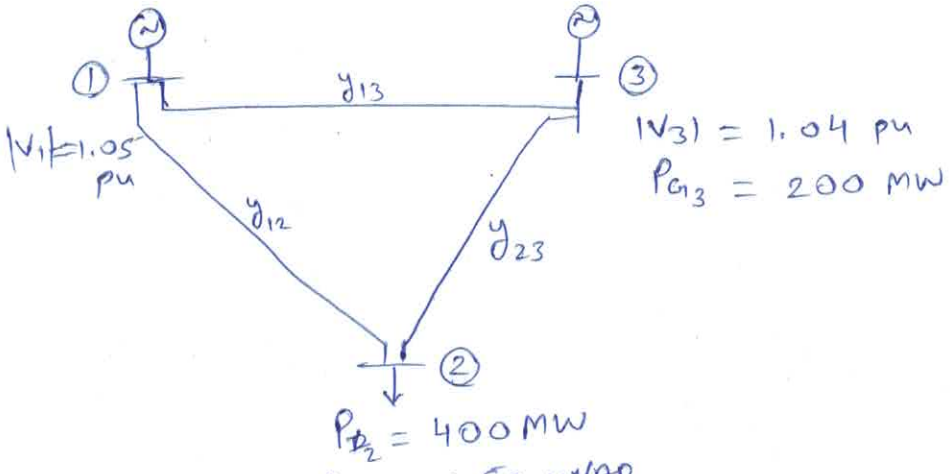
Tutorial for Load Flow



$y_{12} = (10 - j20)$ pu $y_{13} = (10 - j30)$ pu $y_{23} = (16 - j32)$ pu

base MVA = 100 ; line charging susceptances are neglected

Solution

1.) form admittance Matrix ; Y bus

$$Y = \begin{bmatrix} y_{12}+y_{13} & -y_{12} & -y_{13} \\ -y_{21} & y_{21}+y_{23} & -y_{23} \\ -y_{31} & -y_{32} & +y_{31}+y_{32} \end{bmatrix}$$

$$= \begin{bmatrix} 20-j50 & -10+j20 & -10+j30 \\ -10+j20 & 26-j52 & -16+j32 \\ -10+j30 & -16+j32 & 26-j62 \end{bmatrix}$$

2.) Specified variable (assume slack bus is ①)

$|V_1| = 1.05$
$|\theta_1| = 0$

$P_2^{spec} = -4$ pu
$Q_2^{spec} = -2.5$ pu
-ve because $P_j = P_j^G - P_j^L$

$|V_3| = 1.04$
$P_3^{spec} = 2$ pu

initial guess

$|V_2|^{(0)} = 1.0$
$\theta_2^{(0)} = 0$; $\theta_3^{(0)} = 0$

total buses = $N = 3$ ⇒ size of $J = 2N - N_G - 1 \Rightarrow 3 \times 3$
Gen. buses = $N_G = 2$

3) construct $J^{(0)}$: J_{11} J_{12}

$$J = \begin{bmatrix} \frac{\partial P_2}{\partial \theta_2} & \frac{\partial P_2}{\partial \theta_3} & \frac{\partial P_2}{\partial |V_2|} \\ \frac{\partial P_3}{\partial \theta_2} & \frac{\partial P_3}{\partial \theta_3} & \frac{\partial P_3}{\partial |V_2|} \\ \frac{\partial Q_2}{\partial \theta_2} & \frac{\partial Q_2}{\partial \theta_3} & \frac{\partial Q_2}{\partial |V_2|} \end{bmatrix}$$

J_{21} J_{22}

write $P(x)$ for all buses except slack

$$P_2(x) = |V_2||V_1|\left(G_{21}\cos(\theta_2-\theta_1) + B_{21}\sin(\theta_2-\theta_1)\right) + |V_2|^2(G_{22}) + |V_2||V_3|\left(G_{23}\cos(\theta_2-\theta_3) + B_{23}\sin(\theta_2-\theta_3)\right)$$

$$P_3(x) = |V_3||V_1|\left(G_{31}\cos(\theta_3-\theta_1) + B_{31}\sin(\theta_3-\theta_1)\right) + |V_3||V_2|\left(G_{32}\cos(\theta_3-\theta_2) + B_{32}\sin(\theta_3-\theta_2)\right) + |V_3|^2 G_{33}$$

write $Q(x)$ for PQ bus

$$Q_2(x) = |V_2||V_1|\left(G_{21}\sin(\theta_2-\theta_1) - B_{21}\cos(\theta_2-\theta_1)\right) + (-|V_2|^2 B_{22}) + |V_2||V_3|\left(G_{23}\sin(\theta_2-\theta_3) - B_{23}\cos(\theta_2-\theta_3)\right)$$

Elements of J

$$\frac{\partial P_2}{\partial \theta_2} = -Q_2(x) - |V_2|^2 B_{22} = -Q_2(x) - |V_2|^2(-52)$$

$$\frac{\partial P_2}{\partial \theta_3} = |V_2||V_3|\left(G_{23}\sin(\theta_2-\theta_3) - B_{23}\cos(\theta_2-\theta_3)\right)$$

$$\frac{\partial P_2}{\partial |V_2|} = \frac{P_2(x)}{|V_2|} + G_{22}|V_2|$$

$$\frac{\partial P_3}{\partial \theta_2} = |V_3||V_2|\left(G_{32}\sin(\theta_3-\theta_2) - B_{23}\cos(\theta_3-\theta_2)\right)$$

$$\frac{\partial P_3}{\partial \theta_3} = -|V_3||V_1|\left(G_{31}\sin(\theta_3-\theta_1) - B_{31}\cos(\theta_3-\theta_1)\right) - |V_3||V_2|\left(G_{32}\sin(\theta_3-\theta_2) - B_{32}\cos(\theta_3-\theta_2)\right)$$

$$= -Q_3(x) - |V_3|^2 B_{33}$$

$$\frac{\partial P_3}{\partial |V_2|} = |V_3|\left(G_{32}\cos(\theta_3-\theta_2) + B_{32}\sin(\theta_3-\theta_2)\right)$$

$$\frac{\partial Q_2}{\partial \theta_2} = P_2(x) - G_{22}|V_2|^2$$

$$\frac{\partial Q_2}{\partial \theta_3} = -|V_2||V_3|\left(G_{23}\cos(\theta_2-\theta_3) + B_{23}\sin(\theta_2-\theta_3)\right)$$

$$\frac{\partial Q_2}{\partial |V_2|} = \frac{Q_2(x)}{|V_2|} - |V_2|B_{22}$$

all the above equations will be calculated at each iteration

now, from the initial guess & specifications,

⇒ Calculate mismatch vector $\Delta P_2^{(0)}$, $\Delta P_3^{(0)}$, $\Delta Q_2^{(0)}$

$$\Delta P_2^{(0)} = P_2^{(0)}(x^0) - P_2^{spec} = -1.14 - (-4) = 2.86$$

$$\Delta P_3^{(0)} = P_3^{(0)}(x^0) - P_3^{spec} = 0.5616 - 2 = -1.4384$$

$$\Delta Q_2^{(0)} = Q_2^{(0)}(x^0) - Q_2^{spec} = (-2.28) - (-2.5) = 0.22$$

⇒ evaluate J at 0^{th} iteration

$$J = \begin{bmatrix} 54.28 & -33.28 & 24.86 \\ -33.28 & 66.04 & -16.64 \\ -27.14 & 16.64 & 49.72 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \Delta\theta_2^{(0)} \\ \Delta\theta_3^{(0)} \\ \Delta|V_2|^{(0)} \end{bmatrix} = -[J]^{-1} \begin{bmatrix} \Delta P_2^{(0)} \\ \Delta P_3^{(0)} \\ \Delta Q_2^{(0)} \end{bmatrix} = -[J]^{-1} \begin{bmatrix} 2.86 \\ -1.43 \\ 0.22 \end{bmatrix}$$

mismatch vector

⇒ solve for $\Delta\theta_2^{(0)}$, $\Delta\theta_3^{(0)}$, $\Delta|V_2|^{(0)}$ & update values

$$\begin{bmatrix} \Delta\theta_2^{(0)} \\ \Delta\theta_3^{(0)} \\ \Delta|V_2|^{(0)} \end{bmatrix} = \begin{bmatrix} -0.045263 \\ -0.007718 \\ -0.026548 \end{bmatrix} \Rightarrow$$

$$\theta_2^{(1)} = 0 - 0.045263 = -0.045263$$

$$\theta_3^{(1)} = 0 - 0.007718 = -0.007718$$

$$|V_2|^{(1)} = 1 + (-0.026548) = +0.97345$$

Calculate $P_2^{(1)}$, $P_3^{(1)}$, $Q_2^{(1)}$ from updated state variables and Power mismatch vector $\Delta P_2^{(1)}$, $\Delta P_3^{(1)}$, $\Delta Q_2^{(1)}$

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \end{bmatrix}^{(1)} = \begin{bmatrix} P_2^{(1)}(x') \\ P_3^{(1)}(x') \\ Q_2^{(1)}(x^{(1)}) \end{bmatrix} - \begin{bmatrix} P_2^{spec} \\ P_3^{spec} \\ Q_2^{spec} \end{bmatrix}$$

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \end{bmatrix}^{(1)} = \begin{bmatrix} -3.9008 \\ 1.9783 \\ -2.4491 \end{bmatrix} - \begin{bmatrix} -4 \\ 2 \\ -2.5 \end{bmatrix} = \begin{bmatrix} 0.0992 \\ -0.0217 \\ 0.0509 \end{bmatrix}$$

after 1 iteration mismatch vector is reduced significantly

now perform next iteration

$$J^{(1)} = \begin{bmatrix} 51.72 & -31.76 & 21.30 \\ -32.98 & 65.65 & -15.37 \\ -28.53 & 17.40 & 48.10 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \Delta\theta_2 \\ \Delta\theta_3 \\ \Delta|V_2| \end{bmatrix}^{(1)} = -[J]^{-1} \begin{bmatrix} 0.0992 \\ -0.0217 \\ 0.0509 \end{bmatrix} = \begin{bmatrix} -0.0016 \\ -0.0009 \\ -0.0016 \end{bmatrix}$$

$$\begin{bmatrix} \theta_2 \\ \theta_3 \\ |V_2| \end{bmatrix}^{(2)} = \begin{bmatrix} -0.045263 \\ -0.007718 \\ 0.97345 \end{bmatrix} + \begin{bmatrix} -0.0016 \\ -0.0009 \\ -0.0016 \end{bmatrix} = \begin{bmatrix} -0.0469 \\ -0.0086 \\ 0.9718 \end{bmatrix}$$

now,

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \end{bmatrix}^{(2)} = \begin{bmatrix} -3.9924 \\ 1.9998 \\ -2.4970 \end{bmatrix} - \begin{bmatrix} -4 \\ 2 \\ -2.5 \end{bmatrix} = \begin{bmatrix} 0.0076 \\ -0.0004 \\ 0.0030 \end{bmatrix}$$

We can stop now since it is reasonably small

$\Rightarrow$ final solution

$V_2 = 0.9718$

$\theta_2 = -0.0469$

$\theta_3 = -0.0086$

Slack bus power

$P_1 = 2.17$ pu

$Q_1 = 1.40$ pu

Q at PV bus

$Q_3 = 1.457$ pu

this is important

$\Rightarrow$ Line flows

$S_{12} = V_1 I_{12}^* = \vec{V}_1 (\vec{V}_1 - \vec{V}_2)^* y_{12}^* = 1.80 + 1.149$ pu

$= 180$ mw & 114.9 mVAR

$S_{21} = V_2 I_{21}^* = \vec{V}_2 (\vec{V}_2 - \vec{V}_1)^* y_{21}^* = -172$ mw & ~~-101.0~~ 98.3 mVAR

S_{losses}^{12} = losses in line 1-2 $= S_{12} + S_{21} = 8$ mw, 16 mVAR

Similarly, other line flows

$S_{13} = 0.3759 + 0.2247$ pu

$S_{31} = -0.3742 - 0.2195$ pu

$S_{23} = -2.3251 - 1.4167$ pu

$S_{32} = 2.4230 - 1.6125$ pu