Structural Engineering

* Difference between Working stress and limit State Philosophy.

Working stress	Limit State
(1) This method is based on elastic theory which assumes that steel & concrete are elastic material and stress-strain curve for both is linear.	(1) This method is based on actual stress-strain relationship (curve) of steel & concrete. for example, the stress strain relationship for Concrete is non-linear.
(2) In this method, factor of safety are applied to yield stress to get permissible stresses.	(2) In this method, partial safety factors are applied to get the design stresses. (1.15 - steel, 1.5 - concrete)
(3) No factor of safety is used for loads.	(3) Design Loads are obtained by multiplying partial safety factors of loads to working Loads.
(4) Exact margin of safety is not known.	(4) Exact margin of safety is known.
(5) This method gives thicker sections so less economical.	(5) This method is economical as it gives thinner sections.
(6) This method assumes that the actual loads, permissible stresses, and factor of safety are known. So it is called as deterministic approach.	(6) This method is based on the probablistic approach which depends upon actual data or experience. Hence, it is called as non-deterministic approach

→ WSM & LSM are design philosophies / methods used for the design of RCC structures.

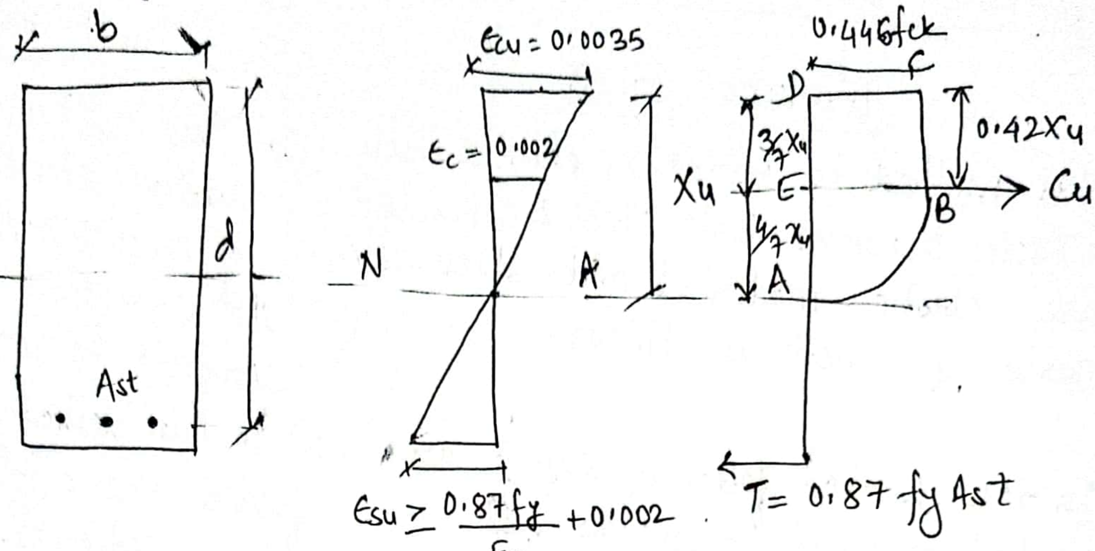
→ WSM is very simple & reliable but as per IS 456: 2000 the WSM is to to be used only if it is not possible to use LSM of design.

→ The object of LSM is based on the concept of achieving an acceptable probability that a structure will not become unserviceable in its lifetime for the use it is intended.

- Limit state of collapse (flexure, compression, shear, torsion)
- Limit state of serviceability (deflection, cracking, vibration)

Analysis of Singly Reinforced Beam

Consider a singly reinforced beam section subjected to bending as shown :



(a) Section (b) Strain diagram (c) stress diagram

fig : Stress and strain distribution in a singly reinforced beam as per IS 456

Strain distribution

→ The assumption of limit state theory gives a linear strain distribution across the cross-section as shown in fig (b).

→ Varies as zero at the neutral axis & maximum at the extreme fibres.

~~Features~~

→ Strain at NA = 0

→ Max^m or ultimate strain in concrete at extreme fibre, $\varepsilon_{cu} = 0.0035$

→ Strain at constant stress of $0.67 f_{ck}$, $\varepsilon_c = 0.002$

→ Ultimate strain in steel corresponding to max^m stress at failure,

$$\varepsilon_{su} = \frac{0.87 f_y}{E_s} + 0.002$$

f_y = Characteristic strength of steel

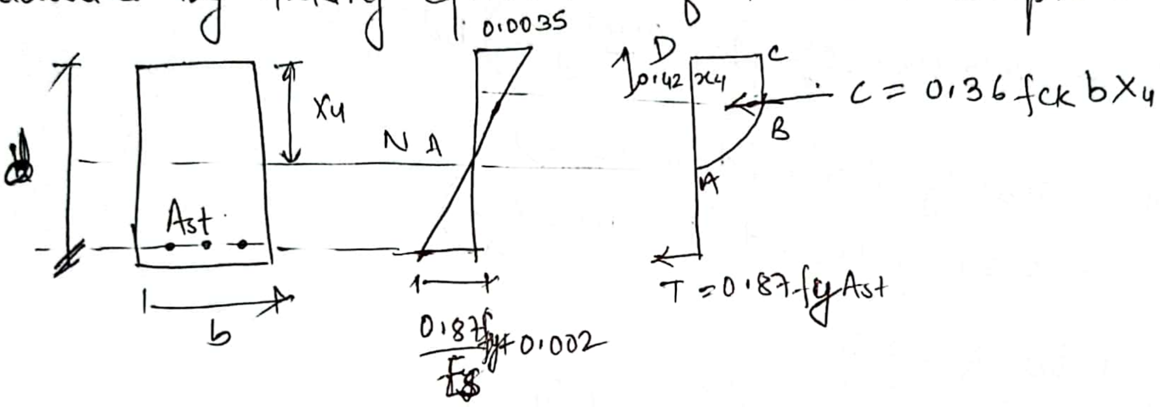
E_s = Modulus of elasticity of steel (2×10^5 N/mm^2)

Stress distribution

- shown in fig (c)
- parabolic shape from A & B & linear from B to C above NA
- Stress ~~of~~ at NA = 0 (Pt. A)
- stress at 0.002 strain = $\frac{0.67 f_{ck}}{1.5} = 0.446 f_{ck}$ (Pt. B)
- Stress at extreme fiber = $0.446 f_{ck}$ (Pt. C)
- Below the NA, the concrete is assumed to be cracked & max^m stress in steel $= \frac{f_y}{1.15} = 0.87 f_y$

Neutral Axis depth (x_u)

→ Calculated by taking equilibrium of tensile & compressive forces



$$\text{Total Tension} = \text{Total Compression}$$

$$0.87 f_y A_{st} = 0.36 f_{ck} x_u b$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b d}$$

Limiting depth of NA ($x_{u\,max}$)

Max^m strain value of concrete at extreme fibre is 0.0035 & the strain in steel at failure should not be less than $\frac{0.87 f_y}{E_s} + 0.002$, this limits the depth of NA to its maximum or limiting values.

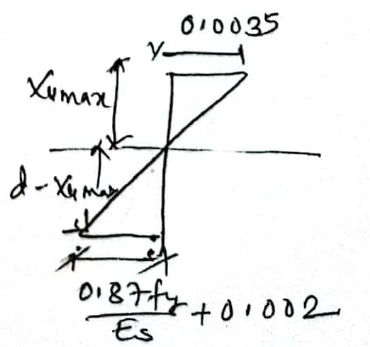


fig : strain diagram

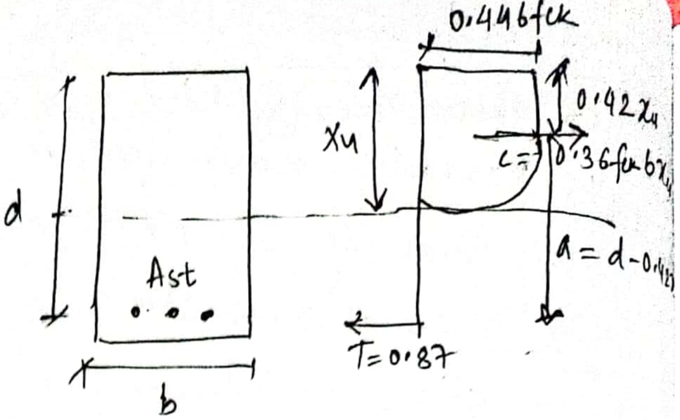
from fig:

$$\frac{x_{u\,max}}{0.0035} = \frac{d - x_{u\,max}}{\frac{0.87 f_y}{E_s} + 0.002}$$

$$\frac{x_{u\,max}}{d} = \frac{0.0035}{\frac{0.87 f_y}{E_s} + 0.0055}$$

Moment of Resistance (M_u)

The MoR is equal to the moment of the couple formed by two equal & opposite forces ie. total compression & total tension.



Ultimate moment of resistance (M_u)

$$M_u = C \times \text{lever arm} = T \times \text{lever arm}$$

$$\text{Lever arm} = d - 0.42 x_u$$

Compression side

$$M_u = 0.36 f_{ck} b x_u \times (d - 0.42 x_u)$$

$$= 0.36 f_{ck} b d x_u \times \left(1 - 0.42 \frac{x_u}{d}\right)$$

$$= 0.36 f_{ck} \cdot \frac{x_u}{d} \left(1 - \frac{0.42 x_u}{d}\right) bd^2$$

Tension side

$$M_u = 0.87 f_y \cdot A_{st} \times (d - 0.42 x_u)$$

$$= 0.87 f_y A_{st} \times d \left(1 - 0.42 \frac{x_u}{d}\right)$$

Putting value of $\frac{x_u}{d}$ in

$$M_u = 0.87 f_y A_{st} \times d \left(1 - \frac{A_{st} f_y}{bd f_{ck}}\right)$$

Percentage of Steel (P_t)

↳ can be found by equating the tensile & compressive forces.

$$T = C$$

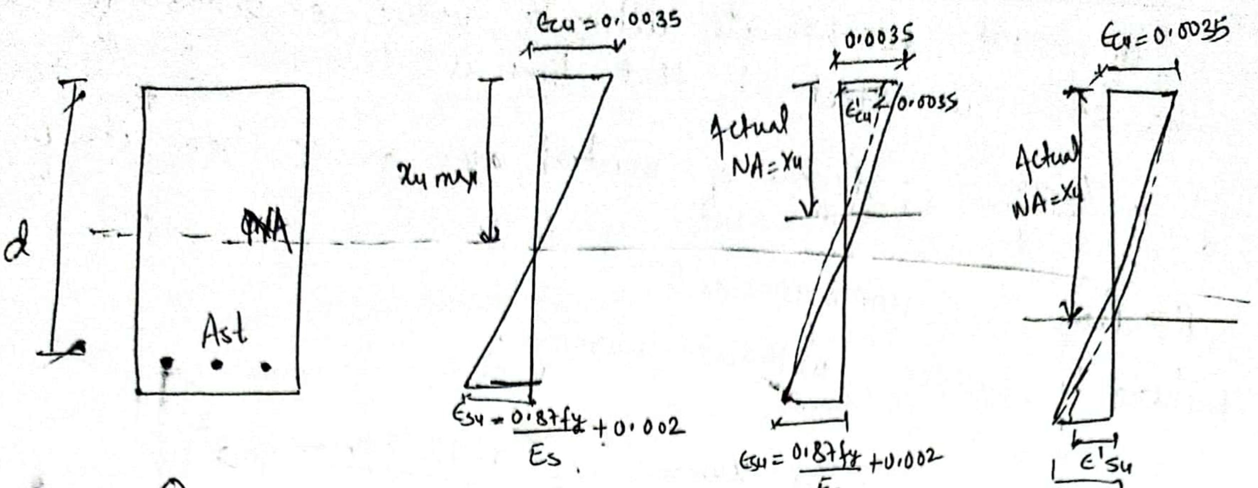
$$0.87 A_{st} \cdot f_y = 0.36 f_{ck} b x_u$$

$$\frac{A_{st}}{b} = \frac{0.36 f_{ck} x_u}{0.87 f_y}$$

$$P_t = \frac{A_{st}}{bd} = \frac{0.36 f_{ck} x_u}{0.87 f_y d}$$

$$P_{t\,lim} (\%) = \frac{0.36 f_{ck}}{0.87 f_y} \times \frac{x_{u\,max}}{d} \times 100$$

Balanced, Under-Reinforced and Over-Reinforced Sections



(a)

(b) Balanced section

(c) Under-reinforced section

(d) Over-reinforced section

I. Balanced section $\left(x_u/d = \frac{x_{u\,max}}{d} \right)$

- In this, the steel reinforcement reaches the yield strain i.e. $\frac{0.87 f_y}{E_s} + 0.002$ at the same time as the concrete reaches the ultimate strain of 0.0035.
- $$\frac{x_u}{d} = \frac{x_{u\,max}}{d}$$
- $P_t = P_{t\,lim}$, limiting percentage of steel
- Moment of resistance is equal to its limiting value $M_{u\,lim}$ & is calculated as.

$$M_{u\,lim} = 0.36 f_{ck} \frac{x_{u\,max}}{d} \left(1 - \frac{0.42 x_{u\,max}}{d}\right) b d^2$$

$$M_{u\,lim} = 0.87 f_y A_{st} d \left(1 - \frac{0.42 x_{u\,max}}{d}\right)$$

II. Under-Reinforced section $\left(x_u/d < \frac{x_{u\,max}}{d} \right)$

- In this, the steel fails first by reaching its yield strain value, although in concrete the ultimate strain has not reached.
- The strain in steel reaches its yield value first i.e. $\frac{0.87 f_y}{E_s} + 0.002$ but at that time the strain in concrete is less than 0.0035.
- $\frac{x_{u\,max}}{d} > \frac{x_u}{d}$ i.e. depth of N.A is less than limiting value.
- $P < P_{lim}$; Section fails in a ductile manner.
- $$M_u = 0.87 f_y A_{st} d \left(1 - \frac{0.42 x_u}{d}\right)$$

III. Over-reinforced section $\left(\frac{x_u}{d} > \frac{x_{u\,max}}{d}\right)$

- Strain in concrete reaches its ultimate value ie. 0.0035 first & the strain in steel at that time is less than $\frac{0.87 f_y}{E_s} + 0.002$.
- $\frac{x_u}{d} > \frac{x_{u\,max}}{d}$; depth of neutral axis greater than its limiting value.
- $P > P_{lim}$; uneconomical.
- failure is sudden, without warning.

$$M_{u\,lim} = 0.36 f_{ck} \frac{x_{u\,max}}{d}\left(1 - 0.42\frac{x_{u\,max}}{d}\right) bd^2$$

* Assumptions of WSM & LSM

WSM

① A section which is plane before bending remains plane after bending. ie. strain above & below NA ∝ distance from NA. Δr shape zero at NA. max^m at extreme fibre

② Steel and Concrete are perfectly bonded.

③ All Tensile stresses are taken up by steel & none by concrete.

④ Stress-strain relationship ~~for~~ of steel and concrete under working loads is a straight line. ie. stress distribution is also linear.

⑤ Modulus of elasticity of steel and concrete are constant.

⑥ Modular ratio has value of $\frac{280}{3\sigma_{cbc}}$, σ_{cbc} - permissible compressive strength of concrete.

⑦ There are no initial stresses in steel & concrete.

LSM

① Plane sections normal to the axis remain plane after bending.

② The max^m ~~comp~~ strain in concrete at the outermost compression fibre is taken as 0.0035 in bending.

③ The relationship between the stress-strain distribution in concrete is assumed to be parabolic. Compressive strength = $0.67 \times f_{ck}$, $\gamma_{mc} = 1.5$

④ The tensile strength of the concrete is ignored.

⑤ The stresses in the reinforcement are taken from the stress-strain curve for the type of steel used.

$\gamma_{ms} = 1.15$

⑥ The max^m strain in steel at failure $\frac{f_y}{1.15 E_s} + 0.002$

Design of Singly Reinforced Beams / Doubly Reinforced Beams.

Basic Rules for Design of Beams.

→ A beam is a member subjected predominantly to bending.

1. Effective Span

The effective span of beams are taken as.

(a) Simply Supported Beam

The eff. span of S.S.B is taken as least of the following:

(i) Clear span plus effective depth of beam
(ii) center to center distance between support.

(b) Continuous Beam

In case of continuous beam, if the width of support is less than 1/12 of clear span, the effective span is taken as in (a).

If the width of support is greater than 1/12 of clear span or 600 mm whichever is less, the effective span is taken as:

(i) For end span with one end fixed & other continuous or for intermediate spans, the eff. span shall be clear span betn the supports.

(ii) For end span with one end free & other continuous, the eff. span shall be clear span plus half the eff. depth of beam or clear span plus half the ~~eff~~ width of discontinous support whichever is less.

(c) Cantilever Beam. – eff. span is taken as

(i) length of over hang plus half the effective depth
(ii) except where it forms the end of a continuous beam where the length up to the center of support is taken.

2. Effective depth

Effective depth of the beam is the distance between the centroid of area of tension reinforcement & topmost Compression fibre. It is equal to the total depth of Beam minus effective cover.

3. Control of Deflection

For beams, vertical deflection limits may be assumed to be satisfied if the span to depth ratios are not greater than the following.

(a) for span up to 10 m

(i) Simply supported beam ; $\frac{\text{Span}}{\text{Eff. depth}} = 20$

(ii) Cantilever beam ; $\frac{\text{Span}}{\text{eff. depth}} = 7$

(iii) Continuous beam ; $\frac{\text{Span}}{\text{eff. depth}} = 26$

(b) for span greater than 10 m, the values given in (a) should be multiplied by 10/span (m), except for cantilever for which is to be calculated the exact deflection.

(c) Value of span to depth ratio can be modified as per modification factor chart depending on area of tension reinforcement & compression reinforcement.

4. Reinforcement

(a) Minimum Reinforcement

$$A_{st} = \frac{0.85\,bd}{f_y}$$

f_y – characteristics strength of reinforcement in N/mm^2

(b) Max^m Reinforcement

$A_{st\,max}$ shall not exceed $0.04\,bD$ ← total depth

(c) Side face Reinforcement. (for depth of web > 750 mm) deep beam.

↳ provided along two faces.

↳ Area of reinforcement ≮ (not less than) 0.1 percent of web area

↳ Spacing { not be more than 300 mm / web thickness } whichever is less (to be distributed equally on both faces.

(d) Transverse Reinforcement for shear.

(i) Min^m shear reinforcement ; $\frac{A_{sv}}{b \cdot S_v} \geq \frac{0.4}{0.87 f_y}$ (design shear strength (τ_c) > nominal shear stress (τ_v))

(ii) shear reinforcement ($\tau_v > \tau_c$)

(a) Vertical stirrups, $V_{us} = \frac{0.87 f_y A_{sv} \cdot d}{s_v}$

(b) bent up bars, $V_{us} = 0.87 f_y A_{sv} \sin\alpha$

(c) Combined system

(e) Spacing of Reinforcement Bars

(i) The horizontal distance between two parallel main bars shall not be less than the greatest of the following:

(a) diameter of the bar if the bar are of same dia.

(b) diameter of larger bar if the diameter are unequal.

(c) 5 mm more than the nominal max^m size of coarse aggregate.

(ii) When the bars are in rows, they should be vertically in line & the min^m vertical distance between the bars shall be greater of following:

(a) 15 mm

(b) 2/3 rd of nominal max^m size of aggregate

(c) Max^m diameter of the bar.

5. Nominal cover to reinforcement.

↳ depth of concrete cover to all steel reinforcement.

↳ not less than diameter of bar in any case.

Nominal cover to meet the durability requirement

Exposure condition	Nominal Cover (mm)
Mild	20
Moderate	30
Severe	45
Very Severe	50
Extreme	75

6. Curtailment of Tension Reinforcement

↳ The reinforcement shall extend at least d or 12ϕ (whichever is greater) beyond the point of theoretical cut off.

↳ As per SP 34, in case of S.S.B, 50% of the main bars can be Curtailed at distance 0.08 l from the face of support.

Design of Singly Reinforced Beams

1. Determine design bending moment (If M_u is not given)

   (i) Assume suitable value of depth and width (to start with, assume depth of beam as $\frac{1}{12}$ to $\frac{1}{15}$ & width of beam as $\frac{1}{2}$ to $\frac{1}{3}$ of the depth).

   (ii) Calculate self-wt. of beam & determine total ~~wt. of beam~~ load (w) by adding imposed loads to self wt.

   (iii) Determine design or factored load (w_u)

   $w_u = \gamma \times w$ (γ = Partial safety factor for loads & is taken as 1.5 for DL + LL)

   $= 1.5 \times w$

   (iv) Calculate the factored moment.

   $$M_u = \frac{w_u l^2}{8}, \text{ for simply supported beams}$$

   $$M_u = \frac{w_u l^2}{2}, \text{ for cantilever beam}$$

2. Determine $\frac{x_{u\,max}}{d}$ for required grade of steel & R_u for required grade of concrete & steel.

   $$R_u = 0.36 f_{ck} \frac{x_{u\,max}}{d}\left(1 - \frac{x_{u\,max}}{d} \times 0.42\right)$$

3. Determine the minimum depth required.

   $$d_{reqd} = \sqrt{\frac{M_u}{R_u \cdot b}}$$

4. Compare d_{reqd} with assumed value of effective depth.

   ① If $d_{assumed} > d_{req}$, then our assumption is correct & provide the assumed depth & calculate overall depth (+ Cover)

   ② If $d_{assumed} < d_{reqd}$, then redesign the section & repeat 2 & 3

5. Determine the area of steel required

   $d_{required} < d_{assumed/provided}$ → so under reinforced section.

   $$M_u = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{b d f_{ck}}\right)$$

CS CamScanner

$Ast > \frac{0.85 d b}{fy}$; Choose suitable dia. of bar

$A\phi = \frac{\pi d^2}{4}$

No. of bars reqd. $= \frac{Ast}{A\phi}$

for balanced section ; $d_{reqd} = d_{provided}$

~~$Ast = 0.87 fy\, Ast$~~

$$Ast = \frac{Mu}{0.87 fy (d - 0.42 x_u)}$$

6/ Check for deflection

i) Calculate service stresses (fs) & Pt

$$fs = 0.58 fy \left[\frac{Ast_{req}}{Ast_{pro}}\right] \quad \& \quad Pt = \frac{100 Ast}{bd}$$

(ii) find out modification factor (K_t) for fs & Pt.

(iii) $(l/d)_{max} = 20 \times K_t$ — for S.S.B

$(l/d)_{max} = 7 \times K_t$ — for cantilever)

if, $(l/d)_{max} > (l/d)_{provided}$; ok

if $(l/d)_{max} < (l/d)_{provided}$; redesign the section.

7/ Design for shear

(i) Calculate Shear force (V_u) ; $V_u = \frac{W_u L}{2}$ for UDL

(ii) Determine nominal shear stress (τ_v) ; $\tau_v = \frac{V_u}{bd}$

(iii) Depending on grade of concrete & % of steel, find out shear strength of concrete (τ_c).

(iv) Compare τ_v & $\tau_{c\,max}$; if $\tau_v > \tau_{c\,max}$, redesign the section

(v) Compare τ_v & τ_c.

(a) If $\tau_v < \tau_c$; Nominal shear reinforcement in the form of vertical stirrups.

$$\frac{Asv}{b \cdot Sv} \geq \frac{0.4}{0.87 fy}$$

(b) If $\tau_v > \tau_c$; shear reinforcement is to be designed.

(i) Calculate V_{us}

$$V_{us} = V_u - \tau_c bd$$

for vertical stirrups,

$$V_{us} = \frac{0.87 f_y A_{sv} \cdot d}{S_v}$$

for bent up bars,

$$V'_{us} = 0.87 f_y A_{sv} \sin\alpha.$$

(v) Spacing of stirrups should not exceed 0.75d or 300 mm whichever is less.

8/ Check for development length

$$\frac{M_1}{V} + l_o \geq L_d ;$$

* Analysis of Doubly Reinforced Beam

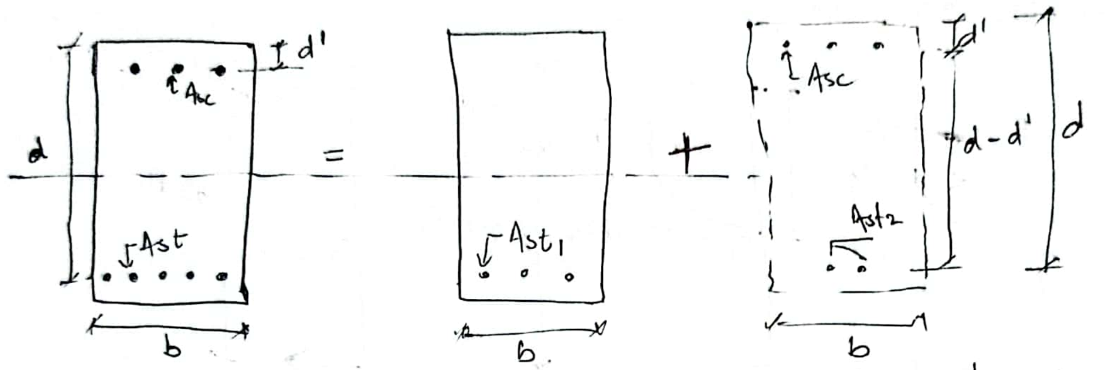
→ A doubly reinforced beam has moment of resistance greater than that of balanced section.

→ Analysed by considering ~~two~~ two sections

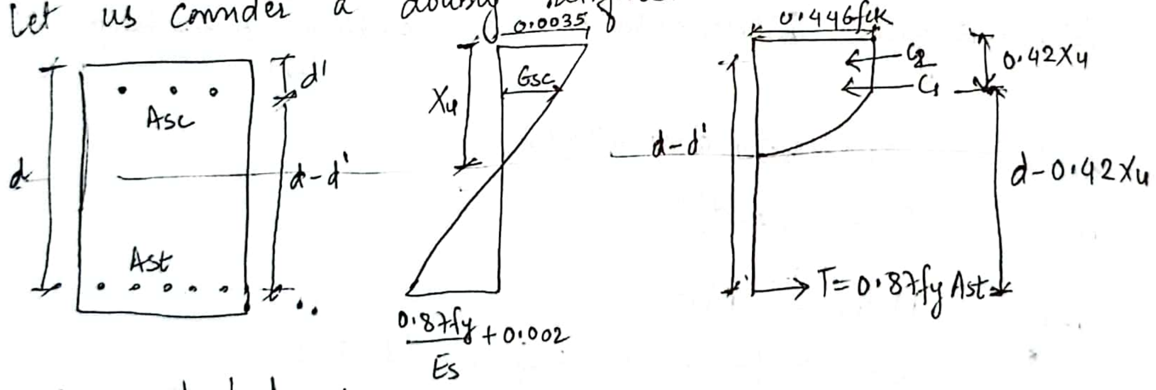
Section 1 — Singly reinforced balanced section having area of steel A_{st1} & Moment of resistance M_{ulim}

Section 2 — consists of compression steel A_{sc} & additional tensile steel A_{st2} corresponding to A_{sc}.

$$M_u = M_{ulim} + M_{u2}$$



Let us consider a doubly reinforced beam as shown.



Depth of Neutral axis. (x_u)

↳ obtained by equating total tension and total compression.

Total compression, $C = C_1 + C_2$

C_1 = force carried by the concrete area

C_2 = Compressive force carried by the Compression steel A_{sc}

$C_1 = 0.36 f_{ck} \cdot b * x_u$

$C_2 = f_{sc} A_{sc} - f_{cc} A_{sc}$

f = stress

$f_{cc} A_{sc}$ - accounts for the loss of concrete area occupied by compression steel.

$C = 0.36 f_{ck} \cdot b \cdot x_u + f_{sc} A_{sc} - f_{cc} A_{sc}$

$= 0.36 f_{ck} \cdot b \cdot x_u + A_{sc}(f_{sc} - f_{cc})$

$T = 0.87 f_y A_{st}$

$C = T$

$0.36 f_{ck} \cdot b \cdot x_u + f_{sc} A_{sc} - f_{cc} A_{sc} = 0.87 f_y A_{st}$

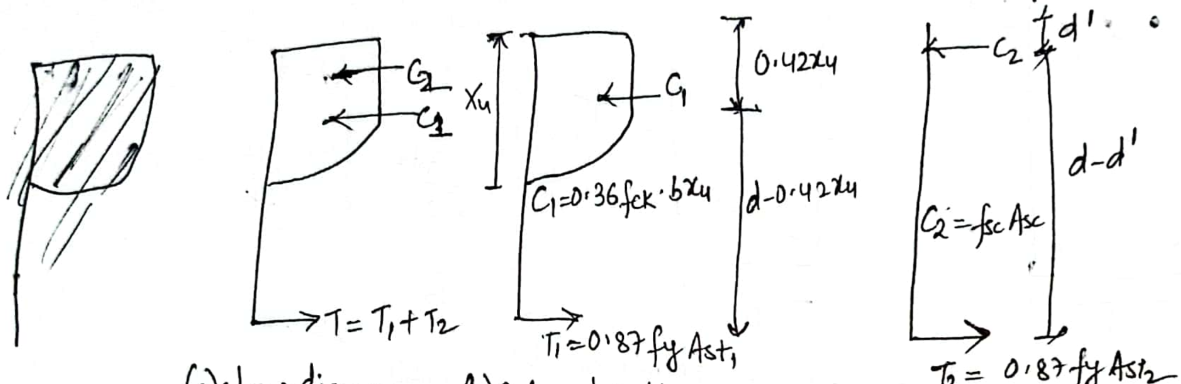
$0.36 f_{ck} \cdot b x_u + A_{sc}(f_{sc} - f_{cc}) = 0.87 f_y A_{st}$

$$x_u = \frac{0.87 f_y A_{st} - A_{sc}(f_{sc} - f_{cc})}{0.36 f_{ck} \cdot b}$$

$$= \frac{0.87 f_y A_{st} - A_{sc} f_{sc}}{0.36 f_{ck} \cdot b}$$

Moment of Resistance (Mu)

↳ Obtained from the stress diagram as shown;



(a) stress diagram (b) Balanced section (c) Additional section having M_{u2}

$M_{ulim} = 0.36 f_{ck} b \cdot x_u (d - 0.42 x_u)$

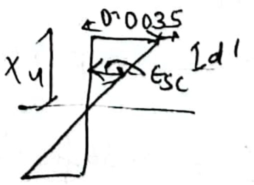
$M_{u2} = f_{sc} A_{sc} \cdot (d - d')$

$M_u = M_{ulim} + M_{u2}$

$= 0.36 f_{ck} \cdot b x_u (d - 0.42 x_u) + f_{sc} A_{sc} \cdot (d - d')$

The value of f_{sc} depends upon the amount of strain in compression steel which is obtained from the strain diagram & d' value

$$\frac{0.0035}{x_u} = \frac{\epsilon_{sc}}{x_u - d'}$$



$$\epsilon_{sc} = \frac{0.0035(x_u - d')}{x_u}$$

$$= 0.0035\left(1 - \frac{d'}{x_u}\right)$$

Corresponding to this ϵ_{sc} value, stress (f_{sc}) in compression steel can be obtained from stress strain curve of given type of steel.

Design of Doubly Reinforced Beams for flexure

↳ Design of doubly reinforced beam generally comprises of determining area of tension and compression steel as the dimensions of the beam are already fixed (or restricted).

1) Determination of Ast

⊛ Determination of Mu lim

$M_u = 0.148\, f_{ck} bd^2$ (250 Fe)

$= 0.138\, f_{ck} bd^2$ (415 Fe)

$= 0.133\, f_{ck} bd^2$ (500 Fe)

Area of steel corresponding to the singly reinforced balanced section (section 1) $= A_{st1}$

$$A_{st_1} = \frac{M_{u\,lim}}{0.87 f_y (d - 0.42 x_{umax})}$$

Area of steel corresponding to section 2 $= A_{st2}$

Moment of resistance of section 2

$$M_{u2} = M_u - M_{u\,lim}$$

$$M_{u2} = 0.87 f_y A_{st_2} (d - d') \quad \text{[Considering tensile Steel]}$$

$$M_{u2} = (f_{sc} A_{sc} - f_{sc} A_{cc})(d - d') \quad \text{[considering compression steel]}$$

$$A_{st_2} = \frac{M_{u2}}{0.87 f_y (d - d')}$$

$$A_{st} = A_{st_1} + A_{st_2}$$

2) Determination of Area of compression steel.

$$M_{u2} = (f_{sc} A_{sc} - f_{sc} A_{cc})(d - d')$$

Neglecting loss of concrete area,

$$M_{u2} = (f_{sc} A_{sc})(d - d')$$

$$A_{sc} = \frac{M_{u2}}{f_{sc}(d - d')}$$

Steps involved in the design of a doubly reinforced beam : ⇒ comprises of determining area of tension and compression steel.

Given : Dimension of the beam ie. $b \times D$ or $b \times d$
Grade of concrete and type of steel
Factored bending moment (M_u)

1. Determine the value of f_{sc} for d'/d ratio for given grade of steel.

2. Determine X_{umax} ie. limiting depth of neutral axis & M_{ulim}

$$M_{ulim} = 0.36 f_{ck}\, b\, x_{umax} (d - 0.42 X_{umax})$$

$M_{ulim} = 0.148 f_{ck} bd^2$ (Fe250)
$= 0.138 f_{ck} bd^2$ (Fe415)
$= 0.133 f_{ck} bd^2$ (Fe500)

3. Determine A_{st1} (Area of steel corresponding to singly reinforced balanced section)

$$A_{st1} = \frac{M_{ulim}}{0.87 f_y \cdot (d - 0.42 X_{umax})}$$

4. Determine M_{u2} & A_{st2}

$$M_{u2} = M_u - M_{ulim}$$

$$A_{st2} = \frac{M_{u2}}{0.87 f_y (d - d')}$$

5. Determine A_{st}

$A_{st} = A_{st1} + A_{st2}$, choose suitable diameter of bar and provide them

6. Determine area of compression steel (A_{sc})

$$A_{sc} = \frac{M_{u2}}{f_{sc}(d - d')}$$

Provide A_{sc} by choosing suitable diameter of the bar.

7. Check for deflection control.

For $P_t = \frac{100 A_{st}}{bd}$ & $f_s = 0.58 f_y \left[\frac{A_{st}\text{ provided}}{A_{st}\text{ required}}\right]$, find k_t

for $P_c = \frac{100 A_{sc}}{bd}$, find k_c.

$$\left(\frac{l}{d}\right)_{max} = 20 \times K_t \times K_c \quad ; \text{ S.S.B}$$

$$= 7 \times K_t \times K_c \quad ; \text{ Cantilever.}$$

If $\left(\frac{l}{d}\right)_{max} > \left(\frac{l}{d}\right)_{provided}$; then ok

If $\left(\frac{l}{d}\right)_{max} < \left(\frac{l}{d}\right)_{provided}$; then redesign the section.

8. Check for shear.

9. Check for development length

$$\frac{M_1}{V_u} + l_0 > L_d$$

* Necessity of Doubly Reinforced Section

(i) When the dimensions ($b \times d$) of the beam are restricted due to any constraints like availability of head room, architectural or space considerations and the moment of resistance of singly reinforced section is less than the external moment.

(ii) When the external loads may occur on either face of the member ie. the loads are alternating or reversing & may cause tension on both faces of the member.

(iii) When the loads are eccentric.

(iv) When the beam is subjected to accidental or sudden lateral loads.

(v) In the case of continuous beams or slab, the sections at supports are generally designed as doubly reinforced sections.

Slabs.

* IS 456 : 2000 Recommendations for design of slabs.

(1) Effective Span

(a) Simply Supported ~~Beam~~ slab

The effective span is taken as smaller of the following:

(i) Center to center of supports.

(ii) Clear distance between the supports plus the effective depth.

(b) Continuous ~~Beam~~ slab

→ width of support $< \frac{1}{12}$ clearspan, effective span is taken as given in (a) of simply supported slab.

→ width of support $> 1/12$ of the clear span or 600 mm, whichever is less, effective span shall be taken as under:

(i) for end span, with one end fixed and the other end continuous or for intermediate spans, the effective span shall be the clear span between supports.

(ii) for end span, with one end free & the other end continuous, the effective span shall be equal to clear span plus the half the effective depth of slab or clear span plus half the width of discontinuous support whichever is less.

(2) Deflection Control

For slabs, the vertical deflection limits are specified by maximum l/d ratio.

(a) for spans up to 10m

	l/d ratio
Cantilever	7
Simply supported	20
Continuous	26

(b) for spans greater than 10m, the above value may be multiplied by 10/span, except for cantilever, for which exact deflection calculations should be made.

(c) for slabs spanning in two directions, the shorter of the two spans shall be used for calculating span to effective depth ratio.

(3) Reinforcement in slabs

(a) Minimum Reinforcement

→ Area of reinforcement in either direction in a slab should not be less than 0.15 percent of the total cross-sectional area in case of mild steel reinforcement.

→ In case of high strength deformed bars, this value can be reduced to 0.12 %.

(b) Maximum Diameter.

→ Max^M diameter of the reinforcing bar in a slab should not exceed $\frac{1}{8}$th of the total thickness of the slab.

(c) Distribution Reinforcement

↳ provided in the longer span of one way slab.

↳ as per minimum reinforcement criteria.

(d) Spacing of Reinforcement

(1) Minimum distance between bars

(i) Minimum horizontal distance between two parallel main bars shall not be less than

- diameter of the bar (largest diameter is to be considered)
- 5 mm more than the nominal maximum size of coarse aggregate

(ii) the vertical distance between two layers of main reinforcement shall be more than:

- 15 mm or
- $\frac{2}{3}$rd the nominal max^M size of aggregate
- Maximum size of the bar

(2) Maximum distance between bars in tension.

(i) Main bars spacing should not exceed the following:

- 3 times the effective depth of slab
- 300 mm

(iv) distribution steel

- 5 times the effective depth of slab
- 450 mm

(e) Cover

↳ Nominal cover to be provided in slab is 20 mm.

(f) Bent up bars

↳ bent up near the support.

↳ alternate bars are bent up at a distance of 0.15l from the center of support.

↳ bar available at the upper face should be more than 0.1 l from the center of support.

(g) Curtailment of bars

In practice, the bars are not curtailed in slabs.

(h) Shear Design

Slabs are safe in shear (nominal shear stress is very low since b is large) therefore no shear reinforcement is provided in slabs except that the alternate bars are bent up near the supports.

Design of one way slab ($l_y/l_x > 2$)

↳ One way slab is designed exactly as a rectangular beam, the only differences are following :

(i) The width of the beam is assumed as one metre.

(ii) The depth of slab can be assumed on the basis of control of deflection.

(iii) In addition to the main tensile reinforcement provided along shorter span, transverse reinforcement or distribution reinforcement is provided.

(iv) Some of the main bars in a slab are bent up near the supports (0.15 l from the center of the support)

(v) Shear is to be checked only. No shear reinforcement is provided.

Two Way Slabs. ($l_y/l_x \leq 2.0$)

(a) Restrained slabs.

→ Corners of the slabs are prevented from lifting.

$$M_x = \alpha_x w l_x^2$$

$$M_y = \alpha_y w l_y^2$$

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* Design steps for two way slab

Known data : Material properties
: Load on slabs.

Step 1 : Calculate the depth of the slab based on deflection control criteria.

$$\frac{\text{Shorter Span}}{\text{depth}} = 35 \text{ to } 40$$

The value should be multiplied by 0.8 for deformed bars.

Step 2 : Calculate the effective length of slab.

$l_{eff.}\ x = c/c$ or $l_{cx} + d$
$l_{eff.}\ y = c/c$ or $l_{cy} + d$
} ~~whichiver~~ whichever is less for both cases.

Step 3 : Calculate the factored loads on slab.

$W_u = 1.5 *$ total load on slab.

Step 4 : Calculate the factored moments. $\alpha_x = \frac{1}{8}\left(\frac{r^4}{1+r^4}\right)$ $\left(r = l_y/l_x\right)$

$M_{ux} = \alpha_x * W_u * l_x^2$

$M_{uy} = \alpha_y * W_u * l_y^2$ $\alpha_y = \frac{1}{2}\left(\frac{r^2}{1+r^4}\right)$

where,

α_x, α_y = BM coefficient. (depends upon l_y/l_x & Conditions of slab - Codal provision)

Step 5 : Depth Verification.

→ Max^m moment $= 0.36 f_{ck}\, b x_{u,max} (d - 0.42 x_{u,max})$

find d.

Adopt the depth of slab = Max^m of (depth from deflection control & moment criteria)

Step 6 : Calculate the area of reinforcement.

$$M_{ux} / M_{uy} = 0.87 f_y Ast_{x/y}\left(d - \frac{Ast_{x/y} f_y}{f_{ck}\, b}\right)$$

* Difference Between Short Column & Long Column

Short Column	Long Column
S.R 1. A column is considered to be short if the ratio of effective length to its least lateral dimension is less than or equal to 12.	1. A column is considered to be long column if the ratio of effective length to its least lateral dimension is greater than 12.
2. The ratio of effective length of a short column to its least radius of gyration is less than or equal to 40.	2. The ratio of effective length of a long column to its least radius of gyration is greater than 40.
3. Buckling tendency is very low.	3. Long & slender colum buckle easily.
4. Load carrying capacity of short column is high as compared to long column of the same cross-sectional area.	4. Load carrying capacity of long column is less as compared to short column of same cross-sectional area.
5. The failure of short column is by crushing.	5. The column generally fails in buckling.

→ Column is a member in upright (vertical) position which supports a roof or floor system and predominantly subjected to compression.

Design of Axially Loaded Columns

Steps for Design of short Axially loaded columns

① Effective length of column, left $= Kl$ - unsupp length

Given : factored load (P_u)
Material- Grade of concrete (f_{ck}) & Steel (f_y)

1. Assume suitable percentage (p) of ~~steel~~ Asc (say 0.8% to 4%)

$$A_{sc} = pA_g$$; A_{sc} = longitudinal reinforcement of column
A_g = gross cross-sectional area

2. Determine A_c in terms of A_g.

$$A_c = A_g - A_{sc}$$ A_c = Area of concrete.

3. Calculate A_g as follows :

$$P_u = 0.4 f_{ck} (A_g - A_{sc}) + 0.67 f_y * A_{sc}$$

Putting $A_{sc} = pA_g$, calculate A_g as all other terms are known

4. Calculate dimensions of column as follows :

for square column, $B^2 = A_g$

for rectangular column, $B * D = A_g$ (assume B & calculate D)

→ Check for slenderness ratio. $\frac{l_{eff}}{b} < 12$, SC; > 12, LC

5. Provide area of reinforcement (A_{sc}) → Min^m no. of bars 4 rect, 6 circular; ɸ ≮ 12 mm

6. Design lateral ties.

→ Check eccentricity $e_{min} = \frac{l}{500} + \frac{b}{30}$; $e_{min} < 0.05D$

(a) Diameter of lateral ties — Should be greater than
  (i) 1/4th of diameter of largest longitudinal bar
  (ii) 6 mm

(b) Pitch of lateral ties - should not be greater than
  (i) least lateral dimension of the column
  (ii) 16 times the diameter of the smallest longitudinal bar
  (iii) 300 mm

(c) Dia of helical reinforcement = dia. of lateral ties

(d) Pitch of helical reinforcement - not greater than
  (i) 75 mm
  (ii) 1/6 th of the diameter of Core of concrete.

helical turns. - not less than ; 25 mm
3 times dia of steel ... forming

Note: If length of the column is given, calculate effective length (l_{eff}) on the basis of end condition & check for slenderness ratio & min^m eccentricity as follows

$$\frac{l_{eff}}{b} < 12$$

$$e_{min} = \frac{l}{500} + \frac{D}{30} \geq 20mm, \ \& \ \frac{e_{min}}{D} \leq 0.05$$

Design of column Subjected to Compression & Uniaxial Bending

(i) Calculate d'/D; Based on f_y & d'/D, select the ~~SP~~ required chart from SP-16.

d' = eff. cover for steel reinforcement
D = Overall depth

(ii) Calculate $\frac{P_u}{f_{ck} b D}$ & $\frac{M_u}{f_{ck} b D^2}$.

(iii) Corresponding to these values, get the value of P/f_{ck} from the charts given.

(iv) Calculate A_{sc} & provide bars.

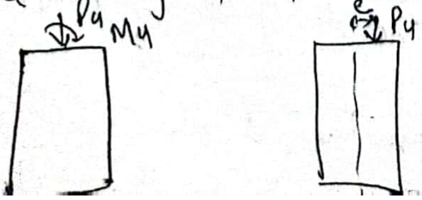
$$A_{sc} = \frac{p \cdot A_g}{100}$$

(v) Design lateral ties.

$e = M_u/P_u$; $e_{min} = l/500 + b/30$; $e > e_{min}$ or $e_{min} > 0.05D$.

It has uniaxial moment.

* Consider a column shown in fig. below subjected to an axial load P_u & a moment M_u. The effect of these forces is equivalent to a load P_u acting at an eccentricity e, such that $M_u = P_u \times e$.



$M_u = P_u \cdot e$.

Steps for design of column subjected to axial load & biaxial bending.

(i) To start with assume some area of reinforcement with its distribution pattern.

(ii) Determine P_{uz} from chart 63 of SP-16 or from following equation

$$P_{uz} = 0.45 f_{ck} A_c + 0.75 f_y \cdot A_{sc}$$

(iii) Find $\frac{P_u}{P_{uz}}$ ratio (P_u - Axial Load)

(iv) Determine M_{ux1} & M_{uy1}, the ultimate/max^m uni-axial moment capacities of the column, with an axial load P_u from suitable design chart from SP-16

(v) Calculate $\frac{M_{ux}}{M_{ux1}}$ & $\frac{M_{uy}}{M_{uy1}}$; $M_u = 1.15 * \sqrt{M_{ux}^2 + M_{uy}^2}$

(vi) from chart 64, find the permissible value of $\frac{M_{ux}}{M_{ux1}}$ corresponding to the $\frac{M_{uy}}{M_{ux}}$ & $\frac{P_u}{P_{uz}}$.

(vii) If $\frac{M_{ux}}{M_{ux1}} < \left(\frac{M_{ux}}{M_{ux1}}\right)_{chart}$, the section is safe & design is adopted.

(viii) If $\left(\frac{M_{ux}}{M_{ux1}}\right) > \left(\frac{M_{ux}}{M_{ux1}}\right)_{chart\text{-}64}$, section is ~~unsface~~ unsafe & hence it is to be re-designed.

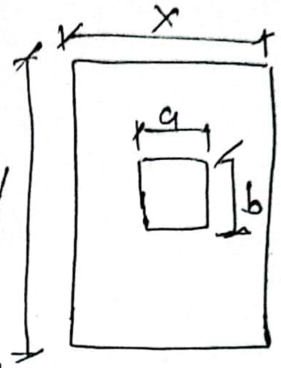
M_{ux} = Moments due to design loads

M_{ux1} = ultimate uniaxial moment capacities.

Design steps for Isolated Rectangular footing.

Given ; Loads on column, safe bearing capacity of soil, grade of concrete and steel

1. Find design constants $\frac{x_{umax}}{d}$ & R_u for given grade of concrete & steel



2. Calculate area of footing as follows:

$$A = \frac{W_c + W_f}{q_o}$$

W_c = load on column

W_f = self wt. of footing + wt. pedestal (usually 10% of W_c)

q_o = safe bearing capacity of soil

3. Calculate the size of footing

(a) for square footing, side of footing $S = \sqrt{A}$, round off.

(b) for rectangular footing, assume one dimension (say x) & calculate the other dimension (say Y) as follows.

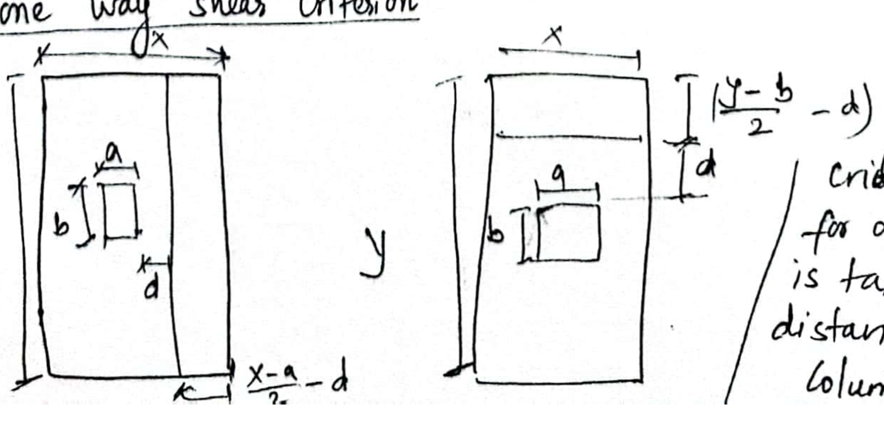
$$Y = \frac{A}{X} \quad \text{round off to nearest 5 or 10 cms.}$$

4. Calculate the soil pressure due to factored column load only, as foll.

$$P_u = \frac{1.5\, W_c}{X \cdot Y}$$

5. Depth of footing is calculated by the following three criteria & highest value so calculated is adopted in the design

(a) By one way shear criterion



Critical section for one way shear is taken at a distance 'd' from column face.

Shear force at the critical section

$$V_u = P_u * X * \left(\frac{y-b}{2} - d\right) \quad \text{---(i)}$$

Shear force resisted by the concrete $= \tau_c * X d$ ---(ii)

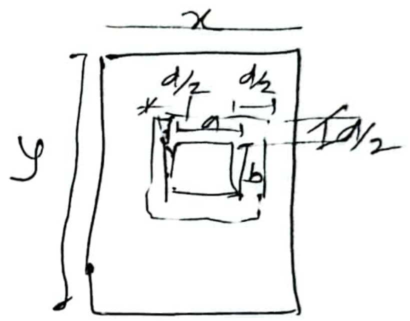
Equating (i) & (ii)

$$\tau_c * d = P_u * X * \left(\frac{y-b}{2} - d\right)$$

τ_c may be assumed as that corresponding to min^m reinforcement.

(b) Two way shear Criterion

→ Critical section for two way shear is taken at a distance $d/2$ from column face.



Perimeter of critical section

$$= 2\left(a + \frac{d}{2} + \frac{d}{2} + b + \frac{d}{2} + \frac{d}{2}\right)$$

$$= 2(a+b+2d)$$

Area of concrete resisting punching shear

$$A = 2(a+b+2d) * d$$

Punching shear on the critical section

$$= P_u * (XY - (a+d) * (b+d)) \quad \text{---(iii)}$$

Punching shear resisted by the section

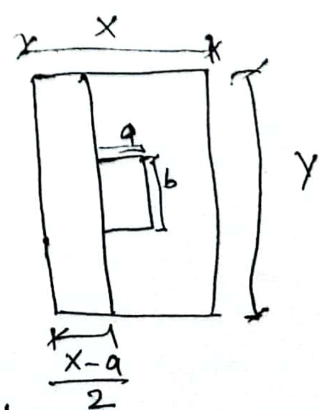
$$= \tau_c * A$$

$$= \tau_c * 2(a+b+2d) * d \quad \text{---(iv)} \quad (\tau_c = 0.25\sqrt{f_{ck}})$$

By equating (iii) & (iv), we can calculate the depth of footing

(c) By bending moment criterion

The critical section for BM is shown in fig:



$$P_u\left(\frac{X-a}{2}\right)\left(\frac{X-a}{4}\right) = \frac{P_u}{8}(X-a)^2 \quad \text{---(v)}$$

$$\text{B.M in } y \text{ direction} = \frac{P_u}{8}(Y-b)^2 \quad \text{---(vi)}$$

Moment of resistance of section

$$= 0.36 f_{ck} * \frac{X_{u\,lim}}{d}\left(1 - \frac{0.42 X_{u\,lim}}{d}\right) Xd^2$$

Equating (v) & (vi) with MoR we get value of d

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The highest value of depth as obtained in steps (a), (b) & (c) above shall be adopted as effective depth of the footing

6. Determine the area of reinforcement required by following equation

$$M_u = 0.87\, f_y\, A_{st}\, d \left(1 - \frac{A_{st} f_y}{x d f_{ck}}\right)$$

7. Check for development length

$$L_d = \frac{0.87\, f_y \phi}{4 \tau_{bd}}$$

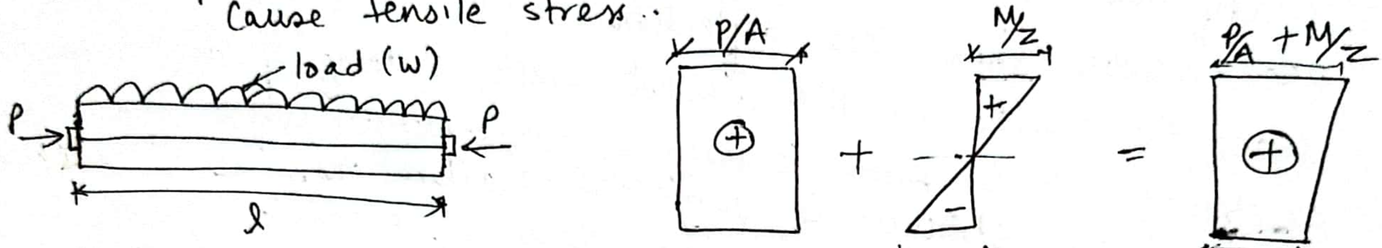
Introduction to Prestressed Concrete

→ Pre-stressed concrete is that type of concrete in which internal stresses of suitable magnitude are introduced so that the stresses resulting from external loadings can be counteracted to a desired degree.

Concept of pre-stressing.

↳ In reinforced concrete, pre-stress is commonly introduced by tensioning the reinforcement.

↳ So, compression is induced in the zones where external load would normally cause tensile stress.



(a) Pre-stressed concrete beam

(b) Compressive pre-stress

(c) Bending stress due to loads

(d) final stress distribution

P = pre-stressing force, w - external load

Compressive pre-stress in beam $= P/A$

Bending stress due to external load $= \pm \frac{M}{I} \cdot y = \frac{M}{Z}$

Net stress $= P/A \pm \frac{M}{I} \cdot y$ (Top fibre = + , Bottom fibre = −)

Materials used in pre-stressed concrete

① Steel – high tensile strength steel is used
- have high ultimate elongation

(a) Tendons (1 - 8 mm) (b) Wire strands or cables (7 to 17 mm) (c) ~~Concrete~~ Bars (10 mm)

② Concrete

Min^m M42 – pre-tensioned system

M35 – post-tensioned System

Principle of Pre-stressing :

→ The main principle of pre-stressing a concrete member consists of inducing sufficient compressive stress in concrete before a member is subjected to loads, in the zones which develops tensile stress due to applied load.

→ The pre-induced compressive stress in concrete neutralises the tensile stress developed due to the external loads, Hence, the zone ultimately will be free from any stress.

In a pre-stressed member, the entire cross-section becomes effective for resisting bending and the danger of cracking, is minimised or even avoided.

Advantages

① It is possible to take full advantage of high compressive strength of concrete and high tensile strength of steel used. ie. the combination of two materials result in most economical section.

② Because of higher strength, prestressed concrete can be safely used for structures having longer span and which are subjected to heavy loads, impact & vibrations.

③ Prestressing eliminates the cracks in concrete under all stages of loading. The entire section therefore becomes effective, whereas in R.C.C. only the portion of section above its neutral axis carries compressive stress.

④ PSC requires only $1/3$rd of concrete required for RCC but of superior quality. Also, the amount of steel required is only $1/4$th of RCC. Thus, there is always a saving of material cost in PSC.

⑤ Reduced dead weight of the superstructure also saves the cost of foundation as the PSC members are comparatively smaller in section.

⑥ There is considerable saving in cost of shuttering and centering in large structures, because PSC members are manufactured in factories.

⑦ There is considerable saving in stirrups, since the shear in PSC members, is reduced by inclination of tendons & the diagonal tension is further minimised by the presence of pre-stress.

⑧ In PSC structures, deflection of beam is considerably reduced.

Advantages of Prestressed Concrete

1. Pre-stressed concrete sections are thinner & lighter than RCC sections, Since high strength Concrete & steel are used in PSC.
2. In PSC, whole concrete area is effective in resisting loads unlike RCC, ~~sections~~ where concrete below the NA is neglected.
3. Thinner sections in PSC results in less self wt. & hence overall economy.
4. Long span bridges & flyovers are made of PSC because of lesser Self wt & thinner section. So, PSC is used for heavily loaded strs.
5. PSC members show less deflection.
6. Since the concrete in PSC does not crack, rusting of steel is minimised.
7. Pre-stressed Concrete ~~mem~~ is used in the structures where tension develops or the structure is subjected to vibrations, impacts & shock like girder, bridges, railway sleepers, electric poles, gravity dams etc.
8. Precast members like electric poles & railway sleepers are produced easily in factories using simple pre-stressing methods.

Dis-advantages of Pre-stressed concrete

1. Pre-stressed concrete construction requires very good quality control & Supervision.
2. Cost of materials used in PSC is very high.
3. Pre-stressing requires specialized tensioning equipments & devices which are very costly.
4. Pre-stressed sections are more brittle because of use of high tension steel.

Methods

1. Internal Prestressing
   - Pre-tensioning – prestress is induced before the Concrete is placed
   - Post tensioning – prestress is induced or tendons are tensioned only after the Concrete has hardened.
2. External Prestressing – by hydraulic jacks.

Linear / Circular. – shapes

* Losses in Pre-stressed Concrete.

→ The pre-stressing force induced in a member doesnot remain constant but there is gradual reduction in pre-stressing force. The amount of force lost after pre-stressing is known as loss of pre-stress and its value generally varies from 15 to 20%.

Losses of pre-stress may be classified into following heads.

1. Loss of pre-stress during tensioning due to friction.

→ There is a loss of pre-stress in tendons due to friction in jacking & anchoring system on the walls of the duct.

Types

(i) Loss due to length effect.

$P_x = P_o e^{-kx}$; P_x = Pre-stressing force in the tendon at any section at a distance x from the tensioning end.

P_o = pre-stressing force. , K = wobble correction factor

(ii) Loss due to curvature effect.

$P_x = P_o e^{-\mu x/R}$; R - radius of curvature of duct ; μ - friction coefficient.

(iii) Loss due to combined effect of length & Curvature

2. Loss of pre-stress at the anchoring stage (due to slip).

$$\Delta f_s = \frac{\Delta e\, E_s}{L}$$

; Δe = effective slip ; E_s = Young's modulus of the tendon. L = Length of the tendon.

3. Loss of pre-stress subsequent to pre-stressing.

(i) Loss due to shrinkage of concrete $= E_s * E_{sh}$ - shrinkage strain of concrete.

Pre-tensioning, $E_{sh} = 3\times10^{-4}$, Post-tensioning, $E_{sh} = \dfrac{2\times10^{-4}}{\log_{10}(T-2)}$ ← days.

(ii) Due to creep of concrete $= (C_e - 1)\, m * f_c$; m - modular ratio $\frac{E_s}{E_c}$; f_c - original pre-stress.

(iii) Due to elastic shortening of concrete

(a) Pre-tensioned member, $\Delta f_s = m \cdot \dfrac{P_o}{A_c}$

(b) Post-tensioned member, = ½ the loss of prestress provided by the first tendon.

(iv) Due to relaxation of stress (creep in steel)

→ 2 to 8% of the average initial stress.