Hydraulics:

(1) **Fluid Pressure:** *

1 Bar = 10^5 N/m^2

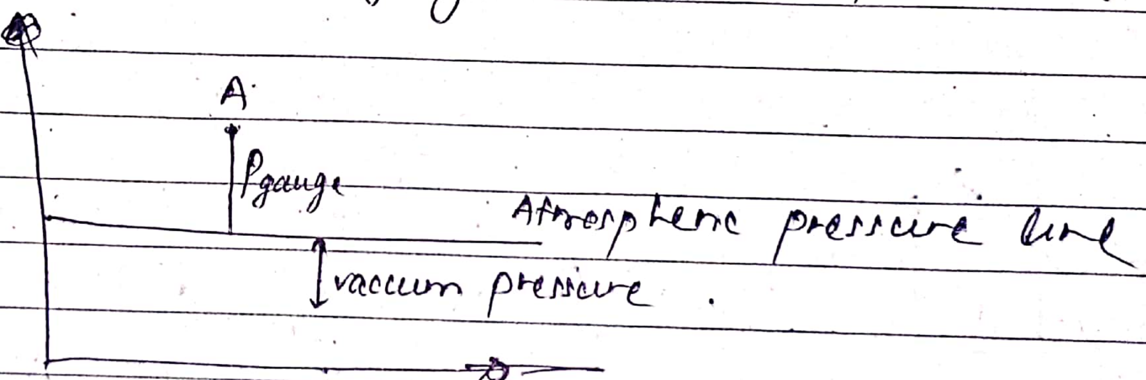
Pascal's Law:

Pressure exerted by a static fluid at a point is equal in all direction.

Hydrostatic Law:

$$\frac{dP}{dz} = \rho g$$

Atmospheric, Absolute, gauge and vaccum pressure:



absolute zero (vaccum pressure) line.

(i) Gauge pressure:

datum is taken as atmospheric pressure
pressure above atmospheric pressure line.

(ii). vaccum pressure:

→ negative pressure
→ below atmospheric pressure line.
above vacum pressure line.

Absolute pressure = atm. pressure + gauge pressure.

$\sigma = \frac{My}{I}$ $\frac{M}{2} =$

*. **Measurement of pressure:**

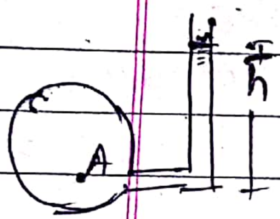
→ manometer ⇒ the device used for measuring pressure at a point of fluid by balancing column of ~~fluid~~ fluid by same or different column of fluid.

Two types

a) simple manometer   b). Differential manometer.

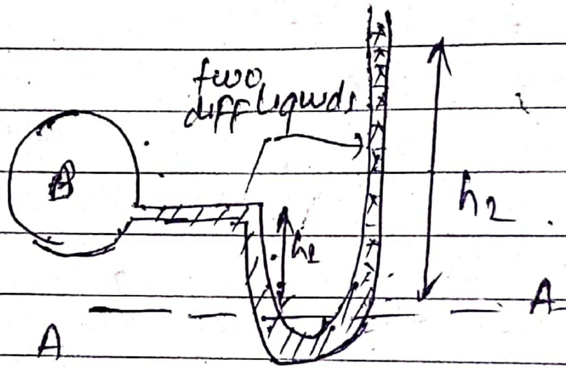
a) **simple manometer:**

(i) Piezometer



$P_A = \rho g h$

(ii). U-tube manometer



- fluid of higher density like mercury is placed beneath the fluid whose pressure is to be measured at point B. The other end is left atmosphere.

$P_{A\,left} = P_B + \rho_1 g h_1$

$P_{A\,right} = \rho_2 g h_2$

(iii) Single column Manometer

→ used to measure pressure in large reservoirs

→ change in liquid level is minimal

→ similar principle as that of ~~yautube~~ U-tube manometer.

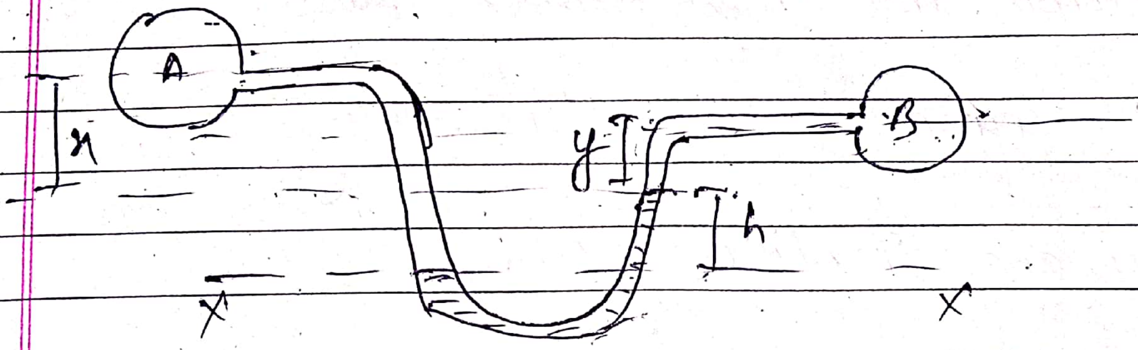
$P_{left} = P_{right}$

$P_B + S_1 g h_1 = S_2 g h_2$

$P_B = g(S_2 h_2 - S_1 h_1)$.

(2) Differential manometer:

→ pressure difference between two points.



$S_2 g h$.

→ consists a heavy fluid whose density is greater than that of the fluid whose pressure is to be measured.

$(P_X)_{left} = P_A + S_1 g(x+h)$

$(P_X)_{right} = P_B + S_2 g(y+h)$

$P_A + S_1 g x + \cancel{S_1 g h} = P_B + S_2 g y + \cancel{S_2 g h}$

$S_1 = S_2$

$P_A - P_B = S_2 g(y - x)$

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* Fluid Kinematics :

→ The study of fluids motion without considering the forces and energy involved.

(i) Ideal fluid
– no friction loss

(ii). Real fluid
– friction loss.

Compressible and Incompressible flow :

↓ variable density

↓ constant density.

Laminar and Turbulent flow :

↓ different layers do not intermix

↓ different layers intermix.

Steady flow
flow conditions like velocity, density, area w.r.t. time at a particular section is same. but can be different at different sections.

$$\frac{dv_A}{dt} = 0$$

Uniform flow
→ flow " "
w.r.t. space at a particular time is same but can be different at different times.

$$\frac{dv}{dx} = 0$$

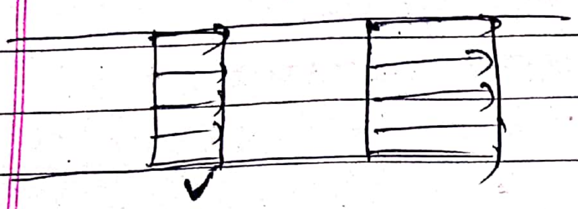
CamScanner

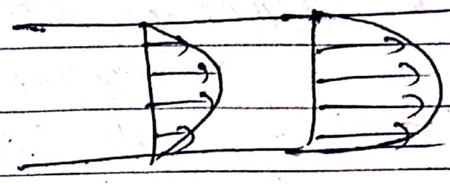
1-D flow

Flow condition at a given instance of time varies only in one direction and not in the cross-section area

2-D flow

Flow conditions vary in direction of flow and in right angles along depth.

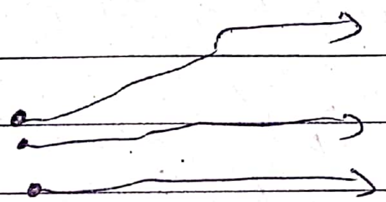




* Path lines:

→ locus of a single particle.
trace made by single particle over a period of time

time varies



Stream line:

→ trace made by particles at a particular instant of time.

Properties

→ stream lines do not cross each other

→ are smooth curves

→ velocity of particle at a point is given by tangent to the streamline

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Streakline:

— Locus of fluid particles that have passed sequentially through a prescribed point of flow.

Dynamics of fluid flow:

→ Study of fluid in motion considering forces and energy.

Forces in a particle

(i) gravity force
(ii). viscous force
(iii). pressure force
(iv). turbulence force
(v). compressibility force.

$$F = (F)_g + (F)_p + (F)_v + (F)_t + (F)_c$$

Reynold's equation of motion
compressible force $(F_c) = 0$

$$F_n = F_g + F_p + F_v + F_t$$

Navier-Stokes Equation:

$F_t = 0$ $\quad F_c = 0$

$$F_n = F_g + F_p + F_v$$

*. Euler's Equation of motion :

$F_v = 0$, $F_t = 0$, $F_c = 0$.

$$\boxed{F_n = F_g + F_p}$$

$$\frac{\partial p}{\rho} + g\,dz + v\,dv = 0 \Rightarrow \text{Euler's equation of motion}$$

$$\int \frac{\partial p}{\rho} + \int g\,dz + \int v\,dv = \text{constant}.$$

If flow is incompressible ρ = constant.

$$\frac{p}{\rho} + gz + \frac{v^2}{2} = \text{constant}.$$

Assumptions of Bernoulli equation :

(i) Fluid is ideal

(ii). Fluid is incompressible (iii) flow is steady and irrotational. (1-D)

* Applications of Bernoulli's Equation :

(i) venturimeter

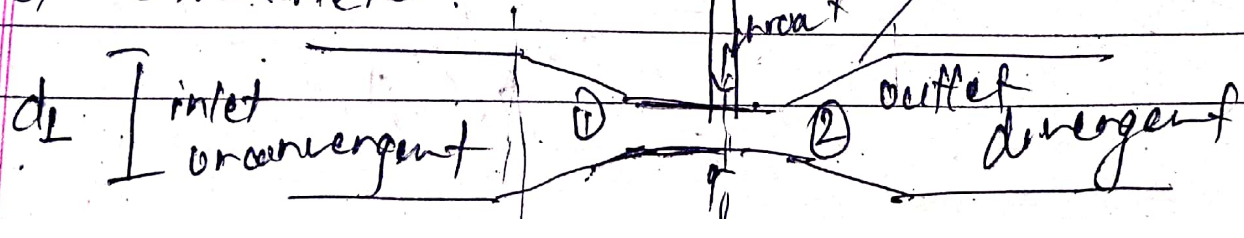
(ii) Orifice meter

(iii). Pitot tube

6-7° ⇒ small angle of divergence s.t. no flow separation

discharge measuring meter.

(i) venturimeter:



Bernoulli's Equation and its application

$$\frac{P_1}{\gamma} + \frac{v_1^2}{2g} + \cancel{z_1} = \frac{P_2}{\gamma} + \frac{v_2^2}{2g} + \cancel{z_2}$$

P.D. $$\frac{v_1^2 - v_2^2}{2g} = \frac{P_2 - P_1}{\gamma}$$

$$\cancel{(v_1^2 - v_2^2) = \frac{P_2 - P_1}{\rho g} \times 2g}$$

$$\frac{P_2 - P_1}{\gamma} = \frac{v_1^2 - v_2^2}{2g}$$

pressure. $$h = \left(\frac{v_1^2 - v_2^2}{2g}\right) \quad \text{--- (1)}$$

#ve

Continuity Equation

$$A_1 v_1 = A_2 v_2$$

$$v_1 = \frac{A_2 v_2}{A_1}$$

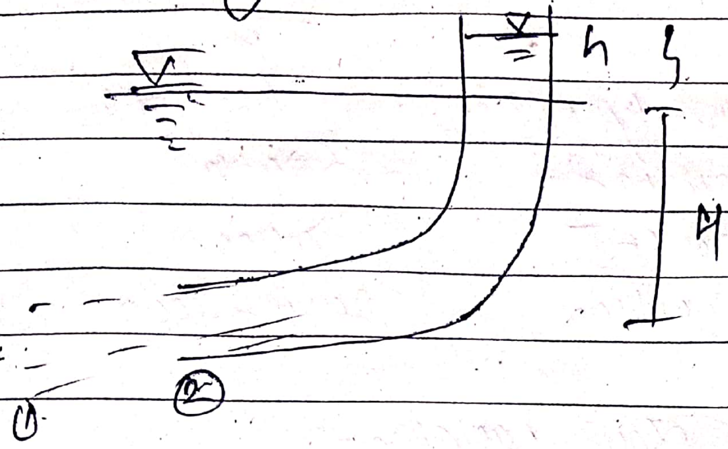
subs. in (1)

$$v_2 = \frac{A_1}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gh}$$

$$Q = a_2 v_2 = \frac{a_1 a_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gh}.$$

Pitot's tube:

→ a device for measuring velocity of flow at any point.



velocity को head में convert करता।

→ principle: if velocity of flow at a point becomes zero pressure increases due to conversion of kinetic energy into potential energy.

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

$$\frac{P_1}{\rho g} = H \qquad \frac{P_2}{\rho g} = H+h$$

$v_2 = 0$

So, $\cancel{H} + \frac{v_1^2}{2g} + 0 = \cancel{H} + h + \cancel{0} \; 0 + 0$

$$v_1 = \sqrt{2gh}$$

Actually $v_1 = C_v\sqrt{2gh}$

Laminar and Turbulent flow in pipes

Reynold's number = $\frac{\text{Inertial force}}{\text{viscous force}}$

$= \frac{\rho V D}{\mu}$

Re flow type (pipe)	Re (pipe)	open channel
Laminar	< 2000	< 400
Turbulent	> 4000	> 800
Transition	2000 - 4000	400 - 800

Uniform flow equation

$\tau = \rho g R S_0$

τ = shear stress (boundary) S_0 = bed slope

Momentum equation

$F = \frac{d(mv)}{dt}$

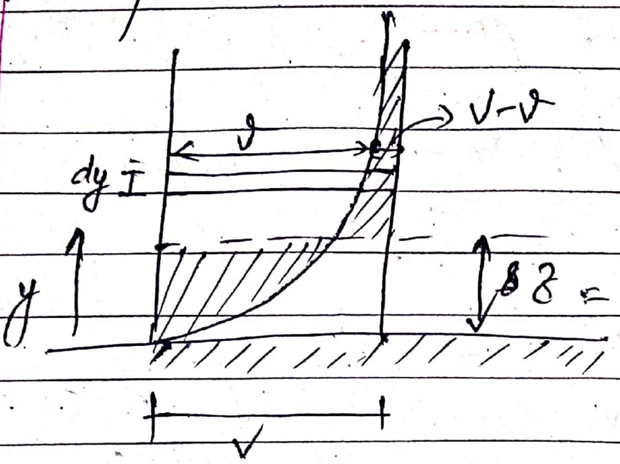
Boundary layer:

When a real fluid flows past a solid boundary a layer of fluid which comes in contact with boundary surface adheres / sticks to it on account of viscosity i.e. the velocity of fluid at boundary is zero. At certain extent of region the fluid undergoes retardation

The region in which velocity gradually increases from zero to mainstream velocity is called boundary layer.

Quantify using 3 perspectives

(1) Displacement thickness



datum माथि सार
datum माथि ideal fluid र real fluid.

The distance by which boundary surface has to be displaced outwards so that total discharge (actual) would be same as that of ideal fluid past displaced boundary

deficit of discharge through an element

$$= (V-v)\,dy$$

deficit of discharge through whole boundary layer

$$= \int_0^{\infty} (V-v)\,dy$$

Equivalent discharge would pass through a layer of depth 'δ'

$$V\delta = \int_0^{\infty} (V-v)\,dy$$

$$\delta = \int_0^\infty \left(1 - \frac{v}{V}\right) dy$$

(ii). momentum thickness:

mass flow through an element $= \rho v \, dy$

deficit of momentum flux $= \rho v \, dy (V - v)$

$$\rho V^2 \theta = \int_0^\infty \rho v (V - v) \, dy$$

$$\theta = \int_0^\infty v/V \left(1 - v/V\right) dy$$

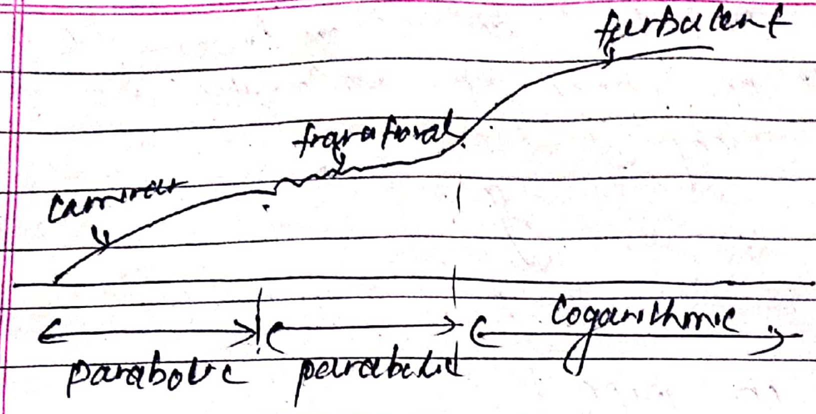
(iii). **Energy thickness:**

$$\text{deficit} = \frac{1}{2} \rho \int_0^\infty (V^2 - v^2) . v \, dy$$

Let δ be energy thickness.

$$\frac{1}{2} \rho V^3 \delta = \frac{1}{2} \rho \int_0^\infty (V^2 - v^2) v \, dy$$

$$\delta = \int_0^\infty \frac{v}{V} \left(1 - \frac{v^2}{V^2}\right) dy$$



Laminar flow :

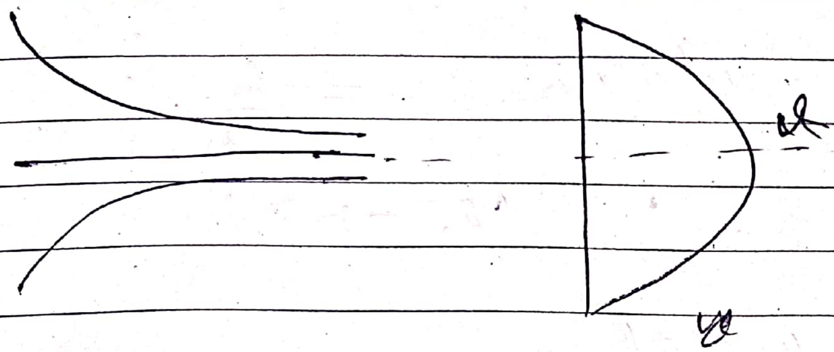
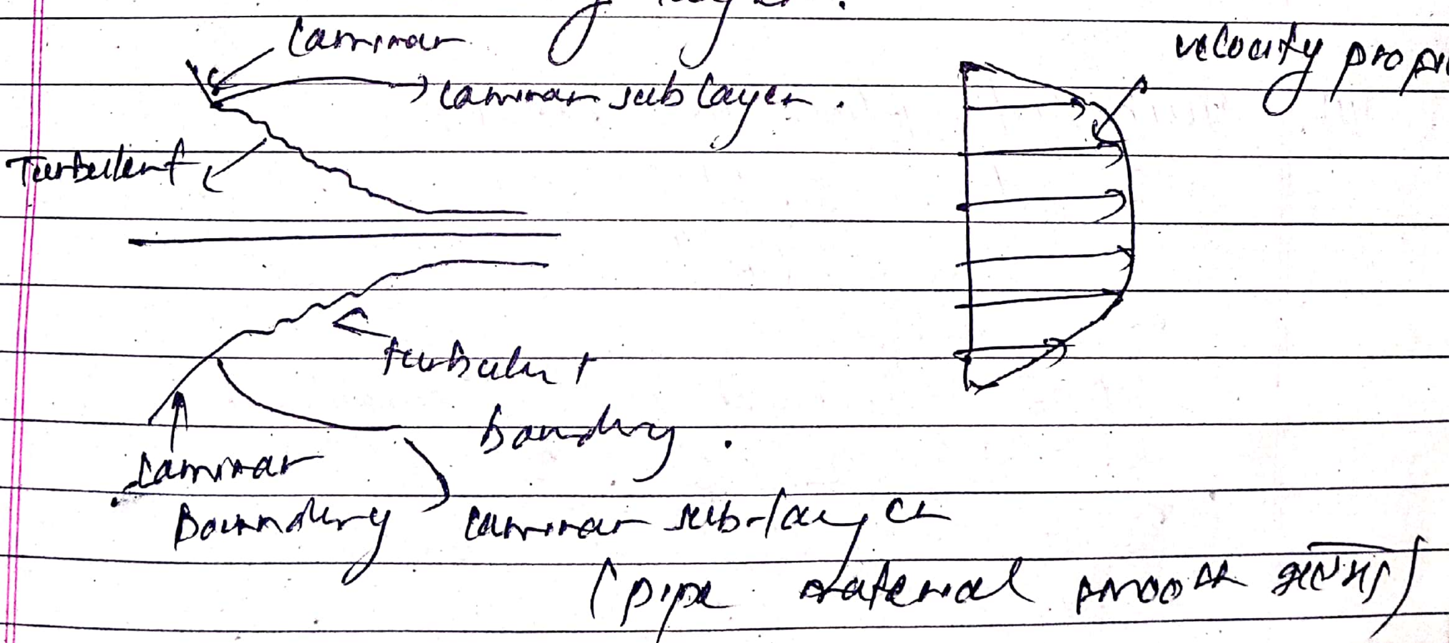
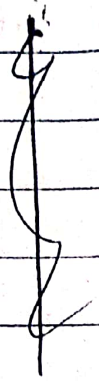


Fig: Laminar Boundary layer.

Turbulent boundary layer.





For turbulent Boundary layer the rate of change of velocity is high i.e. steep $\frac{dv}{dy} = d$

Laminar flow in pipe:

$$\tau = -\frac{\partial p}{\partial x} \cdot \frac{r}{2}$$

velocity distribution $v = \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)(R^2 - r^2)$

$$r = 0.707R$$

$f = \frac{64}{Re}$ $\quad$ $f \Rightarrow$ Darcy's friction factor.

Turbulent flow in pipes:

$$f = \frac{0.316}{Re^{1/4}}$$

$$\tau = \tau_{viscosity} + \tau_{turbulence}$$

$$\tau = \rho l^2 \left(\frac{dv}{dy}\right)^2$$

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$l \Rightarrow$ mixing length $\Rightarrow$ distance required for full development of turbulent flow.

$$\frac{v}{V} = 5.75 \log_{10}\left(\frac{V \cdot y}{\nu}\right) + 5.5$$

* Gradually varied flow (GVF): ok

→ steady non-uniform flow in which depth of flow varies gradually.

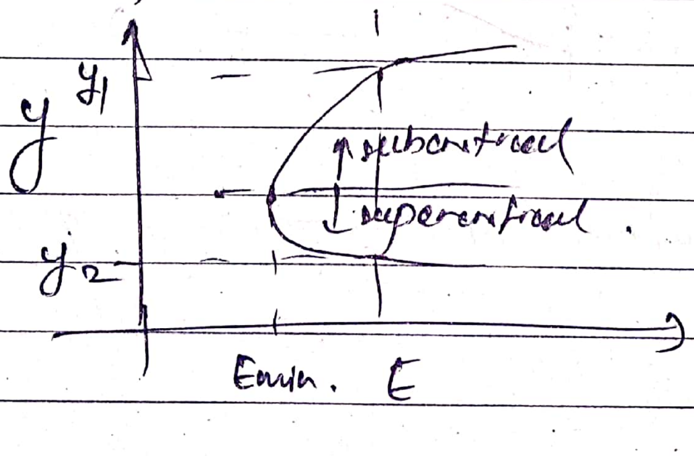
$$Fr^2 = \frac{Q^2 T}{g A^3} \qquad T = \text{surface width.}$$

Open channel flow: V_{max} near the top not at surface

* Specific energy:

→ ~~total energy~~ $\quad E = y + v^2/2g$.

Critical depth.



→ Depth at which specific energy is minimum.

alternate depth :

for a particular specific energy two depths are possible for called alternate depths.

(i) Supercritical ~~depth~~ flow:

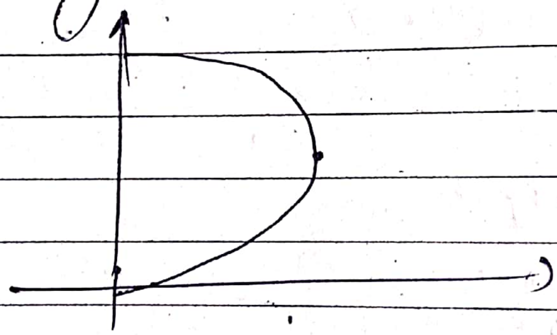
$F_r > 1 \quad y < y_c$ (velocity high)

ii). Sub-critical flow

$F_r < 1 \quad y > y_c$ (velocity low)

For a given specific energy curve. Q is max. when $y = y_c$. ~~Vc~~

(?)



General equation of Gradually varied flow,

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - F_r^2}$$

$S_0 \Rightarrow$ bed slope

$S_f \Rightarrow$ slope of energy line.

y_n = normal depth ⇒ depth of flow when channel bottom slope equal to ~~HGL~~.

* Slopes:

(i) critical slope: The channel bottom slope is said critical when, $S_0 = S_c$

S_c ⇒ critical slope ⇒ when ~~normal~~

$$y_n = y_c.$$

(ii) ~~So~~ mild slope: $S_0 < S_c$

$$y_n > y_c.$$

(iii) steep slope:

$$S_0 > S_c$$

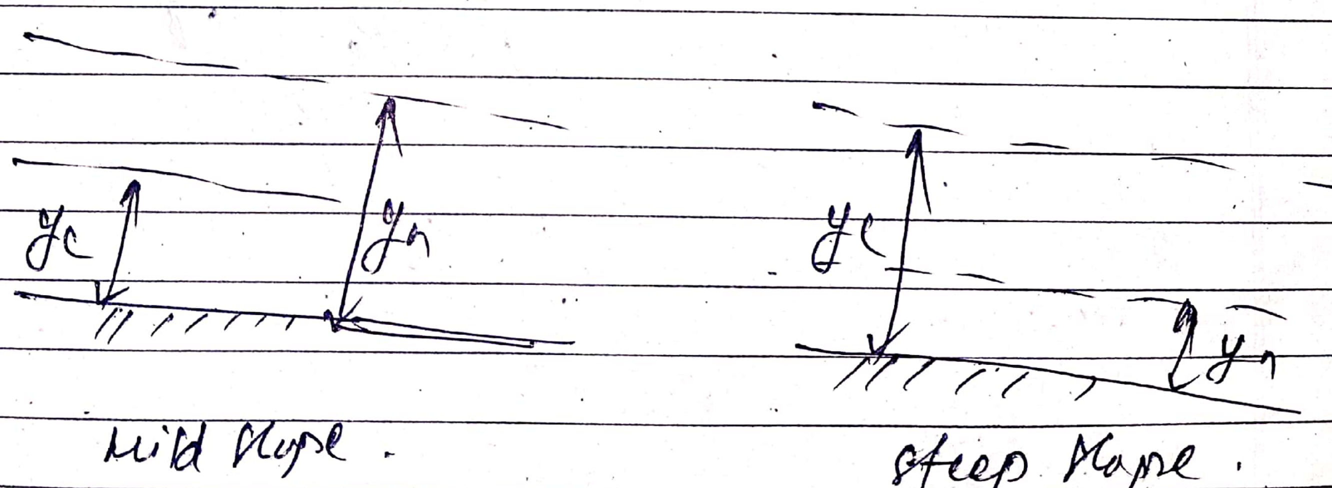
$$y_n < y_c$$

(iv). Horizontal slope.

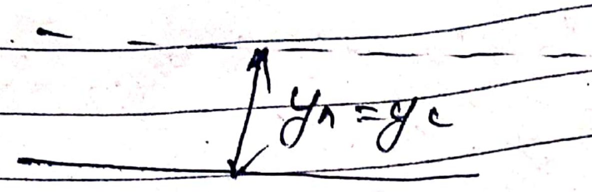
$S_0 = 0$, $y_n = \infty$

(v). Adverse slope

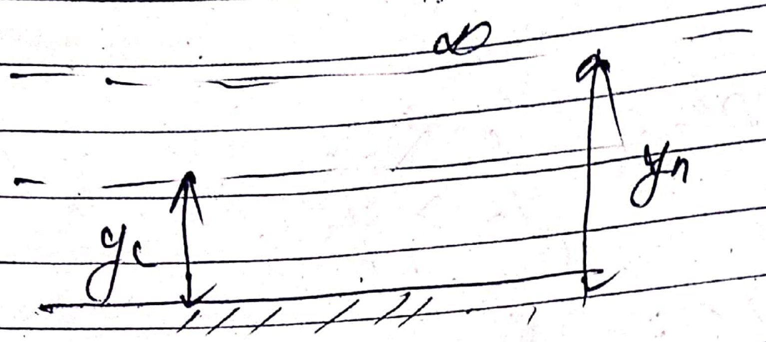
$S_0 < 0$, $y_n \rightarrow$ imaginary.



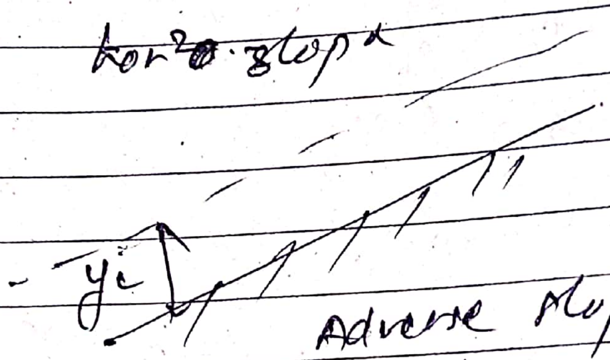
$1-Fr^2$ $\frac{dy}{dx}$



critical slope.



horiz. slope



Adverse slope
Yn imaginary.

$$\frac{dy}{dx} = S_0 \frac{1-\left(\frac{y_n}{y}\right)^3}{1-\left(\frac{y_c}{y}\right)^3}$$

* Imp

<1

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1-Fr^2}$$

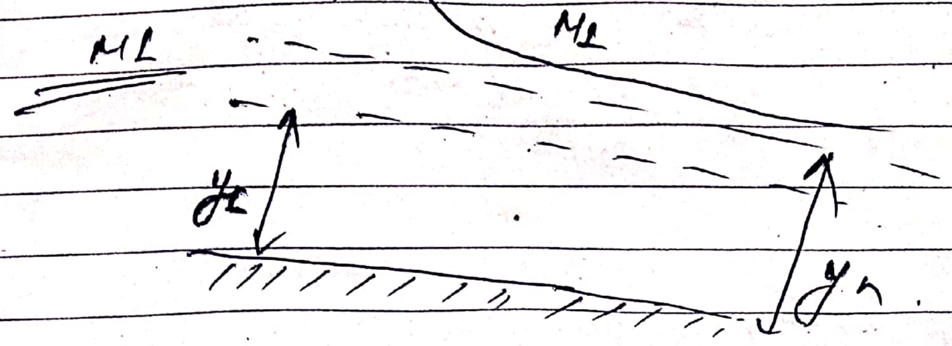
$$dy/dx = \frac{S_0 - S_f}{1-Fr^2}$$

Surface Profile classification.

Flow profiles :

(1) For mild slope

M1

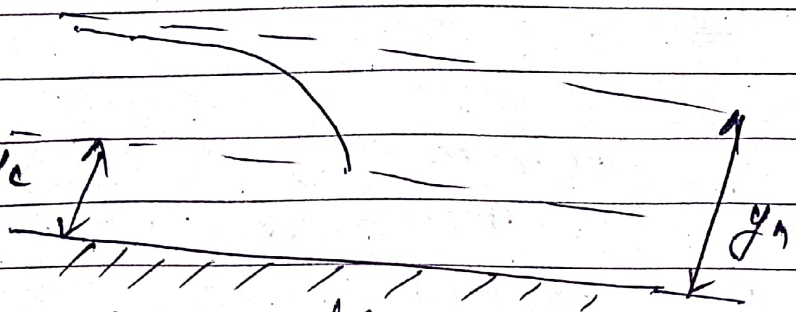


$y > y_n > y_c$

$y \to y_n \quad dy/dx \to 0$

$y \to \infty \;, \quad dy/dx \to S_0$

M2.

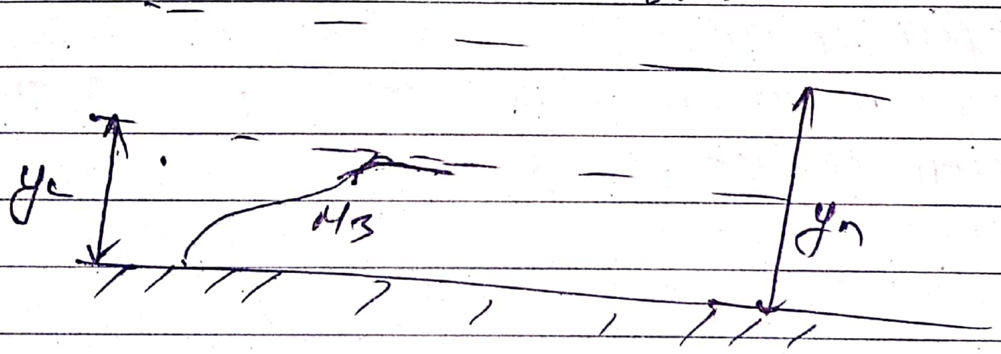


$y_n > y > y_c$

$y \to y_n \quad dy/dx \to 0$

$y \to y_c \quad dy/dx \to -\infty$

M3.



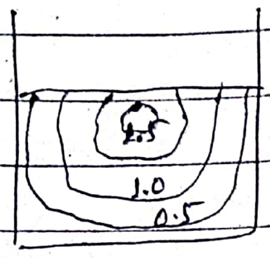
$y \to y_n \;, \quad dy/dx \to 0$

$y \to \infty \;, \quad dy/dx \to S_0$

$$\frac{dy}{dx} = S_0 \frac{\cdots}{1-(y_c/y_0)}$$

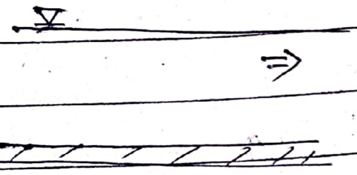
GVF concepts

→ open channel flow



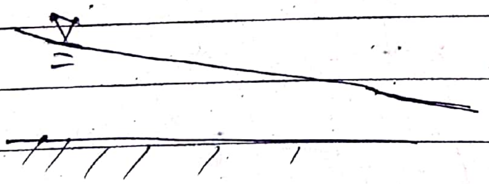
→ mostly turbulent
V_{max} near top but not at surface

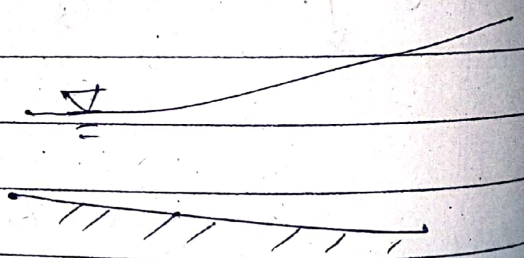
Uniform and steady flow
→ depth doesn't vary with distance ⇒ uniform flow i.e. V constant.


uniform flow

Eg: Bed slope mild (so) uniform for stone s.t. gravity force (accelerates) balanced by friction force (~~retards~~)

non-uniform flow
→ occurs when change in slope ~~and~~ or cross-section.





Accelerated non-uniform flow

(Eg:

Retarded non-uniform flow.

(Eg: dam s/l)

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Steady: ~~d~~ flow at a particular section doesn't vary with time. i.e. depth of a section remains constant with time.

unsteady: - - -

$c = \sqrt{gy}$ $\quad Fr = \sqrt{\dfrac{\text{Inertia force}}{\text{Gravitational force}}} = \dfrac{V}{\sqrt{gy}}$

$Fr = 1 \Rightarrow$ critical

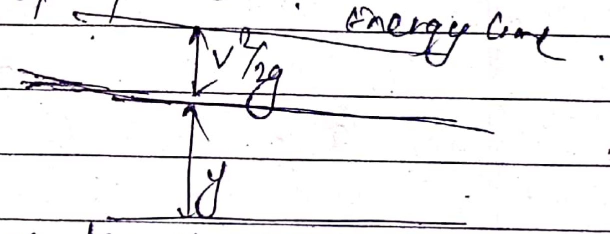
$Fr > 1 \Rightarrow$ super-critical / ~~tranquil~~ rapid flow

$Fr < 1 \Rightarrow$ sub-critical / ~~rapid~~ tranquil flow

(Rember the wave propagation video).

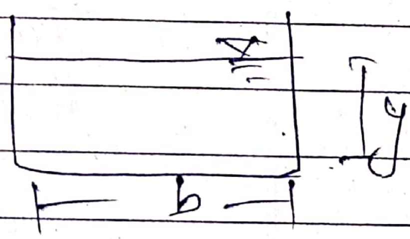
Specific energy = sum of KE and PE per unit weight of fluid.

$$E = \frac{V^2}{2g} + y$$



aka specific head as unit is length. Datum.

$$E = \frac{Q^2}{2gA^2} + y$$

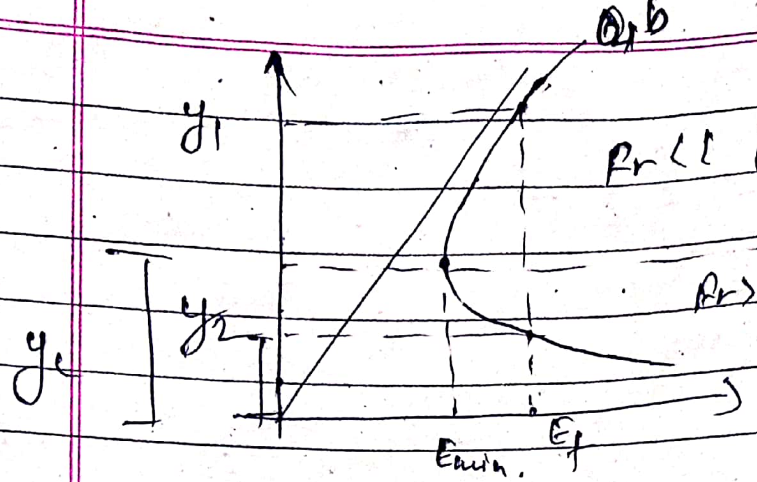


for rectangular section $A = by$

$$E = \frac{Q^2}{2gb^2y^2} + y$$

If Q is const E

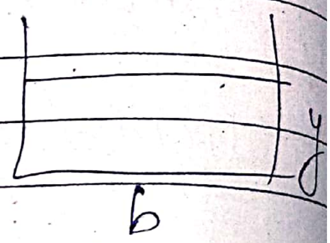
If $Q = 0$ $\quad E = y$ (PE only)



$\frac{dE}{dy} = 0$ $\quad y_c = \frac{2}{3} E_c$

Best Hydraulic section

$V = \frac{1}{n} R^{2/3} S_0^{1/2}$



$Q/A = \frac{A^{2/3} S_0^{1/2}}{n P^{2/3}}$

$Q = \frac{A^{5/3} S_0^{1/2}}{n P^{2/3}}$

Q is max for minm P

$\frac{dP}{dy} = 0$ for a constant Area

for rectangle

$A = by$ $\quad P = b + 2y$

$= \frac{A}{y} + 2y$

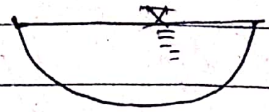
$\frac{dP}{dy} = -A/y^2 + 2 = 0$

$A/y^2 = 2$

$2y^2 = by$ $\boxed{y = b/2}$

lining cost also saved + Q also high.

Ideally semi-circular flowing full is best.
But not practical



Best hydraulic section.

* Critical slope:

c/d slope with which for a particular cross-section the flow depth becomes critical.

$S_0 \to S_c$ $y \to y_c$

$S_0 < S_c$ => ~~supercritical~~ tranquil / sub-critical ~~(rapid)~~

$S_0 = S_c$ => critical f

$S_0 > S_c$ => rapid / supercritical.

* Manning equation for average velocity at a section applicable only for uniform, steady flow.

Also, Chezy. $v = C\sqrt{RS}$

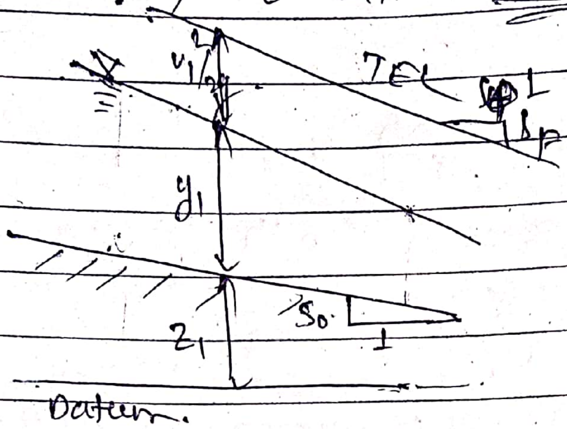
$C = \sqrt{8g/f}$ f = Darcy's friction factor

$$v = \frac{Q\, R^{2/3} S^{1/2}}{n}$$

* GVF :

$$v = \frac{R^{2/3} S_0^{1/2}}{n} \Rightarrow$$ ~~manning's eqn for~~ steady uniform flow.

when any parameter A, S_0, n changes v varies. So depth varies as well. Such flow called GVF.



~~HGL=~~

OCF ⇒ HGL represented by surface of fluid.

$HGL \Rightarrow P/\gamma + Z$

$TEL = HGL + v^2/2g$ may not be parallel in GVF. ~~when not parallel ⇒ GVF~~

Profile equation व्युत्पत्ति :

GVF मा depth vary हुन्छ so velocity pani vary हुन्छ i.e. velocity head change हुन्छ।

*
$$\boxed{\frac{dy}{dx} + \frac{d\left(\frac{v^2}{2g}\right)}{dx} = S_0 - S_f}$$

rect. for rectangular section.

$$v = Q/A = \frac{Q}{by}$$

$$\frac{dy}{dx} + \frac{d\left(\frac{Q^2}{2gb^2y^2}\right)}{dx} = S_0 - S_f$$

or, $$\frac{dy}{dx} + \frac{d}{dx}\frac{Q^2}{2gb^2y^2} = S_0 - S_f$$

or, $$\frac{dy}{dx} + \frac{-2\times Q^2}{2gb^2y^3}\frac{dy}{dx} = S_0 - S_f$$

or, $$\frac{dy}{dx} - \cancel{2}\frac{V^2}{\cancel{2}gy}\frac{dy}{dx} = S_0 - S_f$$

or, $$\frac{dy}{dx} - \frac{Fr^2}{\cancel{B}}\frac{dy}{dx} = S_0 - S_f$$

or, $$\frac{dy}{dx}\,\cancel{B}\left(1 - Fr^2\right) = S_0 - S_f$$

or, $$\boxed{\frac{dy}{dx} = \frac{S_0 - S_f}{1 - Fr^2}}$$

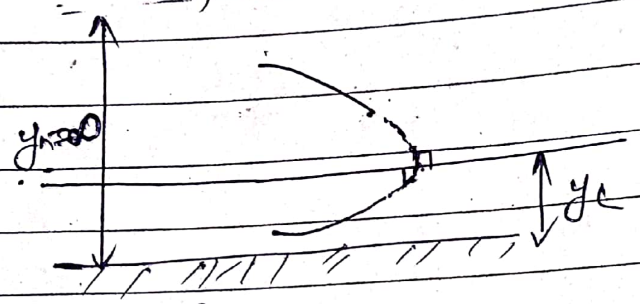
$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - Fr^2}$$

12 possible flow surface profile for bed slope cases

(i) Horizontal. main ध्यान actual flow depth
(ii) Mild और uniform / normal flow depth का ध्यान
(iii) Critical y_n 2 y_c
(iv) Steep : higher value of y
(v) Adverse. zone ① → Intermediate values of y
② → lower " "
③ →

(i) Horizontal : $y_n = \infty$

normal depth or uniform flow depth



$S_0 = 0$

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - Fr^2} \quad \text{when } y > y_c \quad Fr < 1$$

$$= \frac{0 - S_f}{1 - Fr^2 \, (+)} = -ve.$$

when $y = y_c$

when $y < y_c$ $\quad Fr > 1$

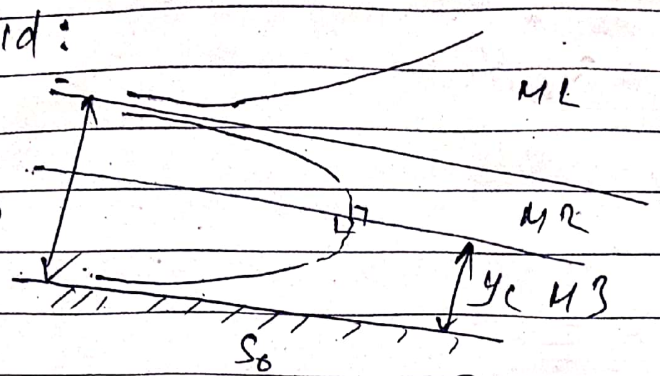
$$\frac{dy}{dx} = \frac{\cancel{S_0}^{0} - S_f}{1 - Fr^2}$$

$$= \frac{0 - S_f}{1 - Fr^2 \, (-ve)}$$

$= +ve.$

when $y = y_c$ $\quad dy/dx = \infty$ $\quad$ slope $= \infty$ $\perp$ ar.

(ii). Mild :



$y = y_n$

$S_0 \Rightarrow S_f = S_0$

$y > y_n$

a) ~~when~~ zone ① $\quad y > y_n > y_c$

$$dy/dx = \frac{S_0 - S_f}{1 - Fr^2} \qquad Fr < 1$$

$$= \frac{+ve}{+ve} = +ve$$

b) zone ② $\quad y_n > y > y_c \quad S_0$

$Fr < 1$

$$dy/dx = \frac{S_0 - S_f}{1 - Fr^2}$$

$$= \frac{-ve}{+ve} = -ve.$$

when $y = y_n$ $\quad S_0 = S_f$

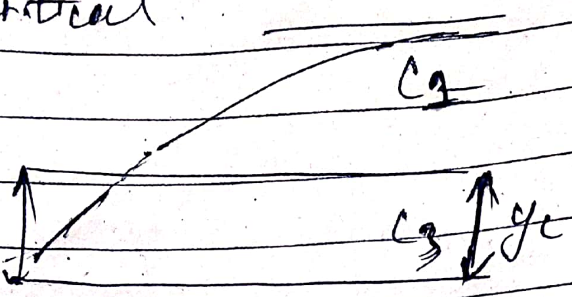
$dy/dx = 0$

c) zone 3 $\quad y < y_c < y_n$ $\quad Fr > 1$

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - Fr^2} = \frac{-ve}{-ve} = +ve.$$

when $y = y_c$ $\quad Fr = 1$ $\quad dy/dx = \infty$

(iii). Critical



$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - Fr^2}$$

Case (a). $y > y_n = y_c$. $Fr < 1$ $S_0 > S_f$

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - Fr^2} \quad \frac{+ve}{+ve} = +ve$$

(b) $y = y_n = y_c$ $Fr = 1$ $S_0 = S_f$

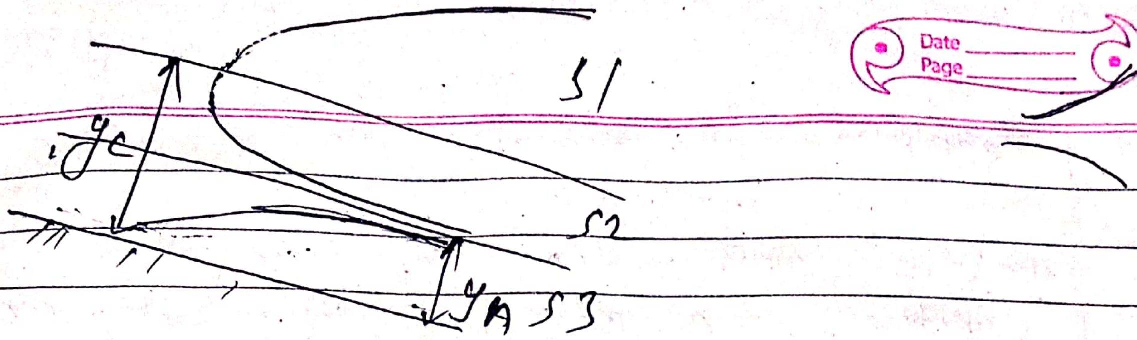
$\frac{dy}{dx}$ = Indeterminate

(c). $y < y_c = y_n$ $Fr > 1$, $S_0 < S_f$

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - Fr^2} = \frac{-ve}{-ve} = +ve$$

$y = 0$

(iv). Steep:



(a). $y > y_c > y_n$ $\quad Fr < 1 \quad S_0 \gtrless S_f$

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - Fr^2} = \frac{+ve}{+ve} = +ve$$

(b). $y = y_c$ $\quad Fr = 1 \quad \frac{dy}{dx} = \infty$

c) $\cancel{y < y_c}$ $\quad y_n < y < y_c, \quad Fr > 1 \quad S_0 > S_f$

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - Fr^2} = \frac{+ve}{-ve} = -ve$$

(d). $y = y_n < y_c$ $\quad S_0 = S_f \quad Fr > 1$

$$\frac{dy}{dx} = \frac{0}{-ve} = 0$$

(e) $y < y_n < y_c$ $\quad Fr > 1, \quad S_0 < S_f$

$$\frac{dy}{dx} = \frac{-ve}{-ve} = +ve.$$

$y = 0 \quad Fr \to \infty \quad 0 \quad \frac{dy}{dx} = -\infty$

(N). Hydraulic Jump and its types:

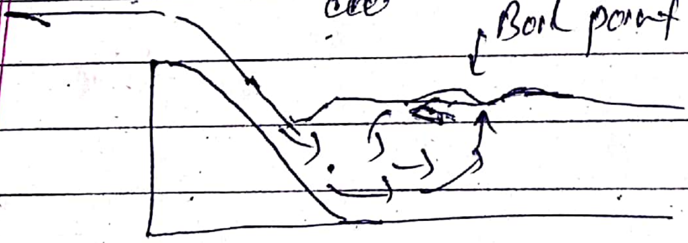
Tailwater depth: depth of flow at the d/s section of a dam: it controls the location of the hydraulic jump.

y_t Tailwater depth low → jump forms away from the dam → Fully developed jump (follows traditional shape and flow patterns).

As ~~to~~ y_t increases jump moves closer to dam (spillway). If y_t is large enough the jump location is at the dam. This condition is known as submerged / drowned jump.

→ submerged jump in low head dams are dangerous for activities like kayaking, rafting, because it creates an area of recirculation immediately d/s of the dam aka keeper region.

Boil point.

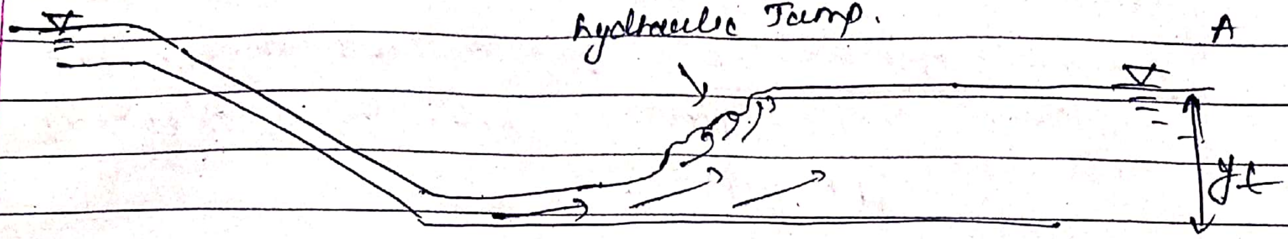


anything within boil point can't escape.

air entrainment reduces buoyancy.

Fully developed jump case A and case B

4 cases of jump (i). case A:

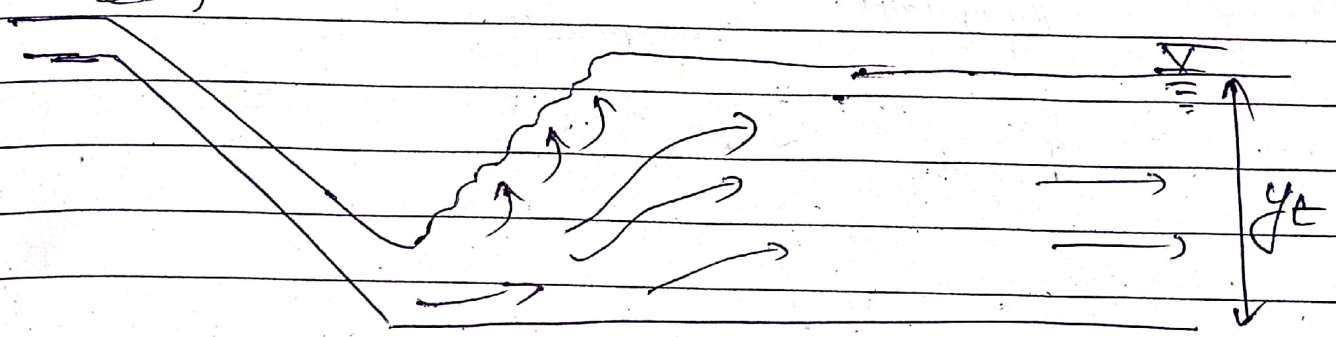


~~Fully developed jump~~ Swept out jump.

→ Falling water has enough momentum to push the jump away from the face of the structure

→ a ~~case~~ type of fully developed jumps.

(ii) Case B:



Optimum Jump

(y_t ~~बढ़ेगा~~ घटेगा case B changes to case A jump / बढ़ेगा → case C)

→ jump form right at the base of the structure.

→ aka optimum jump because minimizes the distance the jump is away from the structure and also forms a safe jump

→ also a type of fully developed hydraulic jumps.

Case A (sweptout

For fully developed jumps: ~~and~~ case B (optimum)

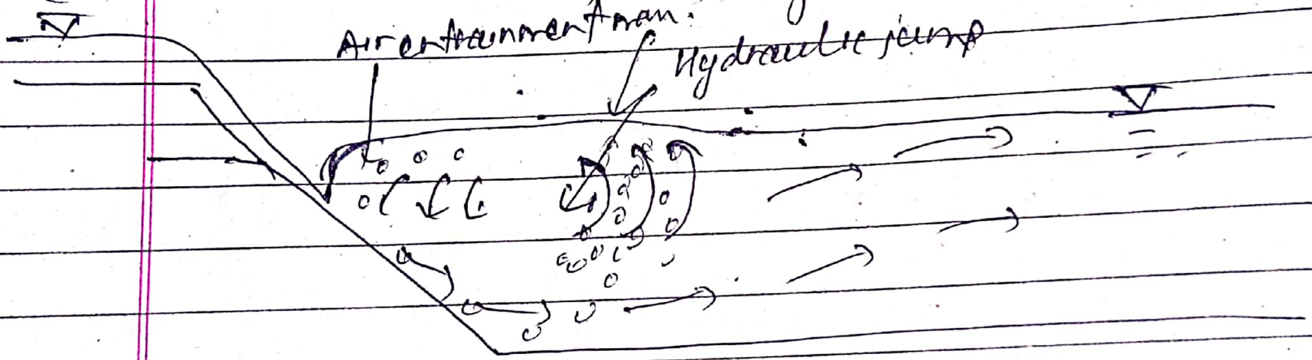
momentum factor of falling water before jump
> momentum factor of water after the jump.

falling water में ज्यादा है tailwater वाले पर
control करता है।

~~(iii). ca~~

(iii) Case C Jump. Boiling point.

Air entrainment mean.



→ case B में if y_t is more / increased it results case C jump.

→ case C is a submerged jump

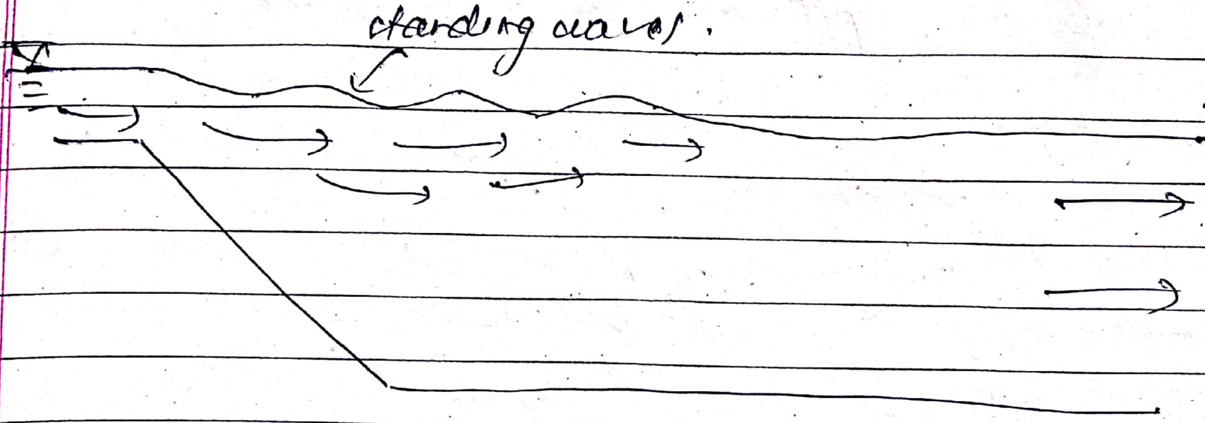
→ air entrained is forced downwards to the bottom the channel where it travel d/s (fig) air then rises to the surface causing large amount of water to follow.

Boiling point left Right

Some water recirculates some escapes d/s.

→ this causes a dangerous condition of no escape

(iv). case D Jump!



→ forms when tailwater is nearly at the same level as the surface of the u/s water level passing the spillway / structure

→ air entrainment is minimum

→ water simply moves a bit faster as it travels over the structure

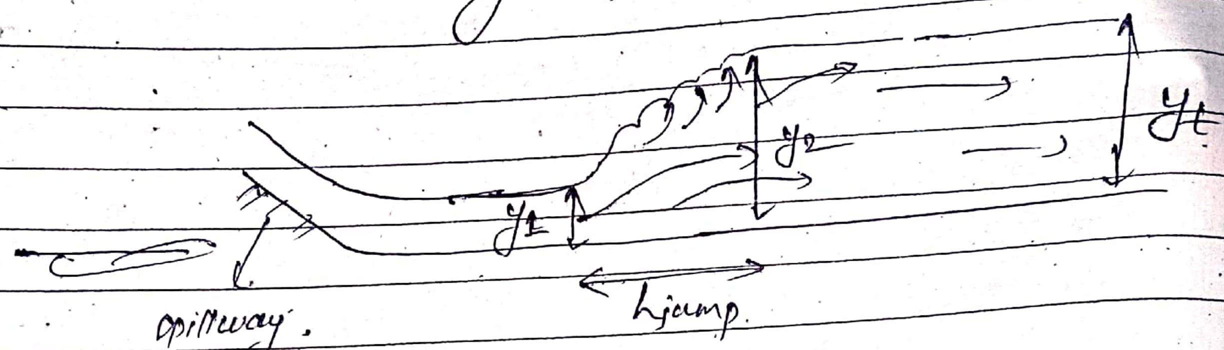
→

$h_L = \frac{(y_2 \ldots)}{4 y_1 y_2}$

What is a hydraulic jump?

→ When rapid flowing water (super-critical flow) makes an abrupt change to tranquil (sub-critical flow) place a hydraulic jump is said to be formed.

→ Occ: It occurs specially at the d/s end of a spillway or undersluice.



→ energy is lost in the form heat due to eddies formation.

$L_{jump} = 5(y_2 - y_1)$ ⇒ length of jump.

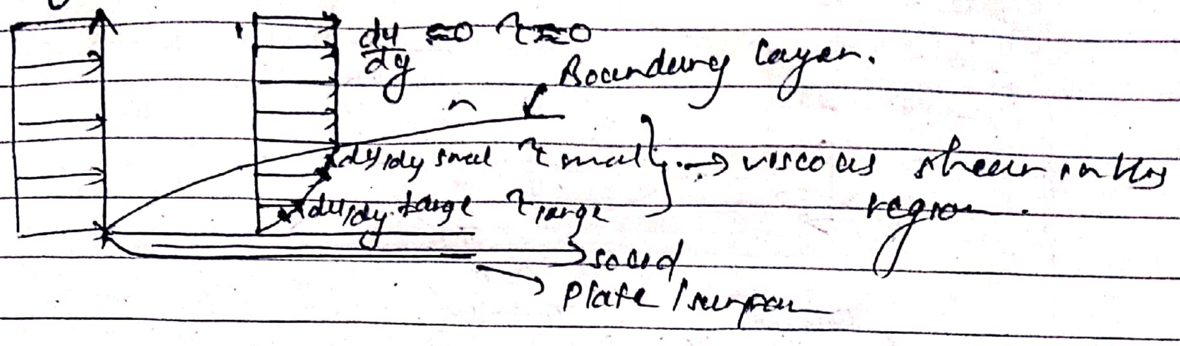
$h_L = \frac{(y_2 - y_1)^3}{4 y_1 y_2}$ ⇒ head loss

$y_2 = \frac{y_1}{2}\left(\sqrt{1 + 8Fr_1^2} - 1\right)$

Also, $y_1 = \frac{y_2}{2}\left(\sqrt{1 + 8Fr_2^2} - 1\right)$

y_1 ⇒ pre-jump depth
y_2 ⇒ post-jump depth

*. Boundary Layers :



Newtonian fluid : $\tau \propto \frac{du}{dy}$ (velocity gradient)

shear stress among fluid layers

$$\tau = \mu \frac{du}{dy}$$

→ fluid in contact with solid boundary adheres to it as τ is maximum $u \approx 0$.

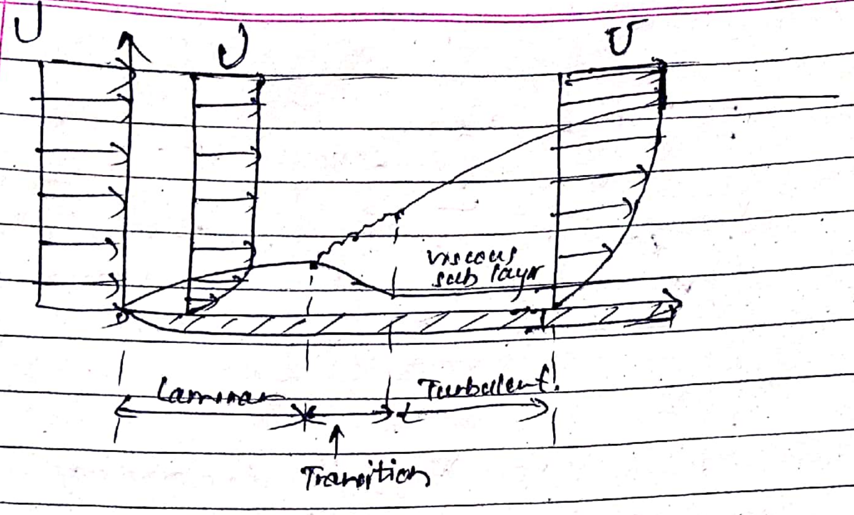
→ subsequent layer above get retarded due to friction. ~~velocity~~

→ this localized region of variable velocity when u changes from 0 at contact point to mainstream velocity is called boundary

→ thickness increases with distance d/s as more and more layer gets disturbed

→ significant in ~~prope~~ design of propeller aeroplane wings, turbine blades, etc.

Boundary layer
→ variable velocity region



Boundary layer can be divided into 3 regions:

a) Laminar flow

→ particles near surface stick while those above retard and are build up in smooth layers further d/s. This layer known as laminar layer.

→ thickness ↑ along d/s as more & more layers retard.

~~B) Turbulent layer:~~

B) Transitional flow:

→ As thickness of boundary layer increase laminar flow breaks down being unstable

→ some particle become turbulent with interlayer movement.

c) Turbulent:

→ mixing of fluid, further increase boundary layer thickness growing rapidly

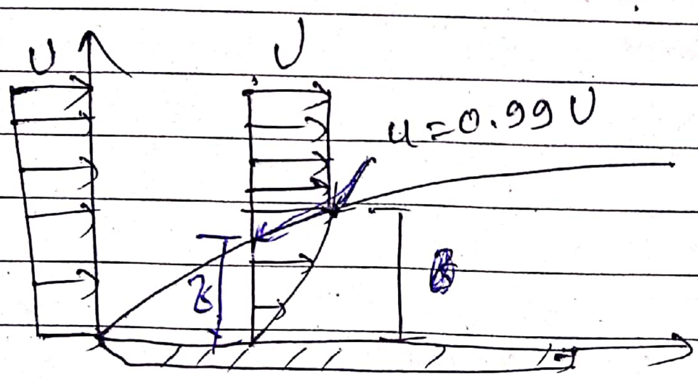
→ viscous sub layer (thin laminary layer) still exists below the turbulent layer (stick चिपकना).

How we quantify the boundary layer thickness?

4 approaches :

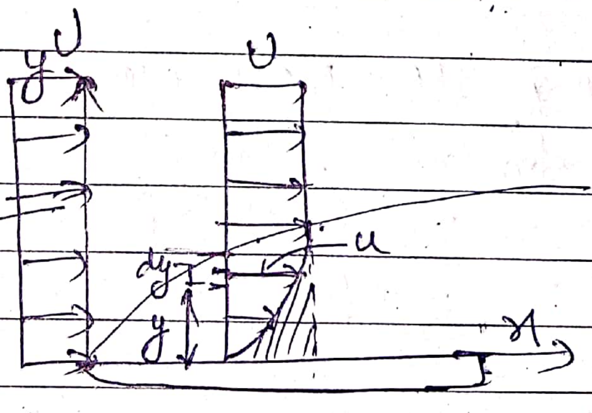
(I). Disturbance thickness :

→ height at which the velocity is certain % of mainstream velocity. Generally when $u = 0.99U$

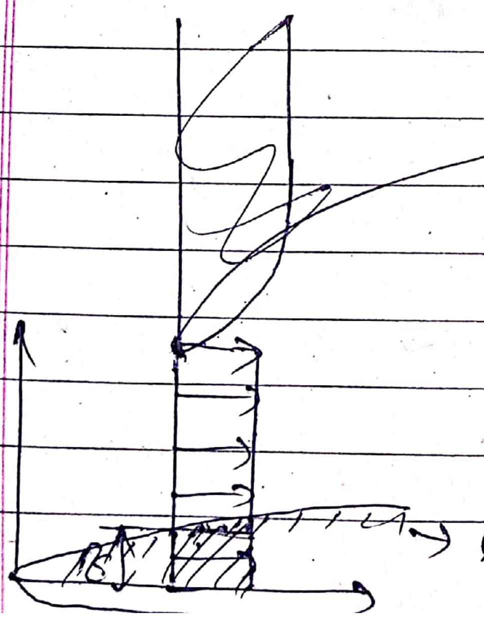


(II). ~~Displacement thickness~~ :

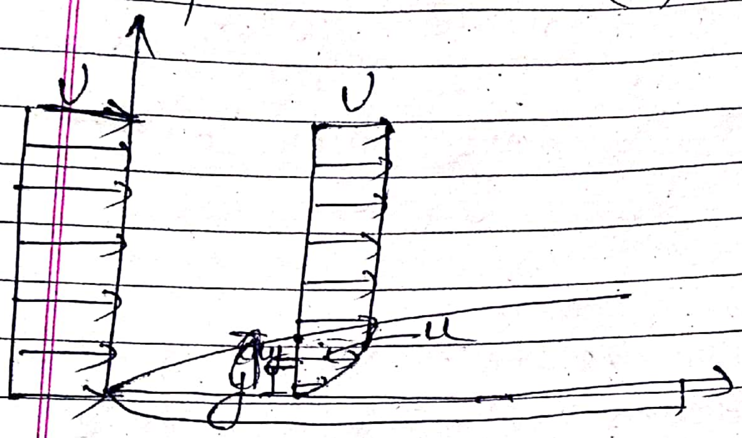
→ ~~defined as~~



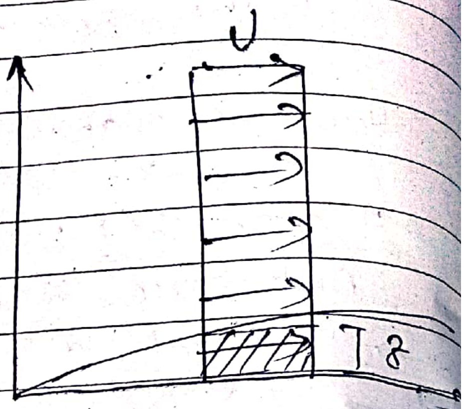
Real fluid condition.



(III). Displacement thickness (δ):



(I) Real fluid condition



(II) Ideal fluid condition with displaced boundary.

→ that thickness δ by which if the boundary is shifted up by considering an ~~real~~ ideal fluid the mass flow with this shifted boundary would be equal to that of the real fluid flow condition.

→ Equating the mass flow deposit.

$$\dot{dm} = \rho dQ$$
$$= \rho . u \, dA$$
$$= \rho u \, b dy$$

If it were an ideal fluid without displacement bound.

$$\dot{dm_0} = \rho U b dy$$

deficit in real flow condition

$= d\dot{m}_0 - d\dot{m}$

$= \rho b dy (U - u)$ — (i)

Total deficit $= \int_0^\infty \rho b dy (U - u)$ — (I)

Deficit in case (II) i.e. displaced boundary :

$\rho U b \delta^*$ — (II)

Cause portion बराबर होगा

$\cancel{\rho} \cancel{b} U \delta = \int_0^\infty \cancel{\rho} \cancel{b} (U - u) \, dy$

$$\boxed{\delta = \int_0^\infty \left(1 - \frac{u}{U} \right) dy}$$

(iii) momentum thickness (θ) :

⇒ height by which the boundary must be displaced so that if the fluid were ideal and flowing with free stream velocity U it would produce the same rate of momentum flow as that by the real fluid

$\cancel{\dot{dm}\, u = \rho dQ\, u}$

$\cancel{= \rho u^2 b dy}$ → momentum deficit

$\cancel{\dot{dm}\, u)_0 = \rho U u b dy}$ → mass deficit

$$\theta = \int_0^\infty \frac{u}{U} \left(1 - \frac{u}{U} \right) dy$$

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(iv) Energy thickness

$$\delta = \int_0^\infty \frac{u}{U}\left(1 - \frac{u^2}{U^2}\right) dy$$

$$\delta = \int_0^\infty \left(1 - \frac{u}{U}\right) dy$$

$$\theta = \int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

$$\Lambda = \int_0^\infty \frac{u}{U}\left(1 - \frac{u^2}{U^2}\right) dy$$

~~to~~ ~~mom~~ displacement ~~and~~, momentum, energy thickness

height by which the boundary must be displaced such that if the fluid were ideal and flowing with mainstream velocity U it would produce the same rates of ~~moment~~ mass, momentum and energy flow ~~per~~ u as that by the real fluid