}, \vec{E}$ and $\vec{P}$.

Here, the equation (ii) is

$$E = \frac{q-q'}{\epsilon_o A} = \frac{1}{\epsilon_o}\left(\frac{q}{A} - \frac{q'}{A}\right)$$

The quantity q/A is called electric displacement vector $\vec{D}$, which is also given by $\vec{D} = \epsilon\,\vec{E}$. $\vec{D}$ is equivalent to free surface charge density and the quantity $\frac{q'}{A}$ is called polarisation vector $\vec{P}$. It represents the capacity of dipole formation due to applied field. It is equivalent to induced surface charge density.

Therefore, $$\vec{E} = \frac{1}{\epsilon_o}(\vec{D} - \vec{P})$$

$$\Rightarrow (\vec{D} - \vec{P}) = \epsilon_o \vec{E}$$

or, $$\vec{D} = \epsilon_o \vec{E} + \vec{P}$$

TYPES OF POLARIZATION

Electronic Polarization

In an atom when there is no applied field, the centre of mass of atom and the charge center coincide with each other. So there is no net dipole moment. But when an external field is applied the negative charge center (electrons) will be shifted in the direction opposite to the direction of applied field. And the positive charge center (nucleus) will be shifted in the direction of applied field. Hence a dipole is formed in atom called atomic dipole. This phenomenon is called polarization.

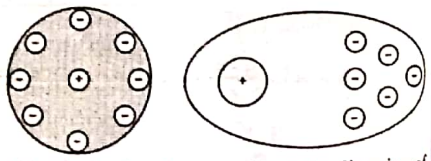

Figure (1): Neutral atom, P= 0, Figure(2) Polarized atom P≠0

Induced dipole moment is proportional to applied electric field.

i.e. $P = \alpha E \Rightarrow \alpha = \frac{P}{E}$ ... (i)

Where α is called polarizibility of the atom. The polarization of neutral atom involves, the shifting of electron so α is called electronic polarizibility or coefficient of electronic polarization:

Since $F = ZeE$ ... (ii)

Also $F = \frac{mv^2}{r} = \frac{m\omega^2 r^2}{r} = m\omega^2 r$ ....(iii)

Where m = mass of electron, ω = angular frequently

Therefore, $ZeE = m\omega^2 r$

$$E = \frac{m\omega^2 r}{Ze}$$

From (1), $\alpha = \frac{P}{E} = \frac{PZe}{m\omega^2 r} = \frac{erZe}{m\omega^2 r} = \frac{Ze^2}{m\omega^2}$

$$\alpha = \frac{Ze^2}{m\omega^2}, \text{ Z = atomic number}$$

Ionic Polarization:

The ionic crystal (crystal having ionic bonding) exhibits ionic polarization. Examples in NaCl, KCl, LiBr etc. The ionic crystal have well defined lattice site in which positive and negative ions are located. Each pairs of positive and negative ion form a dipole. Since these dipoles are lined one after another so there is no net dipole moment. $P_+ - P_- = 0$

When an external field is applied, the negative ions move in the direction opposite to applied field and positive ions move in the direction of applied field. Hence P_+ increases and P_- decreases so that $P_+ - P_- \neq 0$. In this way the ionic crystal is polarized.

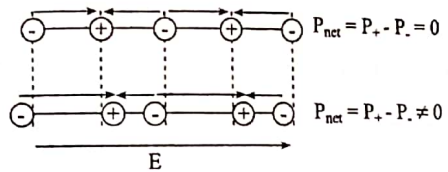

Figure: Na^+Cl^- chain in the absence and presence of field

CLAUSIUS-MASSOTTI EQUATION

The actual field experienced by a molecule in dielectric is called local field E_{local}. This is the sum of applied field and field due to polarization.

$$E_{local} = E + \frac{P}{3\epsilon_o}$$

And induced polarization is now given by,

$$p = \alpha E_{local}$$

Hence, polarization due to N, molecules

$$P = Np = N\alpha E_{local} = N\alpha\left(E + \frac{P}{3\epsilon_o}\right)$$

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$$\Rightarrow \quad P = N\alpha E + \frac{N\alpha P}{3\epsilon_o}$$

$$P\left(1 - \frac{N\alpha}{3\epsilon_o}\right) = N\alpha E$$

$$P = \frac{N\alpha E}{1 - \frac{N\alpha}{3\epsilon_o}} \quad \ldots (i)$$

Also, $P = \epsilon_o \chi E$ .....(ii)

Comparing equation (i) and (ii)

$$\epsilon_o \chi E = \frac{N\alpha E}{1 - \frac{N\alpha}{3\epsilon_o}}$$

$$\epsilon_o \chi \left(1 - \frac{N\alpha}{3\epsilon_o}\right) = N\alpha$$

$$\epsilon_o \chi - \frac{N\alpha\chi}{3} = N\alpha$$

$$N\alpha + \frac{N\alpha\chi}{3} = \epsilon_o \chi$$

$$N\alpha\left(1 + \frac{\chi}{3}\right) = \epsilon_o \chi$$

$$N\alpha\left(\frac{\chi + 3}{3}\right) = \epsilon_o \chi$$

$$\frac{N\alpha}{3\epsilon_o} = \frac{\chi}{\chi + 3} = \frac{\epsilon_r - 1}{\epsilon_r - 1 + 3} = \frac{\epsilon_r - 1}{\epsilon_r + 2}$$

$$\frac{N\alpha}{3\epsilon_o} = \frac{\chi}{\chi + 3} = \frac{\epsilon_r - 1}{\epsilon_r + 2} \quad \ldots (iii)$$

Equation (iii) is called Clausius - Massotti equation, which relates relative permittivity and polarizibility more accurately.

The Clausius-Massotti equation for electronic and ionic polarization is given by

$$\frac{N\alpha_e}{3\epsilon_o} = \frac{\epsilon_r - 1}{\epsilon_r + 2} \text{ and } \frac{N\alpha_i}{3\epsilon_o} = \frac{\epsilon_r - 1}{\epsilon_r + 2} \text{ respectively.}$$

For electronic polarization, the Clausius-Massotti equation can be written in terms of optical frequency as,

$$\frac{N\alpha_e}{3\epsilon_o} = \frac{n^2 - 1}{n^2 + 2} \quad \text{[Since, } \epsilon_r = n^2\text{, n is the optical frequency]}$$

Note

α_i is more than ten times α_e, so ionic solids have larger dielectric constant.

Solved Example

Example 1: A 100 μF capacitor is charged to a potential difference of 50 volts. The charging battery is then disconnected, the capacitor is then connected to second capacitor in parallel. If the measured potential drop to 35 volts. What is the capacitance of second capacitor.

Solution:

Before connection, V = 50 volts, C = 100 μF, Q = CV.

After connection let V_o be the common potential, Q_1 be the charge on first capacitor and Q_2 be the charge on second capacitor.

Therefore,

$$Q_1 = C_1 V_1 = CV_o$$

$$Q_2 = C_2 V_2 = C_2 V_o$$

Also,

$$Q = Q_1 + Q_2$$

$$CV = CV_o + C_2 V_o$$

$$\Rightarrow \quad C_2 = \left(\frac{V - V_o}{V_o}\right) . C$$

$$= \frac{(50 - 35) \times 100}{35} = 42.86\ \mu F$$

Example 2: A parallel plate capacitor with air as dielectric is charged to a potential V. It is then connected to an uncharged parallel plate capacitor filled with wax of dielectric constant K. Calculate common potential of both capacitor.

Solution:

Before connection let, Q and C are the charge and capacitance of capacitor.

Here K = 1 and Q = CV

After connection, let V_o be the common potential

For first capacitor, $Q_1 = C_1 V_1 = CV_o$. For second capacitor, $Q_2 = C_2 V_2 = C_2 V_o = KCV_o$

[Since, $\frac{C_2}{C_1} = K$]

Also,

$$Q = Q_1 + Q_2$$

$$CV = CV_o + KCV_o$$

$$V = V_o (1 + K)$$

$$V_o = \frac{V}{1 + K}$$

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Example 3: An air-filled parallel plate capacitor has a capacitance of 10^{-12}F. The separation between the plates is doubled and wax is inserted between them. Which increases the capacitance to 2×10^{-12} F. Calculate the dielectric constant of wax.

Solution:

Here, $C_1 = 10^{-12}$ F, $d_1 = d$, $A_1 = A$ and $\epsilon_1 = \epsilon_o$

$$C_1 = \frac{\epsilon_1 A_1}{d_1} = \frac{\epsilon_o A}{d} \quad \ldots (i)$$

When d is increased to 2d and wax is introduced

$C_2 = 2 \times 10^{-12}$F, $d_2 = 2d$, $A_2 = A$ and $\epsilon_2 = \epsilon$ (say)

We have,

$$C_2 = \frac{\epsilon_2 A_2}{d_2} = \frac{\epsilon A}{2d} = \frac{K\epsilon_o A}{2d} \quad \ldots (ii)$$

$$\Rightarrow \frac{C_2}{C_1} = \frac{K}{2} \Rightarrow K = \frac{2C_2}{C_1}$$

$$\Rightarrow K = \frac{2 \times 2 \times 10^{-12}}{10^{-12}} = 4$$

Example 4: If the charge on a capacitor is increased by 2 coulomb, the energy stored in it increases by 21%. Find the original charge on the capacitor.

Solution:

Since, $U_1 = \frac{q_1^2}{2C}$ and $U_2 = \frac{q_2^2}{2C}$

Therefore, $\frac{U_2 - U_1}{U_1} = \frac{q_2^2 - q_1^2}{q_1^2}$

Given,

$$\frac{U_2 - U_1}{U_1} = 21\% = 0.21$$

$$\Rightarrow \frac{q_2^2 - q_1^2}{q_1^2} = 0.21 \Rightarrow q_2^2 - q_1^2 = 0.21\, q_1^2$$

$$q_2^2 = 1.21\, q_1^2$$

$$q_2 = 1.1\, q_1 \quad \ldots (1)$$

Also from question, $q_2 = q_1 + 2C$

$$\Rightarrow 1.1 q_1 = q_1 + 2$$

$$q_1 = \frac{2}{0.1} = 20\text{ C}$$

Therefore, original charge on capacitor, $q_1 = 20$ C.

Example 5: The plates of a parallel plate capacitor are 10 cm apart and have area equal to $2m^2$. If the charge on each plate is 8.85×10^{-10}C. Find the electric field at a point between the plates and the capacitance of the capacitor.

Solution:

Here, $d = 10$ cm $= 10^{-2}$m, $A = 2m^2$, $q = 8.85 \times 10^{-10}$ C.

We have, $E = \frac{q}{\epsilon_o A} = \frac{8.85 \times 10^{-10}}{8.85 \times 10^{-12} \times 2} = 50$ N/C

and, $C = \frac{\epsilon_o A}{d} = \frac{8.85 \times 10^{-12} \times 2}{10^{-2}} = 1.77 \times 10^{-9}$ F

Example 6: A leaky parallel plate capacitor is completely filled with a material having dielectric constant K and conductivity σ. Show that the time constant of capacitor is $\frac{K\epsilon_o}{\sigma}$.

Solution:

Let A be the plate area and 'd' the plate separation. The resistivity of material is given by, $\rho = \frac{RA}{d}$ where R is the resistance of the material.

The conductivity σ is, $\sigma = \frac{1}{\rho} = \frac{d}{RA}$ or $R = \frac{d}{\sigma A}$

The capacitance of capacitor, $C = \frac{\epsilon A}{d} = \frac{K\epsilon_o A}{d}$

Therefore, time constant, $\tau = RC = \frac{d}{\sigma A} \cdot \frac{K\epsilon_o A}{d} = \frac{K\epsilon_o}{\sigma}$.

Example 7: Show that capacitance due to a charged sphere of radius 'r' is $4\pi\epsilon_o r$. The total charge on the sphere is supposed to be concentrated at the centre.

Solution:

We have, $V = \frac{q}{4\pi\epsilon_o r}$

Since, $C = \frac{q}{V}$

$$\Rightarrow C = \frac{q}{q} \times 4\pi\epsilon_o r = 4\pi\epsilon_o r$$

Example 8: If n drops, each of capacitance C, coalesce to form a single big drop. Find the capacitance of big drop.

Solution:

The mass of small drop, $M_1 = \frac{4\pi}{3} r^3 \times \rho$

The mass of big drop, $M_2 = \frac{4\pi}{3} R^3 \times \rho$

Since $M_2 = nM_1$

$$\Rightarrow \frac{4\pi}{3} R^3 = n.\frac{4\pi}{3} r^3 \Rightarrow R^3 = nr^3$$

$$R = n^{1/3} r.$$

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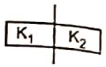
Capacitance of big drop $= 4\pi\epsilon_o R = 4\pi\epsilon_o . n^{1/3} . r$

$= 4\pi\epsilon_o r\, n^{1/3}$

$= n^{1/3}\, C$

Example 9: A parallel plate capacitor is filled with two dielectrics as shown in figure. Show that the capacitance is given by,

$$C = \frac{\epsilon_o A}{d}\frac{(K_1 + K_2)}{2}$$



Solution:

We have,

$$C_1 = \frac{\epsilon_1 (A/2)}{d} = \frac{K_1 \epsilon_o A}{2d} \qquad \left[\text{Since, } K_1 = \epsilon_r = \frac{\epsilon}{\epsilon_o}\right]$$

$$\text{And } C_2 = \frac{\epsilon_2 (A/2)}{d} = \frac{K_2 \epsilon_o A}{2d}$$

The given parallel plate capacitor is equivalent to a combination of two capacitors in parallel combination.

$$\text{Therefore, } C = C_1 + C_2 = \frac{K_1 \epsilon_o A}{2d} + \frac{K_2 \epsilon_o A}{2d} = \frac{\epsilon_o A}{d}\left(\frac{K_1 + K_2}{2}\right)$$

Example 10: A parallel plate capacitor is filled with two dielectrics as shown in figures. Show that the capacitance is given by $C = \frac{2\epsilon_o A}{d}\left(\frac{K_1 K_2}{K_1 + K_2}\right)$

Solution:

$$\text{We have, } C_1 = \frac{K_1 \epsilon_o A}{(d/2)} = \frac{2K_1 \epsilon_o A}{d}$$

$$\text{And } C_2 = \frac{K_2 \epsilon_o A}{(d/2)} = \frac{2 K_2 \epsilon_o A}{d}$$

The given capacitor is equivalent to a combination of two capacitors in series.

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d}{2K_1 \epsilon_o A} + \frac{d}{2K_2 \epsilon_o A} = \frac{d}{2\epsilon_o A}\left(\frac{1}{K_1} + \frac{1}{K_2}\right) = \frac{d}{2\epsilon_o A}\left(\frac{K_1 + K_2}{K_1 K_2}\right)$$

$$C = \frac{2\epsilon_o A}{d}\left(\frac{K_1 K_2}{K_1 + K_2}\right)$$

Example 11: A dielectric slab of thickness 'b' is inserted between the plates of a parallel plate capacitor of plate separation 'd'. (b < d). Show that the capacitance is given by $C = \frac{K \epsilon_o A}{Kd - b(K - 1)}$.

Solution:

The capacitor can be considered as an arrangement of two capacitors in series, one consisting of a dielectric K with thickness b and area A and another with air gap of thickness (d - b) and area A.

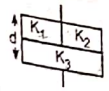
$$\text{Therefore, } C_1 = \frac{K\epsilon_o A}{b} \text{ and } C_2 = \frac{\epsilon_o A}{d - b}$$

The total capacitance C is given by,

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{b}{K\epsilon_o A} + \frac{d - b}{\epsilon_o A} = \frac{b + dK - bK}{K \epsilon_o A}$$

$$C = \frac{K\epsilon_o A}{b + dK - bK} = \frac{K \epsilon_o A}{Kd - b(K - 1)}$$

Example 12: A parallel plate capacitor of area A. plate separation d and capacitance C is filled with three different dielectric materials having dielectric constants K_1, K_2 and K_3 as shown in figure. Find C.



Solution:

We have,

$$C_1 = \frac{K_1 \epsilon_o (A/2)}{(d/2)} = \frac{K_1 \epsilon_o A}{d}$$

$$C_2 = \frac{K_2 \epsilon_o (A/2)}{d/2} = \frac{K_2 \epsilon_o A}{d}$$

$$\text{and, } C_3 = \frac{K_3 \epsilon_o A}{(d/2)} = \frac{2 K_3 \epsilon_o A}{d}$$

The capacitors, C_1 and C_2 are in parallel and their equivalent capacitance is

$$C' = C_1 + C_2 = \frac{(K_1 + K_2)\epsilon_o A}{d}$$

This combination is in series with C_3, therefore, total capacitance C is given by

$$\frac{1}{C} = \frac{1}{C'} + \frac{1}{C_3} = \frac{d}{(K_1 + K_2)\epsilon_o A} + \frac{d}{2K_3 \epsilon_o A}$$

$$= \left(\frac{1}{K_1 + K_2} + \frac{1}{2K_3}\right)\frac{d}{\epsilon_o A} = \frac{(K_1 + K_2 + 2K_3)}{2K_3(K_1 + K_2)}\frac{d}{\epsilon_o A}$$

$$C = \frac{\epsilon_o A}{d}\frac{2K_3 (K_1 + K_2)}{(K_1 + K_2 + 2K_3)} = \frac{2\epsilon_o A K_3 (K_1 + K_2)}{d(K_1 + K_2 + 2K_3)}$$

Example 13: A parallel plate capacitor has a capacitance of 100×10^{-12} F, a plate area of 100 cm² mica is used as a dielectric. At 50 Volts potential difference, Calculate electric field intensity and magnitude of induced charge?

Solution:

Here,

$K = 5.4$, $C = 100 \times 10^{-12}$ F, $A = 100\ cm^2 = 100 \times 10^{-4}\ m^2$

$V = 50$ Volt

$q = CV = 100 \times 10^{-12} \times 50 = 5 \times 10^{-9}$ C

We have,

$$E = \frac{q}{K \epsilon_0 A}$$

$$= \frac{5 \times 10^{-9}}{5.4 \times 8.85 \times 10^{-12} \times 100 \times 10^{-4}}$$

$$= 1.05 \times 10^4 \text{ V/m}$$

$$\text{Now, Induced charge, } q' = q\left(1 - \frac{1}{K}\right) = 5 \times 10^{-9}\left[1 - \frac{1}{5.4}\right] = 4.07 \times 10^{-9}\text{ C}$$

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Example 14: A parallel plate capacitor has circular plates of 8 cm radius and 1 mm separation. What charge will appear on the plates if a potential difference of 100 V is applied?

Solution:

$V = 100\ V,\ r = 8\ cm = 8 \times 10^{-2}\ m.$

$A = \pi r^2 = \pi (8 \times 10^{-2})^2 = 0.020\ m^2$

$d = 1\ mm = 1 \times 10^{-3}\ m$

Then,

$$C = \frac{\varepsilon_0 A}{d} = \frac{(8.85 \times 10^{-12}\ F/m) \times 0.020\ m^2}{1 \times 10^{-3}\ m} = 1.78 \times 10^{-10}\ F$$

Now,

$q = CV = 1.78 \times 10^{-10} \times 100 = 1.78 \times 10^{-8}\ C$

Example 15: A parallel plate capacitor has a capacitance of 100 μμF, a plate area of 100 cm² and mica as dielectric. At 50 volts potential difference calculate a) E in mica, b) the free charge on the plates and (c) the induced surface charge (Given K = 5.4 for mica).

Solution:

Here, $K = 5.4,\ C = 100\ \mu\mu F = 100 \times 10^{-12} F,\ A = 100 cm^2 = 10^{-2} m^2\ V = 50$ volts.\

We have, $q = CV = 100 \times 10^{-12} \times 50 = 5 \times 10^{-9} C$

The induced surface charge is $q' = q\left(1 - \frac{1}{K}\right) = 5 \times 10^{-9}\left(1 - \frac{1}{5.4}\right) = 4.1 \times 10^{-9}\ C$

Electric field in mica is,

$$E = \frac{q}{K \epsilon_0 A}$$

$$= \frac{5 \times 10^{-9}}{5.4 \times 8.85 \times 10^{-12} \times 10^{-2}}$$

$$= 1.05 \times 10^4 V/m$$

Example 16: Two capacitors 2μF and 4 μF are connected in parallel across 300 volts pd. Calculate the total energy in the system.

Solution:

Here,

$C_1 = 2\mu F = 2 \times 10^{-6}\ F$

$C_2 = 4\ \mu F = 4 \times 10^{-6}\ F$

$V = 300$ volts.

Since C_1 and C_2 are connected in parallel, so their equivalent capacitance is given by.

$C = C_1 + C_2 = 6 \times 10^{-6}\ F$

$U = \frac{1}{2} CV^2$

$= \frac{1}{2} \times 6 \times 10^{-6} \times 300^2 = 0.27$ Joule

Example 17: A capacitor of capacitance C is discharged through a resistor of resistance R. After how many time constant is the stored energy becomes one fourth of initial value.

Solution:

We have, $U = \frac{q^2}{2C} = \frac{q_0^2}{2C} e^{-2t/RC} = U_0 e^{-2t/RC}$

According to question, $U = \frac{U_0}{4}$

$\frac{U_0}{4} = U_0 e^{-2t/RC}$

$e^{2t/RC} = 4$

$t = \frac{ln(4)}{2} RC = 0.693\ RC$

$\Rightarrow t = 0.693\ \tau$

Example 18: A capacitor of capacitance C is charged through a resistor R. Calculate the time at which the potential across the resistor is equal to the potential across the capacitor.

Solution:

Here,

$V_R = V_C \Rightarrow IR = \frac{q}{C}$

$RI_0 e^{-t/RC} = \frac{q_0}{C}(1 - e^{-t/RC})$

$\Rightarrow \frac{R\varepsilon}{R} I_0 e^{\frac{-t}{RC}} = \frac{\varepsilon C}{C}\left(1 - e^{\frac{-t}{RC}}\right)$

$\Rightarrow e^{-t/RC} = 1 - e^{-t/RC}$

$2e^{-t/RC} = 1$

$e^{t/RC} = 2$

$t = ln(2) . RC$

$t = 0.693\ RC$

Example 19: Obtain the charging time constant of a capacitor in a RC circuit such that current through the resistor is decreased by 50% of its peak value in 5 seconds.

Solution:

$I = \frac{I_0}{2}$, $t = 5$ sec, $\tau = RC = ?$

We have,

$I = I_0 e^{-t/RC}$

or, $\frac{I_0}{2} = I_0 e^{-t/\tau}$

or $e^{-5/\tau} = \frac{1}{2}$

or $e^{5/\tau} = 2$

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or $\frac{5}{\tau} = ln\,2$

$\therefore \quad \tau = \frac{5}{ln\,2} = 7.21$ sec.

Example 20: Obtain the charging time constant of a capacitor in a RC circuit such that current through the resistor is decreased by 50% of its peak value in 10 seconds.

Solution:

$I = \frac{I_0}{2}$ $\qquad t = 10$ sec

We have, $I = I_o e^{-t/\tau}$

or, $e^{-t/\tau} = \frac{1}{2}$

or, $\frac{-t}{\tau} = ln\left(\frac{1}{2}\right) = -0.693$

or, $\tau = \frac{10}{0.693} = 14.43$ sec.

Example 21: The electronic polarizibility of the Ar atom is 1.7×10^{-40} Fm^2. What is the static dielectric constant of solid Ar if it's density is 1.8 gcm^{-3}? Given $N_A = 6.02 \times 10^{23}$ mol^{-1}, Atomic mass of Ar = 39.95 g mol^{-1}.

Solution:

Here, $\alpha_e = 1.7 \times 10^{-40}$ Fm^2, density d = 1.8 gcm^{-3}

Number of atoms per unit volume, $N = \frac{dN_A}{Mat}$

$= \frac{6.02 \times 10^{23} \times 1.8}{39.95}$

$= 2.71 \times 10^{22}$ cm^3

$= 2.71 \times 10^{28}$ m^{-3}

From Calusius-Massotti equation,

$$\frac{N\alpha_e}{3\epsilon_o} = \frac{\epsilon_r - 1}{\epsilon_r + 2}$$

$$\Rightarrow \frac{N\alpha_e\epsilon_r}{3\epsilon_o} + \frac{2N\alpha_e}{3\epsilon_o} = \epsilon_r - 1$$

$$\epsilon_r\left(\frac{N\alpha_e}{3\epsilon_o} - 1\right) = -\left[1 + \frac{2N\alpha_e}{3\epsilon_o}\right]$$

$$\epsilon_r = \frac{1 + \frac{2N\alpha_e}{3\epsilon_o}}{1 - \frac{N\alpha_e}{3\epsilon_o}} = 1.63$$

Example 22: Consider a pure Si crystal that has $\epsilon_r = 11.9$ a). What the electronic polarizibility due to valence electrons per Si atom b). Suppose that a voltage is applied across Si crystal sample. By how much is the local field greater than the applied field? Given, $\epsilon_o = 8.85 \times 10^{-12}$ Fm^{-1}, $N = 5 \times 10^{28}$ atoms per m^3.

Solution:

Here, $\epsilon_r = 11.9$, $\epsilon_o = 8.85 \times 10^{-12}$ Fm^{-1}, $N = 5 \times 10^{28}$ m^{-3}

a. From Clausius - Mossotti equation

$$\frac{N\alpha_e}{3\epsilon_o} = \frac{\epsilon_r - 1}{\epsilon_r + 2} \Rightarrow \alpha_e = \frac{3\epsilon_o}{N}\left(\frac{\epsilon_r - 1}{\epsilon_r + 2}\right)$$

$$\alpha_e = \frac{3\,(8.85 \times 10^{-12})}{(5 \times 10^{28})}\left(\frac{11.9 - 1}{11.9 + 2}\right) = 4.17 \times 10^{-40}\ \text{Fm}^2$$

b. The local field is

$$E_{local} = E + \frac{P}{3\epsilon_o}$$

Also, $P = \epsilon_o \chi E = \epsilon_o (\epsilon_r - 1) E$

Substituting for P,

$$E_{local} = E + \frac{1}{3}(\epsilon_r - 1) E$$

$$\frac{E_{local}}{E} = \left(1 + \frac{\epsilon_r}{3} - \frac{1}{3}\right) = \frac{\epsilon_r + 2}{3} = 4.63$$

Example 23: The optical index of refraction and the dielectric constant for glass are 1.45 and 6.5 respectively. Calculate the percentage of ionic polarizibility.

Solution:

Here, optical index of refraction, n = 1.45

Dielectric constant, $\epsilon_r = 6.5$

We have from Clausius - Massotti equation

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N(\alpha_e + \alpha_i)}{3\epsilon_o} \qquad \ldots(1)$$

At optical frequencies, ϵ_r in Clausius-Massotti relation for electronic polarization is replaced by n^2

$$\frac{n^2 - 1}{n^2 + 2} = \frac{N\alpha_e}{3\epsilon_o} \qquad \ldots(2)$$

Now, dividing equation (2) by (1)

$$\frac{n^2 - 1}{n^2 + 2} \times \frac{\epsilon_r + 2}{\epsilon_r - 1} = \frac{\alpha_e}{\alpha_e + \alpha_i}$$

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So, percentage of ionic polarizibility is

$$\frac{\alpha_i}{\alpha_e + \alpha_i} \times 100 = \left[1 - \frac{\alpha_e}{\alpha_e + \alpha_i}\right] \times 100$$

$$= \left[1 - \left(\frac{n^2 - 1}{n^2 + 2}\right) \times \left(\frac{\epsilon_r + 2}{\epsilon_r - 1}\right)\right] \times 100$$

$$= \left[1 - \left(\frac{1.45^2 - 1}{1.45^2 + 2}\right) \times \left(\frac{6.5 + 2}{6.5 - 1}\right)\right] \times 100$$

$$= 58.47\%$$

Example 24: Assuming that the polarizibility of an argon atom is equal to 1.43×10^{-40} Fm², calculate the relative dielectric constant at 0°C and 1 atmosphere.

Solution:

Here, $\alpha_e = 1.43 \times 10^{-40}$ Fm², $\epsilon_r = ?$, $T = 0°C = 273$ K

$P = 1$ atm $= 1 \times 10^5$ N/m²

We have from Clausius-Mossotti equation

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N\alpha_e}{3\epsilon_o}$$

$$3\epsilon_r\epsilon_o - 3\epsilon_o = N\alpha_e\epsilon_r + 2N\alpha_e$$

$$\epsilon_r(3\epsilon_o - N\alpha_e) = 2N\alpha_e + 3\epsilon_o$$

$$\epsilon_r = \frac{2N\alpha_e + 3\epsilon_o}{3\epsilon_o - N\alpha_e}$$

From gas equation PV = nRT

$$P = \frac{\text{Number of atoms per unit volume (N)}}{\text{Number of atoms per mole } (N_A)} \times RT$$

$$P = \frac{N}{N_A} . RT$$

$$N = \frac{PN_A}{RT}$$

$$= \frac{1 \times 10^5 \times 6.02 \times 10^{23}}{8.31 \times 273}$$

$$= 2.65 \times 10^{25}/m^3$$

Therefore, $\epsilon_r = \frac{(2 \times 2.65 \times 10^{25} \times 1.43 \times 10^{-40}) + (3 \times 8.85 \times 10^{-12})}{(3 \times 8.85 \times 10^{-12}) - (2.65 \times 10^{25} \times 1.43 \times 10^{-40})}$

$$\epsilon_r = 1.00043$$

Example 25: If the refractive index of a medium with respect to free space, n, is defined as the ratio of the velocity of electromagnetic wave in free space to that in the medium, show that the Clausius - Mosotti equation becomes, $\frac{N\alpha}{3\epsilon_o} = \frac{n^2 - 1}{n^2 + 2}$

Solution:

We have Clausius-Mosotti equation

$$\frac{N\alpha}{3\epsilon_o} = \frac{\epsilon_r - 1}{\epsilon_r + 2} \quad ...(1)$$

The velocity of electromagnetic wave in free space and in medium is given by

$$C = \frac{1}{\sqrt{\mu_o \epsilon_o}}$$

And in medium it is given by

$$\text{And } V = \frac{1}{\sqrt{\mu\epsilon}}$$

For a material that is not magnetic, the permeability is μ_o

$$\text{Therefore, } V = \frac{1}{\sqrt{\mu_o\epsilon}}$$

$$\text{Refractive index, } n = \frac{C}{V} = \sqrt{\frac{\mu_o\epsilon}{\mu_o\epsilon_o}} = \sqrt{\epsilon_r}$$

$$\epsilon_r = n^2$$

Hence, Clausius - Mossotti equation becomes

$$\frac{N\alpha}{3\epsilon_o} = \frac{n^2 - 1}{n^2 + 2}$$

THEORETICAL ANSWER QUESTIONS

1. Define capacitance. Obtain an expression for capacitance of parallel plate capacitor. Explain the effect of dielectric on the capacitance of capacitor.
2. What is capacitor? Write down the expression of capacitance of cylindrical capacitor which has inner and outer radius a and b respectively. Explain why induced charge is always less in magnitude than free-charge.
3. State Gauss law in free space. Prove that the relation $q' = q\left(1 - \frac{1}{k}\right)$ where q is free charge, q' is polarized charge and k is dielectric constant.
4. Show that energy per unit volume in electric field is proportional to square of electric field.
5. Prove that capacitance per unit length of cylindrical capacitor varies inversely with logarithm of ratio of external and internal radii.
6. State and explain Gauss law in dielectrics. Explain the meaning of three electric vectors $\vec{P}$, $\vec{E}$ and $\vec{D}$. And show that, $\vec{D} = \epsilon_0\vec{E} + \vec{P}$.

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7. A capacitor is made of two concentric spherical plates of radii a and b of inner and outer sphere respectively. If the outer plate is charged positively and inner plate is earthed, prove that the capacitance of such capacitor is given as $C = 4\pi\epsilon_0\left(\frac{b^2}{b-a}\right)$

8. A spherical capacitor consists of two concentric spherical shell of radii a and b (with b > a). Show that the capacitance, $C = 4\pi\epsilon_0\frac{ab}{b-a}$

9. Differentiate between polar and nonpolar dielectrics. Using Gauss law in dielectrics establish relation of electric field with displacement vector and polarization vector.

10. Define capacitance. Give a general method to calculate capacitance of a capacitor. Apply it to find capacitance of spherical capacitor.

11. State Gauss law in free space. How the law is modified if the dielectric materials are present? Then prove the relation $\vec{D} = \epsilon_0\vec{E} + \vec{P}$. Where symbols carry their usual meaning.

12. Write the circuit equation for a charging RC circuit. Solve it to find charge and current. Explain the meaning of time constant. Provide qualitative sketch for charge and current varying with time.

13. Show that capacitance of any capacitor depends on the geometry of the capacitor.

14. Define electric flux. Calculate the capacitance of cylindrical capacitor.

15. Obtain expressions for growth and decay of charges in the RC circuits. Explain how you will measure experimentally the capacitance of given capacitor.

16. Explain polarization in detail. Show that there is no polarization in free space or vacuum.

17. Derive Clausius-Massotti equation showing the relation between electronic polarization and relative permittivity.

18. Explain mathematically how relative permittivity is related with electronic polarization using Clausius-Massoti equation.

19. What is electronic polarization? Derive the mathematical relation showing the relation

20. between electronic polarization and relative permittivity, using Clausius-Massotti equation.

21. Derive Clausius-Massotti equation $\left(\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N\alpha_i}{3\epsilon_0}\right)$ for ionic polarization.

NUMERICAL PROBLEMS

1. A 16μF capacitor is charged to a potential difference of 100 volts. The charging battery is then disconnected, the capacitor is then connected to second capacitor in parallel. If the measured potential drop to 80 volts, what is the capacitance of the second capacitor. [Ans: 4μF]

2. What distance should the plates each of area 0.1m × 0.2m of an air capacitor be placed in order to have same capacitance as a spherical capacitor of radius 0.5m. [Ans: 3.183 × 10^{-1} m]

3. A parallel plate capacitor having plate area 100 cm^2 and separation 1.0 mm holds a charge of 0.12 μC when connected to a 120 V battery. Find the dielectric constant of material filling the gap. [Ans: 11.5]

4. A parallel plate capacitor has a capacitance of 100 × 10^{-12} F, a plate area of 100 cm^2 mica (k = 7) is used as a dielectric at 50 volts potential difference. Calculate the electric field intensity and magnitude of induced charge. [Ans: 6.917 × 10^3 V/m, 4.285 × 10^{-9} C]

5. An air filled parallel plate capacitor has a capacitance of 1.1 PF. The separation of the plate is halved and wax is inserted between them. The new capacitance is 8.8 PF. Find the dielectric constant of wax. [Ans: 4]

6. A long cylindrical conductor has length 1m and is surrounded by a co-axial cylindrical shell with inner radius double that of long cylindrical conductor. Calculate the capacitance assuming that there is vacuum in space between cylinders. [Ans: 80 × 10^{-12} F]

7. Calculate the capacitance of the earth, viewed as spherical conductor of radius 6400 km. [Ans: 7.117 × 10^{-4} F]

8. A parallel plate capacitor has circular plates of 8.2 cm radius and 1.3 mm separation in air. (i) Calculate the capacitance (ii) What charge will appear on the plates if potential difference of 120 V is applied. [Ans: 1.438 × 10^{-10} F, 1.725 × 10^{-8} C]

9. In an RC circuit ε = 12 V, R = 1.40 MΩ and C = 1.80 μF (i) Calculate the time constant (ii) Maximum charge that will appear on the capacitor during charging. (iii) How long does it take for the charge to build up to 16 μC? [Ans: 2.52 Sec, 2.16 × 10^{-5} C, 3.404 Sec]

10. Obtain the charging time constant of a capacitor in a RC circuit such that current through the resistor is decreased by 50% of its peak value in 15 seconds. [Ans: 21.65 Sec]

11. Derive an expression of capacitance of a co-axial cable used in transmission line having inner radius 'a', outer radius 'b' and length 'ℓ'. If a = 0.1mm and b = 0.6mm. Calculate capacitance per metre of the cable. Assume that the space between the conductors is filled with polystyrene of dielectric constant 2.6. [Ans: 8.069 × 10^{-11} F/m]

12. Two dielectric slabs are inserted between the plates of parallel plate capacitor. The thickness of the slab with dielectric constant k_1 is t_1 and of other slab having dielectric constant k_2 is t_2. The plates of capacitors are $t = t_1 + t_2$ apart, with area of each plate being A. Find the capacitance of new arrangement. [Ans: $\epsilon_0 A\left(\frac{k_1k_2}{t_1k_2 + t}\right)$]

13. A capacitor of capacitance 400 μF is initially charged by a source of 50 V. After fully charging, it is disconnected from the source and is joined to another capacitor of 200 μF capacitance. What is the loss of energy due to redistribution of charge? [Ans: 0.1666 J]

14. What is capacitance of a spherical capacitor made of two concentric spheres of radii 6 and 6.01 cm? The space between them is filled with a medium with relative permittivity equal to 8. [Ans: 3.208 × 10^{-8} F]

15. A capacitor C is discharged through a resistor, R
    i. After how many time constant does its charge fall to half of its initial value.
    ii. After how many time constant does the stored energy drop to half of its initial value. [Ans: 0.693 RC, 0.346 RC]

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(A) ELECTROMAGNETIC OSCILLATION

INTRODUCTION

Consider a circuit consist of capacitor 'C' and inductor 'L'. The energy stored in the electric field of the capacitor at any time is

$$U_E = \frac{q^2}{2C} \quad \ldots (i)$$

where q is the charge on capacitor at that time, 'C' is capacitance of the capactor.

The energy stored in the magnetic field of inductor at any time is.

$$U_B = \frac{L I^2}{2}$$

Where I is current through inductor at that time. L is inductance of inductor.

Consider the capacitor is fully charged initially. At this condition the charge in capacitor is maximum and hence U_E is maximum. And the current 'I' through the inductor is zero hence U_B is zero.

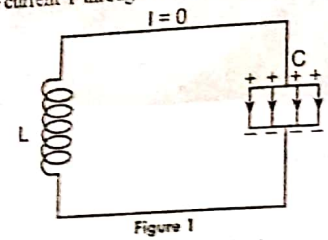


Figure 1

The capacitor now starts to discharge through inductor. Finally the capacitor losses all of it's charge. Now the charge on capacitor is zero hence U_E is zero, and the current through the inductor is maximum hence U_B is maximum as shown in figure (2).

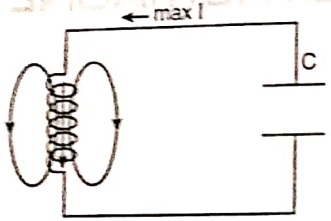


Figure 2

The current then begins to decrease since the flowing charge starts to accumulate in the next plate of the capacitor. Energy now flows from the inductor back to capacitor. The capacitor is now charged oppositely as shown in figure (3).

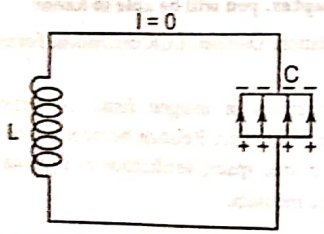


Figure 3

Afterwards the capacitor plates starts to discharge in opposite direction. This situation is like that in figure (2) except that the current is in opposite direction as shown in figure (4),

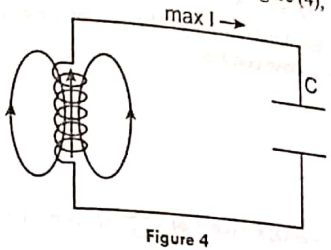


Figure 4

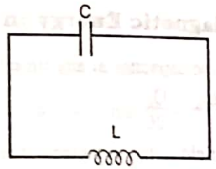
This process of transfer energy of between capacitor and inductor continues and the circuit returns to initial situation. The whole cycle repeats over and over. Thus the process is an oscillatory circuit. The resulting oscillation of capacitor's electric field and the inductor's magnetic field is called electro magnetic or LC oscillation.

LC Oscillation: (Quantitatively)

(To find the frequency of oscillation of an LC circuit):

The total energy at any instant in an oscillating LC circuit is the sum of electric and magnetic energies i.e.

$$U = U_{electric} + U_{magnetic} = \frac{1}{2}\frac{Q^2}{C} + \frac{LI^2}{2} \quad \ldots (i)$$



This implies that, the energy in LC circuit is stored partly as the magnetic field in inductor and partly as the electric field in the capacitor.

Since the circuit has no resistance which indicate there is no energy loss and the total energy is constant with time.

$$\frac{dU}{dt} = O \quad \ldots (ii)$$

$$\Rightarrow \frac{d}{dt}\left(\frac{Q^2}{2C} + \frac{LI^2}{2}\right) = 0$$

$$\Rightarrow \frac{2Q}{2C}\frac{dQ}{dt} + \frac{2LI}{2}\cdot\frac{dI}{dt} = 0$$

$$\Rightarrow \frac{Q}{C}\cdot I + LI\frac{dI}{dt} = 0$$

$$\Rightarrow \frac{Q}{C} + L\frac{dI}{dt} = 0 \text{ or } L\frac{d}{dt}\left(\frac{dQ}{dt}\right) + \frac{Q}{C} = 0$$

$$\Rightarrow L\frac{d^2Q}{dt^2} + \frac{Q}{C} = 0$$

$$\frac{d^2Q}{dt^2} + \frac{Q}{LC} = 0 \quad \ldots (iii)$$

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This is the differential equation that describes the oscillations of a resistance less LC circuit.

The solution of this equation is $Q = Q_o \sin(\omega t + \phi)$

Where, Q_o is maximum value (amplitude) of the charge variation ω is the angular frequency of electromagnetic oscillation, and ϕ is phase constant.

Now, $\frac{dQ}{dt} = Q_o\,\omega \cos(\omega t + \phi)$

and $\frac{d^2Q}{dt^2} = -\omega^2 Q_o \sin(\omega t + \phi)$

Therefore, equation(3) reduces to, $-\omega^2 Q_o \sin(\omega t + \phi) + \frac{1}{LC} Q_o (\sin \omega t + \phi) = 0$

$$\Rightarrow \quad -\omega^2 + \frac{1}{LC} = 0 \Rightarrow \omega^2 = \frac{1}{LC}$$

$$\omega = \frac{1}{\sqrt{LC}} \qquad \ldots \text{(iv)}$$

Hence an LC circuit oscillate with constant frequency given by, equation (iv).

Note

In actual LC circuit, the oscillation will not continue indifinitely i.e. it is no longer undamped because there is always some resistance present that will drain the energy from the electric and magnetic field and dissipate it as thermal energy. Though LC oscillation is considered as undamped but undamped oscillation are idealistic.

Oscillation of Electric and Magnetic Energy in an LC Circuit:

The energy stored as the electric field in the capacitor at any time t is.

$$U_E = \frac{Q^2}{2C} = \frac{[Q_o \sin(\omega t + \phi)]^2}{2C} = \frac{Q_o^2}{2C}\sin^2(\omega t + \phi) \ldots \text{(v)}$$

Similarly, the energy stored as magnetic field in the inductor at the same instant is,

$$U_B = \frac{LI^2}{2} = \frac{1}{2}L\left(\frac{dQ}{dt}\right)^2 = \frac{L}{2}[\omega Q_o \cos(\omega t + \phi)]^2$$

$$U_B = \frac{Q_o^2 L}{2} . \omega^2 \cos^2(\omega t + \phi)$$

$$U_B = \frac{Q_o^2}{2} .L. \frac{1}{LC}\cos^2(\omega t + \phi) = \frac{Q_o^2}{2C}\cos^2(\omega t + \phi)$$

$$U_B = \frac{Q_o^2}{2C}\cos^2(\omega t + \phi) \qquad \ldots \text{(vi)}$$

When we take, $\phi = 0$ then at time, $t = n\frac{T}{2}$, $n = 0, 1, 2, 3, \ldots$

$$U_E = 0 \text{ and } U_B = \frac{Q_o^2}{2C}$$

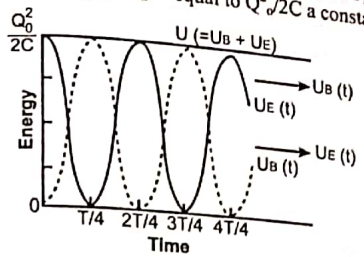
i.e. U_E = minimum, U_B = maximum.

And when we take, $\phi = 0$, then at time, $t = \frac{(2n+1)T}{4}$, $n = 0, 1, 2, 3, \ldots$

$$U_B = O \text{ and } U_E = \frac{Q_o^2}{2C}.$$

i.e. U_B = minimum and U_E = maximum

Note that, the maximum values of U_E and U_B are both $Q_o^2/2C$. When U_E is maximum, U_B is zero and conversely. At any instant the sum of U_E and U_B is equal to $Q_o^2/2C$ a constant.



The Electrical - Mechanical Analogy

(Analogy between LC oscillation and SHM):

The analogy between LC oscillation and simple harmonic motion can be summarized as given in table below.

The correspondences in table suggest that in an LC oscillator, the capacitor is mathematically like the spring in a block - spring system, and the inductors is like the block

	Block spring system	LC oscillators
Energy (Two kinds of energy are involved in both case)	Potential = $1/2\, kx^2$ (spring) Kinetic = $\frac{1}{2} mv^2$ (block)	Electric = $Q^2/2C$ (capacitor) Magnetic = $\frac{LI^2}{2}$ (Inductor)
Element	Spring (Spring is like capacitor)	Capacitor
	Block	Inductor
Parameter (compairing the energy of both system we can see that these parameters are equivalent)	x (displacement)	Q (charge)
	v = dx/dt (Velocity)	$I = \frac{dQ}{dt}$ (current)
	Spring constant (k)	$\frac{1}{C}$ (C is capacitance)
	Mass of spring (m)	Inductance (L)
Differential equations describing SHM and LC oscillation	$m\frac{d^2x}{dt^2} + kx = 0$ $\Rightarrow \frac{d^2x}{dt^2} + \frac{k}{m}x = 0$	$L\frac{d^2Q}{dt^2} + \frac{Q}{C} = 0$ $\Rightarrow \frac{d^2q}{dt^2} + \frac{q}{LC} = 0$
Solution's of differential equations	$x = A \sin(\omega t + \phi)$ (displacement) (This describe spring block oscillation)	$Q = Q_o \sin(\omega t + \phi)$ (Charge) (This describes LC oscillation)
Angular frequency of oscillation	$\omega = \sqrt{\frac{k}{m}}$	$\omega = \frac{1}{\sqrt{LC}}$

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LC OSCILLATION WITH RESISTANCE

Damped oscillation in an RLC circuit

Consider a circuit consist of an inductor (with inductance L), a capacitor (with capacitance C) and a resistor (with resistance R), as shown in figure (1).

consider a capacitor is given a charge Q_o initially. Due to the presence of resistance, a part of the energy is converted into thermal energy. Because of this loss of energy, the oscillation of charge, current and potential difference continuously decrease in amplitude as shown in figure (2) Such oscillations are called damped oscillation. Using Kirchoff's law in the circuit of figure (1)

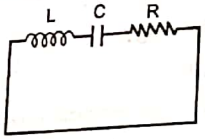

Figure (1)

$$L\frac{dI}{dt} + IR + \frac{Q}{C} = 0$$

$$L\frac{d}{dt}\left(\frac{dQ}{dt}\right) + R\frac{dQ}{dt} + \frac{Q}{C} = 0$$

$$\Rightarrow L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0 \qquad \ldots(i)$$

This is the differential equation for damped LCR Oscillation.

Comparing this equation with differential equation for mechanical damped harmonic oscillator equation.

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0 \qquad \ldots(ii)$$

We get,

$$m = L,\ b = R\ \&\ k = \frac{1}{C}$$

The angular frequency of damped oscillation (LCR circuit) will be.

$$\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2} = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} \qquad \ldots(iii)$$

The solution of equation (ii) is

$$x = Ae^{-\frac{bt}{2m}}\sin(\omega t + \phi)$$

Therefore the solution of equation (1) is given by

$$Q = Q_o e^{-\frac{Rt}{2L}}\sin(\omega t + \phi) \qquad \ldots(iv)$$

Therefore, oscillation of damped (RLC) oscillation is a sinusoidal oscillation with an exponentially decaying amplitude $Q_o e^{-Rt/2L}$.

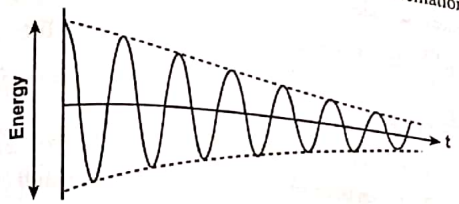

Figure (2)

Equation (iii) shows that the angular frequency of the damped oscillation is always less than the angular frequency of undamped oscillation. Equation (iii) is

$$\omega = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} = \sqrt{\frac{1}{LC} - \left(\frac{R^2}{4L^2}\right)}$$

From this result, we conclude that

1. When $\frac{1}{LC} > \frac{R^2}{4L^2} \Rightarrow \frac{4L}{C} > R^2$, ω is positive, the discharge (of capacitor) is oscillatory.
2. When, $\frac{1}{LC} = \frac{R^2}{4L^2} \Rightarrow \frac{4L}{C} = R^2 \Rightarrow \omega = 0$, the discharge is critically damped.
3. When $\frac{1}{LC} < \frac{R^2}{4L^2} \Rightarrow \frac{4L}{C} < R^2$, ω is -ve, the discharge is non oscillatory.

FORCED ELECTROMAGNETIC OSCILLATION

Since there is always some resistance present in an electric circuit. So some energy is converted into heat energy, causing damped oscillation. Therefore a periodic input of power is given to bring it to behave as an oscillatory circuit. Such oscillation are called forced em oscillation.

Figure shows, LCR Series circuit with an AC frequency generator of emf $\varepsilon = \varepsilon_o \sin \omega t$. Using kirchoff's law (consesration of energy) in the circuit of figure (1).

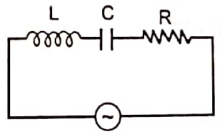

Figure (3)

$$L\frac{dI}{dt} + IR + \frac{Q}{C} = \varepsilon_o \sin \omega t$$

$$\Rightarrow L\frac{d}{dt}\left(\frac{dQ}{dt}\right) + R.\frac{dQ}{dt} + \frac{Q}{C} = \varepsilon_o \sin \omega t$$

$$\Rightarrow L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = \varepsilon_o \sin wt \qquad \ldots(i)$$

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This is the differential equation for forced LCR oscillation. Comparing this equation with differential equation of forced mechanical oscillation.

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_o \sin \omega t \quad \ldots \text{(ii)}$$

We get $m = L$, $b = R$, $k = \frac{1}{C}$, $F_o = \varepsilon_o$

The solution of (ii) is, $x = A \sin(\omega t + \phi)$ ... (iii)

Therefore, solution of (i) is, $Q = Q_o \sin(\omega t + \phi)$

Since,

$$A = \frac{F_o/m}{\sqrt{(\omega_o^2 - \omega^2)^2 + \left(\frac{b\omega}{m}\right)^2}}$$

$$Q_o = \frac{E_o/L}{\sqrt{(\omega_o^2 - \omega^2)^2 + \left(\frac{R\omega}{L}\right)^2}}$$

where, $\omega_o = \sqrt{\frac{1}{LC}}$

Current is given by differentiating equation (3) w.r.t time

$$I = \frac{dQ}{dt} = Q_o\,\omega \cos(\omega t + \phi)$$

This can be written as, $I = I_o \cos(\omega t + \phi)$

Where, $I_o = Q_o\omega$

Substituting Q_o for I_o

$$= \frac{\frac{\omega\varepsilon_o}{L}}{\sqrt{(\omega_o^2 - \omega^2)^2 + \left(\frac{R\omega}{L}\right)^2}}$$

$$I_o = \frac{\frac{\omega}{L}\varepsilon_o}{\sqrt{\left(\frac{\omega}{L}\right)^2\left(\frac{L^2}{\omega^2}(\omega_o^2 - \omega^2)^2 + R^2\right)}}$$

$$= \frac{\frac{\omega}{L}\varepsilon_o}{\frac{\omega}{L}\sqrt{\left(\frac{L\omega_o^2}{\omega} - \frac{L}{\omega}.\omega^2\right)^2 + R^2}}$$

$$= \frac{\varepsilon_o}{\sqrt{\left(\frac{L}{\omega}\cdot\frac{1}{LC} - L\omega\right)^2 + R^2}}$$

$$= \frac{\varepsilon_o}{\sqrt{\left(\frac{1}{C\omega} - L\omega\right)^2 + R^2}}$$

$$I_o = \frac{\varepsilon_o}{\sqrt{R^2 + \left(\frac{1}{C\omega} - L\omega\right)^2}} = \frac{\varepsilon_o}{\sqrt{R^2 + (X_C - X_L)^2}}$$

$$I_o = \frac{\varepsilon_o}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{\varepsilon_o}{Z} \quad \ldots \text{(iv)}$$

Here, $Z = \sqrt{R^2 + (X_L - X_C)^2}$ is the impedence of LCR circuit.

Where, $X_L = L\omega = 2\pi f.L$ is called inducting reactance and $X_C = \frac{1}{C\omega} = \frac{1}{2\pi f.C}$ is called capacitive reactance.

Resonance

Once started, the charge, potential difference and current in undamped LC circuit oscillate at angular frequency $\omega_o = \frac{1}{\sqrt{LC}}$. Such oscillation are said to be free oscillation, and the angular frequency 'ω_o' is said to be the circuit's natural angular frequency.

When an external alternating emf is connected to RLC circuit, the oscillation of charge, potential difference, and current are said to be diriven or forced. What ever the natural angular frequency 'ω_o' of the circuit may be, forced oscillation of charge, current and potential difference in the circuit always occur at the driving angular frequency ω.

When the driving frequency matches to natural frequency, the amplitudes of current in the circuit becomes maximum, such condition is called resonance. The frequency at which amplitude of current becomes maximum is called resonance frequency.

From equation (iv) it is seen that, the current in the circuit will be maximum when 'Z' is minimum, and Z is minimum when $X_L = X_C$.

$$\Rightarrow \omega L = \frac{1}{C\omega} \Rightarrow \omega^2 = \frac{1}{LC}$$

$$\Rightarrow \omega = \frac{1}{\sqrt{LC}} \Rightarrow 2\pi f = \frac{1}{\sqrt{LC}}$$

Hence, Resonance frequency, $f = \frac{1}{2\pi\sqrt{LC}}$

At resonance, $\omega = \omega_o = \frac{1}{\sqrt{LC}}$

Quality Factor

It is defined as the ratio of the voltage drop across inductor (L) or across capacitor (C) to the voltage drop across resistor 'R' at resonance.

i.e. $Q = \frac{V_L \text{ (or } V_C)}{V_R}$

$$\Rightarrow Q = \frac{V_L}{V_R} = \frac{X_L}{R} = \frac{L\omega}{R}$$

or, $Q = \frac{V_C}{V_R} = \frac{X_C}{R} = \frac{1}{C\omega R}$ [Since at resonance, $X_L = X_C$ or $L\omega = \frac{1}{C\omega}$]

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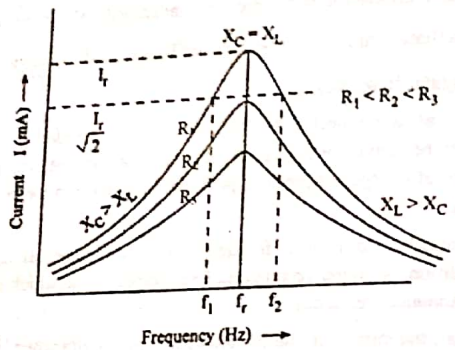
It is also defined as the ratio of the energy stored in inductor or (capacitor) to the energy dissipated across the resistance.

i.e. $Q = \frac{\text{Energy stored}}{\text{Energy dissipated}} = \frac{I^2 X_L t}{I^2 Rt} = \frac{X_L}{R} = \frac{\omega L}{R} = \frac{1}{C\omega R}$

The quality factor is also defined interms of lower and upper half power frequencies f_1 and f_2. These are frequencies lying below and above the resonance frequency where the power dissipation is half of that at resonance frequency. At these frequencies current in the circuit is $\frac{I_r}{\sqrt{2}}$, where I_r is current at resonance frequency.

i.e. $Q = \frac{2\pi f_r}{f_2 - f_1}$

The quantity $f_2 - f_1$ is also called band width of oscillatory circuit.



The quality factor characterize the sharpness of resonance. Since current at resonance $I_R = \varepsilon_o/R$ and $Q = \frac{L\omega}{R}$, this means, current at resonance is directly proportional to Q. Therefore if Q is large resonance curve is sharp.

Quality factor is also the figure of merit that enables us to compare different coils.

INDUCED MAGNETIC FIELD

The Faraday's law deals how a changing magnetic field induces an electric field. i.e.

$$\oint \vec{E}.\vec{dl} = -\frac{d\phi_B}{dt} \quad \ldots (i)$$

Here $\vec{E}$ is the induced electric field along a closed loop by changing magnetic flux ϕ_B encircled by that loop.

Symmetrically, the Maxwell's law of induction deals how a changing electric field induces magnetic field i.e.

$$\oint \vec{B}.\vec{dl} = \mu_o \epsilon_o \frac{d\phi_E}{dt} \quad \ldots (ii)$$

Here $\vec{B}$ is induced magnetic field along a closed loop produced by changing electric flux ϕ_E in the region encircled by that loop.

equation (i) is similar to equation (ii), but they are differ in following two ways.

a. The equation (ii) has two extra symbols μ_o and ϵ_o. They are inserted to make equation (ii) dimensionally correct.

b. Equation (ii) lacks the minus sign, this is because the magnetic field is induced in opposite direction to electric field induced as according to equation (i).

Ampere - Maxwell law

According to Ampere's law, the magnetic field is produced by current (Change of charge with time)

$$\text{i.e. } \oint \vec{B}.\vec{dl} = \mu_o I \quad \ldots (i)$$

The Maxwell's law of induction specifies that magnetic field can be produced by changing electric field (Electric flux)

$$\text{i.e. } \oint \vec{B}.\vec{dl} = \mu_o \epsilon_o \frac{d\phi_E}{dt} \quad \ldots (ii)$$

This means, the Ampere's circuital law is incomplete.

Thus there are at least two ways of setting up a magnetic field, by means of a steady current (According to Amperes law) and by means of changing electric field (According to Maxwell's law of induction).

Therefore we can combine two equations into a single equation, which is known as Ampere - Maxwell law.

$$\oint \vec{B}.\vec{dl} = \mu_o \epsilon_o \frac{d\phi_E}{dt} + \mu_o I \quad \ldots (iii)$$

When there is a current (steady) but no change in electric flux (eg: wire carrying a constant current), the first term of equation (iii) is zero and Ampere - Max well law reduces to Amperes law.

When there is a change in electric flux but no current (eg: A charging capacitor) the second term in right side of (iii) is zero, and Ampere-Max well law reduces to Maxwell law of induction.

Displacement Current

The Ampere-Maxwell law is

$$\oint \vec{B}.\vec{dl} = \mu_o \epsilon_o \frac{d\phi_E}{dt} + \mu_o I \quad \ldots (i)$$

If we compare the two terms on right side of this equation it is seen that the product $\epsilon_o \frac{d\phi_E}{dt}$ must have the dimension of current. This product is considered as fictitious current associated with changing electric field $\vec{E}$ between the plates of capacitor is called the displacement current (I_d). Although no charge actually moves across the gap between the plate, the ficitious displacement current can be considered simply to be a continuation of the real current through the capacitor. In other words, the changing electric field between the plates during charging - discharging of capacitor can be considered equivalent to a current called dispalcement current (I_d).

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We can now rewrite Ampere - Maxwell law as

$$\oint \vec{B}.\vec{dl} = \mu_o (I + I_d) \quad \ldots \text{(ii)}$$

$I_d = \frac{\epsilon_0 d\phi_E}{dt}$ is the displacement current

The charge stored in a parallel plate capacitor is q = CV.

or, $q = \frac{\epsilon_0 A}{d}.V$ [since capacitance $C = \frac{\epsilon_0 A}{d}$]

Where, A = area of each plate, d is distance between them

Therefore, $q = \epsilon_0 A.\frac{V}{d} = \epsilon_0 A E$

Now, the real current, $I = \frac{dq}{dt} = \epsilon_0 A \frac{dE}{dt}$ ... (iii)

Here, displacement current is:

$$I_d = \epsilon_0 \frac{d\phi_E}{dt} = \epsilon_0 \frac{d}{dt}(EA) \quad [\text{Since } E = \phi/A \Rightarrow \phi = EA]$$

$$I_d = \epsilon_0 A \frac{dE}{dt} \quad \ldots \text{(iv)}$$

[Here, we assume, the Electric field $\vec{E}$ between the plates of capacitor is uniform, so $\phi_E = EA$]

From equation (3) and (4), it is seen that, the real current 'I' during charging and discharging of the capacitor is equal to displacement current 'I_d' between the plates.

So we can consider the fictitious displacement current simply to be a continuation of the real current through the capacitor.

Solved Example

Example 1: A radio tuner has a frequency range from 500 KHz to 5 MHz. If its LC circuit has an effective inductance of 400 μH, what is the range of its variable capacitor?

Solution:

The frequency of oscillation of an LC circuit is

$$f = \frac{1}{2\pi\sqrt{LC}} \Rightarrow f^2 = \frac{1}{4\pi^2 LC} \Rightarrow C = \frac{1}{4\pi^2 f^2 L}$$

Here, L = 400 μH = 400 × 10^{-6} H

For f = 500 KHz = 500 × 10^3 Hz

$$C = \frac{1}{4\pi^2 \times (500 \times 10^3)^2 \times (400 \times 10^{-6})} = 2.535 \times 10^{-10} \text{ F}$$

For f = 5 mHz = 5 × 10^6 Hz.

$$C = \frac{1}{4\pi^2 \times (5\times10^6)^2 \times (400\times10^{-6})} = 2.535\times10^{-12} \text{F}$$

Example 2: You are given an inductor of 1 mH. If you are asked to make it oscillate with a frequency of 1MHz, how can you make such an oscillatory device.

Solution:

Here, L = 1 mH = 1 × 10^{-3} H, f = 1 MHz = 1 × 10^6 Hz

We have, $f = \frac{1}{2\pi\sqrt{LC}} \Rightarrow C = \frac{1}{4\pi^2 f^2 L}$

$$C = \frac{1}{4 \times 3.14^2 \times (1 \times 10^6)^2 \times 1 \times 10^{-3}} = 4 \times 10^{-10} \text{ F}$$

By connecting a capacitor of capacitance 4 × 10^{-10} F we can make an oscillatory device of frequency 1MHz.

Example 3: What inductance must be connected to a 17 PF capacitor in an oscillator capable of generating 550 nm Electromagnetic wave?

Solution:

Here, C = 17 PF = 17 × 10^{-12} F, λ = 550 × 10^{-9} m

We have, speed of e.m wave = fλ

$$3 \times 10^8 = f\lambda$$

$$f = \frac{3 \times 10^8}{550 \times 10^{-9}} = 5.45 \times 10^{14} \text{ Hz}$$

Again, $f = \frac{1}{2\pi\sqrt{LC}} \Rightarrow L = \frac{1}{4\pi^2 f^2 C}$

$$\therefore \quad L = 2.95 \times 10^{-10} \text{ H}$$

Example 4: A 40 mH inductor and 1000 μF capacitor form an oscillating circuit. What is the peak value of current if the initial charge is 40 μC?

Solution:

L = 40 mH = 40 × 10^{-3} H

C = 1000 μF = 1000 × 10^{-6} F

Q_0 = 40 μC = 40 × 10^{-6} C

We have,

$$\omega = \frac{1}{\sqrt{LC}}$$

The charge varies sinusoidally as,

$$Q = Q_0 \sin(\omega t + \phi)$$

Therefore, current,

$$I = \frac{dQ}{dt} = \frac{d}{dt}(Q_0 \sin(\omega t + \phi))$$

$$= Q_0 \omega \cos(\omega t + \phi)$$

Therefore, $I_{peak} = Q_0 \omega$

$$= \frac{Q_0}{\sqrt{LC}} = \frac{40 \times 10^{-6}}{\sqrt{40 \times 10^{-3} \times 1000 \times 10^{-6}}}$$

$$I_{peak} = 6.32 \times 10^{-3} \text{ A} = 6.32 \text{ mA}$$

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Example 5: If 10 mH inductor and two capacitors of 5 μF and 2 μF are given, find the two resonant frequencies that can be obtained by connecting these elements in different ways.

Solution:

Here $L = 10$ mH, $C_1 = 5\ \mu F = 5 \times 10^{-6}$ F and $C_2 = 2\mu F = 2 \times 10^{-6}$ F

i. When the capacitors are connected in series, $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{5 \times 2}{5 + 2} = 1.43\ \mu F = 1.43 \times 10^{-6}\ F$$

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{10 \times 10^{-3} \times 1.43 \times 10^{-6}}} = 1.33\ KHz$$

ii. When the capacitors are connected in parallel, $C = C_1 + C_2$

$$\Rightarrow C = 5 + 2 = 7\ \mu F = 7 \times 10^{-6}\ F$$

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{10 \times 10^{-3} \times 7 \times 10^{-6}}} = 0.602\ KHz$$

Example 6: An alternating voltage (in Volts) varies with time t (in Seconds) as $V = 100 \sin(5\pi t)$. Calculate the peak value of voltage, the rms value of voltage and the frequency.

Solution:

Here, $V = 100 \sin(5\pi t)$

Comparing this equation with, $V = V_0 \sin \omega t$

i. The peak value of voltage, $V_0 = 100$ Volts.

ii. The rms value of voltage, $V_{rms} = \frac{V_0}{\sqrt{2}}$

$$= \frac{100}{\sqrt{2}}$$

$$= 70.71 \text{ Volts.}$$

iii. Here, $\omega = 5\pi \Rightarrow f = \frac{5\pi}{2\pi} = 2.5$ Hz.

Example 7: A 1.5 μF capacitor is charged to 57 Volts. The charging battery is disconnected and a 12 mH coil is connected in series with capacitor so that LC oscillation occur. What is the maximum current in the coil? Assuming that the circuit contains no resistance.

Solution:

Here, $C = 1.5\ \mu F = 1.5 \times 10^{-6}$ F, $V = 57$ volts, $L = 12$ mH $= 12 \times 10^{-3}$ H.

We have, $q = q_0 \sin(\omega t + \phi)$

$$I = \frac{dq}{dt} = q_0\, \omega \cos(\omega t + \phi)$$

Therefore, $I_{max} = q_0\, \omega = \frac{CV}{\sqrt{LC}} = V\sqrt{\frac{C}{L}} = 57\sqrt{\frac{1.5 \times 10^{-6}}{12 \times 10^{-3}}} = 0.64$ A

Example 8: Find whether the discharge of a condenser through the following inductive is oscillatory. $C = 0.1\ \mu F$, $L = 1$mH and $R = 200\ \Omega$. If the circuit is oscillatory, calculate it's frequency.

Solution:

Here $C = 0.1 \times 10^{-6}$ F, $L = 1 \times 10^{-3}$H, $R = 200\ \Omega$

Now, $\frac{1}{LC} = \frac{1}{0.1 \times 10^{-6} \times 1 \times 10^{-3}} = 1 \times 10^{10}$

and $\frac{R^2}{4L^2} = \frac{200^2}{4 \times (1 \times 10^{-3})^2} = 1 \times 10^{10}$

Since, $\frac{1}{LC} = \frac{R^2}{4L^2}$, the discharge is critically damped and frequency, $\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} = 0$

Example 9: If the supply voltage were 10 V, the frequency 1 KHz and capacitance 2 μF, what value of R in the circuit would allow a current of 0.1 A to flow?

Solution:

Here, $V = 10$ Volts, $f = 1000$ Hz, $C = 2 \times 10^{-6}$F, $I = 0.1$A

We have,

$$V = IZ \quad \Rightarrow Z = \frac{V}{I}$$

$$\Rightarrow \sqrt{R^2 + X_c^2} = \frac{V}{I}$$

$$\Rightarrow R^2 = \frac{V^2}{I^2} - X_c^2 = \frac{10^2}{0.1^2} - \left(\frac{1}{2\pi \times 1000 \times 2 \times 10^{-6}}\right)^2$$

$R = 60.5\ \Omega$

Example 10: In an oscillating LC circuit, what value of charge expressed in terms of maximum charge is present on the capacitor when the energy is shared equally between the electric and magnetic fields? At what time will this condition occur, assuming the capacitor to be fully charged initially? Assume that $L = 10$ mH and $C = 1.0\ \mu F$.

Solution:

Here, $L = 10\text{mH} = 1 \times 10^{-2}$ H, $C = 1\ \mu F = 1 \times 10^{-6}$ F

i. According to question, $U_E = U_B$

Since $U_E + U_B = U_{max}$

$$2U_E = U_{max}$$

$$U_E = \frac{1}{2} U_{max}$$

$$\frac{q^2}{2C} = \frac{1}{2}\frac{q_0^2}{2C} \quad [q_0 = \text{maximum value of charge}]$$

$$q^2 = \frac{q_0^2}{2}$$

$$q = \frac{q_0}{\sqrt{2}}$$

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ii. According to question, at $t = 0$, $q = q_o \Rightarrow \phi = 0$

$q = q_o \sin \omega t$

$$\frac{q_o}{\sqrt{2}} = q_o \sin \omega t \Rightarrow \sin \omega t = \frac{1}{\sqrt{2}} = \sin \frac{\pi}{4}$$

$$\omega t = \frac{\pi}{4} \Rightarrow t = \frac{\pi}{4\omega} = \frac{\pi\sqrt{LC}}{4}$$

$$t = \frac{3.14 \times \sqrt{1 \times 10^{-2} \times 1 \times 10^{-6}}}{4} = 2.5 \times 10^{-4} \text{ sec}$$

Example 11: A series RLC circuit has L = 12 mH, capacitance C = 1.6 μF and resistance R = 1.5 Ω i) At what time the amplitude of the charge oscillation will be 50% of it's initial value ii) To how many periods of oscillation does this happen.

Solution:

Here, $L = 12 \text{ mH} = 12 \times 10^{-3}$ H, $C = 1.6 \mu F = 1.6 \times 10^{-6}$ F, $R = 1.5 \Omega$

i) We have for damped L.C.R. circuit,

$Q = Q_o e^{-Rt/2L} \sin(\omega t + \phi)$

Amplitude of charge oscillation, $Q_m = Q_o e^{-Rt/2L}$

According to question, $Q_m = 50\%$ of $Q_o = \frac{Q_o}{2}$

Therefore, $\frac{Q_o}{2} = Q_o e^{-Rt/2L} \Rightarrow e^{Rt/2L} = 2$

$$\frac{Rt}{2L} = \ln(2) \Rightarrow t = \frac{2L}{R} \cdot \ln(2) = \frac{2 \times 12 \times 10^{-3} \times \ln(2)}{1.5}$$

$t = 0.011$ sec.

ii) Since $\omega = \frac{1}{\sqrt{LC}} \Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{LC}$

$$= 2 \times 3.14 \sqrt{12 \times 10^{-3} \times 1.6 \times 10^{-6}} = 8.7 \times 10^{-4} \text{ sec}$$

$$\text{Number of periods} = \frac{t}{T} = \frac{0.011}{8.7 \times 10^{-4}} = 13$$

Example 12: What resistance 'R' should be connected in series with an inductance 220 mH and capacitance 12 μF for the maximum charge on the capacitor to decay to 99% of its initial value in 50cycles.

Solution:

Here, $L = 220 \times 10^{-3}$ H, $C = 12 \times 10^{-6}$ F.

$$t = 50\,T = 50\left(\frac{2\pi}{\omega_o}\right) = 100\,\pi\sqrt{LC} = 0.5104 \text{ Seconds}$$

We have, $Q = Q_o e^{-Rt/2L} \sin(\omega t + \phi)$

$Q_{max} = Q_o e^{-Rt/2L}$, where Q_o is charge at $t = 0$

$$e^{Rt/2L} = \frac{Q_o}{Q_{max}}$$

$$R = \frac{2L}{t} \ln\left(\frac{Q_o}{Q_{max}}\right)$$

From question, $Q_{max} = 99\%$ of $Q_o = 0.99\,Q_o$

Therefore,

$$R = \frac{2 \times 220 \times 10^{-3}}{0.5104} \ln\left(\frac{Q_o}{0.99\,Q_o}\right)$$

$$= 8.66 \times 10^{-3}\ \Omega$$

Example 13: A 1200 PF capacitor is charged by a 500V battery. It is disconnected from the battery and is connected to a 75 mH inductor at $t = 0$. Determine i) the initial charge on the capacitor ii) the maximum current iii) the frequency and period of oscillation iv) the total energy oscillating in the system.

Solution:

Here, $C = 1200 \text{ PF} = 1200 \times 10^{-12}$ F, $V = 500$ V, $L = 75 \times 10^{-3}$ H

i. $Q_o = CV = 1200 \times 10^{-12} \times 500 = 6 \times 10^{-7}$ C $= 0.6 \mu C$.

ii. Since, $Q = Q_o \sin(\omega t + \phi)$

$$I = \frac{dQ}{dt} = Q_o\,\omega \cos(\omega t + \phi)$$

$$I_{max} = Q_o\omega = \frac{Q_o}{\sqrt{LC}} = 6.3 \times 10^{-2} \text{ A}$$

iii. $f = \frac{1}{2\pi\sqrt{LC}} = 12{,}000$ Hz $= 12$KHz and $T = \frac{1}{f} = 6 \times 10^{-5}$ sec

iv. Total Energy $= \frac{Q_o^2}{2C} = 1.5 \times 10^{-4}$ J

Example 14: Calculate the resonating frequency and quality factor of a circuit having 0.02 μF capacitance, 8 mH inductance and 0.25 Ohm resistance.

Solution:

Here, $C = 0.02 \mu F = 0.02 \times 10^{-6}$ F, $L = 8\text{mH} = 8 \times 10^{-3}$H and $R = 0.25\ \Omega$

$$\text{We have, } \omega = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} \Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

$$F = \frac{1}{2\pi}\sqrt{\frac{1}{8 \times 10^{-3} \times 0.02 \times 10^{-6}} - \frac{0.25^2}{4 \times (8 \times 10^{-3})^2}}$$

$$= \frac{1}{2\pi}\sqrt{6.25 \times 10^9 - 2.44 \times 10^2} = 12.59 \text{ KHz.}$$

$$\text{and } Q = \frac{L\omega}{R} = \frac{2\pi f L}{R} = \frac{2\pi \times 12.59 \times 10^3 \times 8 \times 10^{-3}}{0.25} = 2530$$

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Example 15: The quality factor of an LCR circuit is 10, 000 and its frequency is 100 Hz. What will be the energy after 15.92 secs?

Solution:

Here, $Q = 10{,}000$, $f = 100$

Since, $Q = \frac{L\omega}{R} \Rightarrow \frac{R}{L} = \frac{\omega}{Q} = \frac{2\pi f}{Q} = \frac{2\pi \times 100}{10{,}000}$

$\frac{R}{L} = 0.0628$

We have, $Q = Q_o e^{-Rt/2L}$

Energy, $U = \frac{Q^2}{2C} = \frac{Q_o^2}{2C} e^{-Rt/L} = U_o e^{-Rt/L}$

$U = U_o e^{-0.0628 \times 15.92}$

$U = 0.34\, U_o$

Example 16: At t = 0, a 40 mH inductor is placed in series with a resistance R = 3Ω and a charged capacitor C = 4.8 μF. Show that this current will oscillate and find the frequency of oscillation. What is the time required for the charge amplitude to drop to half its starting value.

Solution:

Here, $L = 40\text{ mH} = 40 \times 10^{-3}$ H, $R = 3\Omega$, $C = 4.8 \times 10^{-6}$ F $\frac{1}{2\pi\sqrt{LC}}$

i. $\frac{1}{LC} = \frac{1}{40 \times 10^{-3} \times 4.8 \times 10^{-6}} = 5.2 \times 10^6$

$\frac{R^2}{4L^2} = \frac{3^2}{4 \times (40 \times 10^{-3})^2} = 1.4 \times 10^3$

Since $\frac{1}{LC} > \frac{R^2}{4L^2}$ so the circuit is oscillatory

$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{40 \times 10^{-3} \times 4.8 \times 10^{-6}}} = 363.4$ Hz

ii. We have, $Q = Q_o e^{-Rt/2L}$.

From question, $Q = \frac{Q_o}{2}$

Therefore, $\frac{1}{2} = e^{-Rt/2L}$

$e^{Rt/2L} = 2$

$\frac{Rt}{2L} = \ln(2)$

$t = \frac{2L}{R} \cdot \ln(2) = \frac{2 \times 40 \times 10^{-3} \times \ln(2)}{3} = 1.85 \times 10^{-2}$ sec

Example 17: An LC circuit is converted into a LCR circuit inserting a resistance of 10 Ω. Calculate the percentage change in frequency in this conversion. Given inductance = 10 mH and capacitance = 10 μF.

Solution.

Here, $R = 10\ \Omega$

$L = 10\text{ mH} = 10 \times 10^{-3}$ G

$C = 10\ \mu\text{F} = 10 \times 10^{-6}$ F

$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \frac{1}{2\pi}\sqrt{\frac{1}{10 \times 10^{-3} \times 10 \times 10^{-6}}} = 503.2$ Hz

For LCR circuit the frequency is,

$f' = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$

$= \frac{1}{2\pi}\sqrt{\frac{1}{10 \times 10^{-3} \times 10 \times 10^{-6}} - \left(\frac{10}{2 \times 10 \times 10^{-3}}\right)^2}$

$= \frac{1}{2\pi}\sqrt{10^7 - 2.5 \times 10^3}$

$= 496.96$ Hz

$\%\text{ Change} = \frac{f - f'}{f} \times 100\% = \frac{503.2\text{ Hz} - 496.96\text{ Hz}}{503.2\text{ Hz}} \times 100\% = 1.23\%$

Example 18: A circuit has L = 10 mH and C = 10 μF. How much resistance should be added to circuit so that frequency of oscillation will 1% less than that of LC oscillation.

Solution:

Here, $L = 10\text{ mH} = 10 \times 10^{-3}$H, $C = 10\ \mu\text{F} = 10 \times 10^{-6}$ F

The free oscillation frequency,

$f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2 \times 3.14\sqrt{1 \times 10^{-2} \times 1 \times 10^{-5}}} = 503.55$ HZ

The damped frequency is,

$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$

$\Rightarrow \frac{1}{LC} - \frac{R^2}{4L^2} = 4\pi^2 f^2$

$\Rightarrow R^2 = 4L^2\left[\frac{1}{LC} - 4\pi^2 f^2\right]$

According to question, $f = f_o - 1\%$ of $f_o = f_o - \frac{f_o}{100} = 498.51$ Hz

$\Rightarrow R^2 = 4 \times (10^{-2})^2\left[\frac{1}{10^{-2} \times 10^{-5}} - 4\pi^2(498.51)^2\right]$

$\Rightarrow R^2 = 75.809$

$\therefore R = 8.706\Omega$

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Example 19: An LCR series circuit is connected to a variable frequency 200 V source. L = 5H, C = 80 μF and R = 40 Ω. What is the (i) Source frequency which drives the circuit at resonance (ii) Impedance of the circuit at resonance (iii) Current amplitude at resonance (iv) rms potential drop across the inductor, capacitor and resistor at resonance.

Solution:

Here, $L = 5\ H$, $C = 80\times10^{-6} F$, $R = 40\ \Omega$, $\varepsilon_{rms} = 200\ V$

i. The resonant angular frequency,

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5\times80\times10^{-6}}}$$

$$\omega_r = 50\ rad\ s^{-1}$$

Resonant frequency, $f_r = \frac{\omega_r}{2\pi} = \frac{50}{2\pi} = 7.96\ Hz$

ii. The impedence is given by, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

at resonance, $\omega_r = \omega$ and $X_L = X_C$

$Z = R = 40\ \Omega$

iii. Current amplitude at resonance is given by,

$$I_o = \frac{\varepsilon_o}{R} = \frac{\sqrt{2}\,\varepsilon_{rms}}{R} = \frac{\sqrt{2}\times200}{40} = 7.07\ A$$

iv. $I_{rms} = \frac{I_o}{\sqrt{2}} = 5A$

The rms potential drop across inductor,

$V_L = I_{rms} X_L = I_{rms} \times \omega_r \times L$

$= 5 \times 50 \times 5 = 1250$ Volts.

The rms potential drop across capacitor, $V_c = I_{rms} X_C = \frac{I_{rms}}{C\omega_r} = \frac{5}{80\times10^{-6}\times50} = 1250$ Volts

The rms potential drop across resistor,

$V_R = I_{rms} \times R = 5 \times 40 = 200$ Volts

Example 20: An LR series circuit consists of inductance $L = \frac{100}{\pi}$ mH and a resistance R = 10 Ω. A sinusoidal voltage $V = V_o \sin(2\pi ft)$ is applied. It is given that $V_{rms} = 200$ V and f = 50Hz a) Find the peak value of the current in the steady state. b) What is the phase difference between the current and voltage?

Solution:

Here,

$L = \frac{100}{\pi} mH = \frac{0.1}{\pi} H$, $R = 10\ \Omega$, $V_{rms} = 200$ Volts $f = 50$ Hz

a. The amplitude of the current in the steady state is given by

$$I_o = \frac{V_o}{Z} = \frac{\sqrt{2}\,V_{rms}}{(R^2 + X_L^2)^{1/2}} = V_{rms}\left(\frac{2}{R^2 + L^2\omega^2}\right)^{1/2}$$

$$= V_{rms}\left(\frac{2}{R^2 + 4\pi^2 f^2 L^2}\right)^{1/2} = 20A$$

b. The phase difference between the current and voltage is given by, $\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{2\pi fL}{R}\right) = \frac{\pi}{4}$

THEORETICAL ANSWER QUESTIONS

1. Describe LC oscillation. Derive an expression for frequency of LC, oscillation and also compare this oscillation with simple harmonic motion.
2. What are natural and damped oscillation? Set up differential equation of LCR damped oscillation and find the frequency of oscillation.
3. What are electromagnetic oscillation? Set up differential equation for undamped oscillation and find the frequency.
4. Define the terms forced oscillation and resonance. Obtain an expression for the energy stored in LC circuit at any time.
5. Compare electromagnetic oscillation with the oscillation of mass spring system. Obtain differential equation for LCR damped oscillation. Write and explain the solution of equation.
6. What are damped and forced oscillation. Develop a differential equation of damped oscillation and find an expression for frequency of oscillator.
7. What do you mean by electromagnetic oscillation? Derive the frequency of electromagnetic oscillation in LCR circuit.
8. Derive a differential equation for LC oscillation. Show that the maximum value of electric and magnetic energy stored in LC circuit is equal.
9. LC oscillations are called em oscillations, why? Derive the differential equation for damped electromagnetic oscillations and find the amplitude and frequency of that oscillation.
10. Derive differential equation for damped LCR oscillation. Obtain an expression for current and frequency of oscillation.
11. Define forced oscillation. Show that total energy of the damped oscillation decreases with increasing time.
12. Obtain differential equation for forced oscillation. Write its solution. Explain the statement quality factor (Q) is a measure of the sharpness of resonance in case of a driven oscillator.
13. Derive a relation for current that flows in an LCR circuit with sinusoidally varying emf. Find the condition for current resonance.
14. Write differential equations of free, damped and forced vibration in an electrical system. Using the solution of free oscillation, prove that total energy is equal to maximum of electric energy or magnetic energy.

NUMERICAL PROBLEMS

1. A circuit has L = 5 mH and C = 2 μF. How much resistance must be inserted in the circuit to reduce the resonance frequency by 5%? [Ans: 31.30Ω]
2. A 10H coil has a resistance of 180 Ω. What size of capacitor must be put in series with it if the condenser is to resonate when connected to a 60 Hz outlet? [Ans: 7.032×10^{-7} F]

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3. A radio can tune over the frequency of 880 KHz to 1200 KHz. If its LC circuit has an effective inductance of 200 pH, What must be the range of its variable capacitor. [Ans: 8.795×10^{-5} F to 1.635×10^{-4} F]

4. A 1 μF capacitor is charged to 50 Volt. The charging battery is then disconnected and a 10 mH coil is connected across the capacitor so that LC oscillation occur. Calculate. (i) The maximum current in the circuit;(ii) Energy stored in the system [Ans: 0.5A, 1.25×10^{-3} J]

5. A 20 mH inductor and 600 μF capacitor form an oscillating circuit. What is the peak value of current if the initial charge is 60 μC? [Ans: 0.0173 A]

6. A circuit contains a charged capacitor of capacitance 500 μF, inductor of inductance 60 mH and resistor of resistance 200Ω. What is the time required to decrease the total energy of circuit by 50%. [Ans: 2.079×10^{-4} Sec]

7. A capacitor of capacity 0.01 μF is charged to a potential difference of one kilovolt and discharged through a resistance of one ohm and an inductance of 0.1 millihenry connected in series. Calculate the frequency and quality factor of the circuit. [Ans: 1.591 10^5 Hz, 100]

8. A 500Ω resistor, 0.4 μF capacitor and 0.2H inductor are connected in series to a voltage source with an amplitude of 160 V and angular frequency of 400 rad S. (i) what is the amplitude of current (ii) what are the value of inductive and capacitive reactances. [Ans: 0.026A, 80 Ω, 6250 Ω]

9. In an oscillating series RLC circuit, find the time required for the maximum energy present in the capacitor to fall to half its initial value. [Ans: 0.693 τ]

10. An LC oscillator contains capacitor of capacitance 300 mF and inductor of self inductance 80 mH. What is the maximum value of current LC oscillations if the capacitor is initially charged to a potential difference of 100 V? [Ans: 193.65 A]

11. What is the phase shift between the current and voltage in a circuit containing 20Ω resistance, 30mH inductance and 100 μF capacitance for 50 Hz AC? [Ans: 131.754°]

12. At what time will the amplitude of charge, oscillation in the series LCR circuit be 50% of its initial value. Where L = 12 mH, C = 1.6 μF and R = 1.5Ω. Also calculate the number of oscillations completed with in this time. [Ans: 0.011 Sec, 13]

13. Circuit containing a 25 mH inductor, 4Ω resistor and 40 μF capacitor has an initial charge of 3 μC. What time is required to decrease charge on capacitor by e^{-1} time? [Ans: 0.0125 Sec]

14. An RLC series circuit has 5.0Ω resistor, 2.0μF capacitor and 1.0 H inductor. The oscillating emf has peak value 30 V. At what angular frequency will the current amplitude have its maximum value and what is this maximum value? [Ans: 707.11 rad/sec, 5.99 A]

What should be the capacitance of a capacitor in a tuned circuit of frequency 10 MHz having an inductance of 0.01 mH? The resistance of the circuit is negligible? [Ans: 2.53×10^{-17} F]

□□□

(B) ELECTROMAGNETIC WAVE

MAXWELL'S EQUATION IN INTEGRAL FORM

Maxwell discovered that the basic principles of electromagnetism can be expressed in term of a set of four equations. Though he developed the concept of dispalcement current, Maxwell did not discover all of these equations him self. But he put them together and recognized their significance particularly in predicting the existence of electromagnetic waves. These equations are called Maxwell's equations and are described below.

1. **Gauss's law for electrostatics**

It states that 'The total flux through a closed surface enclosing a charge 'q' is equal to $1/\epsilon_o$ times the magnitude of charge enclosed.

i.e. $$\oint \vec{E}.d\vec{A} = \frac{q}{\epsilon_o}$$

This relates the electric field and charge distribution that creates it. It confirms the existence of single charge.

2. **Gauss's law for magnetism**

It states that, the net magnetic flux through a closed surface is zero. i.e.

$$\oint \vec{B}.d\vec{A} = 0$$

It confirms that the magnetic monopole does not exist.

3. **Faraday's law of induction**

It states that 'The induced emf in the circuit equals the negative of the time rate of change of magnetic field.

i.e. $$\oint \vec{E}.d\vec{l} = -\frac{d\phi_B}{dt}$$

It says that changing magnetic field with time produces an electric field.

Note

The current produced due to changing magnetic field is called induced current. The work done per unit charge to produce that current (to move the conduction electrons that constitute the current) is called induced emf. The process of producing the current and emf is called induction.

4. **Ampere Maxwell law**

It is the modification of Ampere's law by Maxwell. It describes that there are at least two ways of setting up a magnetic field 1) by means of steady current (According to Amperes law, $\oint \vec{B}.d\vec{l} = \mu_o I$) 2) by means of changing electric field (According Maxwell's law of induction, $\oint \vec{B}.d\vec{l} = \mu_o \epsilon_o \frac{d\phi_E}{dt}$)

The combined form of these two law is Ampere-Maxwell law. i.e.

$$\oint \vec{B}.d\vec{l} = \mu_o I + \mu_o \epsilon_o \frac{d\phi_E}{dt} = \mu_o (I + \epsilon_o \frac{d\phi_E}{dt})$$

or, $$\oint \vec{B}.d\vec{l} = \mu_o (I + I_d),\ I_d = \frac{\epsilon_o d\phi_E}{dt}$$ is displacement current.

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CONVERSION OF MAXWELL'S EQUATION IN DIFFERENTIAL FORM

1. Maxwell's first equation: $(\nabla.\vec{E} = \rho/\epsilon_o)$

From Gauss's law of electrostatistics, we have,

$$\oint \vec{E}.d\vec{A} = \frac{q}{\epsilon_o}$$

From Gauss divergence theorem

$$\oint \vec{E}.d\vec{A} = \oint \nabla.\vec{E}\,dV$$

The charge, q can be expressed interms of charge density as $q = \oint \rho dv$

Therefore, $\oint (\nabla.\vec{E})\,dv = \frac{1}{\epsilon_o}\oint \rho dv$

or, $\oint (\nabla.\vec{E})\,dv = \oint \frac{\rho}{\epsilon_o} dv$

$\nabla.\vec{E} = \rho/\epsilon_o$, where, ρ is charge density.

2. Maxwell's second equation $(\nabla.\vec{B} = 0)$

$\Rightarrow$ From Gauss's law of magnetism we have, $\oint \vec{B}.d\vec{A} = 0$

Using Gauss divergence theorem, $\oint \vec{B}.d\vec{A} = \oint (\nabla.\vec{B})\,dV$

Therefore, $\oint (\nabla.\vec{B})\,dV = 0$

$\Rightarrow \nabla.\vec{B} = 0$

This shows that magnetic field is solenoidal.

3. Maxwell's third equation, $\left[\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}\right]$

According to Faraday's law of electromagnetic induction we have, $\oint \vec{E}.d\vec{\ell} = -\frac{\partial \phi_B}{\partial t}$

Using Stokes theorem

$$\oint \vec{E}.d\vec{\ell} = \oint (\nabla \times \vec{E})\,d\vec{A}$$

$$\frac{\partial \phi_B}{\partial t} = \frac{\partial}{\partial t}\int \vec{B}.d\vec{A} = \oint \frac{\partial \vec{B}}{\partial t}.d\vec{A}$$

Therefore, $\oint (\nabla \times \vec{E})\,d\vec{A} = -\oint \frac{\partial \vec{B}}{\partial t}.d\vec{A}$

Since $\phi_B = \oint \vec{B}.d\vec{A}$ is the magnetic flux

or, $\oint (\nabla \times \vec{E})\,d\vec{A} = \oint -\frac{\partial \vec{B}}{\partial t}.d\vec{A}$ [Area is independent of time]

Therefore, $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

4. Maxwell's fourth equation, $\left[\nabla \times B = \mu_o\left(J + \epsilon_o \frac{d\vec{E}}{dt}\right)\right]$

According to Ampere - Maxwell law we have

$\oint \vec{B}.d\vec{l} = \mu_o(I + I_d)$ Where, displacement current $I_d = \epsilon_o \frac{d\phi_E}{dt}$

From stokes theorem

$$\oint \vec{B}.d\vec{l} = \int \left(\nabla \times \vec{B}\right) d\vec{A}$$

Here, displacement current $I_d = \epsilon_o \frac{d\phi_E}{dt} = \epsilon_o \frac{d}{dt}\oint \vec{E}.d\vec{A}$

or, $I_d = \oint \epsilon_o \frac{d\vec{E}}{dt}.d\vec{A}$

And real current, $I = \oint \vec{J}.d\vec{A}$ ($\vec{J}$ = current density)

Therefore, $\int \left(\nabla \times \vec{B}\right) d\vec{A} = \mu_o\left[\oint \vec{J}.d\vec{A} + \oint \epsilon_o \frac{d\vec{E}}{dt} d\vec{A}\right]$

$$\int \left(\nabla \times \vec{B}\right) d\vec{A} = \int \mu_o\left(J + \varepsilon_0 \frac{dE}{dt}\right) dA$$

$$\nabla \times \vec{B} = \mu_o\left[J + \epsilon_o \frac{d\vec{E}}{dt}\right]$$

Wave Equation in Free Space

In free space, the charge density 'ρ' and current density 'J' are zero, therefore the Maxwell equations become.

$$\nabla.\vec{E} = 0 \quad \ldots(i)$$

$$\nabla.\vec{B} = 0 \quad \ldots(ii)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \ldots(iii)$$

$$\nabla \times \vec{B} = \mu_o \epsilon_o \frac{\partial \vec{E}}{\partial t} \quad \ldots(iv)$$

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Taking the curl of equation (iii).

$$\nabla \times \nabla \times \vec{E} = -\nabla \times \frac{\partial \vec{B}}{\partial t} = -\frac{\partial}{\partial t}\left(\nabla \times \vec{B}\right)$$

$$\Rightarrow \quad \nabla(\nabla.\vec{E}) - (\nabla.\nabla)\vec{E} = -\frac{\partial}{\partial t}\left(\mu_o \epsilon_o \frac{\partial \vec{E}}{\partial t}\right)$$

$$\left[\text{Since } \vec{a} \times \vec{b} \times \vec{c} = \vec{b}\left(\vec{a}.\vec{c}\right) - \vec{c}\left(\vec{a},\vec{b}\right)\right]$$

$$0 - \nabla^2 \vec{E} = -\mu_o \epsilon_o \frac{\partial^2 \vec{E}}{\partial t} \qquad [\text{Since } \nabla.\vec{E} = 0]$$

$$\nabla^2 \vec{E} = \mu_o \epsilon_o \frac{\partial^2 \vec{E}}{\partial t^2} \qquad \ldots (v)$$

Similarly, taking the curl of equation (iv) and proceeding in similar way,

we get, $\nabla^2 \vec{B} = \mu_o \epsilon_o \dfrac{\partial^2 \vec{E}}{\partial t^2}$ $\qquad \ldots (vi)$

Equation's (v) and (vi) are the equation of electromagnetic wave motion in fress space they are sometime called 'D' Alembert's equation".

The one dimensional solution of wave equations (v) and (vi) can be written as:

$$\vec{E} = E_o \sin(\vec{k}.\vec{x} - \omega t) \qquad \ldots (vii)$$

$$\vec{B} = B_o \sin(\vec{k}.\vec{x} - \omega t) \qquad \ldots (viii)$$

Comparing equation (v) and (vi) with general wave equation

$$\nabla^2 y = \frac{1}{v^2}\frac{d^2y}{dt^2} \text{ we get,}$$

$$\frac{1}{v^2} = \mu_o \epsilon_o \Rightarrow v = \frac{1}{\sqrt{\mu_o \epsilon_o}}$$

$$v = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8.85 \times 10^{-12}}} = 3 \times 10^8 \text{ m/sec}$$

Which is same as the velocity of light in vacuum.

Wave Equations in Non-conducting (Dielectric) Medium

In non conducting medium, the charge density 'ρ' and the current density 'J' are zero. If the medium have permittivity ∈ and permeability 'μ', then the Maxwell's equations become.

$$\nabla.\vec{E} = 0 \qquad \ldots (i)$$

$$\nabla.\vec{B} = 0 \qquad \ldots (ii)$$

$$\nabla \times \vec{E} = -\partial \vec{B}/\partial t \qquad \ldots (iii)$$

$$\nabla \times \vec{B} = \mu \epsilon \frac{\partial E}{\partial t} \qquad \ldots (iv)$$

Taking curl of equation (iii) we have.

$$\nabla \times \nabla \times \vec{E} = \nabla \times -\partial \vec{B}/\partial t = -\frac{\partial}{\partial t}(\nabla \times \vec{B})$$

$$\Rightarrow \quad \nabla(\nabla.\vec{E}) - (\nabla.\nabla)\vec{E} = -\frac{\partial}{\partial t}\left(\mu \epsilon \frac{\partial \vec{E}}{\partial t}\right) \quad [\text{using (iv)}]$$

$$-\nabla^2 \vec{E} = -\mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} \qquad [\text{Since } \nabla.\vec{E} = 0]$$

$$\nabla^2 \vec{E} = \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} \qquad \ldots (v)$$

Similarly, taking the curl of equation (4) and proceeding in the same way as above we get,

$$\nabla^2 \vec{B} = \mu \epsilon \frac{\partial^2 \vec{B}}{\partial t^2} \qquad \ldots (vi)$$

Equation (v) and (vi) are the equations of electromagnetic wave motion in non conducting medium. The equation (v) and (vi) in one dimension can be written as

$$\frac{\partial^2 E}{\partial x^2} = \frac{m\epsilon\, \partial^2 E}{dt^2} \text{ and } \frac{\partial^2 \vec{B}}{\partial x^2} = \mu \epsilon \frac{\partial^2 \vec{B}}{\partial t^2}$$

Comparing these equations with general wave equation.

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \text{ we get ,}$$

$$\frac{1}{v^2} = \mu \epsilon \Rightarrow v^2 = \frac{1}{\mu \epsilon} \Rightarrow v = \frac{1}{\sqrt{\mu \epsilon}}$$

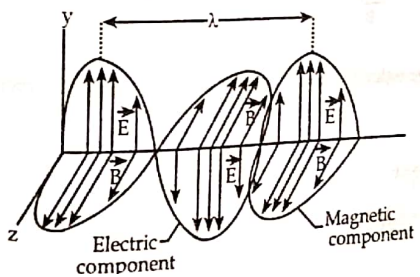
Therefore, the velocity of em wave in non-conducting medium is

$$v = \frac{1}{\sqrt{\mu \epsilon}}.$$

Electromagnetic Wave

Consider a magnetic field varying sinusoidally. It induces (according to Faradays law of em induction) a perpendicular electric field that also varies sinusoidally. Since the electric field is varying sinusoidally, it induces (according to Maxwell law of induction) a perpendicular magnetic field that also varies sinusoidally and so on.

The two fields continuously create each other via induction and the resulting sinusoidal variation in the fields travel as a wave called the electromagnetic wave.



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The features of em wave are.

1. The electric field ($\vec{E}$) and magnetic field ($\vec{B}$) are always perpendicular to each other and the direction of travel of wave. Thus the wave is transverse in nature.

2. The cross product ($\vec{E} \times \vec{B}$) gives the direction of travel of wave.

3. The $\vec{E}$ and $\vec{B}$ fields varies sinusoidally with same frequency and in phase with each other.

On the basis of these features we can assume that an em wave is travelling along +ve x-direction with electric field oscillating parallel to the y - axis and the magnetic field oscillating parallel to z - axis (using right hand rule). Then we can write electric field and magnetic field as sinusoidal function of position 'x' and time 't'

i.e. $E = E_o \sin(kx - \omega t)$

$B = B_o \sin(kx - \omega t)$

Where, E_o and B_o are amplitudes of electric and magnetic field. 'ω' and 'k' are angular frequency and wave number of the wave. Speed of em wave is given by, $v = \frac{\omega}{k} = \frac{2\pi f}{2\pi/\lambda} = f\lambda$.

Speed of Electro Magnetic Wave: (In Terms of Amplitude of Fields)

Consider an electromagnetic wave travelling along the +ve x - axis. A rectangle of width 'dx' and height 'h' is placed on the path of em wave such that it's height is parallel to oscillating electric field. As the electromagnetic wave passes, the magnetic flux through the rectangle changes and electric field is induced,

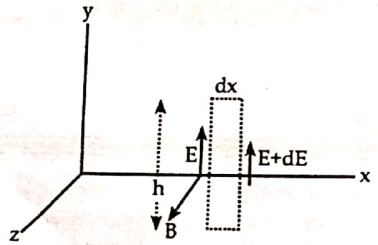

Figure 1: Snapshot of Electromagnetic Wave

According to Faraday's law of induction.

$$\oint \vec{E}.d\vec{\ell} = -\frac{d\phi_B}{dt} \quad \ldots (i)$$

Let $\vec{E}$ and $\vec{E} + d\vec{E}$ are the induced fields along the two long side of the rectangle as shown in figure. Therefore,

$$\oint \vec{E}.d\ell = (E + dE)h - Eh$$

$$= hdE$$

And magnetic flux through rectangle,

$$\phi_B = \oint \vec{B}.d\vec{A} = B(hdx)$$

$$\frac{d\phi_B}{dt} = \frac{d}{dt}[Bhdx]$$

$$\frac{d\phi_B}{dt} = hdx\frac{dB}{dt}$$

Using the value of $\oint \vec{E}.d\vec{\ell}$ and $\frac{d\phi_B}{dt}$ in equation (i)

$$hdE = -hdx\frac{dB}{dt}$$

$$\Rightarrow \frac{dE}{dx} = -\frac{dB}{dt} \quad \ldots (ii)$$

Similarly, taking a rectangle of same dimension as in figure above and applying Maxwell law of induction

$$\int \vec{B}\,d\vec{\ell} = \mu_o\epsilon_o\frac{d\phi_E}{dt} \quad \text{we get,}$$

$$\frac{dE}{dt} = -\frac{1}{\mu_o\epsilon_o}\frac{dB}{dx} \quad \ldots (iii)$$

Though, both E and B are functions of two variables x and t. However in evaluating dE/dx we assume that t is constant and 'x' as a constant to evaluate dB/dt. Same reasons are true for evaluating dB/dx and dE/dt. It is because the figure above is a snapshot and we are dealing with change of fields at a particular place. Under these situation the equation (ii) and (iii) can be written into partial derivatives.

i.e. $$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t} \quad \ldots (iv)$$

$$\frac{\partial E}{\partial t} = -\frac{1}{\mu_o\epsilon_o}\frac{\partial B}{\partial x} \quad \ldots (v)$$

Since, $E = E_o \sin(kx - \omega t) \Rightarrow \frac{\partial E}{\partial x} = k E_o \cos(kx - \omega t)$

$$\frac{\partial E}{\partial t} = -\omega E_o \cos(kx - \omega t)$$

and $B = B_o \sin(kx - \omega t) \Rightarrow \frac{\partial B}{\partial x} = kB_o \cos(kx - \omega t)$

$$\frac{\partial B}{\partial t} = -\omega B_o \cos(kx, \omega t)$$

Using, these relations in equation (4), $k E_o \cos(kx - \omega t) = -[-\omega B_o \sin(kx - \omega t)]$

$$\Rightarrow \frac{E_o}{B_o} = \frac{\omega}{k}$$

Therefore, speed of em wave $v = \frac{\omega}{k} = \frac{E_o}{B_o}$ $\quad \ldots (vi)$

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Similarly, from equation (v)

$$-\omega E_o \cos(kx - \omega t) = -\frac{1}{\mu_o \epsilon_o}[k B_o \cos(kx - \omega t)]$$

$$\Rightarrow \frac{E_o}{B_o} = \frac{1}{\mu_o \epsilon_o} \cdot \frac{k}{\omega}$$

$$\Rightarrow \frac{E_o}{B_o} \cdot \frac{\omega}{k} = \frac{1}{\mu_o \epsilon_o}$$

$$\Rightarrow \left[\text{Since } v = \frac{\omega}{k}, \text{ also } v = \frac{E_o}{B_o}\right]$$

Therefore, $v^2 = \frac{1}{\mu_o \epsilon_o}$

$$\Rightarrow v = \frac{1}{\sqrt{\mu_o \epsilon_o}}$$

Therefore, velocity of E M wave is,

$$v = \frac{E}{B} = \frac{E_o}{B_o} = \frac{1}{\sqrt{\mu_o \epsilon_o}} = C \text{ (Speed of light)}$$

Solved Example

Example 1: What is the displacement current for a capacitor having radius 5 cm with variable electric field 8.9×10^{12} Volts/m.S?

Solution:

Here,

$r = 5\text{cm} = 5 \times 10^{-2}\text{m}$,

$\frac{dE}{dt} = 8.9 \times 10^{12}$ Volts/m.S.

We have,

$$I_d = \epsilon_o \frac{d\phi_E}{dt} = \epsilon_o A \frac{dE}{dt} = \epsilon_o \pi r^2 \frac{dE}{dt}$$

$$I_d = 8.85 \times 10^{-12} \times \pi \times (5 \times 10^{-2})^2 \times 8.9 \times 10^{12} = 0.62 \text{ A}$$

Example 2: A parallel plate capacitor has capacitance 20 μF. At what rate the potential difference between the plates must be changed to produce a displacement current of 1.5 A?

Solution:

Here,

$C = 20\ \mu F = 20 \times 10^{-6}$ F,

$I_d = 1.5$ A, $\frac{dV}{dt} = ?$

We have,

$$I_d = \epsilon_o \frac{d\phi_E}{dt} = \epsilon_o A \frac{dE}{dt} = \epsilon_o A \frac{d}{dt}\left(\frac{V}{d}\right) = \frac{\epsilon_o A}{d}\frac{dV}{dt}$$

$$I_d = C\frac{dV}{dt}$$

$$\frac{dV}{dt} = \frac{I_d}{C} = \frac{1.5}{20 \times 10^{-6}} = 7.5 \times 10^4 \text{ Volts/ sec}$$

Example 3: A sinusoidally varying voltage is applied across a 8 μF capacitor. The frequency of the voltage is 3 KHz and the voltage amplitude is 30 V. Find the maximum value of displacement current.

Solution:

Here $V = V_o \sin \omega t$

$V_o = 30$ V

$f = 3$ KHz $= 3 \times 10^3$ Hz and

$C = 8\ \mu F = 8 \times 10^{-6}$ F

We have, $I_d = C\frac{dV}{dt} = C V_o \omega \cos \omega t$

$I_d(\text{max}) = C V_o \omega = CV_o \times 2\pi f$

$= 8 \times 10^{-6} \times 30 \times 2\pi \times 3 \times 10^3 = 4.52$ A

Example 4: Calculate the displacement current between the square plates of capacitor having one side 1.0 cm if the electric field between the plate is changing at the rate of 3×10^6 V/metre - S

Solution:

Here, length of one side of plate, a = 1 cm

Area of the plate, $A = a^2 = 1\text{ cm}^2 = 1 \times 10^{-4}\text{ m}^2$

and $\frac{dE}{dt} = 3 \times 10^6$ V/mS

We have, $I_d = \epsilon_o A \frac{dE}{dt} = 8.85 \times 10^{-12} \times 1 \times 10^{-4} \times 3 \times 10^6 = 2.655 \times 10^{-9}$ A

Example 5: Using maxwell's equation, prove that $C = \frac{E_m}{B_m}$ where symbols carry their usual meaning.

Solution:

We have third Maxwell equation as

$$\nabla \times E = \frac{dB}{dt}$$

In one dimension it can be expressed as $\frac{dE}{dx} = -\frac{dB}{dt}$ ...(i)

Since,

$E = E_m \sin(kx - \omega t)$

$\frac{dE}{dx} = k.\ E_m \cos(kx - \omega t)$

And

$B = B_m \sin(kx - \omega t)$

$\frac{dB}{dt} = (-\omega) B_m \cos(kx - \omega t)$

From equation (1)

$k.\ E_m = \omega B_m$

$\frac{E_m}{B_m} = \frac{\omega}{k} = C$

Therefore, $C = \frac{E}{B} = \frac{E_m}{B_m}$

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Example 6: Prove the charge conservation theorem

OR - Derive continuity equation

OR - Show that $\nabla.\vec{J}+\frac{d\rho}{dt}=0$. Where symbols carry their usual meaning.

Solution:

We have fourth Maxwell equation as

$$\nabla\times\vec{B}=\mu_0(\vec{J}+\epsilon_0\frac{d\vec{E}}{dt})$$

Taking divergence on both sides

$$\nabla.(\nabla\times\vec{B})=\mu_0(\nabla.\vec{J}+\epsilon_0\frac{d(\nabla.\vec{E})}{dt})$$

Since, divergence of curl of any vector is zero, i.e. $\nabla.(\nabla\times\vec{B})=0$

and we know, $\nabla.\vec{E}=\frac{\rho}{\epsilon_0}$

$$0=\nabla.\vec{J}+\epsilon_0\frac{d}{dt}\left(\frac{\rho}{\epsilon_0}\right)$$

$$\nabla.\vec{J}+\epsilon_0\frac{d}{dt}\left(\frac{\rho}{\epsilon_0}\right)=0$$

$$\nabla.\vec{J}+\frac{d\rho}{dt}=0$$

Example 7: A parallel plate capacitor with circular plates 20 cm in diameter is being charged. The current density of the displacement current in the region between the plates is uniform and has a magnitude of 20 A/m². i) Calculate the curl of magnetic flux density in space between the plates ii) Calculate the rate of change of electric field in this region.

Solution:

Here,

$$d=20\text{ cm}\Rightarrow r=\frac{20}{2}=10\text{ cm}=0.1\text{m}$$

Current density,

$$J_d=20\text{ A/m}^2$$

Displacement current,

$$I_d=\frac{J_d}{\pi r^2}=\frac{20}{\pi\times0.1^2}=637\text{ A}$$

i. We have, $\nabla\times B=\mu_0\epsilon_0\frac{dE}{dt}$

$$\nabla\times B=\frac{\mu_0}{A}\epsilon_0 A\frac{dE}{dt}$$

$$=\frac{\mu_0 I_d}{A}=\mu_0 J_d=4\pi\times10^{-7}\times20=2.512\times10^{-5}\text{ Wb/m}^2$$

ii. Again,

$$I_d=\epsilon_0 A\frac{dE}{dt}$$

$$\Rightarrow\frac{dE}{dt}=\frac{I_d}{\epsilon_0 A}=\frac{J_d}{\epsilon_0}=\frac{20}{8.85\times10^{-12}}=2.26\times10^{12}\text{ V/mS}$$

Example 8: At some distance from transmitter of radio station, the magnitude of em wave emitted by radio station is found to be 1.6×10^{-4} T. If frequency of broadcast is 1020 KHz then find speed, wave length and maximum electric field of em wave.

Solution:

Here, $f=1020\text{ KHz}=1020\times10^3\text{ Hz}$, $B_m=1.6\times10^{-4}\text{ T}$

i. $C=\frac{1}{\sqrt{\mu_0\epsilon_0}}=\frac{1}{\sqrt{4\pi\times10^{-7}\times8.85\times10^{-12}}}=3\times10^8\text{ m/s}$

ii. $\lambda=\frac{C}{f}=\frac{3\times10^8}{1020\times10^3}=294.12\text{ m}$

iii. $E_m=CB_m=3\times10^8\times1.6\times10^{-4}$
$=4.8\times10^4\text{ V/m}$

THEORETICAL ANSWER QUESTIONS

1. What is displacement current? Compare the physical significance of Maxwell third and fourth equation.
2. Write Maxwell equations in free space. Prove that the ratio of magnitudes of electric and magnetic field is equal to the speed of light.
3. Write Maxwell equations in integral form and convert them into differential form.
4. Derive Maxwell's equation:

$$\nabla\times\vec{B}=\mu_0\epsilon_0\frac{d\vec{E}}{dt}+\mu_0\vec{J}.$$

5. Show that velocity of electromagnetic wave in free space is given by $C=\frac{1}{\sqrt{\mu_0\epsilon_0}}$.
6. Derive the velocity of electro magnetic wave using Maxwell's equation.
7. Prove the relation: $\nabla^2\vec{E}=\mu_0\epsilon_0\frac{\partial^2\vec{E}}{dt^2}$.
8. Explain the conduction current and displacement current.
9. Write Maxwell equations in differential form and convert them in to integral form.
10. Prove the charge conservation theorem $\frac{\partial\rho}{\partial t}+\nabla.\vec{J}=0$
11. What are maxwell's equations? Using maxwell's equations derive wave equation of electromagnetic waves in a conducting medium in terms of electric and magnetic field
12. Write down Maxwell equations? Prove that $\nabla\times\vec{B}=\mu_0(\vec{J}+\epsilon_0\frac{d\vec{E}}{dt})$.
13. Starting from Maxwell's equation derive equation of continuity.

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14. Write Maxwell equations in differential form. Show that $\frac{E}{B} = C$, where symbols carry their usual meaning.
15. Write Maxwell equations both in integral and differential forms and point out the physical significance of each of the equation.
16. Derive electromagnetic wave equations in free space and dielectric medium.
17. Prove that speed of electromagnetic wave is equal to velocity of light in free space.
18. What is displacement current? Show that in a given circuit the conduction current is equal to displacement current.
19. Explain how is displacement current produced? Prove that the displacement current density $J_d = \epsilon_0 \frac{d\vec{E}}{dt}$.
20. Why and how Maxwell modified Amperes law in magnetism.
21. What are Maxwell's equations in differential form? Use them to obtain wave equation for $\vec{E}$ and $\vec{B}$ in homogeneous isotropic non conducting medium. Also find their solutions.
22. Prove that for electromagnetic wave, the energy stored per unit volume in electric field and magnetic field are equal in free space.
23. Prove that displacement current in parallel plate capacitor can be written as, $I_d = C\frac{dV}{dt}$ where C and V are capacitance of the capacitor and potential difference across the plates respectively.

NUMERICAL PROBLEMS

1. A parallel plate capacitor with circular plates of 10 cm radius is charged producing uniform displacement current of magnitude 20A calculate $\frac{dE}{dt}$ in this region. [Ans: 2.3×10^{12} V/ms]
2. You are given a 1μF capacitor. How would you establish an instantaneous displacement current of 1A in the space between its plates? [Ans: 10^6 V/sec]
3. Find the displacement current for a capacitor having radius 2 cm with variable electric field 2×10^7 V/ms? [Ans: 0.008 A]
4. An electromagnetic wave travels in a vacuum along z-direction. If the frequency of the wave is 30 MHz, what is its wave length? [Ans: 10 m]
5. The electric field part of the electromagnetic wave is given by $\vec{E}$ = (3.1 V/m) Sin [(1.8 rad m^{-1})x + (5.4×10^8 rad S^{-1}) t]. What is wavelength, frequency, and amplitude of the magnetic field part of the wave? [Ans: 3.49m, 85.96 MHz, 10.33 nT]
6. A capacitor made of two circular plates each of radius 12 cm and separated by 5 cm. The charging current is constant and equal to 0.15 A. Calculate the capacitance and the rate of change of potential difference between the plates. [Ans: 8.01pF, 1.873×10^{10} V/s]
7. The relative permeability and relative permittivity of the medium is 1 and 2.25 respectively. Find the speed of the electromagnetic wave in this medium. [Ans: 2 m/s]

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7 Chapter

QUANTUM MECHANICS

CHAPTER OUTLINE

After completion of this chapter, you will be able to know:

* Inadequacy of classical mechanics, Importance of quantum mechanics, Matter wave (de-Broglie equation), Wave function and its significence. Energy and momentum operator, Time independent and time dependent Schrodinger wave equations, Application of Schrodinger wave equation for the electron in metal, Normalized wave function describing the motion of an electron inside in an infinite potential well.

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INTRODUCTION

By the end of the 19th century, what we now refer to as classical mechanics, Maxwell's electromagnetic theory, and thermodynamics could reasonably explain all known physical events. Lagrange, Jacobi, and Hamilton are principally responsible for the current structure of classical mechanics, which was first established by the ground-breaking work of Newton and Galileo. By the turn of the twentieth century, a number of circumstances forced physicists to reevaluate the ideas presented above. Although there were instances in macroscopic systems when the results of experiments could not be explained using the concepts of classical physics, the departures from the predictions of classical physics were more obvious in the microscopic systems (such as systems of atoms or molecules sizes).

The inadequacy of classical mechanics are:

i. It does not hold in the region of atomic dimensions.

ii. It could not explain the observed spectra of black body radiation.

iii. The variation of the specific heat of metal and gases is also not explained.

iv. It could not explain the observed spectrum of black body radiation.

v. It could not explain the origin of the discrete spectra of atoms. This is so because according to classical mechanics the energy changes are always continuous.

vi. Classical mechanics could not explain a large number of phenomenon. Some of them are photoelectric effect, Compton effect, Raman effect etc.

The shortcomings of classical mechanics made space for quantum mechanics, which offer accurate solutions to the majority of real-world problems. It not only helps in overcoming the restrictions of classical, but it also provides significant solutions to problems.

QUANTIZATION OF ENERGY

In black body radiation, it was considered that the radiation was emitted continuously. In 1902, Planck had shown that, the energy from the body was emitted in separate packets of energy. The energy of each packet is equal to hf, called a quantum energy i.e. $E = hf$, where $h = 6.63 \times 10^{-34}$ J S is called Planck's constant and f is the frequency of radiation. This theory is called Planck's quantum theory.

Again in 1905, Einstein explained photoelectric effect using quantum theory. He suggested that light of frequency 'f' consist of packets or quanta of energy hf.

In general for 'n' number of photons, the total energy is

$$E_n = nhf, \; n = 0, 1, 2, 3 \ldots\ldots$$

This means, energy is emitted in the form of discrete amount i.e. quanta (singular form "quantum"). Energy is multiple of fundamental energy hf, so energy is quantized.

ELECTRONS AND MATTER WAVES: DE-BROGLIE EQUATION

The successful explanation of compton effect, photoelectric effect and black body radiation shows that electromagnetic radiation travels not in the form of a continuous stream of energy but in the form of tiny packets or bundles of energy. These packets of energy were called 'quanta'. For electromagnetic radiation these were called photon.

On the other hand, the successful explanation of phenomenon like, reflection, refraction, interference, diffraction and polarization cannot be explained unless wave nature of electromagnetic radiation is assumed.

Louis de-Broglie in 1924 Suggested his hypothesis that there is 'wave particle dualism'. The wave associated with material particles is called 'matter wave'. To show the wave particle dualism he made the use of Planck's theory of quantum radiation and Einstein theory of relativity.

According to Planck's theory of quantum radiation, Energy of photon is given by,

$$E = hf = \frac{hc}{\lambda} \qquad \ldots (i)$$

where, h is Planck's constant and f is frequency of radiation.

And according to Einstein mass energy relationship, energy of photon is given by,

$$E = mc^2 \qquad \ldots (ii)$$

c is the velocity of light

Comparing (i) and (ii)

$$\frac{hc}{\lambda} = mc^2$$

$$\lambda = \frac{h}{mc} = \frac{h}{p}$$

[where $p = mc$ = momentum of photon]

According to de-Broglie the wave length 'λ', of the wave associated with a moving particle having momentum $p = mv$ is given by,

$$\lambda = \frac{h}{mv} \qquad \ldots (iii)$$

The de-Broglie Electron Wave

According to de-Broglie if an electron is accelerated to various velocities at various potential, different waves having different wavelengths are produced.

For de-Broglie matter wave we have,

$$\lambda = \frac{h}{p} = \frac{h}{mv} \qquad \ldots (i)$$

If an electron of mass 'm' is accelerated through the potential 'V' and velocity v then,

$$\frac{1}{2}mv^2 = eV \Rightarrow v = \sqrt{\frac{2eV}{m}} \qquad \ldots (ii)$$

Therefore, $\lambda = \dfrac{h}{m\sqrt{\dfrac{2eV}{m}}}$

Wave length of electron wave, $\lambda = \dfrac{h}{\sqrt{2meV}}$.

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Characteristics of de-Broglie Matter wave

1. Two different velocities are associated with a matter wave: one is mechanical velocity or group velocity (v) and other is velocity of each component wave called wave or phase velocity (u).
2. The wave velocity of de- Broglie wave is greater than the velocity of electro magnetic wave.
3. The velocity of de-Broglie wave is different for different matter (i.e. not constant as velocity of electromagnetic wave).
4. The wave length of de-Broglie matter wave varies inversely with the mass of the particles.

WAVE AND GROUP VELOCITY

The wave packet accompanies the particle and tell us where the particle may be found. To describe a localized particle we must use a wave packet. According to de-Broglie, each particle of matter in motion is associated with a group of wave differ slightly in wavelength and frequency called wave packet. The velocity of this group of wave or wave packet which is quantitatively equal to mechanical velocity of the particle is called group velocity.

Let us mix two traveling waves that differ slightly in wavelength and frequency.

$$\psi_1 = A \sin (kx - \omega t)$$

$$\psi_2 = A \sin [(k + \Delta k) x - (\omega + \Delta\omega) t]$$

where, $\Delta k << k$ and $\Delta\omega << \omega$

The resulting wave is given by

$$\psi (x, t) = \psi_1 + \psi_2$$

$$\psi (x, t) = A \sin(kx - \omega t) + A \sin [(k + \Delta k) x - (\omega + \Delta\omega) t]$$

$$= 2A \sin \frac{(kx - wt + kx + \Delta k . x - wt - \Delta w.t)}{2} \cos \frac{(kx - wt - kx - \Delta k.x + wt + \Delta w.t)}{2}$$

$$= 2A \cos \frac{(\Delta w.t - \Delta k.x)}{2} \sin \frac{(2kx + \Delta k.x - 2wt - \Delta w.t)}{2}$$

since $\cos(-A) = \cos A$, and $\Delta k << k$ and $\Delta\omega << \omega$

$$\psi (x, t) = 2A \cos \frac{(\Delta k.x - \Delta\omega.t)}{2} \sin (kx - \omega t) \qquad \ldots (i)$$

The resulting wave is thus the product of two traveling waves. The second term of equation (i) represents a wave having nearly the same frequency and wavelength as the original waves. The first term of equation (i) represents a wave having mach larger wave length (since $\lambda = \frac{2\pi}{\Delta k}$, Δk is very small) and much smaller frequency. We can consider ψ as a wave similar to the original ones except that its amplitude, $2A \cos \frac{(\Delta k.x - \Delta\omega.t)}{2}$, is periodically varying forming the envelopes as shown in figure.

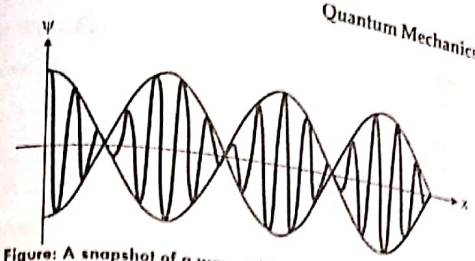

Figure: A snapshot of a wave with a periodically varying amplitude.

Each envelopes includes a group of wave or a wave packet. Such a ψ could be used to describe a beam of particles with one particle in each group.

This group of wave consist of different component waves. The velocity with which each component wave propagates is called phase or wave velocity. It is given by, $v = \frac{\omega}{k}$.

The envelopes (or the wave packets) travel with a velocity called the group velocity and is given by

$$v_{group} = \frac{\Delta w/2}{\Delta k/2} = \frac{\Delta\omega}{\Delta k} = \frac{d\omega}{dk} \qquad \ldots (ii)$$

Since, $E = hf = \frac{h}{2\pi} . 2\pi f = \hbar\omega$ and $p = \frac{h}{l} = \frac{h}{2p} . \frac{2p}{l} = \hbar k$

Equation (2) can be rewritten as

$$v_{group} = \frac{d\omega}{dk} = \frac{d(\hbar w)}{d(\hbar k)} = \frac{dE}{dP} \qquad \ldots (iii)$$

We know that, for a particle of mass 'm' and velocity v,

$$E = \frac{1}{2} mv^2 = \frac{m^2 v^2}{2m} = \frac{P^2}{2m}$$

Therefore, $v_{group} = \frac{dE}{dP} = \frac{d}{dP}\left(\frac{P^2}{2m}\right) = \frac{2P}{2m} = \frac{P}{m} = \frac{mv}{m}$

$$v_{group} = v$$

Therefore, the group velocity is equal to the particle velocity. From this result it can be concluded that the wave packet moves along with the particle.

Relation between Group Velocity and Phase Velocity

We know

Wave vector, $k = \frac{2\pi}{\lambda}$

$$\frac{dk}{d\lambda} = \frac{-2\pi}{\lambda^2}$$

and, $\omega = 2\pi f = \frac{2\pi u}{\lambda}$ [Since, phase velocity, $u = f\lambda$]

$$\frac{d\omega}{d\lambda} = \frac{-2\pi u}{\lambda^2} + \frac{2\pi}{\lambda}\frac{du}{d\lambda}$$

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Now, Group velocity, $v = \frac{d\omega}{dk}$

$$= \frac{d\omega/d\lambda}{dk/d\lambda}$$

$$= \frac{-\lambda^2}{2\pi}\left(\frac{-2\pi u}{\lambda^2} + \frac{2\pi}{\lambda}\frac{du}{d\lambda}\right)$$

$$v = u - \lambda\frac{du}{d\lambda}$$

This is the relation between wave velocity (u) and group velocity v. This shows that wave velocity is always greater than group velocity.

Wave Function and its Significance

A simple harmonic wave is represented by the equation, $y = a\sin(\omega t - kx)$. In such type of wave motion there is only the transfer of energy, but incase of matter wave, there is transfer of momentum (particle) in addition to the energy. The suitable function to represent wave function for matter wave is

$$\psi(x,t) = Ae^{-i(\omega t - kx)} \quad \ldots (i)$$

Where A is called normalizing constant.

Since $E = \hbar\omega \Rightarrow \omega = \frac{E}{\hbar}$ and $P = \hbar k \Rightarrow k = \frac{P}{\hbar}$

Therefore, equation (1) can be rewritten as,

$$\psi(x,t) = A\,e^{-\frac{i}{\hbar}(Et - Px)} \quad \ldots (ii)$$

The wave function ψ has no physical meaning itself. When it is operated by Schrodinger wave equation, it describes the motion of the particle associated with it, as done by second law of motion in classical mechanics. The only quantity having the physical meaning is the square of its magnitude. The quantity $P = |\psi|^2$ evaluated at a particular point at a particular time is proportional to the probability of finding the particle there at that time.

The probability of finding a particle in the volume element dx dy dz is $|\psi|^2$ dx dy dz or $|\psi|^2$ dv.

Since the total probability of finding the particle in the entire space is unity.

$$\int |\psi|^2 dv = \int \psi\psi^* dv = 1.$$

Where, Ψ^* is complex conjugage of ψ.

The wave function satisfying this condition is called normalized wave function. Every acceptable wave function can be normalized by multiplying it with an appropriate constant called normalizing constant.

If the product of a function $\psi_1(x)$ and the complex conjugate $\psi_2^*(x)$ of another function $\psi_2(x)$ vanishes when integrated with respect to x over a interval $a \le x \le b$, i.e. if,

$$\int_a^b \psi_1(x)\psi_2^*(x)\,dx = 0$$

Then $\psi_1(x)$ and $\psi_2(x)$ are said to be orthogonal in the interval (a, b).

Characteristics of Wave Function

1. It must be normalized.
2. It must be single valued and continuous.
3. If $\psi_1(x)$, $\psi_2(x)$ .....$\psi_n(x)$ are solution of Schrödinger wave equation (SWE) then the linear combination, $\psi(x) = a_1\psi_1(x) + a_2\psi_2(x) + \ldots + a_n\psi_n(x)$ must be solution of SWE.
4. The wave function $\psi(x)$ must approach zero as $x \to \pm\infty$.

Energy and Momentum Operator

An equation of the form, $\hat{A}\psi = a\psi$ is called eigen value equation.

Here $\hat{A}$ is an operator

ψ is called eigen function

And 'a' is called eigen value.

An operator is a function over a space of physical states to another space of physical states. The operators must yield real eigen values, since they are values which may come up as the result of the experiment.

We have the wave function associated with quantum mechanical particle as:

$$\psi = A\,e^{-\frac{i}{\hbar}(Et - Px)}$$

Differentiating with respect to 't'

$$\frac{\partial\psi}{\partial t} = \frac{-i}{\hbar}E\psi$$

$$E\psi = \frac{-\hbar}{i}\frac{\partial\psi}{\partial t}$$

$$E\psi = i\hbar\frac{\partial\psi}{\partial t}$$

$$E\psi = \hat{E}\frac{\partial\psi}{\partial t}$$

Where, $\hat{E} = i\hbar\frac{\partial}{\partial t}$ is called Energy operator

Again differentiating ψ with respect to 'x'

$$\frac{\partial\psi}{\partial x} = \left(\frac{-i}{\hbar}\right)(-P)\psi = \frac{iP}{\hbar}\psi$$

$$P\psi = \frac{\hbar}{i}\frac{\partial\psi}{\partial\pi} = \frac{-i^2\hbar}{i}\frac{\partial\psi}{\partial x}$$

$$P\psi = -i\hbar\frac{\partial\psi}{\partial x}$$

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$$P\psi = \hat{P}\frac{\partial\psi}{\partial x}$$

Where, $\hat{P} = -i\hbar\frac{\partial}{\partial x}$ is called momentum operator.

The operators are used to predict the respective parameters i.e. energy and momentum of the particle.

SCHRÖDINGER WAVE EQUATION

Time Independent Schrödinger Equation

Schrödinger wave equation describes the motion of quantum mechanical particle as Newton's second law in classical mechanics.

The wave function for quantum mechanical particle is given by.

$$\psi(x, t) = A\,e^{-i(\omega t - kx)} = Ae^{-\frac{i}{\hbar}(Et - Px)} \quad \ldots (i)$$

Differentiating this equation w.r.t. x

$$\frac{d\psi}{dx} = A\frac{iP}{\hbar}e^{-\frac{i}{\hbar}(Et - Px)} = \frac{iP}{\hbar}\psi$$

or, $$P\psi = \frac{\hbar}{i}\frac{d\psi}{dx} = \frac{-i^2\hbar}{i}\frac{d\psi}{dx} = -i\hbar\frac{d\psi}{dx}$$

Momentum operator $\hat{P} = -i\hbar\frac{d}{dx}$ ...(ii)

Similarly, differentiating equation (1) two times with respect to x we get

$$\frac{d^2y}{dx^2} = \left(\frac{iP}{\hbar}\right)^2\psi$$

$$P^2\psi = -\hbar^2\frac{d^2\psi}{dx^2} \quad \ldots (iii)$$

Consider a particle of mass 'm' and potential energy 'V', moving with velocity 'v', then the total energy is given by.

$$E = \frac{1}{2}mv^2 + V = \frac{1}{2}\frac{(mv)^2}{m} + V = \frac{P^2}{2m} + V$$

Multiplying this equation with ψ,

$$\left(\frac{P^2}{2m} + V\right)\psi = E\psi$$

Substituting the value of $P^2\psi$ from equation (iii)

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx} + V\psi = E\psi$$

$$\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + (E - V)\psi = 0$$

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V)\psi = 0$$

This is the time independent Schrödinger wave equation in one dimension.

Time Dependent Wave Equation

The wave function describing the quantum mechanical particle is given by,

$$\psi(x, t) = Ae^{-\frac{i}{\hbar}(Et - Px)}$$

Differentiating with respect to time 't'

$$\frac{d\psi}{dt} = \frac{-iE}{\hbar}Ae^{-\frac{i}{\hbar}(Et - Px)} = -\frac{iE}{\hbar}\psi$$

$$\Rightarrow E\psi = -\frac{\hbar}{i}\frac{dy}{dt} = \frac{i^2\hbar}{i}\frac{dy}{dt}$$

or, $$E\psi = i\hbar\frac{d\psi}{dt} \quad \ldots (iv)$$

Consider a particle of mass m and potential energy V, moving with a velocity, v then the total energy of the system is

$$E = \frac{1}{2}mv^2 + V = \frac{P^2}{2m} + V$$

Multiplying this equation both sides by ψ

$$E\psi = \frac{P^2\psi}{2m} + V\psi \quad \ldots (v)$$

From (4) and (5)

$$\left(\frac{P^2}{2m} + V\right)\psi = i\hbar\frac{d\psi}{dt}$$

Substituting the value of $P^2\psi$ from equation (3)

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi = i\hbar\frac{d\psi}{dt} \quad \ldots (vi)$$

The term $\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V\right]$ is recognized as Hamiltonian operator $(\hat{H})$

i.e. $$\hat{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V$$

Equation (6) can be rewritten as, ...(vii)

$$H\psi = i\hbar\frac{d\psi}{dt}$$

which is the time dependent Schrodinger wave equation

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PARTICLE IN AN ONE DIMENSIONAL INFINITELY DEEP POTENTIAL WELL

Consider a particle restricted to move along the x-axis between $x = 0$ and $x = \ell$. The potential energy V of the particle is zero inside the box, but rises to infinity on the outside.

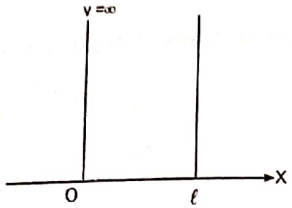

Figure 1: Infinite potential well

i.e. $V = 0$ for $0 < x < \ell$

and $V = \infty$ for $x \le 0$ and $x \ge \ell$

In such case the particle is said to be moving in an infinitely deep potential well.

The Schrödinger wave equation for the particle with in the box ($V = 0$) is.

$$\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0 \qquad \ldots \text{(i)}$$

Let, $$k^2 = \frac{2mE}{\hbar^2} \qquad \ldots \text{(ii)}$$

Therefore, $$\frac{d^2\psi}{dx^2} + k^2\psi = 0$$

The solution of the equation is,

$$\psi(x) = A \sin kx + B \cos kx \qquad \ldots \text{(iii)}$$

where A and B are constants to be determined using boundary conditions.

Since the particle cannot have infinite energy, it cannot exist out side the box. Therefore, the wave function ψ must be zero outside the box, or ψ must be zero at the walls, i.e. at $x = 0$ and $x = \ell$

$$\psi(x) = 0 \text{ at } x = 0$$

From equation (3)

$$\Rightarrow \quad 0 = 0 + B$$

$$\Rightarrow \quad B = 0$$

Now equation (iii) becomes, $\psi(x) = A\sin kx$.

Again, $\psi(x) = 0$ at $x = \ell$

$$\Rightarrow \quad 0 = A \sin k\ell$$

$$\Rightarrow \quad \sin k\ell = 0$$

$$\Rightarrow \quad \sin k\ell = \sin n\pi, \; n = 0, 1, 2, 3\ldots$$

$$\Rightarrow \quad k\ell = n\pi$$

$$\Rightarrow \quad k = \frac{n\pi}{\ell} \qquad \ldots \text{(iv)}$$

From equation (ii) and equation (iii), $\frac{2mE}{\hbar^2} = \frac{n^2\pi^2}{\ell^2}$

$$\Rightarrow \quad E = \frac{n\pi^2\hbar^2}{2m\ell^2}, \; n = 1, 2, 3, 4, \ldots \qquad \ldots \text{(v)}$$

This means the energy of particle in potential well is quantized. Each value of energy given by (iv) is called eigen value and corresponding function is called eigen function.

Now substituting, $B = 0$ and $k = n\pi/\ell$ in equation (iii), the allowed solutions of Schrödinger equation are,

$$\psi_n(x) = A \sin \frac{n\pi x}{\ell}$$

The coefficient A is called normalizing constant and can be determined using normalizing condition.

$$\int_0^l \psi\psi^* \, dx = 1$$

$$A^2 \int_0^l \sin^2 \frac{(n\pi x)}{\ell} dx = 1$$

$$A^2 \int_0^l \frac{1}{2}\left\{1 - \cos 2\frac{(n\pi x)}{\ell}\right\} dx = 1$$

$$A^2 \frac{1}{2}\left[\int_0^\ell dx - \int_0^\ell \cos \frac{2n\pi x}{\ell} dx\right] = 1$$

$$A^2 . \frac{\ell}{2} = 1$$

$$A = \sqrt{\frac{2}{\ell}}.$$

The normalized wave functions of the particles are therefore

$$\psi_n(x) = \sqrt{\frac{2}{\ell}} \sin \frac{n\pi x}{\ell}.$$

The first three eigen functions, ψ_1, ψ_2, ψ_3 together with the probability densities $|\psi_1|^2$, $|\psi_2|^2$, $|\psi_3|^2$ are shown in figure below.

It is obvious that the wave mechanical result is very different from the classical result. Classical mechanics predicts the same probability for the particle being any where in the box. But quantum mechanics predicts that the probability is different at different point.

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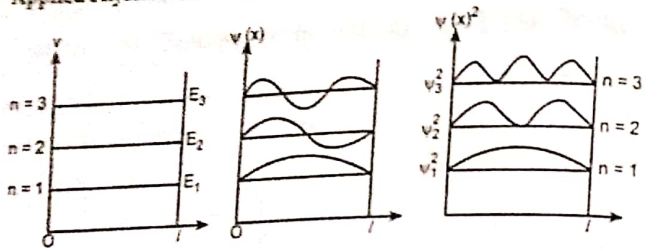

Figure: First three energy states, wave functions and probability densities for a particle in one dimensional box

Solved Example

Example 1: A ball of mass 10 gm has velocity 100 cm/sec. Calculate the wave length associated with it. Why does not this wave nature show up our daily observations. Given, $h = 6.62 \times 10^{-34}$ JS.

Solution:

Here, $m = 10$ gm $= 10 \times 10^{-3}$ kg, $h = 6.62 \times 10^{-34}$ Js

We have,

$$\lambda = \frac{h}{mv}$$

$$= \frac{6.62 \times 10^{-34}}{10 \times 10^{-3} \times 1}$$

$$= 6.62 \times 10^{-32} \text{ m}$$

This wavelength is much smaller than the dimensions of the balls therefore in such cases wave-like properties of matter can not be observed in our daily observations.

Example 2: Calculate the wave length associated with an electron subjected to a potential difference of 1.25 KV.

Solution:

We have,

$$\frac{1}{2}mv^2 = eV$$

$$\Rightarrow v = \sqrt{\frac{2eV}{m}}$$

Now,

$$\lambda = \frac{h}{mv}$$

$$= \frac{h}{\sqrt{2meV}}$$

$$= \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 1.25 \times 10^{3}}}$$

$$\lambda = 0.347 \text{Å}$$

Example 3: Show that wave velocity is greater than velocity of light

Solution:

We have, wave velocity, $u = f\lambda$

From Planck's law and Einstein's mass energy relation.

$$hf = mc^2$$

$$f = \frac{mc^2}{h}$$

Substituting f for u

$$u = \frac{mc^2}{h} \cdot \lambda = \frac{mc^2}{h} \cdot \frac{h}{mv} = \frac{c^2}{v}$$

$$c^2 = uv$$

Since particle velocity (v) must be less than velocity of light (c), so wave velocity (u) is greater than velocity of light.

Example 4: Find the energy of the neutron in units of electron - volt whose de - Broglie wave length is 1Å. Given mass of neutron $= 1.674 \times 10^{-27}$ kg, $h = 6.62 \times 10^{-34}$ joule-sec.

Solution:

Here, $\lambda = 1$ Å $= 1 \times 10^{-10}$ m, $m = 1.674 \times 10^{-27}$kg, $h = 6.62 \times 10^{-34}$ Joule sec.

We have, $E = \frac{1}{2}mv^2$

Also, $\lambda = \frac{h}{mv} \Rightarrow v = \frac{h}{m\lambda}$

Therefore,

$$E = \frac{1}{2}m\frac{h^2}{m^2\lambda^2} = \frac{h^2}{2m\lambda^2}$$

$$= \frac{(6.62 \times 10^{-34})^2}{2 \times 1.674 \times 10^{-27} \times (1 \times 10^{-10})^2}$$

$$= 1.3 \times 10^{-20} \text{ joule}$$

$$= \frac{1.3 \times 10^{-20}}{1.6 \times 10^{-19}} = 8.13 \times 10^{-2} \text{ eV}$$

Example 5: Show that group velocity is equal to particle velocity.

Solution:

The group velocity is given by

$$v_g = \frac{d\omega}{dk} = \frac{d(\hbar\omega)}{d(\hbar k)} = \frac{dE}{dP} \quad \left[\text{Since } E = hf = \frac{h}{2\pi} \cdot 2\pi f = \hbar\omega \text{ and } p = \frac{h}{\lambda} = \frac{h}{2\pi} \cdot \frac{2\pi}{\lambda} = \hbar k\right]$$

Since, $E = \frac{P^2}{2m}$ for free particle

Since, $v_g = \frac{d}{dp}\left(\frac{P^2}{2m}\right) = \frac{2P}{2m} = \frac{mv}{m} = v$ = particle velocity.

Example 6: An electron is confined to an infinite height box of size 0.1 nm. Calculate the ground state energy of the electron. How this electron can be put to the third energy level.

Solution:

The energy of the particle in one dimensional rigid box of side l is given by

$$E_n = \frac{n^2\pi^2h^2}{2ml^2}$$

$$= \frac{n^2h^2}{8ml^2}$$

$$= \frac{(6.62\times10^{-34})^2\times n^2}{8\times9.1\times10^{-31}\times(10^{-10})^2}$$

$$= 6.03\times10^{-18}n^2 \text{ joule}$$

$$= 37.7\, n^2 \text{ eV}$$

In the ground state, $n = 1$, $E_1 = 37.7$ eV

For third energy level, $n = 3$, $E_3 = 37.7\times3^2 = 37.7\times9$ eV

$\therefore$ $E_3 - E_1 = (9-1)\times37.7 = 301.5$ eV

Hence to put the electron to third energy level an extra energy of 301.5 eV is to be given.

Example 7: What voltage must be applied to an electron microscope to produce electrons of wave length 0.50 Å? Given, $e = 1.6\times10^{-19}$ Coulomb, $m = 9.0\times10^{-31}$ kg, $h = 6.62\times10^{-34}$ Joule.sec.

Solution:

We have, $\frac{1}{2}mv^2 = eV$

$$V = \frac{mv^2}{2e}$$

The de-Broglie wavelength is given by, $\lambda = \frac{h}{mv} \Rightarrow v = \frac{h}{m\lambda}$

Therefore,

$$V = \frac{m}{2e}\cdot\frac{h^2}{m^2\lambda^2} = \frac{h^2}{2me\lambda^2}$$

$$= \frac{(6.62\times10^{-34})^2}{2\times9.1\times10^{-31}\times1.6\times10^{-19}\times(0.5\times10^{-10})^2}$$

$$= 602 \text{ Volts.}$$

Example 8: The wave function of a particle confined in a box of length l is $\psi(x) = \sqrt{\frac{2}{l}}\sin\frac{\pi x}{l}$. Calculate the probability of finding the particle in the region $0 < x < \frac{l}{2}$.

Solution:

The probability of finding the particle in the length 0 to $\frac{l}{2}$ is

$$P = \int_0^{l/2}(\psi)^2\,dx = \frac{2}{l}\int_0^{l/2}\sin^2\frac{\pi x}{l}\,dx = \frac{1}{l}\int_0^{l/2}\left(1-\cos\frac{2\pi x}{l}\right)dx = \frac{1}{l}\cdot\frac{l}{2} = \frac{1}{2} = 0.5$$

Example 9: Normalize the one dimensional wave function

$$\Psi = A\sin\left(\frac{\pi x}{a}\right),\ 0 < x < a$$

$$= 0 \quad , \text{outside}$$

VVP

$$\psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi}{L}x\right) \quad n = 1,2,3\ldots$$

$$E_n = \frac{n^2h^2}{8mL^2},\ n = 1,2,3\ldots$$

Solution:

We have normalizing condition

$$\int_{-\infty}^{\infty}|\Psi|^2\,dx = 1$$

or $$\int_0^a A^2\sin^2\frac{\pi x}{a}\,dx = 1$$

or $$\frac{A^2}{2}\int_0^a\left(1-\cos\frac{2\pi x}{a}\right)dx = 1$$

or $$\frac{A^2}{2}\cdot a = 1$$

$$\therefore\ A = \sqrt{\frac{2}{a}}$$

Hence the normalized wave function is $\sqrt{\frac{2}{a}}\sin\frac{\pi x}{a}$.

Example 10: An electron moving is a wave has wave function $\Psi(x) = 2\sin 2\pi x$. Find the probability of finding the electron in the region $x = 0.25$ to 0.5 m.

Solution:

The probability of finding the electron in given region is,

$$P = \int_{0.25}^{0.5}\Psi\Psi^*\,dx = \int_{0.25}^{0.5}4\sin^2 2\pi x\,dx$$

$$P = 2\int_{0.25}^{0.5}2\sin^2 2\pi x\,dx = 2\int_{0.25}^{0.5}(1-\cos 4\pi x) = 2\left[\int_{0.25}^{0.5}dx - \int_{0.25}^{0.5}\cos 4\pi x\,dx\right]$$

Board $$= 2\left[(x)\Big|_{0.25}^{0.5} - \frac{\sin 4\pi x}{4\pi}\Big|_{0.25}^{0.5}\right] = 2\,[(0.5-0.25)-0] = 2\,[0.25-0] = 0.5$$

Example 11: A particle is moving in 1–D box of infinite potential. Evaluate the probability of finding the particle within range 1 Å at the centre of box when it is in lowest energy state.

Solution:

$L = 1$ Å $= 1\times10^{-10}$ m

We have, wave function for the particle in infinite potential well

$$\Psi(x) = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}$$

At the centre, $x = \frac{L}{2} = 0.5$ Å and for lowest energy state, n = 1.

Then,

$$\text{Probability} = \left(\sqrt{\frac{2}{L}}\sin\frac{\pi x}{L}\right)^2 = \left|\sqrt{\frac{2}{L}}\sin\frac{\pi}{2}\right|^2 = \frac{2}{L}$$

The probability in the interval Δx is, $P = |\Psi_1(x)|^2 \Delta x = \left(\frac{2}{L}\right)\Delta x = \frac{2 \times 0.5 \times 10^{-10}}{1 \times 10^{-10}\,m} = 1$

THEORETICAL ANSWER QUESTIONS

1. What is wave function? Describe it's significance. Derive Schrodinger time dependent wave equation for a free particle like electron.
2. Determine the total energy of a particle using Schrodinger equation, when the potential energy has value V = 0 for 0 < x < a and V = ∞ for x ≤ 0 and x ≥ a.
3. Prove that energy levels are quantized, when the electron is confined in an infinite potential well of width 'a'.
5. Derive an expression for the energy of a particle in an one dimensional infinite deep potential well.
6. What do you mean by operator in quantum mechanics? Explain energy and momentum operator.
7. An electron is trapped in an one dimensional infinite potential well of width 'a' such that.
   V = ∞ for x ≤ 0 and x ≥ a; V = 0 for 0 < x < a

   Using boundary condition, prove that the total potential energy of system is $E = \frac{n^2\pi^2\hbar^2}{2mr^2}$. where symbols carry their usual meaning.
8. Discuss the significance of the wave function and deduce the time independent schrodinger equation.
9. A particle is confined in a box of width L. Find an expression for energy eigen value to show that the particle can have only discrete energy and momentum.
10. Define tunneling effect and derive the expression for reflection and transmission coefficient for a barrier of width 'a' and potential of height V_0.
11. Why quantum mechanics is superior to the classical mechanics? Explain.

NUMERICAL PROBLEMS

1. Calculate the de-Broglie wavelength of neutron of energy 28.8eV, h = 6.62×10^{-34} Joule-sec, m = 1.67×10^{-27} kg. [Ans: 5.34×10^{-12} m]
2. Calculate the energy in electron volt of an electron wave of $\lambda = 3\times10^{-2}$ m, Given h = 6.62×10^{-34} JS. [Ans: 1.68×10^{-13} eV]
3. Calculate the de Broglie wave length of an α - particle accelerated through a potential difference of 2000 volts. Given mass of proton = 1.67×10^{-27} kg; Plancks constant = 6.62×10^{-34} Joule-sec. [Ans: 6.41×10^{-13} m]
4. Calculate the energy difference between the ground state (n = 1) and the first excited state for an electron in one dimensional rigid box of length 1 Å. [Ans: 1.808×10^{-17} J]
5. Calculate the velocity and kinetic energy of an electron of wavelength 1.66 Å. [Ans: 4.39×10^6 m/s, 54.7 eV]
6. Calculate the velocity and kinetic energy of a neutron having de-broglie wavelength of 1 Å. [Ans: 3.97×10^3 m/s; 0.0825 eV]
7. An electron has a wave length 0.2 nm. What are the momentum and energy of electron. [Ans: 7.36×10^{-25} kgm/s; 2.976×10^{-18} J]

Find the minimum energy of an electron in a box of width 2Å. [Ans: 7.36×10^{-23} kgm/s]

The energy of an electron constrained to move in one dimensional box of length 4.0 Å is 9.664×10^{-17} J. Find out the order of excited state and the momentum of electron in that state. [Ans: 16; 1.32×10^{-23} kgm/s]

□□□

8 Chapter

FUNDAMENTALS OF THERMODYNAMICS AND HEAT TRANSFER

CHAPTER OUTLINE

After completion of this chapter, you will be able to know:

- Concepts and definition: applications of thermodynamics, properties and state of substance, thermodynamics properties and types, processes (definition, characteristics and examples): reversible and irreversible process.
- Laws of thermodynamics: first law of thermodynamics, first law for closed system, internal and stored energy, Joules law, enthalpy, specific heat, application of first law for closed system, Related problems on closed system, second law of thermodynamics, heat engine (four components of refrigerator and heat pump, COP of refrigerator and heat pumps), Kelvin-Planck and Clausius statement of second law.
- Heat transfer: modes of heat transfer (conduction, convection and radiation), statement and assumption of Fourier's law of thermal conductivity, one dimensional steady state heat conduction through plane wall, basic laws of radiation (Emissive power and emissivity, Stefan-Boltzmann's law), Concept of black bodies.

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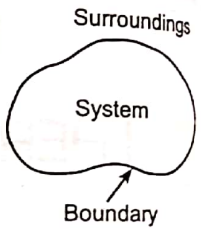
(A) CONCEPT AND DEFINITION

INTRODUCTION

The term thermodynamics is derived from the Greek words therme (heat) and dynamis (power). Thermodynamics is a branch of science that studies the relationship between heat and other types of energy. The same name is used to refer to all aspects of energy and energy transformations, including power production, refrigeration, and interactions between the properties of matter. A variety of topics of engineering and science are dependent on the concept of thermodynamics. Since energy and matter interact in all natural processes, it is difficult to think of a field that does not somehow incorporate thermodynamics. As a result, gaining a solid understanding of thermodynamics' fundamental concepts has long been a crucial component of engineering education. Thermodynamics play an important role in the design and analysis of automotive engines, rockets, jet engines, conventional or nuclear power plants, solar collectors, water, crude oil, and natural gas transportation, city water distribution systems, and the design of vehicles ranging from ordinary cars to airplanes. The energy-efficient home is built with the goal of minimizing heat loss in the winter and heat gain in the summer. After a thermodynamic, heat transfer, and fluid flow analysis of the computer, the size, location, and power input of the fan are also selected.

A **system** is defined as a quantity of matter or a region in space that has been chosen for investigation. The surroundings are the mass or regions that exist outside of the system. The boundary is the real or imaginary surface that separates the system from its surroundings, as shown in figure. The boundary of a system can be fixed or movable. The boundary is the contact surface shared by both the system and the surroundings. Mathematically, the boundary has zero thickness, and thus it can neither contain any mass nor occupy any volume in space.



Types of thermodynamic system

On the basis of mass and energy transfer the thermodynamic system is divided into three types.

i. **Closed system or control mass:** A system in which the energy transfer can take place but mass cannot across the boundary is called a closed system. The mass inside the closed system remains constant. This system is also known as control mass. Piston-Cylinder device

For example: Boiling of water in a closed vessel. Since the water is boiled in closed vessel so the mass of water cannot escapes out of the boundary of the system but heat energy continuously entering and leaving the boundary of the vessel.

ii. **Open system or control volume:** A system in which the transfer of both mass and energy takes place is called an open system. This system is also known as control volume. Air compressor

For example: Boiling of water in an open vessel is an example of open system because the water and heat energy both enters and leaves the boundary of the vessel.

iii. **Isolated system:** A system in which the transfer of mass and energy cannot takes place is called an isolated system. Closed rigid insulated container

For example: Tea kept in a thermos flask. In this the heat and the mass of the tea cannot cross the boundary of the thermos flask. Hence the thermos flak is an isolated system.

Macroscopic and Microscopic Points Of View

Thermodynamic studies are undertaken by the following two different approaches.

Macroscopic approach	Microscopic approach
1. In this approach a certain quantity of matter is considered without taking into account the events occurring at molecular level. This approach in thermodynamics is concerned only with overall behavior.	1. The approach considers that the system is made up of a very large number of discrete particles known as molecules. These molecules have different velocities are energies. The effect of molecular motion is considered.
2. This approach is used in the study of classical thermodynamics.	2. This approach is used in the study of statistical thermodynamics.
3. A few properties are required to describe the system.	3. Large numbers of variables are required to describe the system.
4. The properties like pressure, temperature, etc. needed to describe the system, can be easily measured.	4. The properties like velocity, momentum, kinetic energy, etc. needed to describe the system, cannot be measured easily.

THERMODYNAMIC PROPERTY

A thermodynamic property is any characteristic or variable of a thermodynamic system. Pressure P, temperature T, volume V, and mass m are all well-known properties. Internal energy, total energy enthalpy, entropy, viscosity, thermal conductivity, modulus of elasticity, thermal expansion coefficient, electric resistivity, and even velocity and elevation can be added to the list.

Thermodynamic properties are generally classified into two groups;

i. **Intensive properties:** The properties which are independent of the mass of the system. For example: Temperature, pressure and density are the intensive properties.

ii. **Extensive properties:** The properties which depend on the size or extent of the system are called extensive properties. For example: Total mass, total volume and total momentum.

Generally, uppercase letters are used to denote extensive properties (with mass m being a major exception), and lowercase letters are used for intensive properties (with pressure P and temperature T being the obvious exceptions). Extensive properties per unit mass are called specific properties. Some examples of specific properties are specific volume (v=V/m) and specific total energy (e=E/m).

THERMODYNAMIC STATE AND THERMODYNAMIC EQUILIBRIUM

The term state refers to a system's condition as described by its properties such as pressure, volume, temperature, mass, and so on. All of a system's properties have fixed values at any given state. If the value of even one property changes, the state will be changed. As a result, each distinct condition of a system is known as a state. The state of the system is the condition of the system because all of the properties can be measured or calculated across the entire system, which gives us a set of properties that completely describes the condition.

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On the basis of the above discussion we can determine if a given variable is a property or not by applying the following tests:

(a) A variable is a property if, and only if, it has a single value at each equilibrium state.

(b) A variable is a property if, and only if, the change in its value between any two prescribed equilibrium states is single valued.

Thermodynamics deals with equilibrium states. The word equilibrium implies a state of balance. In an equilibrium state there are no unbalanced potentials (or driving forces) within a system. For a system, the properties describing the state will be constant if the system is not allowed to interact with the surroundings or it the system is allowed to interact completely with unchanging surroundings. Such a state is termed an equilibrium state and the properties are equilibrium properties. A system will be in a state of thermodynamic equilibrium, if the condition for the following three types of equilibrium are satisfied

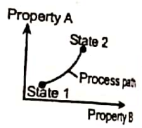
a) **Mechanical equilibrium:** A system is in mechanical equilibrium if there is no change in pressure at any point of the system with time.

b) **Thermal equilibrium:** A system is in thermal equilibrium if the temperature is the same throughout the entire system.

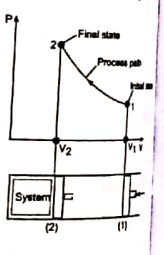
c) **Chemical equilibrium:** A system is in chemical equilibrium if its chemical composition does not change with time, that is, no chemical reactions occur.

THERMODYNAMIC PROCESS: (REVERSIBLE AND IRREVERSIBLE PROCESS)

A process is any change that a system undergoes from one equilibrium state to another, and the series of states through which a system passes during a process is called the path of the process.



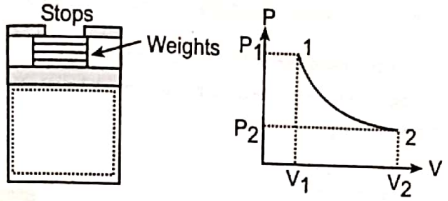
When a process proceeds in such a manner that the system remains very close to an equilibrium state at all times, it is called a quasi-static, or quasi- equilibrium, process. A quasi-equilibrium process can be viewed as a sufficiently slow process and also called **reversible process.** It should be pointed out that a quasi-equilibrium process is an idealized process and is not a true representation of an actual process. But many actual processes closely approximate it. It is important because:



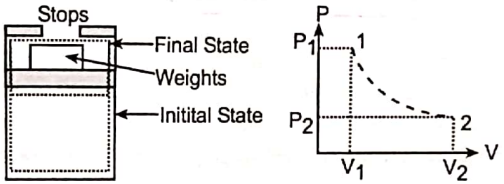
a) they are easy to analyze .

b) serves as standards to which actual process can be compared.

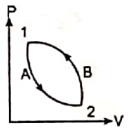
Let us consider a system of gas contained in a cylinder. The system initially is in equilibrium state, represented by the properties P_1, V_1, T_1. The weight on the piston just balances the upward force exerted by the gas. If the weights are removed one by one very slowly from the top of the piston, at any instant of the upward travel of the piston, considering gas system is isolated, every state passed through by the system will be an equilibrium state. Such a process, which is locus of all the equilibrium points passed through by the system, is known as quasi-static or quasi-equilibrium process. A quasi-static process is has a succession of equilibrium states and represented by a continuous line.



If the weight removed suddenly , there will be an unbalanced force between the system and the surroundings, and under gas pressure, the piston will move up till it hits the stops. The system again comes to an equilibrium state, being described by the properties P_2, V_2, T_2. Thus, the intermediate states passed through by the system are non equilibrium states which cannot be described by thermodynamic properties. such process is called non quasi-equilibrium process or irreversible process denoted by a dashed line between initial and final states.



When a process or processes are performed on a system in such a way that the final state is identical with the initial state, it is the known as a thermodynamic cycle or cyclic process. In Figure, 1 – A – 2 and 2 – B – 1 are processes whereas 1 – A – 2 – B – 1 is a thermodynamic cycle.



Note

The prefix iso- is often used to designate a process for which a particular property remains constant. An isothermal process is a process during which the temperature T remains constant; an isobaric process is a process during which the pressure P remains constant; and an isochoric (or isometric) process is a process during which the specific volume v remains constant.

SOME COMMON PROPERTIES

1) Specific Volume

A substance's specific volume (v) is defined as its volume per unit mass and is measured in m^3/kg. Thus, mathematical definition of specific volume is $v = V/m$

Hence, specific volume is the reciprocal of the density, i.e., $v=1/\rho$

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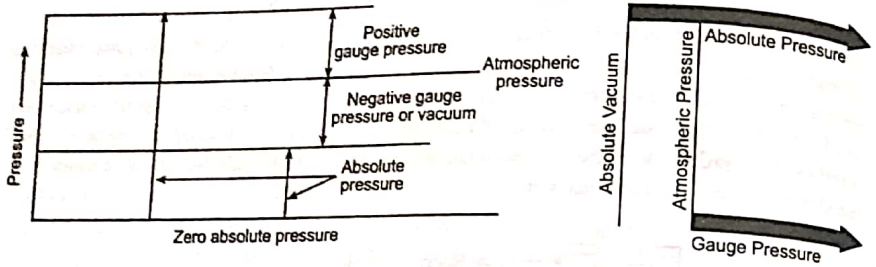
2. Pressure

It is defined as a force per unit area. It is denoted by P. The fundamental SI unit of pressure is N/m^2 or pascal (Pa). Sometimes pressure is also expressed in bar or atm 1 bar= 10^5 N/m^2 = 10^5 Pa. 1atm= 1.01325 bar = 0.76 m (or 760 mm) of Hg.



There are three pressure terms in thermodynamics atmospheric pressure(P_{atm}), gauge pressure(P_{gauge}) and absolute pressure(P_{abs}). The actual pressure at a given position is called the absolute pressure, and it is measured relative to absolute vacuum (i.e., absolute zero pressure). The difference between absolute pressure and local atmospheric pressure is called gauge pressure. Gauge pressure may be positive or negative.

$$P_{abs} = P_{atm} + P_{gauge}$$

$$P_{vac} = P_{atm} - P_{abs}$$

Vacuum is defined as the absence of pressure. A perfect vacuum is obtained when absolute pressure is zero, at this instant molecular momentum is zero. A barometer is a scientific instrument used to measure atmospheric pressure, also called barometric pressure. Barometer gives atmospheric pressure in terms of height of the mercury column h_{baro} as,

$$P_{atm} = \rho_{Hg} g h_{baro}$$

where ρ_{Hg} is the density of mercury and g is acceleration due to gravity.

Where as manometer gives the gauge pressure of a system in terms of height of certain manometric fluid h_{mn} as,

$$P_{gauge} = \rho_{mn} g h_{mn}$$

where ρ_{mn} is the density of manometric fluid and g is acceleration due to gravity.

3. Temperature

Temperature is the thermodynamic magnitude that shows the thermal energy of one body to that of another. This thermodynamic property only describes a macroscopic state, which determines a body's degree of hotness or level of heat intensity. If a body has a high level of heat intensity, it is said to be at a high temperature or hot. Similarly, a body is said to be at a low temperature or cold if its heat intensity is low. The instrument used to measure temperature is known as a thermometer. The three most common scales for measuring temperature are celcius (°C), farenheit (°F) and Kelvin (K).

All temperature scales are based on some easily reproducible states such as the freezing and boiling point of water. The freezing point and boiling point of water in three different scale are $\frac{0°C}{32}$ $\frac{°F}{273K}$ and $\frac{100°C}{212}$ $\frac{°F}{373K}$ respectively.

These three scale can be related as

$$\frac{C-0}{100-0} = \frac{F-32}{212-32} = \frac{K-273}{373-273}$$

ENERGY

Energy is a general term embracing energy in stored energy and transition energy.

1. **Stored Energy:** The stored energy of a substance may be in the forms of mechanical energy and internal energy.

   i. **Internal Energy:** The molecular internal energy or simple internal energy is the sum of all the microscopic forms of energy of a system. The microscopic energy mode refers to the energy stored in the molecular and atomic structure of the system.
   For an ideal gas, there are no intermolecular forces of attraction and repulsion, and the internal energy depends only on temperature.
   i.e., U= f(T) This is Joule's law.

   ii. **Potential Energy:** Potential energy is the energy of the system due to its position. Potential energy is classified into two types in thermodynamics: elastic potential energy and gravitational potential energy. The energy stored in a deformable body, such as an elastic solid or a spring, is referred to as elastic potential energy. Gravitational potential energy is the energy that a system possesses due to its elevation in a gravitational field with respect to a reference.
   Gravitational potential energy (PE) is given by the relation

   $$PE = mgh$$

   where, m is the mass of the system (kg), g is acceleration due to gravity (m/s^2), and h is the elevation of the system. It is due to macroscopic phenomena.

   iii. **Kinetic Energy:** Kinetic energy is the energy possessed by a system as a result of its motion. Kinetic energy (KE) is given by the relation $KE = \frac{1}{2} m\bar{V}^2$ where, m is the mass of the system (kg), $\bar{V}$ is velocity of the system (m/s).It is also due to macroscopic phenomena.

   Total Energy: The total energy of a system is the sum of the potential, kinetic, and internal energies. Thus, the total energy (E) is expressed as

   $$E = U + PE + KE$$

   $$E = U + mgh + \frac{1}{2} m\bar{V}^2$$

   The specific total energy is the ratio of total energy and mass

   That is, $e = \frac{E}{m} = u + gh + \frac{1}{2}\bar{V}^2$

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2. **Transition energy:** Heat and work are the forms of energy in transition. These are the only forms in which energy can cross the boundaries of a system. Neither heat nor work can exist as stored energy.

i. **Work:** Work is a transient quantity that appears at the boundary when a system changes its state due to the movement of a part of the boundary under the action of a force.

Figure (a) illustrates this with the traditional piston and cylinder arrangement, with the heavy line defining the system's boundary.

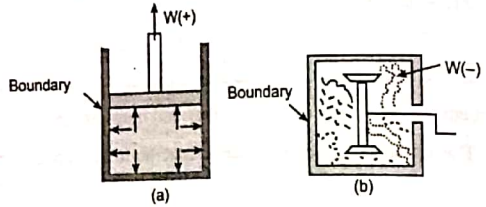


Figure (b) illustrates yet another way that work can be applied to a system. A force is exerted by the paddle as it changes the momentum of the fluid, and since this force moves during rotation of the paddle work is done.

If the work is done by the system on the surroundings, e.g., when a fluid expands pushing a piston outwards, the work is said to be positive.

i.e., Work output of the system = + W

If the work is done on the system by the surroundings, e.g., when a force is applied to a rotating handle, or to a piston to compress a fluid, the work is said to be negative.

i.e., Work input to system = – W

iii. **Heat:** Heat is a transient quantity which appears at the boundary when a system changes its state due to a difference in temperature between the system and its surrounding. If the heat flows into a system from the surroundings, the quantity is said to be positive

i.e. Heat received by the system = + Q.

and if heat flows from the system to the surroundings it is said to be negative.

i.e. Heat rejected or given up by the system = – Q.

Zeroth Law of Thermodynamics

The zeroth law of thermodynamics states when two systems are at the same time in thermal equilibrium with a third system, they are in equilibrium with each other. If an object with a higher temperature comes in contact with an object of lower temperature, it will transfer heat to the lower temperature object. The objects will approach the same temperature and in the absence of loss to other objects, they will maintain a single constant temperature. Therefore, thermal equilibrium is attained.

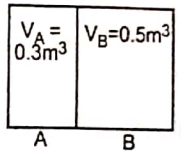
A	B	C

If objects 'A' and 'C' are in thermal equilibrium with 'B', then object 'A' is in thermal equilibrium with object 'C'. Practically this means all three objects are at the same temperature and it forms the basis for comparison of temperatures.

Solved Example

Example 1: A container has two compartments A and B having steam with specific volume $7m^3/kg$ and $8m^3/kg$ respectively. The volume of the compartment A and B are 0.3 m^3 and 0.5 m^3. If the membrane breaks and steam comes to uniform state, find the resulting specific volume.

Solution:



Volume of compartment A (V_A) = $0.3m^3$

Volume of compartment B(V_B) = $0.5m^3$

Specific volume of compartment A (V_A) = $7m^3/kg$

Specific volume of compartment B(V_B) = $8m^3/kg$

Then, mass of compartment A(m_A) = $\frac{V_A}{V_A} = \frac{0.3}{7} = 0.428$ kg

Mass of compartment B(m_B) = $\frac{V_B}{V_B} = \frac{0.5}{8} = 0.0625$ kg

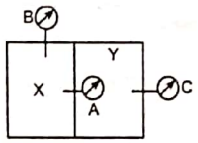
For final state,

Total volume (V) = $V_A + V_B = 0.3 + 0.5 = 0.8\ m^3$

Total mass (m) = $m_A + m_B = 0.0428 + 0.0625 = 0.153$ kg

∴ Resulting specific volume (V) = $\frac{V}{m} = \frac{0.8}{0.1053} = 7.597\ m^3/kg$

Example 2: A large chamber is separated into two compartments which contains air and are maintained different pressure as shown in figure. Pressure gauge A reads 180 kPa and pressure gauge B reads 120 kPa. If the barometric pressure is 100 kPa, determine the absolute existing in the compartments and the reading of gauge C.



Solution:

Atmospheric pressure (P_{atm}) = 100 kPA

Gauge pressure for gauge (P_A) = 180 kPA

Gauge pressure for gauge B(P_B) = 120 kPa

Here,

Absolute pressure of compartment X $(P_{abs})_X = P_{atm} + P_B$

$(P_{abs})_X = 100 + 120 = 220$ kPa

The gauge pressure between the compartments is the difference between absolute pressure

So, $(P_{abs})_X - (P_{abs})_Y = P_A$

$220 - (P_{abs})_Y = 180$

$(P_{abs})_Y = 220 - 180 = 40$ kPa

For gauge pressure C $(P_{abs})_Y = P_{atm} + P_C \Rightarrow P_C = 40 - 100 = 60$ kPa

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Example 3: A pressure gauge connected to a cylinder reads 400 kPa at a location where the atmospheric pressure is 100kPa. Determine the absolute pressure in the cylinder.

Solution:

Gauge pressure (P_{gauge}) = 400 kPa

Atmospheric pressure (P_{atm}) = 100 kPa

Then the absolute pressure in the cylinder is given by

$$P_{abs} = P_{atm} + P_{gauge}$$

$$= 10 + 400 = 500 \text{ kPa}$$

Example 4: A vessel of cylindrical shape having 50 cm in diameter and 75 cm high. It contains 4 kg of a gas. The pressure measured with manometer indicates 62 mm of Hg above atmosphere when barometer reads 760 mm of Hg. Find

(i) the absolute pressure of the gas in the vessel in bar

(ii) Specific volume and density of the gas.

Solution:

Diameter of the vessel d = 50 cm = 0.5 m

Height of the vessel (ℓ) = 75 cm = 0.75 m

Mass of gas (m) = 4 kg

Nanometer reading = 620 mm of Hg

Barometer reading = 760 mm of Hg

Now volume of the vessel $= \frac{\pi d^2}{4} \times \ell = \frac{\pi}{4} \times (0.5)^2 \times (0.75) = 0.147 \text{ m}^3$

(i) Total pressure in the vessel = 760 + 620 = 1380 mm of Hg

$= 1380 \times 133.4 \text{ N/m}^2$ [$\because$ 1 mm of Hg = 133.4 N/m^2 As P = ρgh]

$= 1.841 \times 10^5 \text{ N/m}^2$

$= 1.841 \times$ bar [$\because$ 1 bar = 10^5 N/m^2]

(ii) Specific volume (V) $= \frac{V}{m} = \frac{0.147}{4}$

$= 0.03675 \text{ m}^3/\text{kg}$

Density (ρ) $= \frac{4}{0.147}$

$= 27.21 \text{ kg/m}^3$

Example 5: A piston cylinder has a diameter of 0.15 m with an outside atmospheric pressure of 100 kPa, determine the piston mass that will create an inside pressure of 200 kPa. (Take g = 9.79 m/s^2)

Solution:

Diameter of the piston (d_P) = 0.15 m

Atmospheric pressure (P_{atm}) = 100 kPa

Absolute pressure of gas (P_{gas}) = 200 kPa

Mass of piston (m_p) = ?

Now at equilibrium,

$$P_{gas} \times A_P = P_{atm} \times A_P + w$$

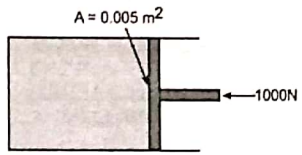
$$P_{gas} = P_{atm} + \frac{m_P g}{\frac{\pi d_P^2}{4}}$$

or, $$200 \times 10^3 = 100 \times 10^3 + \frac{m_P \times 9.79 \times 4}{\pi \times (0.15)^2}$$

or, $$m_P = \frac{100 \times 10^3 \times \pi \times (0.15)^2}{9.79 \times 4}$$

$$= 180.414 \text{ kg}$$

Example 6: For the piston cylinder device shown in figure below determine the absolute pressure inside the device (Take P_{atm} = 100 kPa).



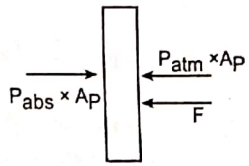
Solution:

Atmospheric pressure (P_{atm}) = 100 kPa

Area of piston (A_P) = 0.005 m^2

Force applied to piston (F) = 1000N

At equilibrium



$$P_{abs} \times A_P = P_{atm} \times A_P + F$$

$$\therefore \quad P_{abs} = P_{atm} + \frac{F}{A_P}$$

$$= 100 \times 10^3 + \frac{1000}{0.005}$$

$$= 100 \times 10^3 + 200 \times 10^3$$

$$= 300 \times 10^3$$

$$= 300 \text{ kPa}$$

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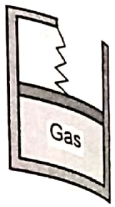
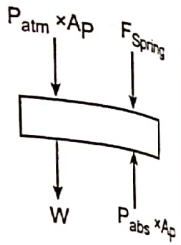
Example 7: A piston cylinder device loaded with a linear spring with a spring with a spring constant of k = 100 kN/m contains a gas initially at a pressure of 100 kPa and a volume of 0.05 m^3 as shown in figure. The cross-sectional area of piston is 0.01m^2. Initially spring touches the piston but exerts no force on it. Heat is supplied to the system until its volume doubles, determine the final pressure.



Solution:



Spring constant (K) = 100 kNm = 100 × 10^3N/m

Initial pressure of gas (P_1) = 100 kPa

Initial volume of gas (V_1) = 0.05m^3

Area of piston (A_p) = 0.01 m^2

Final volume of gas (V_2) = $2V_1$ = 2 × 0.05 = 0.1 m^3

Final pressure of gas (P_2) = ?

At equilibrium

$$Pass = P_{atm} + \frac{W}{A_P} + \frac{F_{spring}}{A_P}$$

Initially spring touches the piston so there is no force on spring.

So, $$P_1 = P_{atm} + \frac{W}{A_p} = 100 \text{ kPa} \quad \ldots (i)$$

Then, length of the gas loaded chamber $X_1 = \frac{V_1}{A_P} = \frac{0.05}{01} = 0.5$ m

When the volume doubleds

$$X_2 = \frac{V_2}{A_P} = \frac{2 \times 0.05}{0.1} = 1\text{m}$$

∴ Change in displacement of piston (X) = $X_2 - X_1$ = 1 – 0.5 = 0.56 m

So,

$$P_2 = P_{atm} + \frac{W}{A_P} + \frac{F_{Spring}}{A_P} = P_1 + \frac{F_{Spring}}{A_P}$$

$$= 100 \times 10^3 + \frac{kx}{A_P}$$

$$= 100 \times 10^3 + \frac{100 \times 0.5 \times 10^3}{0.1}$$

= 600 kPa.

Example 8: A new scale N of temperature is deviced in such a way what the freezing point of water is 20 N and boiling point is 200° N, what will be the temp. reading on °C, °F and K scales when the temperature on the new scale is 160 N.

Solution:

For new scale

Freezing point = 20 N

Boiling Point = 20 N and N = 160° N

We have,

$$\frac{N-20}{200-20} = \frac{C-O}{100-0} = \frac{F-32}{212-32} = \frac{K-273}{373-273}$$

or, $$\frac{N-20}{180} = \frac{C}{100} = \frac{F-32}{180} = \frac{K-273}{100}$$

then,

$$\frac{N-20}{180} = \frac{C}{100}$$

or, $$\frac{160-20}{180} = \frac{C}{100}$$

or, $$C = \frac{140 \times 100}{180} = 77.7° \text{ C}$$

Also, $$\frac{N-20}{180} = \frac{F-32}{180}$$

or, $$F = N - 20 + 32$$
$$= 160 - 20 + 32$$
$$= 172° \text{ F}$$

Again,

$$\frac{N-20}{180} = \frac{K-273}{100}$$

or, $$\frac{160-20}{180} = \frac{K-273}{100}$$

or, $$K = \frac{140 \times 100}{180} + 273$$
$$= 350.78 \text{ k}$$

THEORETICAL ANSWER QUESTIONS

1. Explain the different types of systems with suitable examples.
2. Define state, property and cycle. Explain the concept of quasi-equilibrium process.
3. Explain how different forms of energy are classified in thermodynamics.
4. Differentiate between macroscopic and microscopic approach in thermodynamics.
5. Define a thermodynamic property. Explain intensive, extensive and specific property with examples.
6. Define thermodynamic equilibrium. Explain the condition required for thermodynamic equilibrium.
7. Explain reversible and irreversible processes on the basis of intermediate state.

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8. Define atmospheric pressure, gauge pressure and absolute pressure. Also write the relationship between them.
9. Define the temperature, Name the different temperature scales in common use and establish a relation between Celsius and Fahrenheit scale.
10. What do you understand by transition energy? Explain the forms of transition energy with sign convention.

NUMERICAL PROBLEMS

1. A vacuum gauge indicates the pressure of air in a cylinder is 15 kPa (vaccum). The local barometer reads 760 mm of Hg. Determine the absolute pressure inside the cylinder take ρ_{Hg} = 13600 kg/m^3 and g = 9.79 m/s^2. [Ans: 86.189 kPa]
2. At the inlet and exhaust of a turbine the absolute steam pressure is 6000 kPa and 6 kPa respectively. Barometric pressure is 76 cm of Hg. Calculate the gauge pressure for entering steam and exhaust steam. (Take ρ_{Hg} = 13600 kg/m^3 and g = 9.79 m/s^2). [Ans: 5898.811 kPa, – 5.189 kPa]
3. The absolute pressure inside a tank is 50 kPa and the surrounding is 100 kPa. What will be the reading on gauge mounted in the tank. [Ans: – 50 kPa]
4. During the operation of the lift, it can be subjected to a maximum pressure of 500 kPa. If it is designed to lift a mass upto 900 kg what should be the diameter of the piston/cylinder. [Ans: 0.15m]
5. A gas is contained in a vertical frictionless piston cylinder. The piston has a mass of 5 kg and cross-sectional of 30 cm^2. A compressed spring above the piston exerts a force of 100N on the piston. If the atmospheric pressure is 100 kPa. Find the pressure inside the cylinder. [Ans: 149.647 kPa]
6. The temperature of a system drops by 54 F during a process. Express this drop in temperature in °C and K. [Ans: 30° C and 30 K]
7. A container having two compartments A and B contains steam. The volume of the compartments are 15m^3 and 5m^3 respectively. If the specific volume of steam is compartment B is 5m^3/kg. The membrane breaks and the resulting specific volume is 8m^3/kg. Find the original specific volume of steam is compartment A. [Ans: 10 m^3/kg]
8. A cylinder with a total volume of 1m^3 has a movable piston. When the piston is at one fourth of the length both sides have same specific volume of 4m^3/kg. Find the specific volumes of both sides when the piston is at middle of the cylinder. [Ans: 8m^3/kg, 2.667 m^3/kg]
9. An oxygen having a volume of 10m^3 initially contains 5 kg of oxygen. Determine the specific volume of oxygen in the cylinder initially. During certain process 3 kg of oxygen is consumed, determine the final specific volume of oxygen in the cylinder. [Ans: 2m^3/kg, 5m^3/kg]

□□□

(B) LAWS OF THERMODYNAMICS

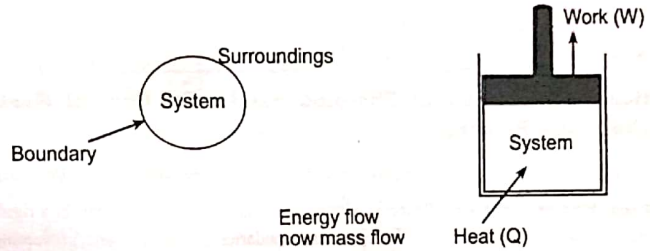
INTRODUCITON

The first law of thermodynamics is based on the principles of conservation. It simply states that *"Energy can be neither created nor destroyed. It can be transferred from object to object and changed into different forms, but the total energy before and after any process is always the same."* It expresses mathematically the effect of interactions between the system and its surroundings on the system's stored energy. Interactions in a closed system (control mass) can only take the form of energy transfer (work and heat transfer). As a result, the first law of thermodynamics for a control mass can be explained only in terms of energy conservation.

In an open system (control volume), interactions can occur through both mass and energy transfer. As a result, the first law of thermodynamics for a control volume is explained using both mass conservation and energy conservation principles.

FIRST LAW OF THERMODYNAMICS FOR CONTROL MASS (CLOSED SYSTEM/NON FLOW PROCESS)

Let us consider a closed system with heat input and work output as shown in figure.



Using Conservation of Energy Equation,

$$\text{Energy entering into the system} - \text{Energy leaving the system} = \text{Change in total energy of the system}$$

or, $E_{in} - E_{out} = \Delta E_s$

$$Q - W = \Delta E_{cm} \qquad \ldots (i)$$

Where, total energy of control mass (ΔE_{cm}) is algebraic sum of Kinetic Energy (KE), Potential Energy (PE) and Internal Energy of the system.

Mathematically,

$$E_{cm} = KE + PE + U$$

$$\Delta E_{cm} = \Delta KE + \Delta PE + \Delta U$$

In a closed system, change in KE and change in PE are negligible as compared to change in U of the system.

Therefore, $\Delta E_{cm} = \Delta U$

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So, from equation (i), we get,

$$Q - W = \Delta U$$

Therefore, $dQ - dW = dU$

Case I : If a closed system undergoes cyclic process

Conservation of Energy Equation can be written as,

$$\oint(dQ - dW) = \oint dU$$

For a cyclic process, $\oint dU = 0$

Hence, Conservation of Energy Equation for a cyclic process becomes

$$\oint(dQ - dW) = 0$$

$$\oint dQ = \oint dW$$

Hence, $Q_{net} = W_{net}$

Hence, the first law of thermodynamics for a cycle states that:

"During any cycle a system (control mass) undergoes, the cyclic integral of the heat is proportional to the cyclic integral of work."

Case II : For any thermodynamic process 1-2,

In a closed system, the conservation of energy equation becomes

$$Q_{12} - W_{12} = U_2 - U_1$$

$$Q_{12} = W_{12} + U_2 - U_1$$

Applications of First Law of Thermodynamics for Control Mass (Closed System/Non Flow Process)

We use $Q_{12} = W_{12} + U_2 - U_1$ to derive the expression for different processes on a control mass.

1) **Constant Volume (Isochoric) Process:** The working substance is contained in a rigid vessel in a constant volume process, making the system's boundaries immovable and preventing any work from being done on or by the system.

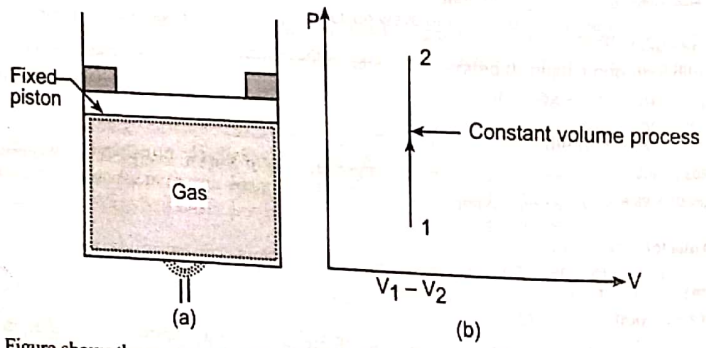


Figure shows the system and states before and after the heat addition at constant volume.

Considering mass of the working substance unity and applying first law of thermodynamics to the process

$$Q_{12} = W_{12} + U_2 - U_1$$

The work done

$$W_{12} = \int_1^2 P\,dV = 0 \text{ as } dV = 0$$

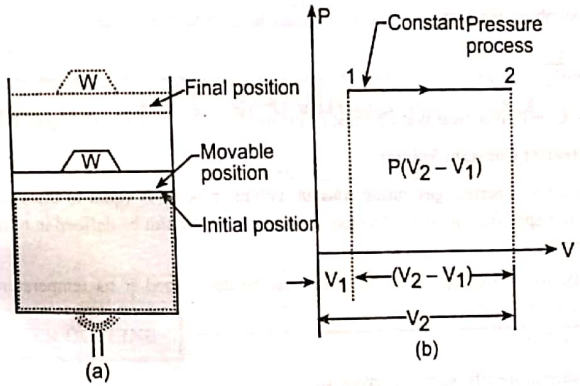
Then, $Q_{12} = U_2 - U_1 = (\Delta U)_{12}$

Hence, heat transferred to a control mass during a constant volume process is equal to the increase in internal energy of the system.

2. **Constant Pressure (Isobaric) Process:** When heat is applied, the boundary must move against an external resistance to undergo a constant pressure process; for example, a gas in a cylinder behind a piston can be made to go through such a process.

Since the piston is pushed through a certain distance by the force exerted by the gas, then the work is done by the gas on its surroundings.

Figure illustrates the system and its states prior to and following the addition of heat under a constant pressure.



Considering unit mass of working substance and applying first law of thermodynamics to the process

$$Q_{12} = W_{12} + U_2 - U_1$$

The wordone $W_{12} = \int_1^2 P\,dV = P(V_2 - V_1)$ where $P_1 = P_2 = P$

Then,

$$Q_{12} = U_2 - U_1 + PV_2 - PV_1$$

$$= U_2 + PV_2 - (U_1 + PV_1)$$

$$Q_{12} = (U + PV)_2 - (U + PV)_1$$

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The expression U + PV occurs so frequently in thermodynamics is known as enthalpy and is denoted by H.

i.e. $H = U + PV$

The enthalpy is defined as the summation of internal energy and product of pressure and volume. It depends only on end states not on the path.

So, $Q_{12} = H_2 - H_1$

Hence, heat transferred to a control mass during a constant pressure process is equal to the increase in enthalpy of the system.

3. **Specific Heat:** Specific heat is defined as the amount of heat required per unit mass to raise the temperature of a substance by one degree.

i.e. $C = \frac{1}{m}\frac{dQ}{dT}$

C = specific heat, and dT = temperature rise.

For a gas, heat required to raise the temperature of unit mass by one degree is different for constant volume and constant pressure process. Therefore two specific heats are defined: specific heat at constant volume (C_V) and specific heat at constant pressure(C_P).

The ratio of specific heat at constant pressure to the specific heat at constant volume is given the symbol γ (gamma).

i.e., $\gamma = \frac{C_P}{C_V}$

Since $C_P = C_V + R$, it is clear that Cp must be greater than Cv for any perfect gas. It follows,

Specific Heat At Constant Volume

Heat supplied to a perfect gas during constant volume process is equal to the change in internal energy. Hence specific heat of an ideal gas at constant volume can be defined in terms of change in internal energy.

The specific internal energy of an ideal gas can be determined if its temperature and specific volume are given,

$$u = u(T, v)$$

Then change in specific energy is given as

$$du = \left(\frac{du}{dT}\right)_v dT + \left(\frac{du}{dv}\right)_T dv$$

For a constant volume process dv=0,

Then $du = \left(\frac{du}{dT}\right)_v dT$

$$du = C_v\, dT$$

where, $C_v \left(\frac{du}{dv}\right)_v$ called specific heat at constant volume and is defined as change in specific internal energy per degree change in temperature.

Specific Heat At Constant Pressure

Heat supplied to a perfect gas during constant volume process is equal to the change in enthapy of the system. Hence specific heat of an ideal gas at constant pressure can be defined in terms of change in enthalpy.

We have for specific enthalpy

$$h = h(T, P)$$

then change in specific enthalpy is given as,

$$dh = \left(\frac{dh}{dT}\right)_P dT + \left(\frac{dh}{dP}\right)_T dP$$

For a constant pressure process, dP = 0

then, $dh = \left(\frac{dh}{dT}\right)_P dT$

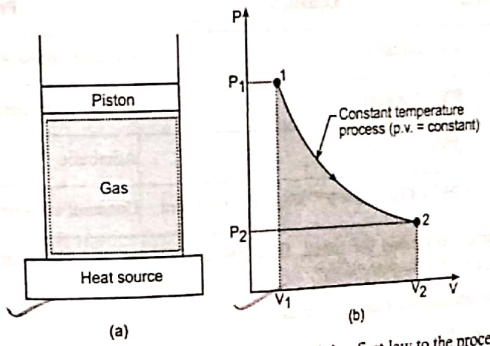
$$dh = C_P\, dT$$

where, $C_P = \left(\frac{dh}{dT}\right)_P$ is called specific heat at constant pressure and is defined as the change in specific enthalpy per degree change in temperature.

4) **Constant Temperature (Isothermal) Process**

An isothermal process is one that takes place at a constant temperature. The temperature tends to drop when a working material in a cylinder behind a piston expands from a high pressure to a low pressure. In an isothermal expansion, heat must be continuously added to maintain the initial temperature. Similar to this, heat must be continuously removed from the working substance during an isothermal compression.

Figure, illustrates the system and states before and after the heat addition at constant temperature.



Considering unit mass of working substance and applying first law to the process,

$$Q_{12} = W_{12} + U_2 - U_1$$
$$= W_{12} + C_V(T_2 - T_1)$$
$$= W_{12} + 0 \qquad \text{As, } T_1 = T_2$$

The work done

$$W_{12} = \int_1^2 P\,dV$$

In this case PV= constant or $P = \frac{C}{V}$ (where C is constant)

$$\text{then, } W_{12} = \int_{V_1}^{V_2} \frac{C}{V} dV = C(\ln V)_{V_1}^{V_2} = C[\ell n(V_2) - \ell n(V_1)] = C\ell n \frac{V_2}{V_1}$$

The constant C can either be written as P_1V_1 or as P_2V_2,

since $P_1V_1 = P_2V_2$ =constant, C

$$\text{then, } Q_{12} = W_{12} = P_1V_1\,\ell n \frac{V_2}{V_1}$$

$$\text{or, } \quad Q_{12} = W_{12} = P_2V_2\,\ell n \frac{V_2}{V_1}$$

Hence, heat transferred to a control mass for an ideal gas during a constant temperature process is equal to work done by the system.

5) **Polytropic Process**

Thermodynamic process which follows the relation PV^n = Constant is called a polytropic process and the index n is called a polytropic index. The index n depends only on the heat and work quantities during the process. The various processes are special cases of polytropic process for a perfect gas.

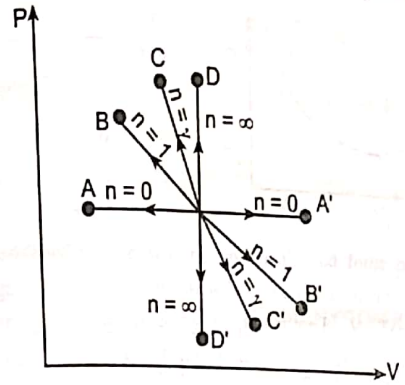
Value of index n	Equation	Process
0	P = Constant	Constant pressure or Isobaric
1	PV = Constant	Constant temperature or Isothermal
Γ	PV^{γ} = Constant	Adiabatic
∞	PV^{∞} = Constant = $P^{1/\infty}$ V i.e. V = Constant	Constant volume or Isochoric

Pressure volume relationship for initial, final and any intermediate state during a polytropic process is given as, $P_1V_1^n = P_2V_2^n = PV^n$

Pressure at any intermediate state is given as

$$P = \frac{P_1V_1^n}{V^n}$$

Figure, illustrates P-V diagram for polytopic process in perfect gas,



Considering unit mass of working substance and applying first law to the process,

$$Q_{12} = W_{12} + U_2 - U_1$$

The work done is $W_{12} = \int_1^2 P\,dV$

$$W_{12} = \int_{V_1}^{V_2} \frac{P_1V_1^n}{V^n} dV = P_1V_1^n \int_{V_1}^{V_2} \frac{dV}{V^n}$$

$$W_{12} = P_1V_1^n \int_{V_1}^{V_2} \frac{dV}{V^n}$$

$$W_{12} = P_1V_1^n \left[\frac{V^{-n+1}}{-n+1}\right]_{V_1}^{V_2}$$

$$W_{12} = \frac{P_1V_1^n}{1-n}(V_2^{1-n} - V_1^{1-n})$$

$$W_{12} = \frac{P_1V_1^nV_2^{1-n} - P_1V_1^nV_1^{1-n}}{1-n}$$

Substituting $P_1V_1^n = P_2V_2^n$

$$W_{12} = \frac{P_2V_2 - P_1V_1}{1-n} \quad \text{for } n \neq 1$$

Also $W_{12} = \frac{P_2V_2 - P_1V_1}{1-n} = \frac{mR(T_2 - T_1)}{1-n}$

and $U_2 - U_1 = (\Delta U)_{12} = mC_v(T_2 - T_1)$

then, $Q_{12} = W_{12} + U_2 - U_1$

$$Q_{12} = \frac{mR(T_2 - T_1)}{1-n} + mC_v(T_2 - T_1)$$

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$$Q_{12} = m\left[\frac{R}{1-n} + C_v\right](T_2 - T_1)$$

$$Q_{12} = mC_n\,(T_2 - T_1)$$

$$Q_{12} = mC_n\,(T_2 - T_1)$$

where, $C_n = \frac{R}{1-n} + C_v$;

C_n is the polytropic specific heat.

SECOND LAW OF THERMODYNAMICS

The first law of thermodynamics is similar to the law of conservation of energy adjusted for thermodynamic processes. It deals with the quantitative aspect of energy and provides that any process is possible as long as the total energy remains constant. But some processes in nature cannot occur although energy conservation principle is satisfied. So, in order to determine whether a process can take place or not, another basic principle (second law) is needed.

The second law of thermodynamics deals with the nature or quality of energy and defines the direction in which the system can proceed.

For example, consider a room is heating using an electric current passing through a resistor. According to the first law, the amount of electric energy supplied to the resistance wires must be equal to the amount of energy transferred to the room air as heat. Let us now try to reverse this process. It should come as no surprise that transferring some heat to the wires does not result in the generation of an equivalent amount of electric energy in the wires. Also, when a shaft rotates on a bearing heat is produced in the bearing due to the friction. But if we heat the bearings of a stationary shaft, it cannot rotate due to heating effect .Heat transfer from a hot container to the cold surroundings is possible but the reverse process is impossible.

Limitations of the First Law of Thermodynamics

- The first law of thermodynamics merely indicates that there is a transformation between the various forms of energy involved in the process, but it gives no information about the feasibility of such a transformation.
- The first law of thermodynamics doesn't provide us anything about the direction that processes will take, whether they are spontaneous or not.

Hence the main feature of the second law of thermodynamics is that it defines the directions of the processes. The second law of thermodynamics defines the process's direction in terms of a system property known as entropy. Entropy is the randomness or disorder of a thermodynamics system. It is an extensive property of a system that is used to measure the amount of energy that is unavailable for work. It is denoted by S.

The equation for the change in entropy, ΔS, is

$$\Delta S = \frac{Q}{T},$$

where Q is the heat that transfers energy during a process, and T is the absolute temperature at which the process takes place.

Total entropy change is equal to the sum of entropy change of system and surroundings.

Total entropy change, $\Delta S_{total} = \Delta S_{surroundings} + \Delta S_{system}$

If the system loses an amount of heat Q at a temperature T_1, which is received by surroundings at a temperature T_2.

So, ΔS_{total} can be calculated

$$\Delta S_{system} = -\frac{Q}{T_1}$$

$$\Delta S_{surrounding} = \frac{Q}{T_2}$$

$$\Delta S_{total} = -\frac{Q}{T_1} + \frac{Q}{T_2}$$

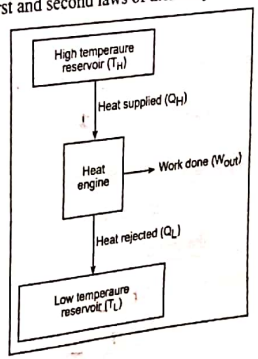
- If ΔS_{total} is positive, the process is spontaneous.
- If ΔS_{total} is negative, the process is non-spontaneous.
- If ΔS_{total} is zero, the process is at equilibrium.

THERMAL RESERVOIR

A thermal reservoir is a large system (very high mass x specific heat value) from which a quantity of energy can be absorbed or added as heat without changing its temperature. The atmosphere and sea are examples of thermal reservoirs. Any physical body whose thermal energy capacity is large relative to the amount of energy it supplies or absorbs can be modeled as a thermal reservoir. A reservoir that supplies energy in the form of heat is called a source and one that absorbs energy in the form of heat is called a sink.

HEAT ENGINE, HEAT PUMP AND REFRIGERATOR

Heat engines are devices that use a cyclic process to generate work where as heat pump and refrigerator are the devices used for heating or cooling of desired space. The first and second laws of thermodynamics can be used to analyze the performance of these devices.



Heat Engine

Heat engines operates on a cyclic process and receive heat from a high-temperature reservoir (source) convert part of this heat to work, and then reject the remaining part of heat to a low-temperature reservoir(sink) during the cycle.

Figure illustrates a schematic representation of heat engine which takes Q_H amount of heat from source at temperature, T_H converts some part into work W and rejects remaining part Q_L to sink at temperature at T_L.

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Performances of heat engine is measured by its efficiency, which is defined as the ratio of work output and the heat suppied.

i.e. $\eta = \frac{W}{Q_H}$

Applying first law of thermodynamics for a cyclic process,

$$\oint dQ = \oint dW$$

Therefore,

$$Q_H - Q_L = W$$

then, $\eta = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H}$

Efficiency for a reversible engine also known as carnot efficiency can also be expressed in terms of temperature as

$$\eta_{carnot} = \boxed{\eta_{rev} = 1 - \frac{T_L}{T_H}}$$

Heat Pump

Heat pump operates on a cyclic process which takes heat from a low-temperature reservoir (surrounding) at T_2 and delivers into the high-temperature reservoir(desired space) at T_1 with the help of external work. Heat pump maintains the temperature of desired space higher than that of surroundings.

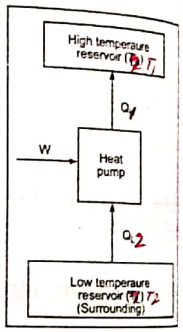

Figure illustrates a schematic representation of heat pump where Q_2 amount of heat takes from surrounding at T_2 and supplied Q_1 to desired space at T_1 with the help of external work W.

Performances of heat pump is measured by its coefficient of performance which is defined as the ratio of desired effect and work supplied.

i.e. $COP = \frac{\text{Desire defect}}{\text{Work input}}$

in case of heat pump the desired effect is the amount of heat supplied to the desired space(Q_1)

then, $(COP)_{HP} = \frac{Q_1}{W}$

Applying first law of thermodynamics for the heat pump,

$$dQ = \oint dW$$

therefore,

$$-Q_1 + Q_2 = -W$$

$$W = Q_1 - Q_2$$

So,

$$(COP)_{HP} = \frac{Q_1}{Q_1 - Q_2} = \frac{1}{1 - \frac{Q_2}{Q_1}}$$

COP for a reversible heat pump can also be expressed in terms of temperature ratio as,

$$(COP)_{rev,HP} = \frac{T_1}{T_1 - T_2} = \frac{1}{1 - \frac{Q_L}{Q_H}}$$

Refrigerator

Refrigerator operates on a cyclic process which takes heat from a low-temperature reservoir (desired space) at T_2 and delivers into the high-temperature reservoir (Surrounding) at T_1 with the help of external work. Refrigerator maintains the temperature of desired space lower than that of surroundings.

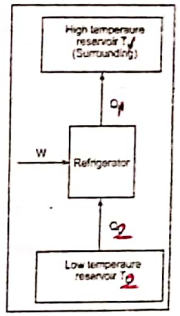

Figure illustrates a schematic representation of refrigerator where Q_2 amount of heat takes from desired space at T_2 and supplied Q_1 to surroundings at T_1 with the help of external work W.

Performances of refrigerator is measured by its coefficient of performance which is defined as the ratio of desired effect and work supplied.

i.e. $COP = \frac{\text{Desire defect}}{\text{Work input}}$

In case of refrigerator the desired effect is the amount of heat taken out from the desired space (Q_2)

then, $(COP)_R = \frac{Q_2}{W}$

Applying first law of thermodynamics for the refrigerator,

$$\oint dQ = \oint dW$$

Therefore,

$$-Q_1 + Q_2 = -W$$

$$W = Q_1 - Q_2$$

So, $(COP)_R = \frac{Q_2}{Q_1 - Q_2} = \frac{1}{\frac{Q_1}{Q_2} - 1}$

COP for a reversible refrigerator can also be expressed in terms of temperature ratio as,

$$(COP)_{rev,R} = \frac{T_L}{T_H - T_L} = \frac{1}{\frac{T_H}{T_L} - 1}$$

Hence $(COP)_{HP} - (COP)_R = 1$

Components of Refrigerator and Heat Pump

A refrigerator and heat pump operates on a cyclic process called refrigeration cycle. The refrigeration cycle is the widely used for heating and cooling of the desired space.

The working substance which under goes cyclic changes in refrigerator is called a refrigerant.

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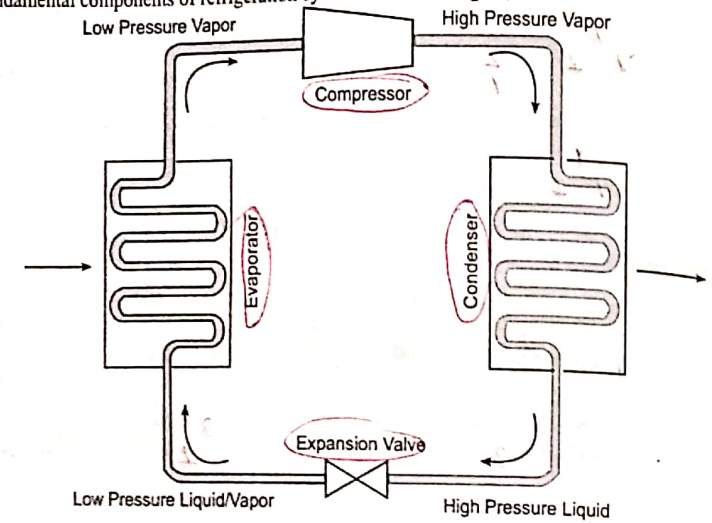
The fundamental components of refrigeration cycle are shown in figure,



1. **Compressor**

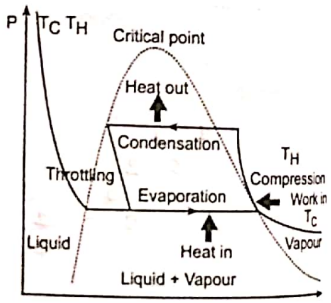
The first step in the refrigeration cycle is compression, and a compressor is used to increase the pressure of the working gas. The refrigeration fluid enters the compressor as a low-pressure, low-temperature gas and exits as a high-pressure, high-temperature gas. The compression process is isentropic process, during the process pressure, temperature and enthalpy of the refrigerant increases while its entropy remains constant.

2. **Condenser**

The high-pressure and high-temperature gas from the compressor enters into the condenser.

The condenser removes heat from the hot refrigerant gas until it condenses into a saturated liquid state at high-pressure and low temperature.

During the condensation process pressure remain constant while enthalpy and entropy of the refrigerant decreases.



3. **Expansion (Throttling) Valve**

The high-pressure and low-temperature liquid from condenser enters into the expansion valve. The expansion valve decreases the pressure of the refrigerant liquid. During the throttling process enthalpy remains constant, pressure and temperature decreases and entropy increases.

4. **Evaporator**

The low-pressure and low-temperature liquid from expansion valve enters into the evaporator. The low temperature refrigerant liquid can take heat from the surrounding inside the evaporator. During the absorption of heat, refrigerant gets evaporated. Evaporation process takes place at constant pressure while the enthalpy and entropy of the refrigerant increases.

After doing so, the refrigerant is sent back to the compressor, where the process restarts.

STATEMENTS OF SECOND LAW OF THERMODYNAMICS

The second law of thermodynamics has been precisely stated by Clausius, Kelvin and Planck in slightly different words although both statements are basically identical. Each statement is based on an irreversible process. Clausius considers transformation of heat between two thermal reservoirs while the transformation of heat into work is considered by Kelvin and Planck.

Clausius Statement

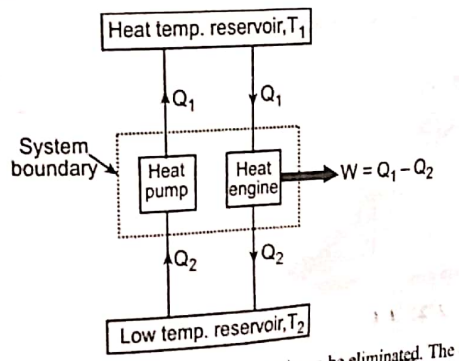
"Heat cannot be transferred from a body at a lower temperature to a body at a higher temperature by a self-acting machine working in a cyclic process without assistance from any external agency." In other words, heat cannot flow from a colder to a hotter body on its own. It implies that COP of heat pump or refrigerator is always less than infinity.

Kelvin-Planck Statement

"It is impossible to build a heat engine that works in a cyclic process that solely converts all heat supplied to it into an equivalent amount of work." It implies that it is impossible to build a heat engine that has thermal efficiency of 100%. These statements appear to be different but for any process if Kelvin and Planck statement is satisfied, Clausius statement will also be satisfied. This implies these two statements are equivalent.

Equivalence of Clausius Statement to the Kelvin-Planck Statement

Consider a higher temperature reservoir T_1 and low temperature reservoir T_2. Let a heat pump shown in figure requires no work and transfers an amount of Q_2 from a low temperature to a higher temperature reservoir which is against the Clausius statement. Let an amount of heat Q_1 ($Q_1 > Q_2$) be transferred from high temperature reservoir to heat engine which develops a net work, $W = Q_1 - Q_2$ and rejects Q_2 to the low temperature reservoir.



Since there is no heat interaction with the low temperature, it can be eliminated. The combined system of the heat engine and heat pump acts then like a heat engine exchanging heat with a single reservoir, which is the against the Kelvin-Planck statement.

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Solved Example

Example 1: A gas is contained is a piston cylinder device initially at a pressure of 150 kPa and a volume of 0.04 m^3 calculate the work done by the gas when it under goes the following process to a final volume of 0.1 m^3.

a. Constant pressure

b. Constant temperature

c. $PV^{1.35}$ = constant

Solution:

Initial state $P_1 = 150$ kPa

$V_1 = 0.04\ m^3$

Final state $V_2 = 0.1\ m^3$

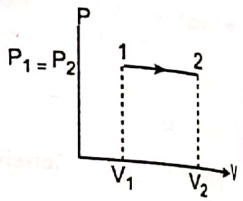
a. For constant pressure

Work done

$W = P_1 (V_2 - V_1)$

$= 150 \times 10^3 (0.1 - 0.04)$

$= 9000$ J

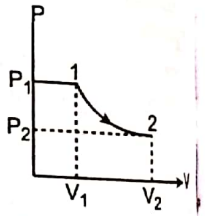


b. For constant temperature process

Work done, $W = P_1V_1 \ln\left(\frac{V_2}{V_1}\right)$

$= 150 \times 10^3 \times 0.04 \ln\left(\frac{0.1}{0.04}\right)$

$= 5497.7$ J



c. For polytropic process ($PV^{1.35}$ = constant)

Now,

$P_1V_1^{1.35} = P_2V_2^{1.35}$

or, $P_2 = P_1\left(\frac{V_1}{V_2}\right)^{1.35}$

$= 150 \times 10^3 \left(\frac{0.04}{0.1}\right)^{1.35}$

$= 43538.38$ Pa

Then, work done during the process

$W = \frac{P_2V_2 - P_1V_1}{(1-n)}$

$= \frac{150 \times 10^3 \times 0.04 - 43538.38 \times 0.1}{1 - 1.35}$

$= \frac{1646.16}{0.35}$

$= 4703.3$J

Example 2: A gas undergoes compression from an initial state of $V_1 = 0.1\ m^3$, $P_1 = 200$ kPa to final state of $V_2 = 0.04\ m^3$, $P_2 = 500$ kPa. If the pressure varies linearly with volume during the process, determine the work transfer.

Solution:

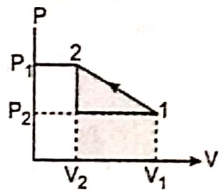
Initial state $V_1 = 0.1\ m^3$, $P_1 = 2300$ kPa $= 200 \times 10^3$ Pa

Final state $V_2 = 0.04\ m^3$, $P_2 = 500$ kPa $= 500 \times 10^3$ Pa.

Here,

The pressure varies linearly with volume during the process .

So,



Work transfer = Area under the curve

= Area of trapezium

$= \frac{1}{2}(P_1 + P_2)(V_1 - V_2)$

$= \frac{1}{2}(200 \times 10^3 + 500 \times 10^3)(0.1 - 0.04)$

$= 21000$ J $= 21$ KJ

Example 3: A control mass containing 0.5 kg of a gas undergoes a process in which there is a heat transfer of 120 kJ from the system to the surroundings work done on the system is 60 kJ. If the initial specific internal energy of the system in 400 kJ/kg, determine its final specific energy.

Solution:

Mass of gas (m) = 0.5 kg

Total heat transfer from the system (Q) = – 120 kJ

Work done on the system (w) = – 60 kJ

Initial specific energy (u_1) = 400 kJ/kg

Final specific energy (u_2) = ?

Total heat transfer is given by

$Q = \Delta U + W$

$= (U_2 - U_1) + W$

$= m(u_2 - u_1) + W$

$-120 = 0.5(u_2 - u_1) - 60$

or, $u_2 - u_1 = \frac{-120 + 60}{0.5}$

or, $u_2 = -120 + 400 = 280$ kJ/kg

Example 4: A gas contained in a piston cylinder device undergoes a polytrophic process for which pressure volume relationship is given by $PV^{2.5}$ = constant. The initial pressure is 400 kPa, the initial volume is 0.2 m^3 and the final volume is 0.4 m^3. The internal volume is 0.2 m^3 and the final volume is 0.4 m^3. The internal energy of the gas decrease by 20 kJ during the process. Determine the work transfer and heat transfer for the process.

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Solution:

Initial state $P_1 = 400$ kPa,

$V_1 = 0.2\ m^3$

Final state $V_2 = 0.4\ m^3$

We have,

$PV^{2.5} = \text{constant}$

Change in internal energy $(\Delta U) = -20$kJ

Then,

$$P_1V_1^{2.5} = P_2V_2^{2.5}$$

$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{2.5}$$

$$= 400\left(\frac{0.2}{0.4}\right)^{2.5}$$

$$= 70.71 \text{ kPa.}$$

Work done during the polytropic process is

$$W = \frac{P_2V_2 - P_1V_1}{1-n}$$

$$= \frac{70.71 \times 0.4 - 400 \times 0.2}{1-2.5}$$

$$= 34.477 \text{ kJ}$$

$\therefore$ Heat transfer during the process $Q = \Delta U + W = -20 + 34.477 = 14.477$ kJ

Example 5: A piston cylinder arrangement contains 0.01 m^3 air at 150 kpa and 300 K. The air is now compressed in a process for which $PV^{1.25}$. To a final pressure 600 kPa Determine the work transfer and heat transfer for the process. [Take R = 287 J/kgK and C_v = 718 J/kgK]

Solution:

Initial state $V_1 = 0.01\ m^3$

$P_1 = 150$ kPa

$T_1 = 27°$ C

$27 + 273 = 300$ K

Final state $P_2 = 600$ kPa.

And process relation $PV^{1.25}$ = Constant

So,

$$P_1V_{1.25}^1 = P_2V_2^{1.25}$$

$$V_2^{1.25} = \left(\frac{P_1}{P_2}\right)V_{1.25}^1$$

$$V_2 = \left(\frac{P_1}{P_2}\right)^{\frac{1}{1.25}} V_1$$

$$= \left(\frac{150}{600}\right)^{0.8} \times 0.01$$

$$= 3.2987 \times 10^{-3}\ m^3$$

Work transfer during the process is

$$W = \frac{P_2V_2 - P_1V_1}{1-n}$$

$$= \frac{600 \times 3.2987 \times 10^{-3} - 150 \times 0.01}{1-1.25}$$

$$= \frac{0.4793}{-0.25}$$

$$= -1.917 \text{ kJ}$$

Also, $P_1V_1 = mRT_1$

Then, mass of air (m) $= \frac{P_1V_1}{RT_1}$

$$= \frac{150 \times 10^3 \times 0.01}{287 \times 300}$$

$$= 0.0174 \text{ kg}$$

Temperature at final state $T_2 = \frac{P_2V_2}{mR}$

$$= \frac{600 \times 10^3 \times 3.2987^{-3} \times 10}{0.0174 \times 287}$$

$$= 395.844 \text{ K}$$

Change in internal energy for the process is given by

$$\Delta U = mC_v(T_2 - T_1)$$

$DU = m C_v (T_2 - T_1)$

$$= 0.0174 \times 718\ (395.844 - 300)$$

$$= 1197.398 \text{ J}$$

$$= 1.1974 \text{ kJ}$$

Total beat transfer for the process is given by

$$Q = \Delta U + W$$

$$= 1.1974 - 1.917$$

$$= -0.7196 \text{ kJ}$$

Example 6: An ideal engine has an efficiency of 25%. If the source temperature is increased by 200° C its efficiency gets doubled, determine it's source and sink temperature.

Solution:

Let, Source temperature = T_H

Sink temperature = T_L

Then, efficiency

$$\eta_1 = 1 - \frac{T_L}{T_H}$$

or, $0.25 = 1 - \frac{T_L}{T_H}$

or, $T_L = 0.75\, T_H$ ... (i)

When source temperature is increased by 200° C (200 K)

Its efficiency gets doubled i.e. $\eta_2 = 2\eta_1 = 0.5$

$$\eta_2 = 1 - \frac{T_L}{T_H + 200}$$

or, $0.5 = 1 - \frac{T_L}{T_H + 200}$

or, $\frac{T_L}{T_H + 200} = 0.5$ ... (ii)

using equation (i) in (ii)

$$\frac{0.75\, T_H}{T_H + 200} = 0.5$$

or, $0.75\, T_H = 0.5\, T_H + 0.5 \times 200$

or, $0.25\, T_H = 100$

$$T_H = \frac{100}{0.25} = 400\ K$$

and $T_L = 0.75 \times 400 = 300$ K

Example 7: A heat pump has a coefficient of performance that is 80% of the theoretical maximum. It maintains hall at 20° C, which leaks energy 1kW per degree temperature difference to the ambient. For a maximum of 1.5kW power input, determine the minimum outside temperature for which the heat pump is sufficient.

Solution:

Higher Temperature (T_4) = 20° C = 293K

Halting rate $\frac{dQ_H}{dt} = 1 \times (T_H - T_L)$

$= (T_H - T_L)$ kW

Power input $\frac{dw}{dt} = 1.5$ kW

Then,

$$(COP)_{actual,\ HP} = 0.8\ (COP)_{rev,\ HP}$$

or, $\frac{\frac{dQ_H}{dt}}{\frac{dW}{dt}} = 0.8 \frac{T_H}{T_H - T_L}$

or, $\frac{T_H - T_L}{1.5} = 0.8 \frac{T_H}{T_H - T_L}$

or, $(T_H - T_L)^2 = 1.2 T_H = 1.2 \times 293 = 351.6$

or, $T_H - T_L = 18.751$

or, $T_L = T_H = -18.751$

$= 393 - 18.751$

$= 274.249$ K.

Example 8: During an experiment, a student claims that, a heat engine receives 300 kJ from a source at 500K converts 160kJ of it into work and rejects heat to the sink at 300K are these data reasonable?

Solution:

Higher temperature (T_H) = 500K

Lower temperature (T_L) = 300K

Heat input (Q_H) = 30 kJ

Work output (W) = 160 kJ

Then, maximum possible efficiency

We have,

$$\eta_{rev} = 1 - \frac{T_L}{T_H} = 1 - \frac{300}{500} = 40\%$$

Also, $\eta_{student} = \frac{W}{Q_H} = \frac{160}{300} = 53.33\%$

Here, $\eta_{student} > \eta_{rev}$, hence these data are not reasonable.

Example 9: A heat engine receives 400 kJ from a source at a temperature of 1000K. It rejects 150 kJ of heat to sink at a temperature of 300K. The engine produce 250 kJ of work output. Is this a reversible cycle, irreversible cycle or an impossible cycle?

Solution:

Higher temperature (T_H) = 1,000K

Lower temperature (T_L) = 300K

Heat rejected (Q_L) = 150kJ

Work output (W) = 250 kJ

Heat input (Q_H) = $Q_L + W = 150 + 250 = 400$ kJ

Then, maximum possible efficiency

$$\eta_{rev} = 1 - \frac{T_L}{T_H}$$

$$= 1 - \frac{300}{1000}$$

$\eta = \eta_{rev}$ ↳ other η_{rev}

And $\eta_{HE} = \frac{W}{Q_H} = \frac{250}{400} = 62.5\%$

Hence, $\eta_{HE} < \eta_{rev}$, the cycle is irreversible.

Here we

Example 10: A heat pump cycle operating between two reservoirs takes energy from the source at $T_L = 270K$ and supplies Q_H to a room temperature at $T_H = 300K$ for each of the following cases, determine whether the cycle operates reversibly, irreversibly or impossible

a. $Q_H = 1000kJ$, $W = 200kJ$ (rev)

b. $Q_H = 2000kJ$, $Q_2 = 1800$ kJ

c. $w = 200kJ$, $Q_L = 2000$ kJ

d. $COP = 8.6$

Solution:

$Q_H = Q_1$ High

$Q_L = Q_L$ low

Higher temperature (T_H) = 300K

Lower temperature (T_L) = 270K

Maximum possible COP of heat pump is

$$(COP)_{rev,HP} = \frac{TH}{T_H - T_L} \qquad \frac{T_1}{T_1 - T_2}$$

$$= \frac{300}{300 - 270}$$

$$= 10$$

a. $Q_4 = 100$ kJ, $W = 200$ kJ

COP of the cycle is

$$(COP)_{HP} = \frac{Q_H}{W} = \frac{1000}{200} = 5 \qquad \frac{Q_1}{Q_1 - Q_L}$$

Here, $(COP)_{HP} < (COP)_{rev,HP}$, Hence it operates irreversibly.

b. $$Q_H = \frac{Q_H}{Q_H - Q_L} = \frac{2000}{2000 - 1800} = 10 \qquad \frac{Q_1}{Q_1 - Q_L}$$

Here, $(COP)_{HP} = (COP)_{rev,HP}$, hence it operates reversibly.

c. $W = 200$ kJ, $Q_L = 2000$ kJ

Then, $Q_H = W + Q_L = 200 + 2000 = 2200$ kJ

Now,

$$(COP)_{HP} = \frac{Q_H}{W} = \frac{2200}{200} = 11 \qquad \text{Same}$$

Here, $(COP)_{HP} > (COP)_{rev}$ HP, hence it is impossible.

d. $COP = 8.6$

Here, $(COP)_{HP} < (COP)_{rev,HP}$, hence it operates irreversibly

THEORETICAL ANSWER QUESTIONS

1. State the first law of thermodynamics and also write the limitations of the law.
2. How are specific heat at constant volume and constant pressure related to change in internal energy and enthalpy respectively?
3. Explain the first law of thermodynamics for a control mass undergoing a cyclic process.
4. State first law of thermodynamics for control mars constant volume (isochoric) process.
5. Explain the first law of thermodynamics for control mass constant pressure (Isobaric) process.
6. Explain the first law of thermodynamics for control mass constant temperature (isothermal) process.
7. Explain the first law of thermodynamics for control mass (non-flow) polytropic process (adiabatic).
8. Define heat engine and its efficiency. Explain how first law is applied to analyse the efficiency of heat engine.
9. Define heat pump and its COP. Explain how first law is applied to analyse the performance of the heat pump.
10. Define refrigerator and its COP. Explain how first law is applied to analyse the performance of the refrigerator.
11. What are the fundamental components of refrigerator and heat pump explain with brief function of each component.
12. Explain second law of thermodynamics. Prove that violation of Kelvin Planck statement leads to violation of Clausius statement.

NUMERICAL PROBLEMS

1. A piston cylinder arrangement contents a gas initially at a pressure of 800 kPa and a volume of $0.1m^3$. Determine the work done by the gas when it undergoes the following processes to a final volume of $0.5\ m^3$.

   a. Constant pressure

   b. Constant temperature

   c. PV^2 = constant (Ans: 320 kJ, 128.755 kJ, 64 kJ)

2. A gas undergoes a polytrophic process from an initial state of 500 kPa and $0.02\ m^3$ to a final state of 100 kPa and $0.05\ m^3$ determine the work transfer. (Ans: 6.614kJ)

3. A piston cylinder device contains 0.2 kg of a gas initially at P_1 = 1000 kPa and $V_1 = 0.02\ m^3$. It undergoes polytropic expansion to a final pressure of 200 kPa during which the relation between pressure and volume is PV^3 = constant. If the specific internal energy of the gas decreases by 160 kJ/Kg during the process, determine the heat transfer for the process. (Ans: – 25.42 kJ)

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4. A piston cylinder device contains 0.3 kg of a gas initially at P_1 = 500 kPa and V_1 = 0.03. It undergoes a polytropic expansion to a final pressure of 200 kPa during which the relations between pressure and volume is $PV^{2.5}$ = constant. If the specific internal energy of the gas decrease by 100 kJ/kg during the process, determine the heat transfer for the process. [Ans: −25.77 kJ]

5. An inventor claims that a heat pump can maintain a building at 20° C, the heat loss from the room occurs at a rate of 1000 kJ/min and the heat pump requires 1 kW of power input. (The surrounding is at 0° C). Evaluate his claim. [Ans: $(COP)_{inv}$ = 14.65 < 16.67 $(COP)_{Carnot}$]

6. During an experiment conducted in a room at 27°C a student measures that a refrigerator consumes 2kW of power and removes 36000 kJ of heat from the desired space at − 23° C. The running time for the refrigerator during the experiment was 30 min. Are these data reasonable. [Ans: $(COP)_{Carnot}$ = 10, $(COP)_{inv}$ = 10.8]

7. A power cycle operating between two reservoirs receives Q_H from high temperature at T_H = 1000 K and rejects energy Q_L to a low temperature sink at T_L = 300K. For each of the following cases, determine whether the cycle operates reversibly, irreversibly or impossible.

   a. Q_H = 800kJ, W = 600kJ

   b. Q_H = 800kJ, Q_L = 240kJ

   c. W = 960kJ, Q_L = 640kJ

   d. η = 50%

   [Ans: η_{rev} = 70%; η_{HE} = 75%, 70%, 60%]

8. An ideal heat engine has the same efficiency for a source and sink at 800K and 400K respectively force source and sink at T_H and 800K Determine T_H in K. [Ans: 1000 K]

9. A heat pump requires a power input of 2.5kW and maintain the temperature of room at 25°C which loses heat at a rate of 20kW to the colder ambient. What is the coefficient of performance of the heat pump? [Ans: 8]

10. A building is maintained at a temperature at 25° by a heat pump having a COP of 2.5. It loses heat at a rate of 1kW per degree temperature difference between the inside and the outside. If the outside temperature is −10°C. Determine the power required to drive the heat pump. [Ans: 14 kW]

11. A refrigerator operates in a room at 22°C. Heat must be taken from the desired space at a rate of 2.5kW to maintain its temperature at − 20°C. What is the minimum power required to drive the refrigerator? [Ans: 0.415 kW]

□□□

(C) HEAT TRANSFER

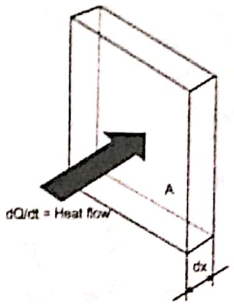
INTRODUCTION

Modes of heat transfer (conduction, convection and radiation), statement and assumption of Fourier's law of thermal conductivity, one dimensional steady state heat conduction through plane wall, basic laws of radiation (Emissive power and emissivity, Stefan-Boltzmann's law), Concept of black bodies.

Heat transfer is defined as the transmission of energy from one region to another another region as a result of temperature difference between them. Heat always flows in the direction of lower temperature. The heat transfer process involve the transfer of energy and obey the first and second law of thermodynamics. It concerns the generation, use, conversion, and exchange of thermal energy(heat) between physical systems. The different modes of heat transfer are : conduction, convection and radiation. All modes of heat transfer require the existence of a temperature difference.

CONDUCTION

Conduction is the transfer of heat from one part of a substance to another part of the same substance, or from one substance to another in physical contact with it, without appreciable displacement of molecules.



Fourier law of thermal conduction states that "The rate of flow of heat through a simple homogeneous solid is directly proportional to the area of the section at right angles to the direction of heat flow, and temperature gradient."

i.e. $\frac{dQ}{dt} \alpha A \frac{dT}{dx}$

$$\frac{dQ}{dt} = -kA\frac{dT}{dx}$$

where, -ve sign indicates that heat flows in the direction of decreasing temperature.

$\frac{dQ}{dt}$ is the rate of heat transfer(Watt)

k is thermal conductivity of the material(Watt/mK)

A is cross-sectional area of the material normal to the direction of heat flow(m^2)

$\frac{dT}{dx}$ is the temperature gradient within the material(K/m)

The temperature gradient $\frac{dT}{dx}$ always negative along positive x direction and therefore the value of $\frac{dQ}{dt}$ becomes +ve.

Thus, the thermal conductivity of a material is defined as "The amount of energy conducted through a body of unit area, and unit thickness in unit time when the difference in temperature between the faces causing heat flow is unit temperature difference".

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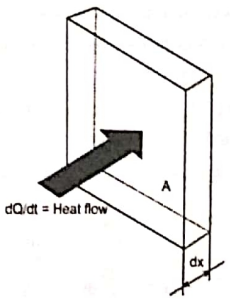

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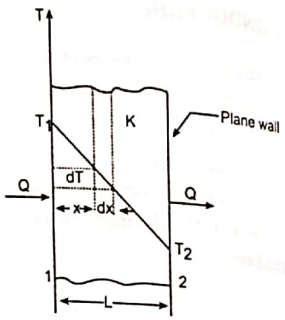
Assumptions:

The following are the assumptions on which Fourier's law is based :

1. Conduction of heat takes place under steady state conditions.
2. The heat flow is unidirectional.
3. The temperatures gradient is constant and the temperature profile is linear.
4. There is no internal heat generation.
5. The bounding surfaces are isothermal in character.
6. The material is homogeneous and isotropic (i.e., the value of thermal conductivity is constant in all directions).

One Dimensional Steady State Heat Conduction Through Plane Wall

Consider a plane wall of thickness L, Cross-sectional area of the wall A and thermal conductivity of the wall material k of homogeneous material through which heat is flowing only in x-direction.



Let, T_1 and T_2 are the temperatures maintained at the two faces 1 and 2 of the wall, respectively. Such that $T_1 > T_2$.

The rate of heat transfer is given by Fourier's law as,

$$\frac{dQ}{dt} = -kA\frac{dT}{dx}$$

or, $$\frac{dQ}{dt}dx = -kAdT$$

Integrating both side,

$$\int_0^L \frac{dQ}{dt}dx = \int_{T_1}^{T_2} -kAdT$$

$$\frac{dQ}{dt}\int_0^L dx = -kA\int_{T_1}^{T_2} -dT$$

$$\frac{dQ}{dt}(L-0) = -kA(T_2 - T_1)$$

$$\frac{dQ}{dt} = -kA\frac{(T_2 - T_1)}{L}$$

$$\frac{dQ}{dt} = \frac{(T_1 - T_2)}{\frac{L}{kA}}$$

Hence thermal resistance $(R_{th}) = \frac{L}{kA}$

CONVECTION

Convection is the transfer of heat within a fluid by mixing of one portion of the fluid with another. It is possible only in a fluid medium and is directly linked with the transport of medium itself. The effectiveness of heat transfer by convection depends largely upon the mixing motion of the fluid.

Magnitude of convection heat transfer is given by Newton's law of cooling,

$$\frac{dQ}{dt} = hA(T_S - T_F)$$

where, $\frac{dQ}{dt}$ is rate of conductive heat transfer(watt)

A is exposed area of solid confining liquid to heat transfer(m^2)

T_s is solid surface temperature(K)

T_F is free stream fluid temperature(K) ,and

h is Coefficient of conductive heat transfer(watt/m^2K)

The coefficient of conductive heat transfer depends thermodynamic and transport properties (e.g., viscosity, density, specific heat etc.), nature of fluid flow, geometry of the surface and the prevailing thermal conditions.

RADIATION

Radiation heat transfer is defined as the transfer of energy across a system boundary using an electromagnetic mechanism. Whereas the heat transfer by conduction and convection takes place only in the presence of medium, radiation heat transfer does not require a medium.

Emissive Power

The emissive power is defined as the total amount of radiation emitted by a body per unit area and time. It is expressed in W/m^2. The emissive power of a blackbody, according to Stefan- Boltzmann, is proportional to absolute temperature to the fourth power.

$$E_b = \sigma T^4 \text{ W/m}^2$$

where σ = Stefan-Boltzmann constant = 5.67×10^{-8} W/m^2 K^4

Emissivity (ε)

It is defined as the ability of the surface of a body to radiate heat. It is also defined as the ratio of the emissive power of anybody to the emissive power of a black body of equal temperature.

i.e. $\varepsilon = \frac{E}{E_b}$ Its values varies for different substances ranging from 0 to 1.

For a black body ε = 1, for a white body surface ε = 0 and for gray bodies it lies between 0 and 1. It may vary with temperature or wavelength.

Stefan-Boltzmann law

Magnitude of rate of radiation heat transfer between two practical bodies at temperature T_1 and T_2 $(T_1 > T_2)$ is given by Stefan-Boltzman law ,

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$$\frac{dQ}{dt} = \epsilon\sigma A (T_1^4 - T_2^4)$$

where,

$\frac{dQ}{dt}$ is rate of conductive heat transfer(watt)

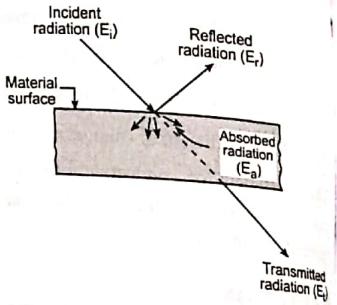
ϵ is the emissivity

A is surface area(m^2) and

σ is Stefan-Boltzmann constant = 5.67×10^{-8} $W/m^2 K^4$.

Radiation Surface Properties

When incident radiation (E_i) strike on a surface, three things happens ; a part is reflected back (E_r), a part is transmitted through (E_t), and the remainder is absorbed (E_a) depending upon the characteristics of the body, as shown in Figure



By the conservation of energy principle,

$$E_a + E_r + E_t = E_i$$

dividing both side by E_i

$$E_a + E_r + E_t = E_i$$

$$\frac{E_a}{E_i} + \frac{E_r}{E_i} + \frac{E_t}{E_i} = \frac{E_i}{E_i} = 1$$

therefore, $\alpha + \rho + \tau = 1$

where α is absorptivity, ρ is reflectivity and τ is transmissivity.

Hence for any surface the sum of absorptivity, reflectivity and transmissivity must be equal to 1.

a. **Black body :** For perfectly absorbing body, $\alpha = 1, \rho = 0, \tau = 0$. Such a body is called a 'black body'

(i.e., a black body is one which neither reflects nor transmits any part of the incident radiation but absorbs all of it).

In practice, a perfect black body ($\alpha = 1$) does not exist. However its concept is very important.

b. **Opaque body:** When no incident radiation is transmitted through the body, it is called an 'opaque body'. For the opaque body $\tau = 0$, and $\alpha + \rho = 1$

c. **White body.** If all the incident radiation falling on the body are reflected, it is called a 'white body'.

For a white body, $\rho = 1, \alpha = 0$ and $\tau = 0$.

d. **Gray body:** If the radiative properties, α, ρ, τ of a body are assumed to be uniform over the entire wavelength spectrum, then such a body is called gray body. A gray body is also defined as one whose absorptivity of a surface does not vary with temperature and wavelength of the incident radiation [$\alpha = (\alpha)_\lambda$ = constant.]

A coloured body is one whose absorptivity of a surface varies with the wavelength of radiation [$\alpha \neq (\alpha)\lambda$].

Concept of Black Bodies

An object that completely absorbs all radiant energy that hits its surface is referred to as a "black body" (for a black body, $\alpha = 1, \rho = 0, \tau = 0$). No real body is completely black; the idea of a black body is an idealization that makes it easy to compare the radiation properties of real bodies.

A black body has the following properties :

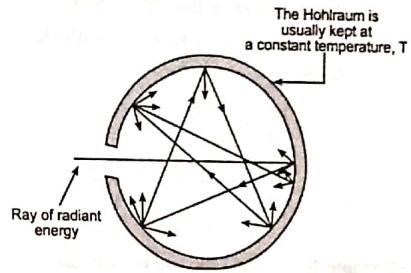
i. It absorbs all the incident radiation falling on it and does not transmit or reflect regardless of wavelength and direction.

ii. It emits maximum amount of thermal radiations at all wavelengths at any specified temperature.

iii. It is a diffuse emitter (i.e., the radiation emitted by a black body is independent of direction).

Consider a hollow enclosure with a very small hole for the passage of incident radiation as shown in Figure.

Incident radiant energy travels through the tiny opening; some of it is absorbed and some of it is reflected by the interior surface. The majority of this energy is, however, absorbed during the second incidence. Once more, only a tiny portion is reflected. Only a very small portion of the original incident energy is reflected back out of the opening after several of these reflections. Due to the fact that all radiant energy entering through a small hole leading into a cavity (Hohlraum) is absorbed, it behaves very similarly to a black body.



Isothermal furnaces, with small apertures, approximate a black body and are frequently used to calibrate heat flux gauges, thermometers and other radiometric devices.

Solved Example

Example 1: An insulting material having a thermal conductive of 0.08 W/mk is used to limit the heat transfer of 80 W/m^2 for a temperature of 15°C across the opposite faces. Find the thickness of the material.

Solution:

Thermal conductivity (k) = 0.08 W/mK

Rate of heat transfer per unit Area

$$\frac{1}{A}\left(\frac{dQ}{dt}\right) = 80\ W/m^2$$

Temp. difference (ΔT) = 15° C = 15K

Thickness of material (L) = ?

$$\frac{1}{A}\frac{dQ}{dt} = \frac{k\Delta T}{L}$$

or, $80 = \frac{0.8 \times 15}{L}$

or, $80 = \frac{0.8 \times 15}{L}$

or, $L = 0.15m$

Example 2: A brick wall 12 cm thick and $5m^2$ surface area exposed to 50° C at one face and 20° C to another face. If the thermal conductivity of the material is 1.5 W/mK. Find the heat transfer rate.

Solution:

Thickness of brick wall (L) = 12 cm = 0.12m

Surface area of wall (A) = $5 m^2$

Temp. of one face (T_1) = 50° C

Temp. of another face (T_2) = 20° C

Thermal conductivity (k) = 1.5 W/mK

We have,

$$\frac{dQ}{dt} = \frac{kA\Delta T}{L}$$

$$= \frac{1.5 \times 5 \times (50 - 20)}{0.12}$$

= 1875 watt.

Example 3: Find the rate of heat loss form a brick wall (k = 0.7 W/mK) of length 5m, height 4 m and 0.25 m thick. The temperature of the inner surface 50° C and that of outer surface is 30°C. Also calculate the distance from the inner surface at which the temperature is 40° C.

Solution:

Thermal conductivity of brick (k) = 0.7 W/mK

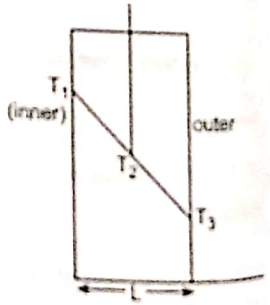
Thickness of wall (L_1) = 0.25 m

Temp. of inner surface (T_1) = 50° C

Temp. of outer surface (T_3) = 30° C

Area of the wall (A) = 5 × 4 = 20 m^2

We have,



Rate of heat loss $\frac{dQ}{dt} = \frac{kA(T_1 - T_3)}{L}$

$$= \frac{0.7 \times 20 \times (50 - 30)}{0.25}$$

= 1120 watt

Now, for distance from the inner surface at which temperature is 40° C.

Again,

$$\frac{dQ}{dt} = \frac{kA(T_1 - T_2)}{L_1}$$

$$L_1 = \frac{0.7 \times 20 \times (50 - 40)}{1120}$$

L_1 = 0.125 m

Example 4: A 1.2m long tube with outer diameter of 4cm having outside temperature of 120° C is exposed to the ambient air at 20° C. If the heat transfer coefficient between the tube surface and the air is 20 W/m^2k. Find the rate of heat transfer from the tube to the air.

Solution:

Length of the tube (L) = 1.2m

Outer Radius (R) = 2 cm = 2×10^{-2} m

Outside temp of tube (T_s) = 120° c

Temp of Air (T_a) = 20° c

Heat transfer coeff (h) = 20 W/m^2k

Area of tube (A) = 2 πRL

$= 2 \pi \times 2 \times 10^{-2} \times 1.2$

$= 0.1508 m^2$

Then,

Rate of heat transfer $\frac{dQ}{dt} = hA(T_s - T_a)$

$= 20 \times 0.1508 \times (120 - 20)$

= 301.593 watt

Example 5: A 2m long 0.35 cm diameter electrical wire in a room at 20° C generating heat and surface temperature of the wire found to be 150° C in steady cooperation. The voltage drop and electric current through the wire are 50 and 2A respectively. Find the convection h eat transfer coefficence. Between the outer surface of the wire and the air in the room.

Solution:

Length of the wire (ℓ) = 2m

Diameter of the wire (d) = 0.35 cm = 0.35×10^{-2} m

Room temp (T_a) = 20 C

Surface temp (T_s) = 150° C

Voltage drop (V) = 50 V

Current (I) = 2 A

Coeff. of convection

Heat transfer (h) = ?

Area (A) = πdl = $\pi \times 0.35 \times 10^{-2} \times 2$

Since, Electric power developed in the wire = 0.02199 m^2 = Rate of heat loss from the wire

i.e. $h = \frac{IV}{A(T_A - T_B)}$

$= \frac{2 \times 50}{0.02199 \times (150 - 20)}$

$= 34.98\ W/m^2K$

Example 6: A room is maintained at 22° C by an air conditioning unit Determine the total rate of heat transfer from the person standing in the roof if the exposed surface area and the average outer surface temperature of the person area 1.5 m^3 and 30° C, respectively, and the convection heat transfer coefficient is 10 W/m^2K – take surface emissivity as 0.95.

Solution:

Room temperature $(T_2) = 22°\ C = 295K$

Expose surface area $(A) = 1.5m^2$

Average outer surface temperature $(T_1) = 30°\ C = 303K$

Convection heat transfer coefficient $(h) = 10\ W/m^2K$

Emissivity $(\varepsilon) = 0.95$

Now,

Total rate of heat transfer $\frac{dQ}{dt} = \frac{dQ_{convection}}{dt} + \frac{dQ_{radiation}}{dt}$

$= hA(T_1 - T_2) + \sigma\varepsilon A\left(T_1^4 - T_2^4\right)$

$= 10 \times 1.5 \times (303 - 295) + 5.67 \times 10^{-8} \times 0.95$

$= 189.126W$

Example 7: A thin metal plate is insulted on the back and exposed its front surface to solar radiation. The exposed surface of the plate has an emissivity of 0.7. If the solar radiation is including on the plate at the rate of 750 W/m^2 and the surrounding air temperature is 20°C, determine the surface temperature of the plate. Assume convection heat transfer coefficient 40 W/m^2K.

Solution:

Emissivity of exposed surface of plate $(\varepsilon) = 0.7$

$\frac{1}{A}\left(\frac{dQ}{dt}\right) = 750\ W/m^2$

Air temperature $(T_2) = 20°\ C = 293K$

Coefficient of convection heat transfer $(h) = 40\ W/m^2K$

Surface temperature of plate $(T_1) = ?$

Here,

Rate of heat incident per unit Area = Rate of heat dissipated per unit area by convection + Rate of heat dissipated per unit area by radiation

or, $\frac{1}{A}\left(\frac{dQ}{dt}\right)_{incident} = h(T_1 - T_2) + \sigma\varepsilon\left(T_1^4 - T_2^4\right)$

$750 = 40(T_1 - 293) + 5.67 \times 10^{-8} \times 0.7\left(T_1^4 - 293^4\right)$

Example 8: A furnace inside temperature of 2250 K has a glass circular viewing of 6 cm diameter. If the transmissivity of glass is 0.08, make calculations for the heat loss from the glass window due to radiation.

Solution:

Temperature of furnace (T) = 2250 k

Diameter (d) = 6 cm = 6×10^{-2} m

Transmissivity $(\tau) = 0.08$

The radiation heat loss from the glass window is given by

$Q = \sigma_b AT^4 \times \tau$

Where τ is the transmissivity of glass

$Q = 5.67 \times 10^{-8} \times \frac{\pi}{4}(0.06)^2 \times 2250^4 \times 0.08$

$= 328.53\ W$

Example 9: **A thin metal plate of 4 cm diameter is suspended in atmospheric air whose temperature is 290 K. the plate attains a temperature of 295 K when one of its face receives radiant energy from a heat source at the rate of 2 W. If heat transfer coefficient on both surfaces of the plate is stated to be 87.5 W/m^2K, workout the reflectivity of the plates.**

Solution:

Heat lost by convection from both sides of the plate $= 2hA\Delta T$

The factors 2 accounts for two sides of the plate $= 2 \times 87.5 \times \left\{\frac{\pi}{4}(0.04)^2\right\} \times (295 - 290)$

$= 1.1W$

For most of solids, the transmissivity is zero.

$\therefore$ Rae of energy lost by reflection $(E_r) = 2 - 1.1 = 0.9W$

Reflectivity $\rho = \frac{E_r}{E_i} = \frac{0.9}{2.0} = 0.45.$

THEORETICAL ANSWER QUESTIONS

1. Explain three different modes of heat transfer.
2. Derive an expression for the one dimensional steady state heat conduction through a plane wall.
3. What do you understand by conduction mode of heat transfer. State the Fourier law of thermal conduction also write the assumption of that law.
4. What is convection mode of heat transfer. State and explain Newton's law of cooling.
5. What is radiation mode of heat transfer. State and explain Stefan-Boltzmann law for two particle bodies at different temperatures.
6. What is black body. Write the properties of black body.

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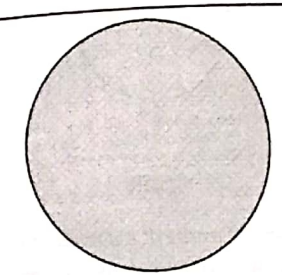
NUMERICAL PROBLEMS

1. The heat transfer rate through a wooden board of 3 c thick for a temperature difference of 24° between the inner and outer surface is 80 W/m^2. Find the thermal conductivity of the board. [Ans: 0.1W/mK]
2. Magnitude of conduction heat transfer through an insulting layer of 0.8 m^2 surface area, 5 cm thick and having a thermal conductivity of 0.25 W/mK is found to be 1600W. Find the temperature difference across the insulting layer. [Ans: 400K]
3. The roof of an electrically heated home is 10 m long, 78 m wide and 0.25 m thick is made of a flat layer of concrete whose thermal conductivity is k = 0.8 W/mK. The temperature of the inner and the outer surface of the roof are measured to be 18º C and 5° C respectively for a period of 12 hrs. Find
   i. The rate of heat loss through the roof and
   ii. The cost of the heat loss to the home owner if the cost electricity is Rs 10/kWhr [Ans: 3328W, Rs. 399936]
4. A plate having a surface area of 5m^2 and temperature of 80º C is exposed to air at 25° C. If the heat transfer coefficient between the surface and air is 20 W/m^{2}K. Find the heat transfer rate from the plate to the air. [Ans: 4400 W]
5. An electric current is passed through a wire 2 mm in diameter and 8 cm long. The wire in submerged in the liquid water. During the boiling of water, temperature of water is 100° C and convection heat transfer coefficient is 4500 W/m^{2}K. Determine the power supplied to the wire to maintain the surface temperature at 120°C. [Ans: 45.239 W]
6. The heat flux at the surface of an electrical heater is 3500 W/m^2, the heater surface temperature is 120° C when it cooled by air at 50° C.
   i. Find the convection heat transfer coefficient.
   ii. Find the heater temperature, if the power is reduced, so the heat flux 2500 W/m^2. [Ans: 50 W/m^{2}K, 100° C]
7. Two very large plates are maintained at 1200° C and 400° C respectively. Calculate the rate of heat transfer due to radiation per unit area. Assume black body properties. [Ans: 255.290 kW/m^2]
8. The inner surface of the walls, floors and the ceiling of the house are observed to be at an average temperature of 5° C in winter and 26° C in summer. Find the rate of radiation heat transfer between a person inside the house and the surrounding surface. If the exposed surface area, the average outer surface temperature and the surface emissivity of the person are 1.4 m^2, 30 C and 0.95 respectively. [Ans: 32.906W]
9. A steel pipe having an outer diameter of 4 cm is maintained at a temperature of 80° C in a room where the ambient temperature is 25°C. The emissive of the surface in 0.8 and the convection heat transfer coefficient between the surface and air is 10 W/m^{2}K. Calculate the total heat loss from the unit length of the pipe. [Ans: 112.671 W/m]
10. A hot plate of length 80 cm, width 50 cm and thickness 4 cm is placed in air stream at 20º C. It is estimated that a total of 300W is lost from the plate surface by radiation when it has an outer surface temperature of 250° C at steady state. If the convection heat transfer coefficient is 25 W/m^{2}K and the thermal conductivity of the plate is 50 W/mK, find the inside surface temperature of the plate. [Ans: 255K]
11. The inner surface of a 2 cm thick 50 cm × 50 cm plate (k = 10 N/mK) is at 400° C. The outer surface dissipates heat by convection and radiation to the air at 27° C. If the plate surface has an emissivity 0.85 and the coefficient of convection heat transfer between the outer surface of plate and air is 20 W/m^{2}K. Find the outer surface temperature of the plate. [Ans: 643.511K]

□□□

APPENDIX

1. Moment of Inertia

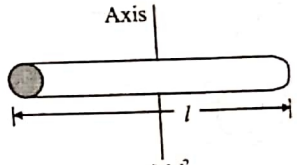


$I = \frac{2}{5}MR^2$

Moment of Inertia of a solid sphere about a

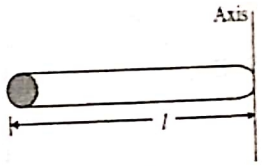
$I = \frac{2}{3}MR^2$

Moment of Inertia of a hollow sphere about a



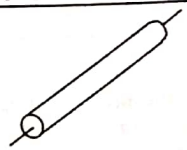
$I = \frac{Ml^2}{12}$

Moment of inertia of thin rod about an axis through center and perpendicular to the rod.



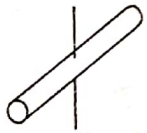
$I = \frac{1}{3}Ml^2$

Moment of inertia of a thin rod about an axis through its end and perpendicular to the rod



$I = \frac{1}{2}MR^2$

Moment of inertia of a solid cylinder about its axis



$I = \frac{1}{4}MR^2 + \frac{1}{12}Ml^2$

Moment of inertia of a solid cylinder about an axis through its center and perpendicular to

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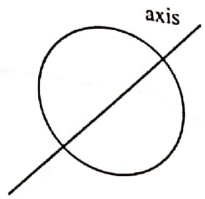
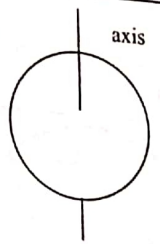


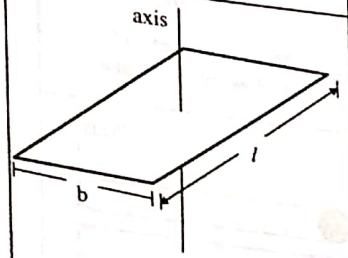
$I = \frac{1}{4} MR^2$

Moment of inertia of a circular disk about any



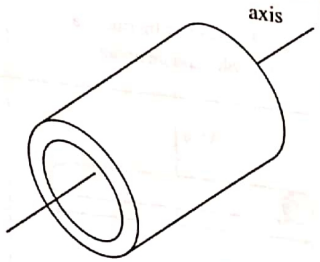
$I = \frac{1}{2} MR^2$

Moment of inertia of a circular disk about an axis through its c.g. and perpendicular to it's plane



$I = \frac{1}{12} M (l^2 + b^2)$

Moment of inertia of a rectangular plate about an axis through its centre and perpendicular to its



$I = \frac{1}{2} M (R_1^2 + R_2^2)$

Moment of inertia of hollow cylinder about its axis

2. Elastic Constant

Substance	Young's Modulus (Y) Dyne/cm² × 10^{11}	Modulus of Rigidity (η) Dyne/cm² × 10^{11}	Bulk Modulus (k) Dyne/cm² × 10^{11}
Aluminum	7.05	2.67	7.46
Brass	9.7 – 10.2	3.5	10.65
Bronze	8.08	3.43	9.52
Copper	12.4 – 12.9	3.9 – 4.0	14.3
Constantan	16.3	6.11	15.5
Glass (crown)	6.5 – 7.8	2.6 – 3.2	4.0 – 5.9
Glass (flint)	5.0 – 6.0	2.0 – 2.5	3.6 – 3.8
Iron (wrought)	19 – 20	7.7 – 8.3	14.6
Iron (cast)	10 – 13	3.5 – 5.3	9.6
Manganin	12.4	4.65	12.1
Steel	19.5 – 20.6	7.5 – 8.9	18.1

1. Universal Constant

Quantity	Symbol	Value	Unit
Absolute zero of temperature	0°K	273.15	°C
Avogadro constant	N_A	6.0221367×10^{23}	mol^{-1}
Boltzmann constant	k	1.380658×10^{-23}	J/K
Charge of proton	e	$1.60217733 \times 10^{-19}$	C
Coulomb's law constant	$k = \frac{1}{4\pi\varepsilon_0}$	8.9874×10^9	Nm^2C^{-2}
Ground state energy of hydrogen atom	E_1	−13.605698	eV
Mass of electron	m_e	$9.1093897 \times 10^{-31}$	kg
Mass of proton	m_p	$1.6726231 \times 10^{-27}$	kg
Mass of neutron	m_n	$1.6749286 \times 10^{-27}$	kg
Mechanical equivalent of heat	J	4.185	Joule Cal^{-1}
Permitivity of vacuum	ϵ_0	$\frac{1}{\mu_0 c^2}$ $= 8.854187817.... \times 10^{-12}$	F/m
Planck constant	h	$6.6260755 \times 10^{-34}$	J–s
Speed of light in vacuum	c	2.99792458×10^8	m/s
Standard atmospheric pressure	P = 1 atm	1.013×10^5	Nm^{-2}
Universal gas constant	R	8.314	Joule $mole^{-1}K^{-1}$
Volume of Ideal gas at 0°C and 1 atm		22.415	litres $mole^{-1}$

4. Resistivity (ρ) at room temperature

Substance	Specific resistance × 10^{-8} Ω m	Substance	Specific Resistance × 10^{-8} Ω m
Aluminum	3.21	Antimony	40.5
Bismuth	119.0	Brass	6 – 9
Cadmium	7.54	Cobalt	9.71
Constantan	49.0	Copper	1.78
Gold	2.42	Lead	20.8
Manganin	44.5	Nichrome	110
Nickel	11.8	Palladium	10.7
Platinum	11.0	Silver	1.64
Tungsten	5.5	Zinc	6.1

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5. Dielectric Constants

Substance	Dielectric Constant	Substance	Dielectric Constant
Vacuum	1.0	Plastics	2.1 – 6.0
Air (at 0° C)	1.00053	Mica	6.0
Parafin wax	2.2	Soda glass	7.5
Paper (dry)	2.6	Porcelain	7.0
Paper (waxed)	2.7	Marble	8.0
Rubber (vulcanized)	3.2	Water	80.0
Paper (oil impregnate)	2.3 – 3.6	Ebonite	3.0

6. Prefixes for SI Units

Factor	Prefixes	Symbol	Factor	Prefixes	symbol
10^1	Deka	Da	10^{-1}	Deci	d
10^2	Hecto	h	10^{-2}	Centi	c
10^3	Kilo	k	10^{-3}	Milli	m
10^6	Mega	m	10^{-6}	Micro	μ
10^9	Giga	G	10^{-9}	Nano	n
10^{12}	Tera	T	10^{-12}	Pico	p
10^{15}	Peta	P	10^{-15}	Femto	f
10^{18}	exa	E	10^{-18}	Atto	a
10^{21}	Zetta	Z	10^{-21}	Zepto	z
10^{24}	Yotta	Y	10^{-24}	Yocto	y

7. Conversion factor

Planne Angle

1° (one degree) = 60′ (sixty minutes)

1′ (one minute) = 60″ (sixty seconds)

π^c (π radian) = 180° (one hundred eighty degree)

Length

1 m = 100 cm, 1 km = 1000 m

1 inch = 2.54 cm, 1 foot = 12 inch

1 mile = 5280 ft, 1 angstrom (1 A°) = 10^{-10} m

1 fermi = 10^{-15} m, 1 fathom = 6 ft, 1 rod = 16.5 ft

1 nautical mile = 1852 m, 1 light year = 9.460×10^{15} m

1 Bohr radius = 5.292×10^{-11} m

1 yard = 3 fit

Area

1 acre = 43,560 ft², 1 barn = 10^{-28} m²

1 hectare = 10^4 m² = 2.471 acres

Volume

1 litre = 1000 cm³ (c.c.)

Mass

1 kg = 1000 gm, 1 slug = 14.59 kg.

1 atomic mass unit (a.m.u. or u) = 1.661×10^{-27} kg

1 ounce = 453.6 gm, 1 ton = 907.2 kg

Time

1 year = 365.25 days, 1 day = 24 hour (h)

1 hour = 60 minutes, 1 minutes = 60 seconds

Force

1 Newton = 10^5 dyne, 1 Pound = 4.448 Newton

1 Kilogram force = mg = 1 × 9.8 = 9.8 N

Pressure

1 atmosphere = 1.013×10^5 N/m² = 760 mm of Hg

1 Pascal = 1 N/m²

1 barr = 10^5 N/m²

1 torr = 1 mm of Hg

Energy, work

1 Joule = 10^7 erg,

1 ev = 1.6×10^{-19} J

1 Calorie = 4.2 Joule,

1 Killowatt hour = 1 × 1000 × 60 × 60 = 3.6×10^6 Joule,

1 horse power hour = 2.685×10^6 Joule.

1 unified atomic mass unit (u) = 1.661×10^{-27} kg = 932 Mev

1 Joule = $\frac{1}{1.6 \times 10^{-19}}$ ev = 6.25×10^{18} ev

1 kg = $1 \times (3 \times 10^8)^2 = 9 \times 10^{16}$ Joule [Since $E = mc^2$]

Power

1 Horse power = 746 watt

Magnetic Field

1 Tesla = 10^4 gauss,

1 milligauss = 10^{-3} gauss

1 Tesla = 1 weber/meter²

Magnetic Flux

1 weber = 10^8 maxwell

8. Specific Rotation (Water solvent)

Substance	Specific rotation (degree)	Substance	Specific rotation (degree)
Can sugar	+66.67	Tartaric acid	+15.6
Dextrose	+52.5	Rochelle salt	+29.73
Glucose	+52.6	Fructose	– 91.0

9. The Greek Alphabet

Alpha	α	Beta	β	Gamma	Γ, γ
Delta	Δ, δ	Epsilon	ϵ	Eta	η
Theta	θ	Lambda	λ	Mu	μ
Nu	ν	Pi	π	Rho	ρ
Sigma	Σ, σ	Tau	τ	Phi	ϕ
Chi	χ	Psi	ψ	Omega	Ω, ω

10. Refractive Index and Dispersive Powers (λ = 5893 A° at 15°C)

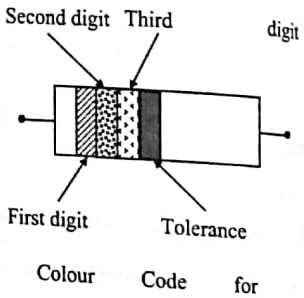
Substance	μ	ω	Substance	μ	ω
Crown	1.50	0.015	Very dense flint	1.72	0.033
Dense crown	1.62	0.018	Calcite ordinary	1.658	0.02
Flint	1.56	0.02	Calcite extra ordinary	1.486	0.013
Dense flint	1.62	0.027	Quartz ordinary	1.544	0.04
Extra dense flint	1.65	0.03	Quartz extra ordinary	1.553	0.015
Cananda Balsam	1.53				

11. Color Code of Resistor

Resistor values are normally shown using colored bands. Each color represents the number as shown in table. The first and second band gives first and second digit.

The third band gives the multiplier. The fourth band is for tolerance of the resistor.

Colour	Code	Multiplier	Tolerance
Black	0	10^0	
Brown	1	10^1	1%
Red	2	10^2	2%
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold			5%
Silver			10%
No colour			20%
Pink			High stability



The resistor color code system is all well and good but need to understand how to apply it in order to get the correct values of the resistor. The left band or the most significant colored band is the band which is nearest to a connecting lead with the color coded bands being read from left to right as follows:

Digit, Digit, Multiplier = Color Code, Color code × $10^{\text{color code}}$ in ohm (Ω)

A resistor has the following colored markings

Yellow Violet Red = 47×10^2

= 4700 Ω

The fourth and fifth bands are used to determine the percentage tolerance of the resistor. Resistor tolerance is a measure of the resistor variation from the specified resistive value and is a consequence of the manufacturing process and is expressed as a percentage of its "nominal" or preferred value.

12. Mathematical Signs and Symbols

= Equals	≈ Equals approximately
~ is the order of magnitude of	≠ is not equal to
> is greater than	>> is much greater than
< is less than	<< is much less than
± Plus or minus	≥ is greater than or equal to
≤ is less than or equal to	∝ is proportional to
Σ the sum of	$\bar{x}$ the average value of x

13. Mathematical Formulas

Geometry

Circle: Circumference = $2\pi r$, area = πr^2

Sphere: Area = $4\pi r^2$, Volume = $\frac{4}{3}\pi r^3$

Cylinder: Area = $\pi r^2 + \pi r^2 + 2\pi r l = 2\pi r^2 + 2\pi r l = 2\pi r\,(r + l)$,

l = Length of cylinder

Volume = $\pi r^2 l$

Quadratic Formula

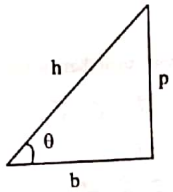
If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometric functions of angle θ

$\sin\theta = \frac{p}{h}$, $\operatorname{cosec}\theta = \frac{h}{p}$

$\cos\theta = \frac{b}{h}$, $\sec\theta = \frac{h}{b}$

$\tan\theta = \frac{p}{b}$, $\cot\theta = \frac{b}{p}$



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Trigonometric Identities

$$\sin(90-\theta)=\cos\theta,\ \cos(90-\theta)=\sin\theta,\ \frac{\sin\theta}{\cos\theta}=\tan\theta$$

$$\sin^2\theta+\cos^2\theta=1,\ \sec^2\theta-\tan^2\theta=1,\ \text{cosec}^2\theta-\cot^2\theta=1$$

$$\sin 2\theta=2\sin\theta\cos\theta,\ \cos 2\theta=\cos^2\theta-\sin^2\theta=2\cos^2\theta-1=1-2\sin^2\theta$$

$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$

$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$

$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$

Binomial Theorem

$$(1+x)^n=1+\frac{nx}{1!}+\frac{n(n-1)x^2}{2!}+\ldots\ldots\ldots\ldots\ (x^2<1)$$

Exponential Expansion

$$e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots\ldots\ldots$$

Trigonometric Expansions

$$\sin\theta=\theta-\frac{\theta^3}{3!}+\frac{\theta^5}{5!}-\ldots\ldots\ldots\ldots$$

$$\cos\theta=1-\frac{\theta^2}{2!}+\frac{\theta^4}{5!}-\ldots\ldots\ldots\ldots$$

$$\tan\theta=\theta+\frac{\theta^3}{3}+\frac{2\theta^5}{15}+\ldots\ldots\ldots\ldots$$

Products of Vectors

⇒ If $\hat{i}$, $\hat{j}$ and $\hat{k}$ be unit vectors in the x, y and z direction. Then,

$$\hat{i}\cdot\hat{i}=\hat{j}\cdot\hat{j}=\hat{k}\cdot\hat{k}=1$$
$$\hat{i}\cdot\hat{j}=\hat{j}\cdot\hat{k}=\hat{k}\cdot\hat{i}=0$$

[since $\hat{i}=(1,0,0)$ $\hat{j}=(0,1,0)$ and $\hat{k}=(0,0,1)$]

⇒ Let, θ be the smaller of the two angles between $\vec{a}$ and $\vec{b}$. Then,

$$\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}=ab\cos\theta$$

and $\vec{a}\times\vec{b}=-\vec{b}\times\vec{a}$

also, $\vec{a}\times\vec{b}=ab\sin\theta$

⇒ If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three vectors, then their scalar triple product is given by

$$\vec{a}\cdot(\vec{b}\times\vec{c})=\vec{b}\cdot(\vec{c}\times\vec{a})=\vec{c}\cdot(\vec{a}\times\vec{b})$$

And the vector triple product is given by

$$\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}$$

Derivatives and Integrals

S. No.	Derivatives	Integrals
1.	$\frac{d}{dx}(\text{constant})=0$	$\int dx=x$
2.	$\frac{d}{dx}x^n=nx^{n-1}$	$\int x^n\,dx=\frac{x^{n+1}}{n+1}\ (n\neq-1)$
3.	$\frac{d}{dx}(\ln x)=\frac{1}{x}$	$\int\frac{dx}{x}=\ln(x)$
4.	$\frac{d}{dx}(u.v)=u\frac{dv}{dx}+v\frac{du}{dx}$	$\int uv\,dx=u\int v\,dx-\int\left[\frac{du}{dx}\int v\,dx\right]dx$
5.	$\frac{d}{dx}e^x=e^x$	$\int e^x\,dx=e^x$
6.	$\frac{d}{dx}\sin x=\cos x$	$\int\sin x\,dx=-\cos x$
7.	$\frac{d}{dx}\cos x=-\sin x$	$\int\cos x\,dx=\sin x$
8.	$\frac{d}{dx}\tan x=\sec^2 x$	$\int\tan x\,dx=\ln\lvert\sec x\rvert$
9.	$\frac{d}{dx}\cot x=-\text{cosec}^2 x$	$\int e^{-nx}\,dx=-\frac{1}{n}e^{-nx}$
10.	$\frac{d}{dx}\sec x=\tan x\sec x$	Note: The letters u and v stand for any function of x, and n is a constant.
11.	$\frac{d}{dx}\text{cosec}\,x=-\cot x\,\text{cosec}\,x$	
12.	$\frac{d}{dx}e^u=e^u\frac{du}{dx}$	
13.	$\frac{d}{dx}\sin u=\cos u\frac{du}{dx}$	

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Pokhara University
Faculty of Science and Technology

Course title: **Applied Physics** (3-1-2)
Nature of the course: Theory and Practical
Year: First
Level: Bachelor

Full marks: 100
Pass marks: 45
Time per period: 1 hour
Total periods: 45
Program: BE

1. **Course Description**

This course covers the fundamental topics of physics and basic principles that are required to study other engineering courses. It develops the ability to identify, formulate and solve engineering physics problems. Moreover, it enables to formulate, conduct, analyze and interpret experiments in engineering physics through tutorials, laboratory work and self-learning activities.

2. **General Objectives**

The general objectives of this course are:

- To equip the students with the fundamental concept and laws of oscillation, electromagnetism and thermodynamics.
- To acquaint the students with waves, laser, optical fiber, quantum mechanics and enlighten the importance of capacitor and dielectrics.

3. **Methods of Instruction**

Lecture, discussion, tutorials, Laboratory work and assignments

4. **Contents in Detail**

Specific Objectives	Contents
Understand mechanical oscillations, solve problems related to different ty of oscillation, familiar with the scope various engineering fields.	**Unit 1: Mechanical Oscillation (6 hrs)** **1.1** Free oscillation, Damped oscillation and Forced oscillation (Physical meaning and equations). **1.2** Compound pendulum, Minimum and maximum time period in compound pendulum, Interchangeability of point of suspension and point of oscillation in compound pendulum, Torsion pendulum, Determination of modulus of elasticity of material using torsion pendulum.
Solve and analyze the problems related to waves	**Unit 2: Wave motion (4 hrs)** Introduction of wave, wave velocity and particle velocity, types of waves and their applications, Speed of transverse wave in stretched string, energy, power and intensity of plane progressive wave, standing wave and resonance, sonometer,.

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Solve the problems related to reverberation in different units of building. Solve the problems related ultra sound.	**Unit 3: Acoustics** **(4 hrs.)** **3.1** Classification of sound waves, Acoustics of building, Reverberation of sound, absorption coefficient, Noise pollution and its control, Sound insulation, Sabine equation. **3.2** Introduction, production and applications of ultrasonic wave. Ultrasonic method in non-destructive testing.
Understand the use of lasers in engineering sciences and solve problems related to laser and fiber optics. Apply the concept of optical fibers in communication system and sensors.	**Unit 4: Photonics** **(6 hrs.)** **4.1** **Laser:** Introduction of laser, Principles of generation of laser light (induced absorption, spontaneous emission, stimulated emission, population inversion, pumping, metastable state), He-Ne laser, Semiconductor laser, Applications of laser. **4.2** **Fiber optics:** Introduction, Types of optical fiber, Principle of propagation of light wave through optical fiber (Acceptance angle), Numerical aperture, Applications of optical fiber in communications, Optical fiber sensors.
Evaluate the capacity of capacitors to store energy with and without dielectrics. Solve problems related to electrostatics.	**Unit 5: Capacitor and Dielectric** **(6 hrs.)** **5.1** **Capacitor:** Introduction, Types of capacitor, Charging and discharging of capacitor. **5.2** **Dielectric:** Introduction, Dielectric constant, electric flux density, Polarization, Polarization in free space, Gauss law in dielectric, Electronic and Ionic polarization (Clausius-Mossotti equation).
Deal with interaction between electric field and magnetic field on matter. Analyze the relationship between electric field, magnetic field and speed of wave.	**Unit 6. Electromagnetism** **(6 hrs.)** **6.1** EM Oscillation: LC oscillation, Damped LCR oscillation, Forced em oscillation, resonance and quality factor (2hrs). **6.2** EM waves: Maxwell equations in integral form, Conversion of Maxwell's equations in differential form, Continuity equation, Relation between electric field, magnetic field and speed of light, wave equations in free space, verification of light wave as an electromagnetic wave, Wave equation in dielectric medium (4hrs).
Apply principles of quantum mechanics to investigate the observable on known wave functions. Solve the problems	**7. Quantum Mechanics** **(5 hrs.)** Inadequacy of classical mechanics, Importance of quantum mechanics, Matter wave (de-Broglie equation), Wave function and its significance, Energy and momentum operator, Time independent and time dependent Schrodinger wave equations, Application of Schrodinger wave equation for the electron in metal, Normalized wave function describing the motion of an electron inside in an infinite potential well.

Acquainted with the laws of thermodynamics and applications. Solve the problems related to thermodynamics and heat transfer.	**8. Fundamentals of thermodynamics and heat transfer (8 hrs.)** **8.1** Concepts and definition: applications of thermodynamics, properties and state of substance, thermodynamics properties and types, processes (definition, characteristics and examples): reversible and irreversible process **(2 hrs.)**. **8.2** Laws of thermodynamics: first law of thermodynamics, first law for closed system, internal and stored energy, Joules law, enthalpy, specific heat, application of first law for closed system, Related problems on closed system, second law of thermodynamics, heat engine (four components of refrigerator and heat pump, COP of refrigerator and heat pumps), Kelvin-Planck and Clausius statement of second law. **8.3** Heat transfer: modes of heat transfer (conduction, convection and radiation), statement and assumption of Fourier's law of thermal conductivity, one dimensional steady state heat conduction through plane wall, basic laws of radiation (Emissive power and emissivity, Stefan-Boltzmann's law), Concept of black bodies **(2hrs)**.

5. Tutorials

1. Solving the problems related to different oscillation.
2. Solving and analyzing the problems related to waves.
3. Determination of standard reverberation time for normal human ear and solving problems related to ultra sound.
4. Determination of angle of acceptance for working of optical fiber and finding population of atoms in different energy states.
5. Solving the problems for different combination of capacitors and finding the charging and discharging time constant for capacitor.
6. Solving the problems related to Gauss law of electrostatics.
7. Determination of frequency of damped and undammed LC oscillation and analyzing the relationship between electric field, magnetic field and speed of wave.
8. Solving the problems related to particle wave using Schrodinger's wave equations.
9. Solving the problems related to thermodynamics and heat transfer.

6. Laboratories (Any Eight):

1. To determine the acceleration due to gravity and radius of gyration of bar pendulum.
2. To determine the value of modulus of rigidity of the material given and moment of inertia of circular disc using torsion pendulum.
3. To determine the acceptance angle of an optical fiber using laser source.
4. To determine the frequency of AC mains by using sonometer apparatus.
5. To determine the wavelength of laser light by using diffraction grating

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6. To determine the capacitance of given capacitor by charging and discharging through resistor.
7. To plot a graph between current and frequency in an LRC series circuit and to find: i) the resonance frequency ii) the quality factor.
8. To determine the dielectric constant of a given material
9. To determine the Planck's constant and photoelectric work functions of the material.
10. To measure the pressure, specific volume and temperature.
11. To find out the efficiency of a compressor.
12. To measure the rate of heat, transfer by conduction
13. To measure the performance of a Refrigeration/ Heat pump

7. **Evaluation system and Students' Responsibilities** Evaluation System

In addition to the formal exam(s), the internal evaluation of a student may consist of qu assignments, lab reports, projects, class participation, etc. The tabular presentation of the inter evaluation is as follows.

External Evaluation	Marks	Internal Evaluation	Weight	Marks
Semester-End examination	50	**Theory**		
		Attendance and Class Participation	10%	3
		Assignments	20%	
		Presentations/Viva/Quizzes	10%	
		Term exam	60%	
		Practical		
		Attendance and Class Participation	10%	2
		Report	10%	
		Viva	20%	
		Exam	60%	
		Total Internal		5
		Full Marks: 50 + 50 = 100		

Student Responsibilities

Each student must secure at least 45% marks in internal evaluation with 80% attendance in the class order to appear in the Semester End Examination. Failing to get such score will be given NOT QUALIFIED (NQ) and the student will not be eligible to appear the Semester-End Examination Students are advised to attend all the classes, formal exam, test, etc. and complete all the assignment within the specified time period. Students are required to complete all the requirements defined for th completion of the course.

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