Experiment No. 8

Date:

TO PLOT A GRAPH BETWEEN CURRENT AND FREQUENCY IN AN LCR SERIES CIRCUIT AND TO FIND THE RESONANT FREQUENCY AND QUALITY FACTOR.

## APPARATUS REQUIRED :

a) Inductance coil (mH) b) Capacitor(μF) c) Resistance box (ohm)

d) Mili-ammeter e) Voltmeter f) Connecting wires.

g) Audio frequency generator (10Hz to 10KHz)

## THEORY :

Inductor, capacitor & resistor are the fundamental circuit components. The series LCR circuit forms a harmonic oscillator for current & resonates in a similar way as on LC circuit. Resistor, increases the decay of LC or free oscillation, known as damped oscillation.

LCR or RLC circuits have many applications as oscillator circuits. Radio receivers & television sets use them from the ambient radio waves.

It is difficult to achieve free oscillation, because there is always some resistance present in an electric circuit. Some energy is converted into heat energy causing damped oscillation. Some energy is converted into heat energy causing damped circuit. such oscillations are called forced em oscillation. An example of this type of circuit is as shown in figure which consist of LCR series circuit with an AC frequency of emf$E = E_0 \sin\omega t$ as input power.

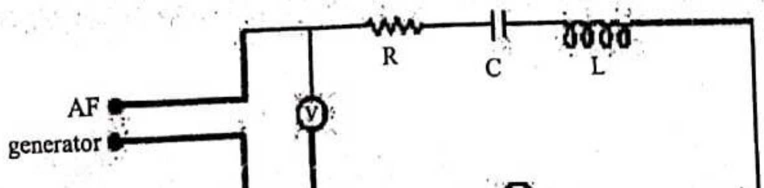


**Fig.(a). LCR series circuit with AC frequency generator**

Using Kirchhoff'svoltage or 2nd law,

Net emf of circuit = P.D. across C + P. D. across R

$$E + \left(-L\frac{dI}{dt}\right) = IR + \frac{Q}{C}$$

or, $L\frac{dI}{dt}+\frac{Q}{C}+IR = E$

or, $\frac{dI}{dt}+\frac{R}{L}I+\frac{Q}{C} = E$

or, $\frac{d^2Q}{dt^2}+\frac{R}{L}\frac{dQ}{dt}+\frac{Q}{C} = E_0 \sin\omega t$ ..... (i)

This is the differential equation for forced LCR oscillation

$$\text{Here, } E = IR + L\frac{dI}{dt}+\frac{Q}{C}$$

$$\text{Since, } I = I_0 \sin\omega t$$

or, $\frac{dI}{dt} = I_0 \cos\omega t.\omega$

$$\text{and } I = \frac{dQ}{dt}\sin\omega t$$

or, $Q = -\frac{I_0}{\omega}\cos\omega t$

$\therefore$ $E = I_0 \sin\omega t. R + L(I_0\omega\ \cos\omega t) + \frac{1}{C}\left(-\frac{I_0}{\omega}\cos\omega t\right)$

or, $E = I_0\left[R \sin\omega t + \left(L\omega - \frac{1}{C\omega}\right)\cos\omega t\right]$

$\therefore$ $E = I_0[R\sin\omega t + (X_L - X_C)\cos\omega t]$

Where, $X_L = L\omega$ is called inductive reactance and

$X_C = \frac{1}{C\omega}$ is called capacitive reactance.

Multiplying and dividing R.H.S. of above equation by

$$\sqrt{R^2+(X_L-X_C)^2}$$

$$\therefore E = I_0\sqrt{R^2+(X_L-X_C)^2}\left[\frac{R}{\sqrt{R^2+(X_L-X_C)^2}}\sin\omega t + \frac{X_L-X_C}{\sqrt{R^2+(X_L-X_C)^2}}\cos\omega t\right]$$

$$\text{Let } \cos\emptyset = \frac{R}{\sqrt{R^2+(X_L-X_C)^2}}$$

or,

$$\sin\phi = \sqrt{1-\cos^2\phi} = \sqrt{1-\frac{R^2}{R^2+(X_L-X_C)^2}}$$

∴

$$\sin\phi = \frac{X_L-X_C}{\sqrt{R^2+(X_L-X_C)^2}}$$

∴

$$E = I_0\sqrt{R^2+(X_L-X_C)^2}.\sin(\omega t+\phi)$$

or,

$$E = E_0 \sin\ (\omega t+\phi) \quad \text{........(ii)}$$

$$\text{Where,} E_0 = \sqrt{R^2+(X_L-X_C)^2}$$

or,

$$I_0 = \frac{E_0}{\sqrt{R^2+(X_L-X_C)^2}} = \frac{E_0}{Z} \qquad \text{... ... (iii)}$$

Where $Z = \sqrt{R^2+(X_L-X_C)^2}$ is called the impedance of LCR circuit.

Here, $\phi$ in equation (ii) is the phase between emf and current is given by

$$\tan\phi = \frac{\sin\phi}{\cos\phi} = \frac{X_L-X_C}{R} = \frac{L\omega-\frac{1}{C\omega}}{R}$$

∴

$$\phi = tan^{-1}\left(\frac{L\omega-\frac{1}{C\omega}}{R}\right) \text{..... (iv)}$$

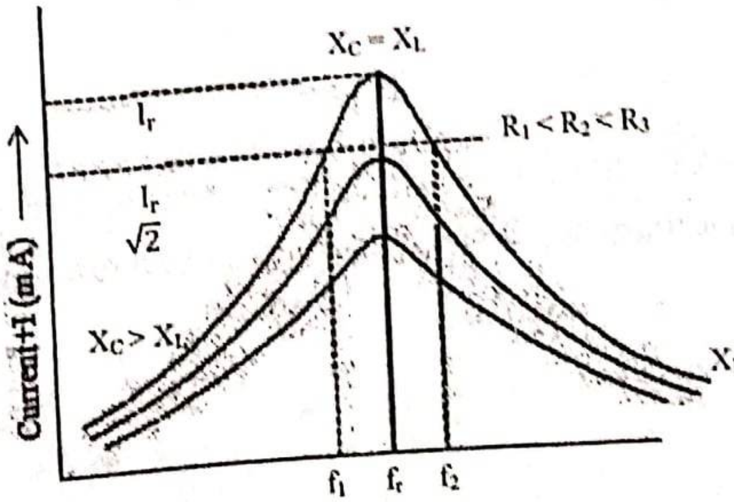


**Fig.(b). Resonance curves at three values of resistances $R_1$, $R_2$ and $R_3$($R_1$<$R_2$<$R_3$) for the forced LCR circuit.**

When $X_C > X_L$, from equation (iv) $\emptyset$ is negative so emf lags the current by the phase angle $\emptyset$. and the circuit is mainly capacitive.

- When $X_L > X_C$, from equation (iv) $\emptyset$ is positive so emf leads the current by the phase angle $\emptyset$. And the circuit is mainly inductive.
- When $X_L = X_C$, $\emptyset = 0$, the emf and current will be in phase, from equation (iii) the impedance in the circuit will be minimum and current will be maximum. This is the condition of resonance.

Once started, the charge, potential difference and current in LC circuit oscillate at angular frequency $\omega = \frac{1}{\sqrt{LC}}$. Such oscillation are said to be free oscillation. And the frequency '$\omega$' is called natural frequency. The oscillation will no longer remain free if there is some resistance present in the circuit. The energy is lost through resistor and the amplitude of oscillation finally becomes zero. To continue the oscillation, an external alternating emf is connected to LCR circuit, such that it recovers the energy lost in damping. Whatever the natural frequency of circuit may be, the oscillation of charge, current and potential difference in the circuit occur at driving frequency. This type of oscillation is called forced oscillation.

Whenthe driving frequency matches to natural frequency, the amplitudes of current in the circuit becomes maximum. Such condition is called resonance. The driving frequency at resonance is called resonance frequency.

From equation (iii) it is seen that, the current in the circuit will be maximum when 'Z' in minimum and Z is minimum when $X_L = X_C$

or, $$\omega L = \frac{1}{C\omega}$$

or, $$\omega^2 = \frac{1}{LC}$$

or, $$\omega = \frac{1}{\sqrt{LC}}$$

or, $$2\pi f = \frac{1}{\sqrt{LC}}$$

Hence, resonance frequency

$$f_r = \frac{1}{2\pi\sqrt{LC}} \quad \ldots\ldots (v)$$

Quality factor: It is defined as the ratio of the voltage drop across inductor (L) or across capacitor (C) to the voltage drop across resistance 'R' at resonance.

$$\text{i.e.} Q_L = \frac{V_L}{V_R} = \frac{L\omega}{R} = \frac{2\pi f_r L}{R}$$

or,

$$Q_C = \frac{V_C}{V_R} = \frac{1}{C\omega R} = \frac{1}{2\pi f_r CR}$$

At resonance, $X_L = X_C$

or,

$$L\omega = \frac{1}{C\omega}$$

$\therefore$

$$Q = Q_L = Q_C$$

The quality factor is also defined in terms of lower and upper half power frequencies $f_1$ and $f_2$. These are the frequencies below and above the resonance frequency at which power dissipation in the circuit is half of that at resonance frequency. At these frequency current in the circuit is $\frac{I_r}{\sqrt{2}}$, where $I_r$ is current at resonance frequency.

$$\text{i.e.} Q = \frac{2\pi f_r}{f_2 - f_1} \quad \text{.... (vi)}$$

The quantity, $f_2$-$f_1$ is called band width of the circuit.

The quality factor characterizes the sharpness of resonance. Since current at resonant frequency is $I_R = \frac{E_0}{R}$ and $Q = \frac{L\omega}{R} \Rightarrow I_R \Rightarrow \frac{E_0}{L\omega} \cdot Q$, this means current at resonance is directly proportional to Q. Therefore if Q is large resonance curve is sharp. Equation (vi) shows that lower the band width better (higher) will be the quality factor of circuit and higher will be the sharpness of resonance.

**PROCEDURE :**

1. First, Inductor L (mH), capacitor (μF) and resistor R(ohms) are connected in series as shown in figure (a).
2. From the given values of L and C calculate natural frequency of the circuit.
3. Adjust the output voltage of A.F. generator should be adjusted at constant value.
4. Adjust the value of frequency of the A.F. generator starting below from natural frequency.

5. Note the corresponding value of current by increasing the value of frequency of A.F. generator.
6. Repeat (5) for atleast three resistances.
7. Plot a graph between current and frequency.

**OBSERVATIONS:**

Inductance (L) =

Capacitance (C) =

Natural frequency $= \frac{1}{2\pi\sqrt{LC}}$

Output voltage of generator =

Measurement of current

| S.N. | $R_1=$ | | $R_2=$ | | $R_3=$ | |
|---|---|---|---|---|---|---|
| | f(Hz) | I(mA) | f(Hz) | I(mA) | f(Hz) | I(mA) |
| 1. | | | | | | |
| 2. | | | | | | |
| 3. | | | | | | |
| 4. | | | | | | |
| 5. | | | | | | |
| 6. | | | | | | |
| 7. | | | | | | |
| 8. | | | | | | |
| 9. | | | | | | |
| 10. | | | | | | |
| 11. | | | | | | |
| 12. | | | | | | |
| 13. | | | | | | |
| 14. | | | | | | |
| 15. | | | | | | |

### Table to Calculate quality factor (Q)

| $R(\Omega)$ | $I_r$ | $\frac{I_r}{\sqrt{2}}$ | $f_1$ | $f_2$ | $f_r$ | $Q = \frac{2\pi f_r}{f_2 - f_1}$ | $\overline{f_r}$ | $\overline{Q}$ | $Q_c = \frac{1}{2\pi f_r CR}$ | $Q_L = \frac{2\pi f_r}{R}$ | $Q_L - Q_C$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $R_1 =$ | | | | | | | | | | | |
| $R_2 =$ | | | | | | | | | | | |
| $R_3 =$ | | | | | | | | | | | |

**RESULTS :**

1. The resonance frequency $\overline{(f_r)}$ =
2. Natural frequency =
3. Percentage error =
4. The quality factor for L.C.R circuit $\overline{(Q)}$=
5. The average of $Q_L - Q_C$ =

**CONCLUSIONS :**

**PRECAUTIONS :**