Questions

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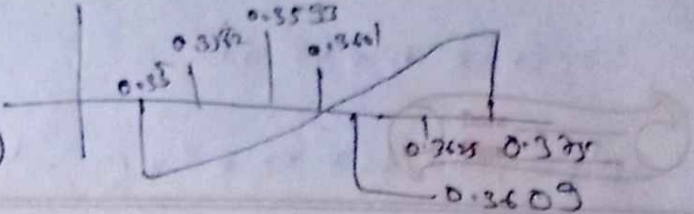


* Solve $3x + \sin x - e^x = 0$ by bisection method correct to 2 d.p.

Sol^n Let $F(x) = 3x + \sin x - e^x$

x	0	0.25	0.5	0.375	0.35
$F(x)$	-1	-0.286	0.330	0.036	-0.026

let $x_0 = 0.35$, $x_1 = 0.375$

$f(x_0) = f(0.35) = -0.026$ $\quad F(x_1) = f(0.375) = 0.036$

$f(x_0) \times f(x_1) < 0$

$$x_2 = \frac{x_0 + x_1}{2} = 0.3625$$

$F(x_2) = f(0.3625) = 0.0052$

$$x_3 = \frac{x_2 + x_0}{2} = 0.3562$$

$F(x_3) = -0.010$ $\quad$ As, $f(x_0) \times f(x_3) > 0$,

$$x_4 = \frac{0.3562 + 0.3625}{2}$$

$= 0.3593$

$f(x_4) = -0.0026$

$$x_5 = \frac{0.3593 + 0.3625}{2} = 0.3609$$

$F(x_5) = 0.001$

$$x_6 = \frac{0.3593 + 0.3609}{2} = 0.3601$$

$f(x_6) = -0.0008$

$\therefore$ Required root correct to dp is $x = 0.36$

Second method

Solve: $3x + \sin x - e^x = 0$ upto 4 dp

Solution:

let $F(x) = 3x + \sin x - e^x$

let $x_0 = 0.36$, $x_1 = 0.37$

$F_0 = F(x_0) = f(0.36) = -0.001055$

$F_1 = F(x_1) = f(0.37) = 0.023880$

$x_2 = x_1 - \frac{F_1 (x_1 - x_0)}{F_1 - F_0} = 0.37 - \frac{0.023880 (x_1 - x_0)}{(0.023880 + 0.001055)}$

$x_2 = 0.360423$

$F_2 = F(x_2) = f(0.360423) = 0.000003$

$x_3 = x_2 - \frac{f_2}{f_2 - f_1} (x_2 - x_1)$

$= 0.360423 - \frac{0.000003 (0.360423 - 0.37)}{(0.000003 - 0.023880)}$

$= 0.360421$

$\therefore$ Required root corrected upto 4 dp is

$x = 0.3604$

* $\log x - \cos x = 0$ upto 4 dp

Solution:

let, $f(x) = \log x - \cos x$

x	1	2	1.43	1.44
$F(x)$	-0.540302	0.717176	0.013004	0.027939

let, $x_0 = 1.43$, $f_0 = 0.015004$

$x_1 = 1.44$, $f_1 = 0.027939$

$x_2 = x_1 - \frac{f_1}{f_1 - f_0} (x_1 - x_0) = 1.44 - \frac{0.027939 (1.44 - 1.43)}{0.027939 - 0.015004}$

$x_2 = 1.418400$

$F_2 = F(x_2) = f(1.418400) = -0.000008$

$x_3 = x_2 - \frac{f_2}{f_2 - f_1} (x_2 - x_1) = 1.418400 - \frac{(-0.000008)(1.418400 - 1.44)}{(-0.000008 - 0.027939)}$

$x_3 = 1.418406$

$\therefore$ Required root $= 1.4184$

* $\tan x + \tanh x = 0$

Solution:

let $F(x) = \tan x + \tanh x$

let $x_0 = 0.1$, $x_1 = 0.01$

$F_0 = F(x_0) = 0.200002$

$F_1 = f(x_1) = 0.020000$

$$x_2 = x_1 - \frac{f_1}{f_1 - f_0}(x_1 - x_0)$$

$$= 0.01 - \frac{0.020000(0.01 - 0.1)}{(0.020000 - 0.200002)}$$

$= 0.000000$

$F_2 = F(x_2) = 0$

$\therefore$ Required root $= 0$

Q. $x - \cos x = 0$

solution:

let $F(x) = x - \cos x$

x	0	1	0.74
$f(x)$	-1	0	0.0015

let $x_0 = 0.74$, $x_1 = 0.75$

$f_0 = F(x_0) = F(0.74) = 0.001531$

$F_1 = F(x_1) = F(0.75) = 0.018311$

$$x_2 = x_1 - \frac{f_1}{f_1 - f_0}(x_1 - x_0)$$

$$= 0.75 - \frac{0.018311(0.75 - 0.74)}{0.018311 - 0.001531}$$

$= 0.739088$

$F_2 = F(x_2) = f(0.739088) = 0.000004$

$$x_3 = x_2 - \frac{f_2}{f_2 - f_1}(x_2 - x_1)$$

$$= 0.739088 - \frac{0.000004(0.739088 - 0.75)}{0.000004 - 0.018311}$$

$= 0.739064$

$\therefore$ Required root $= 0.7390$

Q. $x^3 + x^2 - 100 = 0$

Solution:

$f(x) = x^3 + x^2 - 100$

x	4	5	4.5	4.34
$f(x)$	-20	50	11.375	0.582104

Let $x_0 = 4.34$, $x_1 = 4.35$

$F_0 = F(x_0) = 0.582104$, $f_1 = F(x_1) = 1.235375$

$$x_2 = x_1 - \frac{f_1}{f_1 - f_0}(x_1 - x_0)$$

$$= 4.35 - \frac{1.235375(4.35 - 4.34)}{1.235375 - 0.582104}$$

$x_2 = 4.331089$

$F_2 = F(4.331089) = 0.002337$

$$x_3 = x_2 - \frac{f_2}{f_2 - f_1}(x_2 - x_1) = 4.331089 - \frac{0.002337(4.331089 - 4.35)}{0.002337 - 1.235375}$$

$x_3 = 4.331053$

$\therefore$ Required root $= 4.3310$

$\log_e = \ln$
$\log_{10} = \log$

Q $x\log_{10}x - 1.2 = 0$
solution

x	1	2	2.5	3	2.75
$f(x)$	-1.2	-0.597940	-0.205149	0.231363	0.008164

let $x_0 = 2.75$
$x_1 = 2.76$
$f_0 = f(x_0) = 0.008164$, $f_1 = f(x_1) = 0.016909$

$$x_2 = x_1 - \frac{f_1}{f_1 - f_0}(x_1 - x_0)$$

$$= 2.76 - \frac{0.016909\,(2.76 - 2.75)}{(0.016909 - 0.008164)}$$

$= 2.740664$

$f_2 = f(x_2) = f(2.740664)$
$= 0.000015$

$$x_3 = x_2 - \frac{f_2}{f_2 - f_1}(x_2 - x_1)$$

$$= 2.740664 - \frac{0.000015\,(2.740664 - 2.76)}{0.000016 - 0.016909}$$

$= 2.740646$

$\therefore$ Required root $= 2.7406$ //

Newton-Raphson Method

* $3x + \sin x - e^x = 0$
solution:
Let $f(x) = 3x + \sin x - e^x$, $f'(x) = 3 + \cos x - e^x$
Let $x_0 = 0.36$
$f_0 = f(x_0) = f(0.36) = -0.00055$
$f_0' = f'(x_0) = f'(0.36) = 2.302567$
we have,
By, Newton-Raphson formula

$$x_{n+1} = x_n - \frac{f_n}{f_n'}$$

$$x_2 = x_1 - \frac{f_1}{f_1'} = 0.360421 + \frac{0.000001}{2.501815}$$

$x_2 = 0.360421$

$f_1 = f(x_1) = f(0.360401) = 0.000001$
$f_1' = f'(x_1) = f'(0.360421) = 2.501815$

$\therefore$ Required root correct to 4 dp $x = 0.3604$

* $x\log_{10}x = 3$
solution:

let $f(x) = x\log_{10}x - 3$

x	1	5	4.5
$f(x)$	-3	0.494850	-0.060543

$F'(x) = x\frac{d}{dx}(\log_{10}x) + \log_{10}x \frac{d}{dx}(x)$
$= x \cdot \frac{1}{x}\log_{10}e + \log_{10}x$
$= 0.434294 + \log_{10}x$

let $x_0 = 4.5$
$F_0 = F(x_0) = F(4.5) = -0.060543$
$F_0' = F'(x_0) = 1.087506$

let $x_1 = 4.555671$
$F_1 = F(x_1) = 0.000147$
$F_1' = F'(x_1) = F'(4.555671) = 1.092846$

$x_2 = x_1 - \frac{f_1}{f_1'} = 4.555671 - \frac{0.000147}{1.092846}$
$x_2 = 4.555536$

$F_2 = F(x_2) = F(4.555536) = 0.0000003$
$F_2' = F'(x_2) = F'(4.555536) = 1.092833$

$x_3 = x_2 - \frac{f_2}{f_2'} = 4.555536 - \frac{0.000000}{1.092833}$
$x_3 = 4.555536$

∴ Root correct to 4 dp is 4.5555

* $3x^2 = 1+\sin x$
solution:
$F(x) = 3x^2 - 1 - \sin x$

x	0	0.5	0.75	0.5
F(x)	-1	-0.729425	0.005861	-0.729425

$F' = 6x - \cos x$

log rate variation

let $x_0 = 0.75$
$f_0 = F(x_0) = F(0.75) = 0.005861$
$F_0' = F'(0.75) = 3.768311$

$x_1 = x_0 - \frac{f_0}{f_0'} = 0.75 - \frac{0.005861}{3.768311}$
$= 0.748444$

$F_1 = F(x_1) = F(0.748444) = 0.000006$
$F_1' = F'(x_1) = 3.757915$

$x_2 = x_1 - \frac{f_1}{f_1'} = 0.748444 - \frac{0.000006}{3.757915}$
$x_2 = 0.748442$

∴ Root = 0.7484

* $\log x - \cos x = 0$
solution:
let $F(x) = \log x - \cos x$

x	0.25	1	1.25	1.5
F(x)	-2.355206	-0.540302	-0.092178	0.334727

let $x_0 = 1.3$, $F_0 = F(x_0) = F(1.3) = -0.005134$

$F'(x) = \frac{1}{x} + \sin x$

$F_0' = F_0'(x) = 1.732788$

$$x_1 = x_0 - \frac{f_0}{f_0'}$$

$$= 1.3 + \frac{0.005134}{1.732788} = 1.302962$$

$f_1 = f(x_1) = -0.000003$

$f_1' = f_1'(x_1) = 1.731828$

$$x_2 = x_1 - \frac{f_1}{f_1'} = 1.302962 + \frac{0.000003}{1.731828}$$

$$= 1.302963$$

$\therefore$ Root $= 1.3029$

Partial Differential Equation

Any equation in which partial derivative is involved is called partial differential equation. For example; $\frac{\partial f}{\partial x} = 1$

Overview of derivative

$$\frac{\partial f}{\partial x} = \frac{f(x+h) - f(x)}{h}$$

$$\frac{\partial f(x_i)}{\partial x} = \frac{f(x_i+h) - f(x_i)}{h} = \frac{f(x_{i+1}) - f(x_i)}{h}$$

$$\frac{\partial}{\partial x} f(x_i) = \frac{f_{i+1} - f_i}{h}$$

$$f''(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

$$f''(x_i) = \frac{f(x_i+h) - 2f(x_i) + f(x_i-h)}{h^2}$$

$$f''(x_i) = \frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1})}{h^2}$$

$$f''(x_i) = \frac{f_{i+1} - 2f_i + f_{i-1}}{h^2}$$

$$\frac{\partial^2 f(x,y)}{\partial x^2} = \frac{f(x+h,y) - 2f(x,y) + f(x-h,y)}{h^2}$$

$x_{i+1} - x_i = h$

$x_{i+1} = x_i + h$

$$\frac{\partial^2}{\partial x^2} f(x_i, y_j) = \frac{f(x_i + h, y_j) - 2f(x_i, y_j) + f(x_i - h, y_j)}{h^2}$$

$$\frac{\partial^2}{\partial x^2} f(x_i, y_j) = \frac{f(x_{i+1}, y_j) - 2f(x_i, y_j) + f(x_{i-1}, y_j)}{h^2}$$

$$\boxed{\frac{\partial^2}{\partial x^2} f(x_i, y_j) = \frac{f_{i+1,j} - 2f_{i,j} + f_{i-1,j}}{h^2}}$$

$$\frac{\partial^2}{\partial y^2} f(x_i, y_j) = \frac{f(x_i, y_j + h) - 2f(x_i, y_j) + f(x_i, y_j - h)}{h^2}$$

$$= \frac{f(x_i, y_{j+1}) - 2f(x_i, y_j) + f(x_i, y_{j-1})}{h^2}$$

$$\boxed{\frac{\partial^2}{\partial y^2} f(x_i, y_j) = \frac{f_{i,j+1} - 2f_{i,j} + f_{i,j-1}}{h^2}}$$

Laplace EQUATION

A partial differential equation of a form $\nabla^2 f = 0$ where, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$

nabla

is called laplace's equation.

or, $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) f = 0$ — (1)

or, $\frac{\partial^2}{\partial x^2} f(x_i, y_j) + \frac{\partial^2}{\partial y^2} f(x_i, y_j) = 0$

or, $\frac{f_{i+1,j} - 2f_{i,j} + f_{i-1,j}}{h^2} + \frac{f_{i,j+1} - 2f_{i,j} + f_{i,j-1}}{h^2} = 0$

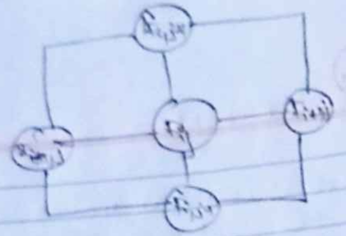
or, $f_{i+1,j} - 4f_{i,j} + f_{i-1,j} + f_{i,j+1} + f_{i,j-1} = 0$

or, $4f_{i,j} = f_{i+1,j} + f_{i-1,j} + f_{i,j+1} + f_{i,j-1}$

or, $f_{i,j} = \frac{1}{4}\left(f_{i+1,j} + f_{i-1,j} + f_{i,j+1} + f_{i,j-1}\right)$

which is laplace five point formula. — (2)

The above formula (2) can be expressed to figure as



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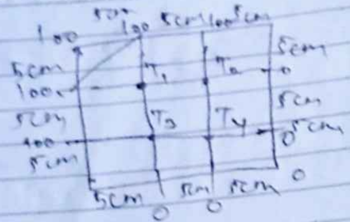

Using laplace formula at

$T_1 = \frac{1}{4}[100 + T_2 + T_3 + 100]$

$\therefore T_1 = 50 + 0.25T_2 + 0.25T_3$ — (1)

$T_2 = \frac{1}{4}[T_1 + 0 + 100 + T_4]$

$\therefore T_2 = 25 + 0.25T_1 + 0.25T_4$ — (2)

$T_3 = \frac{1}{4}[100 + T_4 + 0 + T_1]$

$\therefore T_3 = 25 + 0.25T_4 + 0.25T_1$ — (3)

$T_4 = \frac{1}{4}[T_3 + 0 + T_2 + 0]$

$T_4 = 0.25T_3 + 0.25T_2$ — (4)

(piradi 4 ta digit rakhne standard)

Let $T_1 = 100$, $T_2 = 50$, $T_3 = 50$, $T_4 = 0$

The table of iteration is formed as below:

No. of iteration	T_1	T_2	T_3	T_4
0	100	50	50	0
1	75	43.75	43.75	21.875
2	~~67.125~~ 71.875	48.4375	48.4375	24.21875
3	74.21875	49.609375	49.609375	24.8046875
4	74.8046875	49.9023438	49.9023438	24.95117031
5	74.9511	49.9755	49.9755	24.9877
6	74.9877	49.9938	49.9938	24.9965

$\therefore T_1 = 74.9877 \approx 75$

$T_2 = 49.9938 \approx 50$

$T_3 = 49.9938 \approx 50$

$T_4 = 24.9965 \approx 25$

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NR continue

* $x^3+x^2-100=0$

soln

let $F(x)=x^3+x^2-100$

$F'(x)=3x^2+2x$

x	0	5	4	4.5	4.25	4.37
$F(x)$	-100	50	-20	11.375	-5.171875	2.550353

let $x_0=4.37$

let $f_0=F(x_0)=F(4.37)=2.550353$

$f_0'=F'(x_0)=f'(4.37)=66.0307$

we have,

$x_1=x_0-\frac{f_0}{f_0'}=4.37-\frac{2.550353}{66.0307}=4.331376$

$f_1=F(x_1)=F(4.331376)=0.020975$

$f_1'=f'(x_1)=f'(4.331376)=64.945206$

$x_2=x_1-\frac{f_1}{f_1'}=4.331376-\frac{0.020975}{64.945206}$

$x_2=4.331053$

$f_2=F(x_2)=F(4.331053)=-0.000000$

$F_2'=F'(x_2)=F'(4.331053)=64.936166$

$x_3=4.331053+\frac{0.000000}{64.936166}=4.331053$

$\therefore$ Required root = 4.3310

* $xe^x-\cos x=0$

solution:

let $F(x)=xe^x-\cos x$

$F'(x)=xe^x+e^x+\sin x$

x	0	1	0.5
$F(x)$	-1	2.177979	-0.053221

let $x_0=0.5$

$f_0=F(x_0)=F(0.5)=-0.053221$

$f_0'=F'(x_0)=F'(0.5)=2.952507$

we have,

$x_1=x_0-\frac{f_0}{f_0'}=0.5+\frac{0.053221}{2.952507}=0.518025$

$f_1=F(x_1)=F(0.518025)=0.000814$

$f_1'=f_1'(x_1)=f_1'(0.518025)=3.043487$

$x_2=x_1-\frac{f_1}{f_1'}=0.518025-\frac{0.000814}{3.043487}=0.517757$

$F_2=F(x_2)=-0.000001$

$F_2'=f'(x_2)=3.042121$

$\therefore x_3=x_2-\frac{f_2}{f_2'}=0.517757+\frac{0.000001}{3.042121}=0.517757$

$\therefore$ Root = 0.5177 //

* $x - \cos x = 0$

Solution.

$f(x) = x - \cos x \qquad f'(x) = 1 + \sin x$

x	0	1	0.5	0.75
$f(x)$	−1	0.459697	−0.377582	0.018311

let $x_0 = 0.75$

$f_0 = f(x_0) = f(0.75) = 0.018311$

$f_0' = f'(x_0) = f'(0.75) = 1.681638$

$x_1 = x_0 - \frac{f_0}{f_0'} = 0.75 - \frac{0.018311}{1.681638} = 0.739111$

$f_1 = f(x_1) = f(0.739111) = 0.000043$

$f_1' = f_1'(x_1) = f'(0.739111) = 1.673631$

$x_2 = x_1 - \frac{f_1}{f_1'} = 0.739111 - \frac{0.000043}{1.673631} = 0.739085$

$f_2 = f(x_2) = f(0.739085) = -0.000000$

$f_2' = f'(x_2) = f'(0.739085) = 1.673611$

$x_3 = x_2 - \frac{f_2}{f_2'} = 0.739085 + \frac{0.000000}{1.673611}$

$= 0.739085$

$\therefore$ Root $= 0.7390$ //

* Solve by fixed point iterative method
* $2x = \cos x + 3$

Solution:

$x = \frac{\cos x + 3}{2} \qquad \therefore g(x) = \frac{\cos x + 3}{2}$

We have, $x_{n+1} = g(x_n)$

$x_1 = g(x_0)$

let $x_0 = 1.53$

$\therefore x_1 = g(1.53) = 1.520392$

$\therefore x_2 = g(1.520392) = 1.525191$

$\therefore x_3 = g(1.525191) = 1.522794$

$\therefore x_4 = g(1.522794) = 1.523991$

$\therefore x_5 = g(1.523991) = 1.523394$

$\therefore x_6 = g(1.523394) = 1.523692$

$\therefore x_7 = g(1.523692) = 1.523543$

$\therefore x_8 = g(1.523543) = 1.523617$

$\therefore x_9 = g(1.523617) = 1.523580$

$\therefore x_{10} = g(1.523580) = 1.523599$

$\therefore$ Root $= 1.5235$ //

Q. Find the square root of 7 by fixed point iterative method correct to 3 dp.

Note: $x^2 = 7$

$x^3 + x^2 + x = 7x + x^2 + x$

$x(x^2+x+1) = 7 + x^2 + x$

$x = \frac{7 + x^2 + x}{x^2+x+1}$

$g(x) = \frac{7+x^2+x}{x^2+x+1} = \frac{6}{x^2+x+1} + \frac{x^2+x+1}{x^2+x+1}$

$g(x) = 1 + \frac{6}{x^2+x+1}$

Solution:

Let $x^2 = 7$ — (i)

$x = 7/x$

$g(x) = 7/x$

$x_{n+1} = g(x_n)$

let $x_0 = 2.64$

$\therefore x_1 = g(x_0) = g(2.64) = 2.651515$

$\therefore x_2 = g(x_1) = g(2.651515) = 2.640000$

$\therefore x_3 = g(x_2) = g(2.640000) = 2.651515$

Now, we entered a loop so, it cannot find fixed point. So, arranging (i) as

$x^2 + x = 7 + x$

$x(x+1) = 7 + x$

$\therefore x = \frac{7+x}{x+1} = \frac{6+x+1}{x+1} = \frac{6}{x+1} + 1$

$\therefore x = 1 + \frac{6}{x+1} \quad \therefore g(x) = 1 + \frac{6}{x+1}$

we have,

$x_{n+1} = g(x_n)$

$x_1 = g(x_0)$

let $x_0 = 2.64$

$\therefore x_1 = g(x_0) = g(2.64) = 2.64835$

$\therefore x_2 = g(2.64835) = 2.644579$

$\therefore x_3 = g(2.644579) = 2.646280$

$\therefore x_4 = g(2.646280) = 2.645512$

$\therefore x_5 = g(2.645512) = 2.645859$

$\therefore$ Root $= 2.645$

* Solve $3x + \sin x - e^x = 0$

Solution,

$3x = e^x - \sin x$

$x = \frac{1}{3}(e^x - \sin x)$

$g(x) = \frac{1}{3}(e^x - \sin x)$

let $x_0 = 0.36$

$x_{n+1} = g(x_n)$

$x_1 = g(x_0)$

$\therefore x_1 = g(0.36) = 0.360351$

$\therefore x_2 = g(0.360351) = 0.360409$

$\therefore$ Root $= 0.360$

Interpolation

$F(x) = 3x^2 - 2x + 1$

x	0	2	4
$F(x)$	1	9	41
	x_0, f_0	x_1, f_1	x_2, f_2

Find lagrange's interpolating polynomial to the following observations and find $F(1)$

Solution:

We have, lagrange's interpolating polynomial as

$$F(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} f_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} f_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} f_2$$

$$= \frac{(x-2)(x-4)}{(0-2)(0-4)} \times 1 + \frac{(x-0)(x-4)}{(2-0)(2-4)} \times 9 + \frac{(x-0)(x-2)}{(4-0)(4-2)} \times 41$$

$$= \frac{x^2-4x-2x+8}{8} + \frac{(x^2-4x)9}{-4} + \frac{(x^2-2x)41}{8}$$

$$= \frac{x^2-6x+8}{8} - \frac{9x^2-36x}{4} + \frac{41x^2-82x}{8}$$

$$= \frac{x^2-6x+8-18x^2+72x+41x^2-82x}{8}$$

$$= \frac{24x^2-16x+8}{8}$$

$$= 3x^2-2x+1$$

$\therefore F(x) = 3x^2-2x+1$

$\therefore F(1) = 2$

* Find lagrange's interpolating polynomial to the following observations, and find $F(3.5)$

x	1	2	3	4	5
$F(x)$	-3	4	23	60	121
	x_0, f_0	x_1, f_1	x_2, f_2	x_3, f_3	x_4, f_4

Solution:

$$F(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} f_0 + \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)} (f_2)$$

$$+ \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)} f_1 + \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)} f_3 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)} f_4$$

$$= \frac{(x-2)(x-3)(x-4)(x-5)}{(1-2)(1-3)(1-4)(1-5)} (-3) + \frac{(x-1)(x-2)(x-4)(x-5)}{(3-1)(3-2)(3-4)(3-5)} (23)$$

$$+ \frac{(x-1)(x-3)(x-4)(x-5)}{(2-1)(2-3)(2-4)(2-5)} 4 +$$

$$\frac{(x-1)(x-2)(x-3)(x-5)}{(4-1)(4-2)(4-3)(4-5)} 60 +$$

$$\frac{(x-1)(x-2)(x-3)(x-4)}{(5-1)(5-2)(5-3)(5-4)} 121$$

$$= \frac{(x^2-3x-2x+6)(x^2-5x-4x+20)}{24} (-3) +$$

$$\frac{(x^2-2x-x+2)(x^2-5x-4x+20)}{4} 23$$

$$+ \frac{(x^2-3x-x+3)(x^2-5x-4x+20)}{-6} 4$$

$$+ \frac{(x^2-2x-x+2)(x^2-5x-3x+15)}{-6} 60$$

$$+ \frac{(x^2-2x-x+2)(x^2-4x-3x+12)}{24} 14$$

solving, x^3-4 //

Poisson's Equation

A partial differential equation of the form

$$\nabla^2 f = g \quad \text{(i)}$$

where, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is called poisson's equation.

or, $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) f = g$

or, $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = g$

or, $\frac{\partial^2}{\partial x^2} f(x_i, y_j) + \frac{\partial^2}{\partial y^2} F(x_i, y_j) = g(x_i, y_j)$

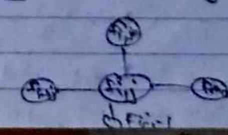
or, $\frac{F_{i+1,j} - 2F_{i,j} + F_{i-1,j}}{h^2} + \frac{F_{i,j+1} - 2F_{i,j} - F_{i,j-1}}{h^2}$

$= g(x_i, y_j)$

or, $F_{i+1,j} - 4F_{i,j} + F_{i-1,j} + F_{i,j-1} = h^2 g(x_i, y_j)$

or, $F_{i+1,j} + F_{i-1,j} + F_{i,j+1} + F_{i,j-1} - h^2 g(x_i, y_j) = 4F_{i,j}$

$$F_{i,j} = \frac{1}{4}\left[F_{i+1,j} + F_{i-1,j} + F_{i,j+1} + F_{i,j-1} - h^2 g(x_i, y_j)\right]$$

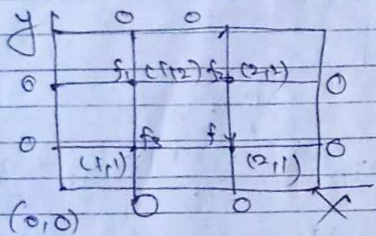


6(b) 2022 New

Solve $\nabla^2 f = -10(x^2+y^2+10)$ over the square with $0 \le x \le 3$, $0 \le y \le 3$ and $f=0$ on the boundary use $h=1$.

Solution:

According to the question the figure is formed as



$g(x,y) = -10(x^2+y^2+10)$

$g(1,1) = -120$

$g(2,1) = -150$

$g(1,2) = -150$

$g(2,2) = -180$

Using poisson's formula at

$$f_1 = \frac{1}{4}\left[0 + f_2 + 0 + f_3 - h^2 g(1,2)\right]$$

$$f_1 = 1/4\left[f_2 + f_3 - g(1,2)\right]$$

$$f_1 = 1/4\left[f_2 + f_3 + 150\right]$$

$$f_1 = 37.5 + 0.25 f_2 + 0.25 f_3 \quad \text{--- (1)}$$

$$f_2 = \frac{1}{4}\left[f_1 + 0 + 0 + f_4 - h^2 g(2,2)\right]$$

$$= \frac{1}{4}\left[f_1 + f_4 + 180\right]$$

$$f_2 = 45 + 0.25 f_1 + 0.25 f_4 \quad \text{--- (2)}$$

$$f_3 = 1/4\left[0 + f_4 + f_1 + 0 - h^2 g(1,1)\right]$$

$$= 1/4\left[f_4 + f_1 + 120\right]$$

$$f_3 = 30 + 0.25 f_1 + 0.25 f_4 \quad \text{--- (3)}$$

$$f_4 = 1/4\left[f_3 + 0 + f_2 + 0 - h^2 g(2,1)\right]$$

$$f_4 = 1/4\left[f_3 + f_2 + 150\right]$$

$$f_4 = 37.5 + 0.25 f_2 + 0.25 f_3 \quad \text{--- (4)}$$

Let $f_1 = 0$, $f_2 = 0$, $f_3 = 0$, $f_4 = 0$

The table of iteration is formed as below:

No. of iterations	f_1	f_2	f_3	f_4
0	0	0	0	0
1	37.5	54.375	39.375	60.9375
2	60.9375	75.4687	80.4687	71.4843
3	71.4843	80.7421	65.7421	74.1210
4	74.1212	82.0605	67.0605	74.7802
5	74.7802	82.3901	67.3901	74.9450
6	74.950	82.4725	67.4725	74.9862

(6)

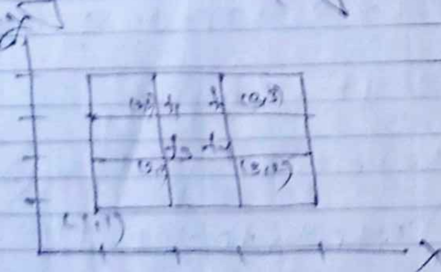
(b) Solution

$\nabla^2 f = 2x^2 + y$

$1 \le x \le 4$, $1 \le y \le 4$ with $f = 0$ on the boundary

$h = k = 1$

According to question, figure is formed as



$g(x,y) = 2x^2 + y$

$g(2,2) = 10$

$g(2,3) = 11$

$g(3,2) = 20$

$g(3,3) = 21$

Using poisson's equation at

$f_1 = \frac{1}{4}[0 + f_2 + 0 + f_3 - 1^2 g(2,3)]$

$f_1 = \frac{1}{4}[f_2 + f_3 - 11]$

$= 0.25 f_2 + 0.25 f_3 - 2.75$

$f_2 = \frac{1}{4}[f_1 + 0 + 0 + f_4 - h^2 g(3,3)]$

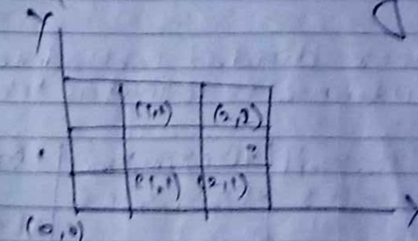
$= \frac{1}{4}[f_1 + f_4 - 21]$

Now solve as previously

6 (a) 2021 Fall

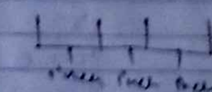
$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = -2$

$\phi = 0$ on the boundary



$h = 1$ (as inches are taken (a))

i.e.



$g(x,y) = -2$

$g(1,1) = -2$

$g(2,1) = -2$

$g(2,2) = -2$

$g(1,2) = -2$

Continuing solve

$F(x) = x^3 - x^2 + 2x + 1$

* Find interpolating polynomial to the following observations.

x	0	3	4	6
$f(x)$	1	25	57	193

Solution:

By Newton's divided method table of divided difference is formed as below.

x_i	f_i	$f[x_i, x_{i+1}]$	$f[x_i, x_{i+1}, x_{i+2}]$	$f[x_i, x_{i+1}, x_{i+2}, x_{i+3}]$
$x_0 = 0$	$f_0 = 1$	$f[x_0, x_1] = 8$	$f[x_0, x_1, x_2] = 6$	$f[x_0, x_1, x_2, x_3] = 1$
$x_1 = 3$	$f_1 = 25$	$f[x_1, x_2] = 32$	$f[x_1, x_2, x_3] = 12$	
$x_2 = 4$	$f_2 = 57$	$f[x_2, x_3] = 68$		
$x_3 = 6$	$f_3 = 193$			

$$f[x_0, x_1] = \frac{f_1 - f_0}{x_1 - x_0} = \frac{25-1}{3-0} = 8$$

$$f[x_1, x_2] = \frac{f_2 - f_1}{x_2 - x_1} = \frac{57-25}{4-3} = 32$$

$$f[x_2, x_3] = \frac{f_3 - f_2}{x_3 - x_2} = \frac{193-57}{6-4} = 68$$

$$f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0} = \frac{32-8}{4-0} = 6$$

$$f[x_1, x_2, x_3] = \frac{f[x_2, x_3] - f[x_1, x_2]}{x_3 - x_1} = \frac{68-32}{3} = 12$$

$$f[x_0, x_1, x_2, x_3] = \frac{f[x_1, x_2, x_3] - f[x_0, x_1, x_2]}{x_3 - x_0} = \frac{12-6}{6-0} = 1$$

We have,

Newton's interpolating polynomial by divided difference method is,

$$f(x) = f_0 + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1) + f[x_0, x_1, x_2, x_3](x - x_0)(x - x_1)(x - x_2)$$

$$= 1 + 8(x-0) + 6(x-0)(x-3) + 1(x-0)(x-3)(x-4)$$

$$= 1 + 8x + 6(x^2 - 3x) + x(x^2 - 7x + 12)$$

$$f(x) = x^3 - x^2 + 2x + 1$$

which is required interpolating polynomial.

Q. Find interpolating polynomial to the following observations

x	3.2	2.7	1	4.8	5.6
$F(x)$	22	17.8	14.2	38.3	51.7

by Newton's divided difference method and find f(5) and f(3)

Vetai rearrange gomo parxa

x_i	f_i	$F[x_i,x_{i+1}]$	$F[x_i,x_{i+1},x_{i+2}]$	$F[x_i,x_{i+1},x_{i+2},x_{i+3}]$	$F[x_i,x_{i+1},x_{i+2},x_{i+3},x_{i+4}]$
$x_0=1$	14.2	$F[x_0,x_1]=2.1176$	$F[x_0,x_1,x_2]=2.8556$	$F[x_0,x_1,x_2,x_3]=-0.5275$	$F[x_0,x_1,x_2,x_3,x_4]=0.25$[illegible]
$x_1=2.7$	17.8	$F[x_1,x_2]=8.4$	$F[x_1,x_2,x_3]=0.8511$	$F[x_1,x_2,x_3,x_4]=0.6493$	
$x_2=3.2$	22	$F[x_2,x_3]=10.875$	$F[x_2,x_3,x_4]=2.7343$		
$x_3=4.8$	38.3	$F[x_3,x_4]=16.75$			
$x_4=5.6$	51.7				

$$F[x_0,x_1]=\frac{f_1-f_0}{x_1-x_0}=\frac{17.8-14.2}{2.7-1}$$

$$F[x_1,x_2]=\frac{f_2-f_1}{x_2-x_1}=\frac{22-17.8}{3.2-2.7}$$

$$F[x_2,x_3]=\frac{f_3-f_2}{x_3-x_2}=\frac{38.3-22}{4.8-3.2}$$

$$F[x_3,x_4]=\frac{f_4-f_3}{x_4-x_3}=\frac{51.7-38.3}{5.6-4.8}$$

$$F[x_0,x_1,x_2]=\frac{F[x_1,x_2]-F[x_0,x_1]}{x_2-x_0}$$

$$=\frac{8.4-2.1176}{3.2-1}=2.8556$$

$$F[x_1,x_2,x_3]=\frac{F[x_2,x_3]-F[x_1,x_2]}{x_3-x_1}$$

$$=\frac{10.875-8.4}{4.8-2.7}=$$

$$F[x_2,x_3,x_4]=\frac{F[x_3,x_4]-F[x_2,x_3]}{x_4-x_2}$$

$$=\frac{16.75-10.875}{5.6-3.2}$$

$$F[x_0,x_1,x_2,x_3]=\frac{F[x_1,x_2,x_3]-F[x_0,x_1,x_2]}{x_3-x_0}$$

$$=-0.5275$$

$$F[x_1,x_2,x_3,x_4]=\frac{F[x_2,x_3,x_4]-F[x_1,x_2,x_3]}{x_4-x_1}$$

$$=0.6493$$

$$f[x_0, x_1, x_2, x_3, x_4] = \frac{f[x_1, x_2, x_3, x_4] - f[x_0, x_1, x_2, x_3]}{x_4 - x_0}$$

$$= \frac{0.6403 + 0.5275}{5.6 - 1}$$

$$= 0.2358$$

We have Newton's interpolating polynomial by divided difference method is

$$f(x) = f_0 + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1) + f[x_0, x_1, x_2, x_3](x - x_0)(x - x_1)(x - x_2) + f[x_0, x_1, x_2, x_3, x_4](x - x_0)(x - x_1)(x - x_2)(x - x_3)$$

$$f(x) = 14.2 + 2.1176(x - 1) + 2.8557(x - 1)(x - 2.7) - 0.5275(x - 1)(x - 2.7)(x - 3.2) + 0.2358(x - 1)(x - 2.7)(x - 3.2)(x - 4.8)$$

which is the required interpolating polynomial

$f(5) = 20.267$

$f(5) =$ ~~41.0532 58.5245~~ 41.0532

---

(5) Find cubic interpolating polynomial to the following observations

x	1	2.7	3.2	4.8	5.6
$f(x)$	14.2	17.8	22	38.3	51.7

x_3 samma matra garne yo case ma $x_4 = 5.6$ ignore garera solve garne

Q. The population of Nepal in previous decades is as below:

No. of years (x)	2040	2050	2060	2070	2080
population $f(x)$	180	200	245	285	305

If the population remain unchanged, population is the function of time (x).

Find population of 2055 by Newton forward interpolating polynomial.

Solution:

The forward difference table is formed as below:

	x_i	f_i	Δf_i	$\Delta^2 f_i$	$\Delta^3 f_i$	$\Delta^4 f_i$
x_0	2040	$f_0 = 180$	$\Delta f_0 = 20$	$\Delta^2 f_0 = 25$	$\Delta^3 f_0 = -30$	$\Delta^4 f_0 = 15$
x_1	2050	$f_1 = 200$	$\Delta f_1 = 45$	$\Delta^2 f_1 = -5$	$\Delta^3 f_1 = -15$	
x_2	2060	$f_2 = 245$	$\Delta f_2 = 40$	$\Delta^2 f_2 = -20$		
x_3	2070	$f_3 = 285$	$\Delta f_3 = 20$			
x_4	2080	$f_4 = 305$				

$h = x_1 - x_0 = 2050 - 2040 = 10$

$s = \frac{x - x_0}{h} = \frac{x - 2040}{10} = \left(\frac{x}{10} - 204\right)$

we have, Newton's forward interpolating polynomial as,

$$F(x) = F_0 + s\Delta f_0 + \frac{s(s-1)}{2!}\Delta^2 f_0 + \frac{s(s-1)(s-2)}{3}\Delta^3 f_0 + \frac{s(s-1)(s-2)(s-3)}{4!}\Delta^4 f_0$$

$$F(x) = 180 + \left(\frac{x}{10} - 204\right)20 + \frac{1}{2}\left(\frac{x}{10} - 204\right)\left(\frac{x}{10} - 205\right)25 + \left(\frac{x}{10} - 204\right)\left(\frac{x}{10} - 205\right)\left(\frac{x}{10} - 206\right)\frac{-1}{6}(30) + \frac{15}{24}\left(\frac{x}{10} - 204\right)\left(\frac{x}{10} - 205\right)\left(\frac{x}{10} - 206\right)\left(\frac{x}{10} - 207\right)$$

$$\therefore F(x) = 180 + 20\left(\frac{x}{10} - 204\right) + \frac{25}{2}\left(\frac{x}{10} - 204\right)\left(\frac{x}{10} - 205\right) - 5\left(\frac{x}{10} - 204\right)\left(\frac{x}{10} - 205\right)\left(\frac{x}{10} - 206\right) + \frac{15}{24}\left(\frac{x}{10} - 204\right)\left(\frac{x}{10} - 205\right)\left(\frac{x}{10} - 206\right)\left(\frac{x}{10} - 207\right)$$

$$F(2055) = 180 + 20(205.5 - 204) + \frac{25}{2}(205.5 - 204)(205.5 - 205) - 5(205.5 - 204)(205.5 - 205)(205.5 - 206) + \frac{15}{24}(205.5 - 204)(205.5 - 205)(205.5 - 206)(205.5 - 207)$$

$= 221.6015$ //

Solve

* Find lagrange's interpolating polynomial to the function,

$F(x) = \sin x$ at grid points $0, \pi/4, \pi/2, 3\pi/4, \pi$

we have,

x	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
$F(x)$	0	0.7071	1	0.7071	0

$f(x) = \frac{1}{4}x^2 - \frac{2}{3}x + 3.6$

By Backward Difference Method

Q. The population of Nepal in previous decades as below

No. of years	2040	2050	2060	2070	2080
population	180	200	245	285	305

Find Newton's backward interpolating polynomial and find population of 2075

Solution:

Table is formed as below:-

x_i	f_i	∇f_i	$\nabla^2 f_i$	$\nabla^3 f_i$	$\nabla^4 f_i$
$x_0=2040$	$f_0=180$				
$x_1=2050$	$f_1=200$	$\nabla f_1=20$			
$x_2=2060$	$f_2=245$	$\nabla f_2=45$	$\nabla^2 f_2=25$		
$x_3=2070$	$f_3=285$	$\nabla f_3=40$	$\nabla^2 f_3=-5$	$\nabla^3 f_3=-30$	$\cancel{\nabla^4 f_1}$
$x_4=2080$	$f_4=305$	$\nabla f_4=20$	$\nabla^2 f_4=-20$	$\nabla^3 f_4=-15$	$\nabla^4 f_4=+15$

$$p=\frac{x-x_n}{h}=\frac{x-2080}{10}=\left(\frac{x}{10}-208\right)$$

We have backward interpolating polynomial

$$F(x)=F_n+p\nabla F_n+\frac{p(p+1)}{2!}\nabla^2 f_n+\frac{p(p+1)(p+2)}{3!}\nabla^3 f_n$$

$$+\frac{p(p+1)(p+2)(p+3)}{4!}\nabla^4 F_n$$

$$F(x)=305+\left(\frac{x}{10}-208\right)(20)-\frac{20}{2}\left(\frac{x}{10}-208\right)\left(\frac{x}{10}-207\right)-\left(\frac{x}{10}-208\right)\left(\frac{x}{10}-207\right)\left(\frac{x}{10}-206\right)\frac{15}{6}+\left(\frac{x}{10}-208\right)\left(\frac{x}{10}-207\right)\left(\frac{x}{10}-206\right)\left(\frac{x}{10}-205\right)\frac{15}{24}$$

which is required interpolating polynomial

$$F(2075)=305+20(207.5-208)-10(207.5-208)(207.5-207)-(207.5-208)(207.5-207)(207.5-206)\frac{15}{6}+(207.5-208)(207.5-207)(207.5-206)(207.5-205)\frac{15}{24}$$

$$F(2075)=297.851$$

Q. The marks scored by students is as below:-

Marks	0-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No. of students	20	15	15	25	15	10	5	2

Find number of students securing the marks between 50 and 55

55 सम्म ल्याउने = 50 भन्दा कम ल्याउने
50 देखि 55 सम्म ल्याउने

Solution:

$f(55) - f(50)$

The given data is rearranged as

Marks(x)	No. of students f(x)
30	20
40	35
50	50
60	75
70	90
80	100
90	105
100	107

We need f_{55}. Here $x = 55$ is near the beginning of forward interpolating polynomial

Now solve

---

* Find missing value from the following observations.

x	0	1	2	3	4	5
f(x)	-3	0	9	?	69	132
	f_0	f_1	f_2	f_3	f_4	

Solution:

Now,
Here, given number of observations = 6
polynomial of degree = 3

$\Delta^3 f_0$ = constant (from theorem)

$\therefore \Delta^4 f_0 = 0$

or, $\Delta^3 \Delta f_0 = 0$

or, $\Delta^3 (f_1 - f_0) = 0$

or, $\Delta^2 \Delta (f_1 - f_0) = 0$

or, $\Delta^2 (\Delta f_1 - \Delta f_0) = 0$

or, $\Delta^2 (f_2 - f_1 - f_1 + f_0) = 0$

or, $\Delta^2 (f_2 - 2f_1 + f_0) = 0$

or, $\Delta \Delta (f_2 - 2f_1 + f_0) = 0$

or, $\Delta (\Delta f_2 - 2\Delta f_1 + \Delta f_0) = 0$

or, $\Delta (f_3 - f_2 - 2(f_2 - f_1) + f_1 - f_0) = 0$

or, $\Delta (f_3 - f_2 - 2f_2 + 2f_1 + f_1 - f_0) = 0$

or, $\Delta (f_3 - 3f_2 + 3f_1 - f_0) = 0$

or, $\Delta f_3 - 3\Delta f_2 + 3\Delta f_1 - \Delta f_0 = 0$

or, $f_4 - f_3 - 3(f_3 - f_2) + 3(f_2 - f_1) - (f_1 - f_0) = 0$

or, $f_4 - f_3 - 3f_3 + 3f_2 + 3f_2 - 3f_1 - f_1 + f_0 = 0$

or, $69 - 4f_3 + 6 \times 9 - 4 \times 0 - 3 = 0$

or, $4f_3 = 120$

$\therefore f_3 = 30$ //

$x = -4$
$y = 2$

* Find best fitting line to the following observations.

x_i	0	1	3	5	7
y_i	-2	2	10	18	26

by least square approximation method.

Solution:

let the fitting curve be

$y = ax + b$ — (1)

when 'a' and 'b' are constant to be determined

The condition for best fitting are.

$\Sigma y_i = a\Sigma x_i + nb$ — (2)

$\Sigma x_i y_i = a\Sigma x_i^2 + b\Sigma x_i$ — (3)

The required table is

x_i	y_i	$x_i y_i$	x_i^2
0	-2	0	0
1	2	2	1
3	10	30	9
5	18	90	25
7	26	182	49
$\Sigma x_i = 16$	$\Sigma y_i = 54$	$\Sigma x_i y_i = 304$	$\Sigma x_i^2 = 84$

$n = 5$

Now (2) and 3 are

$84 = 16a + 5b$ — (4)

$304 = 84a + 16b$ (5)

$\therefore a = 4$

and $b = -2$

~~$y = -4x + 2$~~

$y = 4x - 2$

* Find best fitting line to the following observations

x_i	0	1	3	5	9	10
y_i	1.2	1.7	2.7	3.7	5.7	6.2

by least square approximation method

Solution:

let the best fitting curve be $y = ax + b$ — (1)

when 'a' and 'b' are constant to be determined

The condition for best fitting are

$\Sigma y_i = a\Sigma x_i + nb$ — (2)

$\Sigma x_i y_i = a\Sigma x_i^2 + b\Sigma x_i$ — (3)

The required table is

x_i	y_i	$x_i y_i$	x_i^2
0	1.2	0	0
1	1.7	1.7	1
3	2.7	8.1	9
5	3.7	18.5	25
9	5.7	51.3	81
10	6.2	62	100
$\Sigma x_i = 28$	$\Sigma y_i = 21.2$	$\Sigma x_i y_i = 141.6$	$\Sigma x_i^2 = 216$

$n = 6$

Now (2) and (3) are

$21.2 = a28 + 6b$ — (4)

$141.6 = 216a + 28b$ — (5)

On solving (4) and (5), we get,

$a = 0.5$ and $b = 1.2$

$\therefore$ eqn (1) will be

$y = 0.5x + 1.2$

Q. Fit a parabolic curve of the following observations

x	0	2	3	5	7
y	5	5.2	8.9	23.5	47.7

Solution

Let the required parabolic curve be

$y = ax^2 + bx + c$ — (1)

The condition for best fitting are

$\Sigma y_i = a\Sigma x_i^2 + b\Sigma x_i + nc$ — (2)

$\Sigma x_i y_i = a\Sigma x_i^3 + b\Sigma x_i^2 + c\Sigma x_i$ — (3)

$\Sigma x_i^2 y_i = a\Sigma x_i^4 + b\Sigma x_i^3 + c\Sigma x_i^2$ — (4)

∴ The required table is

x_i	y_i	x_i^2	$x_i y_i$	x_i^3	$x_i^2 y_i$	x_i^4
0	5	0	0	0	0	0
2	5.2	4	10.4	8	20.8	16
3	8.9	9	26.7	27	80.1	81
5	23.5	25	117.5	125	587.5	625
7	47.7	49	333.9	343	2337.3	2401
$\Sigma x_i = 17$	$\Sigma y_i = 90.3$	$\Sigma x_i^2 = 87$	$\Sigma x_i y_i = 485.5$	$\Sigma x_i^3 = 503$	$\Sigma x_i^2 y_i = 3027.5$	$\Sigma x_i^4 = 3123$

Replacing the value in (2), (3) and (4), we get

$90.3 = a\,87 + 17b + 5c$ — (5)

$488.5 = 503a + 87b + 17c$ — (6)

$3027.5 = 3123a + 503b + 87c$ — (7)

Solving (5), (6) and (7), we get,

$a = 1.2$

$b = -2.3$

$c = 5$

$\therefore y = ax^2 + bx + c$

$y = 1.2x^2 - 2.3x + 5$

Q. By method of least square method fit a parabola $y = a + bx + cx^2$ to the following data

x	2	4	6	8	10
y	3.07	12.85	31.47	57.38	91.29

Solution:

The required curve is

$y = cx^2 + bx + a$ — (1)

The condition for best fitting are

$\Sigma y_i = c\Sigma x_i^2 + b\Sigma x_i + na$ — (2)

$\Sigma x_i y_i = c\Sigma x_i^3 + b\Sigma x_i^2 + a\Sigma x_i$ — (3)

$\Sigma x_i^2 y_i = c\Sigma x_i^4 + b\Sigma x_i^3 + a\Sigma x_i^2$ — (4)

x_i	y_i	x_i^2	x_i^3	x_i^4	$x_i y_i$	$x_i^2 y_i$
2	3.07	4	8	16	6.14	12.28
4	12.85	16	64	256	51.4	205.6
6	31.47	36	216	1296	188.82	1132.92
8	57.38	64	512	4096	459.04	3672.32
10	91.29	100	1000	10000	912.9	9129
$\Sigma x_i = 30$	$\Sigma y_i = 196.06$	$\Sigma x_i^2 = 220$	$\Sigma x_i^3 = 1800$	$\Sigma x_i^4 = 15664$	$\Sigma x_i y_i = 1618.3$	$\Sigma x_i^2 y_i = 14152.12$

Now eqⁿ (2), (3) and (4) becomes

$196.06 = 220c + 30b + 5a$ - (5)

$1618.3 = 1800c + 220b + 30a$ - (6)

$14152.12 = 15664c + 1800b + 220a$ - (7)

Solving (5), (6) and (7), we get,

$c = 1$

$b = -0.85$

$a = 0.696$

(calculation may be wrong)

$\therefore y = x^2 - 0.85x + 0.696$ //

Q. Fit a curve of $y = \frac{1}{a+bx}$ to the following observations

x	0	1	2	4
y	0.2	0.1428	0.111	0.0769

Solution:

The required curve is:

$$y = \frac{1}{a+bx}$$

$$\text{or, } \frac{1}{y} = a + bx \quad \text{- (i)}$$

The condition for best fitting are

$$\sum \frac{1}{y_i} = na + b\sum x_i \quad \text{- (2)}$$

$$\sum \frac{x_i}{y_i} = a\sum x_i + b\sum x_i^2 \quad \text{- (3)}$$

x_i	y_i	x_i^2	x_i/y_i	$1/y_i$
0	0.2	0	0	5
1	1/7	1	7	7
2	1/9	4	18	9
4	1/13	16	52	13
$\sum x_i = 7$	$\sum y_i = 0.530891$	$\sum x_i^2 = 21$	$\sum x_i/y_i = 77$	$\sum 1/y_i = 34$

$\therefore$ Equation (2) and (3) becomes

$34 = 4a + 7b$ - (i)

$77 = 7a + 21b$ - (ii)

Solving (i) and (ii), we get,

$a = 5$ and $b = 2$

$$\therefore y = \frac{1}{5+2x} \text{ //}$$

Note:- $y = ax + bx^2$

or, $y = x(a+bx)$

or, $\frac{y}{x} = a + bx$

$Y = a + bx$ where $Y = \frac{y}{x}$

The condition for best fitting

$\sum y_i/x_i = na + b\sum x_i$ - (i)

$\sum y_i = a\sum x_i + b\sum x_i^2$ - (ii)

$a \times e^{10}$
$\ln a = \ln e^{10}$
$\ln a = 10$

* If the relationship between voltage V and time t is of the form $V = ae^{kt}$ using the least square method fit for the following observations.

Time(t)	0	2	4	6	8
Voltage(V)	150	63	28	12	5.6

Find V when t=2.6

Solution:

Since the equation is in the form of

$V = ae^{kt}$ - (1)

We can apply fitting transdental function.

Taking ln on both side of eqn (1)

$\ln V = \ln(ae^{kt})$

$\ln V = \ln a + \ln e^{kt}$

$\ln V = \ln a + kt \ln e$

$\ln V = \ln a + kt$ - (2)

The condition for best fitting are:-

$\Sigma \ln V_i = n \ln a + k \Sigma t_i$ - (3)

$\Sigma \ln V_i t_i = \ln a \Sigma t_i + k \Sigma t_i^2$ - (4)

The required table is:-

t_i	V_i	$\ln V_i$	$\ln V_i t_i$	t_i^2
0	150	5.006	0	0
2	63	4.1431	8.2862	4
4	28	3.3322	13.3288	16
6	12	2.4849	14.9094	36
8	5.6	1.7227	13.7816	64
$\Sigma t_i = 20$	$\Sigma V_i = 258.6$	$\Sigma \ln V_i = 16.6935$	$\Sigma \ln V_i t_i = 50.306$	$\Sigma t_i^2 = 120$

no. of observations(n) = 5

Now, equation (3) and (4) becomes

$16.6935 = 5 \ln a + 20k$ - (5)

$50.306 = \ln a \, 20 + 120k$ - (6)

Solving (5) and (6), we get,

$\ln a = 4.9855$

$a = e^{4.9855} = 146.2766$

$k = -0.4117$

Now, eqn (1) becomes

$V = 146.2766 \, e^{-0.4117t}$

which is required model

when t=2.6

$V = 146.2766 \, e^{-0.4117 \times 2.6}$

$V = 50.1530$ v

Q. Fit a power function model of the form $y = ax^b$ to the following observations.

x	1	2	4	6	8
y	0.5	2	8	18	32

% log nai ho

Solution:

The given model is

$y = ax^b$ — (1)

$\log y = \log a + \log x^b$ — (2)

$\log y = \log a + b\log x$ — (3)

The condition for best fitting are:

$\sum \log y_i = n\log a + b\sum \log x_i$ — (4)

$\sum \log x_i \log y_i = \log a \sum \log x_i + b\sum \log x_i \times \log x_i$ — (5)

The required table is formed as below:

x_i	y_i	$\log x_i$	$\log y_i$	$\log y_i \log x_i$	$\log x_i \times \log x_i$
1	0.5	0	-0.3010	0	0
2	2	0.3010	0.3010	0.0906	0.0906
4	8	0.6020	0.9030	0.5436	0.3624
6	18	0.7781	1.2552	0.9766	0.6054
8	32	0.9030	1.5051	1.3591	0.8154
$\sum x_i = 21$	$\sum y_i = 60.5$	$\sum \log x_i = 2.5841$	$\sum \log y_i = 3.6633$	$\sum \log y_i \log x_i = 2.9699$	$\sum \log x_i \log x_i = 1.87$

Now eq^n (4) and (5) becomes

$3.6633 = 5\log a + 2.5841 b$ — (6)

$2.9699 = 2.5841\log a + 1.8738 b$ — (7)

Solving (6) and (7) we get,

$\log a = -0.3010$ $\therefore a = 10^{-0.3010}$

~~a = 1 e^{-0.3010}~~ $a = 0.5$

$b = 2.0001$

$\therefore$ eq^n (1) becomes

$y = 0.5x^2 = \frac{1}{2}x^2$

Q. If the relation between time t (in seconds) and Temperature T is related as $T = a + be^{t/4}$ then fit for the following observation,

t	0	2	4	7
T	3.5	5.44	8.65	17.76

Ans: $T = 0.543e^{t/4}$

Q. The following table shows pressure and specific volume of dry saturated steam

V	38.4	20	8.59	4.44	3.03
P	10	20	50	100	150

Fit a curve of the form $PV^n = k$ by using least square method.

Solution:

Given model is

$$PV^{\alpha} = \beta \quad (1)$$

$$\log(PV^{\alpha}) = \log\beta$$

$$\log P + \log V^{\alpha} = \log\beta$$

$$\log P + \alpha\log V = \log\beta$$

$$\log P = \log\beta - \alpha\log V$$

$$\log P = \log\beta + (-\alpha)\log V \quad (2)$$

$$Y = A + bX$$

The condition for best fitting are

$$\Sigma\log P_i = n\log\beta - \alpha\Sigma\log V_i \quad (3)$$

$$\Sigma\log P_i \times \log V_i = \log\beta\,\Sigma\log V_i - \alpha\Sigma\log V_i \times \log V_i \quad (4)$$

V	P	$\log V_i$	$\log P_i$	$\log P_i \times \log V_i$	$\log V_i \times \log V_i$
38.4	10	1.5843	1	1.5843	2.5100
20	20	1.3010	1.3010	1.6926	1.6926
8.51	50	0.9299	1.6989	1.5798	0.8647
4.44	100	0.6473	2	1.2946	0.4189
3.03	150	0.4814	2.1760	1.0475	0.2317
		$\Sigma\log V_i =$ 4.9439	$\Sigma\log P_i =$ 8.1759	$\Sigma\log P_i \times \log V_i =$ 7.1988	$\Sigma\log V_i \times \log V_i$ = 5.7179

Now eqn (3) and (4) becomes

$$8.1759 = 5\log\beta - 4.9439\alpha \quad (5)$$

$$7.1988 = 4.9439\log\beta - 5.7179\alpha \quad (6)$$

Solving (5) and (6), we get,

$$\log\beta = 2.6905$$

$$\beta = 10^{2.6905} = 490.34$$

$$\alpha = 1.0673$$

$\therefore$ Eq (1) becomes

$$490.34 = PV^{1.0673}$$

Numerical Derivative and Integration

For example:

If $f(x) = x^2$, find $f'(1)$ by forward difference quotient, backward difference quotient, central difference quotient and compare the result with exact value taking $h = 0.01$

Soln

By forward difference quotient, we know that

$$f'(x) = \frac{f(x+h) - f(x)}{h} \text{ formula}$$

for exact derivative, $f'(1)$

$f'(x) = 2x = 2 \times 1 = 2$

$$f'(1) = \frac{f(1+h) - f(1)}{h} \approx \frac{(1.01)^2 - (1)^2}{0.01}$$

$$= \frac{f(1+0.01) - f(1)}{h} \quad f'(1) = 2.01$$

$$= \frac{f(1.01) - f(1)}{h}$$

By backward difference quotient,

$$f'(x) = \frac{f(x) - f(x-h)}{h}$$

$$f'(1) = \frac{f(1) - f(1-h)}{h}$$

$$= \frac{f(1) - f(1-0.01)}{0.01} \quad \therefore f'(1) = 1.99$$

$$= \frac{(1)^2 - (0.99)^2}{0.01}$$

By central difference quotient

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h}$$

$$f'(1) = \frac{f(1+h) - f(1-h)}{2h}$$

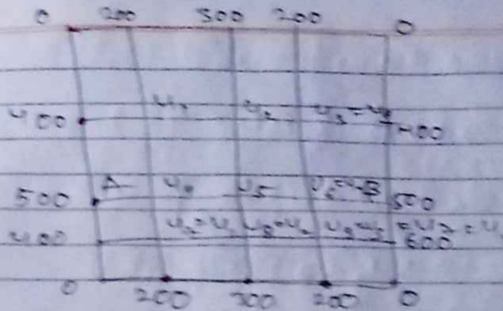
$$f'(1) = \frac{f(1+0.01) - f(1-0.01)}{2 \times 0.01}$$

$$\text{or, } f'(1) = \frac{f(1.01) - f(0.99)}{0.02}$$

$$f'(1) = 2$$

Therefore central difference quotient is more accurate than forward difference quotient and backward difference quotient.

2023 5 b)



Since given distribution is symmetric about AB, so
$u_7 = u_1$, $u_8 = u_2$, $u_9 = u_3$

The distribution is also symmetric about CD, so

Since given distribution is symmetric about CD

For initial guessing

$$u_5 = \frac{1}{4}(300+300+500+500) = 400$$

$$u_1 = \frac{1}{4}(0+400+500+300) = 300$$

$$u_2 = \frac{1}{4}(300+300+300+400) = 325$$

$$u_4 = \frac{1}{4}(500+400+500+300) = 375$$

$\therefore$ To solve the above system, it is sufficient to solve u_1, u_2, u_4, u_5

Using Laplace equation formula,

$u_1 = \frac{1}{4}[400 + u_2 + 200 + u_4]$

$\therefore u_1 = 150 + 0.25u_2 + 0.25u_4$ — (1)

$u_2 = \frac{1}{4}[300 + u_5 + u_1 + u_1]$

$\therefore u_2 = 75 + 0.5u_1 + 0.25u_5$ — (2)

$u_4 = \frac{1}{4}[500 + u_5 + u_1 + u_1]$

$\therefore u_4 = 125 + 0.5u_1 + 0.25u_5$ — (3)

$u_5 = \frac{1}{4}[u_4 + u_2 + u_2 + u_4]$

$= 0.5u_4 + 0.5u_2$ — (4)

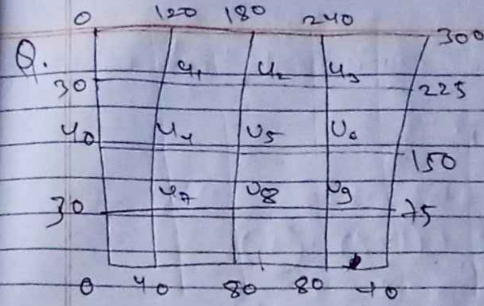
The table of iteration is formed as below:-

Solve

No. of iterations	u_1	u_2	u_4	u_5
0	300	325	375	400
1	325	337.5		

Symmetric बनाएको आए
पा सक्दै

Q.



For initial guessing

$u_5 = \frac{1}{4}(180 + 80 + 40 + 150) = 112.5$

$u_1 = 75$

$u_3 = 232.5$

$u_7 = 35$

$u_9 = 77.5$

$u_6 = \frac{1}{4}(u_5 + 150 + u_3 + u_9)$

$u_4 =$

$u_2 =$

$u_8 =$

No. of iterations	u_1	u_2	u_3	u_4	u_5
0					
1					
2					

u_6	u_7	u_8	u_9

Second iteration
स्तेम गर्देर
answer
vandene

Using laplace formula

$u_1 = \frac{1}{4}[30 + u_2 + 120 + u_4]$

$u_2 = \frac{1}{4}[u_1 + u_3 + 180 + u_5]$