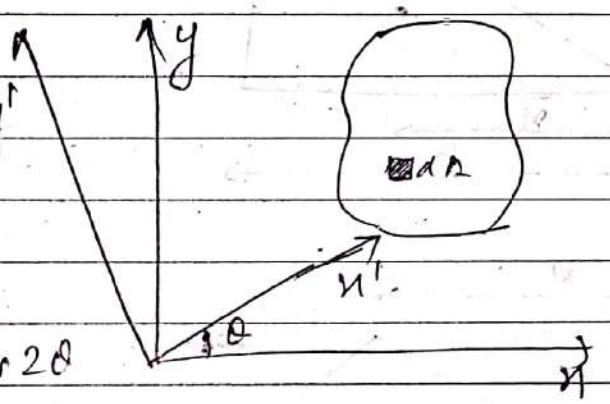
$x' = x\cos\theta + y\sin\theta$
$y' = y\cos\theta - x\sin\theta$

Structural Engineering.

1.1. Center of gravity, MOI and Radius of gyration:

MOI about inclined axes

$x' = x\cos\theta + y\sin\theta$
$y' = y\cos\theta - x\sin\theta$



$$I_{x'} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$I_{y'} = \frac{I_x + I_y}{2} - \left(\frac{I_x - I_y}{2}\right)\cos 2\theta + I_{xy}\sin 2\theta$$

$$I_{x'y'} = \left(\frac{I_x - I_y}{2}\right)\sin 2\theta + I_{xy}\cos 2\theta$$

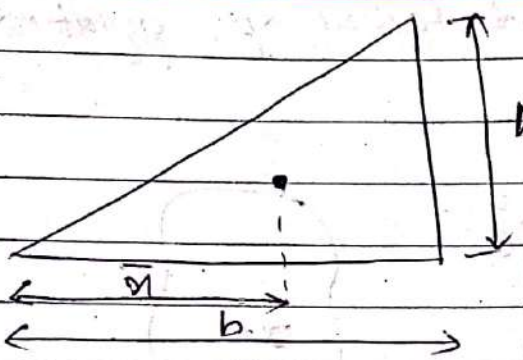
Principal MOI.

$$\frac{dI_{x'}}{d\theta} = 0$$

$$\tan 2\theta_p = \frac{-I_{xy}}{(I_x - I_y)/2}$$

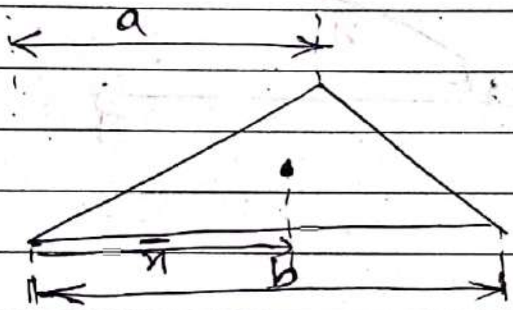
$$I_{\substack{max\\min}} = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$$

Shape

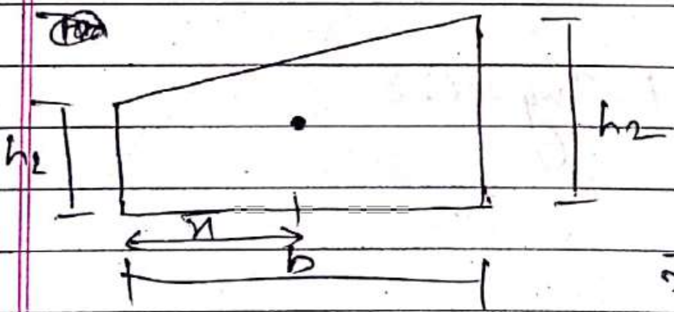


$$A = \frac{bh}{2}$$

$$\bar{x} = \frac{2}{3} b$$



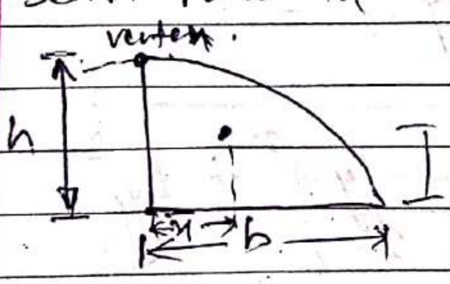
$$\bar{x} = \frac{a+b}{3}$$



$$A = \frac{1}{2}(h_1 + h_2)\, b$$

$$\bar{x} = \frac{b\,(h_1 + 2h_2)}{3\,(h_1 + h_2)}$$

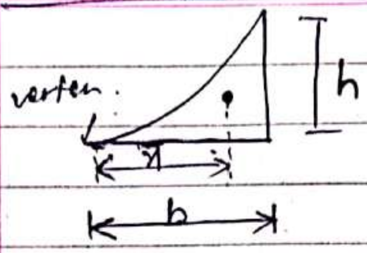
Semi-Parabola



$$A = \frac{2}{3} bh \qquad \bar{x} = \cancel{\tfrac{3}{8}}\ \frac{3b}{8}$$

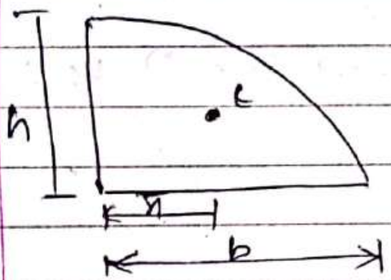
$$\bar{y} = \frac{2h}{5}$$

Parabolic spandrel



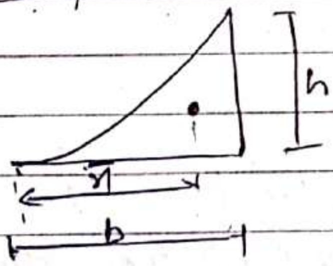
$A = \frac{bh}{3}$ $\qquad \bar{x} = \frac{3b}{4}$

cubic



$A = \frac{3}{4} bh$ $\qquad \bar{x} = \frac{2}{5} b$

cubic spandrel



$A = \frac{bh}{4}$ $\qquad \bar{x} = \frac{4}{5} b$

General :
for spandrel

$A = \frac{bh}{n+1}$ $\qquad \bar{x} = \frac{(n+1)\, b}{(n+2)}$

$n =$ degree.

Centroid : arithmetic mean of position of all the points in the figure.

Modulus of elasticity is same in comp. and Tension.

Radius of gyration (r) :
→ unit of length

$$r = \sqrt{\frac{I}{A}}$$

Product MOI

$$I = \int xy\,dA$$

– used to determine maximum and min^m MOI of an area

\# Theory of Torsion & Flexure.

<u>Flexure formula</u>

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

This formula can only be applied when bending occurs about axes that represent the <u>principal</u> <u>axes of inertia</u> for the cross-section

always oriented along the axis of symmetry and ⊥ar to it.

Compatibility Equation: equation obtained by relating applied loads and reaction to the displacement or slope at different points of structure.

· members should be properly constrained by their supports.

Improper constraint situation

a) Partial constraint

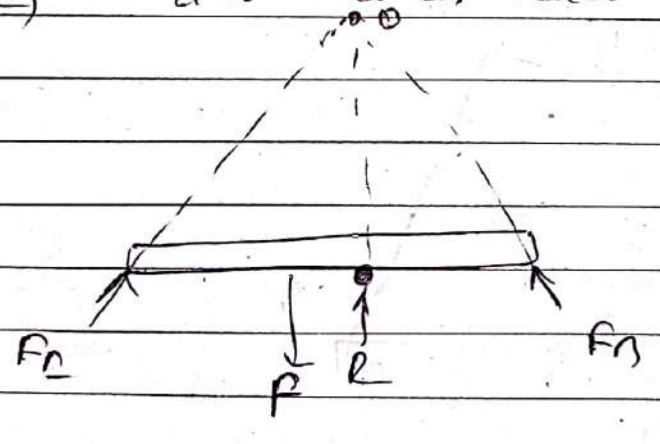
→ fewer reactive forces than equation of equilibrium.

→ unstable

b) Improper constraint

→ occurs when support reactions are concurrent at a point O

→ also when reactive forces are all parallel.



$\Sigma M_O \neq 0$

so, unstable.

internal roller ⇒ permits rotation as well as translation
$\Sigma F_p = 0$ and $\Sigma M_p \neq 0$

* Analysis of beams and frames: BM, SF and deflection
→ Shear ऊपर left of section consider नीचे (↑ +ve). (↓ -ve)

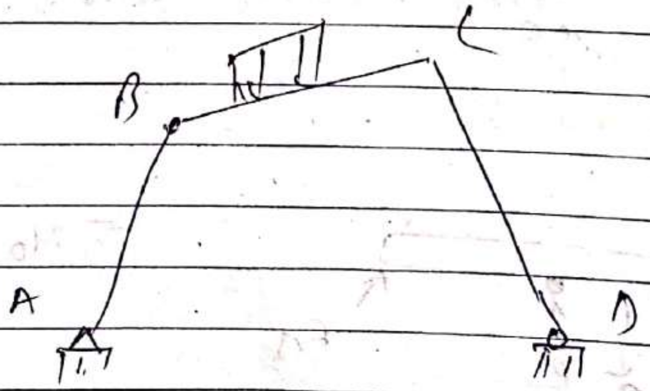
→ Elastic curve aka Qualitative deflected shape.

$\frac{dV}{dx} = -w \quad V = \frac{dM}{dx}$

Statically determinate:
if all internal forces & external rxn can be determined using static eqn equation.

$6m + r = 3(m+j) + e_c$
$3m + r = 3j + e_c$
↑ equation of condition.

Eg



6 unknown: each member में 6 act unknown + unknown rxn, (3)
$= 6m + r$

Eqn available ⇒ 3 member equilibrium eqn + Joint eq
$= 3m + 3j$

e_c = equation of condition
Eg: internal hinge, roller. for determinacy

$\therefore$ $6m + r = 3(m + j)$

$3m + r = 3j + e_c$ ⇒ determinate
$3m + r < 3j + e_c$ ⇒ unstable
$3m + r > 3j + e_c$ ⇒ indeterminate

} necessary but not sufficient.

for truss $m + r = 2j + e_c$ ⇒ determinacy . . . .

Note : Free ends and supports are treated as joints (j)

* Concept of stability :

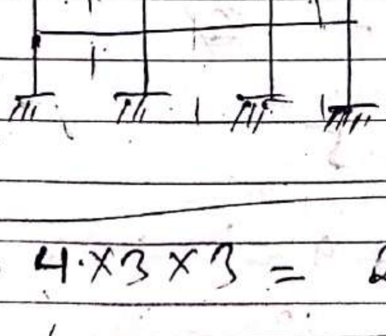
(1) Internal stability :

→ if it maintains its shape and remains rigid when detached from supports.

→ internally unstable if large deformations under small disturbance when not supported externally.

Total indeterminacy ⇒ internal + external

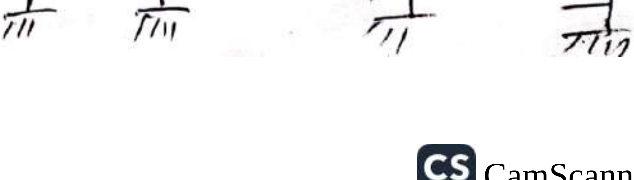
#



External indeterminacy
= 3×3 = 9

Internal = 9×3 = 27
(no. of closed loops)

or # imaginary line to make each section determinate



Total ⇒ 4×3×3 = 27

(2).

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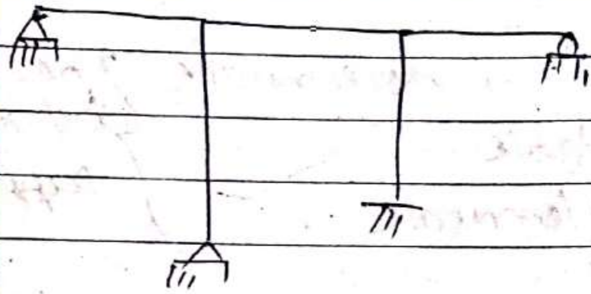
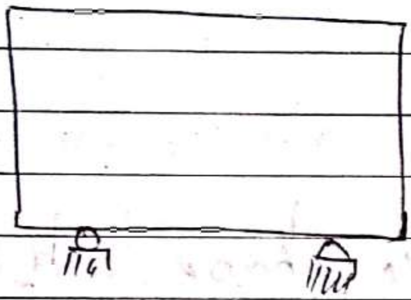


$I_e = 5$

$I_{int} = 0$
no. closed loop.



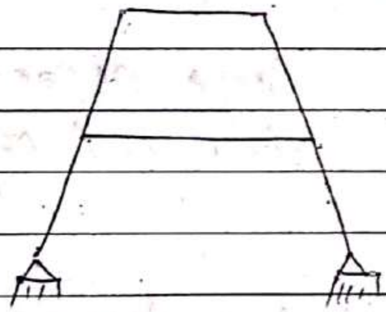
$I_e = 0$

$I_i = 3$

$3\times6+4 = 3\times6+e_c$

$22 = 18$

4 #

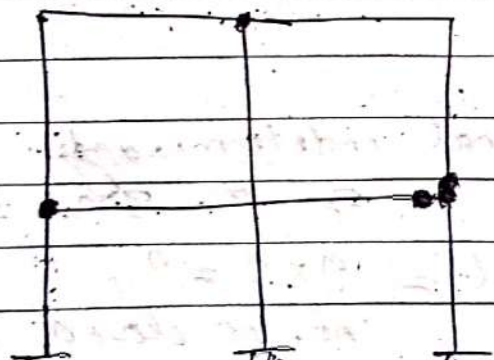


$I_e = 1$

$I_i = 3$

Total = 4 #

$3m+r = 3j+e_c$



$30+9 = 3\times9+2+1$

$I_e = \cancel{0}\ 6$

$I_i = 3\times2 = 6$

$e_c = \cancel{0}\ 2+2+2 = 6$

$= 32$

7 #

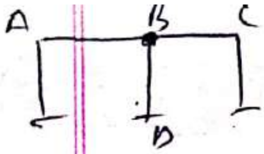
-6

$= 1$

$m = 10 \quad j = 9 \quad r = 9 \quad e_c = 5$

$3m+r-(3j+e_c) = 3\times10+9-(5+3\times9)$

$= 39-32 = 7$ #



Internal hinge at

$M_H^{BC} = 0$, $M_H^{AB} = 0$

$M_H^{BD} = 0$

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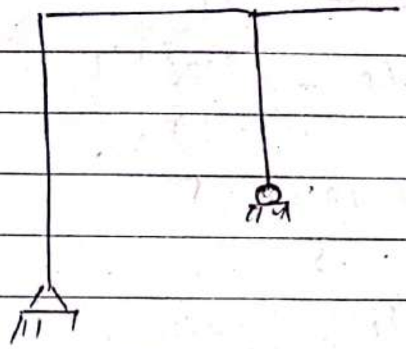
satisfying any 2 eqn^s automatically the 3^rd one
so only 2 eqn^s are available not 3.
i.e. e_c = no. of members coming in − 1 #

*.



$I_e = 0$

$I_i = 0$

⇒ statically determinate.

OR. formula

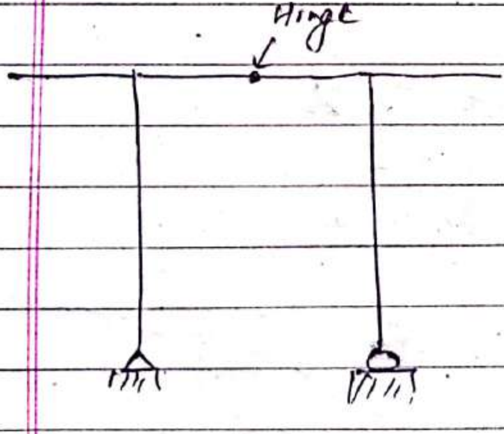
$m = 4 \quad j = 5 \quad r = 3$

$e_c = 0$

$3m + r - (3j + e_c)$

$= 3 \times 4 + 3 - (3 \times 5 + 0)$

$= 15 - 15 = 0$ #



$m = 5 \quad r = 3 \quad j = 6$

$e_c = 1$

$3m + r - (3j + e_c)$

$= 3 \times 5 + 3 - (3 \times 6 + 1)$

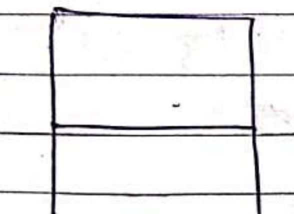
$= 18 - 19$

$3m + r < 3j + e_c$

So, determinate, but unstable

6

#



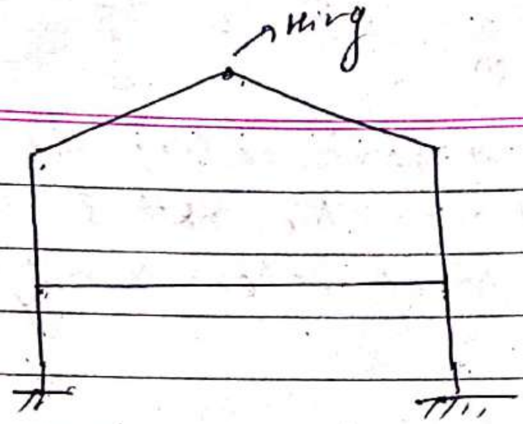
$m = 7 \quad r = 3 \quad j = 6$

$3m + r = 24$

$3j + e = 18$

$I_i = 6$

$I_e = 0$



#1.

$m = 6 \quad r = 6$

$j = 6 \quad e_c = 1$

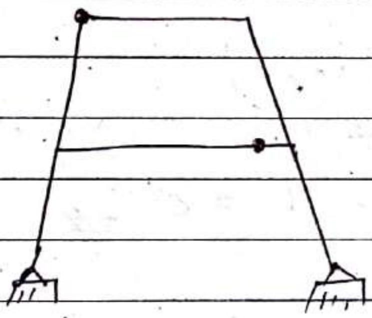
$3m + r = 3 \times 6 + 6$
$= 24$

$3j + e_c = 3 \times 6 + 1$
$= 19$

$I_{total} = 24 - 19 = 5$

$I_e = 3$

$I_i = 2$ #



$I_e = 1$

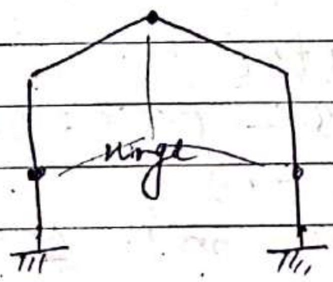
$I_i = 3$

$e_c = 1 + 1 = 2$

$I_{total} = 2$

$I_e = 1$

$I_i = 1$



~~$I_{total} =$~~

$m = 3 \quad j = 4$

$e_c = 3$

$r = 6$

$3m + r = 9 + 6 = 15$

$3j + e_c = 12 + 3 = 15$

$\Rightarrow$ Determinate

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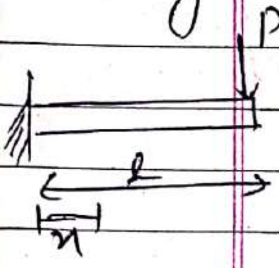
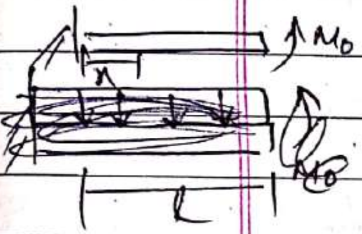
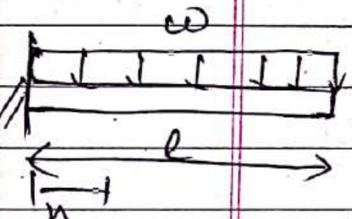
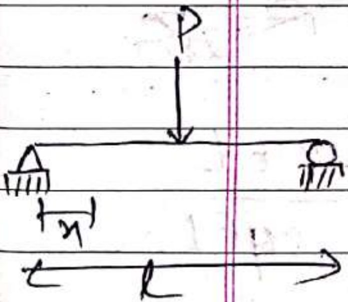
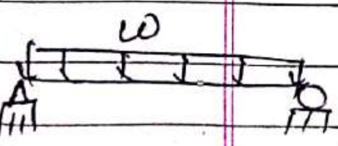
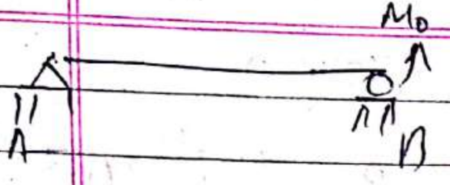
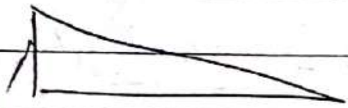
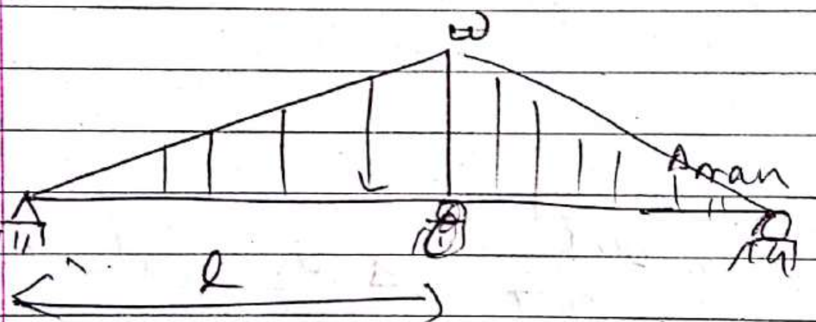
Beam Deflections and slope

Loading	Δ + ↑	θ ↺ +	Equation
	$\Delta_{max} = -\frac{PL^3}{3EI}$ at $x = L$	$\theta_{max} = -\frac{PL^2}{2EI}$ at $x = l$	$\Delta = \frac{P}{6EI}(x^3 - 3Lx^2)$
	$\Delta_{max} = \frac{M_0 l^2}{2EI}$ at $x = l$	$\theta_{max} = \frac{M_0 l}{EI}$ at $x = l$	$\Delta = \frac{M_0}{2EI} x^2$
	$\Delta_{max} = \frac{-wl^4}{8EI}$ at $x = L$	$\theta_{max} = \frac{-wl^3}{6EI}$ at $x = L$	$\Delta = \frac{-w}{24EI}(x^4 - 4lx^3 + 6l^2x^2)$
	$\Delta_{max} = -\frac{Pl^3}{48EI}$ at $x = L/2$	$\theta_{max} = \pm\frac{Pl^2}{16EI}$ at $x = 0, l$	$\Delta = -\frac{P}{48EI}(4x^3 - 3l^2x)$ $0 \le x \le l/2$
	$\Delta_{max} = \frac{5wl^4}{384EI}$	$\theta_{max} = \pm\frac{wl^3}{24EI}$ At $x = 0, l$	$\Delta = -\frac{wx}{24EI}(x^3 - 2x^2 l + l^3)$

	Δ_{max}	θ_{max}	Δ
	$\dfrac{M_0 l^2}{9\sqrt{3}\, EI}$	$\theta_B = \dfrac{M_0 l}{3EI}$ at $x = 0, l$; $\theta_A = \dfrac{M_0 l}{6EI}$	$= -\dfrac{M_0 x (l^2 - x^2)}{6EIl}$

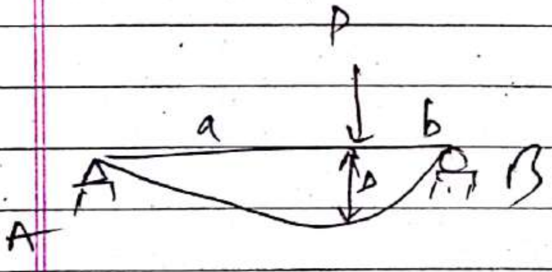


$\dfrac{wl^4}{30\,EI}$ $\qquad$ $\dfrac{wl^3}{24\,EI}$

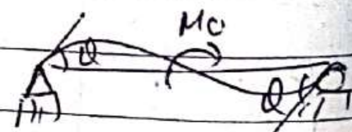


$\Delta_{max} = \dfrac{5\,wl^4}{120\,EI}$ midspan

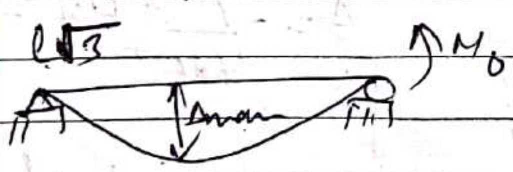
$\theta_{max} = \pm \dfrac{5\,wl^3}{192\,EI}$



$\Delta = \dfrac{P a^2 b^2}{3EIl}$



$\theta = \dfrac{M_0 l}{24 EI}$



$\Delta_{max} = \dfrac{M_0 l^2}{9\sqrt{3} EI}$

$\theta_B = \dfrac{M_0 l}{3EI}$

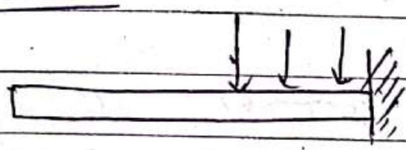
$\theta_A = \dfrac{M_0 l}{6EI}$

ILD

* Absolute max^m shear and moment.

→ Location on the beam and the loading position

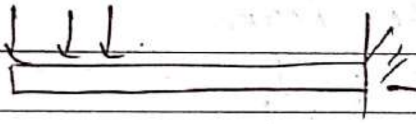
Cantilever



shear

max^m shear just next to the fixed support and loading case as above.

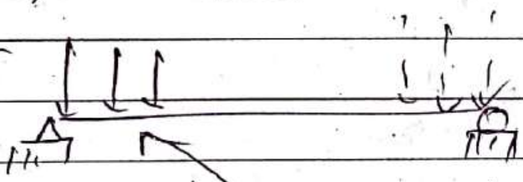
Absolute max. BM



→ at fix support when load at the farthest free end.

Simply supported

a) shear



OR.

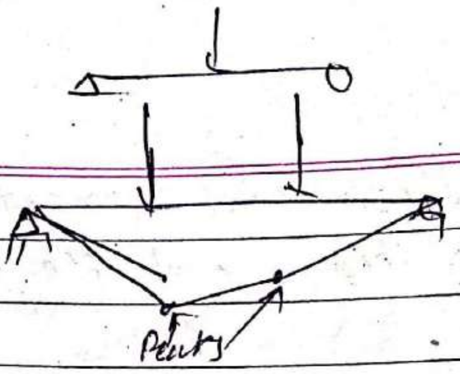
max shear position ⇒ just next to one of the support
& loadposition ⇒

b) moment

⇒ difficult to determine by simple inspection.

⇒ one thing for sure. ∵ moment diagram for point loads have peaks under the load.

Eg:



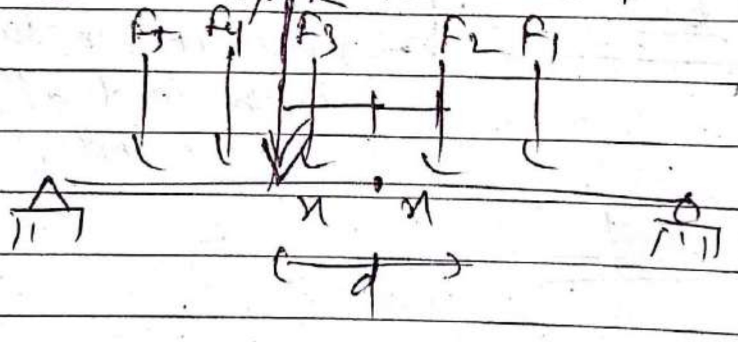
~~Let us assume~~ ~~Peaks~~

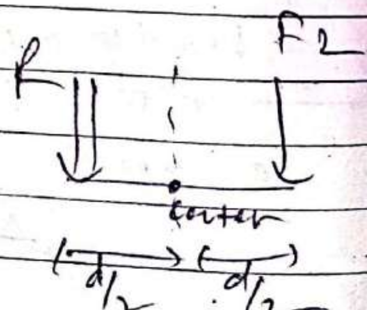
⇒ ~~*~~ So absolute maximum moment occurs under one of these loads. Let's assume under F_2

Steps

→ Determine resultant of the force system.

→ Now place the force system in such a way that force F_2 and R are equidistant from the center of R the beam.





$x = d/2$

* Envelope of maximum influence line values.
  - → difficult to determine absolute BM and V for beams other than simple and cantilever. So, determined at each point along beam to develop an envelope of maximum

Simple / cantilever बाहेक अरु अवस्था case

⇒ computer used

$\theta \quad \frac{d\theta}{dx}$. $\frac{E}{R} = \frac{M}{I}$ $\frac{dM}{dx} = V$ $\frac{d^2M}{dx^2} = w$ $\frac{dw}{dx}$ $\frac{1}{R}$ $\frac{d^2y}{dx^2}$

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Conjugate Beam

$$\frac{dV}{dx} = -w \qquad \frac{d^2M}{dx^2} = w$$

$$\frac{d\theta}{dx} = \frac{M}{EI} \qquad \frac{d^2y}{dx^2} = \frac{M}{EI}$$

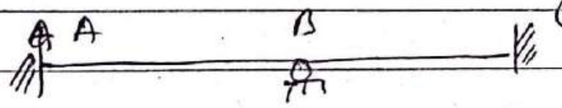
Similarities of these equation.

$$V = \int w\,dx \qquad M = \int\left(\int w\,dx\right)dx$$

$$\updownarrow \qquad\qquad \updownarrow$$

$$\theta = \int \frac{M}{EI}\,dx \qquad y = \int\left(\int \frac{M}{EI}\,dx\right)dx$$

Distribution Factor

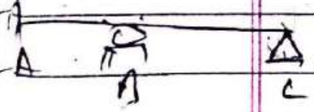


Sum of DFs = 1

For Joint A $\quad DF_{AB} = \dfrac{K_{AB}}{K_{AB} + \infty(\text{wall})} = 0$

$$D_{CB} = \frac{K_{CB}}{K_{CB} + \infty} = 0$$

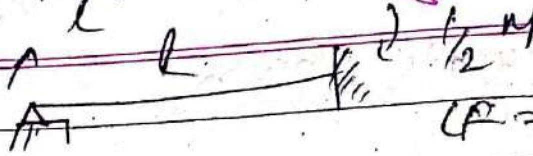
Meaning: Fin end का moment बाँटता है
An end का सारा member को
contribute नहीं करता है।



$DF_{CB} = 1 \Rightarrow$ C का moment सारा CB को जाएगा।

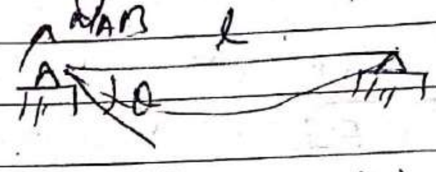
$$M = \frac{4EI}{l}\theta$$

$K = \dfrac{4EI}{l}$ for



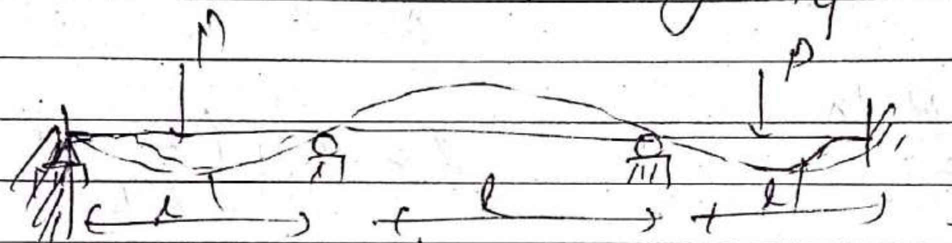
this case only

$K = \dfrac{3EI}{L}$ for



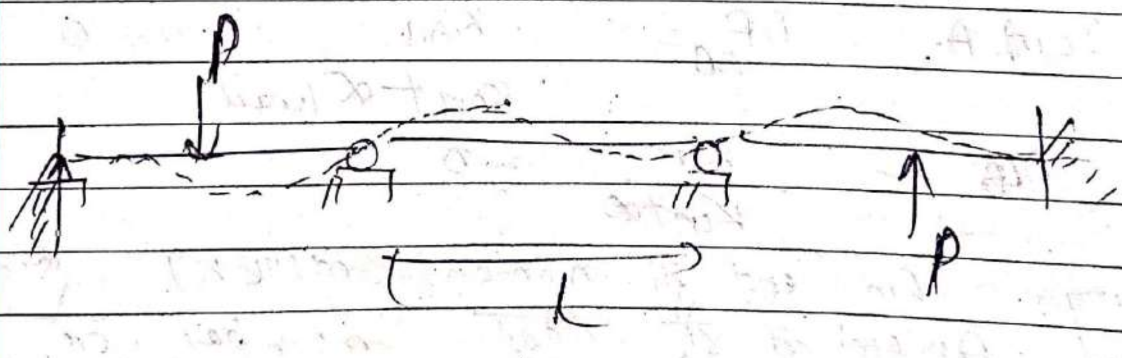
CF = 0 ∵ B doesn't carry a moment

So, if pin support replaces fix end $K = \dfrac{4EI}{L}$ modified by $3/4$.



For symm beam and loading

$$\boxed{K_{\text{mid span center}} = \frac{2EI}{l}}$$



Symm. beam with anti-symm. loading

$$\boxed{K = \frac{6EI}{l}}$$ center span

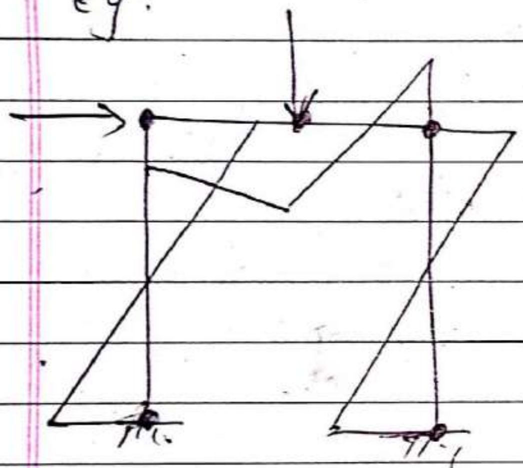
$M_P = K\lambda$
$M = K_0$
$r = 4e^5$

$m - n$. $(n+1)$

Plastic Analysis :

m = no. of possible plastic hinges possible.
n = no. of redundancy
N = no. of independent mechanism
$n+1$ = total no. of hinges required for collapse
$N = m - n$

Eg:

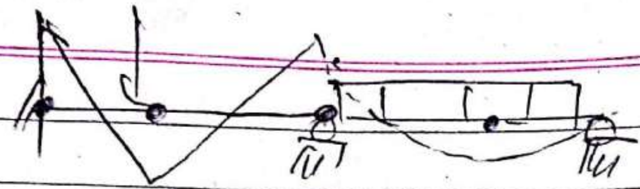


$m = 5$
$n = 3$
$n+1 = 4$
→ 4 hinges required for complete collapse.

$N = 5 - 3 = 2$ no. of independent mechanism.

Partial collapse
no. of hinges formed $< n+1$

complete collapse
~~...~~ no. of hinges $\geq n+1$
3 cond^n req^n for complete
– Mechanism $(n+1)$
– Equilibrium
– Yield σ_y
i.e $BM_{max} \leq M_p$



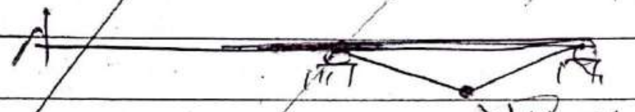
$m = 4$

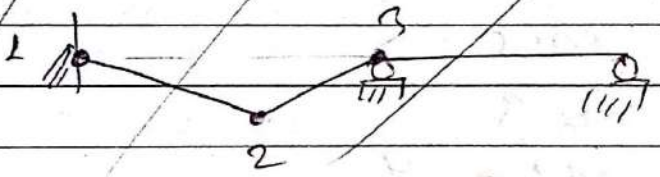
$n = 2$

$m+1 = 3$ no. of hinges req for complete collapse.

$N = m - n = 4 - 2 = 2$ no. of independent mechanism.

Independent mechanism I



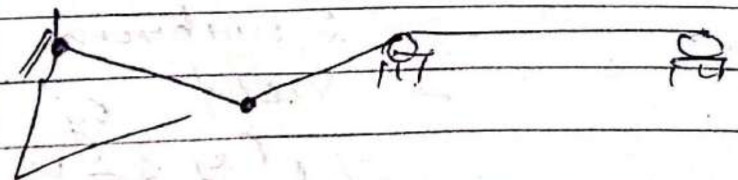


com

Independent mechanism I.

Partial no. of hinges $< n+1$

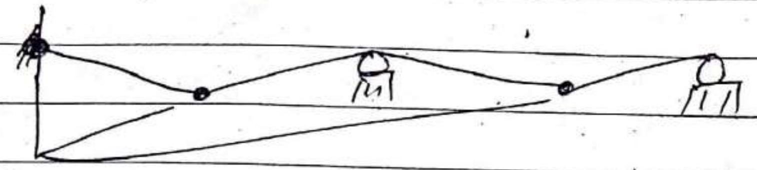
Independent mechanism 2 :



Partial

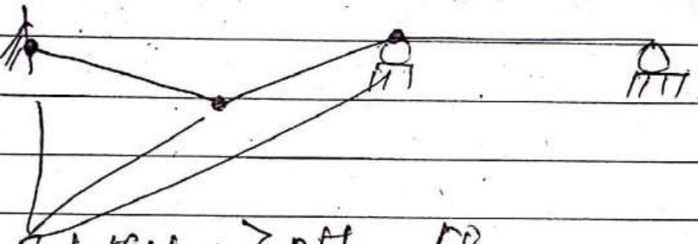
2 hinges.

combined mechanism 3



3 hinges   no. of hinge $\geq n+1$ complete collapse.

combined mechanism 4



complete collapse

3 hinges $\geq n+1$ no

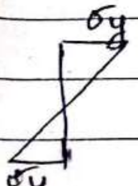
~~So, how does the comple~~

So what is the actual collapse mechanism?

$\Rightarrow$ using the 3 cond^n

- Equilibrium ✓
- mechanism ✓ — calculate collapse load for each mechanism
- yield ? — lowest one is the ultimate load; check to see if $|M| > M_p$

$M \Rightarrow$ moment corresponding to working load. Stresses are within proportional limit

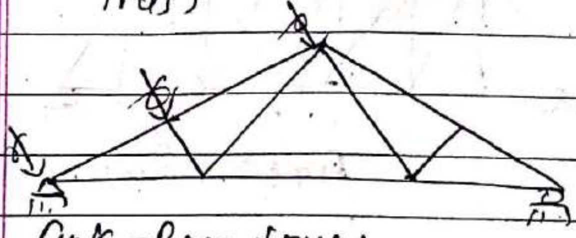


$M_y \Rightarrow$ moment at which the section develops yield stress

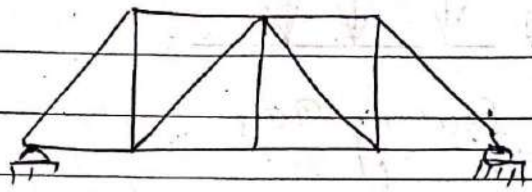
$M_p \Rightarrow$ entire section under yield stress.

Zero force members are used to increase stability of truss during construction and added support if loading changes.

Truss



Fink-Roof truss

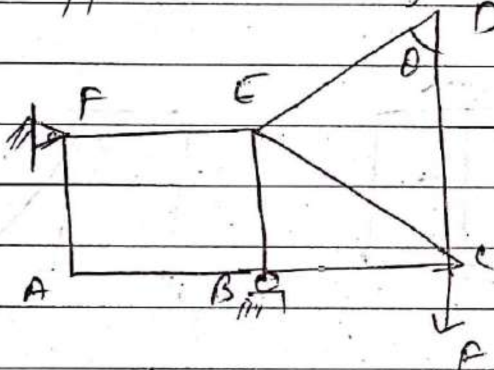


Bridge truss

How to identify 0 force member in truss?

=) (1) If only two non-collinear members form a truss joint and no external load or support reaction is applied to the joint.

Eg:



Zero force members

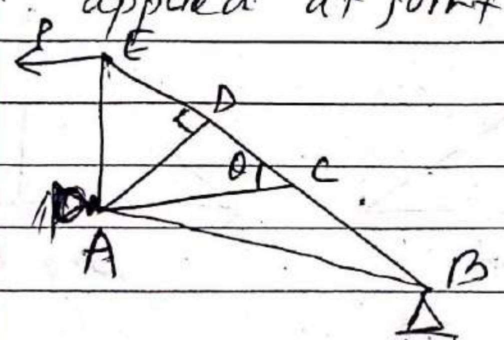
AB and AF =) as only two non-collinear members and no load / reaction.

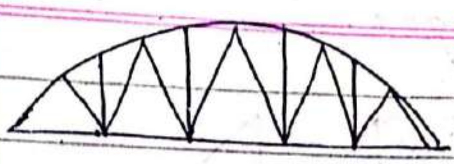
DE and DC =)

(2). If 3 members form a joint where 2 are collinear and then the 3rd one is a zero force member provided no external load/rx? applied at joint.

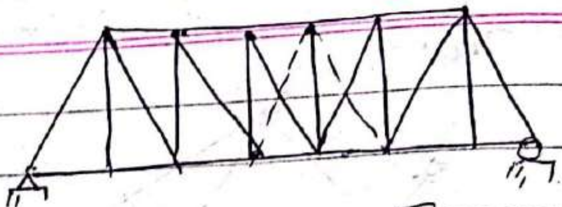
Zero force member

AD, AC

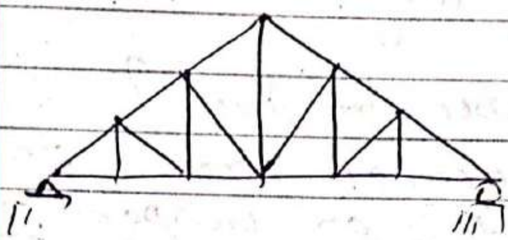




Bowstring Truss

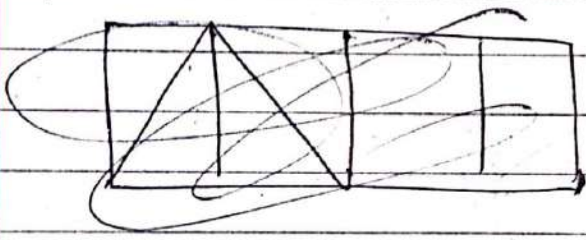


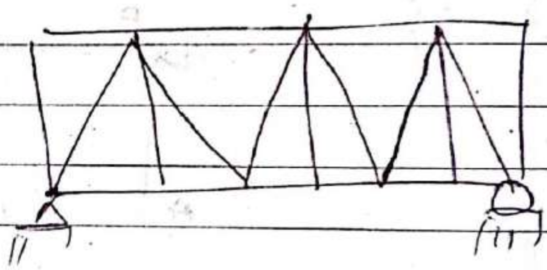
Pratt Truss



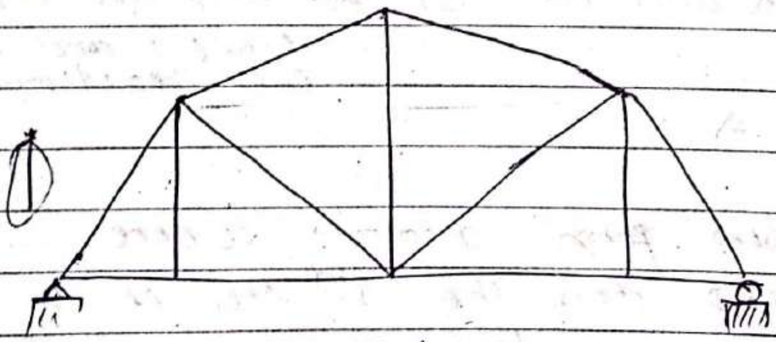
Howe

Modified pink

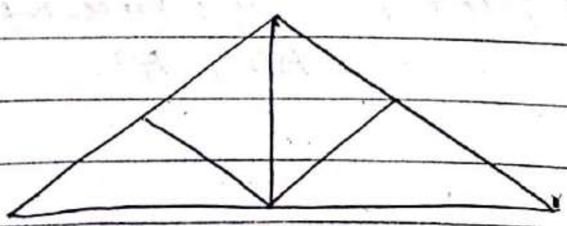




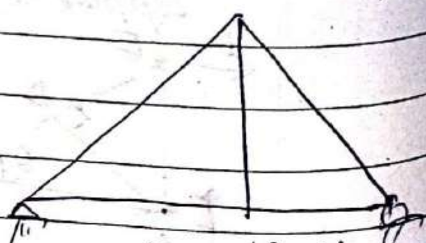
warren truss



Gambrel Truss



Queen Post



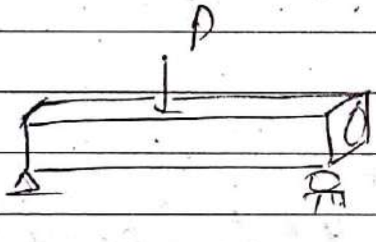
King Post

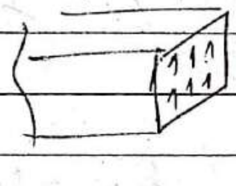
* Notes

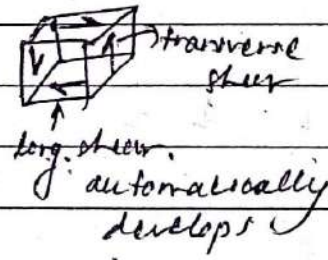
→ In General BM and V function are discontinous or their slopes are discontinous at points where concentrated loads are applied or distributed load charges.

→ Flexure formula is valid ~~when~~ if X-section symmetric about an axis ⊥ar to the neutral axis and M acts along neutral axis i.e symmetric bending.

actually applicable only for pure symm. bending.

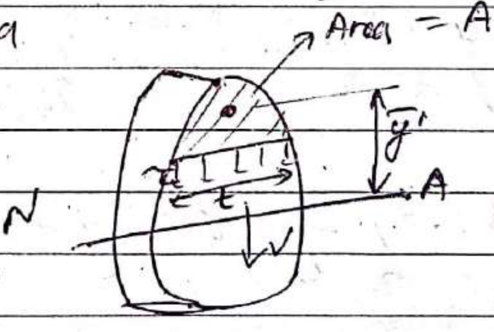






→ Shear Formula

$$\tau = \frac{VQ}{It}$$



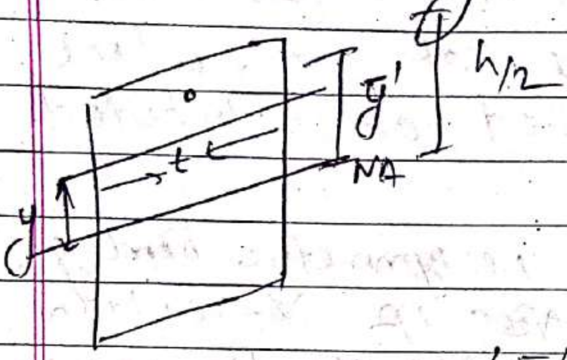
$Q = \bar{y}' A'$ I = MOI about NA

t = width of member's X-section at the point where τ is to be determined.

→ assumes uniform shear stress over t.

→ Shear force causes non-linear shear-strain distribution over X-section causing it to warp.

→ doesn't apply for sudden X-section change.



$$\tau = \frac{VQ}{It}$$

$$t = b$$

$$I = \frac{bh^3}{12}$$

$$Q = A'\bar{y}'$$

$$A' = \underset{(b)}{t} * \left(\frac{h}{2} - y\right) \qquad \bar{y}' = \cancel{y} \; y + (\ldots$$

$$\bar{y}' = y + \frac{1}{2}\left(\frac{h}{2} - y\right)$$

$$Q = b\left(\frac{h}{2} - y\right)\left[y + \frac{1}{2}\left(\frac{h}{2} - y\right)\right]$$

$$= b\left[y\left(\frac{h}{2} - y\right) + \frac{1}{2}\left(\frac{h}{2} - y\right)^2\right]$$

$$= b\left(\frac{h}{2} - y\right)\left(y + \frac{1}{2}\left(\frac{h-2y}{2}\right)\right)$$

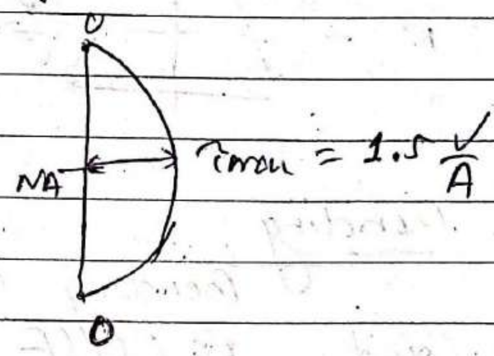
$$= b\left[\left(\frac{h-2y}{2}\right)\left(\frac{4y + h - 2y}{4}\right)\right]$$

$$= b\left(\left(\frac{h-2y}{2}\right)\left(\frac{h+2y}{2}\right)\right)$$

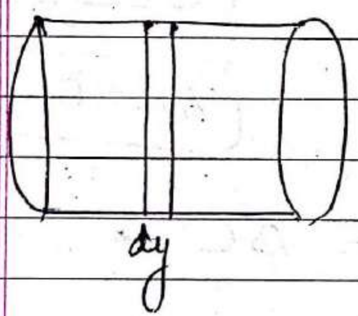
$$= \frac{b}{2}\left(\frac{h^2}{4} - y^2\right)$$

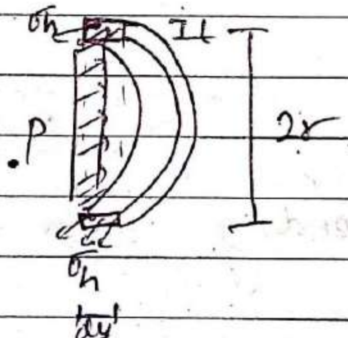
$$\tau = \frac{VQ}{It} = \frac{6V}{bh^3}\left(\frac{h^2}{4} - y^2\right)$$

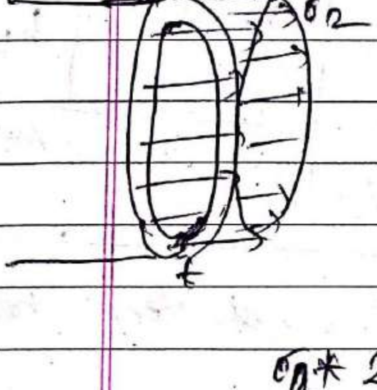
i.e. parabolic shear distribution along the rectangle.



hoop stress







$$\cancel{2}(\sigma_h * t * \cancel{dy}) = p * \cancel{2}r * \cancel{dy}$$

$$\sigma_h = \frac{pr}{t}$$

$$\sigma_h = \frac{pd}{2t}$$

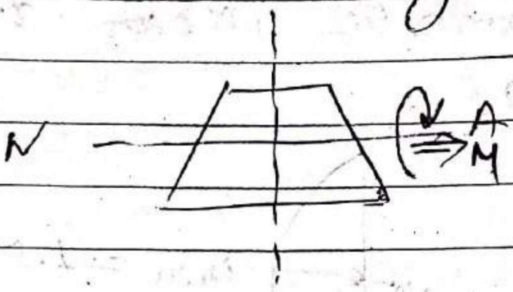
$$\sigma_l * 2\cancel{\pi}\cancel{r} * t = p\cancel{\pi}r^{\cancel{2}}$$

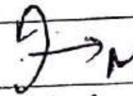
$$\boxed{\sigma_h = 2\sigma_l}$$

$$\sigma_l = \frac{pr}{2t} = \frac{pd}{4t}$$

or, Bending occurs about NA

* Symmetric Bending :: the neutral axis
→ moment acts along ~~plane of symmetry~~.
X-section symm. about an axis ⊥ar to NA

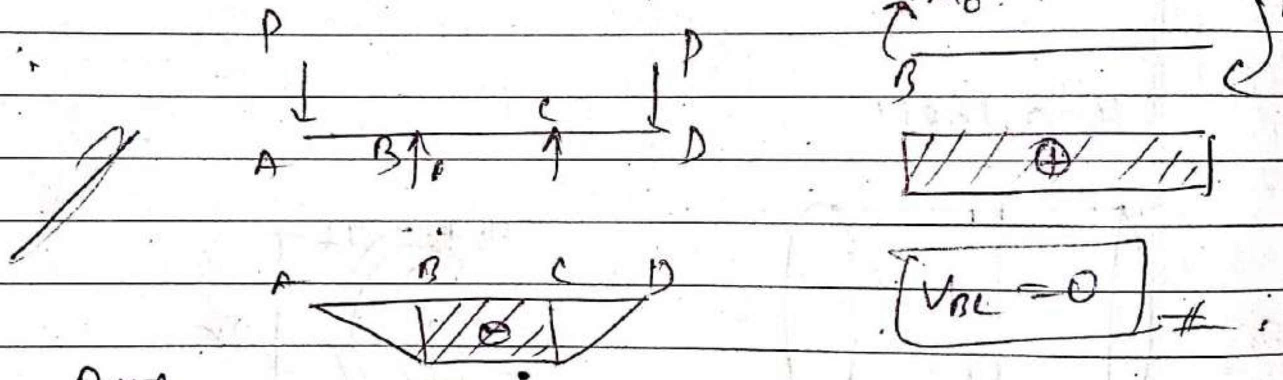




(Right hand rule)

* Pure Bending :
Bending of beam under constant BM and ≠ 0 SF (V), SF = 0

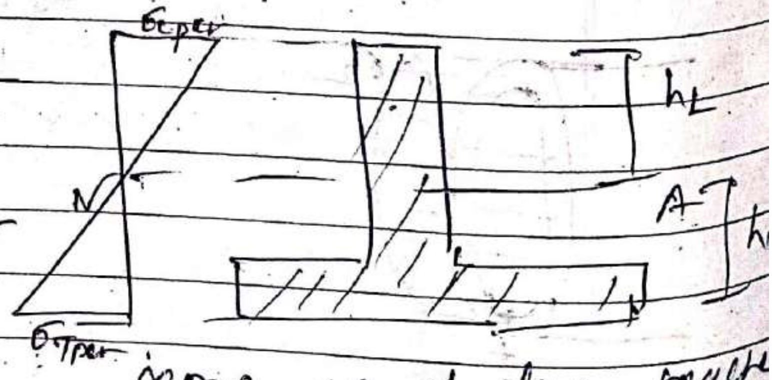
Eg.:



$\boxed{V_{BC} = 0}$

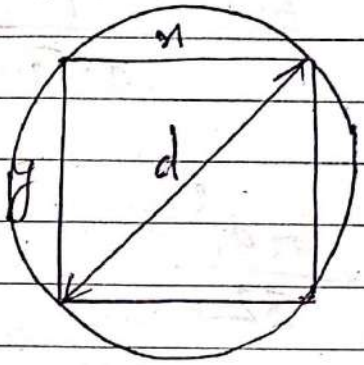
→ Pure Bending in portion BC.

* For economic material utilization
$$\frac{\sigma_{cper}}{\sigma_{tper}} = \frac{h_1}{h_2}$$
section design s.t. $\frac{h_1}{h_2}$ =



i.e. max. corresponding comp. and tens. reached simultaneously.

#1. Find the strongest section that can be cut from a circle of dia 'd'



$$Z = \frac{I}{y} = \frac{bd^2}{6} = \frac{xy^2}{6}$$

$$y = \sqrt{d^2 - x^2} \qquad \therefore Z = \frac{x(d^2-x^2)}{6}$$

For strongest $\frac{dz}{dx} = 0$

$$\frac{dz}{dx} = \frac{d^2 - 3x^2}{6} = 0$$

$$3x^2 = d^2$$

or, $x^2 = \frac{d^2}{3}$ $\quad x = \frac{d}{\sqrt{3}}$

$$y^2 = d^2 - x^2 = d^2 - \frac{d^2}{3}$$

$$y^2 = \frac{2}{3} d^2$$

$$y = \sqrt{\frac{2}{3}}\, d$$

$$x = \frac{d}{\sqrt{3}} \qquad y = \sqrt{\frac{2}{3}}\, d$$

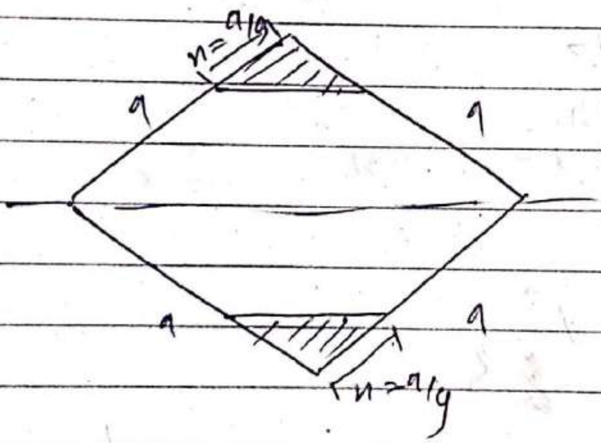
Thus a rectangle with sides $\frac{d}{\sqrt{3}}$ and $\sqrt{2}\frac{d}{\sqrt{3}}$ #.

$$Z_{max} = \frac{d/\sqrt{3} \times 2/3\, d^2}{6} = \frac{d^3}{9\sqrt{3}}$$

~~else circle का~~

$$\cancel{Z_0 = \frac{\pi d^3}{32}}$$

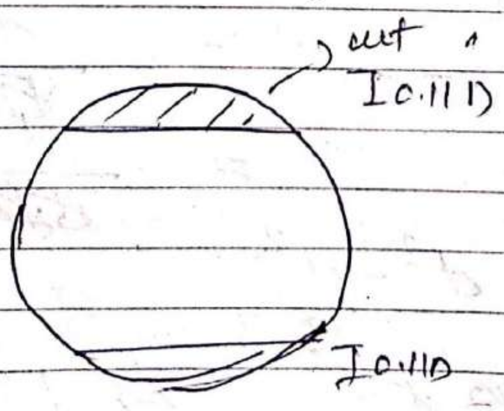
Some interesting result.



$x = a/9$ भी काटेंर

i.e dep m op 0.0785a

beam strength increases by 5.35%. about x-x bending.

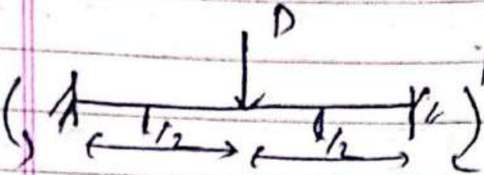


BM ↑ by 0.7%.

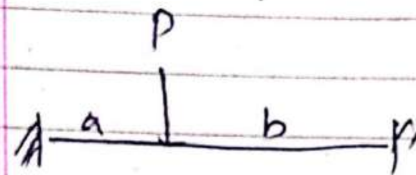
* Point of inflection => Point where deflected shape changes curvature BM = 0 at this section.

Point of contraflexure => BMD changes sign. BM = 0 at this section

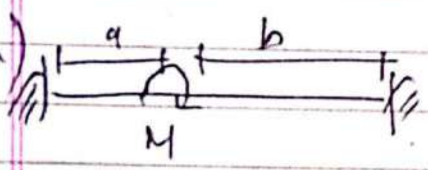
Fixed End Moments



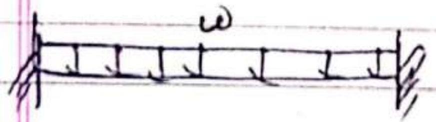
$\frac{Pl}{8}$ $\qquad$ $\frac{Pl}{8}$



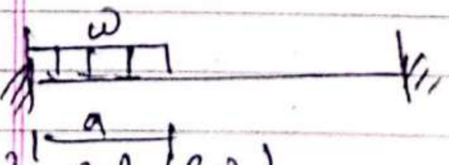
$\frac{Pab^2}{l^2}$ $\qquad$ $\frac{Pa^2b}{l^2}$



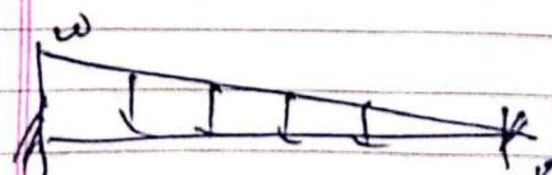
$\frac{Ma(b-2a)}{l^2}$ $\qquad$ $\frac{Mb(2b-a)}{l^2}$



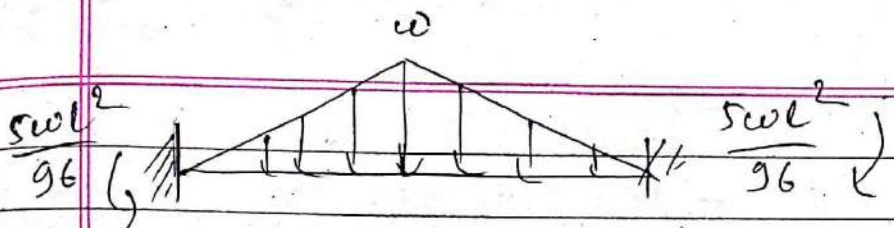
$\frac{wl^2}{12}$ $\qquad$ $\frac{wl^2}{12}$

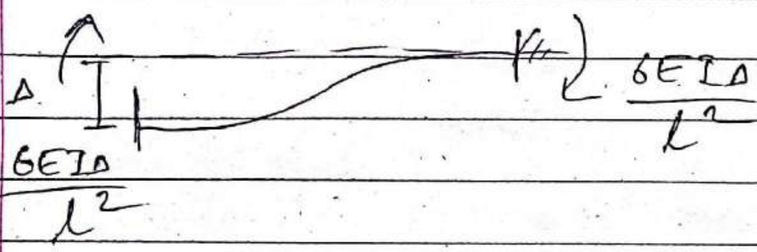


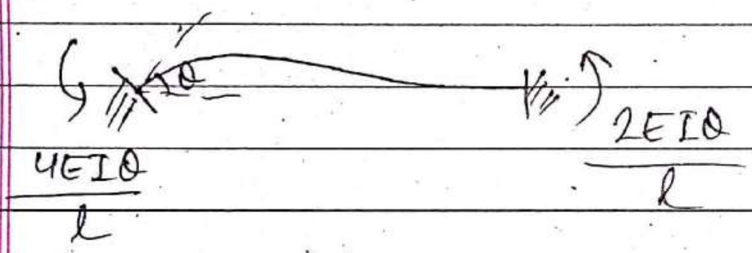
$\frac{wa^2}{12l^2}(6l^2-8al+3a^2)$ $\qquad$ $\frac{wa^3}{12l^2}(4l-3a)$



$\frac{wl^2}{20}$ $\qquad$ $\frac{wl^2}{30}$


w
5wl²/96
5wl²/96


Δ
6EIΔ/l²
6EIΔ/l²


θ
4EIθ/l
2EIθ/l

History of mechanics of materia: Galileo studied effects of loads on rod.
19th century :- advances by Saint-venant, Poisson, Lamé Navier mostly in France.

Stress and strain:

stress: Internal resisting force per unit area against deformation. rigid body ⇒ no internal stress
only deformable stress.

Types Direct

Normal stress | shear stress | Bearing stress.
Tension | | Localized
compression | | compressive stress.
| | contact betn two surface.

$$\sigma_z = \lim_{\Delta A \to 0} \frac{\Delta F_z}{\Delta A}$$

Indirect

Bending
- due to moment induced by tension and compression

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

Torsional
- twisting moment.

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\phi}{L}$$

Strain

change in dimension to original dimension

(i) longitudinal strain
$$\epsilon = \frac{\Delta L}{L}$$ along longest dimension

(ii). ~~Normal strain~~ lateral
$$\epsilon = \frac{\text{change in lateral dimension}}{\text{original lateral dimension}}$$

For non-porous material (ν) = 0.25 – 0.355

Shear strain (γ) = change in angle (radians)

Poisson's ratio (ν)

$$= \frac{\text{lateral strain}}{\text{longitudinal strain}}$$

$$= \frac{\text{transverse strain}}{\text{linear strain}}$$

$\nu \ngtr 0.5$ $\qquad$ $\nu = 0.5$ for ~~perfectly incompre~~ssible isotropic material deformed elastically.

limiting value of poisson's ratio (-1 to 0.5) $\qquad$ $\nu_{max} = 0.5$

For highly compressible materials $\simeq 0$

" incompressible materials $\simeq$ 0.5 (rubber)

For concrete $\Rightarrow$ 0.12 – 0.2

metal $\Rightarrow$ 0.25 – 0.3

steel = 0.25 – 0.33

non-dilatant material, $\mu \simeq 0.6$

Elastic constants

E, G, K

valid within elastic limit (proportional)

volumetric

Force in all three direction must be equal.

Note: Poisson's ratio applies within elastic range.
most materials $\Rightarrow \nu = 0 - 0.5$

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Relationship between elastic constants,

(i) $E > G > K$

(ii) $E = 2G(1+\nu)$

(iii). $E = 3K(1-2\nu)$

(iv). $E = \frac{9KG}{3K+G}$

\# For an isotropic, homogeneous and elastic material obeying Hooke's Law no. of independent elastic constants are:

a) 1 b) 2 ✓ c) 3 d) 4

→ same in all direction except perpendicular

\#. For an orthotropic material no. of independent elastic constants are

a) 3 b) 6 ✓ c) 9 d) 12.

Eg: cold rolled steel.

\#. For an anisotropic material no. of elastic constants are

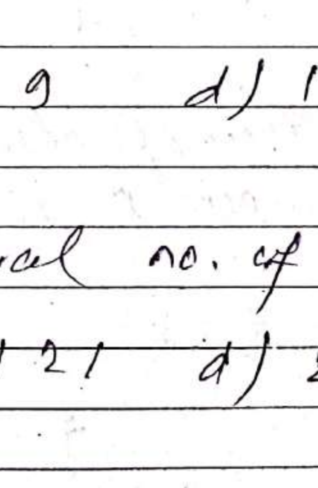
a) 15 b). 18 c) 21 d) 25

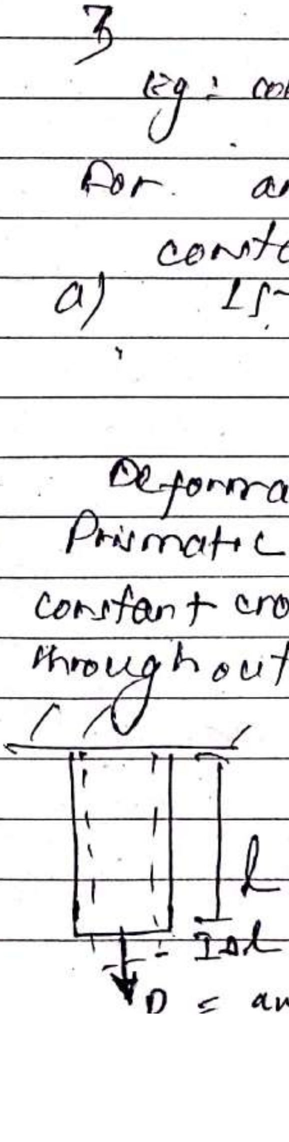
* Deformation produced due to loading

a) Prismatic bar

constant cross-section throughout the length

b) conical bar

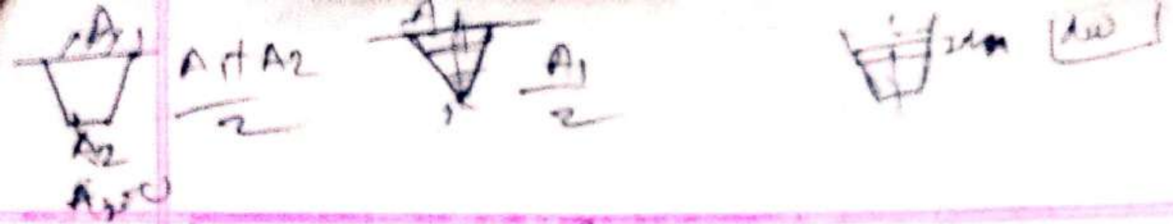




$\Delta l = \frac{Pl}{AE}$ total weight.

Due to self weight

$\Delta l = \frac{(w)l}{2AE} = \frac{\gamma l^2}{2E}$

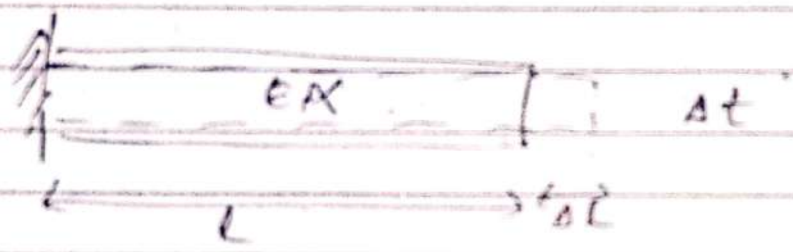


b) Conical Bar.

self-weight $= \dfrac{\omega l}{6AE} = \dfrac{\gamma l^2}{6E}$

$= \frac{1}{3}$ def due to prismatic bar.

* Thermal stress



Thermal stress $= 0$
depends on support condition.

$\Delta t = 0$

Thermal stress $= E\alpha\Delta t$
for $\Delta t \Rightarrow +ve$
$\sigma_t =$ compressive
$\Delta t \rightarrow -ve \Rightarrow \sigma_t \rightarrow$ tension.

composite material :

combination of two or more material acts

(i) as a single unit (r) strain compatibility holds

$\epsilon_1 = \epsilon_2 = \epsilon_3$

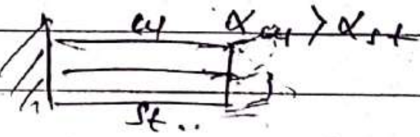
(ii). Load sharing

$P = P_1 + P_2 + P_3$

Eg : RCC .

A composite material composed of steel and copper due to rise in temperature copper undergoes

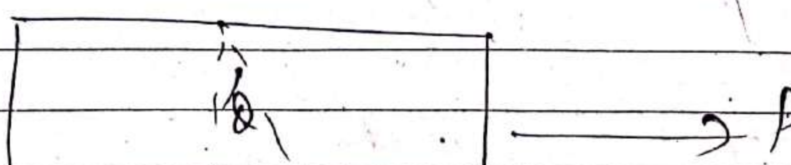
a) compression b) tension c) both d) none .



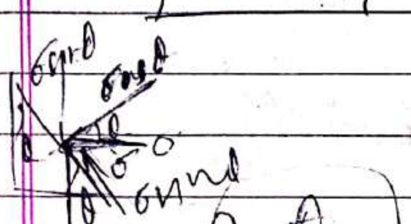
A stress produced in inclined plane when element is subjected to uniaxial direct loading.

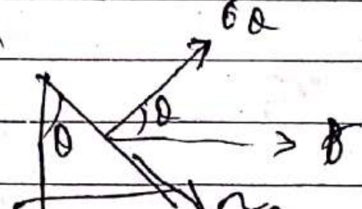
$\sigma_\theta = \sigma \cos^2\theta$

$2 \sin\theta\cos\theta = \sin 2\theta$



$\sigma_\theta = \frac{P\cos\theta}{A_\theta}$





$= \frac{P\cos\theta}{\frac{A}{\cos\theta}}$

$= P/A \cos^2\theta$

$\tau = \frac{A}{\cos\theta}$

$\frac{2P\sin\theta\cos\theta}{2A}$ $\frac{P\sin\theta}{2A\cos^2\theta}$

$\sigma_\theta = \sigma\cos^2\theta$

$\tau_\theta = \dfrac{P\sin\theta}{A/\cos\theta}$

$= \dfrac{P}{2A} 2\sin\theta\cos\theta$

$= \dfrac{P\sin 2\theta}{2A}$

$= \dfrac{\sigma}{2}\sin 2\theta$

maximum stress in inclined plane occurs when $\theta = 45^0$

$\tau_\theta = \sigma/2 \times \sin(2\times 45^0)$

$= \sigma/2 = \tau_{max}$.

$\sigma_\theta = \tau_\theta$

$\sigma\cos^2\theta = \sigma/2 \sin 2\theta$

$2\cos^2\theta = \sin 2\theta$

$2\cos^2\theta = 2\cos^2\theta - 1$

or,

At $\theta = 45^0$

$\sigma_\theta = \sigma/2$ $\quad \tau_\theta = \sigma/2$

$\therefore \sigma_{45} = \tau_{45}$ at $\theta = 45^\circ$ #

* Principle plane and principle stress

A plane is principle if

(i) shear stress along the plane is zero (ii) only normal stress acting

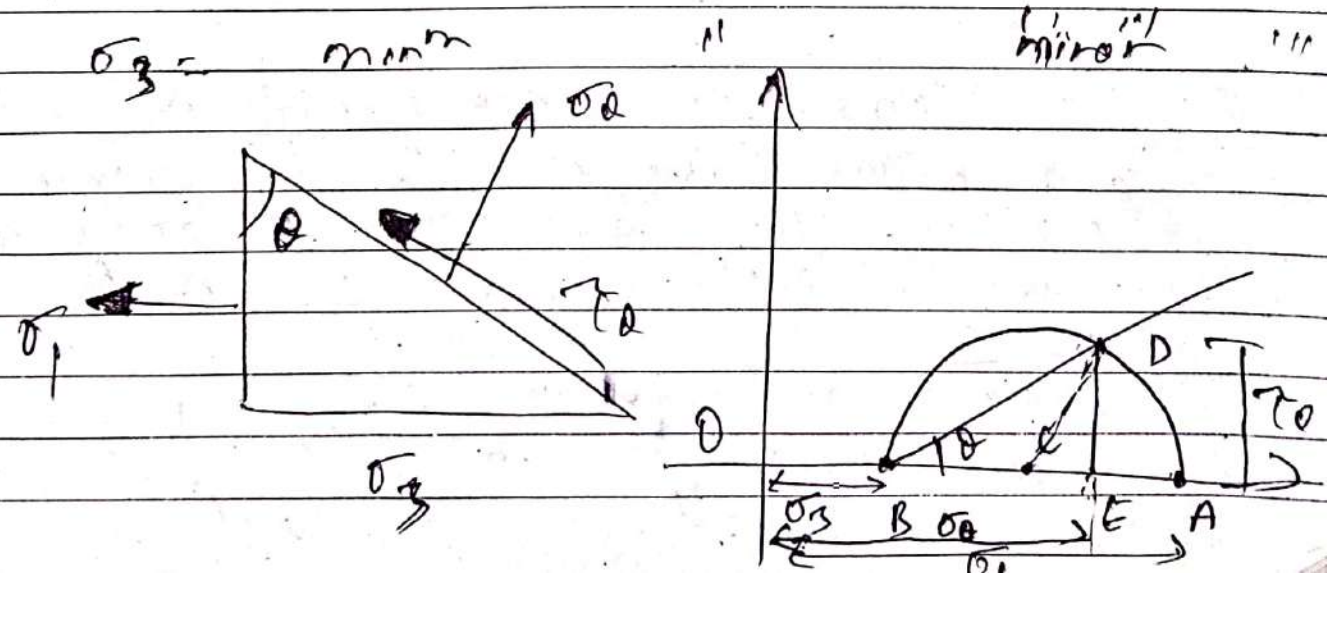
Major principle plane → only maximum normal stress → $\tau = 0$

Minor principal plane → min^m normal stress → $\tau = 0$ #

Principal stress:

σ_1 = max. normal stress acting along major principal

σ_3 = min^m " minor " "



Salient Feature

Diameter of Mohr's circle = $\left(\sigma_1 - \sigma_3\right)$

Radius = $\frac{\sigma_1 - \sigma_3}{2}$

$\sigma_\theta = OC + CE = \frac{\sigma_1 + \sigma_3}{2} + \left(\frac{\sigma_1 - \sigma_3}{2}\right) \cos 2\theta$

$\tau_\theta = \left(\frac{\sigma_1 - \sigma_3}{2}\right) \sin 2\theta$

#. Maximum shear stress on Mohr's circle is given by:

a) Radius b) Diameter c) both
d) none

#. An element is subjected to two like and equal direct tensile stress (σ) on two mutually ⊥ar stress in which shear stress is 0, will be

a) circle and radius (σ)
b) ~~circ~~ circle of dia ($\sigma/2$).
c) A point d) all.

CamScanner

MOI => a dynamic property of matter.

(15) ~~Unlike~~ (3) on Q (2) replace, by like then like

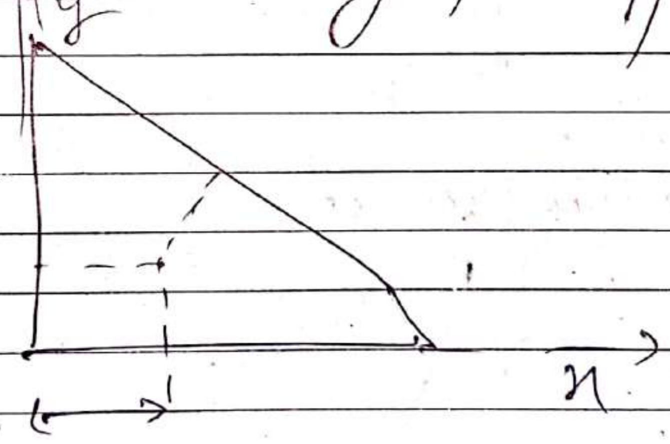
" " unequal σ_1 and σ_2.

Centroid and moment of inertia:

centroid => It is the center of geometrical area about which whole area of the figure will be assumed to be concentrated.

CG => A point through which whole weight must pass through CG.

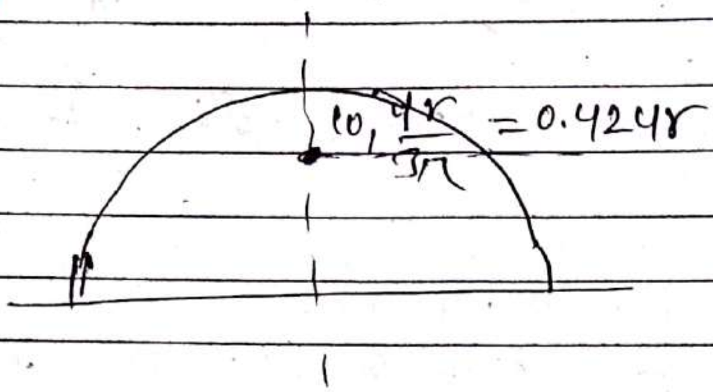
Triangle: at intersecting point of median.



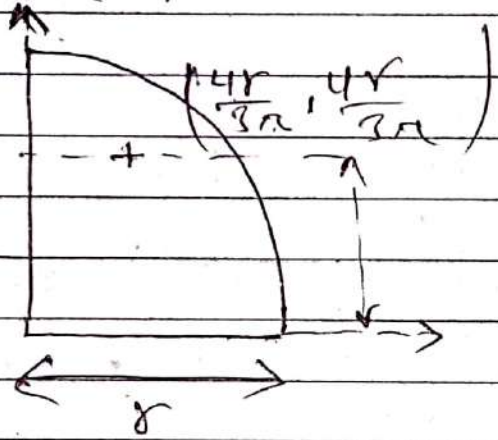
$\bar{x} = x/3$

$\bar{y} = y/3$

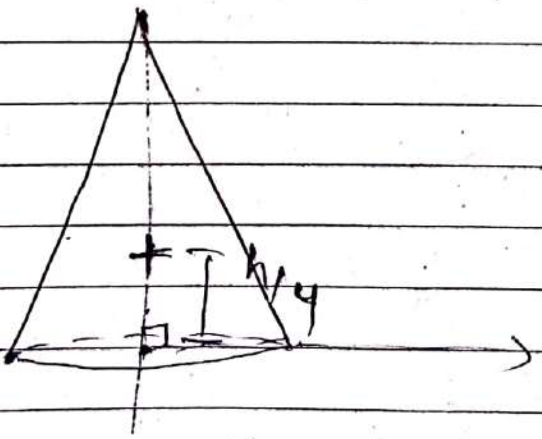
(ii) Semi-circle.



(iii) quarter circle.



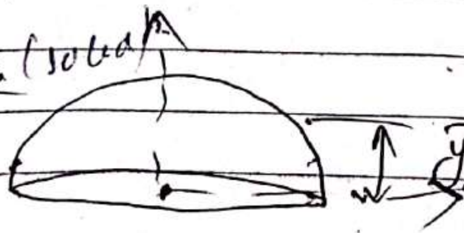
(iv). Right circular cone (solid)



hollow
$\bar{y} = h/3$ H.

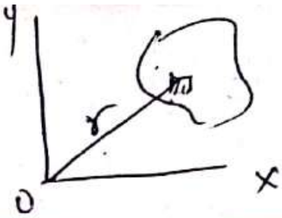
hollow = r/2

(v). Hemisphere (solid)



$\bar{y} = \frac{3r}{8}$

Polar MOI about origin or z-axis $\Rightarrow$



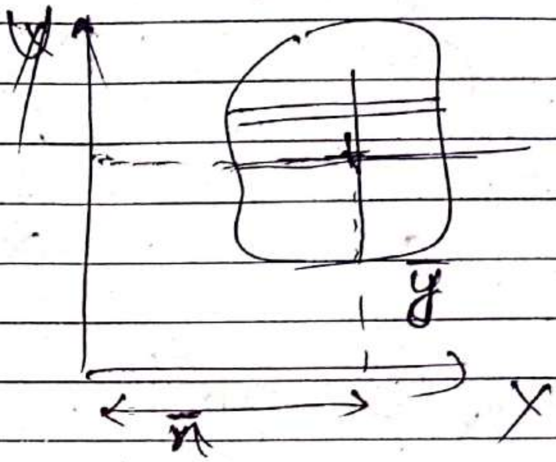
$\int r^2 dA$

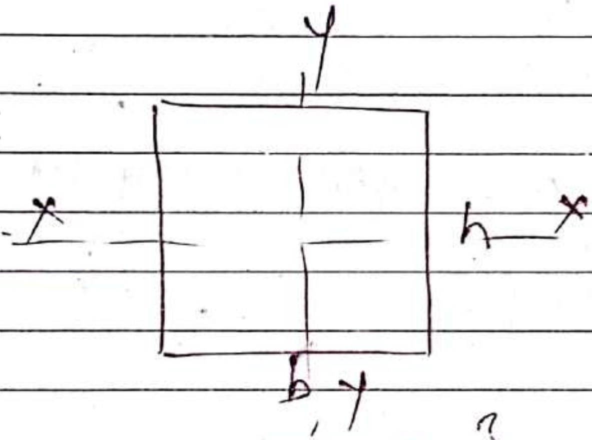
$= I_{xx} + I_{yy}$ $(r^2 = x^2 + y^2)$

Moment of Inertia:

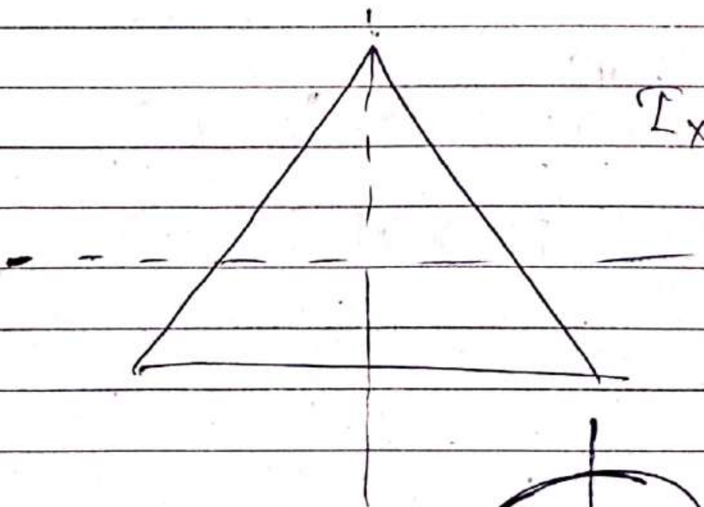
$I_{xx} = \int y^2 dA$

$I_{yy} = \int x^2 dA$





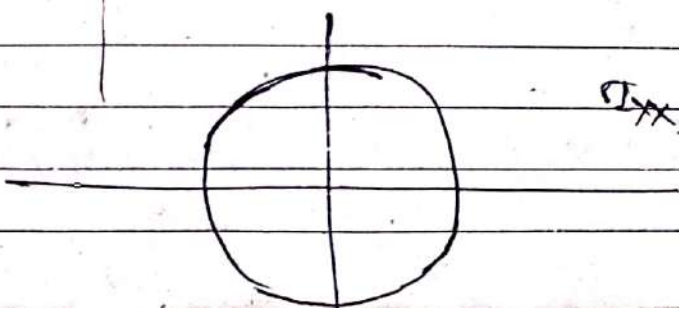
$I_{xx} = \frac{bh^3}{12}$ $\quad I_{yy} = \frac{hb^3}{12}$



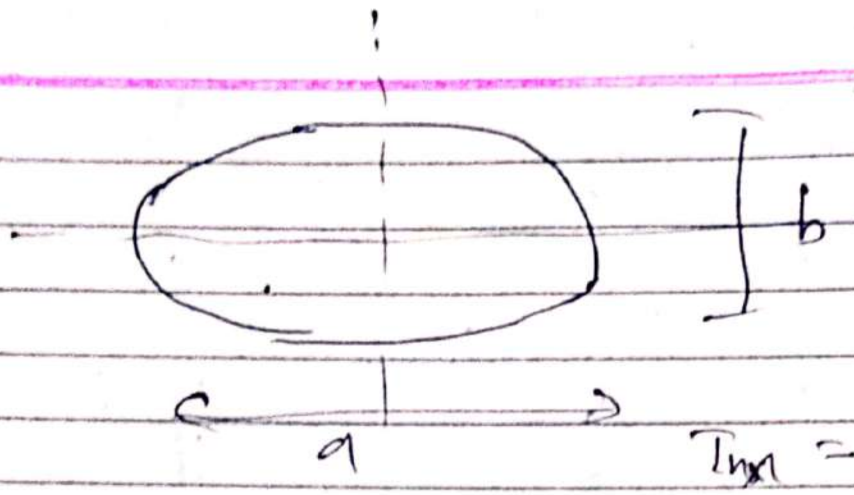
$I_{xx} \quad \frac{bh^3}{36}$

$I_{yy} = \frac{hb^3}{36}$

Circle:



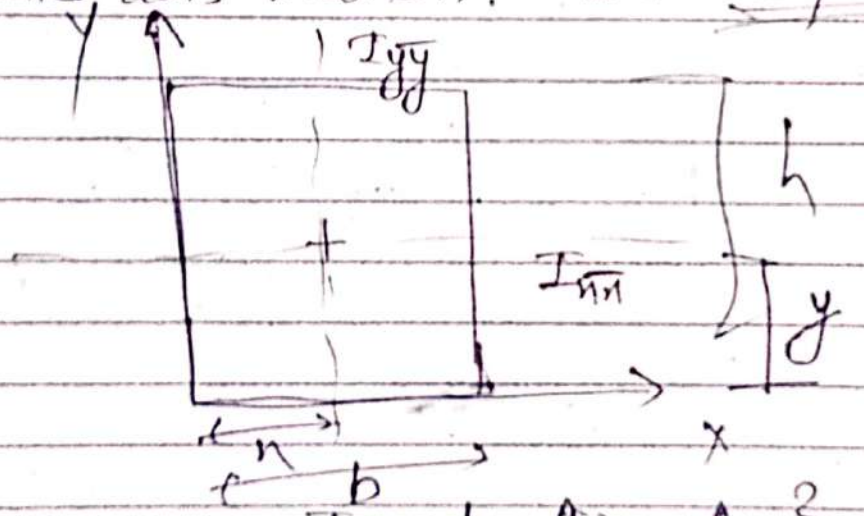
$I_{xx} = I_{yy} = \frac{\pi D^4}{64}$



$$I_{xx} = \frac{\pi a b^3}{64}$$

$$I_{yy} = \frac{\pi a^3 b}{64}$$

* Parallel axis theorem! aka transfer theorem



$$I_{xx} = I_{\bar{x}\bar{x}} + A y^2$$

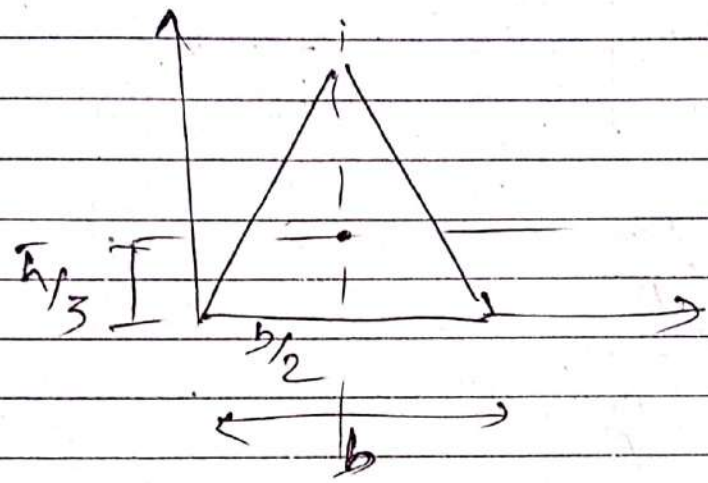
$$I_{yy} = I_{\bar{y}\bar{y}} + A x^2$$

If $x = b/2$ $\quad y = h/2$

$$I_{xx} = \frac{bh^3}{12} + bh \times h^2/4$$

$$= bh^3/12 + \frac{bh^3}{4}$$

$$= \frac{bh^3 + 3bh^3}{12} = \frac{bh^3}{3}$$

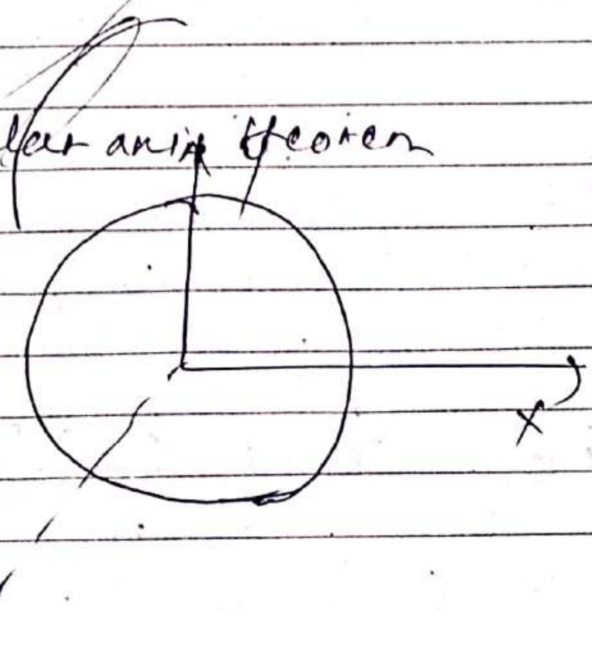


$$I_{xx} = \frac{bh^3}{36} + \frac{1}{2} bh \times \frac{h^2}{9}$$

$$= \frac{bh^3}{36} + \frac{bh^3}{18}$$

$$= \frac{bh^3}{12}$$

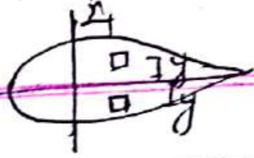
* Perpendicular axis theorem



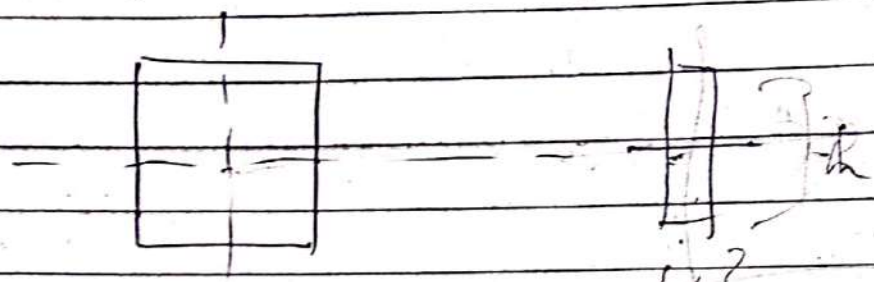
$I_z = I_x + I_y$

* Product of inertia = $\int_A xy\,dA$ may be ...

If area is symm. about x or y product of inertia is 0



Radius of gyration



$r_x = \sqrt{\frac{I_{xx}}{A}}$ $\quad \frac{bh^3}{12} \quad \frac{hb^3}{12}$

$r_y = \sqrt{\frac{I_{yy}}{A}}$

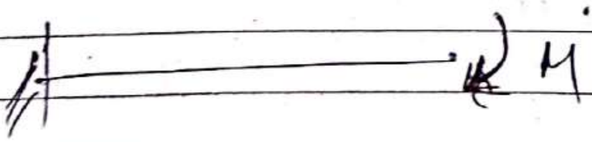
Buckling along least radius of gyration. ~~about~~

of gyration. $A r^2$

$= I_{xx}$

* Bending theorem of Flexure:

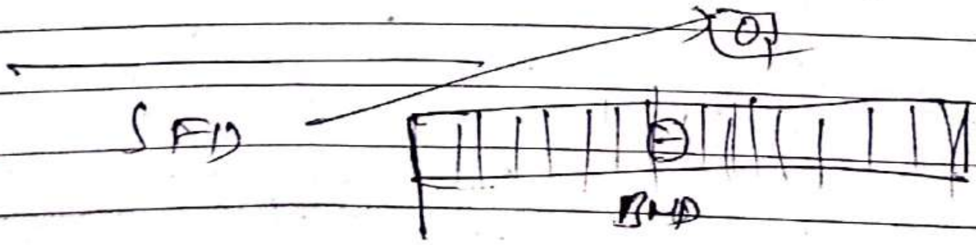
Simple Bending and pure bending



Pure Bending (0)

SFD

BMD



neutral axis should pass through centroid of the section

Simple Bending equation

$$\frac{M}{I} = \frac{\sigma}{y_{max}} = \frac{E}{R}$$

bending stress

neutral axis

radius of curvature

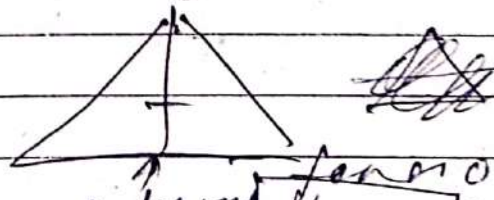
Assumption:
- static loading
- $\frac{M}{I} = \frac{E}{R}$ or, $\frac{1}{R} = \frac{M}{EI}$

or, curvature $= \frac{M}{(EI)}$

$(EI) \Rightarrow$ flexural rigidity constant for a given beam.

If constant moment acts along a given beam the Beam bends in
$\Rightarrow$ circular shape.

extreme comp^n



extreme tension

y_{max}

* section modulus:

$$Z = \frac{I}{y}$$

. Triangle

$$Z_N = \frac{\frac{bh^3}{36}}{\frac{2}{3}h}$$

$$= \frac{bh^2 \times 3}{2 \times 36_{12}} = \frac{bh^2}{24}$$

$\frac{2}{3}h$

$$\frac{\frac{d^2y}{dx^2}}{\left[1+\left(\frac{dy}{dx}\right)^2\right]}$$

Circular

$$\frac{\frac{\pi D^4}{64}}{D/2} = \frac{\pi D^3}{32}$$

curvature ⇒ rate of change of slope.

$$\frac{1}{R} = \frac{d\theta}{dx}$$

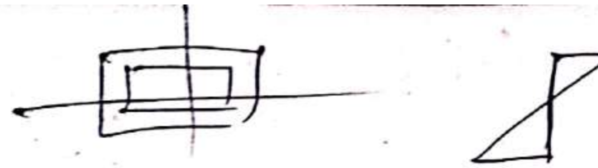
$$= \frac{d^2y}{dx^2}$$

$$M = EI\frac{d^2y}{dx^2}$$

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

or, $\cancel{\frac{dy}{dx}}$ $d^2y = \frac{M}{EI}\,dx^2$

$$M = EI\frac{d^2y}{dx^2}$$

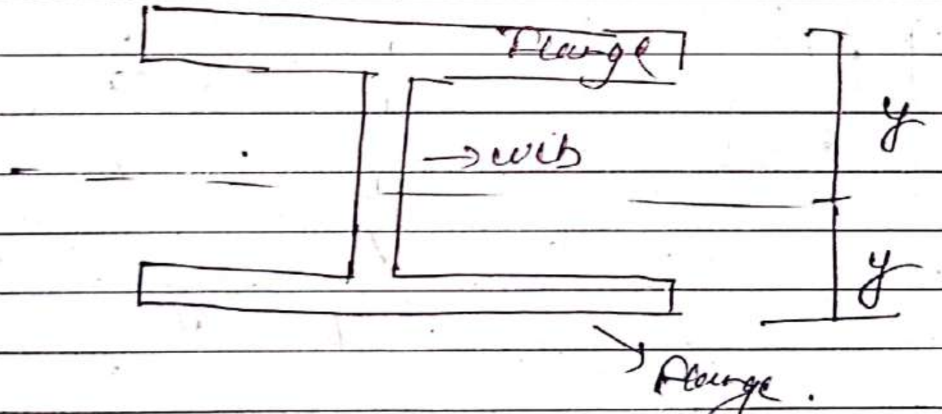


$$V = \frac{dM}{dx} = EI\frac{d^3y}{dx^3}$$

$$\omega = -\frac{dV}{dx} = -EI\frac{d^4y}{dx^4}$$

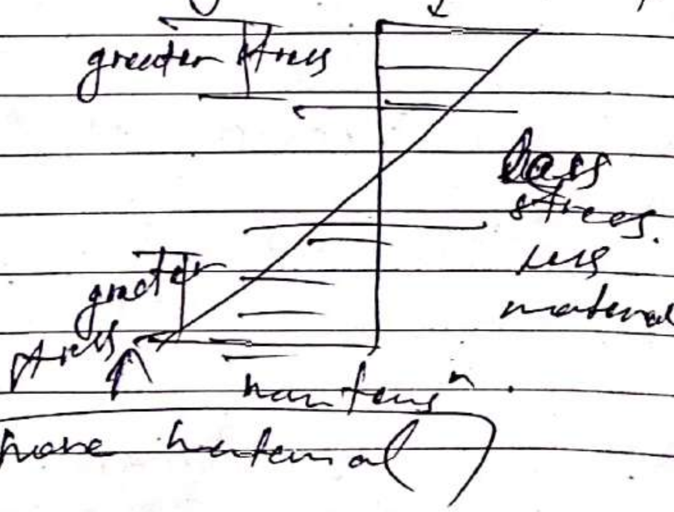
#. Which section is most efficient for carrying bending moment?

a) ✓ I-section b) channel (c) Hollow circular section (d) All



$$\frac{M}{I} = \frac{\sigma_b}{y}$$

$$\sigma_b = \frac{My}{I}$$



* variation of shear stress :

Horizontal shear stress (τ) = $\frac{V S_c}{Ib}$

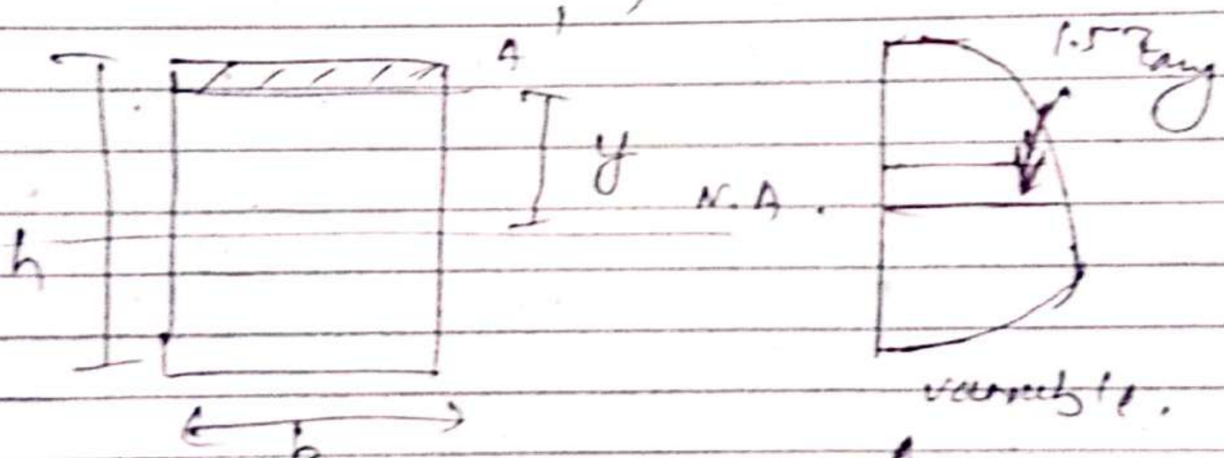
where,

V = vertical shear force

S_c = static moment = $A\bar{y}$

I = MOI about neutral axis

b = width of beam.



$$\tau = \frac{V}{2I}\left[\frac{h^2}{4} - y^2\right]$$

=> shear stress distribution in a beam is parabolic in nature

condition (i)

when $y = 0$, At N.A

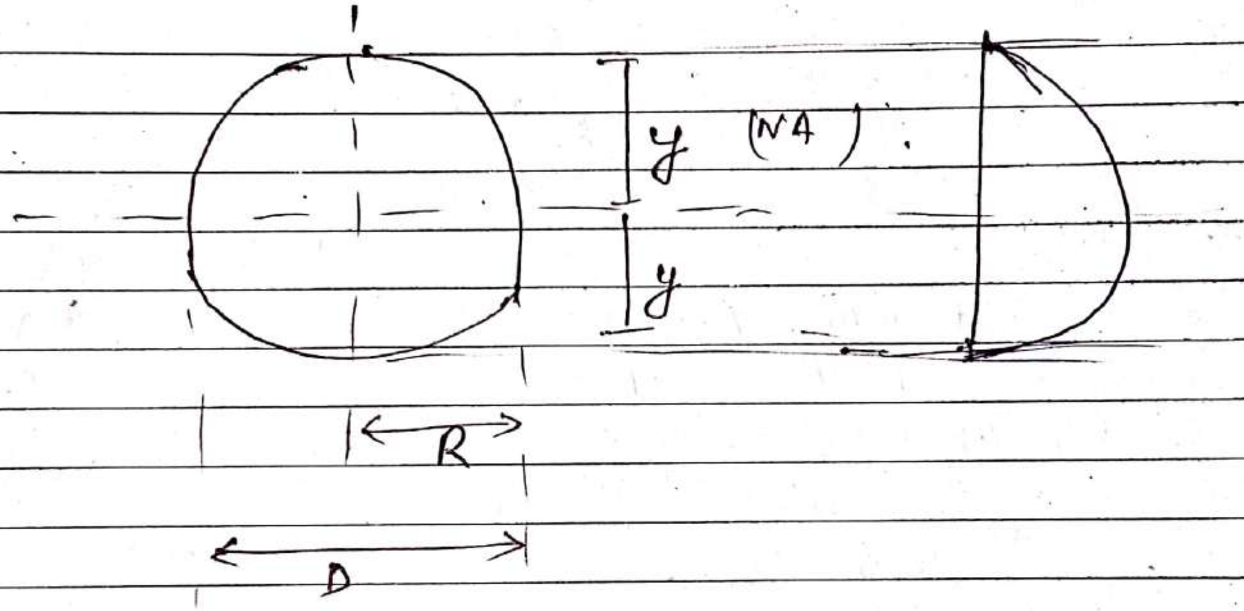
$$\tau_{max} = \frac{3}{2}\frac{V}{bh}$$

$$\therefore \tau_{max} = \frac{V}{bh}\ \tau_{avg}.$$

condition (ii) when $y = h/2$ at extreme fiber.

$\tau =$

* For circular section.



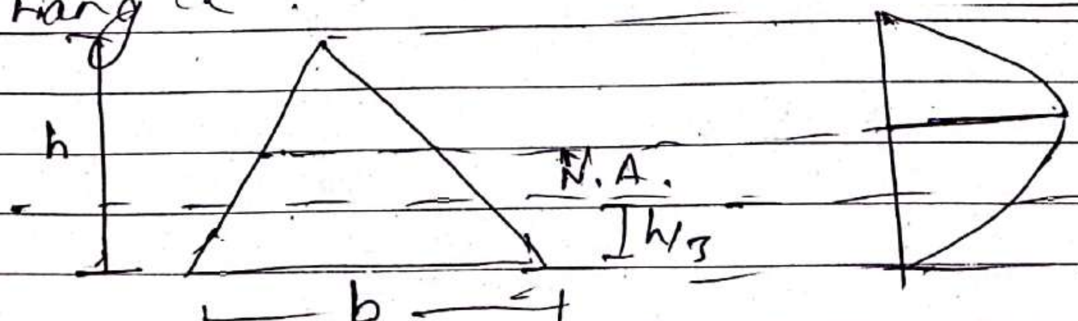
when $y = 0$, At NA, $\tau = \tau_{max} = \frac{4}{3}\tau_{avg}$

when $y = R$ at extreme fibre.

$\left(\tau_{avg} = \frac{V}{\pi R^2}\right)$

$\tau = \tau_{min} = 0$

* Triangle.



At NA, $\tau_{NA} = \frac{4}{3}\tau_{avg}$.

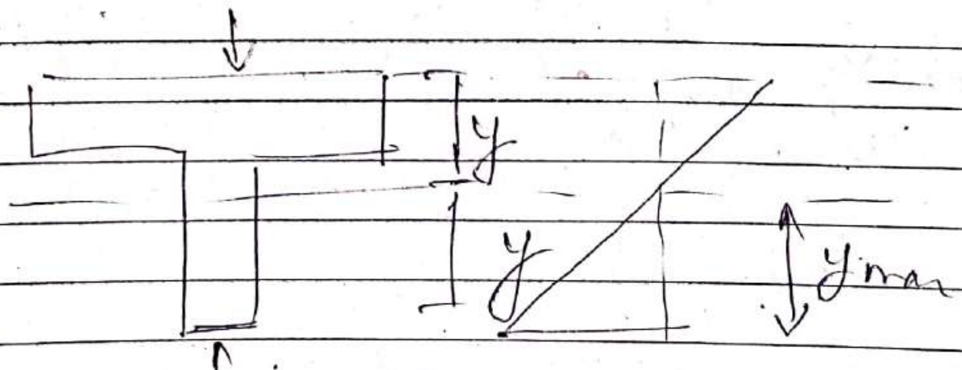
At mid $\tau_{max} = \frac{3}{2}\tau_{avg}$.

$\tau = \tau_{min} = 0$

$\tau_{avg} = \frac{V}{\left(\frac{bh}{2}\right)}$.

*. Maximum Bending stress in T = beam section will be occurs at:

a) extreme flange portion
b) extreme web portion
c) junction of flange and web
d) all.

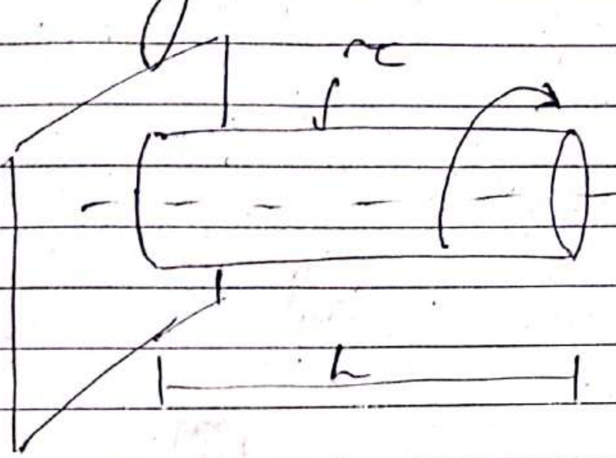


extreme web.

Que.2. In T-section almost all shear resisted by
⇒ web portion.

pure torsion => only torsional moments no BM and axial stress.

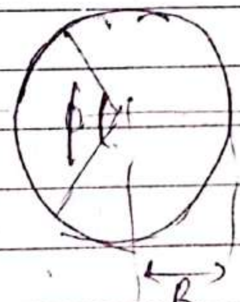
Twisting or Torsion



Torsion => moment about longitudinal axis.

Assumption
=> Angle of twist small

Fig: Solid circular shaft.



$$J = \frac{\pi D^4}{32}$$ => polar MOI

ϕ varies along the shaft length $\phi(x)$. $\phi = 0$ at fixed end
given at free end.

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\phi}{L}$$

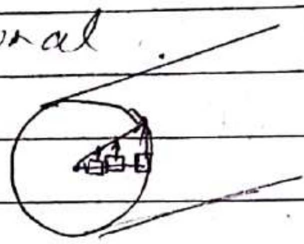
=> not valid for impact load.
valid within elastic limit.

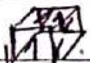
$$\frac{T}{J} = \frac{G\phi}{L}$$

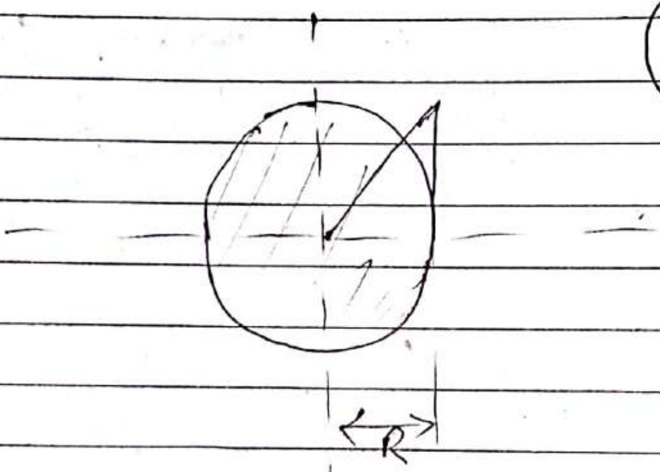
or, $$T = \left(\frac{GJ}{L}\right)\phi$$

Q when $l = 1$ $\frac{GJ}{l}$ = torsional rigidity per unit length.

$\tau \propto R$ linear proportional







Axial direction mea in shear develo $G\theta/l$ |

→ complementary proper of shear

Fig : shear stress (τ)

=) shear strain also varies linearly along radial line.

τ_{max} at outer periphery

$\tau_{min} = 0$ at $R = 0$

$$\frac{T}{J} = \left(\frac{\tau}{R}\right)$$

$$T \Rightarrow \text{strength} = \left(\frac{J}{R}\right)\tau$$

Strength of shaft (Torsional moment)

$$\frac{T}{J} = \frac{\tau}{R} \qquad T = \left(\frac{J}{R}\right)\tau$$

$$T = \left(\frac{\pi D^4/32}{D/2}\right) \times \tau$$

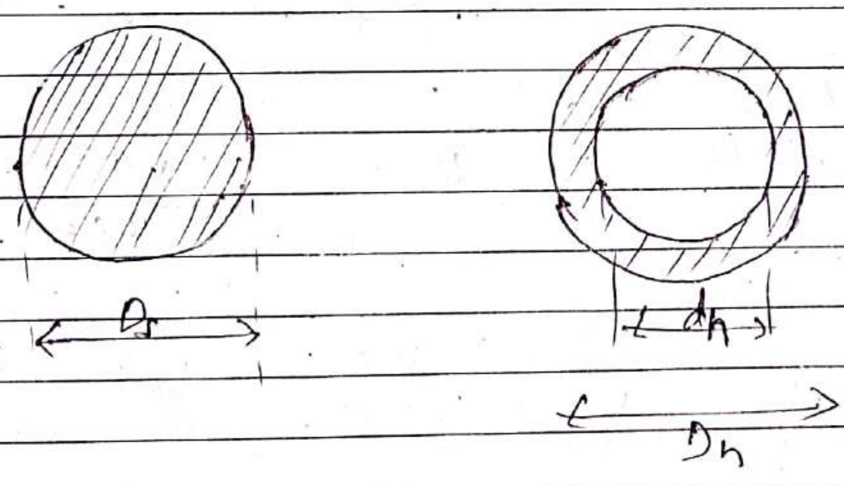
$$= \left(\frac{\pi D^3}{16}\right)\tau$$

Power transmitted by shaft:

$P = T\omega$ $\qquad$ ω = Angular velocity $= 2\pi f$ (rad/s)

f = frequency cycle/sec, Hertz.

* comparison of solid and hollow circular shaft.



$$n = \frac{D_h}{d_h}$$

(a) Strength comparison

For same length (l) and weight (ω)

$$\frac{T_h}{T_s} = \left(\frac{n^2+1}{n\sqrt{n^2-1}}\right)$$

Date
Page

Conclusion:
Hollow shaft are more stronger than solid shaft in case of torque transmission.

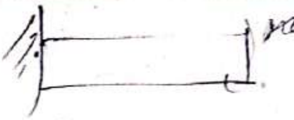
Weight comparison:
same material subjected to same torque (τ), max. shear stress produced also equal.

$$\left(\frac{w_h}{w_s}\right) = \frac{n^2-1}{\left(\frac{n^4-1}{n}\right)^{2/3}}$$

when, $n = 2$
$D_h = 2d_h$

$w_h = 0.78\, w_s$

Conclusion:
Hollow shaft are more economical than solid shaft in case of torque transmission.



$T = \left(\frac{J}{R}\tau\right)$ $\sigma = \frac{My}{I}$

$BM = M$

#. A shaft is simultaneously subjected to BM (M) and twisting moment (T). Ratio of max^m σ_B to τ will be

(4). $\Rightarrow \dfrac{2M}{T}$ #.

$$\frac{\frac{My}{I}}{\frac{TR}{J}} = \frac{\frac{M \times R}{\frac{\pi D^4}{64}}}{\frac{T \times R}{\frac{\pi D^4}{32}}}$$

$$= \frac{\cancel{M}R}{\left(\cancel{\pi D^4}/\cancel{64}\,2\right)} \times \frac{\left(\cancel{\pi D^4}/\cancel{32}\right)}{T\cancel{R}}$$

$$= \frac{2M}{T}$$

Note: $M_{equivalent}$ for Bending + Twisting

$$M_{eq} = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right]$$

$$T_{eq} = \sqrt{M^2 + T^2}$$

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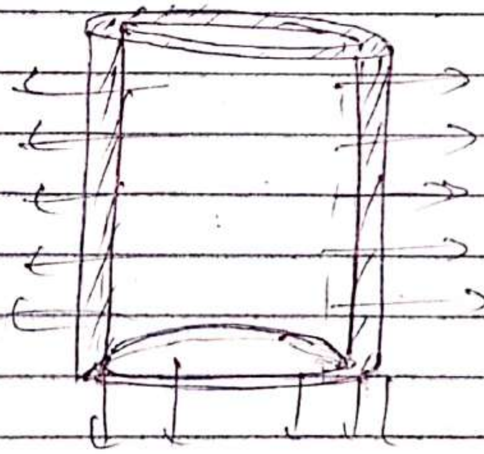
Thin cylinder and sphere:

condition: $\left(\frac{t}{d}\right) < \left(\frac{1}{10}\right)$

$\frac{d}{t} > 20$

In many books

Thin cylinder:



(c) Internal fluid pressure (P):

a) Hoop or circumferential stress (σ_h):

→ tangential to the circumference of the cylinder.

$\sigma_h = \frac{Pd}{2t}$ [Bursting stress ∠ resisting capacity of wall thickness]

Tensile uniform stress.

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For thin cylinder, wall thickness is small so the variation of stress is neglected
σ_h = constant throughout wall.

b). Longitudinal stress (σ_L).
acting along longitudinal direction

$$\sigma_L = \left(\frac{Pd}{4 \times t}\right)$$ tensile uniform stress

* Relation Between σ_h and σ_L
$\sigma_h > \sigma_L$
$\sigma_h = 2\sigma_L$

σ_h $\perp^{ar}$ σ_L
↳ major principal stress
↳ minor principal stress

max. value of shear stress
= radius of Mohr's circle.

$$\tau_{max} = \frac{\sigma_h - \sigma_L}{2}$$

$$= \frac{\frac{pd}{2t} - \frac{pd}{4t}}{2}$$

$$= \frac{pd}{8t}$$

CamScanner

$$\epsilon_h = \frac{1}{E}\left[\sigma_h - \mu\sigma_L\right] = \frac{pd}{4tE}\left(2-\mu\right)$$

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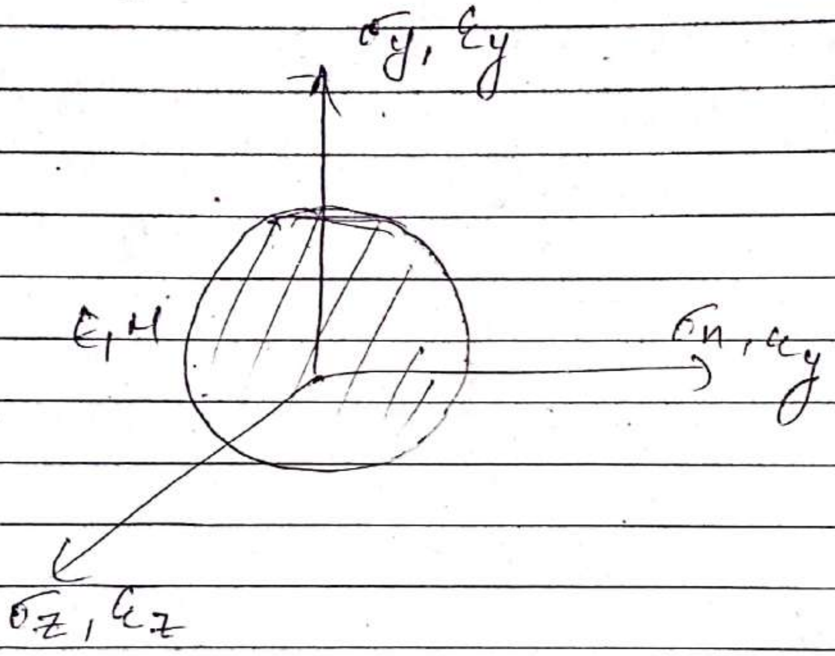
Strain

$$\text{Hoop strain } (\epsilon_h) = \frac{1}{E}\left[\sigma_h - \frac{\sigma_L}{2}\right]$$



$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \mu(\sigma_y + \sigma_z)\right]$$

$$\epsilon_y = \frac{1}{E}\left[\sigma_y - \mu(\sigma_x + \sigma_z)\right]$$

$$\epsilon_z = \frac{1}{E}\left[\sigma_z - \mu(\sigma_x + \sigma_y)\right]$$

Thin sphere :

$$\sigma_h = \frac{pd}{4t} = \sigma_l$$

$$\epsilon_h = \frac{1}{E}\left[\sigma_h - \mu\sigma_l\right]$$

$$= \frac{1}{E}\frac{pd}{4t}\left(1-\mu\right)$$

$$\boxed{\epsilon_v = 3\epsilon_h}$$

ϵ_v = volumetric strain

what stress would you consider to design a thin cylinder ?

a) Hoop stress b) Longitudinal

c) Both d) avg of two

A cylindrical shell with hemi-spherical end at both end, then thickness of cylindrical portion is ...... that of spherical end.

a) more than b) less than c) equal

d) none.

$$t_{cyl.} = 2\, t_{sph.}$$

Column and strut :

Column : A compression member generally in vertical position. ~~and fixed~~ (d)
~~Both on~~

Type
a) Short b) Intermediate c) Long

$$\lambda = SR = \left(\frac{L_{eff}}{r_{min}}\right)$$

Short : $\lambda < 8\phi$ ⇒ for circular
$\lambda < 32$
⇒ fails by crushing $\sigma = \left(P/A\right)$

Intermediate :
$8\phi \le \lambda \le 30\phi$
$32 \le \lambda \le 120$

⇒ fails due to crushing and buckling.

$\sigma =$ Direct stress $\pm$ Bending stress

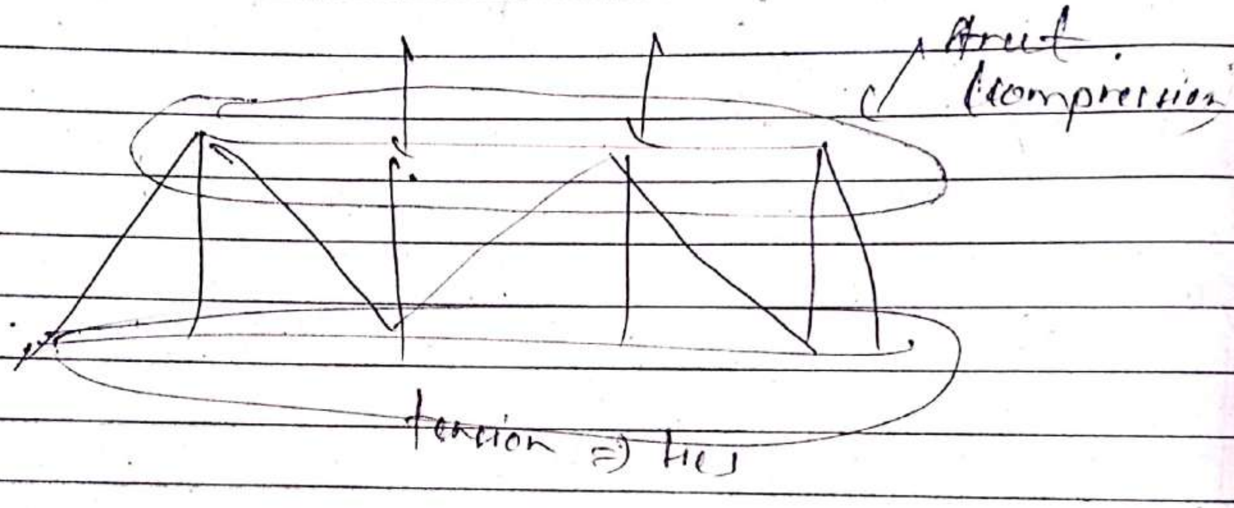
Long :
$\lambda > 30\phi$ ⇒ circular.
$\lambda > 120$ $\sigma = \frac{M}{Z}$

⇒ fails mainly due to buckling

Strut :

→ compression member other than vertical position.

$\lambda > 20$



Crippling load or critical load or buckling load. (P_{cr})

a) Euler's column theory:

→ applicable only for long column.

b) $\lambda \geq 80$ (mild steel)

$$P_{cr} = \frac{\pi^2 EI}{l_{eff}^2}$$

where, I = least MOI.

l_{eff} = theoritical effective length.

Crippling stress (σ_{cr}) $= \dfrac{P_{cr}}{A}$

$$= \frac{\pi^2 E \times r_{min}^2 A}{l_{eff}^2 \times A} = \frac{\pi^2 E}{\lambda^2}$$

Effective length

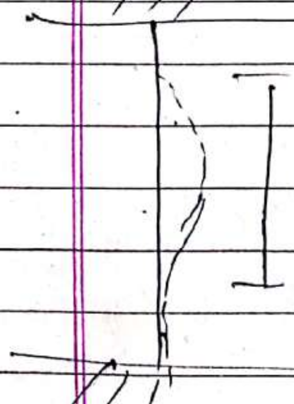
Theoritical

⇒ Distance between point of inflection in deffected shape of column.

→ different for different support condition.

Recommended

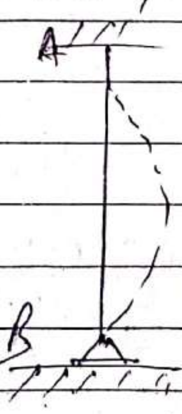
a) Both end fixed.



$l_{eff} = l/2$

$$P_{cr} = \left(\frac{\pi^2 EI}{l^2}\right) \times 4$$

b) One end fixed other hinge.

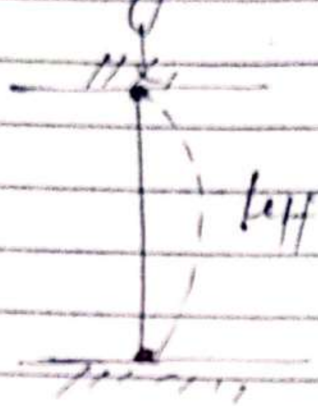


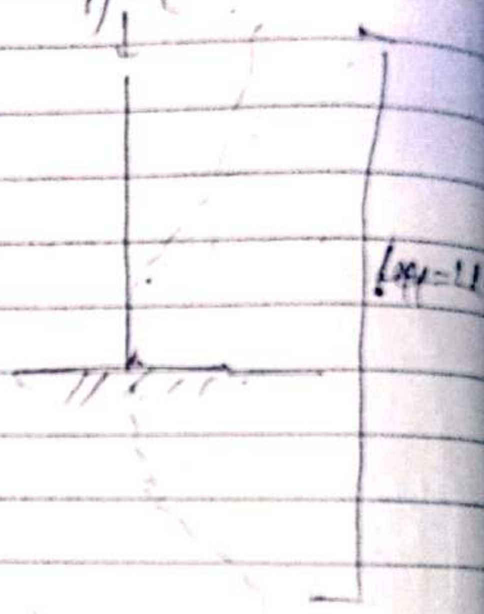
$l_{eff} = \dfrac{l}{\sqrt{2}}$

$= 0.707\, l$

$$P_{cr} = \left(\frac{\pi^2 EI}{l^2}\right) \times 2$$

c) Both ends hinged. (d). one fixed other free





$$P_{cr} = \frac{\pi^2 EI}{l^2}$$

$$P_{cr} = \frac{\pi^2 EI}{4l^2}$$

max^m load resisting capacity in both end fixed.

+ Rankine's Theorem:
  - → applicable for all types of column
  - → Based upon experimental investigation

$$\frac{1}{P_{rank}} = \frac{1}{P_{euler}} + \left(\frac{1}{P}\right)$$

crushing load

(iv) Rankine - Gordon Theorem:

- applicable for intermediate column only.

#. Effective length of chimney of 20 m height is

a) 20 m b) 10 m c) 40 m d) 60 m

$l_{eff} = 2L$ (one fix other free)

#. Slenderness ratio of compression member is zero when its length is

a) equal to radius of gyration b) supported throughout its length

c) ~~supported~~ zero d) none

Theory of structure I:

Analysis of statically determinate beam, truss, frame, arch and cable structure:

simply supported beam   cantilever Beam   overhanging

(i) Energy method
→ used to calculate slope and deflection of beam.

Principle of conservation of energy:

a) Strain Energy Method
b) Real work method
c) virtual work
d) Betti's theorem =>
e) Maxwell's Reciprocal Theorem.
f) Castigliano's Theorem.

A) Strain Energy (U):
→ energy stored in a body due to its internal work done.

Resilience :

⇒ strain energy stored within the proportional limit

Proof resilience :

max^m energy stored within prop. limit.

modulus resilience :

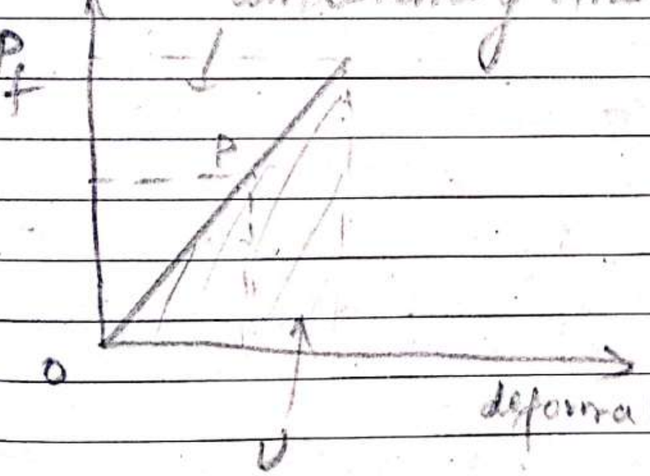
proof resilience per unit volume

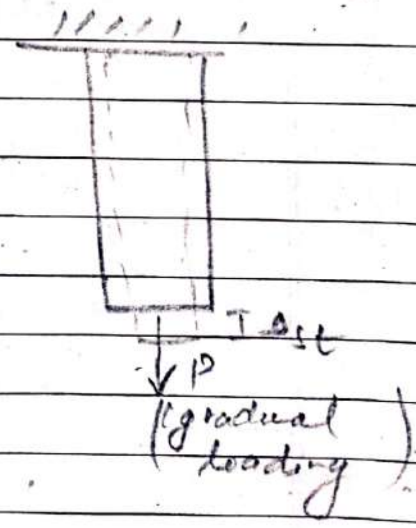
$= \frac{U}{V} = \frac{1}{2} \times \sigma_{prop} \times \epsilon$

*. Strain Energy stored due to different loading condition.

(i) Axial loading :

complementary strain energy.





$\Delta_{st} = \frac{PL}{AE}$

U = Area under P_{Δ} curve

$= \frac{1}{2} P \Delta_{st} = \frac{1}{2} \frac{P^2 L}{AE}$

$$= \frac{P^2 l}{2AE}$$

$$U_{total} = \Sigma \left(\frac{P^2 l}{2AE} \right)$$

$$MOR = \frac{U_{max}}{V}$$

$$= \frac{\frac{P^2 l}{2AE}}{A \times l}$$

$$= \frac{P^2}{2A^2E}$$

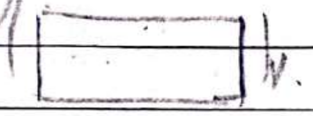
$$= \frac{\sigma_u^2}{2E}$$

$$= \frac{1}{2} \frac{\sigma_u^{\cancel{2}}}{\cancel{\sigma}/E}$$

$$= \frac{1}{2} \sigma \times \epsilon_{strain}$$

(ii) Shear force:

$$U_v = K \int \frac{V^2}{2GA} \, dx$$

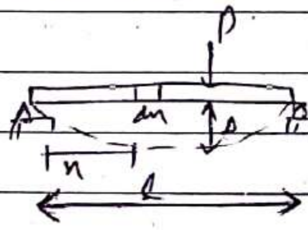


$K \Rightarrow$ form factor $= 1.2$ for rectangular section

$= 10/9$ for circular

(iii) Bending:

$$U_m = \int \frac{M^2}{2EI} \, dx$$



or $dU = \frac{1}{2} M \, d\theta$

$$U = \int_0^l \frac{1}{2} M \, d\theta = \int_0^l \frac{1}{2} M \frac{M}{EI} \, dx = \int_0^l \frac{M^2}{2EI} \, dx$$

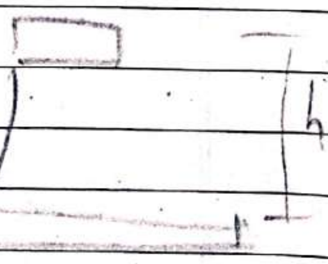
(iv) Torsional

$$U_T = \sum \frac{T^2 L}{2 I G}$$

* Real work Method:

Impact loading:

$$\Delta_d = \Delta_{st} \left(1 + \sqrt{1 + \frac{2h}{\Delta_{st}}} \right)$$



Impact factor. Δ_{max}.

$$\Delta = \Delta_{st} \sqrt{1 + }$$

* For suddenly applied load:

$k = 2$

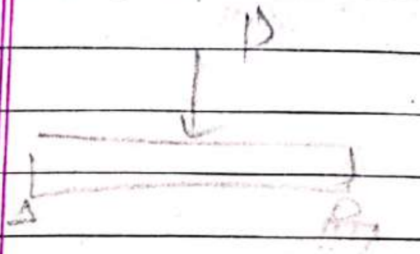
$IF = 2$ $\boxed{\Delta_d = 2\Delta_{st}}$

Ratio of maximum instantaneous stress due to suddenly applied loading to gradually applied loading.

a) 1 b) 2 c) 3 d) 4

#. Strain energy stored in simply supported beam having length L, flexural rigidity EI due to a point load at mid span will be:

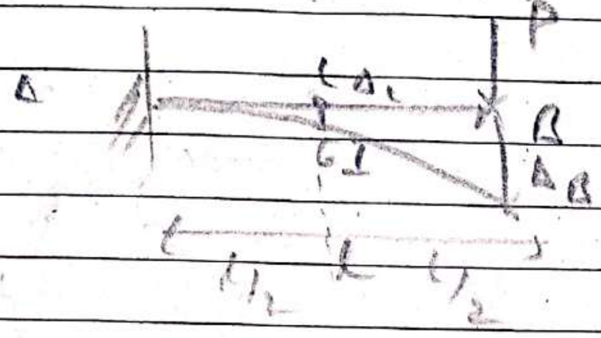
a) $\frac{P^2 L^3}{96EI}$



$\Delta = \frac{Pl^3}{48EI}$

$U = \frac{1}{2} P\Delta = \frac{1}{2} \times P \times \frac{Pl^3}{48EI} = \frac{1}{2} \frac{P^2 l^3}{48EI}$

(II) Real work method:



$$U = W = \frac{1}{2} P \Delta_B$$

or, $$\Delta_B = \frac{2U}{P}$$

$$U = W = \frac{1}{2} M \theta_B$$

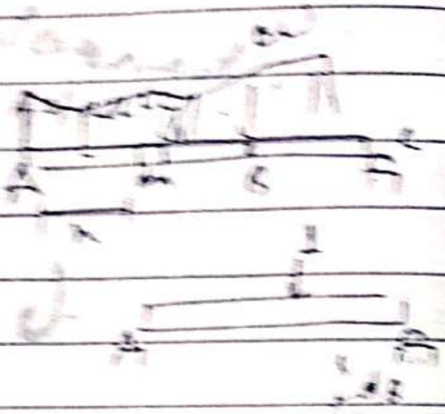
$$\theta_B = \frac{2U}{M}$$

* Limitation:
  ⇒ slope and deflection can't be deter-
  -mined at points other than

* Virtual work:
  John Bernoulli in 1717 A.D.
  - highly convenient, popular method.
  - apply unit virtual loading at the point where deflection / slope is to be determined.

Real load & virtual displacement

virtual load & real displacement



For beam and frame

$W = 1$

$d\theta = \frac{M\,dx}{EI}$

$\theta = \int \frac{M m\,dx}{EI}$

$\Delta = \int \frac{M m\,dx}{EI}$

where,

$M \Rightarrow$ due to real loading

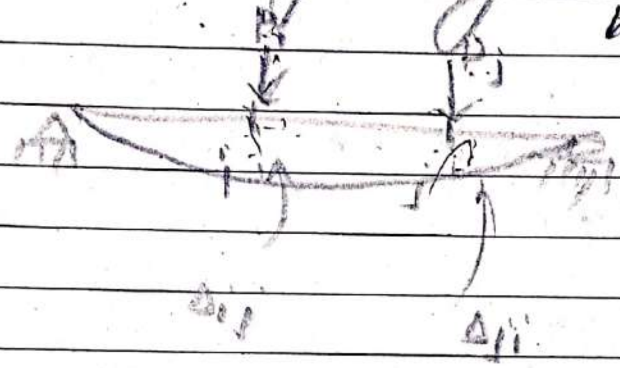
$m \Rightarrow$ virtual loading

For Truss

$$\Delta = \sum \frac{P P' L}{AE} + \sum P'(\alpha L\,\Delta t) + \sum P' \times \Delta$$

(iii) Bettys Theorem:

In a linearly elastic material in which temperature remains constant and support are unyielding. Then



Wbg P system forces through displacement by Q system of force = Work by Q system of forces through displacement by P system of forces.

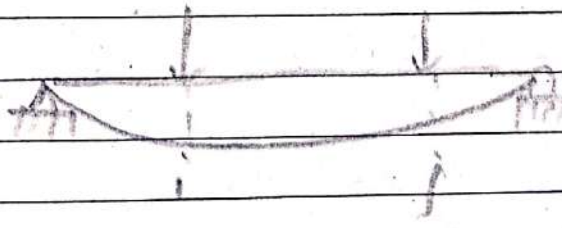
$P_i \Delta_{ij} = P_j \times \Delta_{ji}$

(iv). Maxwell's Reciprocal Theorem

" " are unyielding. Then

$P_i = 1 \quad P_j = 1$

=) Specific form of Bettis Law.



$\delta_{ij} = \delta_{ji}$

$\delta_{12} = \delta_{21}$

$\frac{\phi^{\prime\prime}m}{\frac{R}{EI}} = \frac{d^2y}{dx^2}$

$\frac{d^2y}{dx^2} \propto \frac{M}{EI}$

$\frac{dV}{dx} = -w$

$\frac{d^2M}{dx^2} = w$

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Castigliano's Theorem:

- also called minimum strain energy method.

I Theorem: In a linearly elastic structure in which supports are unyielding and temp remains constant.

$$P_i = \frac{\partial U}{\partial(\Delta_i)}$$

Principle of superposition is applicable only for:
Statically determinate
Indeterminate
superposition valid
none

Second Theorem:
Linearly elastic or non linear elastic structure, support are non-yielding

$$\Delta = \frac{\partial U}{\partial(P_i)}$$

$$\Delta = \int \frac{\partial M}{\partial P} \frac{M}{EI} dx$$

$$\Delta = \sum \frac{\partial F}{\partial P} \frac{FL}{AE}$$

Truss.

$V = \int w\,dx$ $\quad M = \int\left(\int w\,dx\right)dx$ $\quad w = \frac{dV}{dx}$

$\theta = \int \frac{M}{EI}\,dx$ $\quad y = \int\left[\int \frac{M}{EI}\,dx\right]dx$ $\quad V = \int w\,dx$

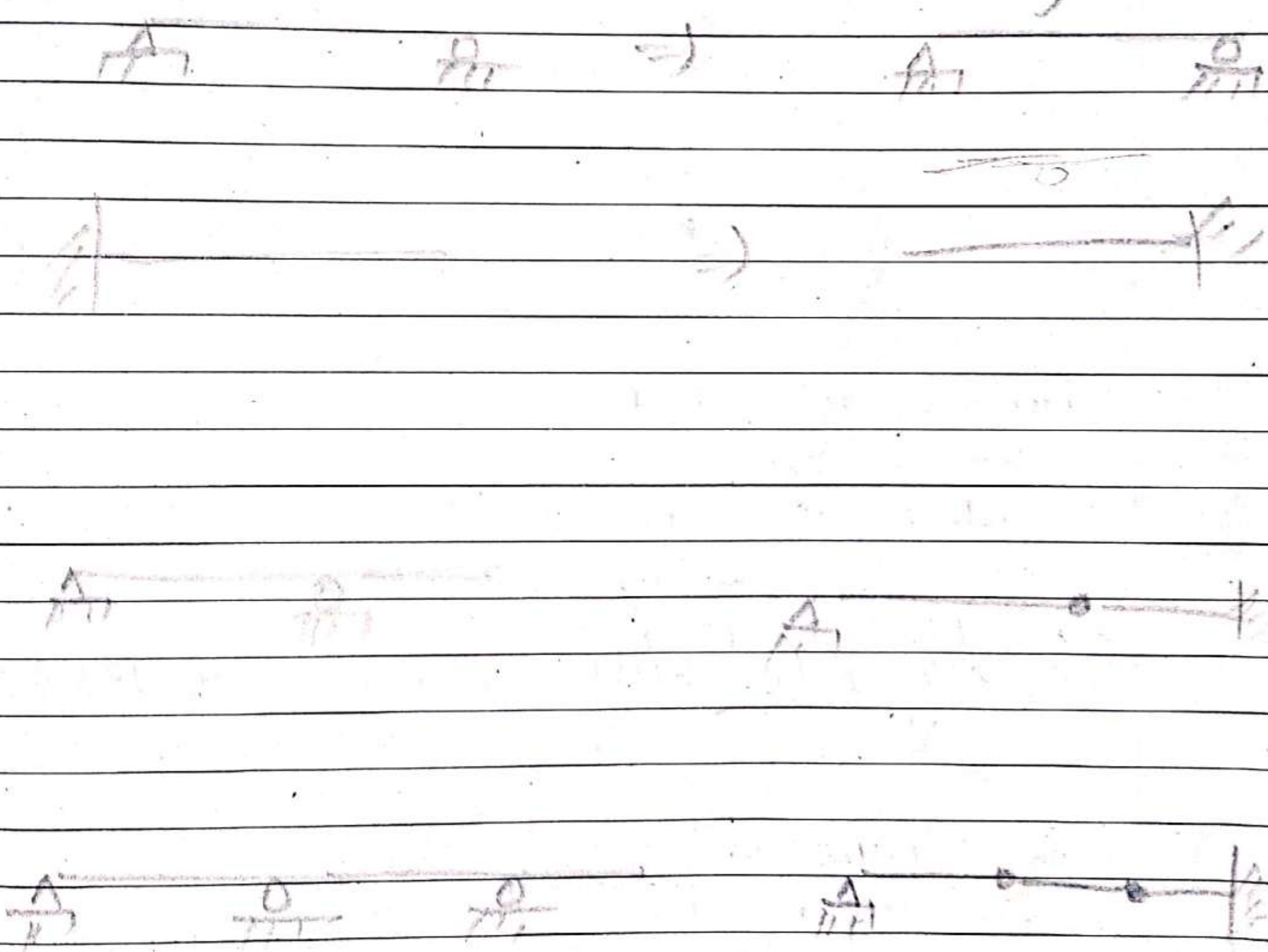
*. Geometric method.

ii) Conjugate Beam Method :

→ Muller Breslau in 1865 A.D

→ used to calculate slope and deflection of prismatic or non-prismatic.

Real $\qquad\qquad$ Conjugate



Loading in conjugate beam is $\frac{M}{EI}$

1st theorem:

Slope of any point in real beam is numerically equal to shear force at corresponding point in conjugate beam.

2nd theorem:

Deflection at a point in real ⇒ moment at corresponding point in conjugate Beam.

* Moment Area Method:

— Semi-graphical method to calculate slope and deflection of beam and frame.

— elastic curve is to be drawn and tangent at desired point.

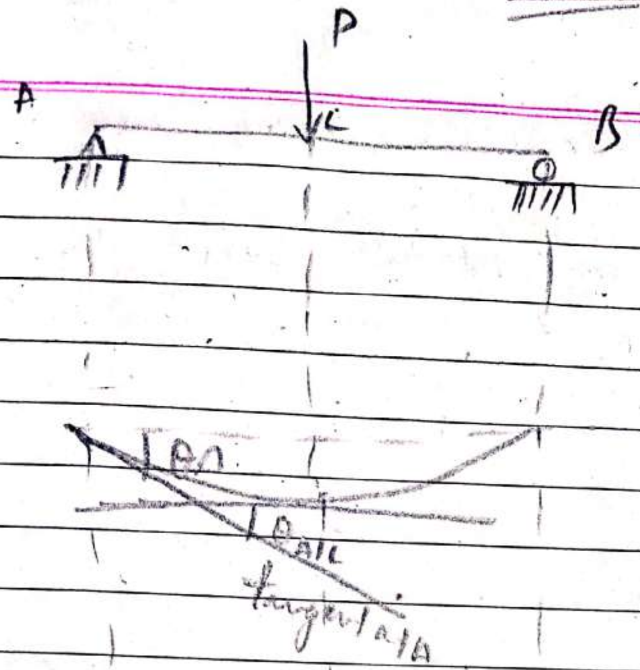
→ Otto Mohr. later Charles E Greene 1873.

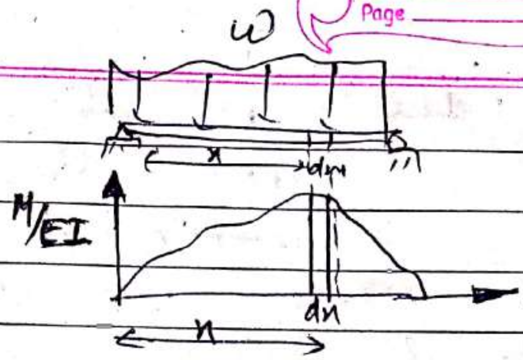
1st theorem:

change in slope between two tangent on elastic curve is numerically equal to area under $\frac{M}{EI}$ diagram.

$$\omega_{BA} = \int_A^B \left(\frac{M}{EI}\right) dx = \text{Area under } \frac{M}{EI} \text{ diagram}$$

tedious method





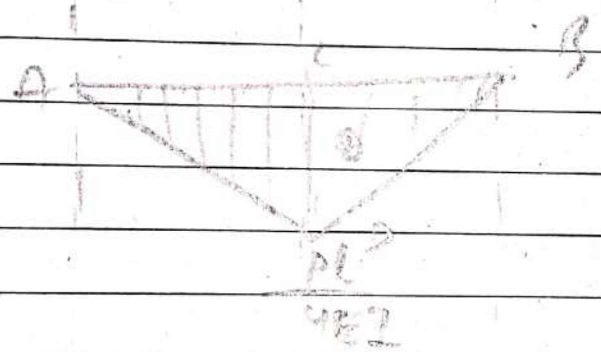
$$\frac{M}{EI} = \frac{d^2y}{dx^2}$$

$$\frac{M}{EI} = \frac{d\theta}{dx}$$

$$\therefore d\theta = \frac{M}{EI}\,dx$$

$$\int_A^B d\theta = \int_A^B \frac{M}{EI}\,dx$$

$\theta_{B/A}$



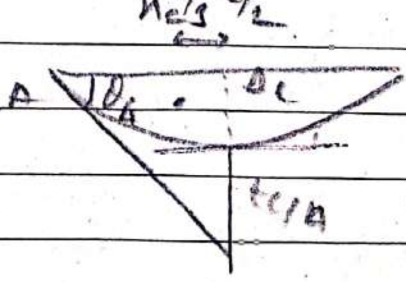
AD

$$\text{Area of } \Delta = \frac{1}{2} \times \frac{L}{2} \times \frac{PL}{4EI}$$

$$= \frac{PL^2}{16EI}$$

2nd Theorem: The vertical deviation of a tangent at a point wrt another point on elastic curve equals the moment of area under $\frac{M}{EI}$ from point A

The vertical intercept (deviation) between elastic curve and only one tangent drawn on



$$t_{C/A} = \int_A^C \left(\frac{M}{EI}\right) \bar{x}\,dx$$

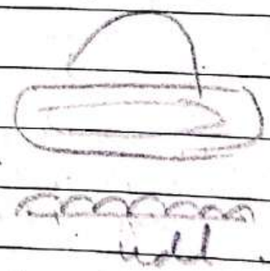
$$\theta_A = \frac{\Delta_C + t_{C/A}}{L/2}$$

Influence Line Diagram (ILD):

* ILD for statically determinate structure
  - → ILD is linear
  - → variation of an internal stress a speci-fied section due to moving load over the member.
  - → used to calculate SF and BM of a specified section due to moving load (series of point load & UDL)

in Bridge ⇒ live load (IRC load vehicle load)

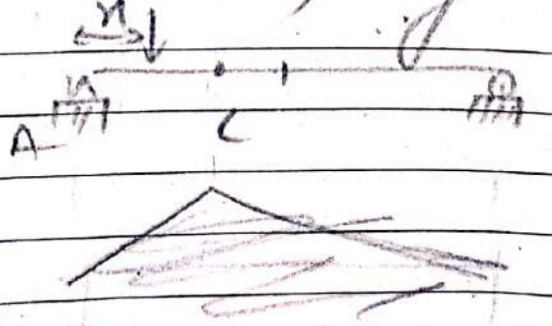
Track load ⇒ army tank,



UDL.

(1) Simply supported Beam (Girder).

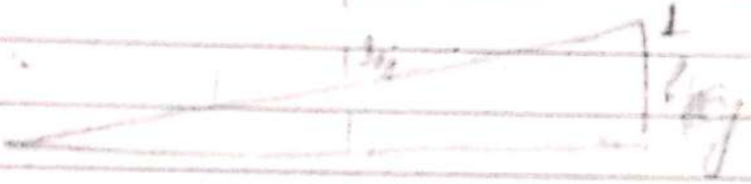
Girder ⇒ higher depth



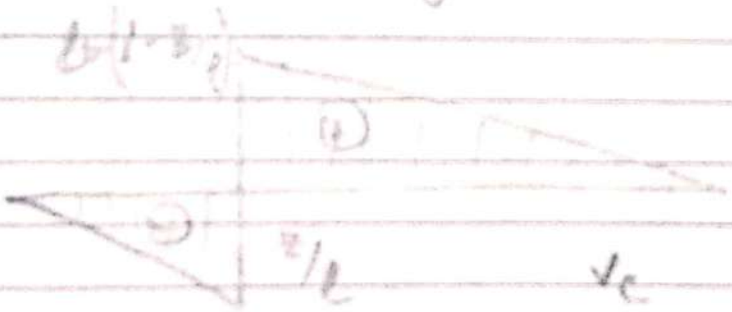
Support reaction (R_{Ay}, R_{By})

when $x = 0$
$R_A = 1$
~~$R_B = 0$~~
when $x = l/2$
$R_{Ay} = 1/2$
At $x = l$
$R_{Ay} = 0$

R_{Ay}



R_{By}

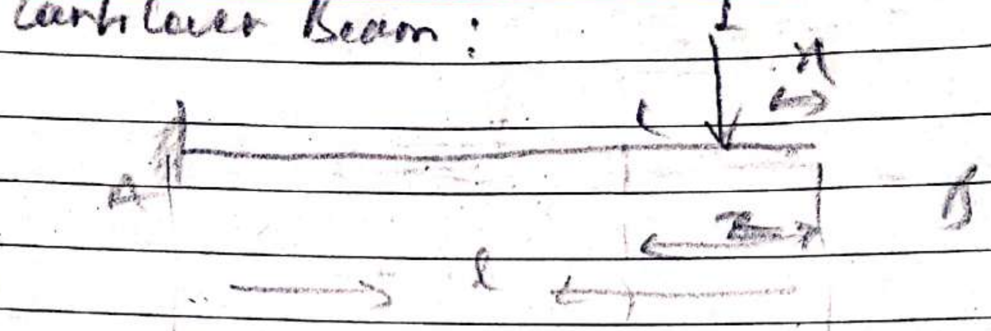


V_c

$z(1-z/l)$

BM_c

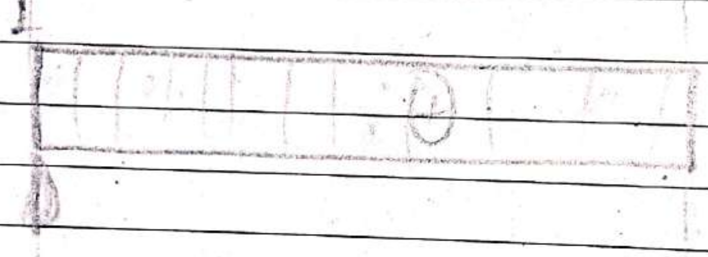
Cantilever Beam :



At $x = 0$ $\quad R_A = 1$.

$x = l/2$ $\quad R_A = 1$

$x = l$ $\quad R_A = 1$

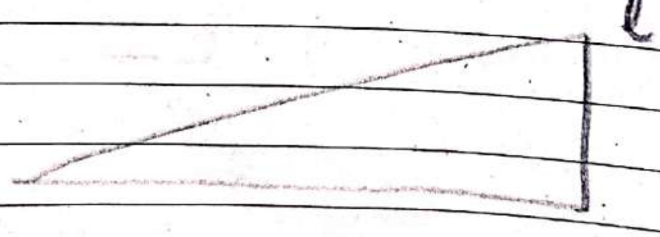


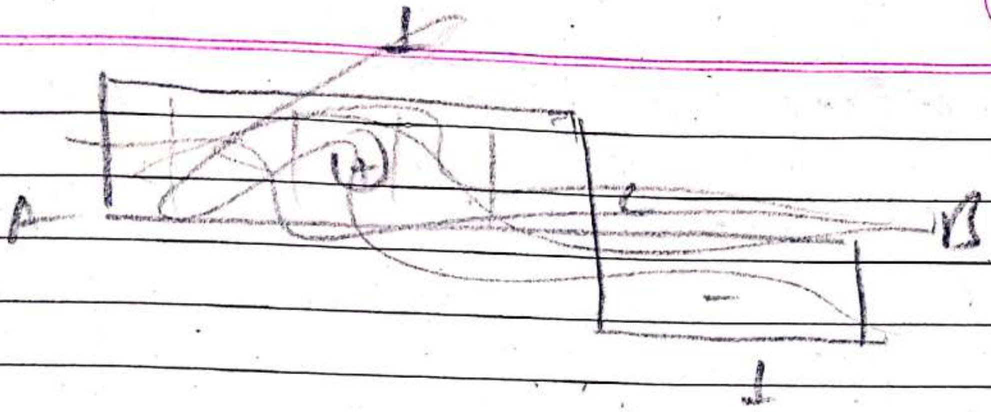
ILD for R_A

At $x = 0$, $\quad M_A = l$

At $x = l/2$ $\quad M_A = l/2$

At $x = 0$ $\quad M_A = 0$





For shear at C

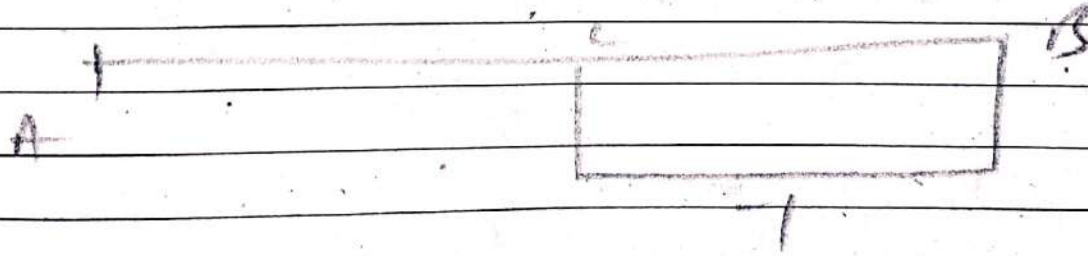
At $x = 0$ $V_C = -1$

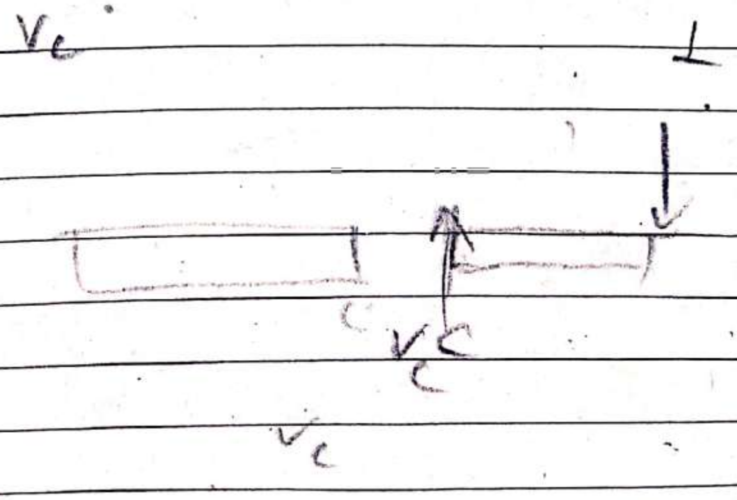
At $x = z$, $V_C = -1$
(just left of C)

At $x = z$ (just right of C), $V_C = 0$

At $x = l/2$, $V_C = 0$

At $x = l$, $V_C = 0$





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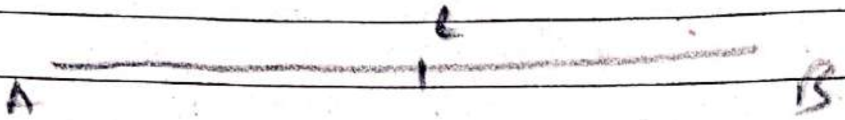
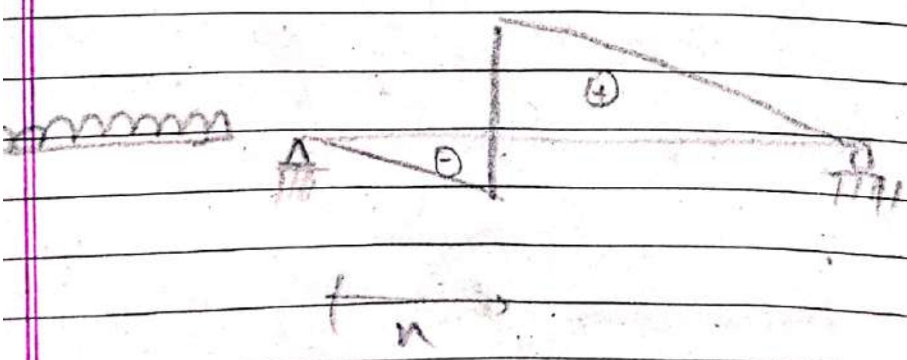


Using ILD calculate maximum value of shear force and BM due to different position of moving load and UDL point

1). length of UDL larger than span of girder.

2) length of UDL shorter than the span of girder

3). A number of concentrate load (a train of point load).



V_c = max ?

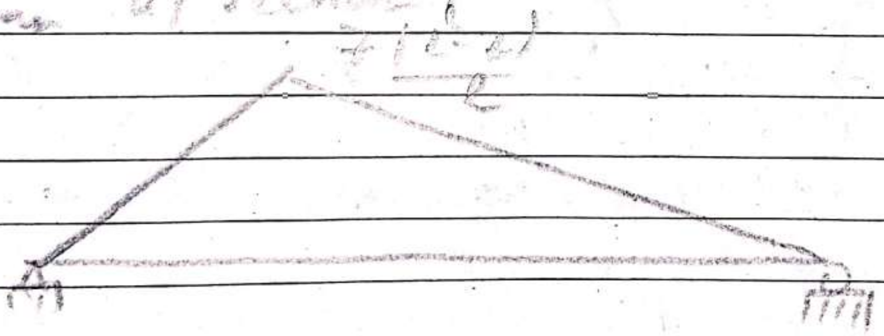
Draw ILD ⇒ position when load of UDL is at position C

$(V_{max})_c$ = w × area under ILD occupied by udl length.

$= w \times \frac{1}{2} \times z \times \left(\frac{z}{l}\right)$

$(V_{max}) = w \times \frac{1}{2} (l-z) \times \left(\frac{l-z}{l}\right)$

M_{max} at section C



position of udl . whole area must be occupied.

M_{max} = Area under curve × intensity of loading

$= \frac{1}{2} \times l \times$

Absolute maximum bending moment.

Maximum BM over the section.
at $x = l/2$

$$\frac{d\left(\frac{z(l-z)}{l}\right)}{dz} = 0$$

or $\frac{l - 2z}{l} = 0$

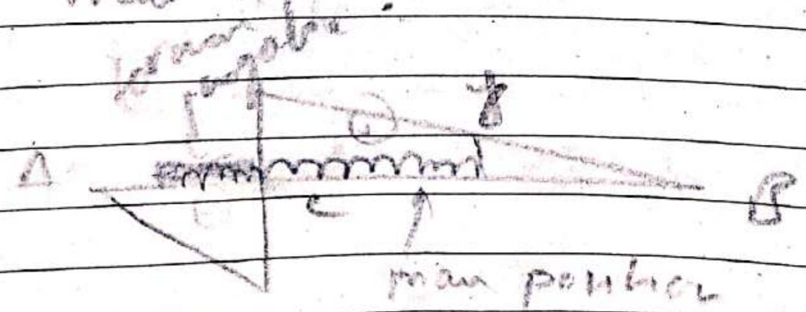
or $l = 2z$

$z = l/2$

co-ordinate of ILD of BM $= \frac{l}{4}$ $\quad \frac{l/2 \times l/2}{l}$

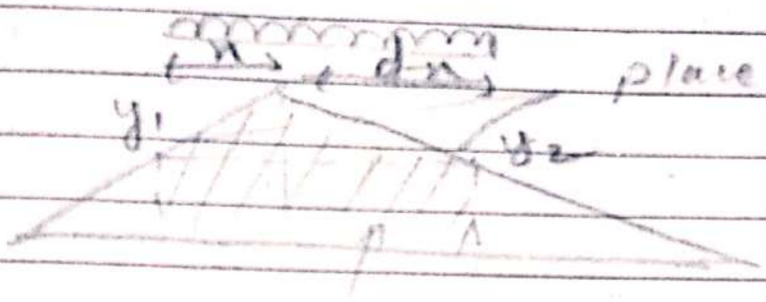
Condition (ii). Length of udl less than span

Shear at



$$V_{max} = w \frac{y - z/L}{z}$$

V_{max} when ~~...~~ Area under ~~...~~ trapezium × load intensity.



place udl I.L. equal ordinate is spanned

maximise this area

First order derivative of BM w.r.t. x

$y_1 = y_2$

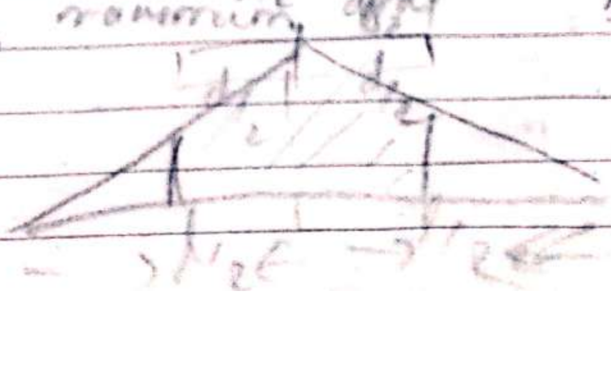
load ordinate = tale ordinate max^m BM occurs.

Determining y_1, y_2 using similar triangle.

$$\frac{x}{d} = \frac{z}{L} \qquad \boxed{x = \frac{z}{L} \times d}$$

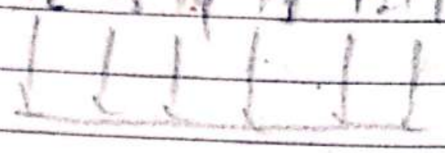
Conclusion: to get max. value of BM at section C UD is positioned, in such a way that section 'c' divide the UDL in the same ratio as it divides the span

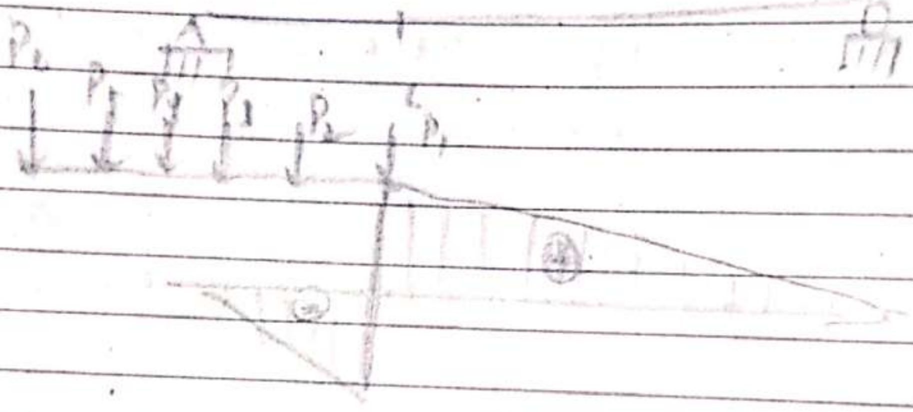
* Absolute maximum BM $\qquad BM_{abs} = \text{Area} \times w$



Condition III : A train of load.

trailing load P_6 P_5 P_4 P_3 P_2 P_1 → leading load





$V_{max}^{-ve} = P_1 y_1 + P_2 y_2 + P_3 y_3 + \dots$

when $P_1 << P_2$

Trial 1 : P_1 at section C

Trial 2 : P_2 at section C ⇒ max

$\left(\frac{R_1}{z}\right)_{left} > \frac{R_2}{(L-z)}$

R_1 ⇒ Resultant of load left of section

R_2 ⇒ right of section

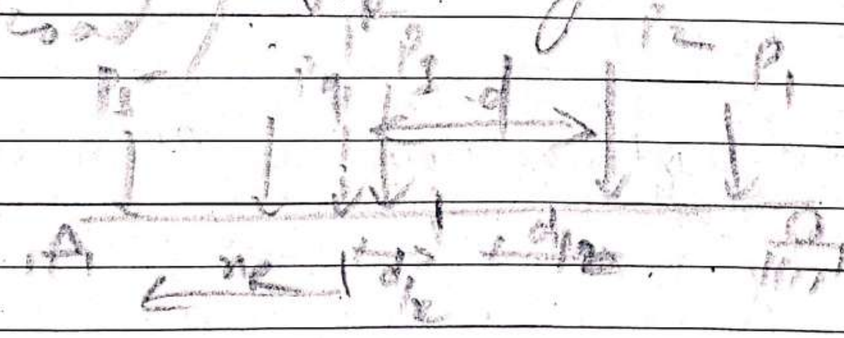
If P_3 is L

$$\left(\frac{R_1}{z}\right)_{\text{left}} < \left(\frac{R_2}{l-z}\right)$$

$$M_{max} = P_1 y_1 + P_2 y_2 + P_3 y_3.$$

* Maximum bending moment under the given load due to a moving train of load

To find max. value of BM under (P) load (particularly giving load)

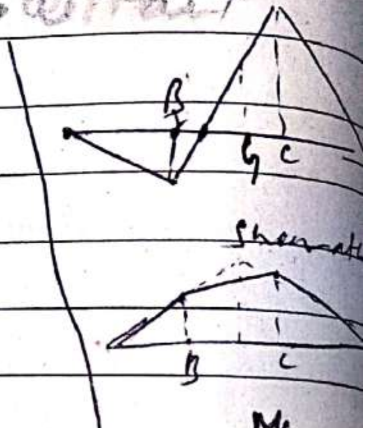


R = Resultant of all point load
$= \Sigma(P_1 + \ldots + P_5)$

d = distance bet^n resultant load (R) and given loading..

For max. BM under P_2

R and P_2 should be equidistant from center.

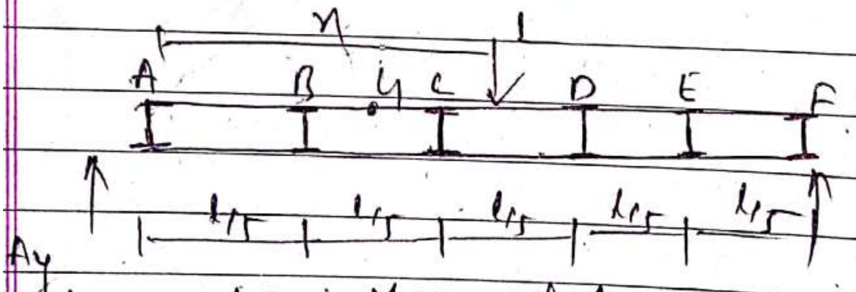


$$x = \left(\frac{l}{2} - \frac{d_1}{2}\right)$$

wrt d = position of giving load (P_2)

ILD for girder.

Just order



$F_y = x/l$

$1 - \frac{x}{l}$

Eg. shear at G

when unit load left of BC section

$S_G = -x/l \qquad 0 \le x \le l/5$

& when unit load on right of BC

$S_G = 1 - x/l$

when betn BC

$S_G = A_y - F_B$

$= 1 - \frac{x}{l} - \frac{2l/5 - x}{l/5}$

Structure . moment application at θ_A, θ_B rotation [illegible] / But usually loading at. So loading into beam

— Heinrich Manderla & Otto Mohr. Later Maney referred.

\+ Slope Deflection Method

→ Based on stiffness approach (displacement method)

→ rotation and displacement of point are unknown parameter.

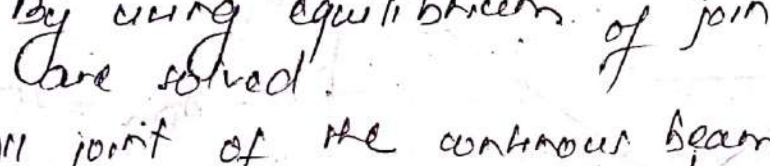
→ By solving the number of simultaneous slope displacement equations, rotation and deflection are determined.

→ By using equilibrium of joint, no. of equation are solved.

→ All joint of the continuous beams and frame are treated as rigid and fixed. All members are interfebible.

$$\boxed{M_N = 2EK(2\theta_N + \theta_F - 3\psi) + (FEM)_N}$$

General slope-deflection eqn — axial & shear neglected.

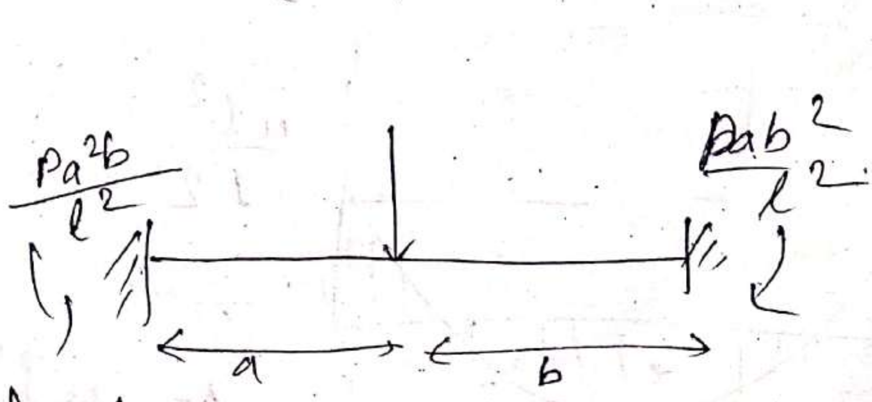


MFEM

$$M_{AB} = (M_{FEM})_{AB} + (M\theta_A)_{AB} + (M\theta_B)_{AB} + (M_\Delta)_{AB}$$

$$M_{BA} = (M_{FEM})_{BA} + (M\theta_B)_{BA} + (M\theta_A)_{BA} - (M_\Delta)_{BA}$$

$\psi = \frac{\Delta}{l}$ → settlement

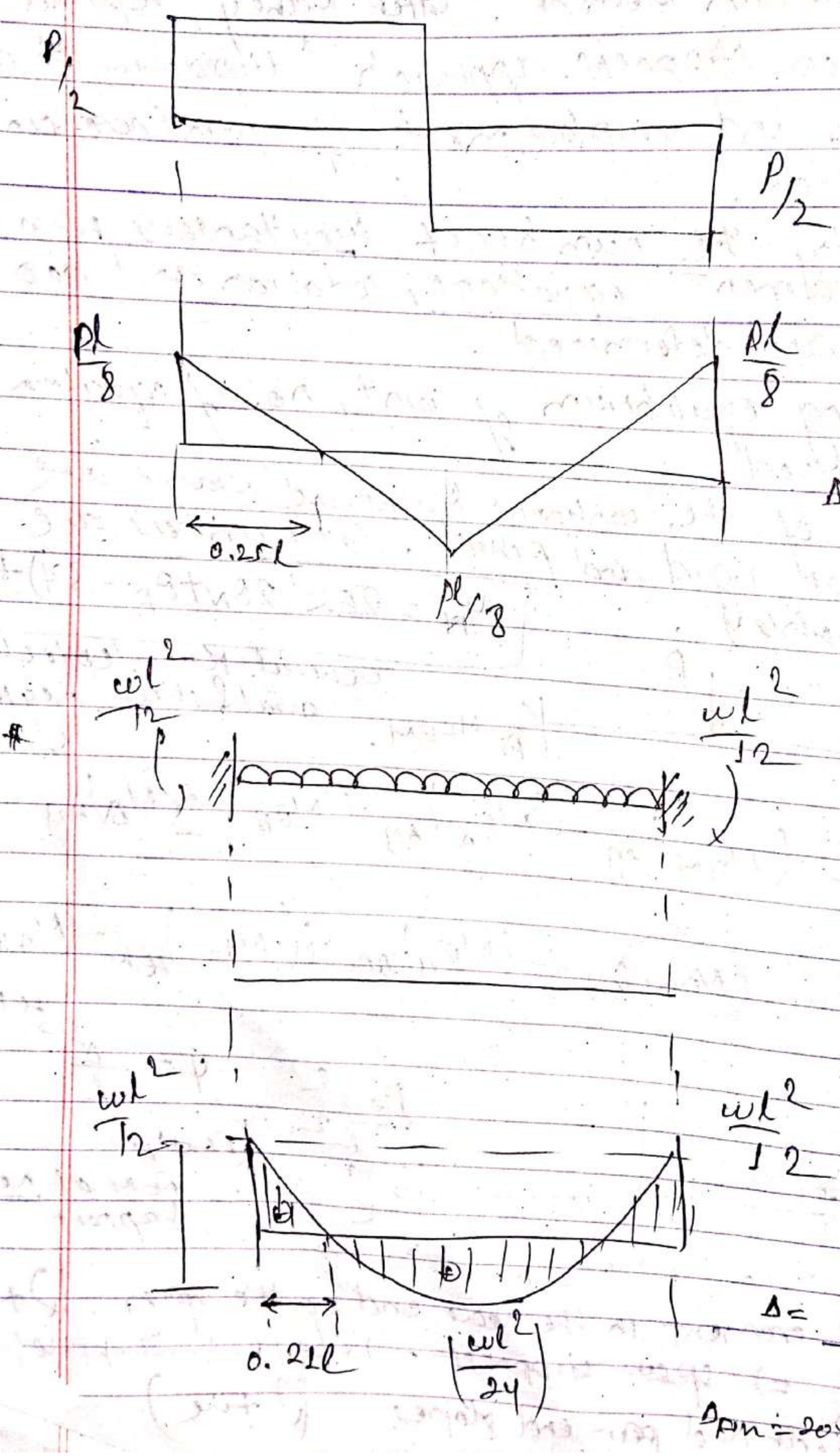


$(FEM)_N$ = FEM at near support

M_N = internal moment in the near end of the span (↻ +ve)

$K = \frac{I}{l}$ ⇒ ~~span stiffness~~ = relative stiffness ;

θ_N, θ_F ⇒ near and far end slopes (↻ +ve)

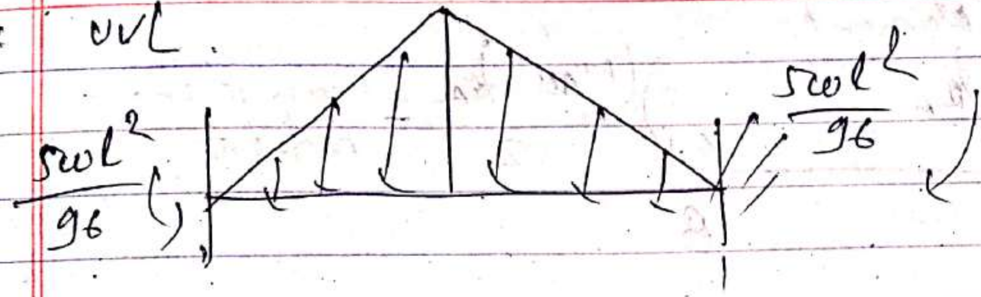


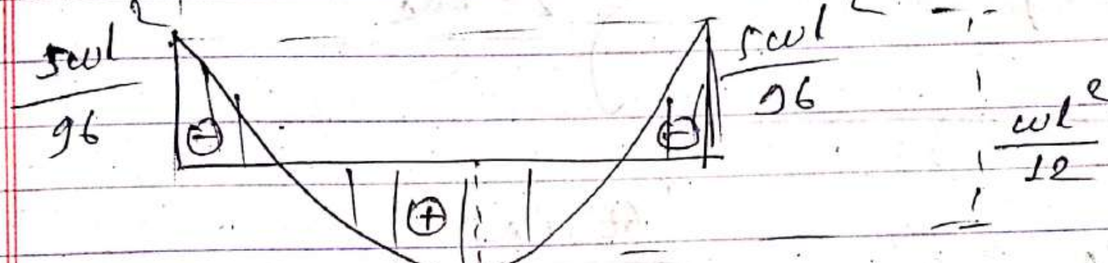
$\Delta_{An} = 25\% \, \Delta_{sm}$

$\Delta = \frac{wl^4}{384EI}$

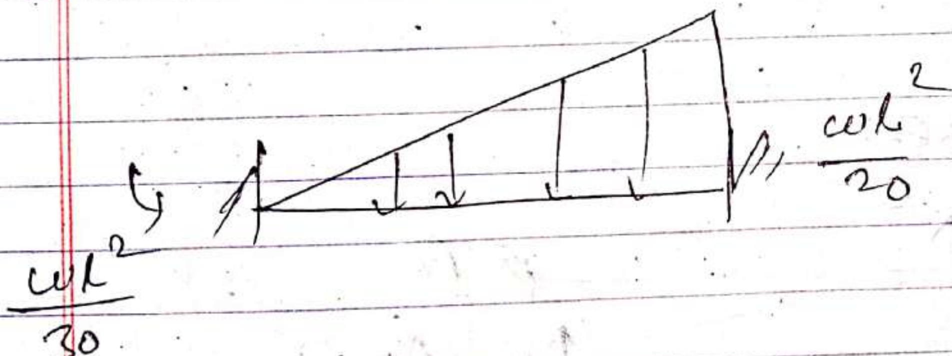
$\Delta_{An} = 20.01\% \, \Delta_{...}$

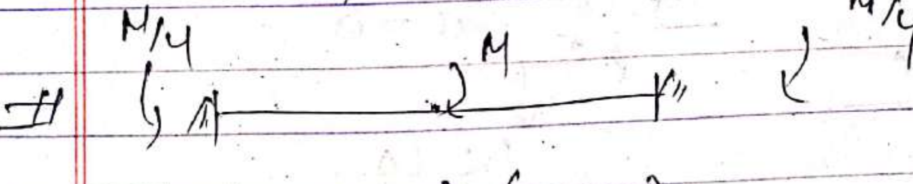
* UVL

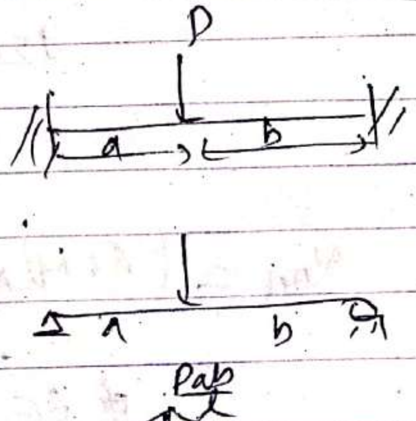




$\frac{wl^2}{32EI}$ $\Rightarrow \left(\frac{wl^2}{12} - \frac{5wl^2}{96}\right)$







Note :: $FEM_{AB} = \frac{2}{l^2}(2g_A - g_B)$

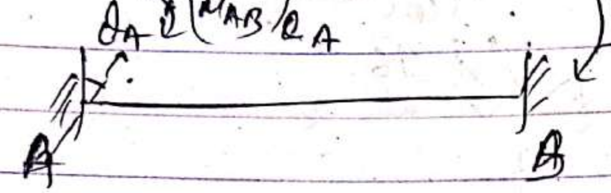
$FEM_{BA} = \frac{2}{l^2}(g_A - 2g_B)$

g_A = moment of area under BM diagram of simple beam.

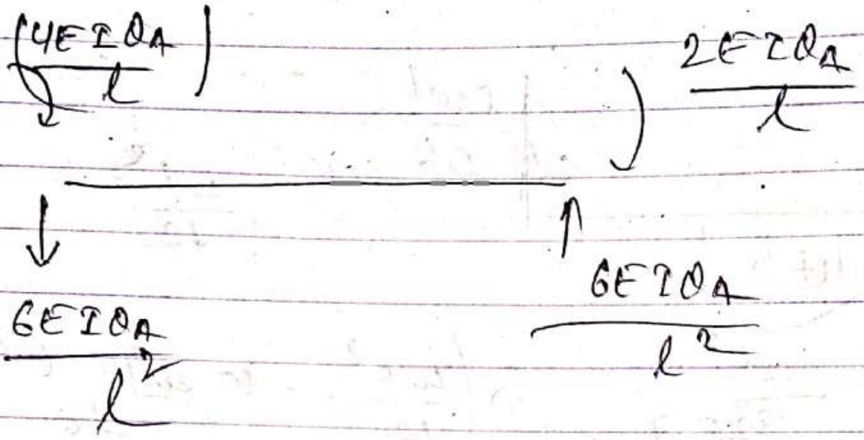
$g_A = \frac{1}{2} a \left(\frac{Pab}{l}\right) \times \frac{2}{3} a$

$g_B = \frac{1}{2} b \left(\frac{Pab}{l}\right) \times \frac{2}{3} b$

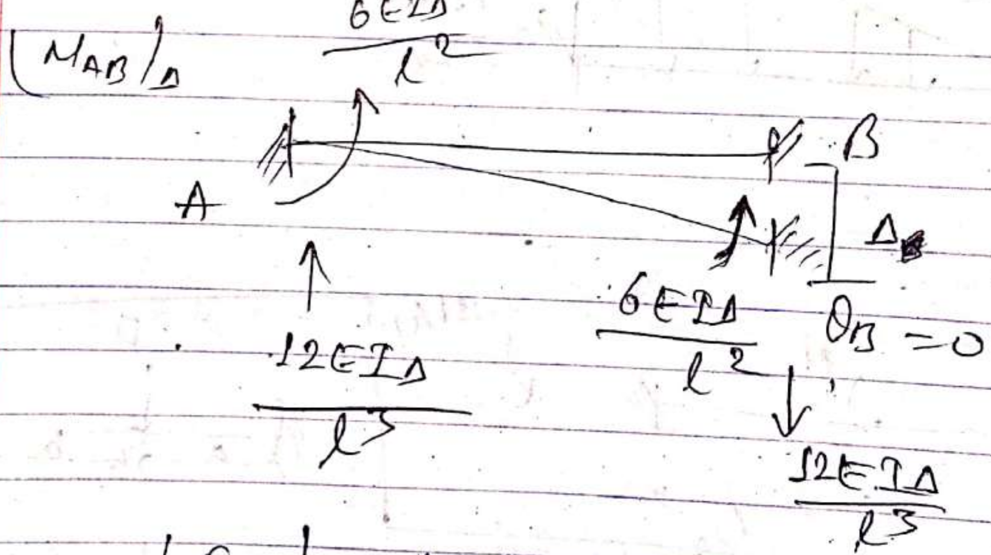
* Rotational Moment :



$\frac{(M'_{AB})_{\theta_A}}{2}$ = carryover moment



Settlement



$$M_{AB} = (FEM)_{AB} + \frac{4EI\theta_A}{l} + \frac{2EI\theta_B}{l} - \frac{6EI\Delta}{l^2}$$

$$M_{BA} = (FEM)_{BA} + \left(\frac{2EI\theta_A}{l}\right) + \frac{4EI\theta_B}{l} - \frac{6EI\Delta}{l^2}$$

In slope deflection equation variation of the joint (deformation) are caused due to

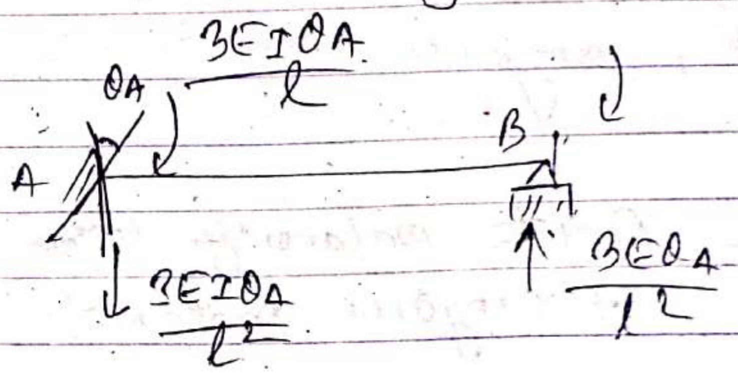
a) BM b) stress only
c) Both d) None

* Moment Distribution Method.

→ developed by Hardy-cross in 1930.
→ iterative process
→ highly popular method to solve continuous frame (high degree of static indeterminacy).
→ iterated until unbalancing moment considerably small (upto desired level of accuracy).
→ successive approx. - locking and unlocking each joint in succession.
→ Relative stiffness

⇒ far end fixed, relative stiffness = I/L

⇒ far end hinged, modified stiffness = $\frac{3}{4}K$



$M = \left(\frac{4EI}{l}\right)\theta_A$

$K = \frac{4EI}{l}$ → member stiffness factor

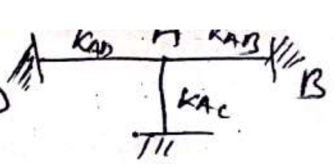
$\boxed{M = K\theta_A}$

* Carry over moment

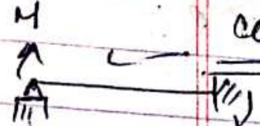
→ when far end is fixed

Joint stiffness factor $K_T = \Sigma K$

$k = \frac{4EI}{L}$ $\quad = K_{AD} + K_{AB} + K_{AC}$



=) amount of M needed to rotate Joint A by 1rad.



carryover factor = $\frac{\text{carry over moment}}{\text{end moment}}$

$M_{AB} = \left(\frac{4EI}{L}\right)\theta_A$ $\quad = \frac{M/2}{M} = +\frac{1}{2}$ → both same direction

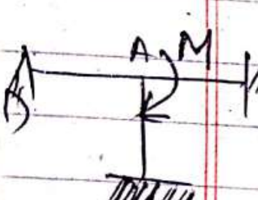
$M_{BA} = \left(\frac{2EI}{L}\right)\theta_A$

∴ $M_{BA} = \frac{1}{2} M_{AB}$ i.e. moment M at pin induces $M' = \frac{1}{2} M$ at fin end. $\frac{1}{2}$ represents fraction of M carried over to M'.

Distribution factor

~~calc from a joint~~)

$DF = \frac{K_i}{\Sigma K}$ → sum of distribution factor at a point.



moment resisting capacity depends upon respective member.

→ Applied moment M Joint A मा . कुन member ले कति resisting moment contribute गर्छ

Process

(i) FEM calculation
(ii) Relative stiffness
(iii) Unbalancing moment
(iv) Balancing moment
(v) DF calculate, distribute
(vi) Carryover factor, carryover moment

$DF_i = \frac{M_i}{M} = \frac{K_i\theta}{\theta\Sigma K_i}$

Same हो गर्छ all member

∴ $DF_i = \frac{K_i}{\Sigma K_i}$

Final moment = FEM + Balancing moment + carryover moment

* Member Relative stiffness factor :

For same material E is const. So.

$DF = \frac{K_i}{\Sigma K_i} = \frac{4E\left(\frac{I_i}{L_i}\right)}{}$ $\quad K_R = \frac{I}{L}$ relative

Internal $= 3c + \cancel{\ldots}$
$(3\times2) \cancel{\ldots} = \cancel{\ldots}\ 6$

$6\cancel{7}$ $\quad 6+2=8$

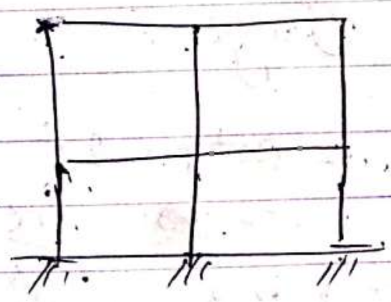
* Degree of static indeterminacy and degree of kinematic indeterminacy.

Force Method or Flexibility

→ Degree of static indeterminacy (DS)

→ Flexibility matrix
Δ → symm. square matrix

$$[\Delta]_{D_S\times 1} = [\delta]_{D_S\times D_S}[P]_{D_S\times 1}$$



$$(3m+r) - 3j = (3\times10+9) - 3\times9$$

$$= 39-27$$
$$= 12$$

$D\ [\delta]_{12\times12} =$

→ tedious. P (~~12~~ redundant)

Displacement method or stiffness method.

→ Degree of kinematic indeterminacy or Degree of freedom (DOF).

$$\{F\}_{D_K\times1} = [K]_{D_K\times D_K}\{\Delta\}_{D_K\times 1}$$

$$D_K = 3j - r - m$$
$$= 3\times9 - 9 - 10$$
$$= 27 - 19$$
$$= 8$$

⇒ good for programming.

(i) Consistent Deformation method.

(ii) Three Moment Theorem

(iii) column - Analogy method.

(i) Slope - Deflection method.

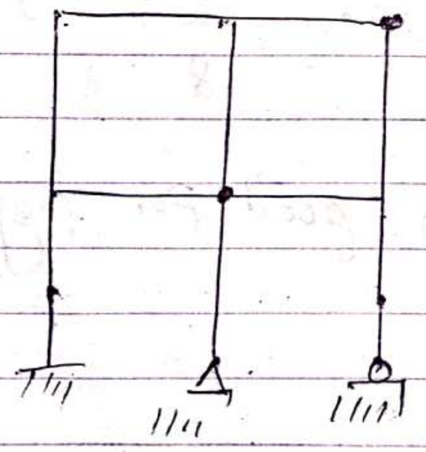
(ii) Moment - distribution method

(iii) Kani's Method

$(3m+r) - 3j - \Sigma(m'-1)$

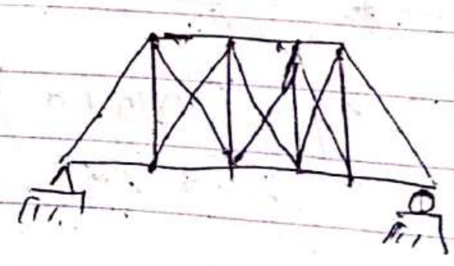
6   6 if space   2

$m' \Rightarrow$ no. of member meeting at internal hinge.



$= m+r-2j$

$= 0$ 3



$3m+r-3j-2$

$3\times10+6-3\times9-2$

$= 36-27-2$

$= 7$

$= 3\times10+6-3\times9-(3+1)$

$= 36-27-4 = 5$ #

$= 0$

$3m+r-3j$

$= 2\times16+3-3\times10$

$= 32+3-30$

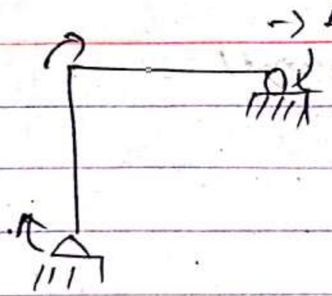
$= 5$

#.

Truss ⇒ members are extensible.

Frame ⇒ inextensible.

Static.



$3m + r - 3j$

$= 3\times2+3 - 3\times3$

$= 0$

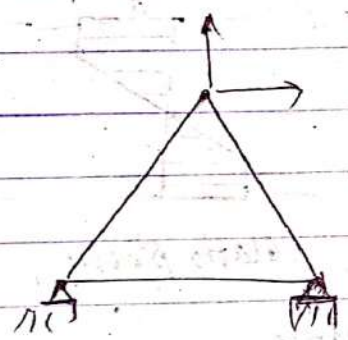
$D_K = \cancel{3}\ 4$

$= 3j - r - m$

$= 4$

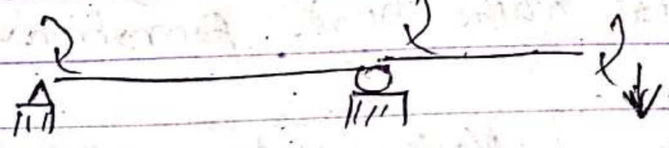
Fixed support

no. of compatibility eq^n = 3



$D_K = 2j - r = 2\times3 - \cancel{8}\ 4$

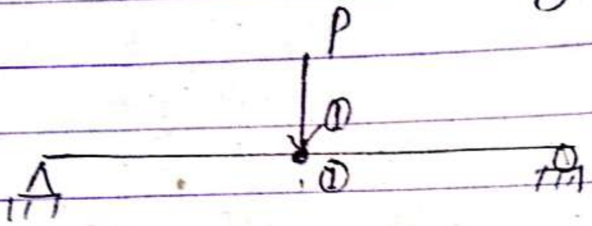
$= \cancel{4}\ \cancel{20}\ 2$

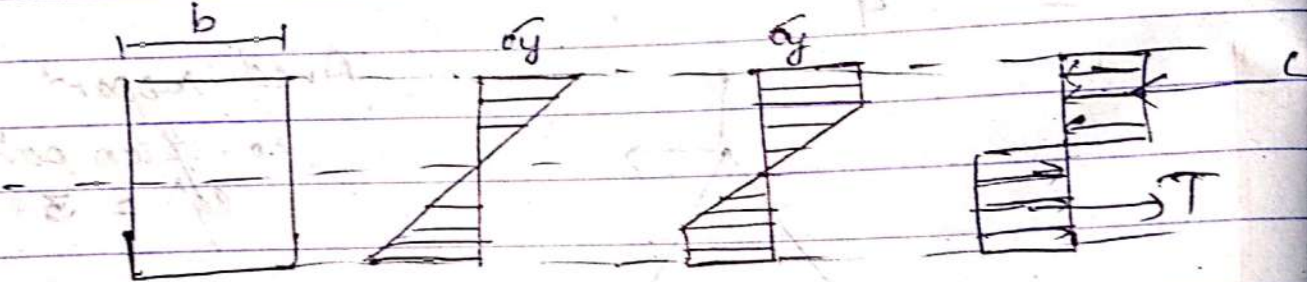


$\Rightarrow 4$ #.

Plastic Analysis of Beam and Frames.

Plastic Moment capacity.





Section ① - ①   elasto-plastic.   pure plastic

(i) when $P < P_{collapse}$

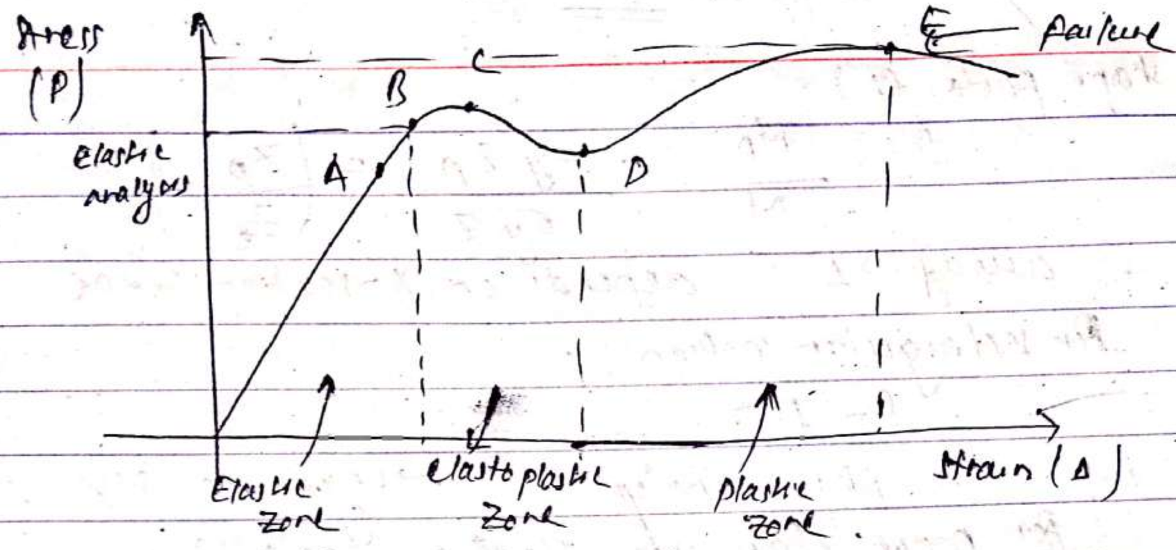
pure elastic stress diagram.

Pure plastic:

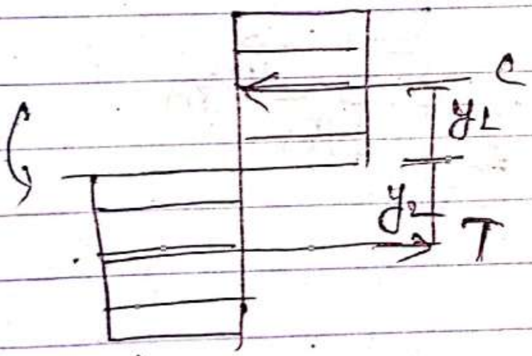
All fibre at a section undergoes yielding. It is in this state that plastic hinge formation starts

Equal area axis , plastic section modulus

Z_p



Plastic section modulus (Z_p)



rectangular section.

$$Z_p = \frac{A}{2}\left[y_1 + y_2\right]$$

$$= \frac{bd}{2}\left[\frac{d}{4} + \frac{d}{4}\right]$$

$$= \frac{bd}{2} \cdot \frac{d}{2}$$

$$= \frac{bd^2}{4} \quad \#$$

obtained by taking static moment of compression area about NA.

Shape factor (S):

$$S = \frac{M_p}{M} = \frac{\sigma_y Z_p}{\sigma_y Z} = \left(\frac{Z_p}{Z_e}\right)$$

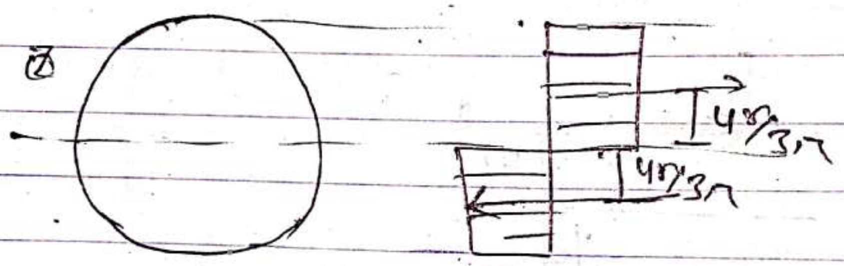
- always > 1 - depends on X-section shape

For rectangular section,

$S = 1.5$

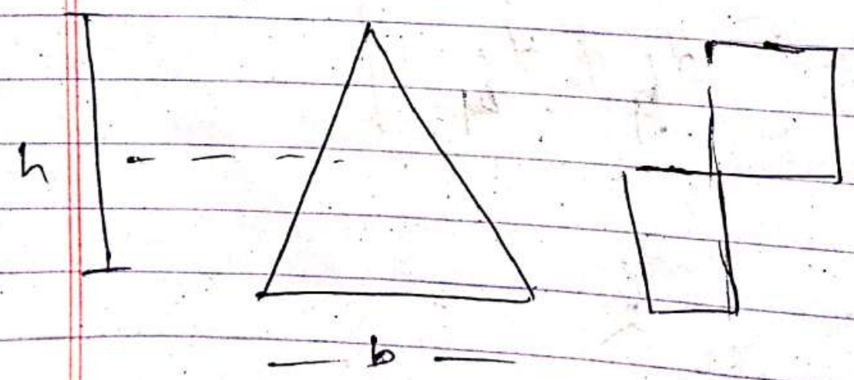
i.e. in plastic analysis the strength is 50% more than in elastic analysis.

b) Circular section:



$S = 1.69 \simeq 1.7$

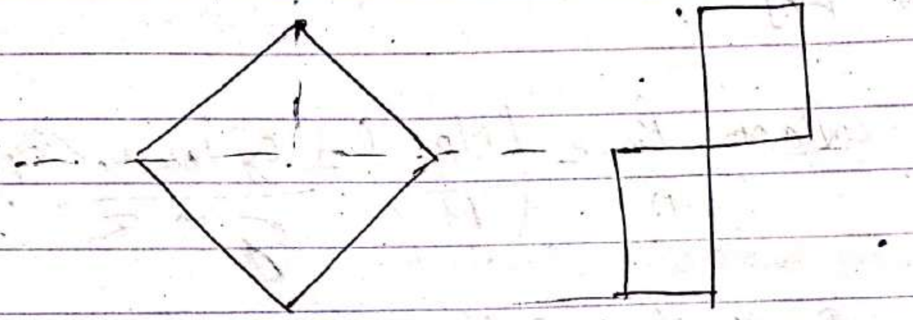
c) Triangular section:



$S = 2.34$

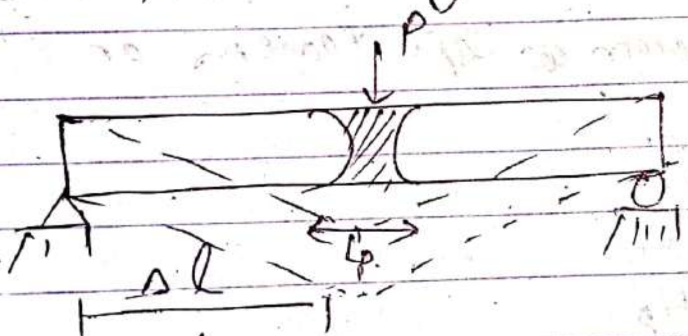
Rolled steel ~~and~~ I-section $\Rightarrow 1.14 - 1.18$.

Diamond shape.

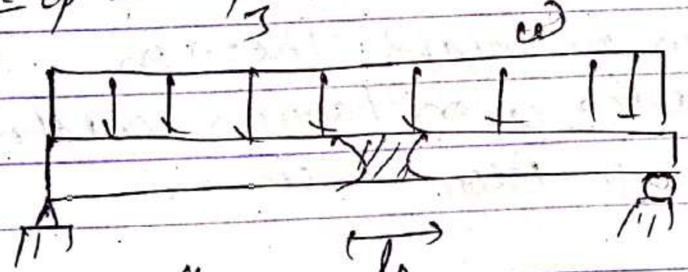


$S = 2$

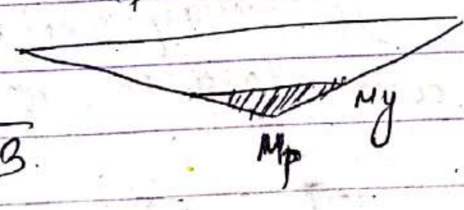
Length of plastic hinge. $\Rightarrow$ in statically indet. structures more than one plastic hinge necessary to develop failure mechanism.



$\Delta l < L_p = \frac{L}{3}$



$l_p = \frac{l}{\sqrt{3}}$



Load Factor (λ):

It is the ratio of collapse load (P_u) to service load (P)

$$\lambda = \frac{P_u}{P}.$$

→ depends on shape as depends on Load Evaluation.

Relationship between shape factor, load factor and FoS :

$$\lambda = \frac{\text{collapse } P_y}{P} = \left(\frac{M_p}{M}\right) = \frac{(\sigma_y)_{plastic} \times Z_p}{\sigma_y \times Z}$$

$=$ FoS × shape factor.

correct mechanism is one which results in the lowest possible load for which M doesn't exceed M_p.

Theorem to calculate collapse Load.

Lower Bound Theorem or Safe theorem or static method :

$P \leq P_y$

$\lambda \leq \lambda_y$

$M \leq M_p$

Upper Bound Theorem or Unsafe Theorem or kinematic Method or mechanism method.

$P \geq P_y$

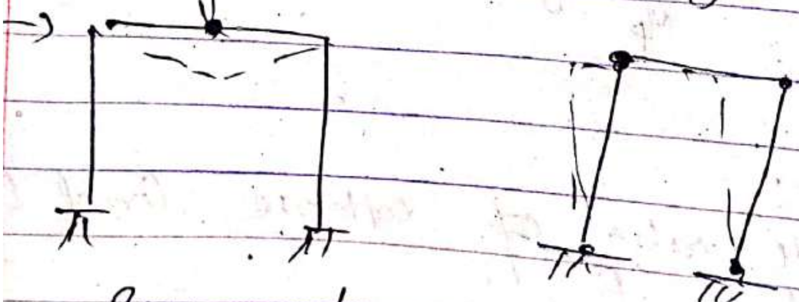
$\lambda \geq \lambda_y$

$M \geq M_y$

Beam mechanism

Sway Mechanism.

→ for structure with large no. of redundants



Beam mechanism.

Sway mechanism

collapse determined from following values.

m = no. of possible hinges $n+1$ = total no. of hinges req. for total collapse.

n = no. of redundancy

N = no. of independent mechanism

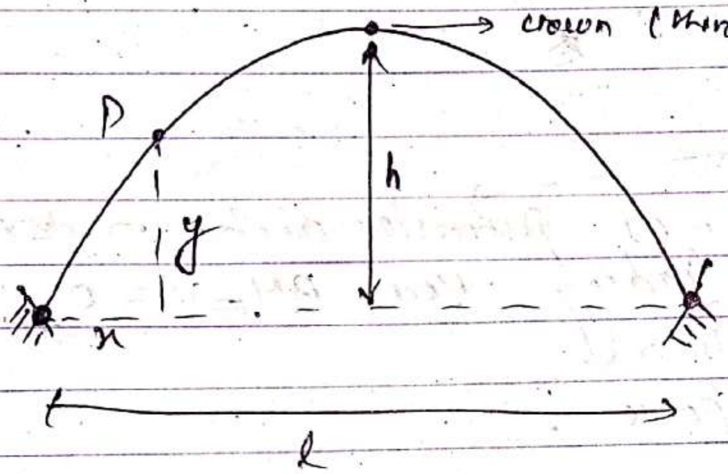
$N = m - n$

* Three hinged arches:

→ statically determinate arch

→ two hinges at support and third any where (generally at crown)

→



$$y = \frac{4hx}{l^2}(l - x)$$

→ For an arch spanning the same length as of a beam the moment induced is much less.

So, $M_{beam} > M_{arch}$.

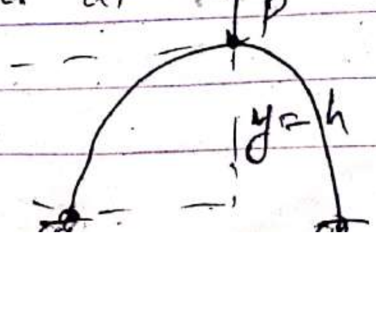
→ arch are economical for long span and heavy loading

$$\boxed{BM_{arch} \text{ at } P = M_{beam \text{ at } P} - H * y}$$

arching effect.

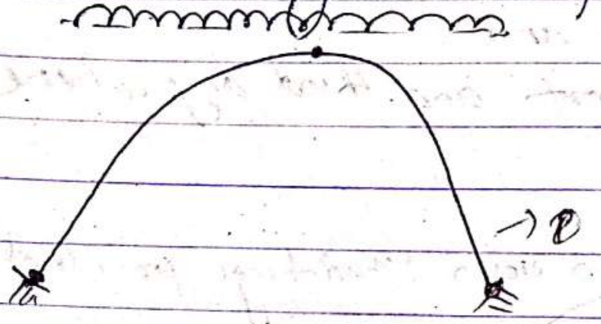
* calc^n of Horizontal Thrust (H)

① Point load at crown.



$$\boxed{H = \frac{M_{beam}}{h}} = \frac{Pl}{4h}$$

(ii) udl load throughout the span :

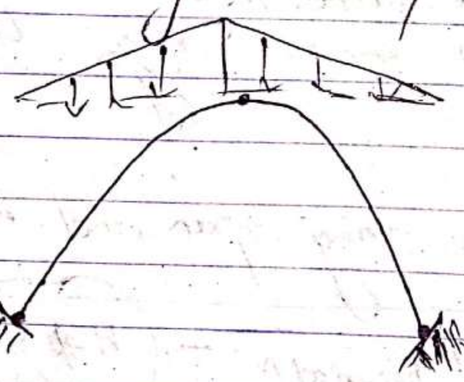


Eq (parabolic)

The arch is known as ~~parabolic~~ circular arch in ~~the~~ this state of loading. Here, BM = V = 0 at every section.

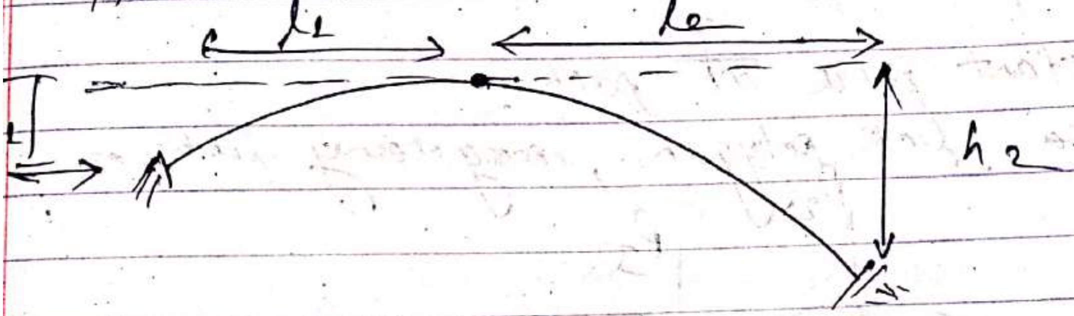
→ only compressive force.

(iii). UVL throughout the span.



$H = wl^2/12h$

Support at different level.



$$l_1 = \frac{l\sqrt{h_1}}{(\sqrt{h_1}+\sqrt{h_2})}$$

$$l_2 = l - l_1$$

$$= \frac{l\sqrt{h_2}}{\sqrt{h_1}+\sqrt{h_2}}$$

(i) Point load at crown.

$$H = \frac{PL}{(\sqrt{h_1}+\sqrt{h_2})^2}$$

(ii). uniform load

$$H = \frac{wl^2}{2(\sqrt{h_1}+\sqrt{h_2})^2}$$

Linear arch:

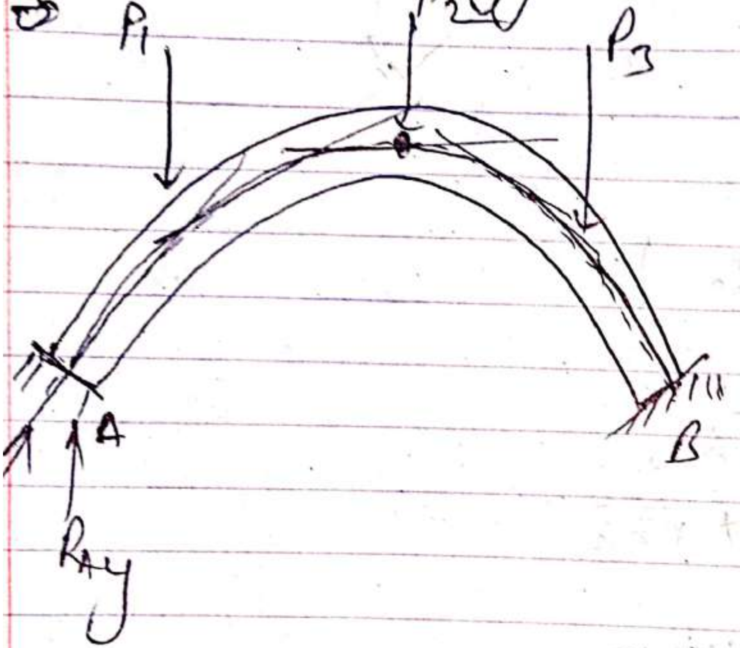
→ arch axis coincides with line of thrust

→ BM = 0 and SF = 0

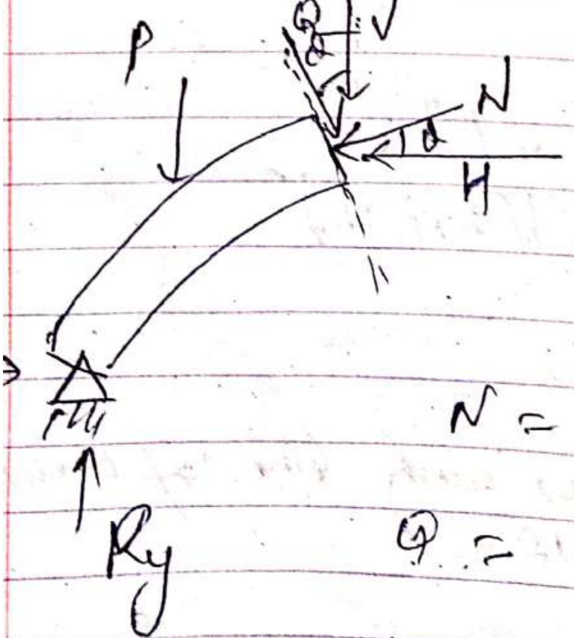
* Line of thrust :

→ Resultant force at path.

→ aka link polygon, imaginary linkage.



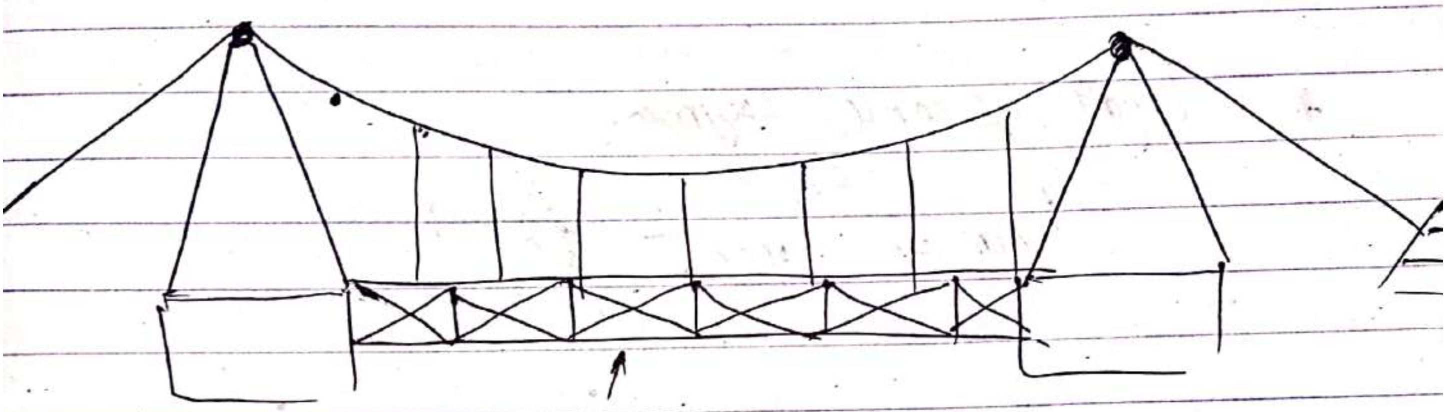
Radial shear (Q) and normal thrust (N)



$$N = H\cos\theta + V\sin\theta$$

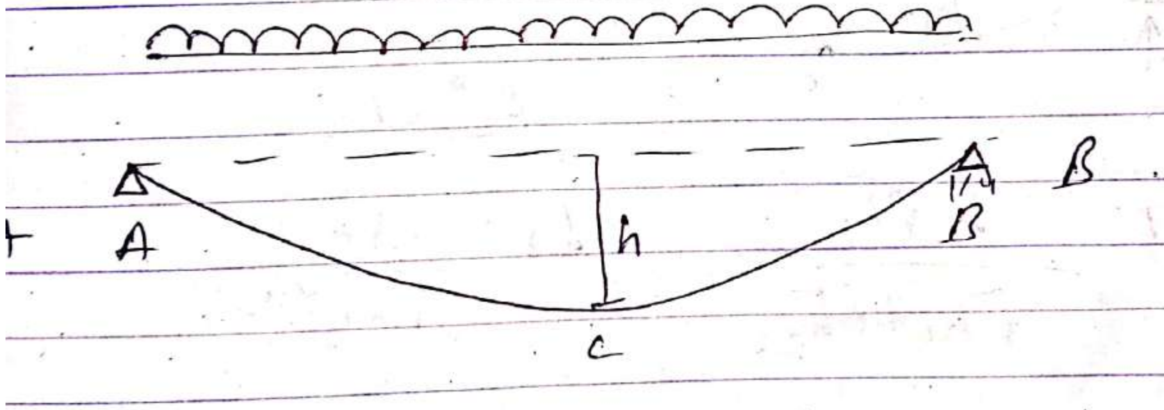
$$Q = V\cos\theta - H\sin\theta$$

suspension cable system:



stiffening girder.

ideally in cable only tension
self-weight (=) cable shape catenary curve
udl (=) parabolic shape



maximum tension at highest point.

$T_{max} = \sqrt{H^2 + R_{Ay}^2} = \sqrt{H^2 + R_{By}^2}$

For udl

$$H = \frac{wl^2}{8h}$$

Minimum tension at lowest point.

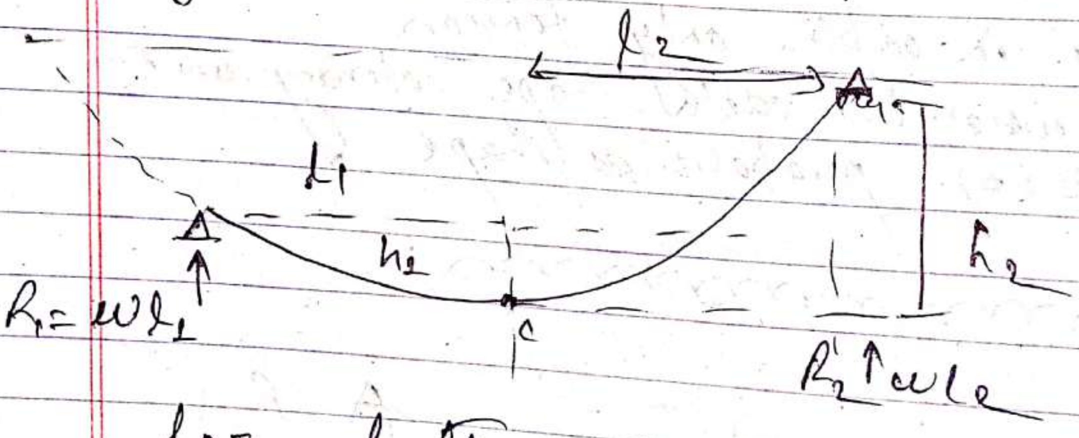
$T_{min} = H$

&

* Length of cable (symm.)

$$l_{ACB} = l_{span} + \frac{8}{3}\left(\frac{h^2}{l}\right)$$

* Unsymmetric arch.



$$l_1 = \frac{l\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}} \qquad l_2 = \frac{l\sqrt{h_2}}{\sqrt{h_1}+\sqrt{h_2}}$$

$$H = \frac{wl^2}{2(\sqrt{h_1}+\sqrt{h_2})^2}$$

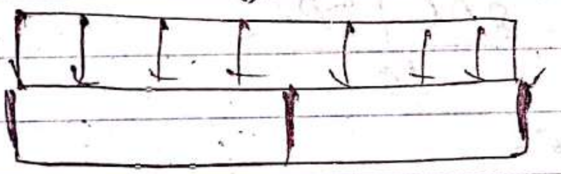
$$L_{total} = L_{span} + \frac{2}{3}\frac{h_1^2}{L_1} + \frac{2}{3}\frac{h_2^2}{L_2}$$

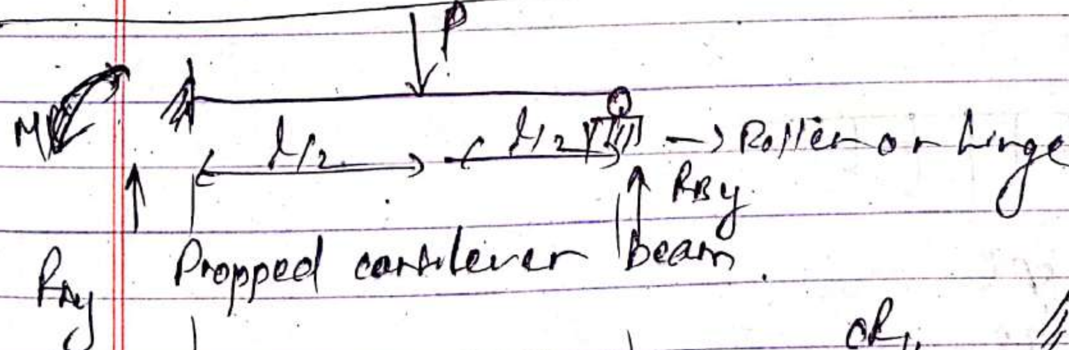
* Stiffening girder:

→ to even out the loading ~~load~~ (into udl)
prevent excessive sag (localized)

→ designed for shear force and BM.

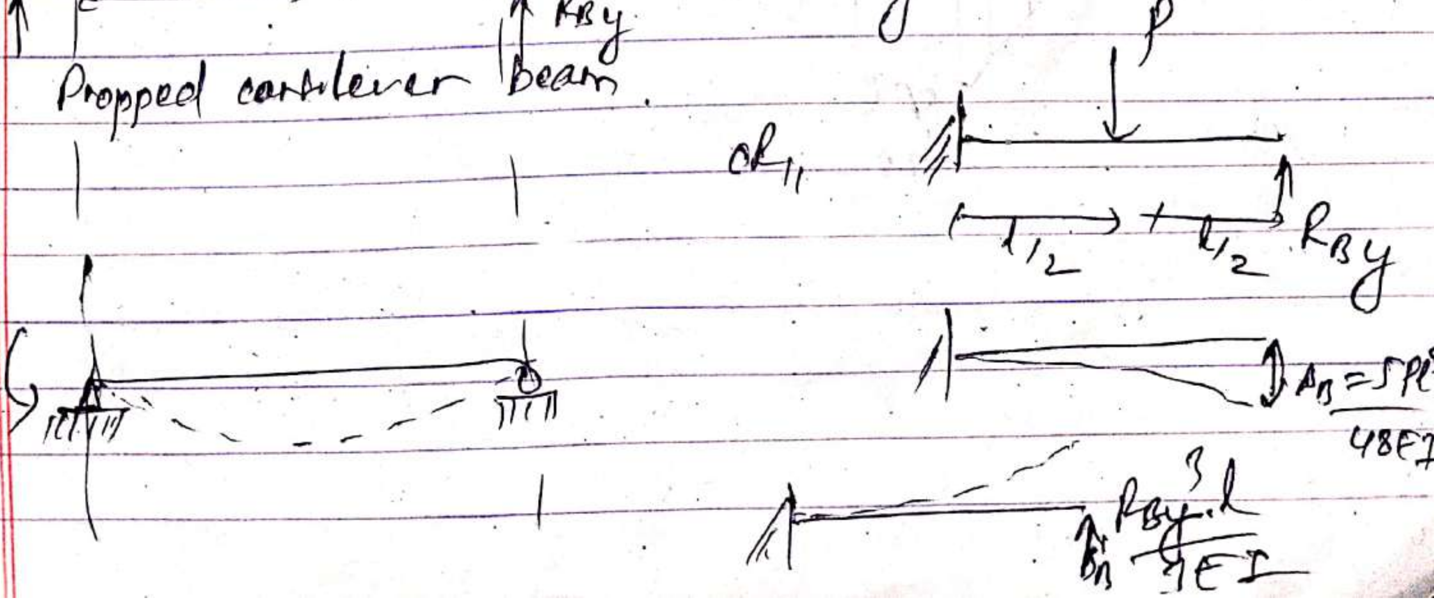
→ ~~dea~~ for stiffening girder ~~load~~ DL is not taken in design. only for LL (~~carton~~ vehicle and pedestrian.





Propped cantilever beam.

$D_1 = L$



$$\frac{5Pl^{\cancel{3}}}{\cancel{48EI}\,16} = \frac{R_{By}\,\cancel{l^3}\,8}{\cancel{3EI}}$$

$$R_{By} = \cancel{40P}\ \frac{5P}{16}$$

$$R_{Ay} = P - \frac{5P}{16} = \frac{11P}{16}$$

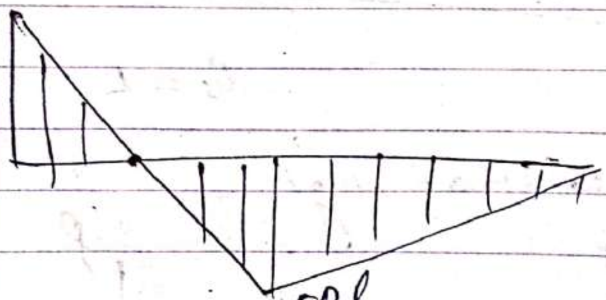
Max. shear in case of point is max^{m} reaction.

$$M_A = -\frac{Pl}{2} + \frac{5P}{16} \times l$$

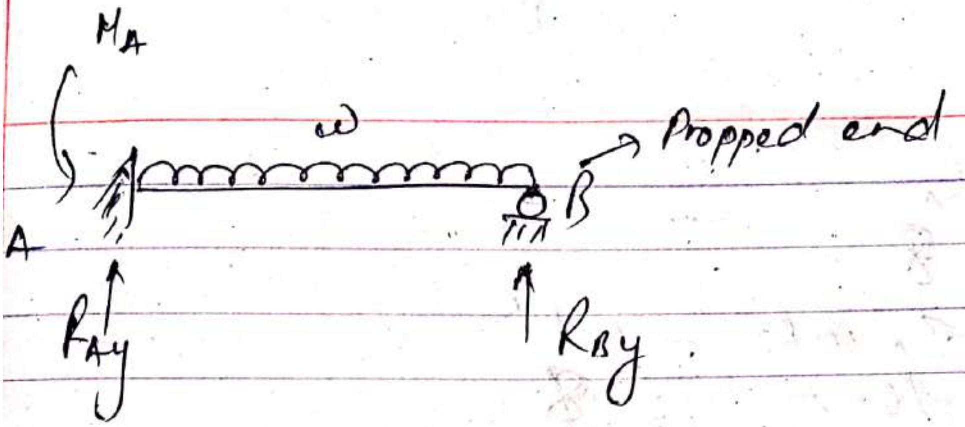
$$= \frac{-8Pl + 5Pl}{16}$$

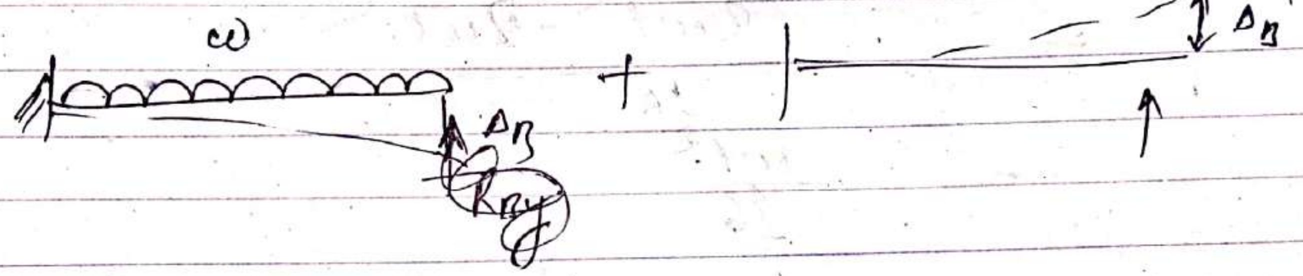
$$= -\frac{3Pl}{16}$$

$\frac{3Pl}{16}$



$\frac{5Pl}{32}$





$$\Delta_B = \frac{wl^4}{8EI} \qquad \Delta_B' = \frac{R_{By}\, l^3}{3EI}$$

$$\text{or,} \quad R_{By} = \frac{3wl}{8}$$

$$R_{Ay} = wl - \frac{3}{8}wl$$

$$= \frac{5wl}{8}$$

$$V_{max} = \frac{5wl}{8}$$

$$(M_{max})_A = \frac{5}{8}w . \frac{3}{8}wl \times l - \frac{wl^2}{2}$$

$$= \frac{3}{8}wl^2 - \frac{wl^2}{2}$$

$$= \frac{3wl^2 - 4wl^2}{8} = -\frac{wl^2}{8}$$

Midspan:

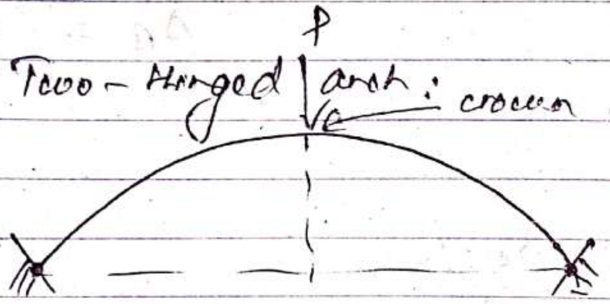
$\frac{3}{8} wl \times \frac{l}{2} - \frac{wl^2}{8}$

$= \frac{3}{16} wl^2 - \frac{wl^2}{8}$

$= \frac{3wl^2 - 2wl^2}{16}$

$= \frac{wl^2}{16}$

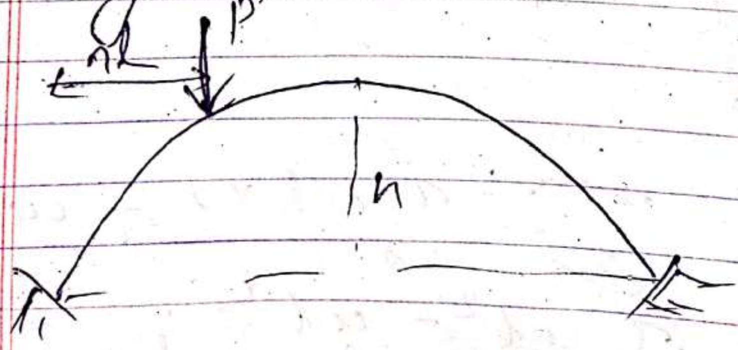
* Two-Hinged arch: crown.



→ statically indeterminate structure.

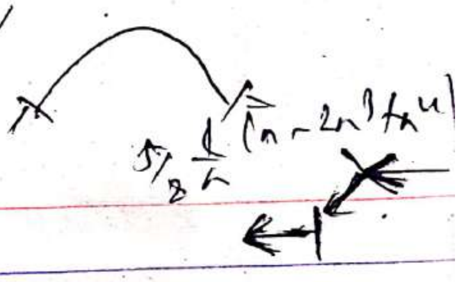
For point load (P) at crown.
same as for three hinged.

Only different if P is unsymm. load.



$\left(n - 2n^3 + n^4\right)$

$\frac{5}{8}\frac{l}{h}\left(n - 2n^3 + n^4\right)$ $\quad$ $\frac{5}{8}\frac{l}{h}\left(n - 2n^3 + n^4\right)$



$$H = \frac{5}{8}\,\frac{l}{h}\left(n - 2n^3 + n^4\right)$$

In 2-hinged arch an increase in temperature induced

no bending moment = 0 in arch

uniform BM in arch

Maxm BM at crown

Minm BM at crown

$\frac{5}{8}\frac{l}{h}\left(n - 2n^3 + n^4\right)$

$\frac{l}{h}\left(n - 2n^3 + n^4\right)$ $\quad$ $\frac{5}{8}\frac{l}{h}\left(n - 2n^3 + n^4\right)$ $\quad$ $\frac{5}{8}\frac{l}{h}\left(n - 2n^3 + n^4\right)$

$\frac{5}{8}\frac{l}{h}\left(n - 2n^3 + n^4\right)$ $\quad$ $\frac{5}{8}\frac{l}{h}\left(n - 2n^3 + n^4\right)$

$\frac{5}{8}\frac{l}{h}\left(n - 2n^3 + n^4\right)$