Experiment No. 1 **Date:**

DETERMINATION OF THE ACCELERATION DUE TO GRAVITY AND RADIUS OF GYRATION OF THE BAR PENDULUM ABOUT AN AXIS PASSING THROUGH ITS CENTRE OF GRAVITY.

APPARATUS REQUIRED:

a) Bar pendulum   b) Stop Watch   c) Meter scale,   d) Knife edge,

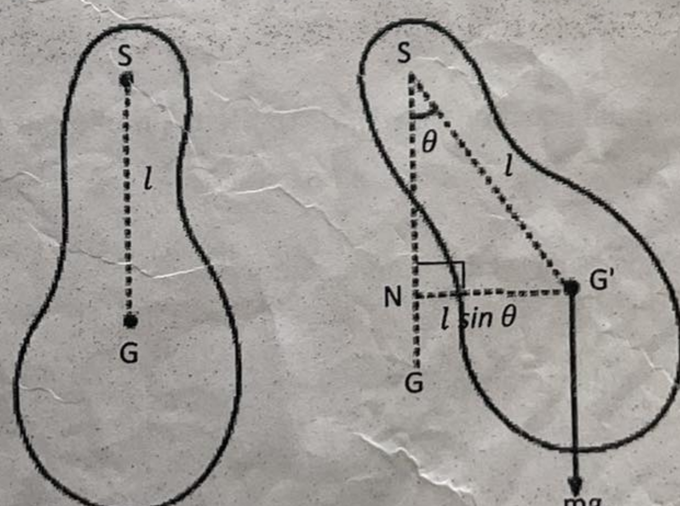
e) Spirit Level   f) Graph Paper.

THEORY:

A compound pendulum is a rigid body of arbitrary shape, capable of being oscillated in a vertical plane about a horizontal axis passing through it. It is also called real of physical pendulum. And bar pendulum is a symmetric compound pendulum usually a rod having equal no. of holes.

Figure (a) shows a compound pendulum free to rotate about a horizontal axis passing through the point of suspension S. In its normal position of rest it'sc.g'G' lies vertically below 'S'. The distance between point of suspension (S) and centre of gravity (G) is called length of pendulum (l).

Let the pendulum be given small angular displacement 'θ' so that it's c.g. takes new position G' as shown in figure (b). Due to the weight mg acting vertically downward at G', it constitutes a restoring torque whose action is to tend to bring the pendulum back into its original position.



(a) **Fig.Compound pendulum** (b)

The restoring torque is,

$$\tau = -mg(G'N) = -mgl\sin\theta \text{ .... (i)}$$

negative sign indicates that torque is oppositely directed to the displacement 'θ'.

If I is the moment of inertia of the pendulum about the axis of suspension and α be its angular acceleration then,

$$\tau = I\alpha = I\frac{d^2\theta}{dt^2} \text{ .... (ii)}$$

From (i) and (ii),

we get, $I\frac{d^2\theta}{dt^2} = -mgl\sin\theta$

and, $\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \text{............},$

For θ to be small, $\sin\theta \sim \theta$

So, $I\frac{d^2\theta}{dt^2} = -mgl\theta$

or, $\frac{d^2\theta}{dt^2} = -\frac{mgl}{I}\theta$

or, $\frac{d^2\theta}{dt^2} + \frac{mgl}{I}\theta = 0$

or, $\frac{d^2\theta}{dt^2} + \omega^2\theta = 0$ ....(iii)

Equation (iii) is the differential equation of S.H.M. Hence the motion of compound pendulum is simple harmonic.

Equation (iii) is also referred as angular harmonic motion.

Here, $\omega = \sqrt{\frac{mgl}{I}}$, is the angular frequency

or, $\frac{2\pi}{T} = \sqrt{\frac{mgl}{I}}$

or, $T = 2\pi\sqrt{I/mgl}$

If k is the radius of gyration of the pendulum. Then, from the theorem of parallel axis, the total moment of inertia of the pendulum about the axis through point of suspension is,

$$I = I_{CG} + ml^2 = mk^2 + ml^2, \text{ where } I_{CG} = mk^2$$

$$= m(k^2 + l^2)$$

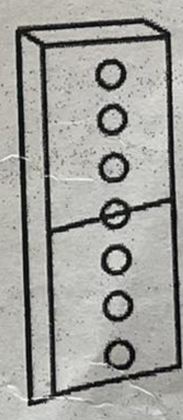
$$\therefore \quad T = 2\pi\sqrt{\frac{m(k^2 + l^2)}{mgl}}$$

$$= 2\pi\sqrt{\frac{\frac{k^2}{l} + l}{g}} \quad \text{.......(iii)}$$

Thus, time period of compound pendulum is same as that of a simple pendulum of length $L = \frac{k^2}{l} + l$.

This length 'L' is therefore called the length of an equivalent simple pendulum or the reduced length of compound pendulum. Since $k^2 > 0$ i.e. $k^2/l > 0$, the length of equivalent simple pendulum (L) is always greater than length of compound pendulum(l).

A bar pendulum is the simplest form of compound pendulum which consists of a uniform metal rod having equally spaced holes drilled along its length on either side of C.G.



$T = 2\pi\sqrt{\frac{L}{g}}$

Fig. Bar pendulum

The time period of bar pendulum is [shown in figure (above)]

$$T = 2\pi\sqrt{\frac{L}{g}} \quad \text{......(iv)}$$

Where, $L = \left(\frac{k^2+l^2}{l}\right)$

Squaring equation (iv) both sides,

$$T^2 = (4\pi^2/g)\left(\frac{k^2+l^2}{l}\right)$$

or, $$lT^2 = \frac{4\pi^2}{g}l^2 + \frac{4\pi^2}{g}k^2 \quad \text{.........(v)}$$

or, $$\frac{4\pi^2}{g}l^2 - lT^2 + \frac{4\pi^2}{g}k^2 = 0$$

Which is the quadratic in l, so it possesses two roots, let they be l_1 and l_2.

$$\therefore \text{sum of roots, } l_1 + l_2 = -\frac{(-T)^2}{\frac{4\pi^2}{g}} = \frac{gT^2}{4\pi^2}$$

or, $$g = \frac{4\pi^2}{T^2}(l_1 + l_2) = \frac{4\pi^2}{T^2}L \quad \text{.................(vi)}$$

and the product of roots, $$l_1.l_2 = \frac{\frac{4\pi^2}{g}k^2}{\frac{4\pi^2}{g}} = k^2$$

$$\therefore \quad k^2 = l_1.l_2$$

$$\therefore \quad k = \pm\sqrt{l_1.l_2} \quad \text{.................(vii)}$$

[Since for $ax^2+bx+c=0$, sum of roots = $-b/a$ and product of roots $=c/a$]

Here, l_1 and l_2 are two values of 'l' for one side of bar pendulum for which the value of time period is same.

[If $l_1 = l$ then $l_2 = k^2/l$; such that $l_1 + l_2 = k^2/l + l = L$]

PROCEDURE:

a) Suspend the bar pendulum in the first hole from side A such that pendulum is hanging parallel to the wall.

b) Set the pendulum into oscillation with small amplitude approximately 5° and note the time taken for 10 complete oscillations.

c) Repeat the procedure (b), again and take the mean, let it be (t).

d) Divide it by 10 to get time period (T).

e) Measure the distance (l) of C.G. of the bar from the point of suspension. Repeat the process by hanging the bar through different holes.

f) Suspend the bar on side B and repeat the observation as above.

g) Plot of graph between l and T is shown in figure below graph

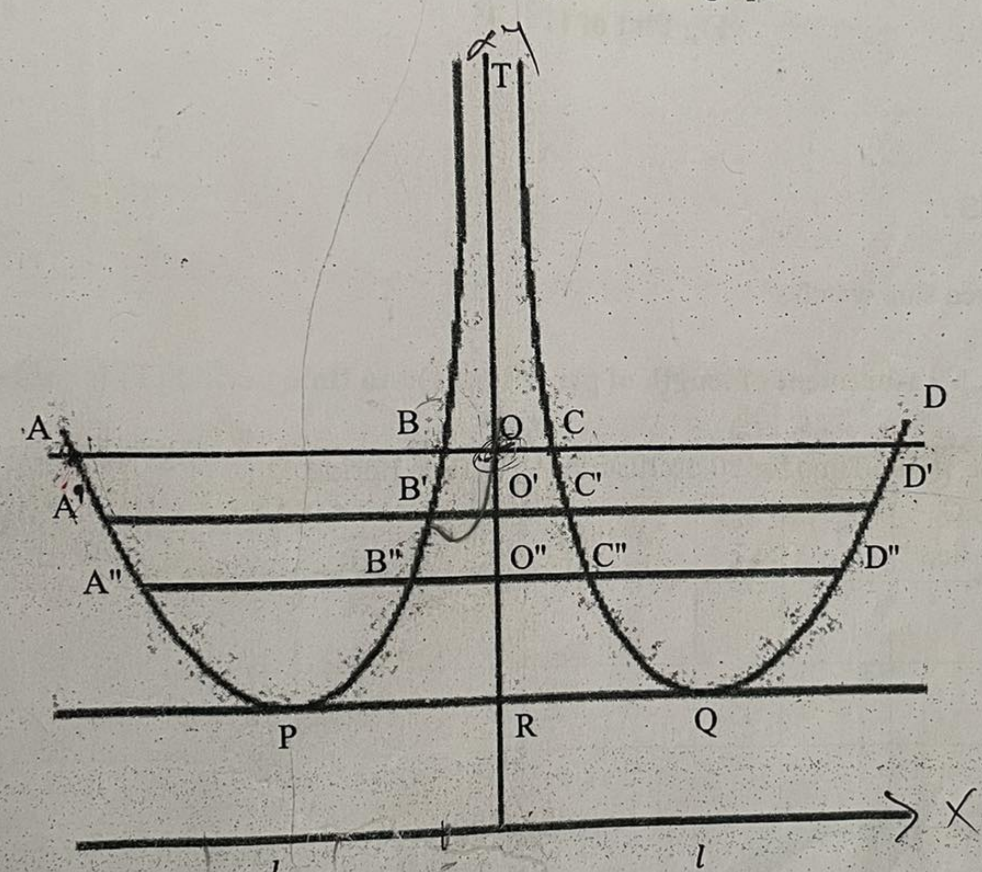


Fig. Plot of time period & length of pendulum

h) Draw horizontal lines ABOCD, A'B'O'C'D', A"B"O"C"D" as shown.

i) Again plot a graph between lT^2 and l^2 for both sides A and B as shown in figure below.

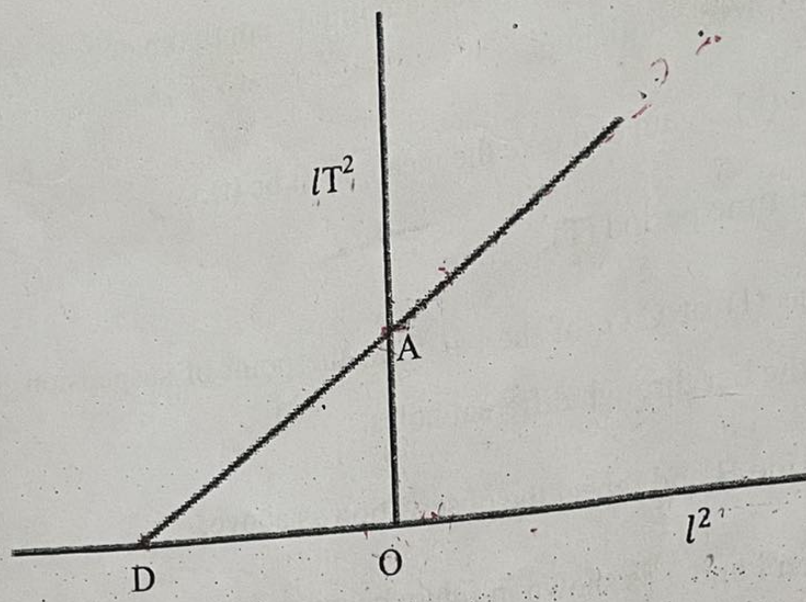


Fig Plot of $lT^2 \sim l^2$

OBSERVATIONS :

L.C. of given stop watch =

Table. 1. Measurement of length of pendulum(l)and time period (T) for side A

S. N.	Distance (cm) between C.G. and Suspension	Time for 10 oscillations			Time period $T = \frac{t}{10}$ (sec)	T^2 $(sec)^2$	lT^2 cm(sec)^2	l^2 $(cm)^2$
		1 (sec)	2 (sec)	Mean(t) (sec)				
1.	45	15.54	15.60					
2.	40	15.32	15					
3.								
4.								
5.								
6.								
7.								
8.								
9.								

Table 2. Measurement of length of pendulum (l) and time period (T) for side B

S. N.	Distance between C.G. and Suspension	Time for 10 oscillations			Time period	T^2	lT^2	l^2
		1	2	Mean(t)	$T = \frac{t}{10}$			
1.								
2.								
3.								
4.								
5.								
6.								
7.								
8.								
9.								

CALCULATIONS:

Table 3. Measurement of g from the plot T~l

S.N.	Straight line	Length of equivalent simple pendulum			Time period (T)	$g = \frac{4\pi^2 L}{T^2}$	$\bar{g}$	$g_i - \bar{g}$	$(g_i - \bar{g})^2$	$\sigma_g = \sqrt{\frac{\Sigma(g_i - \bar{g})^2}{n(n-1)}}$
		(1)	(2)	Mean (L)						
1.	ABCD	AC=	BD=							
2.	A'B'C'D'	A'C'=	B'D'=							
3.	A"B"C"D"	A"C"=	B"D"=							
4.										
5.										

Table No.2. Measurement of K from the plot of T~l

S.N.	l_1	l_2	$K = \sqrt{l_1 l_2}$	$\overline{K}$	$K_i - \overline{K}$	$(K_i - \overline{K})^2$	$\sigma_K = \sqrt{\frac{\Sigma(K_i - \overline{K})^2}{n(n-1)}}$
1.	AO=	OC=					
2.	OD=	OB=					
3.	A'C'=	O'C'=					
4.							
5.							
6.							
7.							
8.							
9.							
10.							

Table No.3. Determination of g and k from the plot of $lT^2 \sim l^2$

S.N.	Side	OA	OD	Slope $= \frac{OA}{OD}$	$g = \frac{4\pi^2}{Slope}$	$k = \sqrt{OD}$
1.	A					
2.	B					

RESULTS:

1. The value of g = i) ii) mean:

2. Standard value of g in Kathmandu valley $= 9.8\left(1 - \frac{2h}{R}\right) =$

   Where, average height of Kathmandu from sea level (h) = 1350m

   radius of earth (R) = 6400 km $= 6400 \times 10^3$

3. Percentage error in g =

4. The value of k =

5. Standard value of k $= \frac{\text{Total length of bar}}{\sqrt{12}} = \frac{1}{\sqrt{12}} = 0.28$

6. Percentage error in K =

CONCLUSION :

PRECAUTIONS: