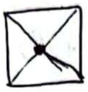
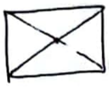
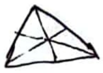
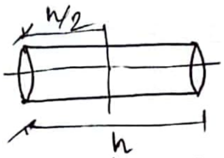
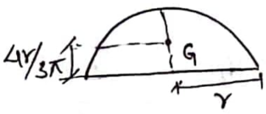
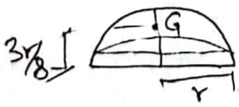
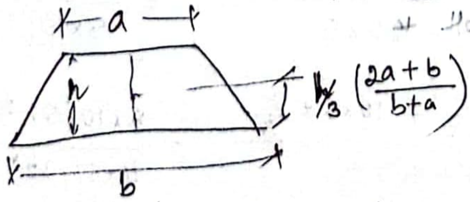
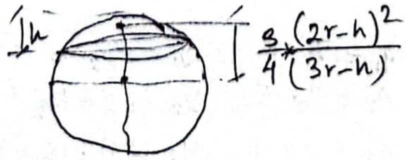
1. Structural Engineering.

Center of gravity (C.G)

→ point through which the whole weight of the body acts, irrespect of its position. ↳ depends upon the shape of the body

→ center of mass ↳ may be inside the body or within the surface.

* Center of area = centroid

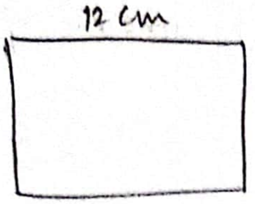
	Standard figures		Center of gravity.
1.	Uniform rod		at its mid point
2.	Square		Intersection of diagonals
3.	Rectangle		Intersection of diagonals.
4.	Triangle		Intersection of medians (line connecting vertex & middle point of opp side) ($\frac{1}{3}h$)
5.	Cylinder		Mid-point of axis, $h/2$
6.	Circular disc & Circular ring		($d/2$, r) Center of disc & center of ring resp.
7.	Hollow sphere.		Center of sphere. (r)
8.	Semi-circle		$4r/3\pi$ from base measured along the vertical radius
9.	Cube		$\frac{1}{2}$ from every face
10.	Hemi-sphere		$3r/8$ from base measured along the vertical radius.
11.	Right Circular solid Cone		$h/4$ from its base. * Hollow cone = h/3 above base
12.	Trapezium		$\frac{h}{3}\left(\frac{2a+b}{a+b}\right)$, measured from side b
13.	Segment of sphere of height h		$\frac{3}{4}\frac{(2r-h)^2}{(3r-h)}$ from center of sphere measured along the height
14.	Equilateral triangle with side a		$\left(\frac{a}{2}, \frac{a}{2\sqrt{3}}\right)$ (centroid (circum center

→ C.G is a point around which the resultant torque due to gravity force vanishes (total gravitational torque on body = 0)

→ C.G of a body may lie inside or outside the body.

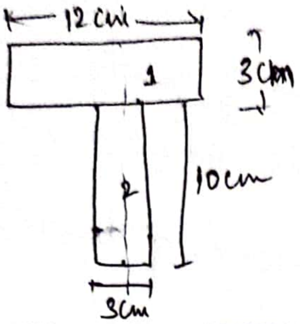
Q.

1.



$$C.G = \left(\frac{12}{2}, \frac{3}{2}\right) = (6, 1.5)$$

2.



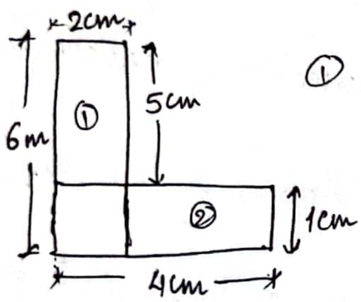
$a_1 = 12 \times 3$, $y_1 = 10 + 3/2 = 11.5 = 23/2$

$a_2 = 10 \times 3$, $y_2 = 10/2$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{12 \times 3 \times 23/2 + 10 \times 3 \times 10/2}{12 \times 3 + 10 \times 3}$$

$$= \frac{3(6 \times 23 + 10 \times 5)}{3(12+10)} = \frac{138 + 50}{22} = \frac{\cancel{188}\ 94}{\cancel{22}\ 11} = 8.54 \text{ cm}$$

3.



① $a_1 = 2 \times 6 = 12$

$x_1 = 2/2 = 1$

$y_1 = 6/2 = 3$

② $a_2 = 2 \times 1 = 2$

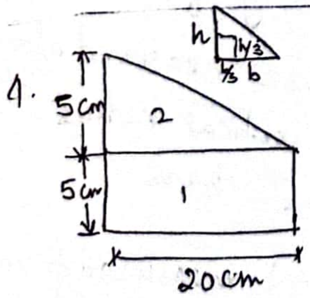
$x_2 = 2 + 2/2 = 3$

$y_2 = 1/2$

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = \frac{12 \times 1 + 2 \times 3}{12 + 2} = \frac{\cancel{18}\ 9}{\cancel{14}\ 7} = 1.286$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{12 \times 3 + 2 \times 1/2}{12 + 2} = \frac{37}{14} = 2.64$$

4.



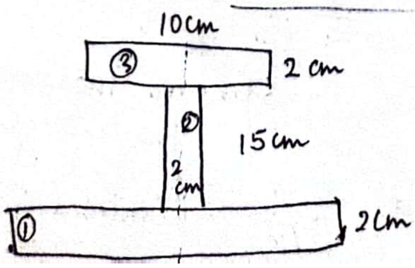
$a_1 = 20 \times 5$, $x_1 = \frac{20}{2} = 10$, $y_1 = \frac{5}{2}$

$a_2 = \frac{1}{2} \times 20 \times 5$, $y_2 = 5 + \frac{5}{3} = \frac{20}{3}$, $x_2 = \frac{20}{3}$

$$\bar{x}_f = \frac{20 \times 5 \times 10 + 10 \times 5 \times \frac{20}{3}}{10 \times 5 + 20 \times 5} = \frac{5 \times 10(20 + 20/3)}{5 \times 10(1+2)} = \frac{80}{3} \times \frac{1}{3} = 8.88 \text{ cm}$$

$$\bar{y}_f = \frac{\overset{10}{\cancel{20}} \times 5 \times \frac{5}{\cancel{2}} + 10 \times 5 \times \frac{20}{3}}{10 \times 5 + 20 \times 5} = \frac{5 \times 10(5 + 20/3)}{10 \times 5(1+2)} = \frac{35}{3} \times \frac{1}{3} = 3.88 \text{ cm}$$

5.



$a_1 = 20 \times 2$, $y_1 = 2/2 = 1$

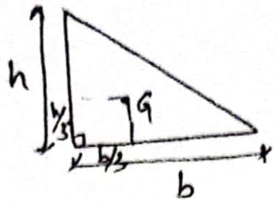
$a_2 = 2 \times 15$, $y_2 = 2 + 15/2 = 19/2$

$a_3 = 2 \times 10$, $y_3 = 2 + 15 + 2/2 = 18$

$$\bar{y} = \frac{40 \times 1 + 2 \times 15 \times \frac{19}{2} + 20 \times 18}{40 + 30 + 20} = \frac{685}{90} = 7.61 \text{ cm}$$

→ For symmetrical objects, the center of gravity lies at intersection of axes,

→ C.G. of right angled triangle with base 'b' & height 'h' and vertex at 90° is at origin

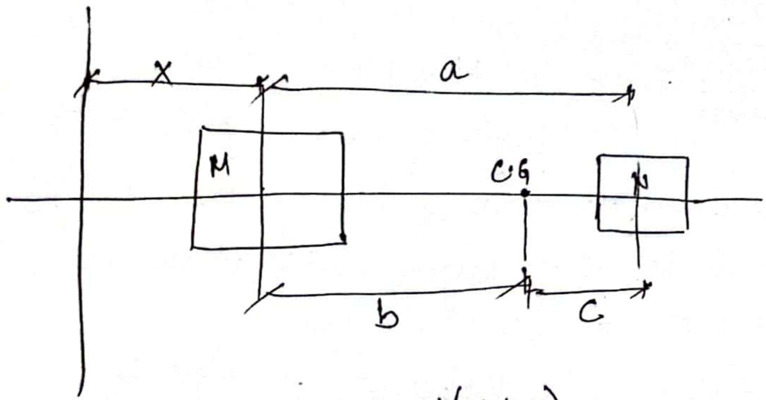


$$G = \left(\frac{b}{3}, \frac{h}{3}\right)$$

→ For the body which is symmetrical to one of its axes, center of gravity lies on the axis of symmetry.

→ If body is suspended, then at center of gravity the body can be balanced.

Q



Two square with M & N are its center of gravity, find the distance between the center of gravity from the square with
(i) center of gravity M ie. $b = ?$
(ii) center of gravity N ie. $c = ?$

$$CG = \frac{Mx + N(x+a)}{M+N}$$

$$\therefore b = C.G - x = \frac{Mx + Nx + Na}{M+N} - x$$

$$= \frac{Mx + Nx + Na - x(M+N)}{M+N}$$

$$= \frac{\cancel{Mx} + \cancel{Nx} + Na - \cancel{Mx} - \cancel{Nx}}{M+N}$$

$$= \frac{Na}{M+N}$$

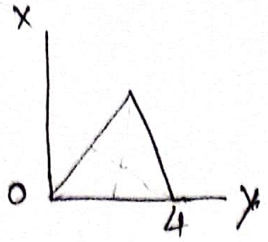
$$c = x + a - CG = x + a - \frac{Mx + N(x+a)}{M+N}$$

$$= \frac{(M+N)(x+a) - [Mx + Nx + Na]}{M+N}$$

$$= \frac{\cancel{Mx} + Ma + \cancel{Nx} + \cancel{Na} - \cancel{Mx} - \cancel{Nx} - \cancel{Na}}{M+N}$$

$$= \frac{Ma}{M+N}$$

Q) Find the C.G co-ordinates ;



$a = 4$

$(x, y) = \left(a/2, \frac{a}{2\sqrt{3}}\right)$

$= \left(4/2, \frac{4}{2\sqrt{3}}\right)$

$= (2, 2/\sqrt{3})$

* Center of mass & center of gravity may or may not coincide with eachother. | Coincides in uniform gravi... or gravity - free space.

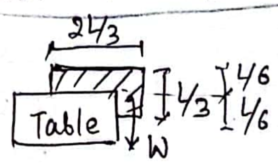
Q) A uniform chain of L length & M mass, two third part of chain is on a frictionless table and one third part is vertically suspended. Work done to pull the whole chain back on table is.

a) $\frac{MgL}{18}$

b) $\frac{MgL}{9}$

c) $\frac{MgL}{6}$

d) $\frac{MgL}{3}$



L = Length of chain
M = Mass of chain

length of hanging part $= L/3$

Since it is uniform, mass of hanging part $= M/3$

The wt. acts at the C.G of hanging part ie. at Midpoint of $L/3$

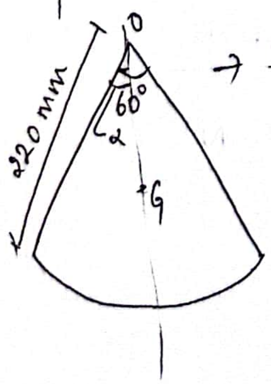
Wt. acts at $= \frac{L/3}{2} = L/6$

Work done $= f \times$ displacement $= Mg \times L/6$

$= \frac{Mg}{3} \times L/6$

$= \frac{MgL}{18}$

Q) Find the center of gravity of Lamina from the point O.



→ Symmetrical about y-y axis.

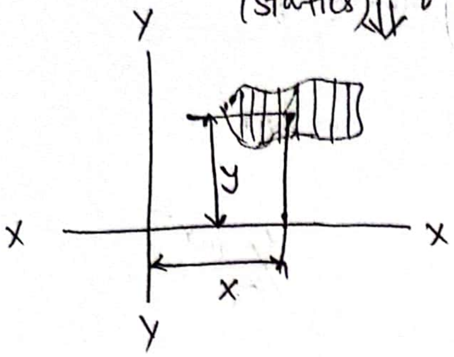
$\bar{y} = \frac{2r}{3} \frac{\sin\alpha}{\alpha}$ $\left(\alpha = \frac{60°}{2} = 30° = \pi/6 \text{ rad}\right)$

$= \frac{2 \times 220}{3} \frac{\sin(30°)}{\pi/6}$ $\left(1° = \pi/180 \text{ rad}\right)$

$= 140$ mm

Moment of Inertia

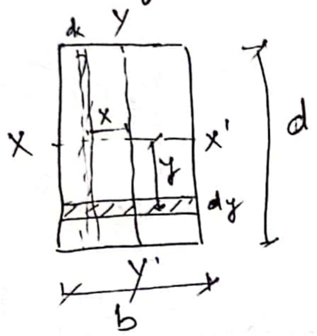
= Moment of the moment of a force | second moment of force
= Second moment of area | second moment of mass
(statics) (m^4) (dynamics) ($kg \cdot m^2$)



$$I_{yy} = \Sigma dA \cdot x^2$$

$$I_{xx} = \Sigma dA \cdot y^2$$

MOI of a Rectangular section



$$I_{xx} = \frac{bd^3}{12}$$

$$I_{yy} = \frac{db^3}{12}$$

through base

$$\frac{bd^3}{3}$$

Theorem of perpendicular axis

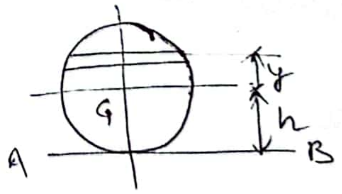
$$\boxed{I_{zz} = I_{yy} + I_{xx}}$$

MOI of Circular section.

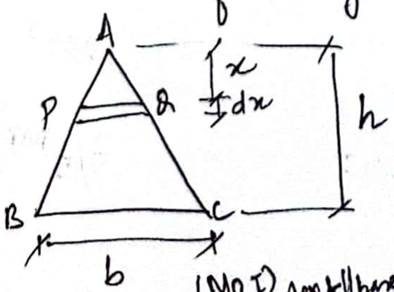
about diameter, $I_{xx} = I_{yy} = \frac{I_{zz}}{2} = \frac{\pi d^4}{64}$, about axis perpendicular to section $= \frac{\pi d^4}{32}$

Theorem of Parallel Axis.

$$I_{AB} = I_G + ah^2$$



MoI of Triangular section



$MoI_{BC} = \frac{bh^3}{12}$; MoI about ~~C.G~~ base.

$(MoI)_{G.xx} = \frac{bh^3}{36}$; MoI about an axis through C.G & (base) parallel to x-x axis/ centroid

$(MoI)_{vertex \parallel base} = \frac{bh^3}{4}$ $I_{yy} = \frac{hb^3}{36}$

MoI of semi-Circle, $I_G = 0.11 r^4$; $\frac{\pi D^4}{128}$

$I_{C.G} = 0.393 r^4$

Quarter-Circle.

$I_{xx} = I_{yy}$

$= 0.055 r^4$

Mass Moment of Inertia

$$I = \sum m_i r_i^2$$

Shape	Axis of Rotation	Moment of Inertia
Ring	passing through center perpendicular to the plane of ring	mr^2
Ring	passing through diameter of ring	$\frac{1}{2} mr^2$
Solid Cylinder	passing through center perpendicular to plane of ring	$\frac{1}{2} mr^2$
Solid Sphere	through center	$\frac{2}{5} mr^2$
Hollow sphere / Spherical shell	through center	$\frac{2}{3} mr^2$
Rod	through mid point (c.g.) perpendicular to rod its length	$\frac{1}{12} ml^2$
Solid Cone	about vertical axis	$\frac{3}{10} mr^2$
Rod	(about parallel axes through one end) perpendicular to rod through one end	$\frac{1}{3} ml^2$
Thin Spherical shell	through center	$\frac{2}{3} MR^2$
disc	passing through center perpendicular to the plane	$MR^2/2$
disc	about center	$MR^2/4$
hollow Cylinder	through center	MR^2
Sphere	about tangential axis	$\frac{7}{5} mr^2$

Section Modulus, $Z = \frac{I}{y_{max}}$

Radius of gyration → distance where the whole mass (or area) of a body is assumed to be concentrated.

↳ measure of the elastic-stability of a cross-section against buckling.

$r = \sqrt{I/A}$ ↳ perpendicular distance from the mass towards the axis of rotation.

→ Radius of gyration is indirectly to the parallel axis theorem. The parallel axis theorem gives the MOI perpendicular to the surface of considerance.

* MOI of elliptical section $= \frac{\pi}{64} BD^3$

* Stresses and Strains

① Strain, $\epsilon = \frac{\Delta l}{L}$

② Stress, $\sigma = P/A$

③ $\sigma \propto \epsilon$

$\sigma = E\epsilon$; E = Modulus of elasticity $= \sigma/\epsilon$

④ Deformation of a body due to force acting on it

$$\Delta l = \frac{PL}{AE}$$

⑤ Deformation of a body due to self-wt.

$$\Delta l = \frac{wl^2}{2E} = \frac{WL}{2AE} \qquad (W = wAL = \text{Total wt})$$

* If P(direct load) = W (wt. of bar)

$$(\Delta l)_{\text{self wt}} = \frac{1}{2}(\Delta L)_{\text{direct load}}$$

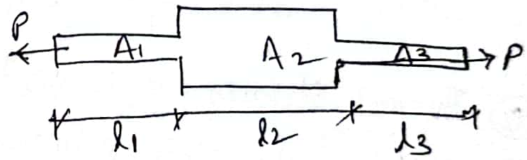
* Elongation of bar due to own wt.

→ Conical bar $= \frac{wl^2}{6E}$

→ Prismatic bar $= \frac{wL^2}{2E}$ (bar of uniform cross-section)

⑥ Stresses in bars of different sections

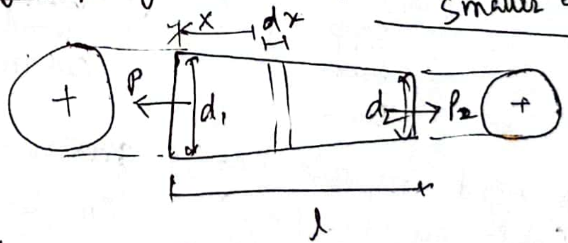
$$\delta l = \frac{P}{E}\left(\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3} + \cdots\right)$$



⑦ Stresses in the bars of uniformly tapering sections (max^m stress at smaller end)

ⓐ Circular sections

$$\delta l = \frac{4Pl}{\pi E d_2 d_1}$$



for $d_1 = d_2 = d$

$$\delta l = \frac{4Pl}{\pi E d^2} = \frac{PL}{\frac{\pi}{4}d^2 E} = \frac{PL}{AE}$$

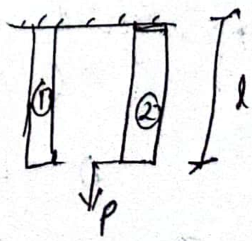
ⓑ Rectangular Sections.

$$\delta l = \int_0^l \epsilon_x \cdot dx = \int_0^l \frac{\sigma_x}{E} \cdot dx \qquad \left(\sigma_x = P/A_x\right)$$

⑧ Stresses in the bars of composite sections.

→ Extension or contraction of the bar being equal, strain i.e. deformation per length is also equal.

→ Total external load, on the bar, is equal to the sum of loads carried by the different materials.



$$P = P_1 + P_2 = \sigma_1 A_1 + \sigma_2 A_2$$

$\sigma_1 = \frac{E_1}{E_2} \times \sigma_2$

$\sigma_2 = \dots$

⑨ Thermal Stresses in Simple bars

$\alpha \uparrow$ - expansion ↑

temp. increase - expansion

temp. decrease - contraction

$$\delta l = l \alpha t$$

$$\epsilon = \frac{\delta l}{l} = \frac{l \alpha t}{l} = \alpha t$$

$$\sigma = \epsilon E = \alpha t E$$

Δ = yield of support,

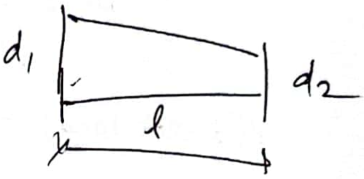
$$\delta l = l \alpha t - \Delta$$

$$\epsilon = \frac{\delta l}{l} = \frac{l \alpha t - \Delta}{l} = \alpha t - \Delta/l$$

$$\sigma = \epsilon E = (\alpha t - \Delta/l) E$$

⑩ Thermal stresses in bars of circular tapering sections

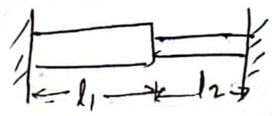
$$\sigma_{max} = \frac{\alpha t E d_1}{d_2}$$



for $d_1 = d_2$, $\sigma = \alpha t E$

⑪ Thermal Stresses in Bars of varying section

$$\delta l = \frac{1}{E} (\sigma_1 l_1 + \sigma_2 l_2)$$



⑫ Thermal Stresses in composite bar.

→ Higher the coeff. of linear expansion (α), greater is the expansion.

$(\alpha)_{\text{copper, brass, bronze}} > (\alpha)_{\text{wrought iron, cast iron}} > \alpha_{steel} > \alpha_{aluminium}$

for rigid composite bar; (while increasing temp.) greater α - subjected to compression

smaller α - subjected to tension

temp. decrease / cooled; greater α - subjected to tensile stress

⑬ Stresses in a Thin Cylindrical shell. (thickness < 1/10th to 1/15th of its diameter)

→ Subjected to atmospheric pressure = resultant pressure on wall of shell is zero.

→ Subjected to internal pressure = walls are subjected to tensile stress.

a) Circumferential/hoop stress, $\sigma_c = \frac{Pd}{2t}$; P = intensity of int. pressure

b) Longitudinal stress, $\sigma_l = \frac{pd}{4t}$

⑭ Stresses in thin spherical shell, $\sigma = \frac{pd}{4t}$.

(P) Elastic Constants

① **Poisson's Ratio** $\left(\frac{1}{m} \text{ or } \mu\right) = \dfrac{\text{Lateral strain}}{\text{Linear strain}}$

Lateral strain:
- Opp. dir^n to stress.
- secondary strain
- opp. strain in every dir^n at right angle to direct stress.

Linear strain $\left(\frac{\delta l}{l}\right)$:
- same dir^n to stress
- Primary strain

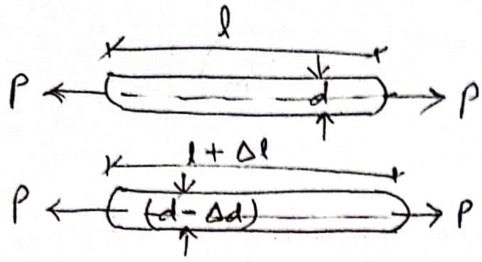


fig: Linear & lateral strain

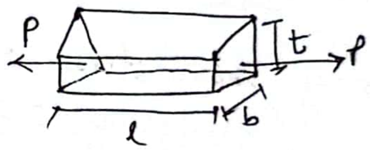
$(\mu)_{steel} = 0.25 \text{ to } 0.33$

$(\mu)_{concrete} = 0.08 \text{ to } 0.18$

② **Volumetric strain** (ϵ_v)

$$= \frac{\text{Change in volume } (\delta V)}{\text{Original volume } (V)}$$

(P) Volumetric strain of a rectangular body subjected to an axial force, P

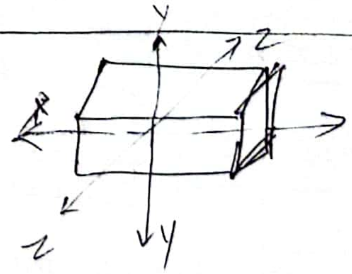


$$\text{Change in volume } (\delta V) = V * \frac{P}{btE}\left(1 - \frac{2}{m}\right)$$

$$\text{Volumetric strain } \left(\frac{\delta V}{V}\right) = \frac{P}{btE}\left(1 - \frac{2}{m}\right)$$

$$= \epsilon\left(1 - \frac{2}{m}\right) \qquad \left(\frac{P}{btE} = \epsilon = \text{strain}\right)$$

(P) Volumetric strain of a rectangular body subjected to three mutually perpendicular forces



$$(X\text{-}X)\ \epsilon_{xx} = \frac{\sigma_x}{E}, \quad (Y\text{-}Y)\ \epsilon_{yy} = \frac{\sigma_y}{E}, \quad (Z\text{-}Z)\ \epsilon_{zz} = \frac{\sigma_z}{E}$$

let ϵ_x, ϵ_y & ϵ_z be the resultant strains,

$$\frac{\delta V}{V} = \epsilon_x + \epsilon_y + \epsilon_z$$

$$= (\sigma_x + \sigma_y + \sigma_z) * \frac{1}{E}\left(1 - \frac{2}{m}\right)$$

③ **Bulk Modulus (K)** $= \dfrac{\text{Direct stress}}{\text{Volumetric Strain}} = \dfrac{\sigma}{\delta V / V}$; Volumetric strain due to change pressure

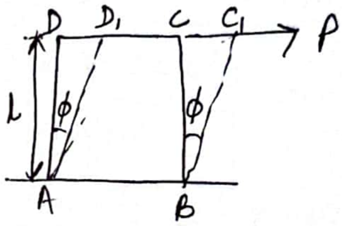
④ **Relation Between Bulk Modulus & Young's Modulus**

$$\boxed{K = \frac{mE}{3(m-2)}} \qquad \boxed{E = 3K\left(1 - \frac{2}{m}\right)}$$

⑤ **Shear Stress**

↳ When a section is subjected to two equal and opposite forces, acting tangentially across the resisting section, as a result of which the body tends to shear off across the section, the stress induced is called Shear stress. The corresponding strain is called shear strain.

Shear strain $= \dfrac{\text{Deformation}}{\text{Original length}} = \dfrac{CC_1}{AL} = \phi$



Shear Stress, $\tau = \dfrac{P}{AB}$

→ "A shear stress across a plane, is always accompanied by a balancing shear stress across the plane & normal to it."

⑥ **Shear Modulus or Modulus of Rigidity (C)** $= \dfrac{\text{Shear stress}}{\text{Shear strain}}$

within elastic limit, $\tau \propto \phi$

$\tau = C\phi$; C = Modulus of rigidity (G or N)

⑦ **Relation between Modulus of Elasticity & Modulus of Rigidity**

$$\boxed{C = \frac{mE}{2(m+1)}} \qquad \boxed{E = \frac{2C(m+1)}{m} = 2C\left(1 + \frac{1}{m}\right)}$$

⑧ **Relation between modulus of elasticity (E), modulus of rigidity (C) & Bulk Modulus (K)**

$$\boxed{E = \frac{9KC}{3K + C}}$$

Strain Energy and Impact Loading.

1. Strain Energy = energy absorbed in a body, when strained within its elastic limit
   = Work done
2. Resilience = total energy (strain) stored in a body due to external loading within elastic limit.
3. Proof Resilience = Max^m strain energy
   = Corresponding stress is called proof stress
4. Modulus of Resilience = proof resilience per unit volume
5. Types of loading
   (a) gradually (b) suddenly (c) with impact

* Strain energy stored in a body, when the load is gradually applied.

$$\boxed{U = \frac{\sigma^2}{2E} * V}$$

$V = Al$, $\boxed{\text{Modulus of resilience} = \sigma^2/2E}$

* Strain energy stored in a body, when the load is suddenly applied

Work done $= P * \delta l$; $U = \frac{\sigma^2}{2E} A l$

$$P * \delta l = \frac{\sigma^2 A l}{2E}$$

$$\left(\delta l = \frac{Pl}{AE} = \sigma l/E \right)$$

$$P * \frac{\sigma l}{E} = \frac{\sigma^2 A l}{2E}$$

$$\boxed{\sigma = 2 * P/A}$$

→ Stress induced in a body when the load is suddenly applied is twice the stress induced when the load is applied gradually.

* Strain energy Stored in a body, when the load is applied with impact.

$$\sigma = P/A \left[1 \pm \sqrt{1 + \frac{2AEh}{Pl}} \right]$$

When δ is very small as compared to h. (height through which the load will fall, before impacting)

$$\boxed{\sigma = \sqrt{\frac{2EPh}{Al}}}$$

* Strain energy stored in a body due to shear stress.

$$\boxed{U = \frac{\tau^2}{2C} * V}$$

* Max^m Shear stress developed in a beam of rect. cross-section = 1.5 * Avg. Shear stress

* $\frac{\text{(Shear Stress) Rect. section}}{\text{(Shear Stress) Circular Section}} = 9/8$

* Theory of flexure

$$\boxed{\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}}$$

M = B.M , I = MoI , E = Young's M
R = radius of curvature
σ = Bending stress in a fibre, at a d
y from N.A

→ Bending stress at NA = 0
→ distribution of bending stress across the section = uniformly

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$M = \sigma_{max} * \frac{I}{y_{max}}$$

$$M = \sigma_{max} * Z \quad ; Z = \frac{I}{y_{max}} = \text{section modulus.}$$

* $(Z)_{\text{rectangular section}} = \frac{bd^2}{6}$
* (Z) circular section $= \pi/_{32}\, d^3$

generally, beam (S.S.B) — Compressive stress above N.A / Tensile stress below NA

Cantilever beam – tensile stress ~~at b~~ above NA & compressive stress below NA

---

* Theory of Torsion

Shaft < one end fixed / other subject to T

$$\boxed{\frac{\tau}{R} = \frac{T}{J} = \frac{C\theta}{l}}$$

τ = Shear stress (max^m)
T = Torque or Twisting moment
J = Polar moment of inertia ($I_{xx} + I$
C = Modulus of rigidity
θ = Angle of twist (radians) on a length l
R = Radius of shaft

τ_{max} – outer surface
$\tau_{\text{centroidal axis}}$ = zero.

$$T/\theta = \text{Torsional rigidity} = \frac{CJ}{l}$$

→ Torque, T transmitted by solid shaft of dia. D, when subjected to shear stress

$$T = \pi/_{16} * \tau * D^3$$

→ $(T)_{\text{hollow shaft}} = \pi/_{16} * \tau * \left(\frac{D^4 - d^4}{D}\right)$ D = external diameter, d = Internal diameter

→ Power transmitted by a shaft, $P = \frac{2\pi NT}{60}$ kW ; N = rev./minute, T = in kN-m

* Analysis of beams and frames: bending moment, shear force and deflection of beams and frames

Bending Moment and Shear force.

Shear Force (S.F)

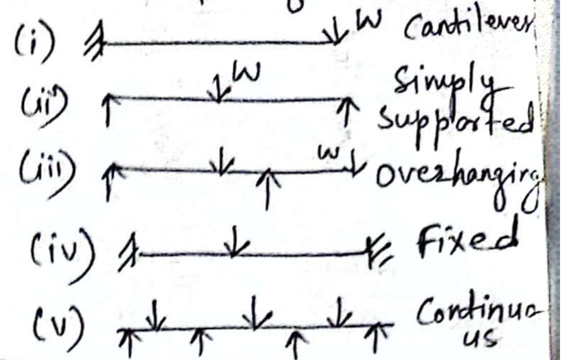
↳ unbalanced vertical force to the right or left of the section.

Bending Moment (B.M

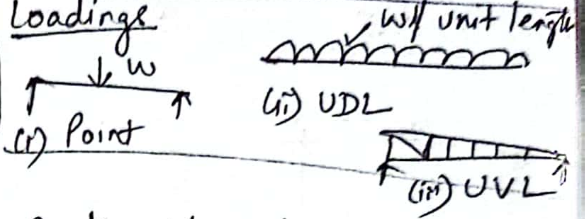
↳ algebraic sum of the moments of the forces.

↳ while calculating BM or S.F, at a section the beam will be assumed to be weightless.

Types of Beams



Loadings



Relation between Loading, Shear Force and Bending Moment

(i) point load at a section on the beam ⇒ S.F suddenly changes (vertical S.F line)
B.M remains same.

(ii) No load betn two points ⇒ S.F does not change (S.F line is horizontal)
B.M changes linearly - (inclined st. line)

(iii) UDL betn two points ⇒ S.F changes linearly (incline st. line)
B.M changes acc. to parabolic law (parabola line)

(iv) UVL betn two points ⇒ S.F changes acc. to parabolic law (parabola line)
B.M changes acc. to cubic law.

Polar Moment of Inertia (J)

① J solid circular shaft, $= \pi/32 * D^4$

② Hollow circular shaft, J. $= \pi/32 \, (D^4 - d^4)$

③ Solid shaft, $Z_p = \pi/16 \, D^3$

④ Hollow shaft, $Z_p = \frac{\pi}{16D} (D^4 - d^4)$

Z_p - polar modulus $= \frac{I_p \,(J)}{R}$

Strain energy due to Torsion

Solid Circular shaft,

$$U = \frac{\tau^2}{4C} * V$$

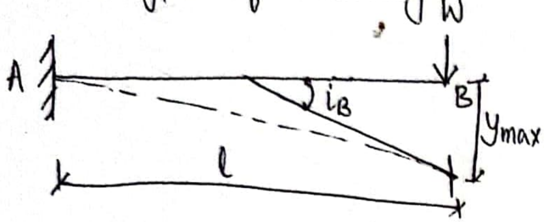
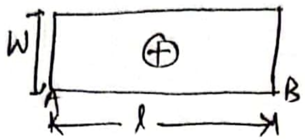
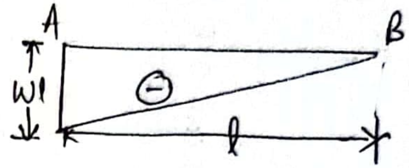
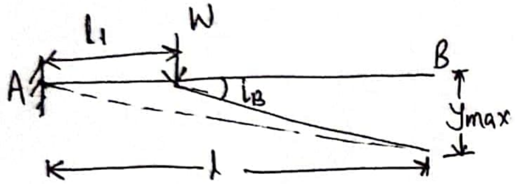
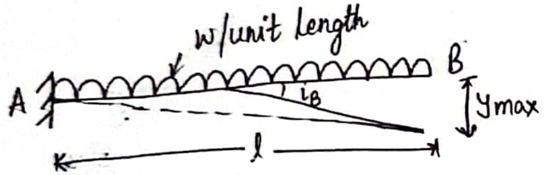
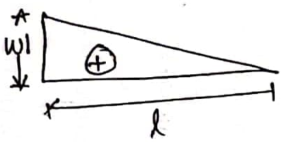
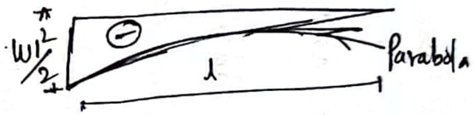
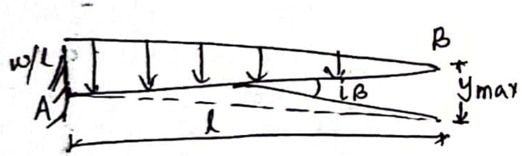
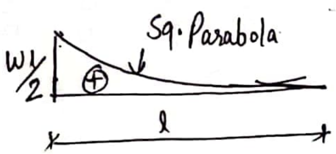
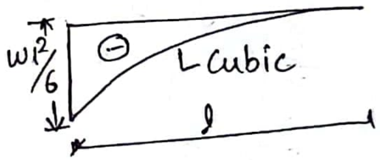
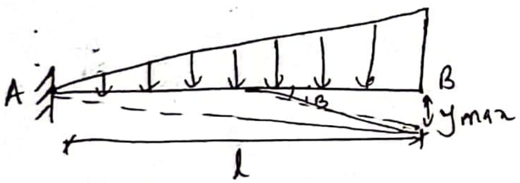
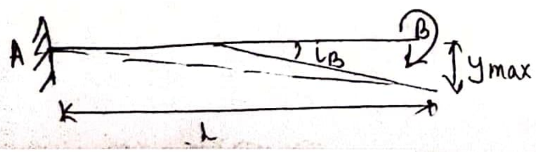
Hollow Circular Shaft:

$$U = \frac{\tau^2}{4C} \left(\frac{D^2 + d^2}{D^2}\right) * V$$

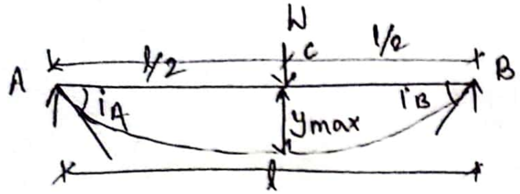
Axial stress & shear stress

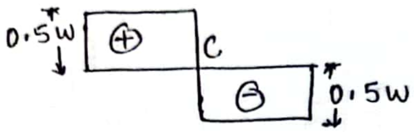
$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{xy}}$$

* Shear stress, $\tau = \frac{Tr}{J} = \frac{16T}{\pi D^3}$

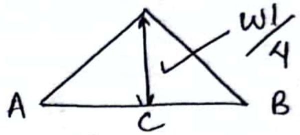
S.N.	Type of Loading	Max^m Shear force	Max^m Bending Moment	Slope	Maximum Deflection
1.				$i_B = -\frac{Wl^2}{2EI}$	$y_{max} = y_B = -\frac{Wl^3}{3EI}$
2.		Rectangle	Triangle	$i_B = \frac{-Wl_1^2}{2EI}$	$y_{max} = \frac{Wl_1^2}{6EI}(3l - l_1)$
3.				$i_B = -\frac{wl^3}{6EI}$	$y_{max} = -\frac{wl^4}{8EI}$
4.				$i_B = -\frac{wl^3}{24EI}$	$y_{max} = -\frac{wl^4}{30EI}$
5.		Sq. Parabola	Cubic Parabola	$i_B = \frac{wl^3}{8EI}$	$y_{max} = \frac{11wl^4}{120EI}$
6.		St. line	Rectangle	$i_B = -\frac{Ml}{EI}$	$y_{max} = -\frac{Ml^2}{2EI}$

7.





Rectangle



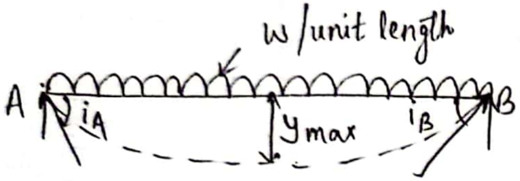
Triangle

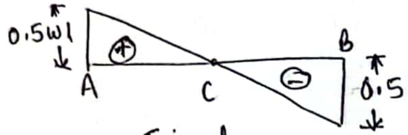
$i_A = -\frac{Wl^2}{16EI}$

$i_B = +\frac{Wl^2}{16EI}$

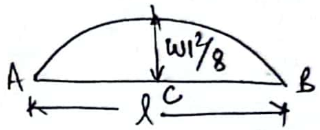
$y_{max} = y_c = \frac{Wl^3}{48EI}$

8.





Triangle



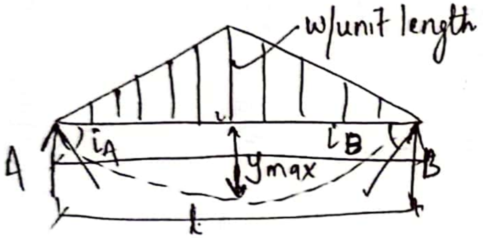
Sq. Parabola

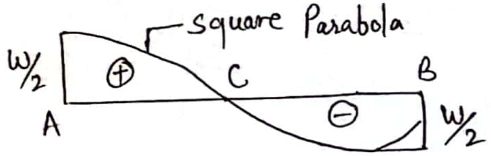
$i_A = -\frac{wl^3}{24EI}$

$i_B = +\frac{wl^3}{24EI}$

$y_{max} = y_c = \frac{5wl^4}{384EI}$

9.





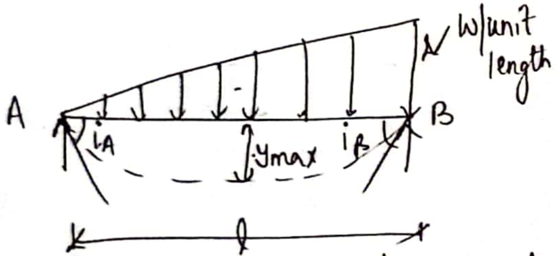
Cubic parabola

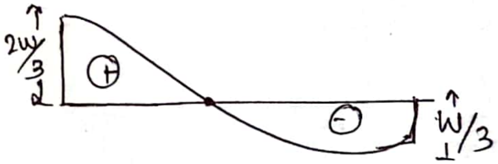
$i_A = -\frac{5wl^3}{192EI}$

$i_B = +\frac{5wl^3}{192EI}$

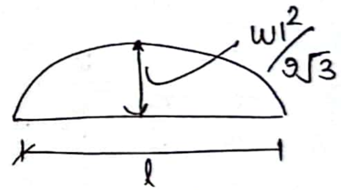
$y_{max} = y_c = -\frac{wl^4}{120EI}$

10.





Sq. parabola



Cubic Parabola

$i_A = -\frac{wl^3}{45EI}$

$i_B = +\frac{7wl^3}{360EI}$

$y_{max} = \frac{0.00652wl^4}{EI}$

$y_c = \frac{0.0064wl^4}{EI}$

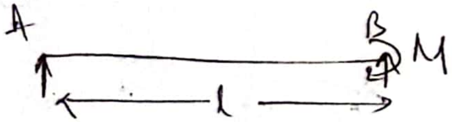
↳ y_{max} occurs at a point 0.522 l, from the end carrying zero load.

16. Couple at center of S.S.B — St. line — Rectangle

$i_A = i_B = \frac{ML}{24EI}$

$y = -\frac{ML^2}{36\sqrt{2}\,EI}$

17.



$i_B = \frac{Ml}{3EI}$, $y_{max} = \frac{Ml^2}{16EI}$

* Conjugate Beam Method

→ Proposed by Prof. O. Mohr in 1868 & later on systematically devel by Prof. H.F.B Muller-Breslau in 1885.

Conjugate Beam

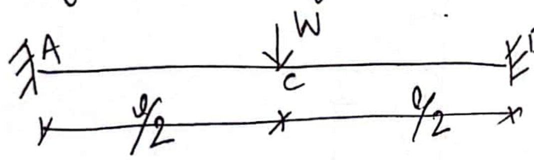
→ Imaginary beam of length equal to that of the original beam. width equal to $1/EI$.

* Shear force of conjugate Beam = Slope of elastic curve of actual beam
* Bending Moment of conjugate beam = Deflection of elastic curve of actual beam

Actual beam, fixed end = Conjugate Beam, free end
" " free end = " " , fixed end
" " , S.S./roller = " " , S.S.
" " , Intermediate hinge = " " , " , Intermediate support

Fixed Beams

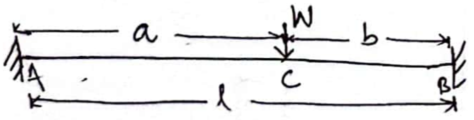
① Fixing Moments of a fixed beam carrying a central Point Load



$$M_A = M_B = \frac{WL}{8}$$

$$\text{Max. deflection}, y_c = \frac{Wl^3}{192EI}$$

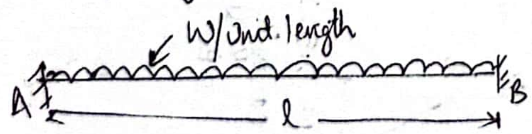
② Eccentric Point Load



$$M_A = \frac{Wab^2}{l^2}, \quad M_B = \frac{Wa^2b}{l^2}$$

$$y_c = \frac{Wa^3b^3}{3l^3EI}, \quad y_{max} \doteq \frac{2}{3}\frac{Wa^3b^2}{(3a+b)^2EI}$$

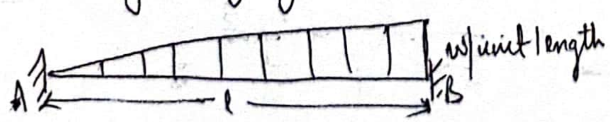
③ Uniformly distributed loads



$$M_A = M_B = \frac{Wl^2}{12}, \quad M_c = \frac{Wl^2}{24}$$

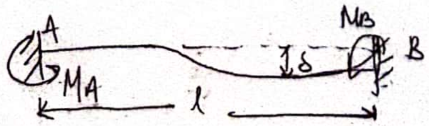
$$y_c = \frac{Wl^4}{384EI}$$

④ Gradually varying load



$$M_A = \frac{Wl^2}{30}, \quad M_B = \frac{Wl^2}{20}$$

⑤ Sinking of Supports



$$M_A = -\frac{6EI\delta}{l^2}$$

$$M_B = +\frac{6EI\delta}{l^2}$$

* Geometric stability and Static Determinancy.

$2j = m + r$; m = number of members
r = number of reaction components.
j = joints.

1. $2j < m+r$ ⇒ statically indeterminate & stable
2. $2j = m+r$ ⇒ Statically determinate & stable
3. $2j > m+r$ ⇒ statically indeterminate & unstable.

Internal redundancy

$I = m - (2j - r)$

$E = R - r$

Space truss

$3j = m + r$

Frame

Perfect frame, $n = 2j - 3$
Deficient frame, $n < 2j - 3$
Redundant frame, $n > 2j - 3$

* Moment - Area Method

① Change of slope bet^n two points = Area of $\frac{M}{EI}$ diagram bet^n two points.

$$\frac{d\theta}{dx} = \frac{M}{EI} \text{ Area}.$$

② Deflection = Moment of the $\frac{M}{EI}$ diagram

→ Valid for single span beams

* Conjugate beam method.

y = deflection ordinates of the elastic curve

$$\frac{dy}{dx} = \theta = \text{slope of the elastic curve}$$

$$\frac{d\theta}{dx} = \frac{d^2y}{dx^2} = \frac{M_x}{EI} \quad ; \text{ Ordinate of BM}$$

$$\frac{dM_x}{EI\,dx} = \frac{d^3y}{dx^3} = \frac{V_x}{EI} \quad ; \text{ Ordinate of S.F}$$

$$\frac{dV_x}{EI\,dx} = \frac{d^4y}{dx^4} = \frac{W_x}{EI} \quad ; \text{ ordinate of loading}$$

Moment Distribution Method

↳ Hardy Cross, 1930

↳ displacement method of analysis

↳ for analysing of statically indeterminate beams and rigid frames.

→ only accounts for flexural effects & not for axial & shear effects.

Stiffness → force/moment required to produce unit displacement.

$K = F/\Delta$ or M/θ

K = Stiffness
F = force required to produce Δ.
M = moment " " " θ.

Stiffness factor

→ moment required to produce unit rotation.

① far end is fixed ; $M = \frac{4EI}{L}$

② Far end is hinged ; $M = \frac{3EI}{L}$

③ far end is free ; $M = 0$

④ far end is guided rollers ; $M = \frac{EI}{L}$

Carry Over Factor

→ ratio of the far end moment & near-end moment.

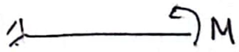
→ also called developed moment or induced moment.

① far end is fixed ; COF $= \frac{1}{2} = 0.5$ →

far end is fixed & moment applied at hinged.

② far end is hinged ; COF $= 0$ → far end is hinged in S.S.B

③ Cantilever Beam ; COF $= -1$ →



for end is fixed for cantilever beam

④ far end is guided by roller ; COF $= -1$

Distribution Factor

→ ratio of stiffness of a member to the total stiffness of members meeting at joint.

① At fixed end ; DF = 0

② At hinged end ; DF = 1

③ Sum of the DF of all members meeting at joint is one (1).

④ Relative Stiffness

① far end is fixed ; R.S = I/L

② far end is hinged ; R.S $= \frac{3I}{4L}$

* Difference between force & Displacement Methods

	Force Method	Displacement Method
→ Types of Indeterminacy	Static	Kinematic
→ Governing equation	Compatibility equations	Equilibrium equations
→ Force displacement relations	Flexibility Matrix	Stiffness matrix
→ Examples	1. Method of consistent deformation 2. The theorem of three moments (1/EI) 3. Column analogy method 4. Flexibility matrix method 5. Castigliano's theorem	1. Slope deflection method 2. Moment distribution method 3. Kani's method 4. Stiffness matrix method

* Theorem of three moments (Clapeyron's Theorem)

→ derived using Mohr's first & 2nd moment theorems

→ describes the relationship among the bending moments at three consecutive supports of a horizontal continuous beam.

Applicable to

- → Continuous beams with over hangs
- → Single span fixed beams
- → Continuous beams with sinking supports & rotating joints

Not applicable to

- → trusses & frames
- → fixed supports

* In case of fixed end of a continuous beam, we assume
  - → imaginary length (zero length)
  - → infinite stiffness
  - → replacing fixed end with simply supported end

* Flexibility Matrix Method

→ Elements of flexibility matrix are displacements.
→ Flexibility is defined as displacement caused due to unit force. (opposite of stiffness)
→ Elements along the diagonal will always be non-zero and positive.
→ follows maxwell reciprocal theorem i.e. $f_{ij} = f_{ji}$
→ only for stable structure in which there is no rigid body displacement.

* Castigliano's theorem

→ based on principle of virtual work energy (conservation of strain energy)

1st Theorem

→ The first partial derivative of the total internal energy (strain energy) in a structure w.r.t. any particular deflection component at a point is equal to the force applied at that point & in the direction corresponding to that deflection component.

→ applicable to linearly or non-linearly elastic structures with constant temperature and un-yielding support.

2nd Theorem

→ -- w.r.t. force applied at any point is equal to the deflection at the point of application of that force in the direction of its line of action.

→ applicable to linearly elastic structures.

→ used to find deflections in beams, trusses, & frames.

* Slope deflection Method

→ rotations of the joints are treated as unknowns. (also displacement)
→ All the joints of the frames are considered rigid.

Degree of freedom

→ No. of joints rotation & independent joint translations in a structure is called degrees of freedom.

Expressed in two types.

- In rotation - for beam or frame.

$$D_r = j - f$$

D_r = degree of freedom
j = no. of joints, including supports
f = no. of fixed support

- In translation - for frame.

$$D_f = 2j - (2f + 2h + r + m)$$

j = no. of joints (each joint has joint translation)
f = no. of fixed support
h = no. of hinged support
r = no. of roller supports
M = no. of supports

$$M_{AB} = \bar{M}_{AB} + \frac{2EI}{L}\left[2\theta_A + \theta_B - \frac{3\Delta}{L}\right]$$

(Rigid frames)

→ deformations are caused by bending moment only.

* Kani's Method

→ analysis of frame
→ extension of slope deflection method

Displacement factor = 1.5
Rotation factor = 0.5

* Strain Energy:

1. Due to axial force, $U_{axial} = \Sigma \frac{P^2 l}{2AE}$

2. Due to bending moment, $U_{bending} = \Sigma \int \frac{M^2 \cdot dx}{2EI}$

3. Due to shear, $U_{shear} = \Sigma \int \frac{K \cdot V^2 \cdot dx}{2GA}$

4. Due to torsion, $U_{Torsion} = \Sigma \int \frac{T^2 dx}{2GJ}$

Where, K = Shear factor
= 1.2 for rect. section

G = Shear modulus of elasticity

J = Polar Moment of inertia

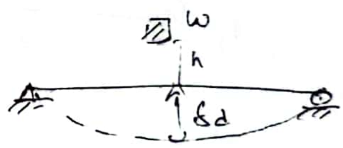
Also,

✓ Strain Energy, $U = \int_V \frac{\sigma^2}{2E} dV$

✓✓ Strain energy density, $u = \frac{\sigma^2}{2E}$ (U per unit volume)

$$\boxed{U = \frac{1}{2} P\delta}$$

* Deflection due to Impact loading



$$\delta d = \delta st \left[1 + \sqrt{1 + \frac{2h}{\delta st}}\right]$$

$$= \delta st * I.F$$

where,

δst = Static deflection

δd = dynamic deflection

I.F = Impact factor

$= \left(1 + \sqrt{\frac{2h}{\delta st}}\right) > 1$.

h = ht. of fall.

Here,

$\delta d > \delta st$ (always)

for suddenly applied load,

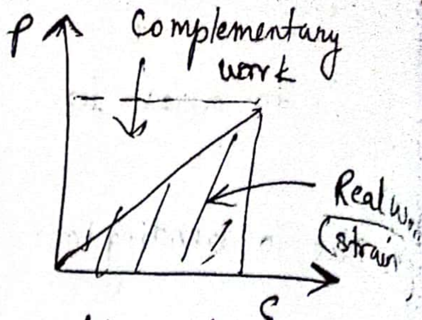
$h = 0$

$\therefore IF = 2$

$$\boxed{\delta d = 2\delta st}$$

CamScanner

* Real Work and Complimentary work



Linear form

→ Real work is given by area under load deflection diagram.

→ The area above the curve is complimentary work.

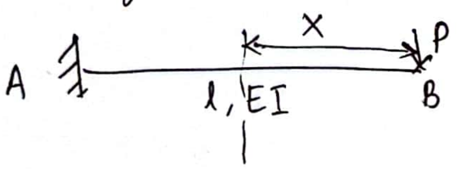
* Deflection by Real Work Method.

→ Can only be used for flexural members in which deflection is required under point of application of load.

Limited to.

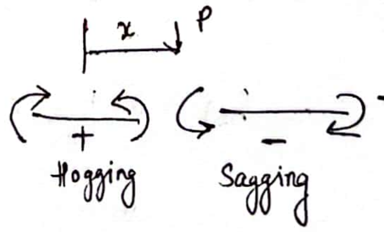
↳ Single deflection due to point load only at the point of application of load.

Q Calculate deflection at free end of cantilever beam.



Sol^n;

→ strain energy nikalne.
↓
(BM निकालने पड़ेंगे आउट्ठ)
⇓
S.E = Work ; deflection aucha.



→ Pair अगेन्स्ट सक्रिय

RS — Clockwise so −ve moment

$$M_x = -P.x$$

Strain energy (U) $= \int_0^L \frac{M_x^2 . dx}{2EI}$

$$= \frac{1}{2EI} \int_0^L (-Px)^2 . dx$$

$$= \frac{P^2}{2EI} * \left[\frac{x^3}{3}\right]_0^L$$

$$= \frac{P^2 L^3}{6EI}$$

Now,

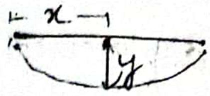
$$U = \text{Work}$$

$$\frac{P^2 L^3}{6EI} = \frac{1}{2} \times P \times \Delta$$

$$\boxed{\Delta = \frac{PL^3}{3EI}}$$

Slope and deflection of Beams.

If y is the deflection of beam at a section x from origin

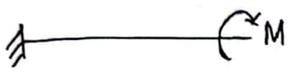


$EI.\frac{d^4y}{dx^4} \propto w$, intensity of load (kN/m)

$EI.\frac{d^3y}{dx^3} \propto V$, Shear force

$EI.\frac{d^2y}{dx^2} \propto M$, Bending Moment

$EI.\frac{dy}{dx} \propto \theta$, Slope

	Type of Beam	Mmax	Slope (θ)	Deflection (Δ)
1.		M	$\theta = \frac{ML}{IE} = \frac{ML}{EI}$	$\Delta = \theta \times L/2 = \frac{ML^2}{2EI}$
2.		WL	$\theta = \frac{ML}{2EI} = \frac{WL^2}{2EI}$	$\Delta = \theta \times 2L/3 = \frac{Wl^3}{3EI}$
3.		$\frac{WL^2}{2}$	$\theta = \frac{ML}{3EI} = \frac{WL^3}{6EI}$	$\Delta = \theta \times \frac{3L}{4} = \frac{Wl^4}{8EI}$
4.		$\frac{Wl^2}{6}$	$\theta = \frac{ML}{4EI} = \frac{Wl^3}{24EI}$	$\Delta = \theta \times \frac{4L}{5} = \frac{Wl^4}{30EI}$

* Virtual Work Method.

It states that "Work done by a real force due to ~~vertical~~ virtual displacement is equal to work done by virtual force due to real displacement".

For beams & frames,

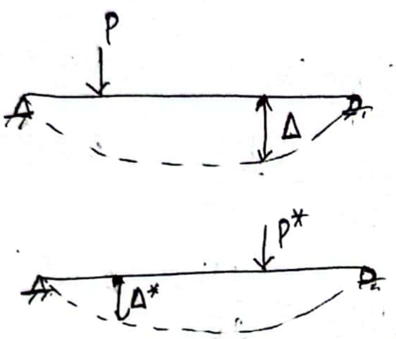
$$\theta = \Sigma \int \frac{M_p \overline{M} \cdot dx}{EI}$$

where, M_p = B.M due to loads

$\overline{M}$ = BM due to unit moment (1

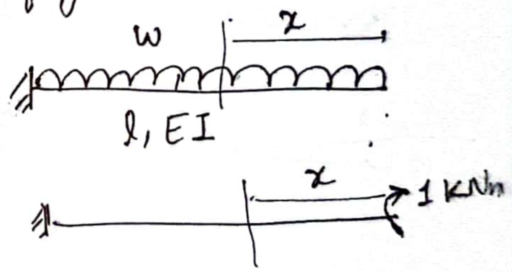
$$\Delta = \Sigma \int \frac{M_p \cdot \overline{M_x} \cdot dx}{EI}$$

where, $\overline{M_1}$ = BM due to unit force (1 kN



$$P\Delta^* = P^* \Delta$$

Q) Calculate slope & deflection at free end of given cantilever beam.



Soln for slope, apply 1 KNm at B,

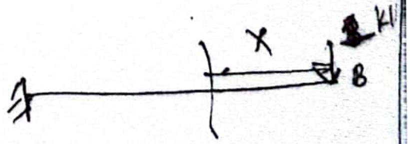
$$M_p = -\frac{wx^2}{2}$$

$$\overline{M} = -1$$

$$\therefore \theta = \int_0^L \frac{M_p \cdot \overline{M} \cdot dx}{EI} = \frac{1}{EI}\int_0^L \frac{wx^2}{2} \cdot dx$$

$$= \frac{w}{2EI} * \left[\frac{x^3}{3}\right]_0^L$$

$$= \frac{wL^3}{6EI}$$



for deflection, apply 1 KN load at B,

$$M_p = -\frac{wx^2}{2}, \quad \overline{M} = -x$$

$$\therefore \Delta = \int_0^L \frac{wx^3}{2EI} \cdot dx = \frac{w}{2EI} \times \left[\frac{x^4}{4}\right]_0^L = \frac{wL^4}{8EI}$$

* Castigliano's Theorem

Theorem I : The partial derivative of strain energy stored due to several loads with respect to particular force gives the deflection at the point where that particular load is applied.

$$\boxed{\Delta_i = \frac{\partial U}{\partial P_i}}$$

We know,

$$U = \Sigma \int \frac{M^2 \cdot dx}{2EI}$$

$$\therefore \int \frac{\partial U}{\partial P_i} = \Sigma \int \frac{\frac{\partial M^2}{\partial P_i} \cdot dx}{2EI}$$

$$\therefore \Delta_i = \Sigma \int \frac{2M \cdot \frac{\partial M}{\partial P_i} \cdot dx}{2EI}$$

$$\boxed{\Delta_i = \Sigma \int \frac{M \cdot \frac{\partial M}{\partial P_i} \cdot dx}{EI}}$$

Theorem II : The partial derivative of strain energy with respect to a displacement gives the value of force induced at that point.

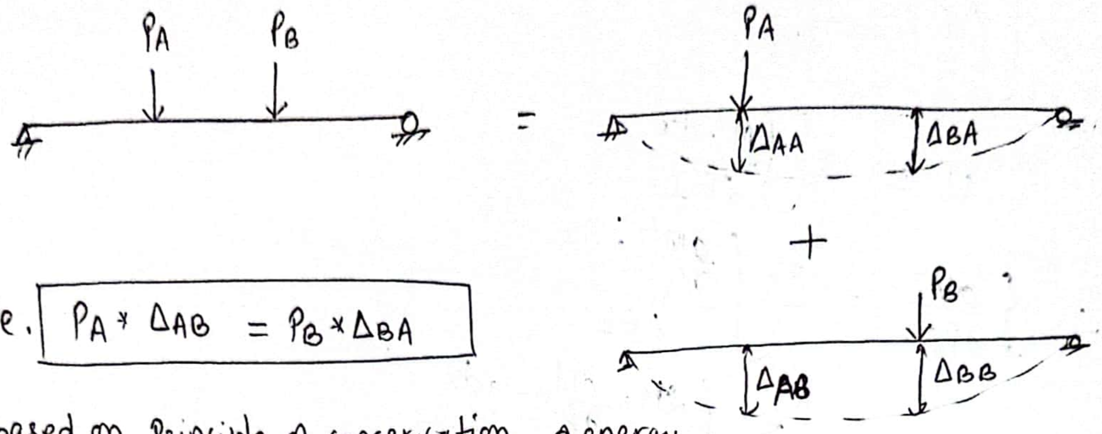
$$\boxed{\frac{\partial U}{\partial \Delta_i} = P_i} \qquad \text{Also, } \boxed{\frac{\partial U}{\partial \theta_i} = M_i}$$

Both the theorems are based on principle of conservation of energy.

Inorder to calculate displacement at a point, force must have been applied at that point.

Betti's law

" The product of first load with displacement due to second load along the direction of first load is equal to product of second load with displacement due to first load along the direction of second load. ~~are equal~~.



ie. $\boxed{P_A \times \Delta_{AB} = P_B \times \Delta_{BA}}$

→ based on Principle of conservation of energy.

Maxwell's Reciprocal Theorem

→ It is special case of Betti's law.

for $P_A = P_B = 1\,kN$

$$\boxed{\Delta_{AB} = \Delta_{BA}}$$

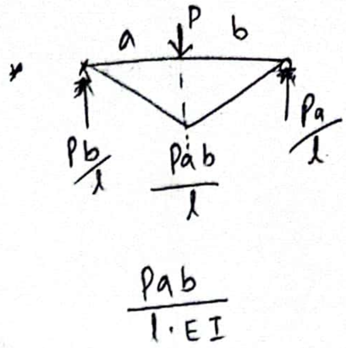
It states " Displacement along A-direction due to unit load along B is equal to displacement along B direction due to unit load along A."

→ It is applicable for calculating stiffness and flexibility matrix.

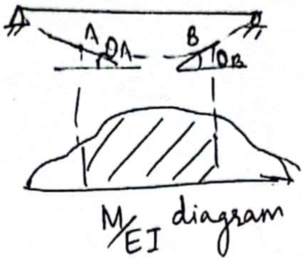
→ It is also used for drawing ILD for indeterminate structures.

Moment Area Method

Theorem I : Difference between slope at two points on elastic line of curve (deflected shape) is equal to area of $\frac{M}{EI}$ diagram between those two points.

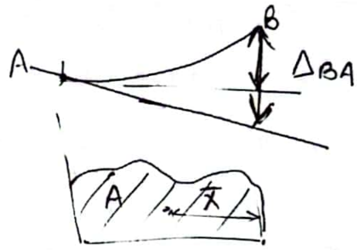


$\theta_A - \theta_B$ = Area of hatched diagram



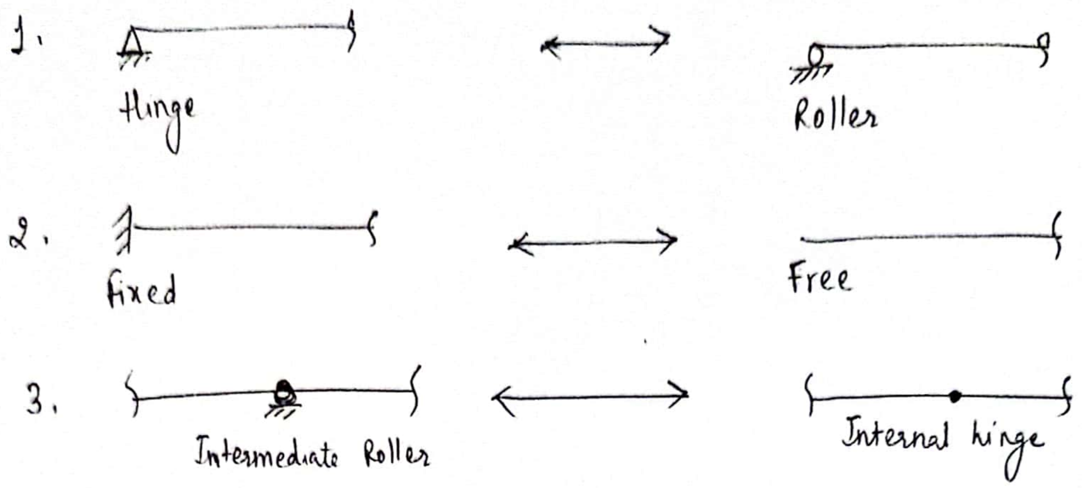
Theorem II : Deflection of a point w.r.t. tangent at other point is equal to moment of area of $\frac{M}{EI}$ diagram between those two points taken about the point where deflection is to be calculated.

$$\Delta_{BA} = A * \bar{x}$$



* Congugate Beam Method

Real Beam | Conjugate Beam



Theorem I : Slope at any point on elastic line of curve of real beam i[s] equal to shear force at the same point of conjugate bea[m] loaded with $\frac{M}{EI}$ diagram.

* Support = Rx^n
Intermediate = SF nikalne.

$$\boxed{\theta \rightarrow V}$$

Theorem II : Deflection at any point on elastic line of curve of real b[eam] is equal to bending moment at the same point of conju[gate] beam loaded with $\frac{M}{EI}$ diagram.

$$\boxed{\Delta \rightarrow B.M}$$

Steps

1. Beam with load given.
2. B.M diagram banaune. (Real beam & Conjugate beam ko conditions apply garne)
3. M/EI diagram.
4. For slope = Jun point ko lagi ho, teha SF nikalne
5. For deflection = Jun " " " ", teha BM nikalne.

* Degree of Static & Kinematic Indeterminacy of Structures

Indeterminacy
- Static
  - External
  - Internal
- Kinematic

* Degree of static indeterminacy of pin jointed plane frame (plane truss)

→ Degree of external indeterminacy $= r-3$ ✓

→ Degree of internal indeterminacy $= m-(2j-3)$ ✓ $\quad (m = 2j-3)$

→ Degree of static indeterminacy $= E.I + I.I$

$$= (r-3) + m-(2j-3)$$

$$= (m+r)-2j$$

* If $m < 2j-3$; internally unstable

$m = 2j-3$, stable/unstable , statically determinate internally if stable internally

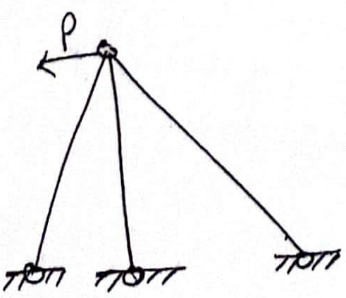
$m > 2j-3$, overstiff , statically indeterminate internally

✓ Alternatively,

Degree of static indeterminacy : $(m+r)-2j$

m = no. of members
j = no. of joints
r = no. of unknown reaction forces

- If $(m+r) < 2j$, unstable
- If $(m+r) = 2j$, stable , statically determinate
- If $(m+r) > 2j$, stable , statically indeterminate



$m = 3, \quad r = 6, \quad j = 4$

$$\therefore DSI = (m+r)-2j$$

$$= (3+6) - 2\times 4$$

$$= 1 .$$

Here, $(m+r) > 2j$, it is stable but statically indeterminate.

* Degree of Static Indeterminacy of Rigid Jointed plane frame

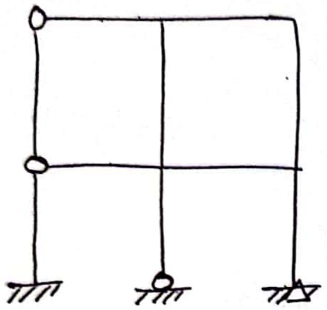
$DSI = (3m+r) - (3j+c)$; c = eqⁿ of condition = No. of members connected to an internal hinge − 1

→ If $(3m+r) < (3j+c)$, unstable.

→ If $(3m+r) = (3j+c)$, stable/unstable, statically determinate if stable

→ If $(3m+r) > (3j+c)$, stable/unstable, statically indeterminate

*



$m = 10$, $j = 9$, $r = 6$, $c = (2-1)+(3-1) = 3$

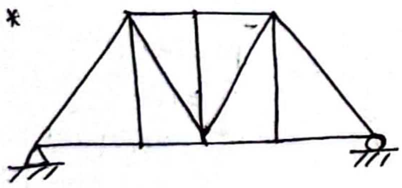
Now,

$$DSI = (3m+r)-(3j+c)$$
$$= (3\times10+6) - (3\times9+3)$$
$$= 36 - 30$$
$$= 6$$

* Degree of Kinematic Indeterminacy of Pin Jointed Plane frame (Plane Truss)

$DKI = 2j - r$

*



$j = 8$, $r = 3$

$$DKI = 2j - r$$
$$= 2\times8-3$$
$$= 13$$

* Degree of Kinematic Indeterminacy of Rigid Jointed Plane Frame (Plane frame)

① If plane frame is extensible, $DKI = 3j - r$

② If plane frame is inextensible & unbraced, $DKI = 3j - (r+m)$

③ If plane frame is inextensible & braced, count degree of freedom by visual inspection.

Static Indeterminacy (D_s)

It is the no. of unknown forces in the structure in excess to number of available equations.

D_s = No. of unknowns − No. of available equations.

Equilibrium Eqn^s — Extra equations (Internal hinge - -)

External Indeterminacy (D_{se}) ⇒ Due to support conditions

Internal Indeterminacy (D_{si}) ⇒ Due to extra redundant members,

* For Plane frames (Rigid jointed Plane frames) (Beams & frames)  No rotation at joints.

$D_{se} = r - 3$ $\qquad$ r = no. of rx^n forces

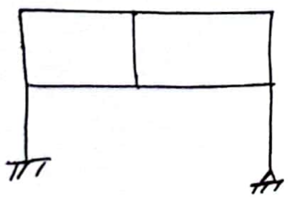
$D_{si} = 3C$ $\qquad$ where, C = No. of cuts required to make open tree configuration
or, No. of closed loops.

$D_s = D_{se} + D_{si}$

① 

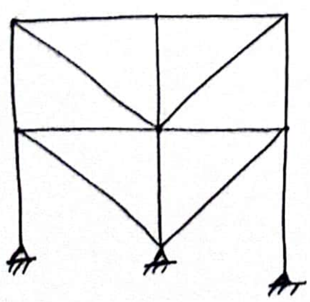
$D_{se} = r - 3 = 6 - 3 = 3$

$D_{si} = 0$

② 

$D_{se} = r - 3 = 5 - 3 = 2$

$D_{si} = 3c = 3 \times 2 = 6$

③ 

$D_{se} = r - 3 = 6 - 3 = 3$

$D_{si} = 3c = 3 \times 6 = 18$

Alternately,

$$D_s = (3m + r) - (3j)$$

* For Pin jointed Plane frame (Trusses)

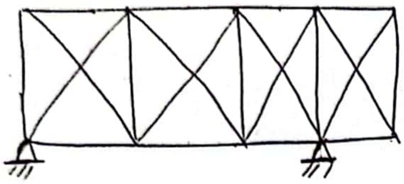
$D_{se} = r - 3$

$D_{si} = m - 2j + 3$

$\therefore D_s = D_{se} + D_{si}$

$= r - 3 + m - 2j + 3$

$= (m + r) - (2j)$



$m = 29$, $r = 4$, $j = 14$

$D_{se} = r - 3 = 4 - 3 = 1$

$D_{si} = m - 2j + 3 = 29 - 2 \times 14 + 3$

$= 32 - 28$

$= 4$

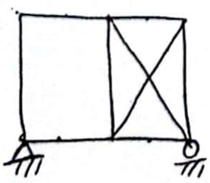
$D_s = 5.$

Ans

Check the stability of truss

1. $m < 2j - 3 \Rightarrow$ Internally unstable
2. $m = 2j - 3 \Rightarrow$ stable and statically determinate
3. $m > 2j - 3 \Rightarrow$ overstiff & statically indeterminate internally.

Check the stability of truss



$m = 11$

$j = 7$

$2j - 3 = 2 \times 7 - 3$

$= 11$

$m = 2j - 3 \Rightarrow$ Stable.

For Hybrid structure.

$D_{se} = r - 3$

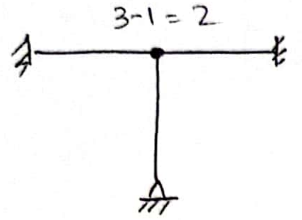
$D_{si} = 3C - \Sigma S$

$D_s = D_{se} + D_{si}$

where, s = effective pin value
$= m' - 1$

m' = No. of members meeting at the hinge.

①

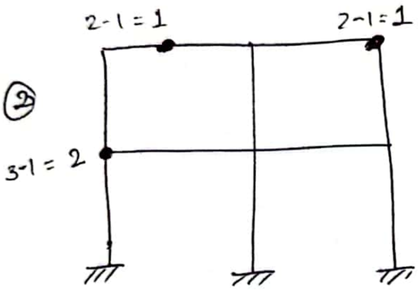


$D_{se} = r - 3 = 8 - 3 = 5$

$D_{si} = 3C - \Sigma S = 3 \times 0 - 2 = -2$

$D_s = 5 - 2 = 3$ #

②

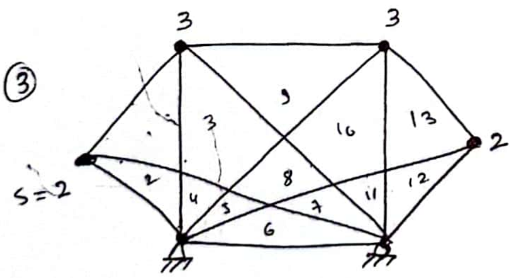


$D_{se} = r - 3 = 9 - 3 = 6$

$D_{si} = 3C - \Sigma S = 3 \times 2 - (1+1+2) = 2$

$D_s = 6 + 2 = 8$

③



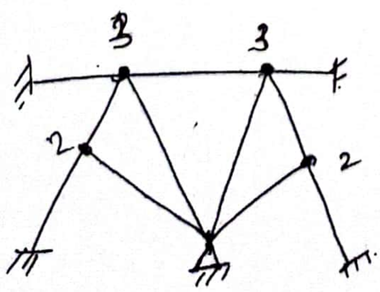
$m = 24$, $j = 12$

$D_{se} = r - 3 = 3 - 3 = 0$

$\cancel{D_{si} = 3C - \Sigma S} = 3 \times 13 - (2+3+3+2)$

$D_{si} = m - 2j + 3 = 24 - 2 \times 12 + 3$
$= 3$

④



$D_{se} = r - 3 = 14 - 3 = 11$

$D_{si} = 3C - \Sigma S = 3 \times 3 - (2 + 3 + 3 + 2)$
$= 9 - 10$
$= -1$

* Kinematic Indeterminacy (Dk)

- It is the degree of freedom for given structure.

For rigid joint, DOF = 3 (1H, 1V, 1θ)

For pin joint, DOF = 2 (1H, 1V)

For plane frames,

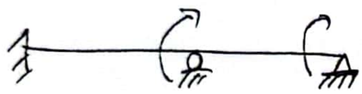
$$D_k = 3j - r \quad \text{[Extensible members]}$$

$$= 3j - (r+m) \quad \text{[Inextensible members]}$$

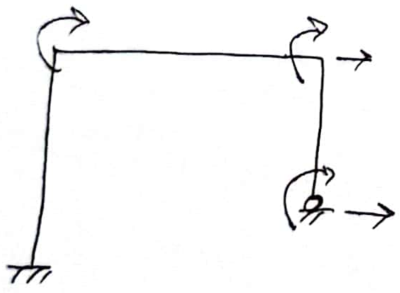
1) 

m = 2, j = 3

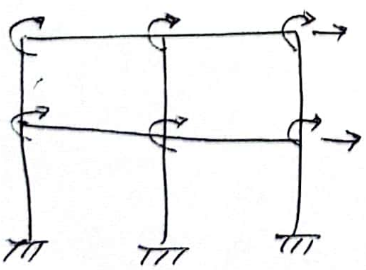
$$D_k = 3j - (r+m)$$
$$= 3 \times 3 - (5+2)$$
$$= 2$$

2) 

DK = 2.

3) 

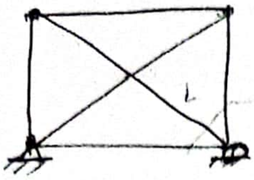
Dk = 5

4) 

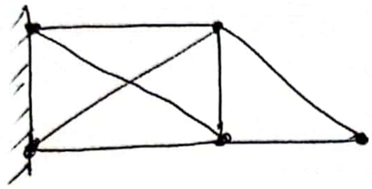
Dk = 8 $[3 \times 9 - (9+10)]$

For Trusses

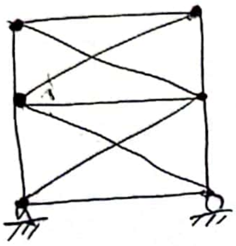
$$D_K = 2j - r$$

1) 

$D_K = 2 \times 4 - 3 = 5$

2) 

$D_K = 2 \times 5 - 4 = 6$

3) 

$D_K = 2 \times 6 - 3 = 9$

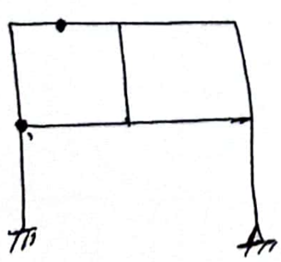
Hybrid Structure

$D_K = 3j - (r + m) + \Sigma S + 4 *$ No. of member hinges

$\Sigma S \rightarrow$ For joint hinge.

$4 \rightarrow$ no. of DoF per member hinge.

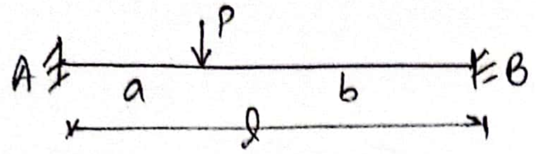
s=2



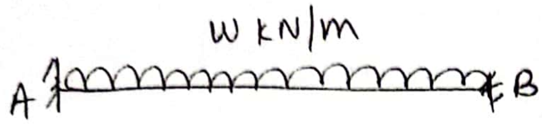
$$D_K = 3 \times 8 - (5 + 9) + 2 + 4 \times 1$$
$$= 24 - 14 + 6$$
$$= 16$$

#

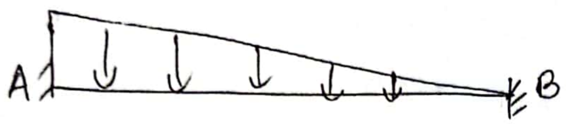
* Fixed End Moments (+ve = Clockwise , -ve = Anticlockwise)

1. 

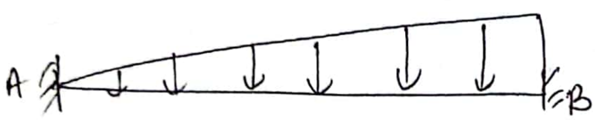
$$FEM_{AB} = -\frac{Pab^2}{l^2} \qquad FEM_{BA} = +\frac{Pa^2b}{l^2}$$

2. 

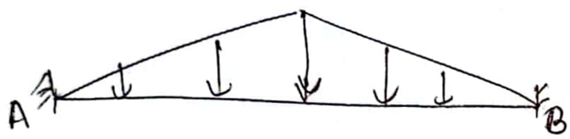
$$FEM_{AB} = \frac{-wl^2}{12} \qquad FEM_{BA} = +\frac{wl^2}{12}$$

3. 

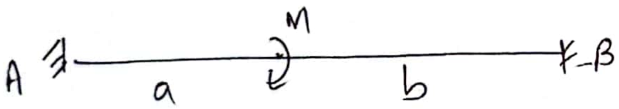
$$FEM_{AB} = \frac{-wl^2}{20} \qquad FEM_{BA} = +\frac{wl^2}{30}$$

4. 

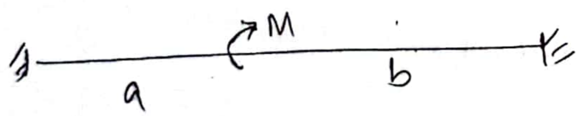
$$FEM_{AB} = -\frac{wl^2}{30} \qquad FEM_{BA} = +\frac{wl^2}{20}$$

5. 

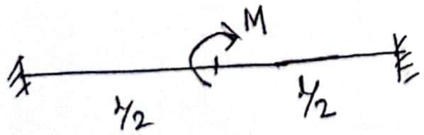
$$FEM_{AB} = -\frac{5wl^2}{96} \qquad FEM_{BA} = +\frac{5wl^2}{96}$$

6. 

$$FEM_{AB} = -\frac{Mb(b-2a)}{l^2}$$

7. 

$$= \quad FEM_{BA} = +\frac{Ma(2b-a)}{l^2}$$

a) 

$$FEM_{AB} = -\frac{M\cdot\frac{l}{2}\left(\frac{l}{2} - 2\times\frac{l}{2}\right)}{l^2}$$

$$= -\frac{M\cdot\frac{l}{2}\cdot -\frac{l}{2}}{l^2}$$

$$= \frac{M}{4}$$

$$FEM_{BA} = -\frac{M}{4}$$

* Slope Deflection Method.

- Developed by GA Maney in 1915
- Suitable for continuous and rigid jointed frames.
- The joints are assumed to be rigid
- Shear deformations are neglected.
- Rotations at different joints are unknown.
- Equilibrium equations are used to calculate unknown rotation

Slope deflection equations are :

$$M_{AB} = FEM_{AB} + \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\Delta}{l}\right]$$

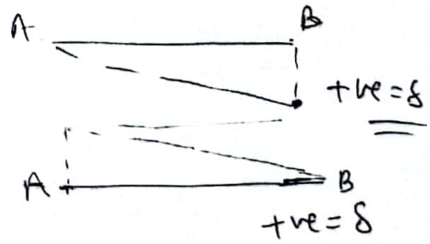
$$M_{BA} = FEM_{BA} + \frac{2EI}{l}\left[\theta_A + 2\theta_B - \frac{3\Delta}{l}\right]$$

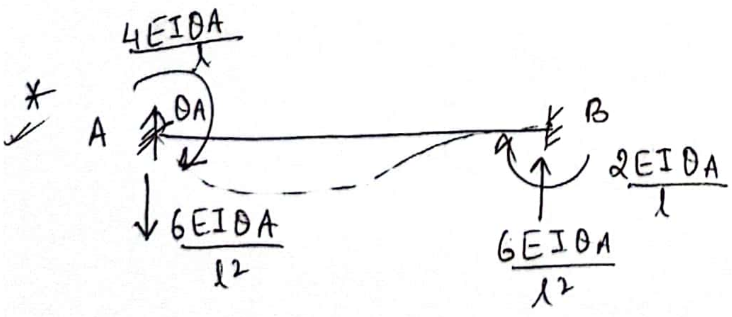
Sign Conventions

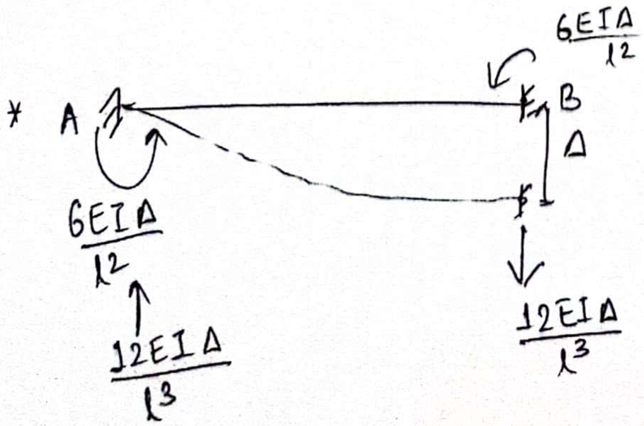
θ_A & θ_B = +ve for Clockwise

= -ve for anticlockwise

Δ = +ve if right support settles down w.r.t left support or vice versa.







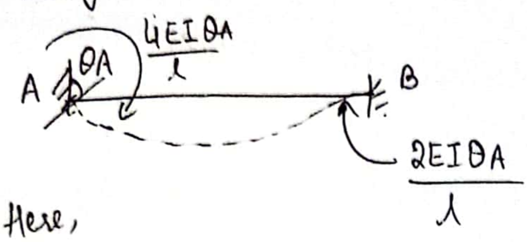
* Moment Distribution Method

→ Developed by Hardy Cross in 1930.

→ It is iterative process.

→ Iteration are applied when carry over moments are negligible.

* Carryover factor.



Here,

$$M_{AB} = \frac{4EI\theta_A}{l}$$

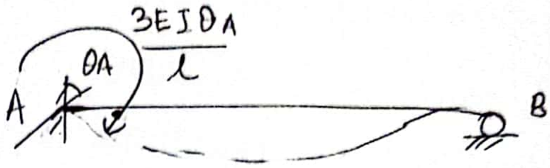
$$M_{BA} = \frac{2EI\theta_A}{l}$$

$$\therefore M_{BA} = \frac{1}{2} M_{AB}$$

The factor $\frac{1}{2}$ is carryover factor for far end fixed.

For far end fixed,

$$\text{Stiffness, } K = \frac{4EI}{l}$$



Here,

$$M_{AB} = \frac{3EI\theta_A}{l}$$

$$M_{BA} = 0$$

$$\therefore M_{BA} = 0 * M_{AB}$$

The factor 0 is called carry over factor for far end hinged or roller or pinned.

For far end pinned,

$$\text{stiffness, } K = \frac{3EI}{l} = \frac{3}{4} * \frac{4EI}{l}$$

∴ Stiffness of member with far end hinged/pinned is $\frac{3}{4}$ (75%) of the member with far end fixed.

* Distribution Factor.

→ It is the fraction of the moment shared by a member of the total applied moment.

Also, it is ratio of stiffness of member to the sum of stiffness of members meeting at that joint.

$$D.F = \frac{K_i}{\Sigma K_i}$$

* Force Method & Displacement Methods.

Force Method	Displacement Method
1. Degree of static indeterminacy is calculated.	1. Degree of kinematic indeterminacy is calculated.
2. Elements of flexibility matrix is calculated.	2. Elements of stiffness matrix is Calculated.
3. Forces are unknown.	3. Displacements are unknown.
4. Compatibility equations are used.	4. ~~Stiffness~~ Equilibrium equations are used.
Types.	Types.
(a) Consistent Deformation Method	(a) Slope deflection Method
(b) Three moment equation	(b) Moment distribution method
(c) Column Analogy method	(c) Kani's Method
(d) flexibility Matrix Method	(d) Stiffness matrix method
(e) Castigliano's Theorem	

Flexibility – It is the displacement produced per unit force applied. or, It is the rotation produced per unit moment applied. $\delta = \Delta/F$ or $\delta = \theta/M$

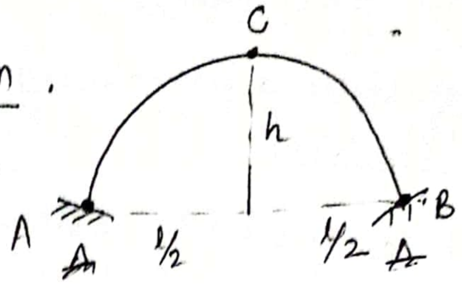
Stiffness – It is the force produced per unit displacement applied, or, It is the moment produced per unit rotation applied. $K = F/\Delta$ or. $K = M/\theta$

Properties of flexibility matrix/stiffness matrix

1. It is symmetric matrix. (Corresponding elements equal) — Square Matrix
2. The elements of main diagonal are always positive.
3. It is system or structure specific (Support, length etc). It doesn't depend upon applied load.

Arches

- The supports are called Springings.
- Topmost point of arch is called crown.
- Span of arch $= l$
- Rise of arch $= h$
- In 3-hinged arch, third hinge can be located anywhere along the axis of arch.



Geometry of Parabolic arch.

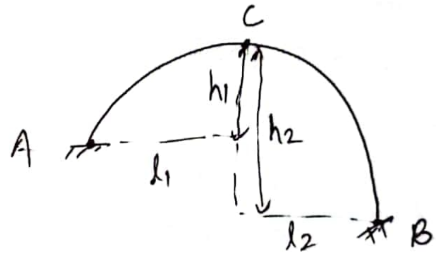
$$y = \frac{4hx}{l^2}(l-x)$$

$$\text{Slope, } \tan\theta = \frac{dy}{dx} = \frac{4h}{l^2}(l-2x)$$

For supports at different levels.

$$l_1 = \frac{\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}} * l$$

$$l_2 = \frac{\sqrt{h_2}}{\sqrt{h_1}+\sqrt{h_2}} * l$$

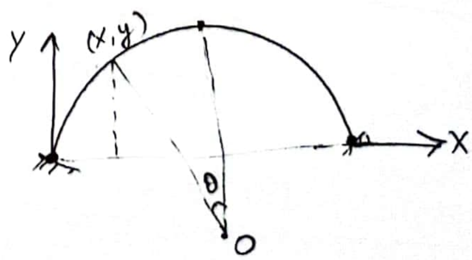


Circular Arch.

$$\text{Radius, } R = \frac{l^2}{8h} + \frac{h}{2}$$

$$x = \frac{l}{2} - R\sin\theta$$

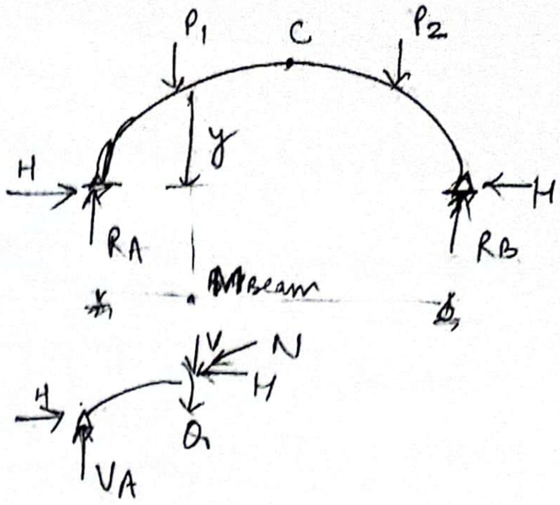
$$y = h - R(1-\cos\theta)$$



* **Three Hinged Arch**

→ Statically determinate arch.

* The internal forces developed at any section of arch are;



B.M, $M_x = M_{Beam} - H * y$

Normal Thrust (acts in tangent with axis of arch)

Axial force (अक्षीय)

$$N = V\sin\theta + H\cos\theta$$

Radial shear ($\perp$r to axis)

$$Q = V\cos\theta - H\sin\theta)$$

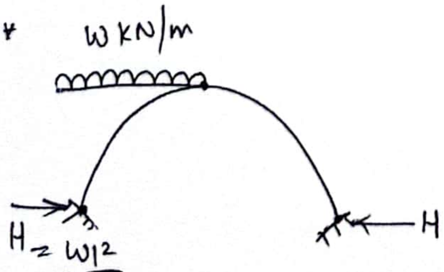
→ for long span structures, arch is preferred in comparison to beam because B.M at a section is very less in comparison to beam by amount Hy. So, lesser depth of arch can carry a larger amount of load due to arching action.

→ In arch, most of the load is dissipated by Normal Thrust.

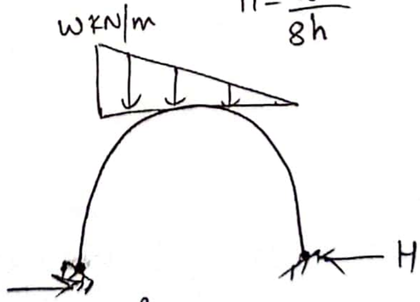
→ BM at any section of arch loaded with UDL on entire span is zero.

$H = \frac{wl^2}{8h}$ Parabolic arch.

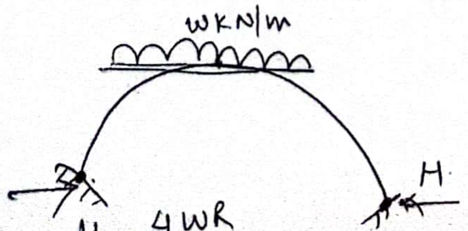
*



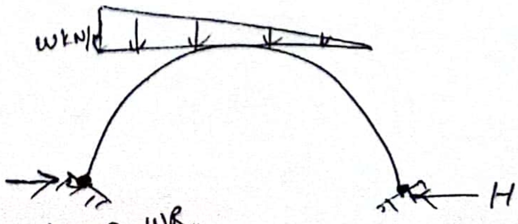
$H = \frac{wl^2}{16h}$ (Parabolic arch)



$H = \frac{wl^2}{16h}$ (Parabolic arch)



$H = \frac{4}{3}\frac{wR}{\pi}$



$H = \frac{2}{3}\frac{wR}{\pi}$

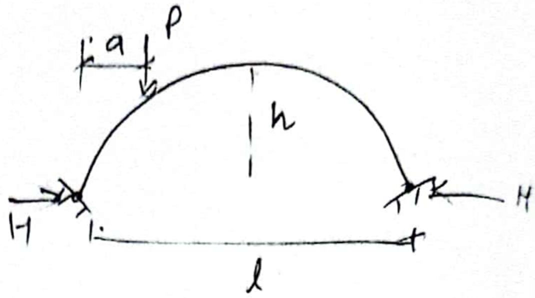
* Two Hinged Arch

- Indeterminate arch with 1 degree of indeterminacy.
- Compatibility equation is used to solve the arch.

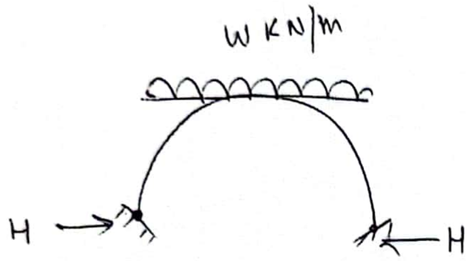
1. For a Single Point Load

Horizontal reaction,

$$H = \frac{5Pa(l-a)(l^2+la-a^2)}{8hl^3}$$



2. For UDL,



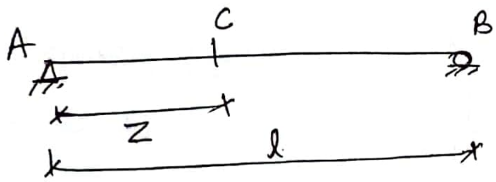
$$H = \frac{wl^2}{8h}$$

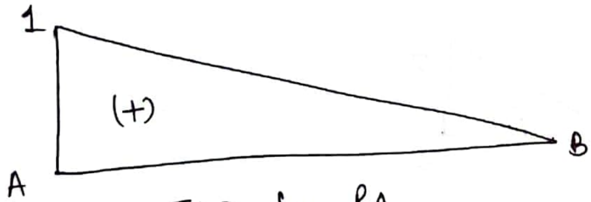
$$H = \frac{wl^2}{16h}$$

* Influence Line Diagram (ILD)

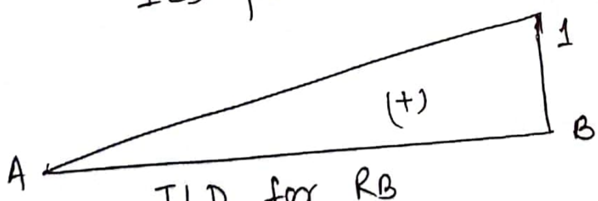
- ILD for a given response at a given section is defined as the diagram obtained by plotting the ordinates when unit load (1 kN) moves from one end to other.
- The ordinate of ILD gives the value of response at the given section when 1 kN load is placed at that point.
- It is system specific ie. it doesnot depend on loads.
- ILD can be drawn for the responses like reactions, S.F & B.M.
- For ILD, load is moving but section is fixed.

1. ILD for Simply supported Beam

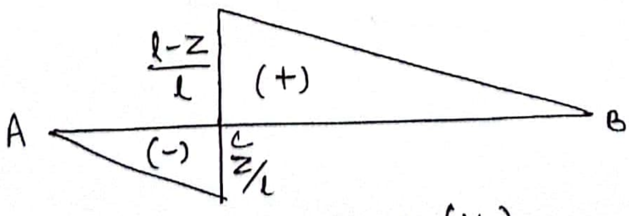




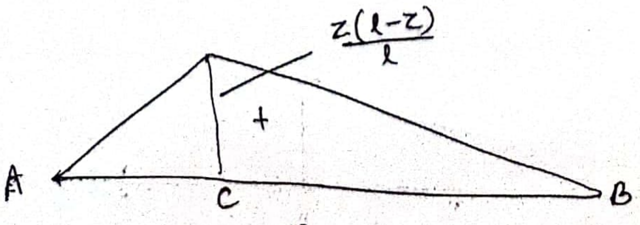
ILD for RA



ILD for RB

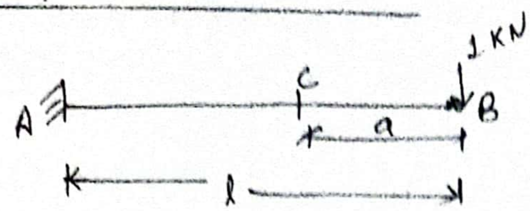


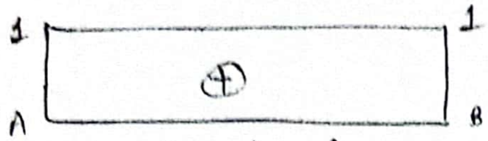
ILD for S.F (V_c)



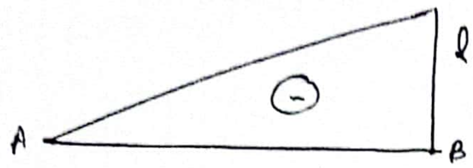
ILD for Mc

2. ILD for Cantilever Beam

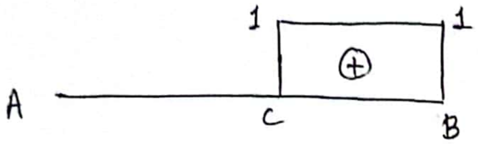




ILD for R_A

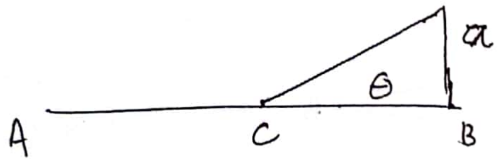


ILD for M_A



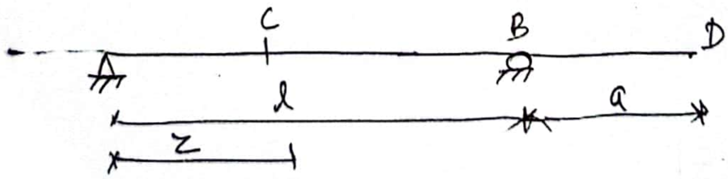
ILD for V_c

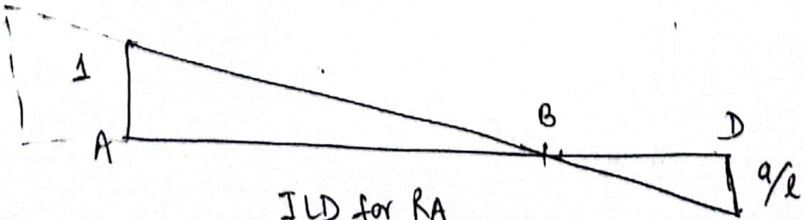
load ; section भन्दा Right
BM, SF aucha
left हुँदा आउँदैन



ILD for M_c

3. ILD for Overhanging beam

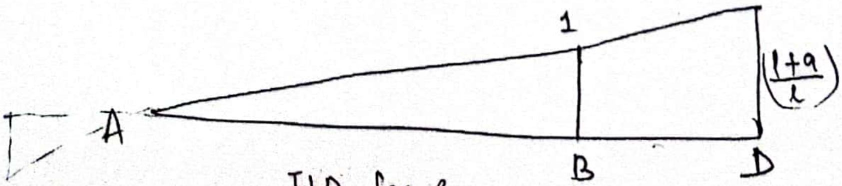




ILD for R_A

$\frac{1}{l} = \frac{x}{a}$

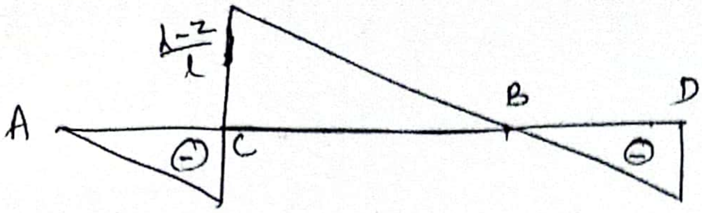
$\Rightarrow x = a/l$



ILD for R_B

$\frac{1}{l} = \frac{x}{a+l}$

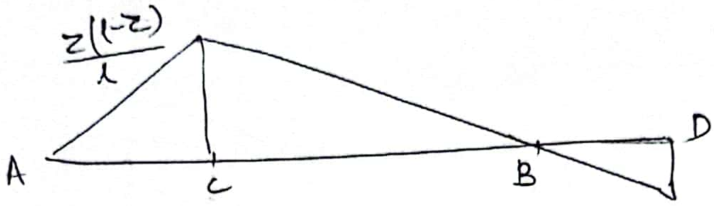
$x = \frac{(l+a)}{l}$



$$\frac{\frac{l-z}{l}}{l-z} = \frac{x}{a}$$

$$x = a/l$$

ILD for V_c



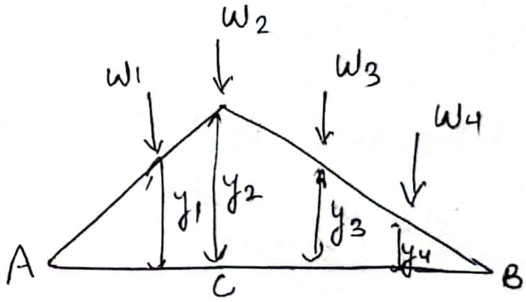
$$\frac{\frac{z(l-z)}{l}}{l-z} = \frac{x}{a}$$

$$x = \frac{az}{l}$$

ILD for M_c.

Uses of ILD.

1. Point Loads.



$$M_c = W_1 y_1 + W_2 y_2 + W_3 y_3 + W_4 y_4$$
$$= \Sigma W_i y_i$$
= Sum of (load * ordinate)

// SF pani same for point Loads)

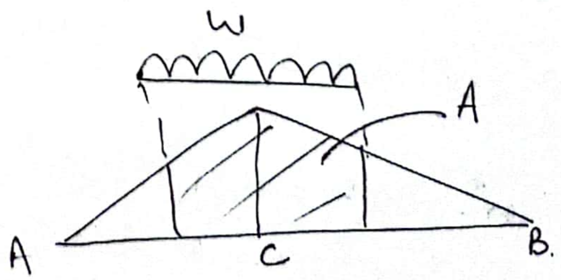
$y_2 = \frac{z(l-z)}{l}$

y_1, y_3, y_4 - Similar Δs nikalne

2. Uniformly distributed Load (UDL)

$M_c = W *$ Area of hatched

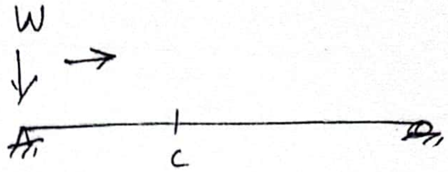
$= W * A$



for SF, rxn too.

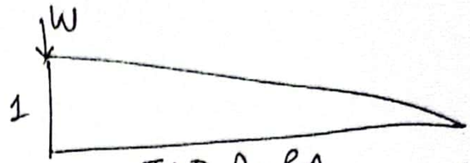
* Moving Loads

1. Single Point Load



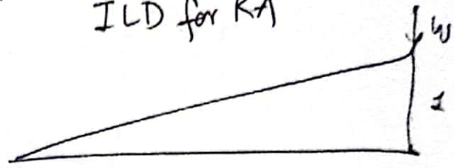
Maximum $R_A = W \times 1$

$= W$



ILD for RA

Maximum $R_B = W \times 1$



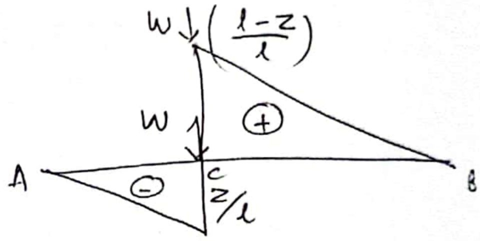
ILD for RB

✓ Maximum -ve SF $= W \times \frac{z}{l}$

(Just left of section)

✓ Maximum +ve SF $= W \times \frac{l-z}{l}$

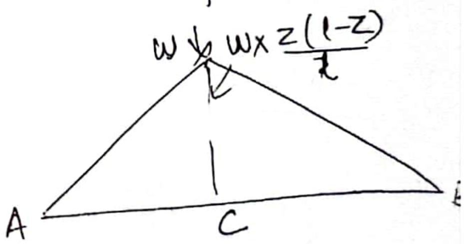
(Just right of section)



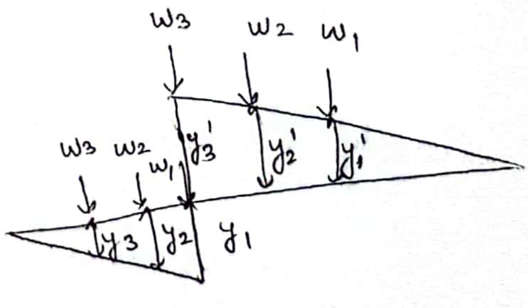
ILD for Vc

✓ Maximum B.M $= W \times \frac{z(l-z)}{l}$

(when load is at section)



2. Series of Point loads (Train of loads)



(a) Max^m +ve S.F & -ve S.F

Max^m -ve S.F $= W_1y_1 + W_2y_2 + W_3y_3$

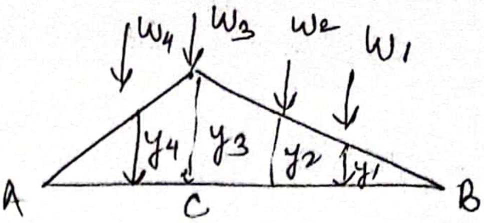
(when 1st load reaches at section C)

Max^m +ve S.F $= W_1y_1' + W_2y_2' + W_3y_3'$

(when last load reaches at section C)

(b) Maximum B.M

load is placed in such a way that



Avg. load on left = Avg. load on right

$$\frac{\Sigma W_{left}}{z} = \frac{\Sigma W_{Right}}{l - z}$$

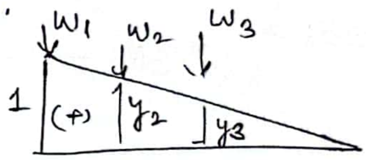
Loads are chossed from the section such that average load changes the sign.

$$\therefore M_{max} = W_1 y_1 + W_2 y_2 + W_3 y_3 + W_4 y_4$$

(c) Absolute maximum -ve S.F & +ve S.F.

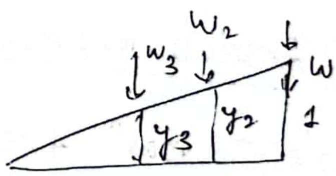
→ determine section where the value is maximum & place the load to cover max^m area

* Absolute max^m +ve S.F occurs at left support.



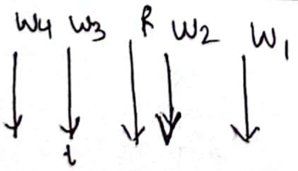
$$\therefore \text{Absolute } max^m \text{ +ve S.F} = W_1 \times 1 + W_2 y_2 + W_3 y_3$$

* Absolute max^m -ve S.F occurs at right support.

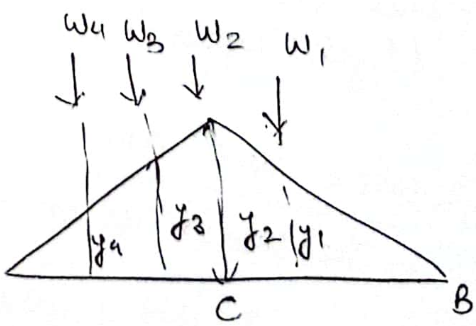


$$\therefore \text{Absolute } max^m \text{ -ve S.F} = W_1 \times 1 + W_2 y_2 + W_3 y_3$$

(d) Absolute max^m B.M



Let R = Resultant of all point loads.
d = distance betn R & max^m load W_2 (assume)



The section for absolute max^m B.M lies at distance.

$$AC = \frac{l}{2} + \frac{d}{2}$$ from left support. (max^m load right of C huda)

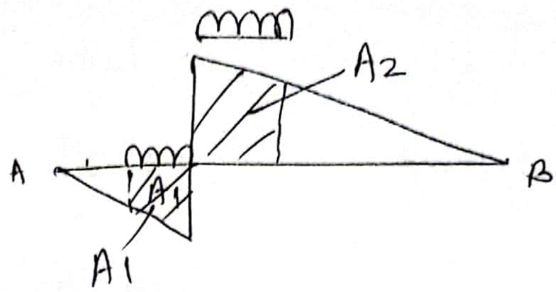
(If max^m load, left of C, $AC = \frac{l}{2} - \frac{d}{2}$)

The loads are placed in such a way that max^m load W_2 lies at section C.

$$\therefore \text{Abs. } max^m \text{ BM} = W_1 y_1 + W_2 y_2 + W_3 y_3 + W_4 y_4$$

3. UDL Shorter than Span

(a) Max^m +ve SF & -ve SF



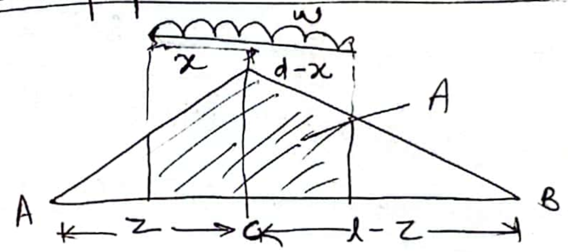
Max^m +ve S.F $= w * A_2$
(tail coincide at the section)

Max^m -ve S.F $= w * A_1$
(head coincide at the section)

(b) Maximum BM

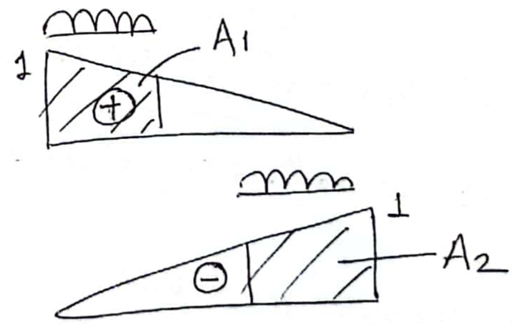
UDL is placed in such a way that section divides the load in the same proportion as it divides the span,

$$\frac{x}{z} = \frac{d-x}{l-z}$$



~~loads~~ Max^m BM $= w * \text{Area}$.

(c) Absolute maximum +ve & -ve SF

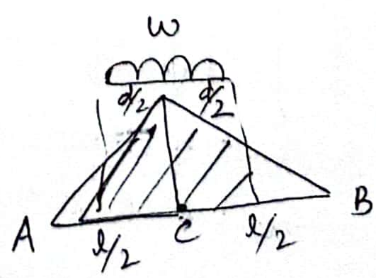


Absolute max^m +ve SF $= w * A_1$
(Left support)

Absolute max^m -ve SF $= w \times A_2$
(Right support)

(d) Absolute max^m BM

i) The section lies at mid-span.

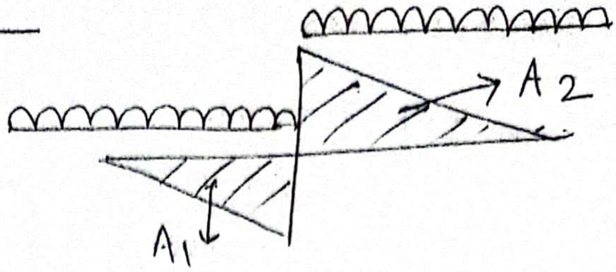


Abs. max^m BM $= w * \text{Area}$

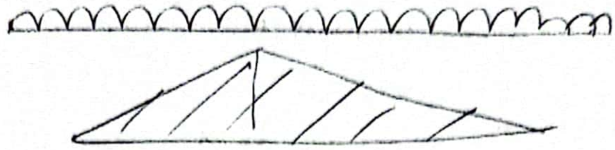
4. UDL longer than span

(1) Max^m -ve SF $= W \times A_1$

(2) Max^m +ve SF $= W \times A_2$



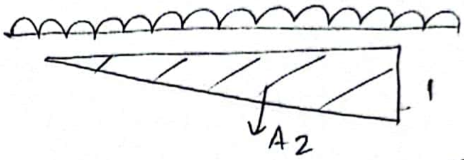
(3) Max^m BM $= W \times \text{Area}$



(4) Absolute Max^m

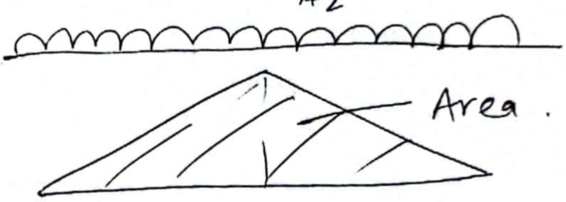
+ve SF $= W \times A_1$ (Left)

-ve SF $= W \times A_2$ (Right)



(5) Absolute Max^m

BM $= W \times \text{Area}$

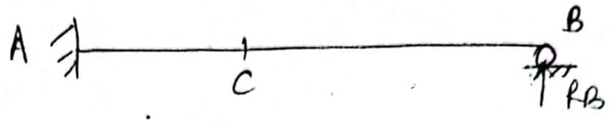


Muller Breslaw's Principle

↳ Applicable for both determinate and indeterminate structures.

↳ It states that "ILD for a given response is the shape obtained by removing the given response and applying unit displacement (for force) or ~~rr~~ rotation (for moment) along the same direction.

↳ Shape of ILD

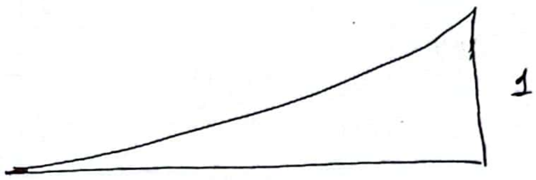


for determinate structure.

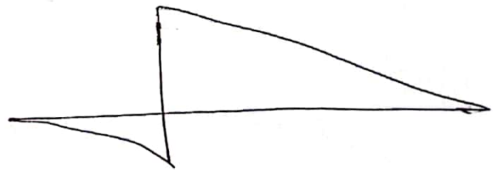
= ILD is always straight line

Indeterminate structure

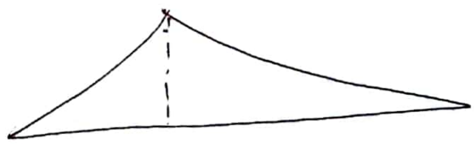
= ILD is always curve shaped. (parabolic or cubic)



ILD for RB



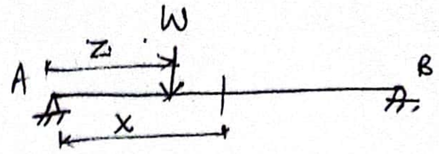
ILD for Vc



ILD for Mc

* Influence lines

→ rolling loads.



① Single Concentrated load

→ Max^m SF = when load is on the section itself.

→ Max^m BM = when the load is over the section.

② UDL longer than the span

→ Max^m -ve SF = when the head of UDL is at the section

Max^m +ve SF = when the tail of the load touches the section

→ Max^m BM = when the load fully occupies the span.

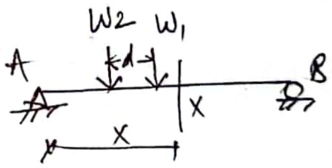
③ UDL Shorter than the span

→ Max^m - ve SF = when the head of UDL touches the section

Max^m +ve SF = when the tail of UDL touches the section

→ Max^m BM = when the position of load is such that the section divides the span & the load in the same ratio.

④ Two Concentrated loads



* Max^m SF

→ when W_1 is at section X & W_2 is bet^n A & X.

→ when W_1 is bet^n X & B & W_2 is at section X.

* Max^m BM = when one of two loads lies on the section.

⑤ Series of Concentrated loads

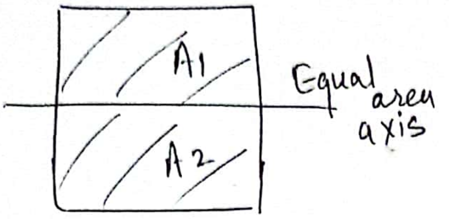
Max^m SF = when one of the loads is at the section itself.

Max^m BM = when load & the resultant of all loads are located equidistant from the center of span ie. centre of beam lies midway between the resultant R & load under consideration

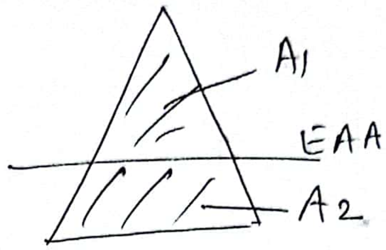
→ Muller - Breslau Principle. - applies to construction of influence lines for force quantities only.

* Plastic Analysis

- → The actual failure mechanism of a structure is calculated.
- → Plastic hinge is the section where all the fibres yields.
- → While plastic hinge is formed, it penetrates towards the depth of section up to equal area axis (yielding of fibres meet)



$A_1 = A_2$

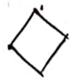


$A_1 = A_2$

→ Shape Factor

→ It is the ratio of plastic moment capacity to the yield moment capacity.

$$S = \frac{M_P}{M_y} \qquad (S > 1 \text{ always})$$

Shape factor for
- Rectangle = 1.5
- Triangle = 2.34
- Circular = 1.698 ≈ 1.7
- Diamond = 2 
- Tubular section = $4/\pi$ 
- I-section = 1.14

→ Load factor

→ ratio of collapse load to the working load.

$$\text{load factor } (\lambda) = \frac{\text{Collapse load}}{\text{Working load}} \qquad (\lambda > 1)$$

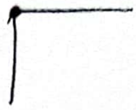
$$\boxed{\text{load factor} = \text{shape factor} * \text{factor of safety}}$$

* Plastic hinge

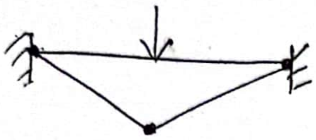
i) formed at

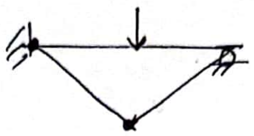
(a) Fixed support

(b) Rigid support

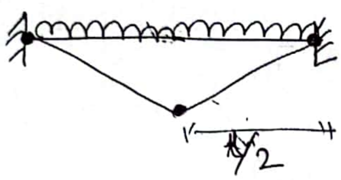


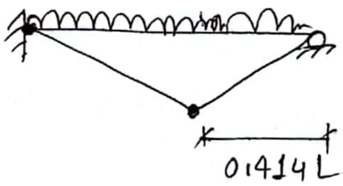
(c) At the point of application of point load.





(d) for UDL





41.4% of length from the ~~hinge~~ hinged end.

\# Mechanisms possible (failure हुआ way)

(a) Beam Mechanism (b) Sway Mechanism (c) Combined Mechanism

ii) No. of hinges formed in sway & combined Mechanism = $D_s + 1$

* Plastic moment capacity is the maximum of all Mps obtained from all mechanism.

* Collapse load is the minimum load obtained from all the mechanisms.

Basic theorem

(a) Lower Bound theorem or Static theorem

(b) Upper Bound theorem or Kinematic theorem

(c) Uniqness theorem