Engineering Economics

9. **Engineering Economics** **(3 marks)**

9.1 Benefit cost analysis, cost classification, sensitivity analysis, internal rate of return, time value of money; economic equilibrium, demand, supply and production, net present value, financial and economic evaluation

Economics

- Economics is science of wealth. – Adam Smith
- Economics is on one side study of wealth; and on the other side a study of human welfare based on wealth. - Marshall
- Economics is the social science that examines how people choose to use limited or scarce resources in attempting to satisfy unlimited wants. –N. Gregory Mankiw
- Economics is a science which studies human behavior as a relationship between ends and scarce means which have alternative uses. – Lionell Robbins

Engineering Economics

- Engineering economics is the application of economic techniques to the evaluation of design and engineering alternatives. – Dr. John M.Watts

- “Engineering economics deals with the methods that enable one to take economics decision towards minimizing the cost or maximizing the benefit to business organization.”

Resources: - 3M/5M

- Manpower
- Machine
- Materials
- Money
- Minute /Management

Example

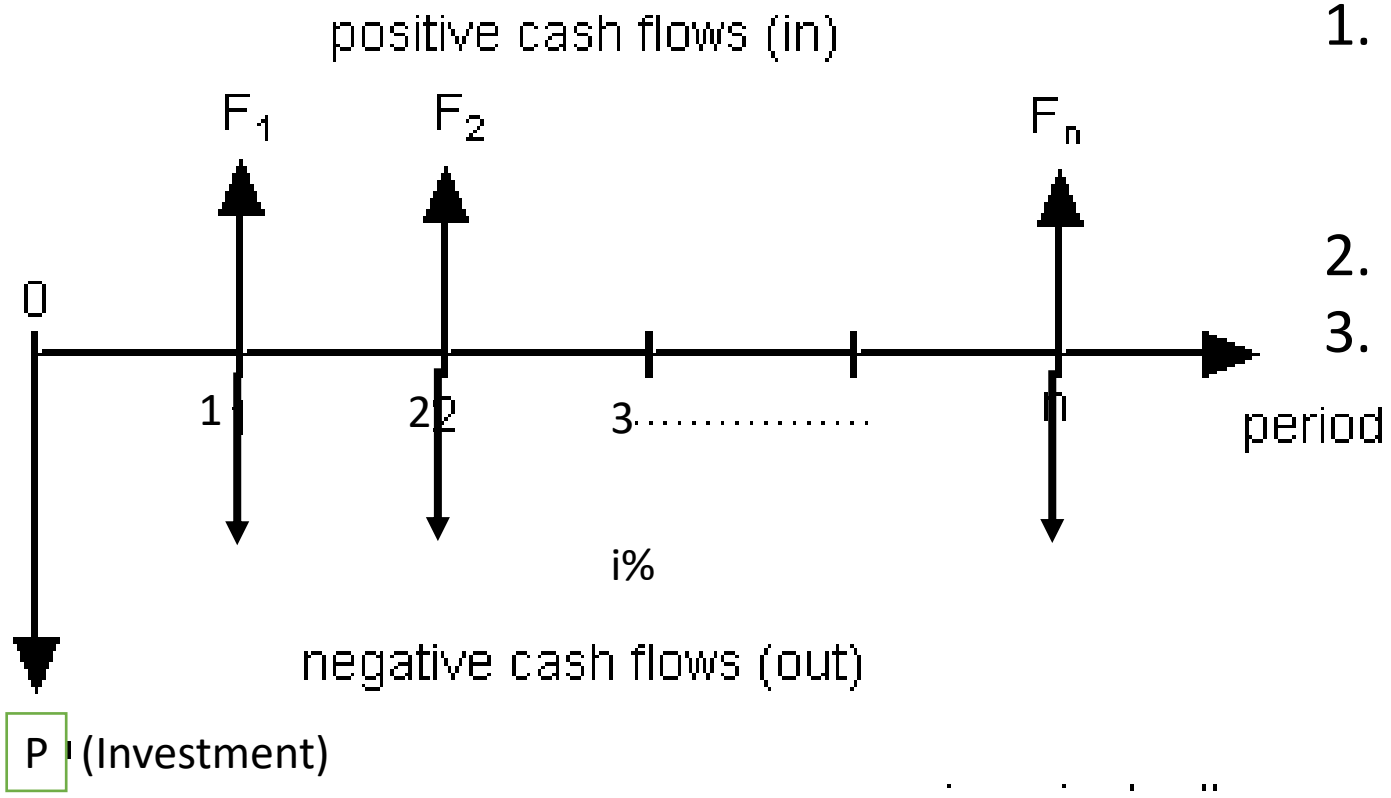
1. Buy a Laptop for online class
2. Specifications of laptop (Core i5), laptop shops, online shopping
3. Dell, Sony, Apple, HP
4. Criteria- upto Rs. 80,000
5. Dell -75000, Sony-120000, Apple-150000, HP-80000
6. Dell
7. Dell-75,000, select Dell.

Cash flow:

Cash inflows are the receipts, revenues, incomes, and savings generated by project and business activity. A plus sign indicates a cash inflow.

- The analysis of events and transaction that affects the cash position of company is termed as cash flow.
- Cash flow is the net amount of cash and cash-equivalents being transferred into and out of a business.
- **Positive cash flow/ Cash inflows** indicates that a company is adding to its cash reserves.
- **Negative cash flow/ Cash outflows** indicates cash outgoing from the firm or project.

Cash Flow Diagram



The Picture should show the following

1. A time interval divided into appropriate no of periods
2. All Cash flow
3. Interest rate

MAJOR PRINCIPLES OF ENGINEERING ECONOMICS

How we take decisions?

- Principle 1: An earlier dollar is worth more than a later dollar.
- Principle 2: All that counts is the differences among alternatives.
- Principle 3: Marginal revenue must exceed marginal cost.
- Principle 4: Additional risk is not taken without expected additional return.

Sources : (Chan S. Park)

Role of Engineers in Decision making:

Economic Decision making

- *Service Improvement*
- *Equipment, project and process selection*
- *Equipment Replacement*
- *New Product and*
- *Product expansion*
- *Risk and uncertainty analysis of the project*

Time value of Money

- The relationship between money and time leads to the concept of time value of money.
- A rupee or dollar in hand is worth more than a rupee or dollar received 'N' years from now.

What do you Prefer ?

A. 100000 today

B. 100000 one year later

C. But why ???

It is a well-known fact that money makes money. The time value of money explains the change in the amount of money over time for funds that are owned (invested) or owed (borrowed). This is the most important concept in engineering economy.

Why Time value of Money?

- Money has time value because the purchasing power of money as well as the earning power of money changes with time.
- During inflation, purchasing power of money decreases over time.
- Money can earn an interest for a period of time.
- Also we can invest money in shares or other platform and earn up to certain amount.
- Even some risk is associated, which demand compensation
- Therefore, both **purchasing power** and **earning power (opportunity cost) of money** should be considered while taking into account the time value of money.
- **Interest is the manifestation of the time value of money.** Computationally, interest is the difference between an ending amount of money and the beginning amount. If the difference is zero or negative, there is no interest.

Simple Interest

- When the interest earned or charged is directly proportional to the initial investment or principal amount (P), the interest rate (i), and number of interest period (N), the interest (I) and the interest rate is said to be simple interest and simple interest rate.
- **I = P*N*i**
- **F= P+I**

- **It is not used in Economic Analysis.**

Compound Interest

- When the interest charge for any interest period (a year) is based on the remaining principal amount plus any accumulated interest charges up the beginning of that period (not withdrawn), the interest is said to the compound.
- Mostly used in Practice

$$F = P(1+i)^N$$

- I = ?

Comparison of Simple and Compound Interest , P= $1000, i=8%. N =3 years

End of Year	Beginning Balance	Interest earned	Ending Balance
0			$1,000
1	$1,000	$80	$1,080
2	$1,080	$80	$1,160
3	$1,160	$80	$1,240

End of Year	Beginning Balance	Interest earned	Ending Balance
0			$1,000
1	$1,000	$80	$1,080
2	$1,080	$86.40	$1,166.40
3	$1,166.40	$93.31	$1,259.71

Nominal Interest rate and Effective Interest rate

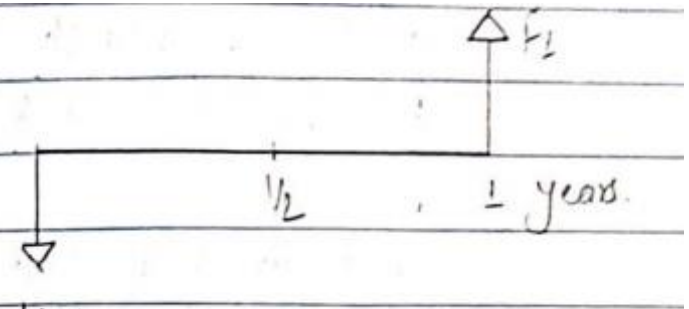
- Interest rate 10% compounded Semiannually (nominal interest rate)
- Interest rate semiannually = 10%/2
  - = 5%

  If P =1000

  Find Accumulated Amount after 1 year?

Interest rate semiannually = $\frac{10\%}{2}$

$= 5\%$



$F_{1/2} = 1000 + \frac{1000 \times 5}{100}$

$= Rs.1050$

$F_1 = Rs.1050 + \frac{1050 \times 5}{100}$

$= Rs.1050 + Rs.52.5$

$F = Rs.1102.5$

$I = F - P$

$= Rs.1102.5 - 1000$

$= 102.5$

Effective interest rate (i) $= \frac{102.5}{1000} \times 100\%$

$\therefore i = 10.25$

Nominal interest rate ?

- In general, interest charged or earned on the principal amount is quoted as ' i % compounded annually or i % per year’.
- Very often, the interest period or time between successive compounding, is less than year
- For example, if the interest rate is 6% per six month, it is customary to quote this rate as '12% compounded semi-annually.
- The basic annual interest rate, 12% in this case, is known as nominal interest rate and denoted by 'r'.

Effective Interest rate

- The actual or exact rate of interest rate earned on the principal during one year is known as effective interest rate and dented by 'i’.

The relationship between effective interest rate 'i' and nominal interest rate 'r' is

$$i = \left(1 + \frac{r}{M}\right)^{M} - 1$$

Where M is number of compounding periods per year.

When M > 1, then i > r

the effective interest rate is useful for describing the compounding effect of interest earned on interest within one year.

Some problems

Question1: Suppose that you invest $1,000 for 1 year at 18% compounded monthly. How much interest would you earn?

Question 2:
Suppose your savings account pays 9% interest compounded quarterly. If you deposit $10,000 for one year, how much would you have at the end of the year ?

Cont

Equating ① & ②

$$P(1+i) = P(1 + r/m)^m$$

$$1+i = (1 + r/m)^m.$$

$$\therefore \boxed{i = (1 + r/m)^m - 1}$$

If, $m = 1$, $i = r$

$m > 1$, $i > r$

If $m \to \infty$ (continuously)

$$i = \lim_{m \to \infty} \left[\left(1 + \frac{r}{m}\right)^m - 1 \right].$$

$$\therefore \boxed{i = e^r - 1}$$

Also,

$$(1+i^{\circ}) = \left(1+\frac{r}{m}\right)^{m}$$

$$\left(1+\frac{r}{m}\right) = (1+i^{\circ})^{1/m}$$

$$\boxed{\frac{r}{m} = (1+i^{\circ})^{1/m} - 1}$$

$$i^{\circ}_{\text{monthly}} = (1+i^{\circ})^{1/12} - 1$$

$$i^{\circ}_{\text{quarterly}} = (1+i^{\circ})^{1/4} - 1.$$

$$i^{\circ}_{\text{semiannually}} = (1+i^{\circ})^{1/2} - 1.$$

Practice problem

Q. What is the effective interest rate of nominal interest 9% per year if the compounding is

(a) yearly (b) quarterly (c) semi-annually

(d) monthly (e) daily (f) continuously ($m \to \infty$)

Soln,

Given, Nominal interest rate (r) = 9% = 0.09

(a) For yearly compounding ; $m = 1$.

$$i = \left(1 + \frac{0.09}{1}\right)^1 - 1 = 0.09 = 9\%$$

(b) For compounding quarterly ; $m = 4$.

$$i = \left(1 + \frac{r}{m}\right)^m - 1 = \left(1 + \frac{0.09}{4}\right)^4 - 1 = 0.093 = 9.3\%$$

(f) For compounding continuously ; $m \to \infty$

$$\therefore i = e^r - 1 = e^{0.09} - 1 = 0.09417 = 9.417\%$$

When is the maximum effective interest rate?

If quarterly interest rate is 3%. find the nominal and effective interest rates.

Solution,

Given. $\frac{r}{m} = 3\% = 0.03$

$m = 4$ for compounding quarterly.

Nominal interest rates $= 3\% \times m = 3\% \times 4 = 12\%$.

$\therefore r = 12\%$.

$$i = \left(1 + \frac{r}{m}\right)^m - 1$$

$$= (1 + 0.03)^4 - 1$$

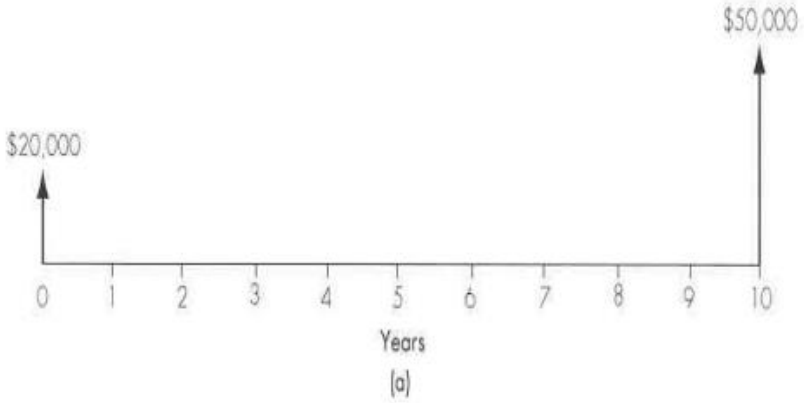
$\therefore i = 0.1255$

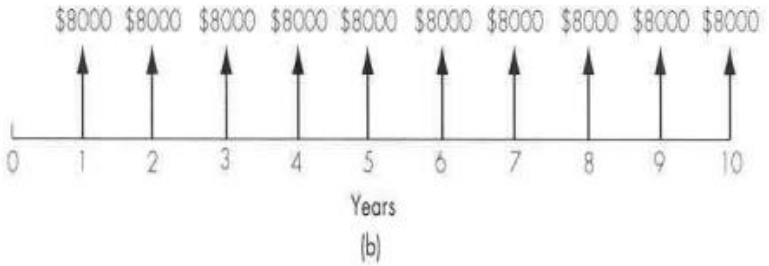
$\therefore i = 12.55\%$.

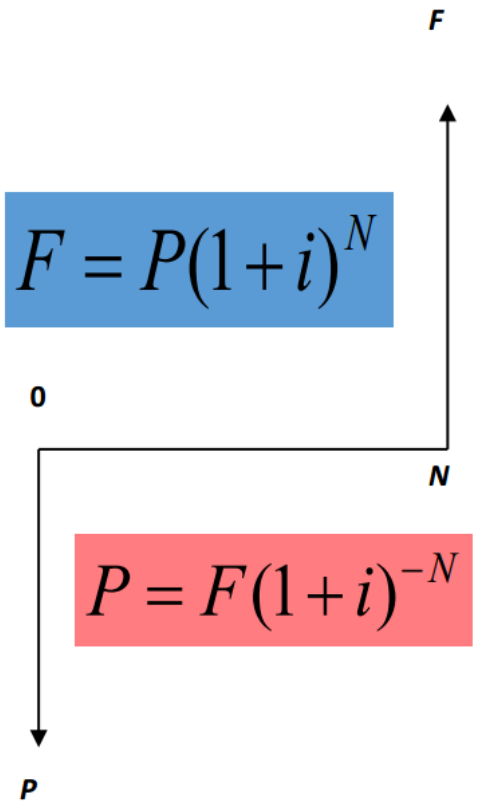
$i > r$ if $m > 1$

Economic Equivalence

- Two things are said to be equivalent when they have the same effect.
- Economic equivalence refers to the fact that a cash flow - whether single payment or series of payments - can be converted to an equivalent cash flow at any point in time.







$$F = P(1+i)^{N}$$

$$P = F(1+i)^{-N}$$

Economic Equivalence

Economic equivalence is a combination of interest rate and time value of money to determine the different amounts of money at different points in time that are equal in economic value.

- PRINCIPLE 1 : - Equivalence calculations made to compare alternatives require common time basis.
- PRINCIPLE 2 : - Equivalence depends on interest rate.
- PRINCIPLE 3 :- Equivalence calculations may require converting multiple cash flow to a single cash flow.
- PRINCIPLE 4 :- Equivalence is maintained regardless of point of view.

Various dollar amounts that will be economically equivalent to $3,000 in 5 years, given an interest rate of 8%

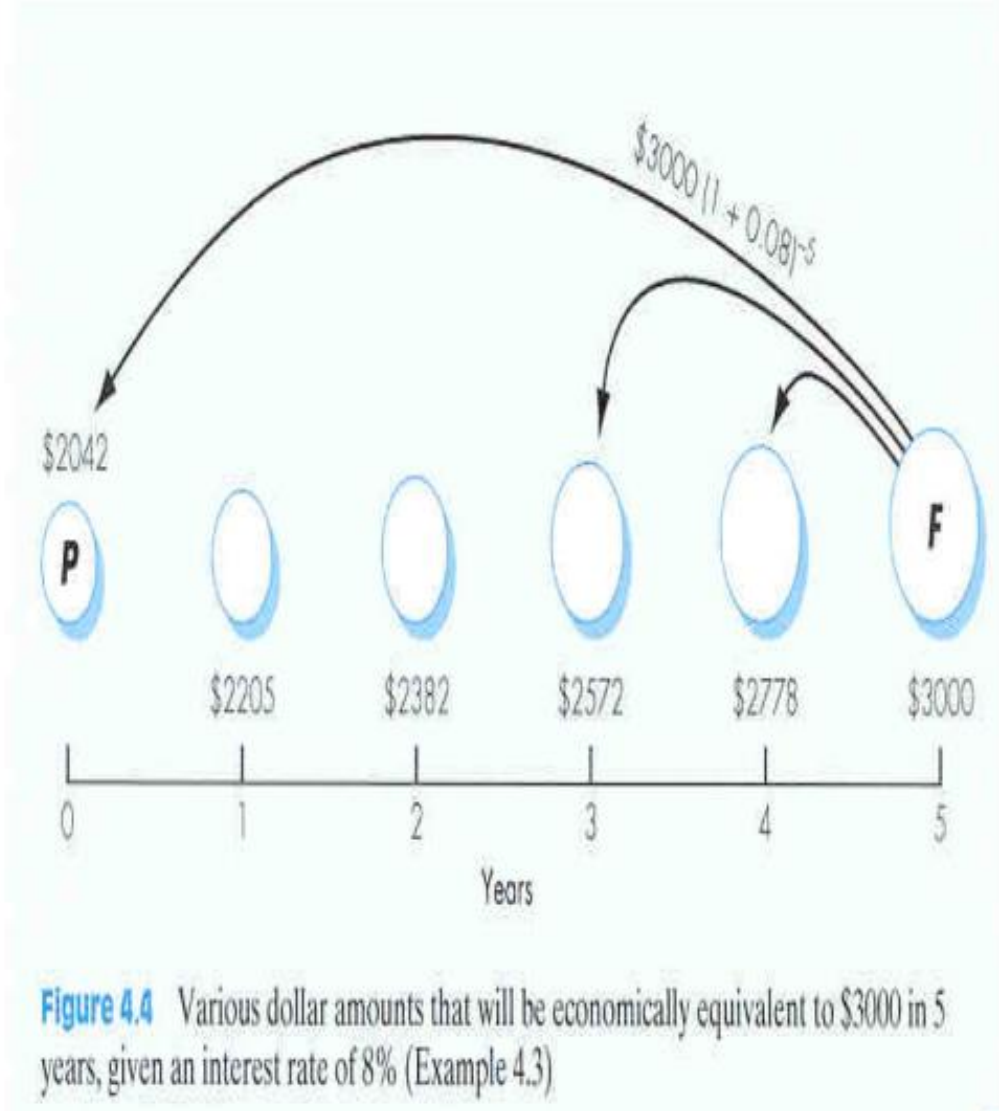


Figure 4.4 Various dollar amounts that will be economically equivalent to $3000 in 5 years, given an interest rate of 8% (Example 4.3)

Equivalent Cash Flows are Equivalent at Any Common Point In Time

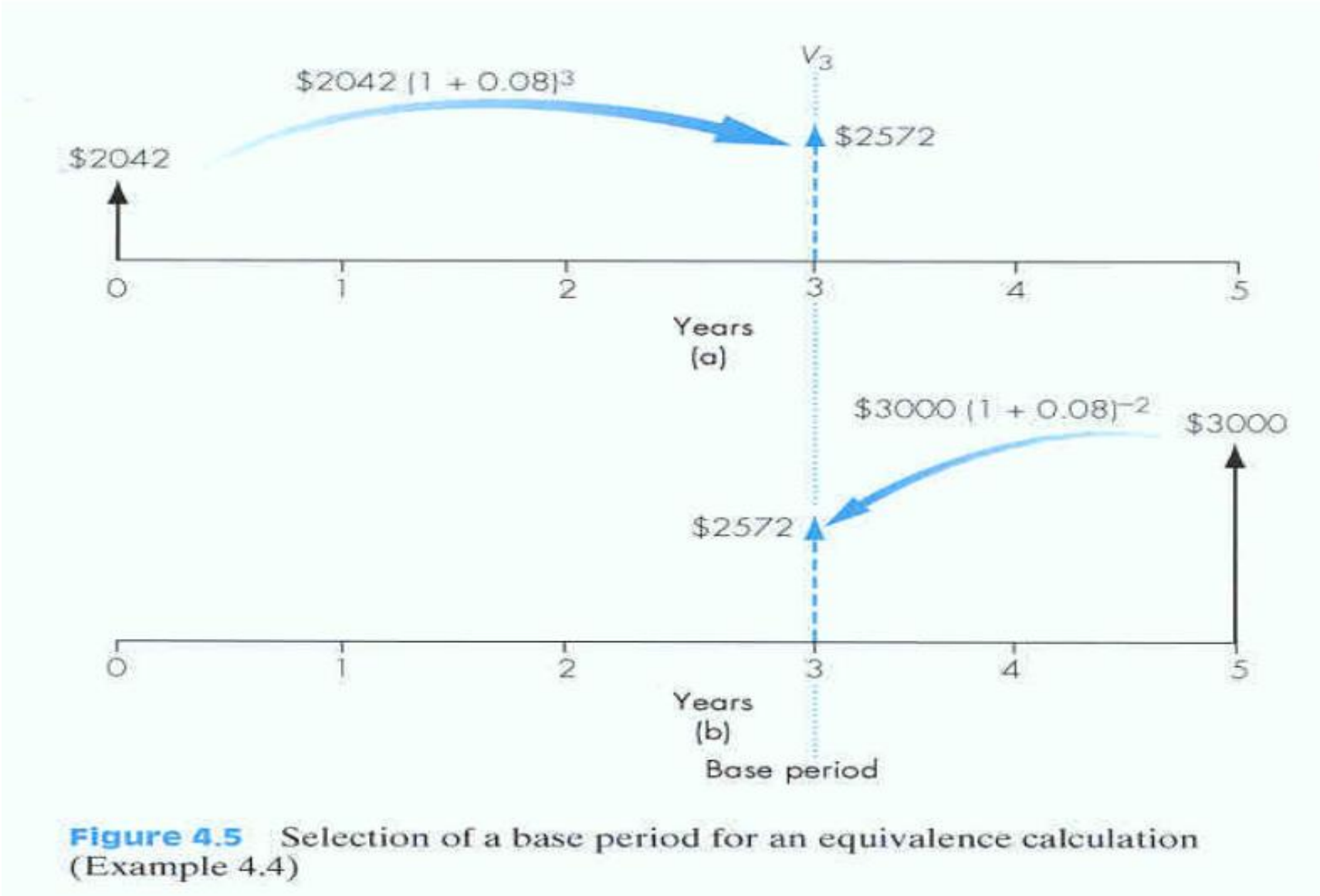


Figure 4.5 Selection of a base period for an equivalence calculation (Example 4.4)

Example 4.5 Equivalence Calculations with Multiple Payments

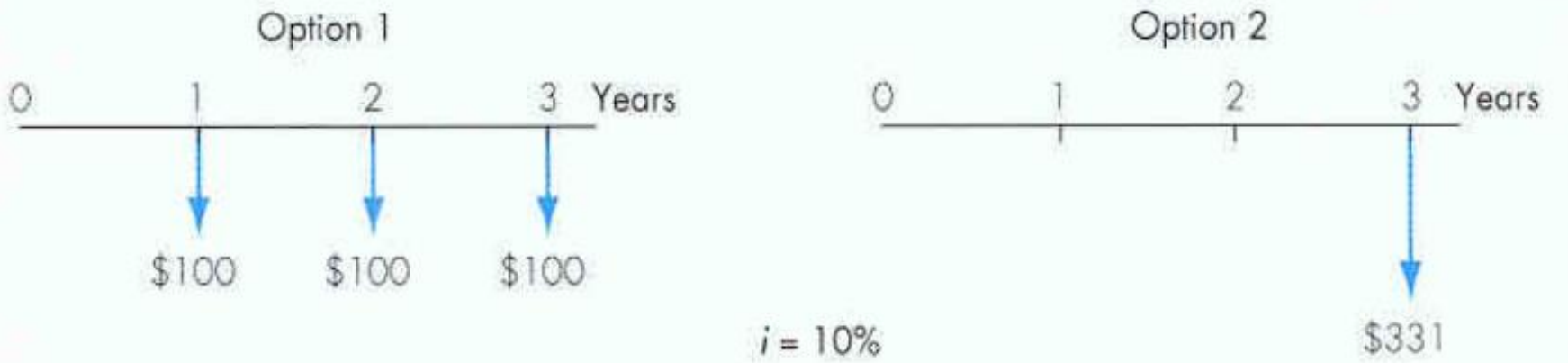


Figure 4.6 Equivalent cash flow diagram for Option 1 and Option 2 (excluding the common principal payment $1,000 at the end of year 3) (Example 4.5)

$$F_3 \text{ for } \$100 \text{ at } n = 1: \$100(1+0.10)^{3-1} = \$121$$

$$F_3 \text{ for } \$100 \text{ at } n = 2: \$100(1+0.10)^{3-2} = \$110$$

$$F_3 \text{ for } \$100 \text{ at } n = 3: \$100(1+0.10)^{3-3} = \$100$$

Types of Cash Flow

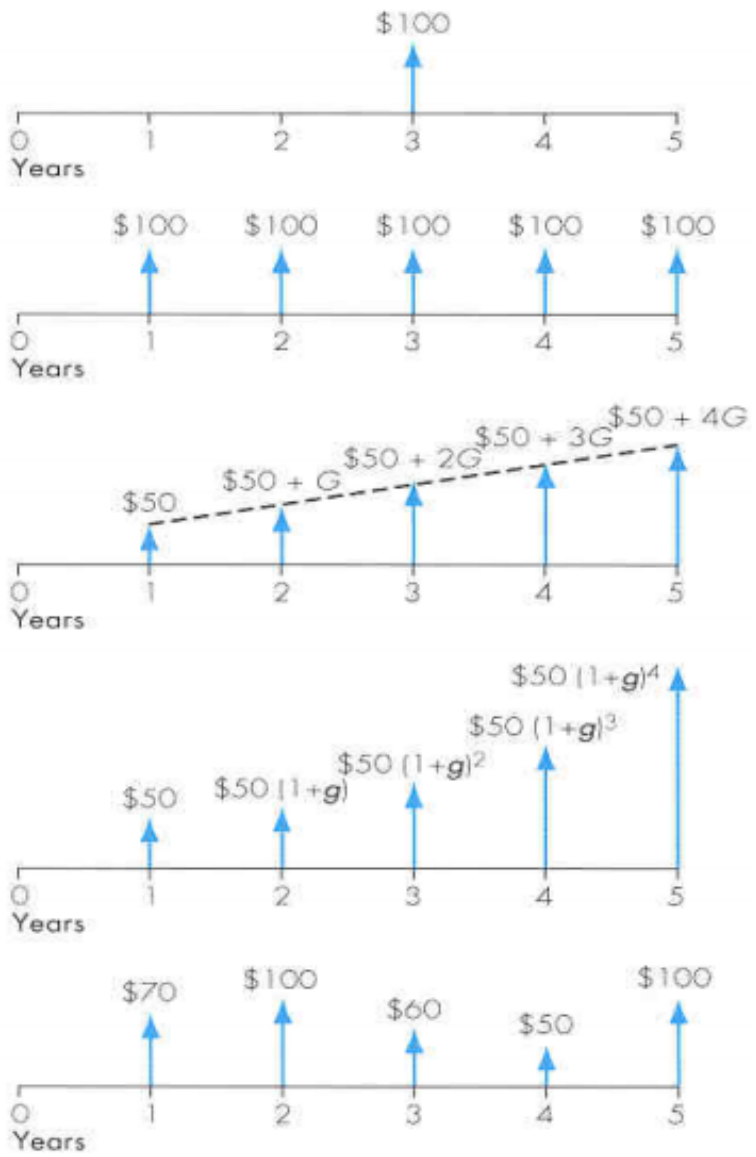
(a) Single cash flow

(b) Equal (uniform) payment series

(c) Linear gradient series

(d) Geometric gradient series

(e) Irregular payment series



Single Cash Flow Formula

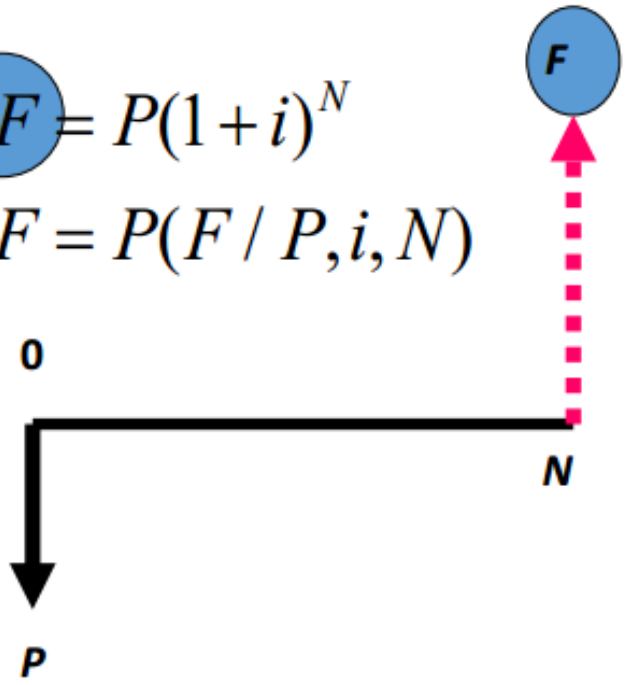
- Single payment compound amount factor (growth factor)
- Given:

$$i = 10\%$$
$$N = 8 \text{ years}$$
$$P = \$2{,}000$$

- Soln:

$$F = \$2{,}000(1+0.10)^8$$
$$= \$2{,}000(F/P,10\%,8)$$
$$= \$4{,}287.18$$

$$F = P(1+i)^N$$
$$F = P(F/P,i,N)$$



Also Called Compounding Factor !!

Atsushi has had $800 stashed under his mattress for 30 years. How much money has he lost by not putting it in a bank account at 8 percent annual compound interest all these years?

Single Cash Flow Formula

- Single payment present worth factor (discount factor)
- Given:

$$i = 12\%$$

$$N = 5 \text{ years}$$

$$F = \$1{,}000$$

- Soln:

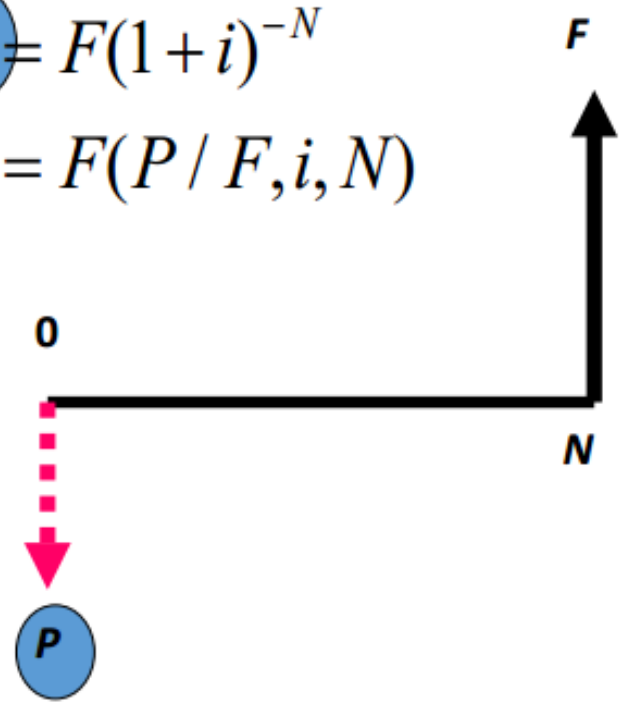
$$P = \$1{,}000(1+0.12)^{-5}$$

$$= \$1{,}000(P/F,12\%,5)$$

$$= \$567.40$$

$$P = F(1+i)^{-N}$$

$$P = F(P/F,i,N)$$



40. The present value of an amount having future value of Rs.800 after 4 years with a annual interest rate of 5% is equal to :

- [ ] 658.16
- [ ] 685.16
- [ ] 972.405
- [ ] 927.405

Uneven Payment Series

$$P_1 = \$25{,}000(P/F,10\%,1)$$
$$P_2 = \$3{,}000(P/F,10\%,2)$$
$$P_4 = \$5{,}000(P/F,10\%,4)$$
$$P = P_1 + P_2 + P_4$$
$$= \$28{,}622$$

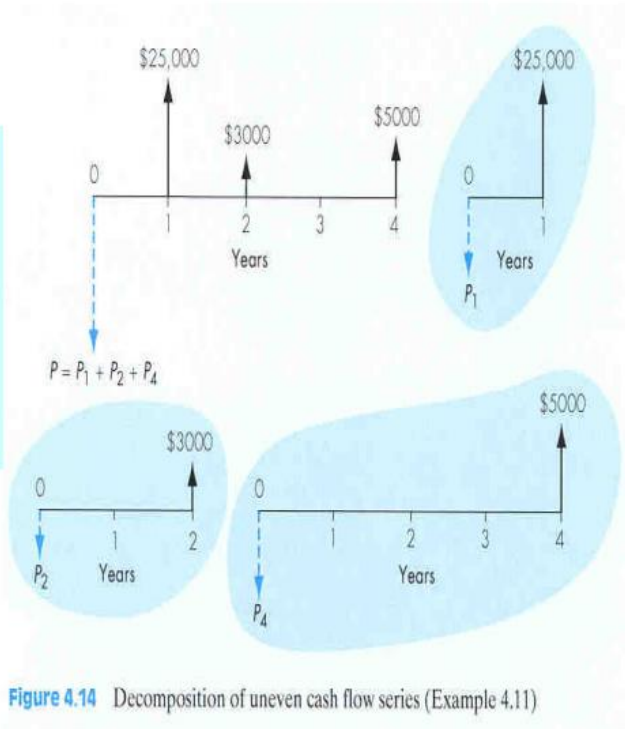
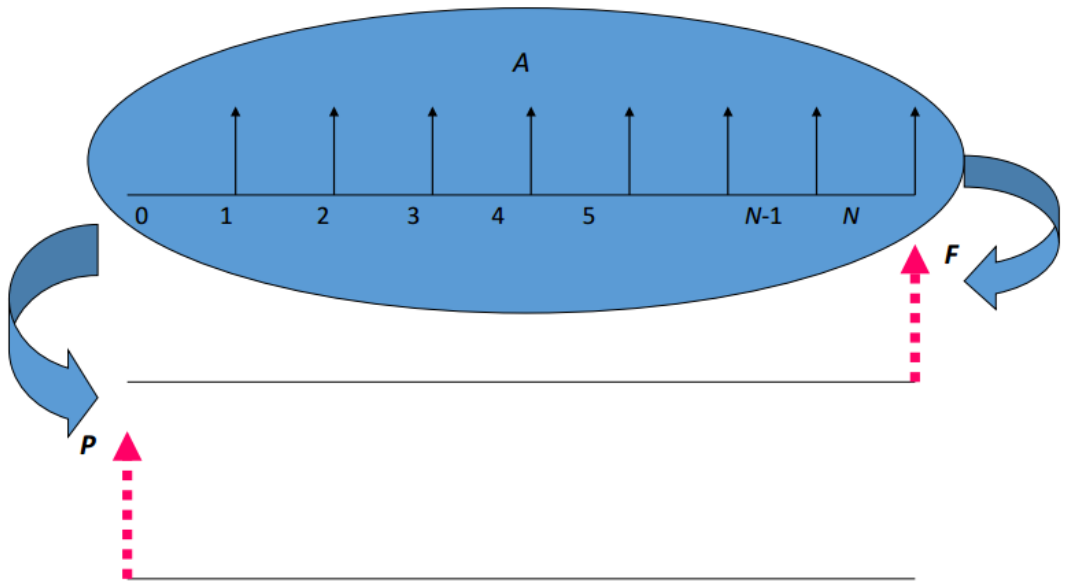
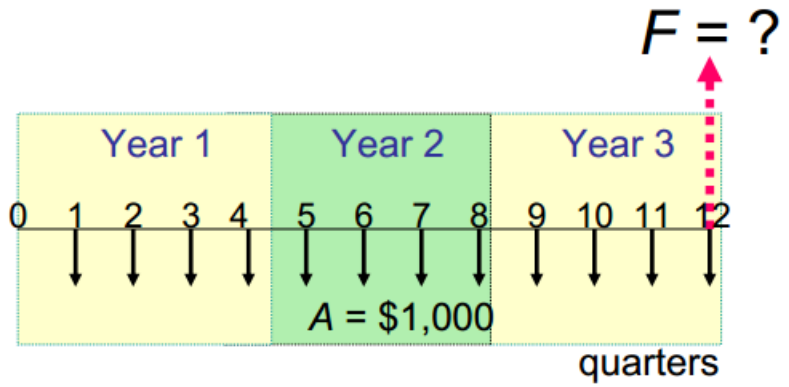


Figure 4.14 Decomposition of uneven cash flow series (Example 4.11)

Equal Payment Series



Suppose you make equal quarterly deposits of $1,000 into a fund that pays interest at 12% compounded monthly. Find the balance at the end of year 3.



Factor Name	Converts	Symbol	Formula
Single Payment Compound Amount	to F given P	$(F/P, i\%, n)$	$(1+i)^n$
Single Payment Present Worth	to P given F	$(P/F, i\%, n)$	$(1+i)^{-n}$
Uniform Series Sinking Fund	to A given F	$(A/F, i\%, n)$	$\frac{i}{(1+i)^n-1}$
Capital Recovery	to A given P	$(A/P, i\%, n)$	$\frac{i(1+i)^n}{(1+i)^n-1}$
Uniform Series Compound Amount	to F given A	$(F/A, i\%, n)$	$\frac{(1+i)^n-1}{i}$

Annuities

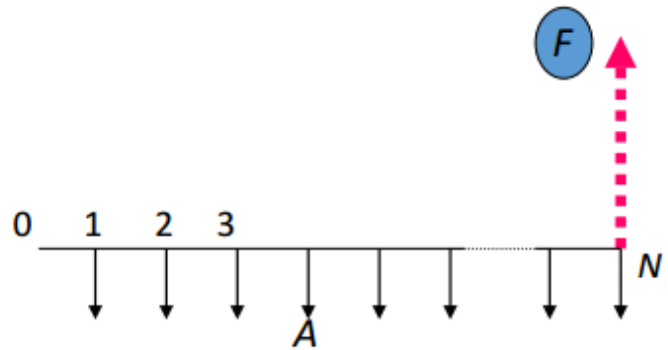
Annuities: are essentially a series of fixed payments required from you or paid to you at a specified frequency over the course of a fixed time period.

- Ordinary Annuity: Payments or receipts occur at the end of each period.
- Annuity Due: Payments or receipts occur at the beginning of each period.
- Perpetuity : Annuities that go forever.
- Deffered Annuity : Student loan (Payments are made after certain gap or interval)

Some Examples of Annuities

Student Loan Payments, Car Loan Payments , Insurance Premiums

Equal Payment Series Compound Amount Factor



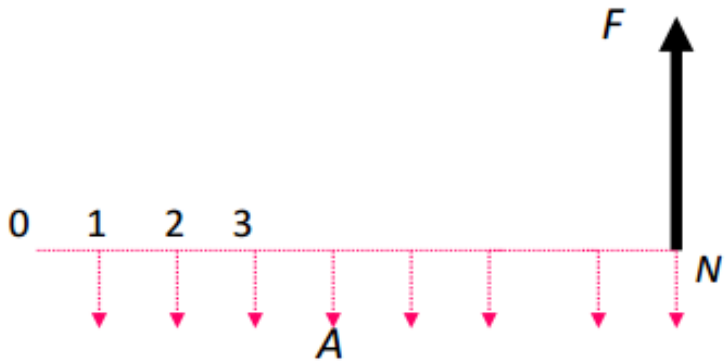
$$F = A\boxed{\frac{(1+i)^N - 1}{i}}$$
$$= A(F/A, i, N)$$

Example 4.13:

- Given: A = \$3,000, N = 10 years, and i = 7%
- Find: F
- Solution: F = \$3,000(F/A,7%,10) = \$41,449.20

Remember, the future worth is always located in the same period as the last uniform-series amount when using the F/A factor.

Sinking Fund Factor



$$A = F\left[\frac{i}{(1+i)^N - 1}\right]$$
$$= F(A/F, i, N)$$

Example 4.15:

- Given: $F = \$5{,}000$, $N = 5$ years, and $i = 7\%$
- Find: A
- Solution: $A = \$5{,}000(A/F, 7\%, 5) = \$869.50$

Numerical

If you deposit Rs. 10,000 at the end of each year for 10 years, How much money he accumulated to your account at the end of 10 years? when i = 9%. compounded annually.

Solution.

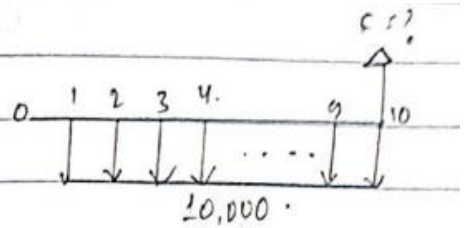
Given.

Annuity (A) = 10,000

N (Nos. of years) = 10

Interest rate (i) = 9% = 0.09

Future worth (F) = ?



By using Uniform series compound Amount factor

$$F = A(F/A, 9\%, 10)$$

$$= 10,000 * \left[\frac{(1+0.09)^{10} - 1}{0.09}\right]$$

$$\therefore F = Rs.\ 1,51,929.30$$

Hence, Future worth after 10 years = Rs. 1,51,929.30.

Capital Recovery factor

The **capital recovery factor**, denoted by $(A/P,i,N)$, gives the value, A, of the equal periodic payments or receipts that are equivalent to a present amount, P, when the interest rate is i and the number of periods is N. The capital recovery factor is easily derived from the sinking fund factor and the compound amount factor:

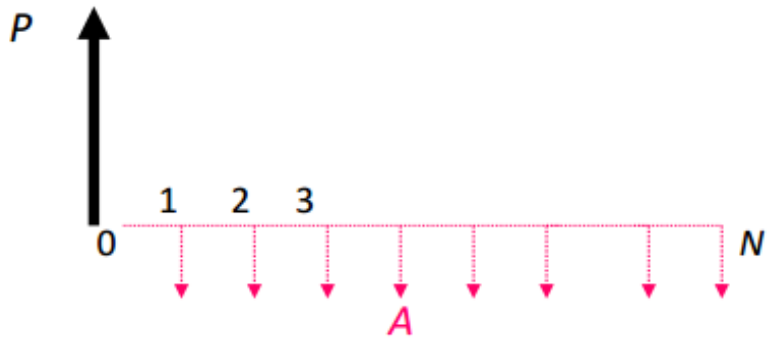
$$(A/P,i,N) = (A/F,i,N)(F/P,i,N)$$

$$= \frac{i}{(1+i)^N - 1}(1+i)^N$$

$$= \frac{i(1+i)^N}{(1+i)^N - 1}$$

The capital recovery factor can be used to find out, for example, how much money must be saved over N future periods to "recover" a capital investment of P today. The

Capital Recovery Factor



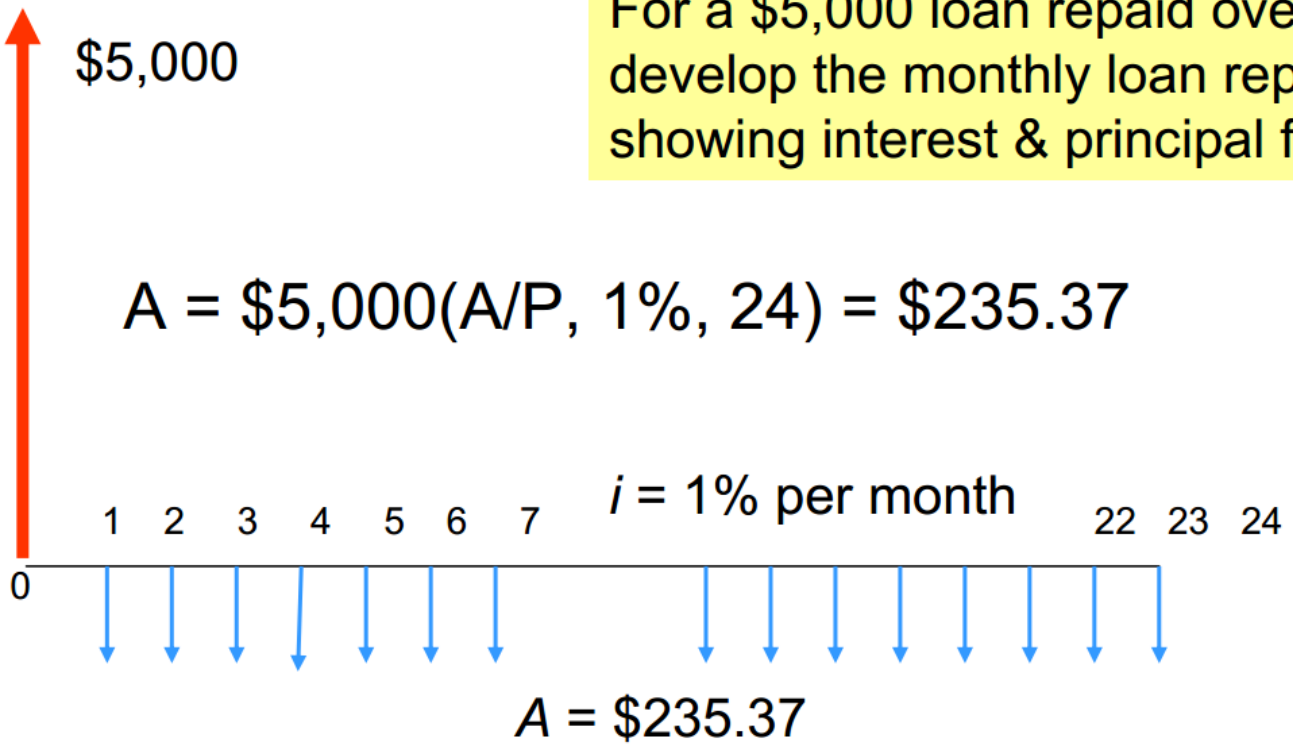
$$A = P\frac{i(1+i)^N}{(1+i)^N - 1}$$

$$= P(A/P, i, N)$$

Example 4.16:

- Given: $P = \$250{,}000$, $N = 6$ years, and $i = 8\%$
- Find: A
- Solution: $A = \$250{,}000(A/P, 8\%, 6) = \$54{,}075$

For a $5,000 loan repaid over 2 years at 12% develop the monthly loan repayment schedule showing interest & principal for each period.



Consider the 7th payment ($235.37)

a. How much is interest?

b. What is the amount of principal payment?

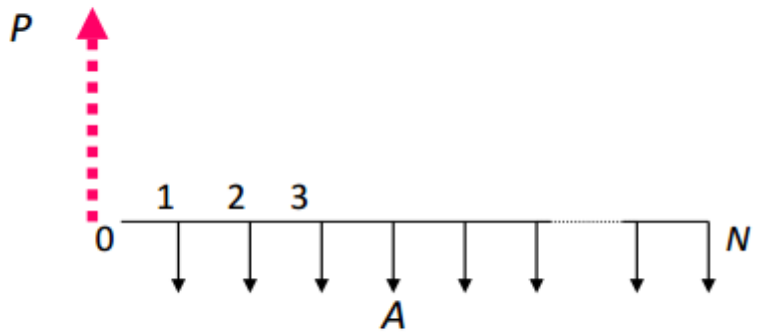
Example 3.7 Loan Repayment Schedule

Contract amount	$ 5,000.00		Total payment		$ 5,648.82
Contract period	24		Total interest		$648.82
APR (%)	12				
Monthly Payment	($235.37)				
	Payment No.	Payment Size	Principal Payment	Interest payment	Loan Balance
	1	($235.37)	($185.37)	($50.00)	$4,814.63
	2	($235.37)	($187.22)	($48.15)	$4,627.41
	3	($235.37)	($189.09)	($46.27)	$4,438.32
	4	($235.37)	($190.98)	($44.38)	$4,247.33
	5	($235.37)	($192.89)	($42.47)	$4,054.44
	6	($235.37)	($194.82)	($40.54)	$3,859.62
	7	($235.37)	($196.77)	($38.60)	$3,662.85
	8	($235.37)	($198.74)	($36.63)	$3,464.11
	9	($235.37)	($200.73)	($34.64)	$3,263.38
	10	($235.37)	($202.73)	($32.63)	$3,060.65
	11	($235.37)	($204.76)	($30.61)	$2,855.89
	12	($235.37)	($206.81)	($28.56)	$2,649.08
	13	($235.37)	($208.88)	($26.49)	$2,440.20
	14	($235.37)	($210.97)	($24.40)	$2,229.24
	15	($235.37)	($213.08)	($22.29)	$2,016.16
	16	($235.37)	($215.21)	($20.16)	$1,800.96
	17	($235.37)	($217.36)	($18.01)	$1,583.60
	18	($235.37)	($219.53)	($15.84)	$1,364.07
	19	($235.37)	($221.73)	($13.64)	$1,142.34
	20	($235.37)	($223.94)	($11.42)	$918.40
	21	($235.37)	($226.18)	($9.18)	$692.21
	22	($235.37)	($228.45)	($6.92)	$463.77
	23	($235.37)	($230.73)	($4.64)	$233.04
	24	($235.37)	($233.04)	($2.33)	$0.00

The **series present worth factor**, denoted by $(P/A,i,N)$, gives the present amount, P, that is equivalent to an annuity with disbursements or receipts in the amount, A, where the interest rate is i and the number of periods is N. It is the reciprocal of the capital recovery factor:

$$(P/A,i,N) = \frac{(1+i)^N - 1}{i(1+i)^N}$$

Equal Payment Series Present Worth Factor



$$P = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right] = A(P/A,i,N)$$

Example 4.18:

- Given: $A = \$32{,}639$, $N = 9$ years, and $i = 8\%$
- Find: P
- Solution: $P = \$32{,}639(P/A,8\%,9) = \$203{,}893$

Remember, the present worth is always located one period prior to the first uniform-series amount when using the P/A factor.

Linear Gradient Series

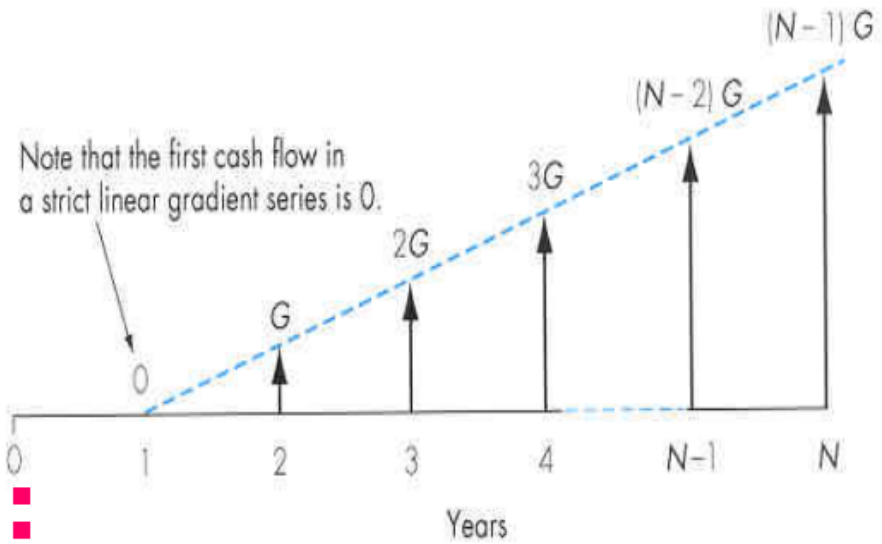
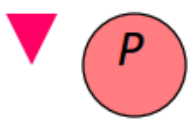


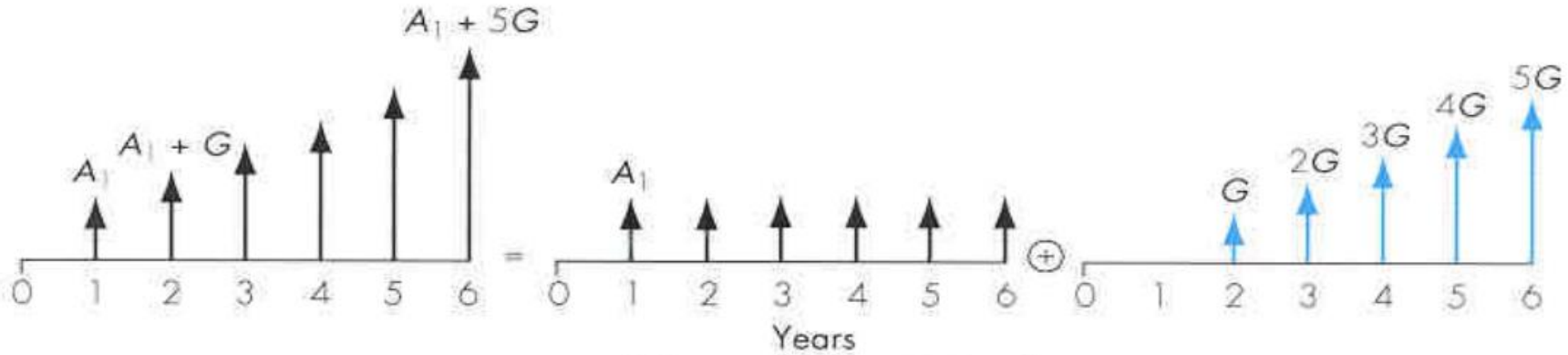
Figure 4.25 Cash flow diagram of a strict gradient series

$$P = G\frac{i(1+i)^N - iN - 1}{i^2(1+i)^N}$$

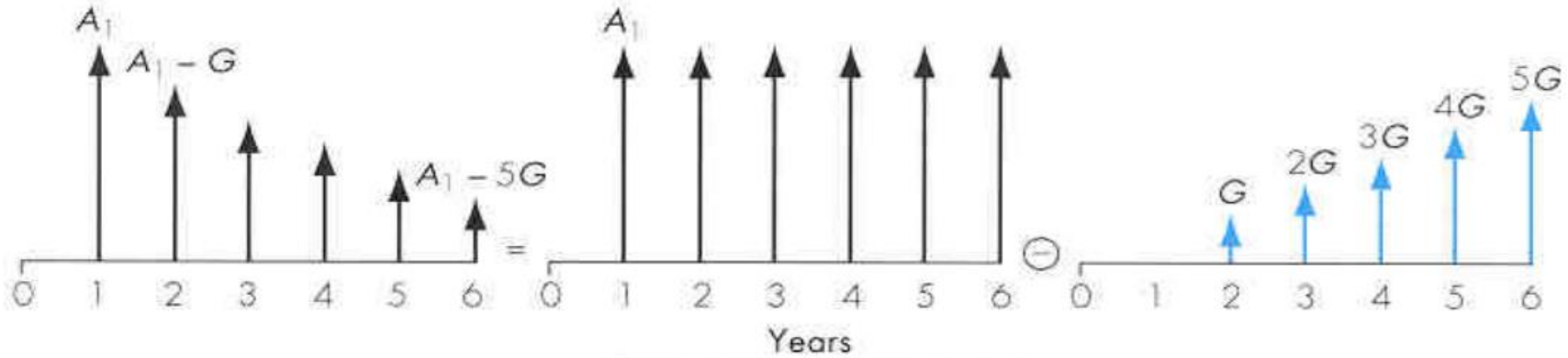
$$= G(P/G, i, N)$$



Gradient Series as a Composite Series

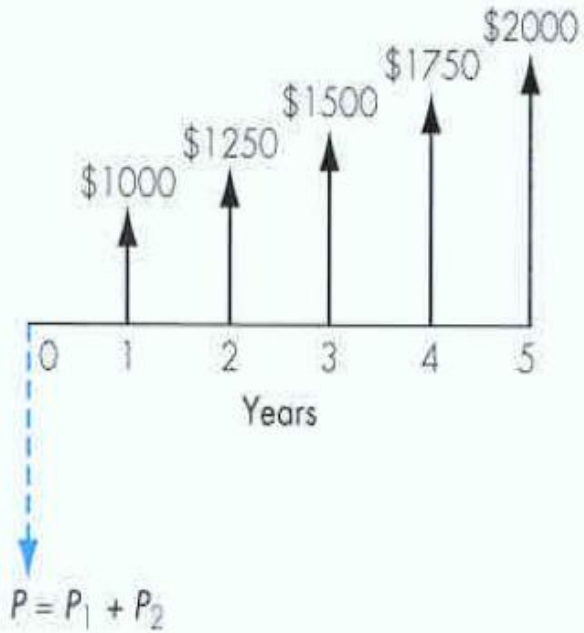


(a) Increasing gradient series



(b) Decreasing gradient series

Figure 4.27 Two types of linear gradient series as composites of a uniform series of N payments of A_1 and the gradient series of increments of constant amount G



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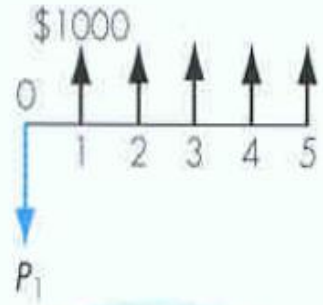
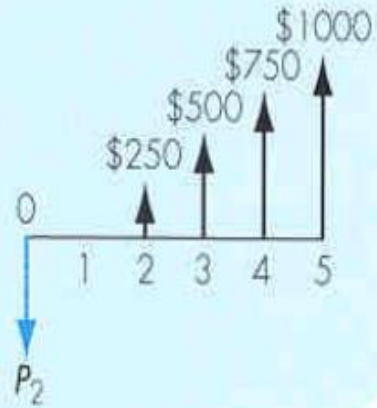
Equal payment series



+

Gradient series



$$P_1 = \$1{,}000(P/A,12\%,5)$$
$$= \$3{,}604.80$$

$$P_2 = \$250(P/G,12\%,5)$$
$$= \$1{,}599.20$$

$$P = \$3{,}604.08 + \$1{,}599.20$$
$$= \$5{,}204$$

Geometric Gradient Series

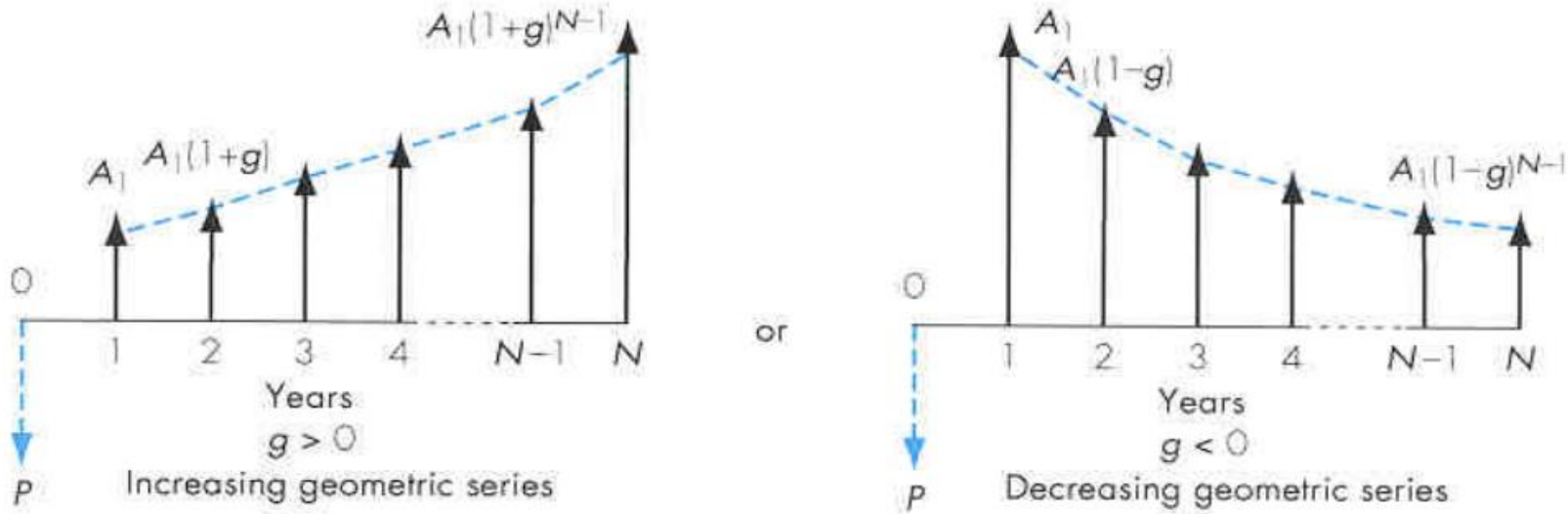
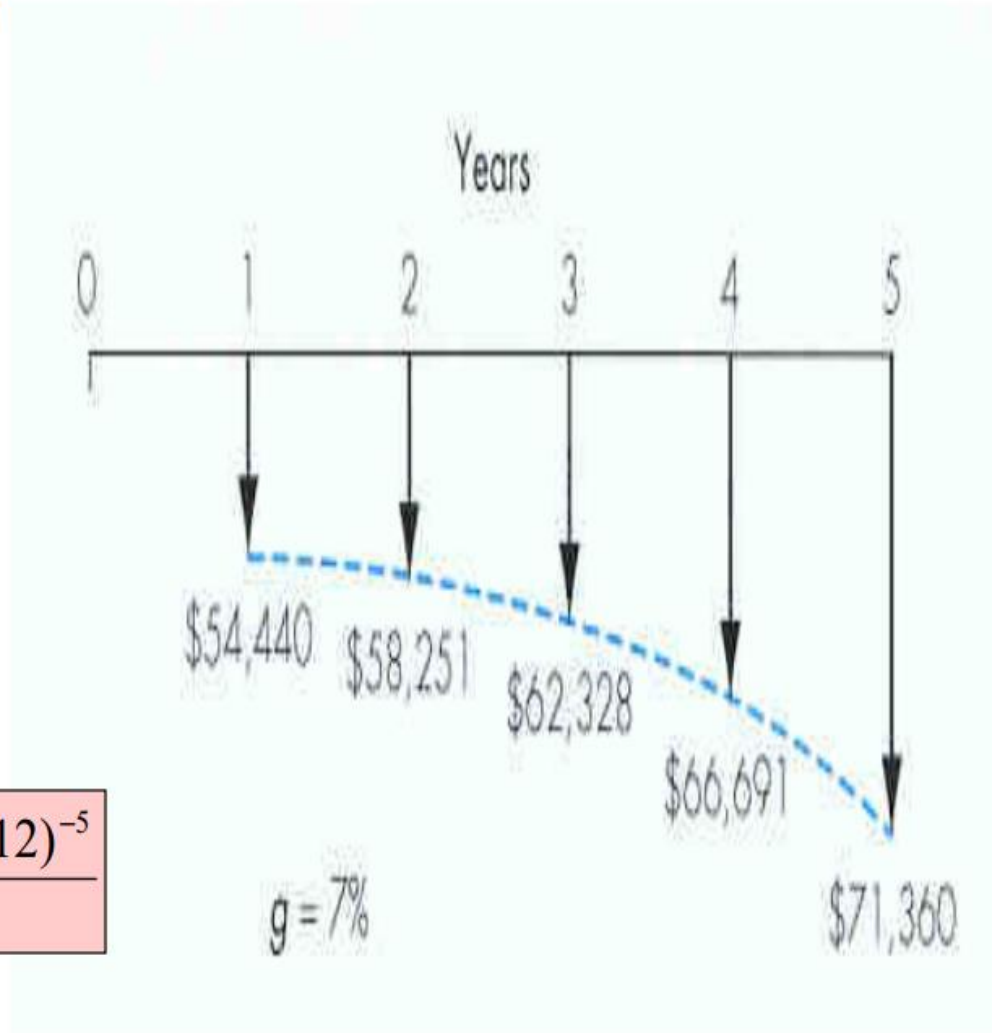


Figure 4.32 A geometrically increasing or decreasing gradient series at a constant rate g

$$P = \begin{cases} A_1 \dfrac{1-(1+g)^N(1+i)^{-N}}{i-g}, & \text{if } i \neq g \\ NA_1/(1+i), & \text{if } i = g \end{cases}$$

Find P, Given A_1, g, i, N

- Given:
  - $g = 7\%$
  - $i = 12\%$
  - $N = 5$ years
  - $A_1 = \$54{,}440$
- Find: P



$$P = \$54{,}440\left[\frac{1-(1+0.07)^{5}(1+0.12)^{-5}}{0.12-0.07}\right]$$

$$= \$151{,}109$$

Four cases of geometric Gradient

1. $i > g > 0$. *Growth is positive, but less than the rate of interest.* The growth-adjusted interest rate, $i°$, is positive. Tables or functions built into software may be used to find the conversion factor.
2. $g > i > 0$. *Growth is positive and greater than the interest rate.* The growth-adjusted interest rate, $i°$, is negative. It is necessary to compute the conversion factor directly from the formula.
3. $g = i > 0$. *Growth is positive and exactly equal to the interest rate.* The growth-adjusted interest rate, $i°$, equals zero. As with any case where the interest rate is zero, the present worth of the series with constant terms, $A/(1 + g)$, is simply the sum of all the N terms:

$$P = N\left(\frac{A}{1+g}\right)$$

4. $g < 0$. *Growth is negative.* In other words, the series is decreasing. The growth-adjusted interest rate, $i°$, is positive. Tables or functions built into software may be used to find the conversion factor.

Numerical

Air plane ticket price will increase 8% in each of the next four years. The cost at the end of the first year will be Rs. 2000. How much should be put away now to cover a passenger travel home at the end of each year for the next four years. Assume i=5%.

Solution

Given: $A_1 = 2000$, $g = 8\%$, $i = 5\%$

Here $i \neq g$

$$P = A_1 \frac{[1-(1+g)^N (1+i)^{-N}]}{i-g}$$

$$= 2000 \frac{[1-(1+0.08)^{-1}(1+0.05)^{-1}]}{0.05-0.08}$$

$$= 20000^{*}\ 3.9759$$

$$= \textbf{Rs. 7951.84 (Ans)}$$

For Check: Without Using Geometric gradient

For year 1: $A_1 = 2000$

For year 2: $A_2 = 2000 + 0.08\ (2000) = 2160$

For year 3: $A_3 = 2160 + 0.08\ (2160) = 2332.8$

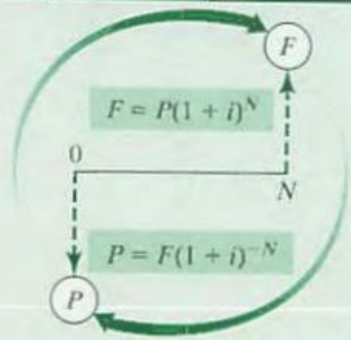
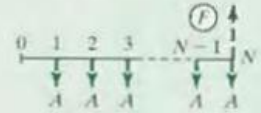
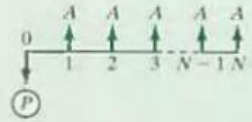
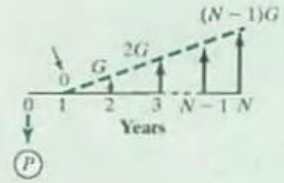
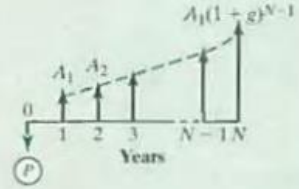
For year 4: $A_3 = 2333.8 + 0.08\ (2332.8) = 2519.24$

$P = 2000\ (P/F, 5\%, 1) + 2160\ (P/F, 5\%, 2) + 2332.8\ (P/F, 5\%, 3) + 2519.24\ (P/F, 5\%, 4)$

$= 2000\ (1+0.05)^{-1} + 2160\ (1+0.05)^{-2} + 2332.8\ (P/F, 5\%)^{-3} + 2519.24\ (1+0.05)^{-4}$

$= 1904.76 + 1959.18 + 2015.16 + 2072.58 = \textbf{7951.684 (Ans)}$

Summary of Interest Factors and Other Useful Formulas

Flow Type	Factor Notation	Goal	Formula	Equivalent Cash Flow Diagrams
Single payment	Future worth $(F/P, i, N)$	Find: F Given: P	$F = P(1+i)^N$	
	Present worth $(P/F, i, N)$	Find: P Given: F	$P = F(1+i)^{-N}$	
Equal payment series	Future worth $(F/A, i, N)$	Find: F Given: A	$F = \left[\frac{(1+i)^N - 1}{i}\right]$	
	Sinking fund $(A/F, i, N)$	Find: A Given: F	$A = \left[\frac{i}{(1+i)^N - 1}\right]$	
	Present worth $(P/A, i, N)$	Find: P Given: A	$P = \left[\frac{(1+i)^N - 1}{i(1+i)^N}\right]$	
	Capital recovery $(A/P, i, N)$	Find: A Given: P	$A = \left[\frac{i(1+i)^N}{(1+i)^N - 1}\right]$	
Linear gradient series	Present worth $(P/G, i, N)$	Find: P Given: G	$P = G\left[\frac{(1+i)^N - iN - 1}{i^2(1+i)^N}\right]$	
	Equal payment $(A/G, i, N)$	Find: A Given: G	$A = G\left[\frac{(1+i)^N - iN - 1}{i(1+i)^N - i}\right]$	
Geometric gradient series	Present worth $(P/A_1, g, i, N)$	Find: P Given: g	$P = \begin{cases} A_1\left[\frac{1-(1+g)^N(1+i)^{-N}}{i-g}\right] \\ A_1\left(\frac{N}{1+i}\right), \text{ if } i = g \end{cases}$	

Continuous Compounding Single Cash Flow

$i = e^r - 1$

e^r is equal to $(1+i)$ (i= effective interest rate)

$e^r = (1+i)$

$i = e^r - 1$

We have $F = P(1+i)^N$

$$F = pe^{rN}$$

$F = P\{F/P, r\%, N\}$

(Continuously compounded compound amount factor for single cash flow)

r% is used to denote the nominal rate and the use of continuous compounding

Substitute $e^r = 1 + i$

$F = P(e^r)^N = e^{rN}$ $\quad F = P * (F/P, r\%, N)$

$F = A\ \frac{e^{rN}-1}{e^r-1}$ $\quad F = A * (F/A, r\%, N)$

$P = A\ \frac{e^{rN}-1}{(e^r-1)*e^{rN}}$ $\quad P = A * (P/A, r\%, N)$

THANK YOU !!

Basic Methodologies of Engineering Economics Analysis and Risk Analysis

Sushil Rijal

MARR (Minimum Acceptable/Attractive Rate of Return)

- MARR is that interest rate at which an investor can accept to earn or borrow money easily. MARR is determined by top management from policy level. So, it may be different from time to time and firm to firm.

MARR is determined by taking into numerous considerations. They are:

- The amount of money available for investment, and the source and cost of these funds (i.e. equity funds and borrowed funds).
- The number of good projects available for investment and their purpose
- The amount of perceived risk that is associated with investment opportunities
- The type of organization involved (i.e., government, public utility, or competitive industry).
- MARR gov < MARR public < MARR private

The Minimum Attractive Rate of Return (MARR) is a reasonable rate of return established for the evaluation and selection of alternatives. A project is not economically viable unless it is expected to return at least the MARR. MARR is also referred to as the hurdle rate, cutoff rate, benchmark rate, and minimum acceptable rate of return.

Financial Analysis	Economic Analysis
Objective: To determine financial feasibility i.e. whether someone is willing to pay for a project & capability to raise the necessary funds.	Objective: To determine if a project represents the best use of resources over the analysis period. i.e. project is justified socio-economic objectives.
Perspective: Evaluation is from the perspective of parties expected to pay their allocated costs	Perspective: Evaluation is from the perspective of many parties i.e. investors, beneficiaries, govt., environment, communities etc.
Cost & Benefits: Consider only controlled price (market price) fixed by government in monetary units.	Cost & Benefits: All tangible/ intangible, primary/secondary/ tertiary effects to society/economy as a whole is taken into consideration at shadow price (modified/adjusted market price to reflect real price/value.

Methods of Engineering Economic Analysis

1. Equivalent Worth Method
   - (a) Present Worth (PW)
   - (b) Annual Worth (AW)
   - (c) Future Worth (FW)
2. Rate of Return Method
   - (a) Internal Rate of Return (IRR)
   - (b) External Rate of Return (ERR)
3. Payback Period Method
   - (a) Simple payback period
   - (b) Discounted payback period
4. Benefit Cost Ratio Method
   - (a) Conventional B/C ratio
   - (b) Modified B/C ratio

Pay Back Period Method

- Payback period is defined as the number of years required to recover the initial investment.
- It focus on liquidity i.e. how fast an initial investment can be recovered (easy recovery). It is not measure of profitability.
- It does not consider cash flows of entire life of project. i.e. ignores cash flow information after payback period.

SIMPLE PAYBACK PERIOD

Simple Payback Period is the payback period which ignores the time value of money. i.e. i = 0.

It does not consider the time value of money.

Simple Payback period

- **EQUAL OR UNIFORM CASH FLOW**
  Simple Payback Period = Initial Investment /Annual net cash flow
  If Calculated Payback Period < Standard Payback Period, Accept the project,
  Or If Calculated Payback Period > Standard Payback Period, Reject the Project

- Initial Investment = 10,000. Annual cash inflow = 5,000. Annual cash outflow = 3,000. Standard Payback Period = 4 years

$$\text{Simple Payback Period} = \frac{\text{Initial Investment}}{\text{Annual net cash flow}}$$

$$\text{Simple Payback Period} = \frac{10{,}000}{5{,}000-3{,}000} = 5 \text{ years}$$

- Since Calculated Payback Period > Standard Payback Period,
  Reject the Project.

Discounted Pay back Period

- Simple Payback Period ignores the time value of money. i.e. i = 0. To remedy this defect of simple payback period, time value of money is considered in the Discounted Payback period.
- Cash flows are discounted at certain MARR and determine the number of years required to recover the initial investment.

If MARR= i =10%, and Standard Pay back period = 3.5 years, evaluate discounted payback period and acceptability of project

EOY	Net Cash Flow
0	-10,000
1	+2,000
2	+3,000
3	+4,000
4	+5,000
5	+6,000
6	+7,000

MERITS OF PAYBACK PERIOD

- Simple to understand
- Easy to calculate
- Inexpensive to use
- Focus on liquidity i.e. how fast an initial investment can be recovered (easy recovery)
- Easy and crude way to tackle/cope with riskiness of investment.
- Based on cash flow information

Demerits of Payback Period

- Simple payback period ignores the time value of money. Use discounted payback period to take into account the time value of money.
- Does not consider cash flows of entire life of project. i.e. ignores cash flow information after payback period
- Is not measure of profitability.
- No rational basis to set/determine a maximum/minimum acceptable standard payback period. It is generally, a subjective decision.
- fails to consider the pattern of cash flow. i.e. timing and magnitude

Comparisons between alternative

Choose the one project among given alternative if MARR = 10%

a)By Simplified Payback method

b)By Discounted Payback method

	Alternative		
EOY	A	B	C
0	-10,000	-10,000	-10,000
1	2,500	4,000	1,000
2	2,500	3,000	2,000
3	2,500	2,000	3,000
4	2,500	1,000	4,000
5	2,500	2,500	2,500
Simple Payback Period	4 Years	4 Years	4 Years
Discounted Payback Period at MARR = 10%	> 5 Years	4.08 Years	> 5 Years

Equivalent Worth (EW) Method

- Equivalent worth methods convert all cash flows into equivalent present, future, or annual amounts at the MARR. If a single project is under consideration,

If EW ≥ 0; Accept the project,
If EW < 0, Rejected the project.

- **Future Worth Method (FW)**
- **Present Worth Method or Present Value Method (PW/PV)**
- **Annual Worth Method (AW)**

Future Worth (FW) Method or Future Value

- Future worth criterion has become popular because a primary objective of all time value of money is to maximize the future wealth of the owners of the firm. i.e. how much it worth at the end of given number of years.
- FW methods convert all cash flows into equivalent future amounts at the MARR. All cash inflows and outflows are compounded forward to a reference point called the future, at the interest period rate MARR.

$$FW = P_0 (1 + i)^{N-0} + P_1 (1 + i)^{N-1} + P_2 (1 + i)^{N-2} + \ldots + P_k (1 + i)^{N-k}$$
$$+ \ldots + P_N (1 + i)^{N-N}$$

Where i = effective interest rate, k = future cash flow at the end of period, N = number of compounding period.

If a single project is under consideration,

If FW ≥ 0; Accept the project,

If FW < 0, Rejected the project

Present Worth Methods or Present Value

- PW methods convert all cash flows into equivalent present amounts at the MARR. All cash inflows and outflows are discounted to the base or beginning point in time at the interest period rate MARR.
- Present worth of project is a measure of how much fund will have to be put aside now to provide all future expenditures during the project period. It is assumed that such fund placed in reserve earns interest rate equal to MARR.

$$PW = F_0 (1 + i)^{0} + F_1 (1 + i)^{-1} + F_2 (1 + i)^{-2} + \ldots + F_k (1 + i)^{-k} + \ldots + F_N (1 + i)^{-N}$$

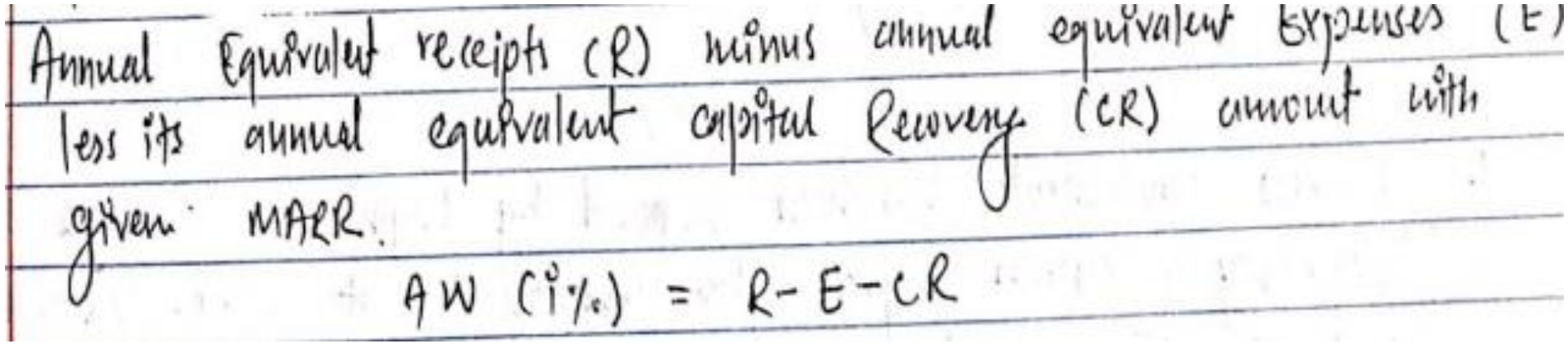
If a single project is under consideration,

If PW ≥ 0; The project is economically justified.
Therefore, accept the project,

If PW < 0, The project is economically not justified. Therefore,
reject the project.

Annual Worth Method (AW)

- Annual worth of a project is uniform series of amount that is equivalent to the cash inflows and outflows occur during the project duration.
- AW methods convert all cash flows into equivalent annual amounts at the MARR.



- If a single project is under consideration,
  If AW ≥ 0; Accept the project,
  If AW < 0, Rejected the project.

Capital Recovery Cost (CR)

- Capital Recovery Cost (CR) of a project is the equivalent uniform cost of the capital invested. It covers both depreciation and interest on invested capital (MARR). It can be calculate by following formulas.
- **CR = I (A/P, i, N) - S (A/F, i, N)**

  AW = R - E - CR
  where R = Annual equivalents receipts or savings, E = annual equivalent expenses, CR = Capital Recovery Cost.
- If CR = R - E
  i.e. Capital to be recovered per year = net annual cash flow. No gain/No loss.
  If CR < R - E
  i.e. Capital to be recovered per year < net annual cash flow. There is gain.
  If CR > R - E
  i.e. Capital to be recovered per year > net annual cash flow. There is loss.

Numerical

- The initial investment is Rs. 25,000, and salvage value is 10% of the initial investment at the end of its useful life 10 years. Annual revenue and expenses are Rs 14,000 and Rs. 10,000 respectively. Evaluate the investment proposal by EW (FW/PW/AW) methods. MARR = 10%.

FW (i%) = - I * (F/P, i, N) + A * (F/A, i, N) + S

AW (i%) = - I * (A/P, i, N) + A + S * (A/F, i, N)

PW (i%) = - I + A * (P/A, i, N) + S * (P/F, i, N)

N	MARR	(F/P, i%, N)	(F/A, i%, N)	(P/A, i%, N)
10	10%	2.5937	15.9374	6.1446

FW (i%) = - I * (F/P, i, N) + A * (F/A, i, N) + S

FW (10%) = - 25000 * (F/P, 10%, 10) + (14,000 - 10,000) * (F/A, 10%, 10) + 2,500

FW (10%) = - 25,000 * 2.5937 + 4,000* 15.9374 + 2,500

FW (10%) = - 64,842.5 + 63,749.6 + 2,500

FW (10%) = + Rs.1,407.1

AW (i%) = - I * (A/P, i, N) + A + S * (A/F, i, N)

AW (10%) = - 25,000 * (A/P, 10%, 10) + (14,000 - 10,000) + 2,500 * (A/F, 10%, 10)

AW (10%) = - 25,000/6.1446 + 4,000 + 2,500/15.9374

AW (10%) = - 4068.61 + 4,000 + 156.86

AW (10%) = + Rs.88.25

PW (i%) = - I + A * (P/A, i, N) + S * (P/F, i, N)

PW (10%) = - 25,000 + (14,000 - 10,000)* (P/A, 10%, 10) + 2,500 * (P/F, 10%, 10)

PW (10%) = - 25,000 + 4,000 * 6.1446 + 2,500/2.5937

PW (10%) = - 25000 + 24578.4 + $963.87

PW (10%) = + Rs.542.27

Rationale for the EW Method

EW=0 → Project's inflows are "exactly sufficient to repay the invested capital and provide the required rate of return."

- Choose between mutually exclusive projects on basis of higher Worth/ Value(Future Worth/ Present Worth / Annual worth)

  - Meets all desirable criteria
    - Considers all CFs
    - Considers TVM
    - Can rank mutually exclusive projects

Rate of Return

- Definition: A relative percentage method which measures the yield as a percentage of investment over the life of a project

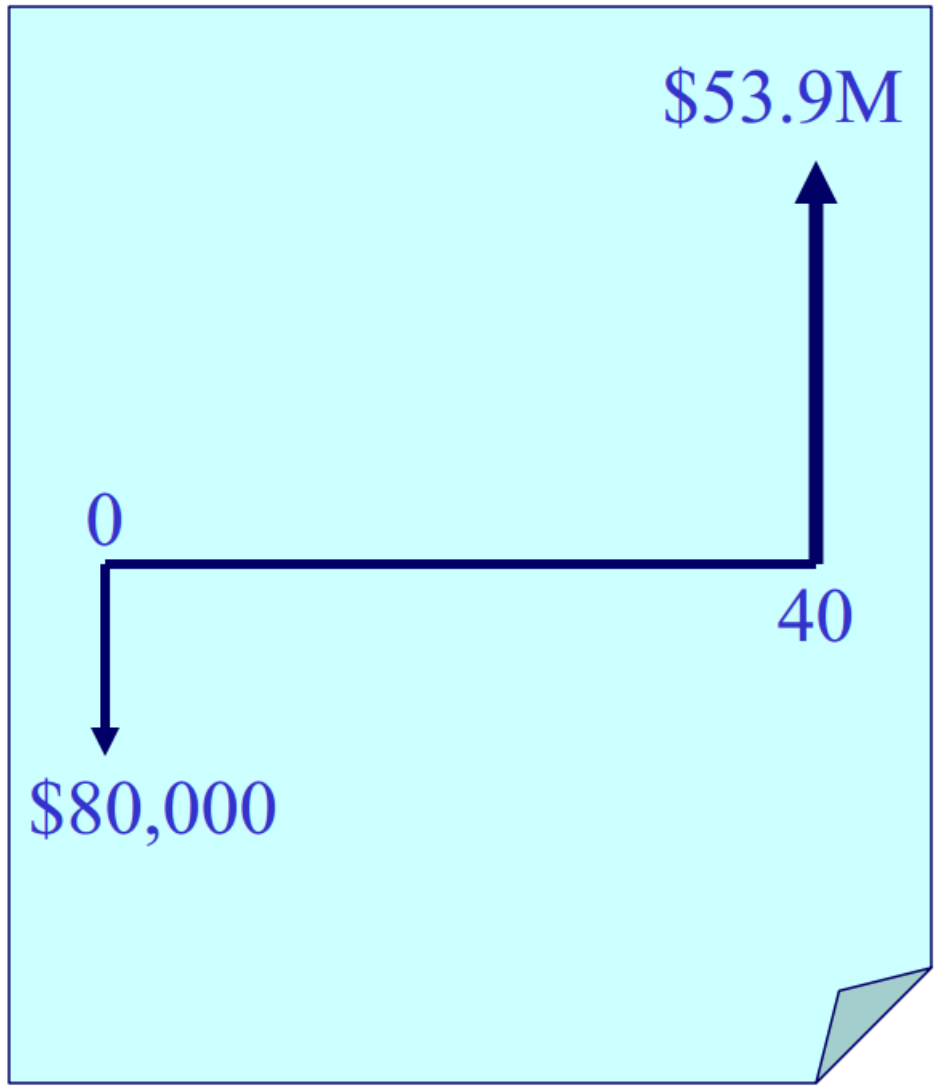
- Example: Vincent Gogh's painting "Irises"
  - John Whitney Payson bought the art at $80,000.
  - John sold the art at $53.9 million in 40 years.
  - What is the rate of return on John's investment?

Rate of Return

- Given: P =\$80,000, F = \$53.9M, and $N = 40$ years
- Find: i
- Solution:

$$\$53.9\,M = \$80{,}000(1+i)^{40}$$

$$i = 17.68\%$$



• Definition 1: Rate of return (ROR) is defined as the interest rate earned on the *unpaid balance* of an installment loan.

• Example: A bank lends $10,000 and receives annual payment of $4,021 over 3 years. The bank is said to earn a *return of 10%* on its loan of $10,000.

• Definition 2: Rate of return (ROR) is the break-even interest rate, i^*, which equates the present worth of a project's cash outflows to the present worth of its cash inflows.

• Mathematical Relation:

$$PW(i^*) = PW(i^*)_{\text{cash inflows}} - PW(i^*)_{\text{cash outflows}}$$
$$= 0$$

Rate of Return Method

- If the return on investment is expressed in terms of rate of return or percentage, even common layman can readily understand.
- In this method, the interest is found out that equates the equivalent worth of an all cash inflows (receipts or savings) and cash outflows (investment and expenditure).
- In other word IRR is the interest rate at which given cash flow becomes zero. IRR can be found out by any of EW (PW or FW or AW) Method.
- Once IRR is computed, it is compared with MARR.
  I**f IRR ≥ MARR,** Accept the project,
  If IRR < MARR, Rejected the project

NET PRESENT VALUE (NPV)

- Net present value (NPV) is **the difference between the present value of cash inflows and the present value of cash outflows over a period of time**.
- Net Present Value (NPV) is the value of all future cash flows (positive and negative) over the entire life of an investment discounted to the present.
- NPV analysis is a form of intrinsic valuation and is used extensively across finance and accounting for determining the value of a business, investment security, capital project, new venture, cost reduction program, and anything that involves cash flow.

***NPV*=TVECF−TVIC**
where:
TVECF=Today's value of the expected cash flows
TVIC=Today's value of invested cash

NPV is ……proportional to discount rate

A) Directly B) Inversely C) Constant D) All of the above

IRR

- IRR is the term for rate of return that stresses the interest earned on the portion of project that is **internally invested.**

IRR = the discount rate(%) that forces

" PV inflows = cost"

➡ **Equating NPV = 0**

NPV: Enter i, solve for NPV

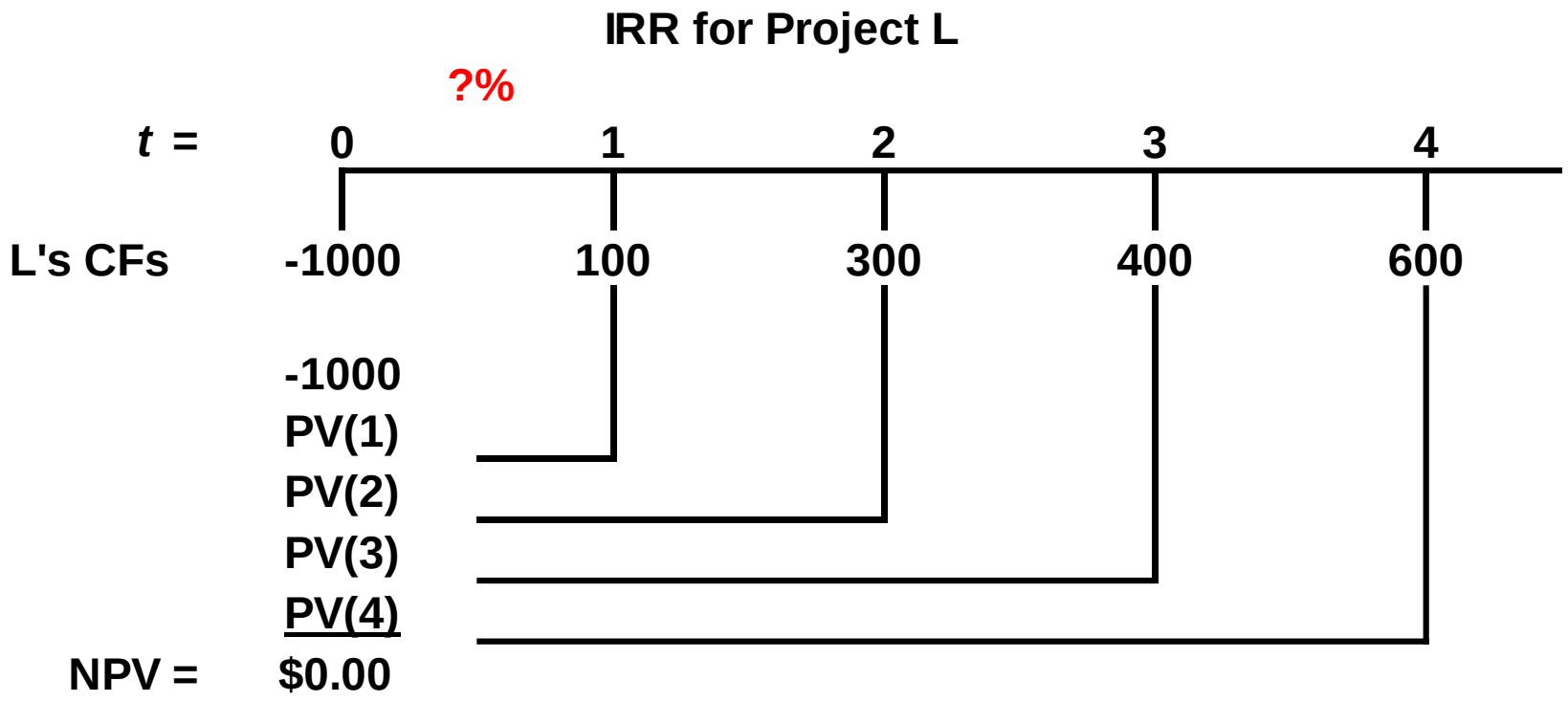
$$\sum_{t=0}^{n} \frac{CF_t}{(1+i)^t} = NPV$$

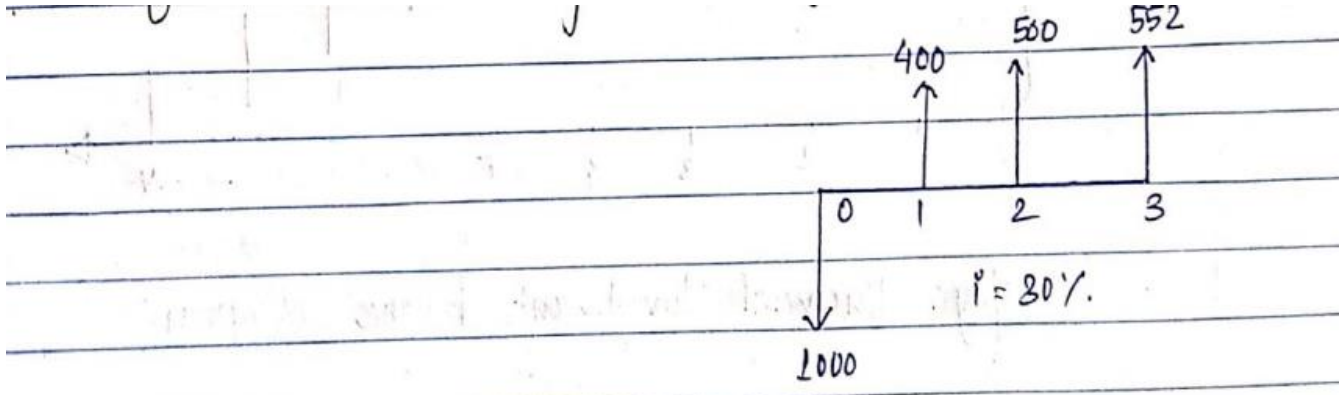
IRR: Enter NPV = 0, solve for x = IRR

$$\sum_{t=0}^{n} \frac{CF_t}{(1+x)^t} = 0$$

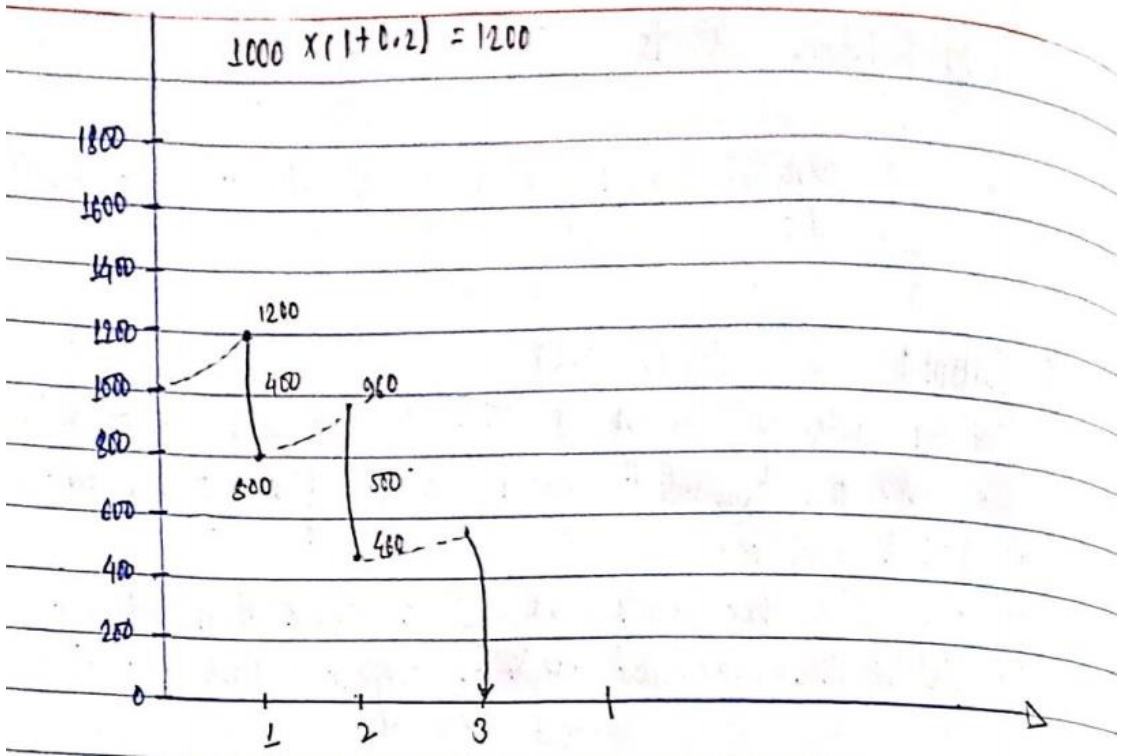
- Methods of Calculations of IRR
  - A) Direct Solution Method
  - B) Trial and Error Method
  - C) Computer Solution Method

Calculate IRR (i = ?)





An Unrecovered Investment balance diagram to illustrate IRR.



Evaluate the IRR of the following project and decide whether the project is acceptable or not. Also draw investment balance diagram.

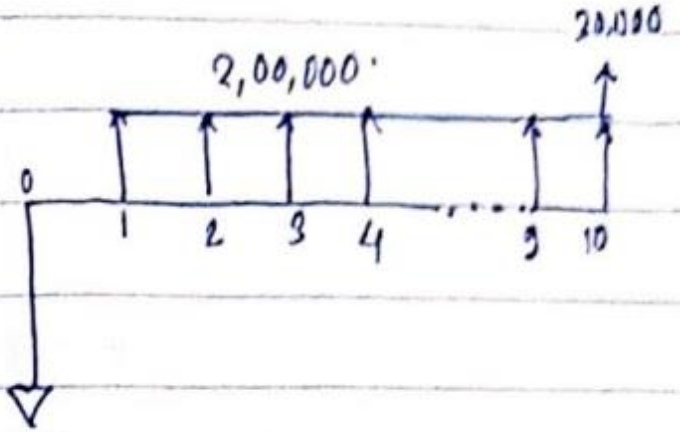
Initial Investment = Rs. 10,00000

Net annual Revenue = Rs. 2,00,000.

Salvage value = Rs. 30,000.

Useful life = 10 years.

MARR = 11%



Soln.

To find IRR

Net present value (worth) = 0 at i%.

$$-10,00,000 + 2,00,000 * (P/A, 9\%, 10) + 30,000 * (P/F, 9\%, 10) = 0$$

$$-10,00,000 + 2,00,000 * \frac{(1+i)^{10} - 1}{i*(1+i)^{10}} + \frac{30,000}{(1+i)^{10}} = 0$$

Hit and Trail Method.

Assume $i = 20\%$.

$$PW(20\%) = -10,000 + 2,00,000 * \left[\frac{(1.2)^{10}-1}{0.2 \times 1.2^{10}}\right] + \frac{30,000}{(1+0.2)^{20}}$$

$$= -1,56,600 \quad \text{(project is in loss)}$$

So, we have to decrease i.

Assume $i = 15\%$.

$PW(15\%) = 11,169.26$ (project is in profit)

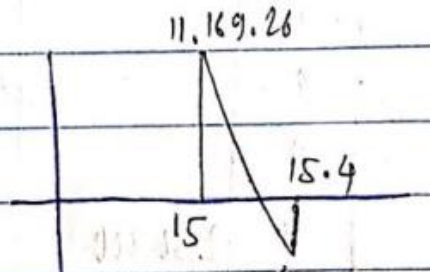
} Range of Answer

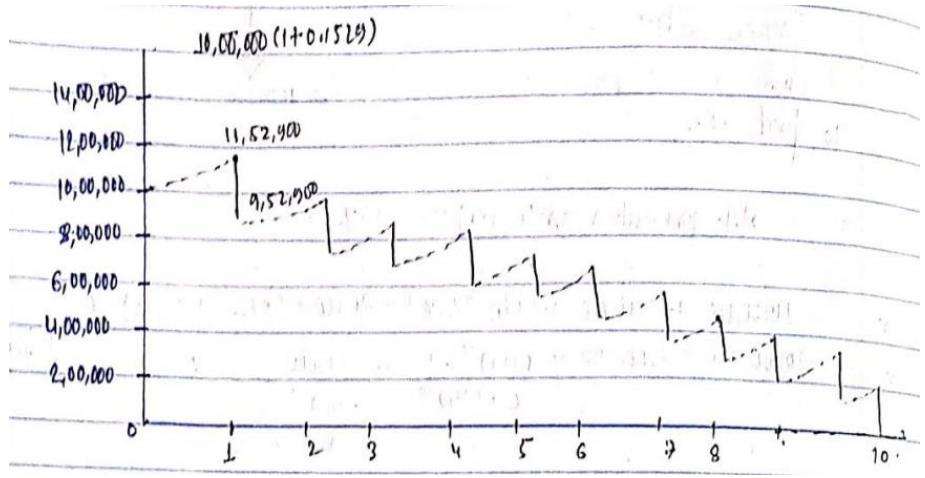
Assume $i = 16\%$.

$PW(15\%) = -26,553.99$

Assume $i = 15.4\%$.

$PW(15.4) = -4,200$





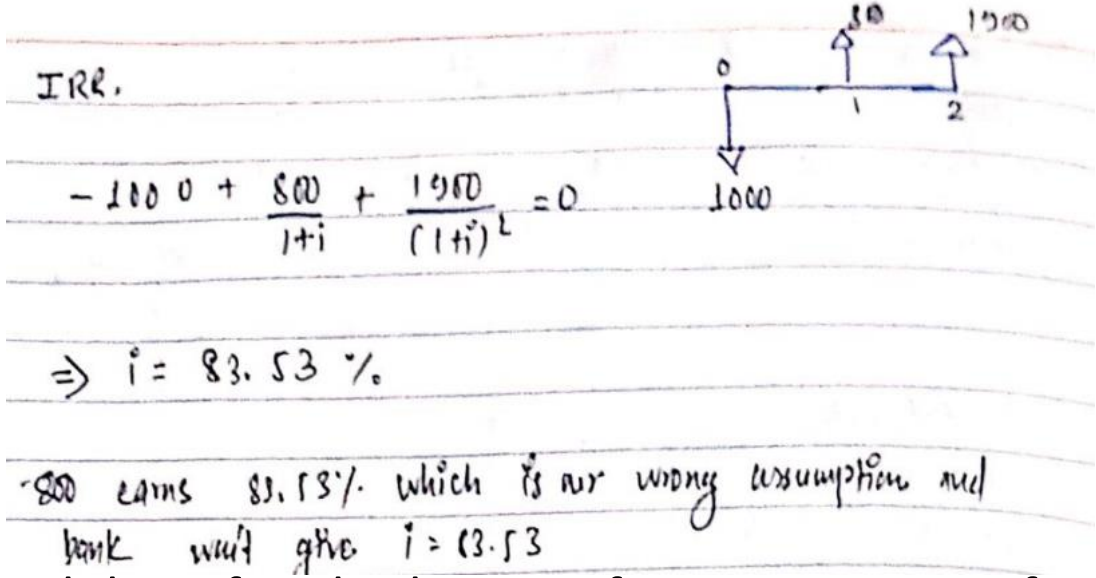
Using Interpolation

$$IRR(i) = 15\% + \frac{11169.26}{11169.26 + 4200} * (15.4 - 15\%)$$

$\therefore i = 15.29\% > 11\%$, project is beneficial

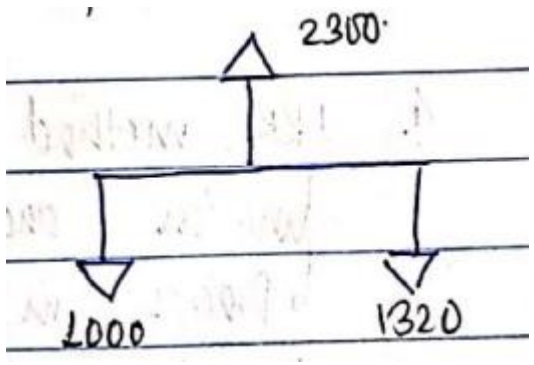
Disadvantages of IRR

- The IRR method assumes that the recovered funds, if not consumed at the end of the year, are reinvested at IRR rather than at MARR. Greater the IRR, at much higher rate of return, it may not be practically possible to reinvest net cash proceeds from the project within the firm.



- IRR may not be uniquely defined. There is possibility of multiple rate of return, in case of non-simple investment (i.e. cash flow stream of a project has more than one changes in sign).
- Find the IRR for the following cashflow:

EOY	0	1
Cashflow	-1,600 +10,000	-10,000



Disadvantages of IRR

- When choosing among mutually exclusive projects, IRR may be misleading:

a. substantial different cash outlays

Project	0	1	IRR	PW(12%)
P	-10000	+20,000	100%	+7858
Q	-50,000	+75,000	50%	+16968

b. different project lives

Project	0	1	2	3	4	5	IRR	PW(12%)
P	-10000	+12,000	-	-	-	-	20%	+909
Q	-10,000	-	-	-	-	20114	15%	+2489

c. different timing of cashflows

Project	Cashflow				IRR	PW	PW	PW	PW
	0	1	2	3	%	5%	10%	20%	30%
P	-1680	1,400	700	140	23	409	276	-53	-125
Q	-1680	140	840	1510	17	520	276	-106	-388

Q is better at less than 10%, P is better at greater than 10%.

Incremental analysis should be conducted (See Chapter 4).

External Rate of Return (ERR) or Modified IRR

- The ERR method takes into account the external reinvestment rate (Ɛ) at which net cash flows generated (or required) by the project over its life can be reinvested (or borrowed) outside the firm.
- (∑ PW of negative net cash flows at Ɛ%) * (1+ERR)**^N** = (∑ FW of positive net cash flows at Ɛ%).
-

Steps of ERR calculation

- All cash outflows are discounted to period zero (present) at ε % per compounding period
- All cash inflows are Compounded to period N at ε %
- ERR is the interest rate that equivalence between the two equation.

Accept /Reject Decision Rule

If $ERR > MARR$, *accept the project*
$ERR = MARR$, *remain indifferent*
$ERR < MARR$, *reject the project*

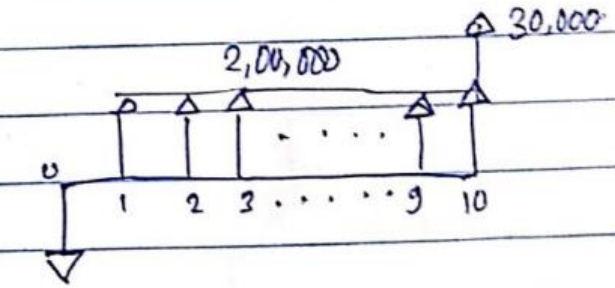
Advantage of ERR over IRR

1. It doesn't need trail and error process to solve for i%.
2. No possibility of multiple rate of return.
3. Use of reinvestment rate.

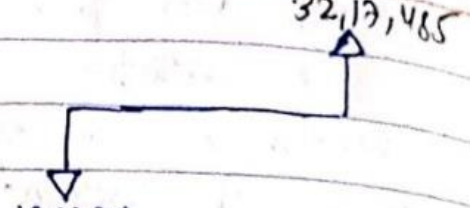
Q. find ERR if external re-investment rate is 10%.

MARR = 11% per year

ERR = ?



Compounding all cash inflows to the year 10 @ 10%.



$F_{10} = 2,00,000 * (F/A, 10\%, 10) + 30,000$

$= 2,00,000 * \frac{(1+i)^N - 1}{i} + 30,000$

$F_{10} = 32,17,485$

$P * (1+i)^N = F$

find i.

Establishing the equivalence between two quantities

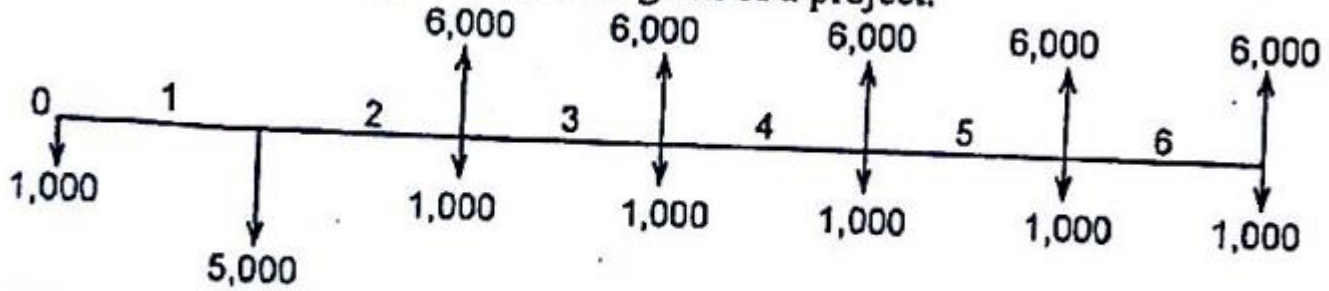
$10,00,000\ [F/P, i\%, 10] = 32,17,485$

$10,00,000\ (1+i)^{10} = 32,17,485$

$\therefore i = 12.84\% > MARR$

Hence, project is feasible $\therefore$ Accept it.

Consider the following cash flow diagram of a project.



Calculate the ERR of the project if MARR = 30% and external reinvestment rate 0 = 15%. Is the project accepted?

Discounting all the cash outflows to the present at 15%;

$$1{,}000 + 5{,}000(P/F, 15\%, 1) = 1{,}000 + 5{,}000(1 + 0.15)^{-1}$$

$$= 5{,}347.82$$

Compounding all the cash inflows to the year 6 at 15%

$$5{,}000(F/A, 15\%, 5) = 5{,}000\left\{\frac{(1 + 0.15)^5 - 1}{0.15}\right\}$$

$$= 33{,}711.90$$

Establishing the equivalence between two quantities

$$5{,}347.82(F/P, i'\%, 6) = 33{,}711.90$$

or, $5{,}347.82(1 + i')^6 = 33{,}711.90$

or, $(1 + i')^6 = 6.303$

or, $i' = 1.3591 - 1 = 0.3591$

Since, ERR (35.91%) > MARR (30%),
Accept the project.

Benefit/ Cost Ratio (BCR)

- Benefit Cost Relation is a special tool of cost Benefit analysis (CBA) and it is used to evaluate public sector projects, although it can be extended to any private sector projects.
- B/C ratio can be defined as the ratio of the equivalent worth of benefits to the equivalent worth of costs.

Conventional B/C ratio

It is the ratio of gross benefits to costs and e

With PW formulation

$$\text{B/C ratio} = \frac{PW(B)}{PW(I) - PW(S) + PW(O \& M)}$$

With FW formulation

$$\text{B/C ratio} = \frac{FW(B)}{FW(I) - FW(S) + FW(O \& M)}$$

With AW formulation

$$\text{B/C ratio} = \frac{AW(B)}{AW(I) - AW(S) + AW(O \& M}$$

Modified B/C ratio

It is the ratio of net benefits to costs

With PW formulation

$$\text{B/C ratio} = \frac{PW(B) - PW(O \& M)}{PW(I) - PW(S)}$$

With FW formulation

$$\text{B/C ratio} = \frac{FW(B) - FW(O \& M)}{FW(I) - FW(S)}$$

With AW formulation

$$\text{B/C ratio} = \frac{AW(B) - AW(O \& M)}{AW(I) - AW(S)}$$

$B/C > 1$, *accept the project*

$B/C = 1$, *remain indifferent*

$B/C < 1$, *reject the project*

Benefit–cost ratio BC(i)	**A relative measure to evaluate public projects by finding the ratio of the equivalent benefits over the equivalent costs.**	**BC(i) > 1**
Profitability index PI(i)	**A relative measure to evaluate projects by calculating the ratio of the equivalent net benefits over the equivalent net investment.**	**PI(i) > 1**

For PSc, we focus on PV only

While the minimum value of b/c ratio for selection of any project is 1.

Internal rate of return	$\mathrm{PW}(i^*) = \frac{A_0}{(1+i^*)} + \frac{A_1}{(1+i^*)} + \cdots + \frac{A_N}{(1+i^*)} = 0$
Benefit–cost ratio	$\mathrm{BC}(i) = \frac{B}{I + C'}$
Profitability index	$\mathrm{PI}(i) = \frac{B - C'}{I}$

B/C ratio is …….. proportional to discount rate

C/B ratio ??

Find both types of B/C ratio using PW, FW and AW method.

Initial investment = Rs 20,000
Revenue / year = Rs 10,000
Expenses/ year = Rs 4,400
Salvage value = Rs 4,000
Useful life = 5 years
MARR = 8%

Using PW method

PW (B) $= 10,000\ (P/A, 8\%, 5) = 10,000\ [\frac{(1.08)^5 - 1}{(1.08)^5 * 0.08}]$

$= Rs\ 39,927$

PW (S) $= 4,000\ (P/F, 8\%, 5)$

$= 4,000\ (1/\ (1.08)^5)$

$= Rs\ 2,722.33$

PW (O&M) $= 4,400\ (P/A, 8\%, 5)$

$= 4,400[\frac{(1.08)^5 - 1}{(1.08)^5 * 0.08}]$

$= Rs\ 17,567.90$

Conventional B/C ratio $= \frac{PW\ (B)}{(I) - PW\ (S) + PW\ (O\&M)}$

$= \frac{39,927}{20,000 - 2,722.33 + 17,567.90}$

= 1.146 > 1 (justified)

Modified B/C ratio $= \frac{PW\ (B) - PW\ (O\&M)}{(I) - PW\ (S)}$

$= \frac{39,927 - 17567.90}{20,000 - 2722.33}$

= 1.294 > 1 (justified)

Using AW method

Conventional B/C ratio $= \frac{AW\ (B)}{CR + AW\ (O\&M)}$

CR $= 20,000\ [A/P, 8\%, 5] - 4,000\ (A/F, 8\%, 5)$

$= 20,000\ [\frac{(1.08)^5 * 0.08}{(1.08)^5 - 1}] - 4,000\ (0.08/\ (1.08)^5 - 1)$

$= Rs\ 4,327$

Conventional B/C ratio = 10,000/4,327+ 4,400

= 1.146 > 1 (justified)

Modified B/C ratio $= \frac{AW\ (B) - AW\ (O\&M)}{(CR)}$

$= 10,000 - 4,400\ /4,327$

= 1.294 >1 (justified)

COMPARISON OF EXCLUSIVE ALTERNATIVES HAVING SAME USEFUL LIFE

- **4.1.1 Payback period and Equivalent Worth method**

❑ **For this method, calculate as earlier and just compare the result of every project**

❑ **project with Equivalent worth < 0 are rejected**

❑ **Project with highest worth among project worth >0 is accepted**

❑ **Payback period > Standard pay back period are rejected**

❑ **Project with smallest Payback period among project payback < Standard pay back period is accepted**

Compare the mutually exclusive alternatives (MARR = 12%.)
(By PW method)

	Option 1	Option 2	Option 3
Investment(rs)	2,69,00	3,19,000	3,30,000
Net Revenue	81,500	86,500	98.300
Useful life	5	5	5
Salvage Value	1,00,000	1,20,000	1,10,000.

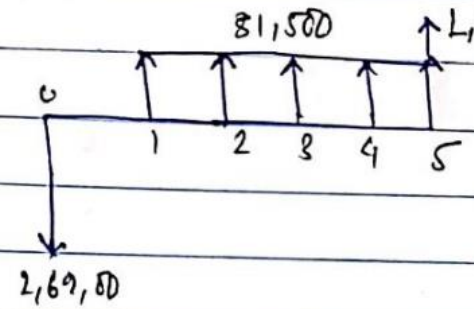
Option 3.

$$PW(12\%) = -3,30,000 + 98,300 * \frac{1.12^5 - 1}{0.12 * 1.12^5} + \frac{1,10,000}{1.12^5}$$

$$= 86,766$$

$PW_3 > PW_1 > PW_2$; Option 3 is most beneficial option.

Option 1



$$PW(12\%) = -2,69,00 + 81,500 * (P/A, 12\%, 5) + 1,00,000 * (P/F, 12\%, 5)$$

$$= -2,69,00 + 81500 * \frac{(1+0.12)^5 - 1}{0.12 * 1.12^5} + \frac{1,00,000}{1.12^5}$$

$$= 81,532$$

Option 2

$$PW(12\%) = -3,19,00 + 86,500 * \frac{1.12^5 - 1}{0.12 * 1.12^5} + \frac{1,20,000}{1.12^5}$$

$$= 60,904$$

Numerical

From the following three mutually exclusive projects, determine which should be chosen using the AW method. MARR is 10% per year and study period is 5 years and there is no limitation on investment fund.

Project	X	Y	Z
Initial investment (Rs.)	10,000	12,000	15,000
Net annual revenue (Rs.)	2,300	2,800	4,068
Salvage value	10,000	0	0

Also compare the projects with both type of Pay back periods if Standard payback is 4 years

AW calculation for each project;

AW for project X = R – CR

= 2,300 – {10,000(A/P, 10%, 5) – 10,000(A/F, 10%, 5)}

= 2,300 – {10,000 (0.2638) – 10,000 (0.1638)}

= 2,300 – (2,638 – 1,638)

= **Rs. 1,300**

AW for project Y = R – CR

= 2,800 – {12,000(A/P, 10%, 5) – 0}

= 2,800 – (3,166)

= – **Rs. 366**

AW for project Z = R – CR

= 4,068{15,000(A/P, 10%, 5) – 0}

= 4,068 – 3,957

= **Rs. 111**

∴ Project X and Z are both satisfactory but project X is highly preferred due to high AW.

Rate of Return Method and BCR Method Steps for Incremental Analysis

1) Calculate IRR or ERR or BCR for each alternative & screen out unfeasible alternatives from the analysis.
2) Arrange the feasible alternatives based on increasing initial investment.
3) Choose the feasible alternative having least initial investment as the base alternative.
4) Incremental analysis is performed between base alternative & alternative with the next higher initial investment. Analysis is aimed to check whether it is worthwhile to increase investment from base alternative to next higher initial investment. Calculate the incremental cash flow & calculate incremental IRR or ERR or BCR as the case may be. If incremental IRR≥ MARR or ERR≥MARR or BCR≥1, then increment of investment to next higher initial investment is justified. Otherwise, return to base alternative.
5) Repeat & select the best alternative

Rate of Return Method and BCR Method

IRR, ERR, BCR

Unlike equivalent worth method, highest figure cannot be selected as best

EOY	A	B	(MARR = 10%)
0	-1000	-5,000	
1	2000	7000	
IRR =	100%	40%	
BCR(10%) =	1.818	1.27	
PW(10%) =	818.18	369.68	

NPW = 0 ; i calc.

Equivalent worth method is absolute method where as IRR, ERR and BCR is relative measure and cannot be applied in same way. To remove the inconsistency, Incremental analysis is done.

Incremental analysis

It evaluate differences, or the 'increment' between two or more mutually exclusive alternatives.

Incremental Investment

n	Project A1	Project A2	Incremental Investment (A2 – A1)
0	-$1,000	-$5,000	-$4,000
1	$2,000	$7,000	$5,000
ROR	100%	40%	25%
PW(10%)	$818	$1,364	$546

- Assuming MARR of 10%, you can always earn that rate from other investment source, i.e., $4,400 at the end of one year for $4,000 investment.

- By investing the additional $4,000 in A2, you would make additional $5,000, which is equivalent to earning at the rate of 25%. Therefore, the incremental investment in A2 is justified.

Incremental Analysis (Procedure)

Step 1: Compute the cash flows for the difference between the projects (A,B) by subtracting the cash flows for the lower investment cost project (A) from those of the higher investment cost project (B).

Step 2: Compute the IRR on this incremental investment (IRR_{B-A}).

Step 3: Accept the investment B if and only if

$$IRR_{B-A} > MARR$$

IRR on Increment Investment: Three Alternatives

n	D1	D2	D3
0	-$2,000	-$1,000	-$3,000
1	1,500	800	1,500
2	1,000	500	2,000
3	800	500	1,000
IRR	34.37%	40.76%	24.81%

Step 1: Examine the IRR for each project to eliminate any project that fails to meet the MARR.

Step 2: Compare D1 and D2 in pairs.

$IRR_{D1-D2}=27.61\% > 15\%$,

so select D1.

Step 3: Compare D1 and D3.

$IRR_{D3-D1}= 8.8\% < 15\%$,

so select D1.

Here, we conclude that D1 is the best Alternative.

Capitalized worth and Externalities

- Capitalized worth is the present worth of an alternative for overall study period.
- CW method is introduced as a special case of PW criterion when revenue and expenses occur over an infinite length of time.
- Perpetuity: Infinite annuity is called perpetuity.
    - CW(i%) = A/I

Externalities

- An externality is referred to as an external effect, is a special class of goods which is not deliberately created by project sponsor but is an incidental outcome of legitimate economic activity.

Positive /Negative externalities:

- The approach road built by a company may improve the transport of that area.
- Nearly constructed airport will increase the sound level of surrounding

Some Qns

1. Benefit cost ratio is the ratio of

- A) PVB/PVC B) PVC/PVB c) FVB/FVC d) FVC/FVB

2. the average cost incurred in supplying a particular benefit over the entire life time of the project is

A) project cost b) Lifecycle cost

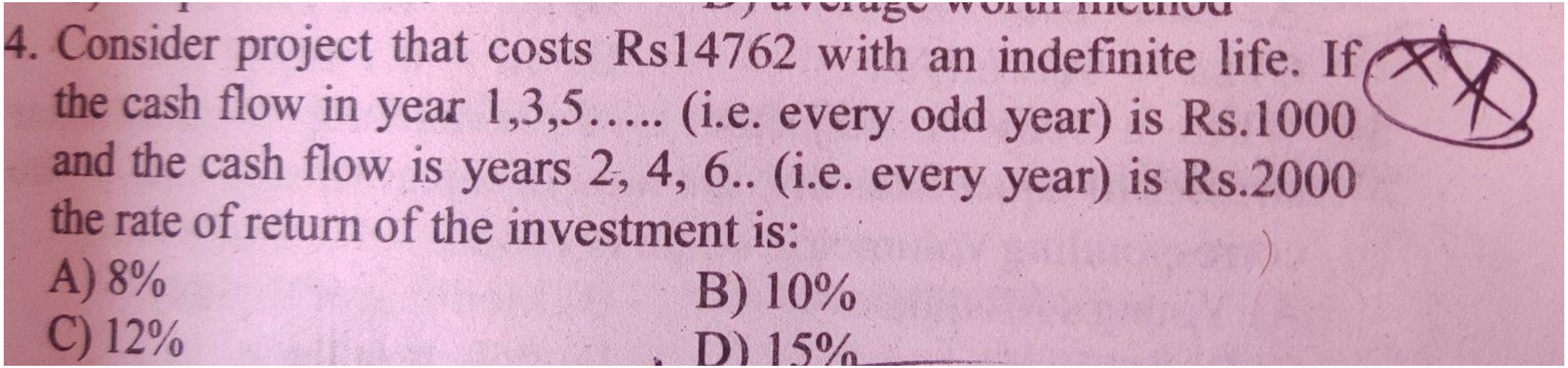
c) Maintenance cost d) none

3. Find the effective interest rate if 10% is compounded 3 times per year

- A) 10.11 b) 10.22 c)10.33 d)10.44

4. Find the effective interest rate if 12% is compounded quarterly

- A) 12.22 b) 12.33 c)12.44 d)12.55


4. Consider project that costs Rs14762 with an indefinite life. If the cash flow in year 1,3,5….. (i.e. every odd year) is Rs.1000 and the cash flow is years 2, 4, 6.. (i.e. every year) is Rs.2000 the rate of return of the investment is:

A) 8%
B) 10%
C) 12%
D) 15%

Compound rate: Rate used to calculate Future value of Present cash flow.

Discount rate: Rate used to calculate present value of series of future cash flow. Inverse of compound rate

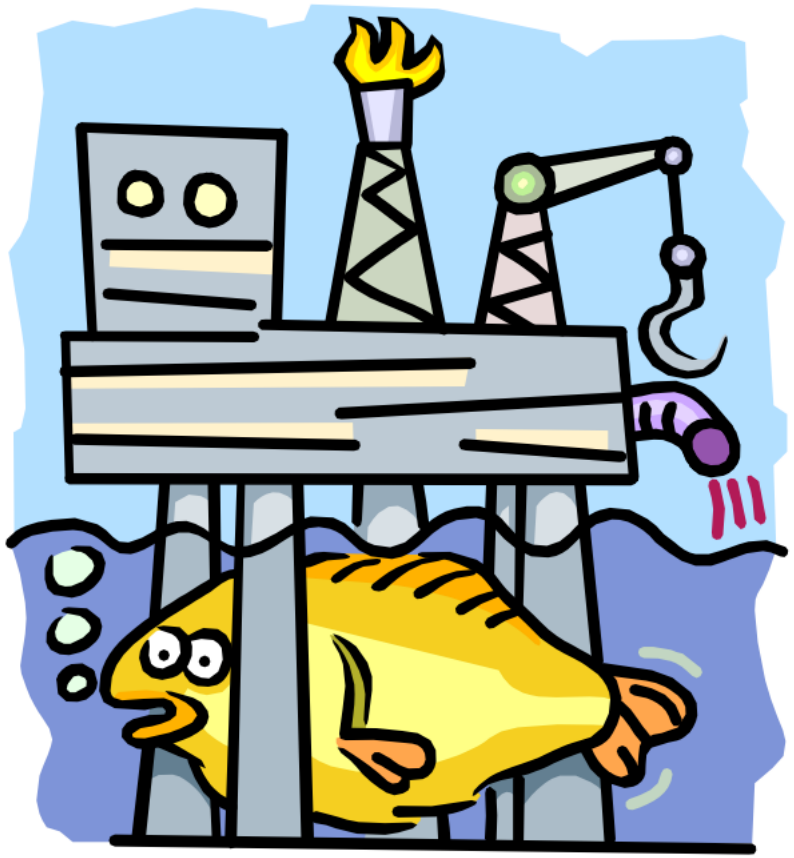
Accounting Rate of return

(does not consider time value of money)

- ARR = Average Income/ Average Investment
- ARR> MARR

Origins of Project Risk

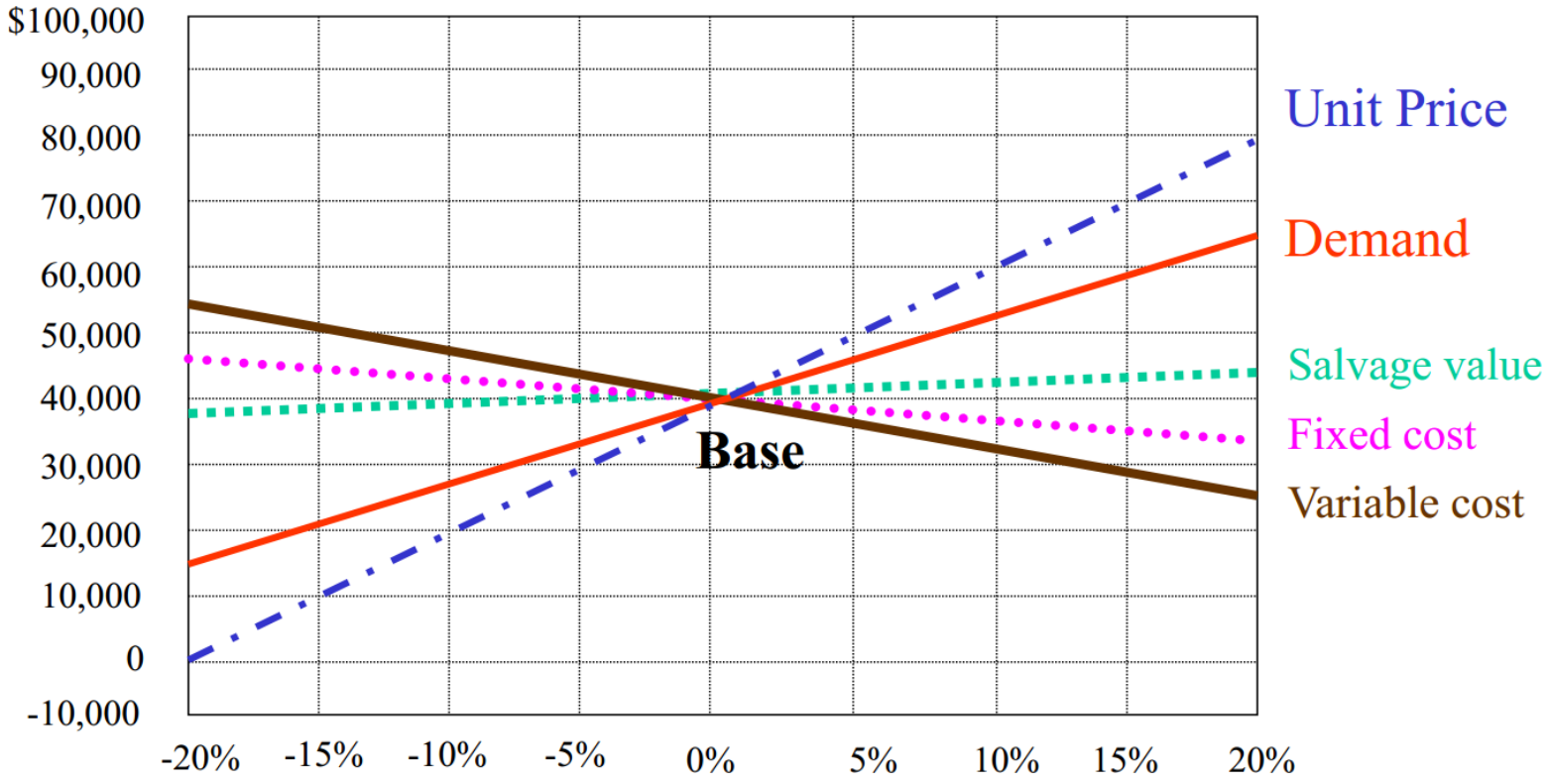
- Risk: the potential for loss
- Project Risk: variability in a project's NPW
- Risk Analysis: The assignment of probabilities to the various outcomes of an investment project



Methods of Describing Project Risk

- Sensitivity Analysis: a means of identifying the project variables which, when varied, have the greatest effect on project acceptability.
- Break-Even Analysis: a means of identifying the value of a particular project variable that causes the project to exactly break even.
- Scenario Analysis: a means of comparing a "base case" to one or more additional scenarios, such as best and worst case, to identify the extreme and most likely project outcomes.

Sensitivity Analysis



- Determining how sensitive our NPV analysis is.
- Can be called Spider plot
- If used for infinite period, it is called Monte Carlo Simulation

Some Definitions

Cost Terminologies

Fixed cost:

- Cost which are associated with fixed factors of production like land, plant, building, equipment etc.
- It remains constant in the production process for certain level of output capacity.

Variable cost:

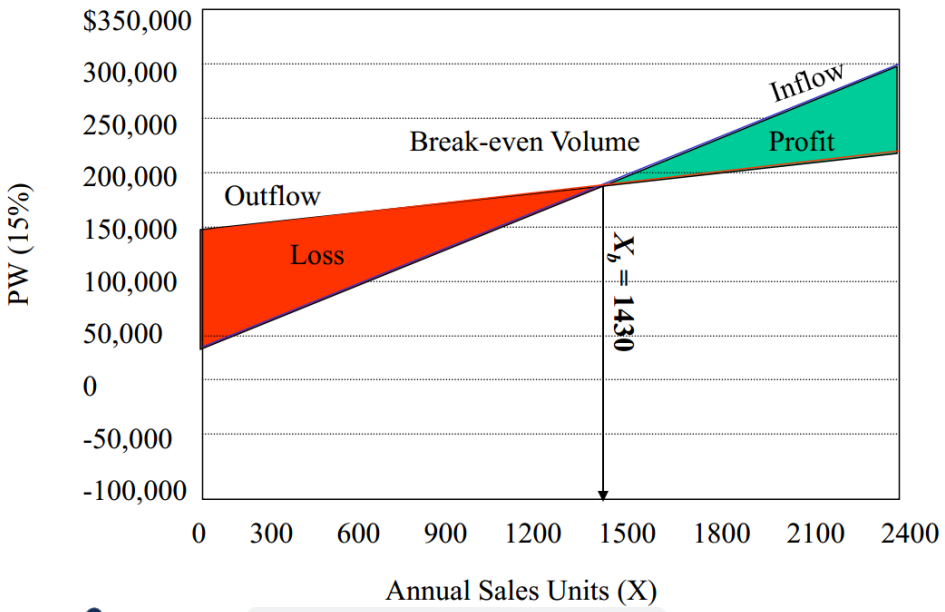
- Cost which are associated with variable factors of production and vary with change in level of output like labour, raw materials, fuel etc.

Total cost/ Sum cost: Sum of fixed and variable cost

Sunk cost:

- All the past cost which cannot be recovered when a firm leave from an industry.
- Cost of engineering design, wages, decoration, depreciation, registration, license etc.

Breakeven Analysis



- Break even quantity
= Fixed costs / (Sales price per unit – Variable cost per unit)
- Total Revenue = Total cost
- Revenue sufficient to cover all expense
- Organization is neither in gain nor in loss.

Other: Export = Import; Supply = Demand;
Cost of two products being equal

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- **Profit** when **Revenue > Total Variable cost + Total Fixed cost**
- **Break-even point** when **Revenue = Total Variable cost + Total Fixed cost**
- **Loss** when **Revenue < Total Variable cost + Total Fixed cost**

Sensitivity Analysis

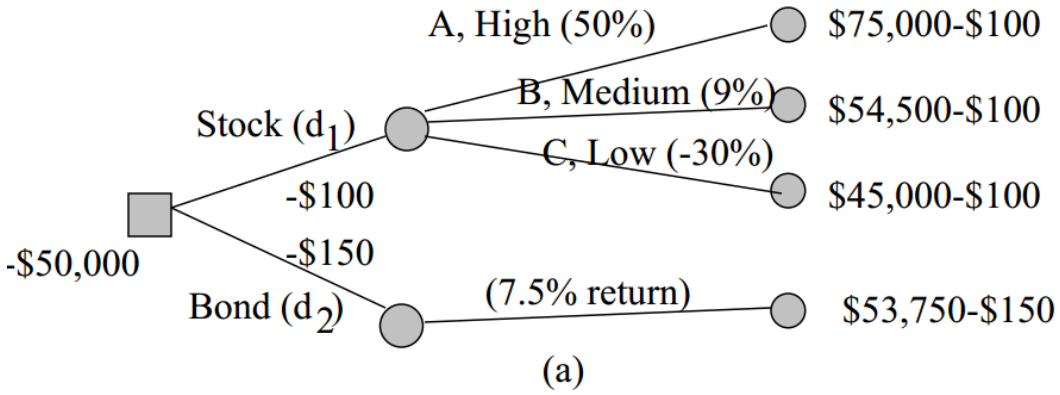
Break even analysis is often a component of sensitivity analysis and scenario analysis performed in financial modeling. Using Goal Seek in Excel, an analyst can backsolve how many units need to be sold, at what price, and at what cost to break even.

https://corporatefinanceinstitute.com/resources/knowledge/modeling/break-even-analysis/

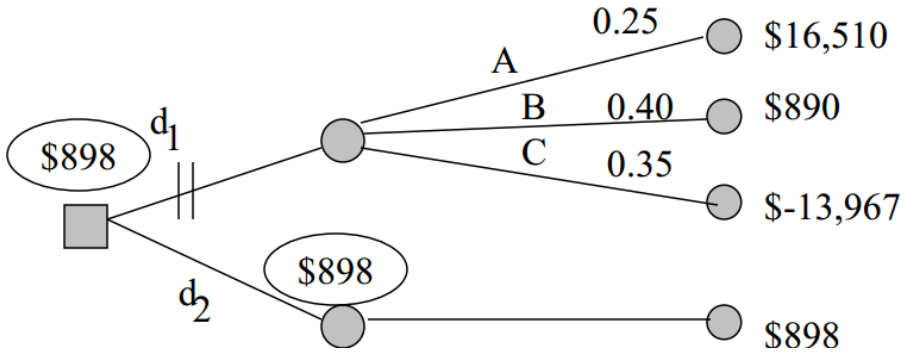
Scenario Analysis

Variable Considered	Worst-Case Scenario	Most-Likely-Case Scenario	Best-Case Scenario
Unit demand	1,600	2,000	2,400
Unit price ($)	48	50	53
Variable cost ($)	17	15	12
Fixed Cost ($)	11,000	10,000	8,000
Salvage value ($)	30,000	40,000	50,000
PW (15%)	-$5,856	$40,169	$104,295

Decision tree Analysis



(a) Relevant cash flows before taxes



(b) Net present worth for Each decision path

COST, RISK ANAlYSIS ,RATIO & Others

SUSHIL RIJAL

Capitalized worth and Externalities

- Capitalized worth is the present worth of an alternative for overall study period.
- CW method is introduced as a special case of PW criterion when revenue and expenses occur over an infinite length of time.
- Perpetuity: Infinite annuity is called perpetuity.

$$CW(i\%) = A/I$$

Externalities

- An externality is referred to as an external effect, is a special class of goods which is not deliberately created by project sponsor but is an incidental outcome of legitimate economic activity.

Positive /Negative externalities:

- The approach road built by a company may improve the transport of that area.
- Nearly constructed airport will increase the sound level of surrounding

Some Qns

1. Benefit cost ratio is the ratio of

A) PVB/PVC B) PVC/PVB c) FVB/FVC d) FVC/FVB

2. the average cost incurred in supplying a particular benefit over the entire life time of the project is

A) project cost b) Lifecycle cost

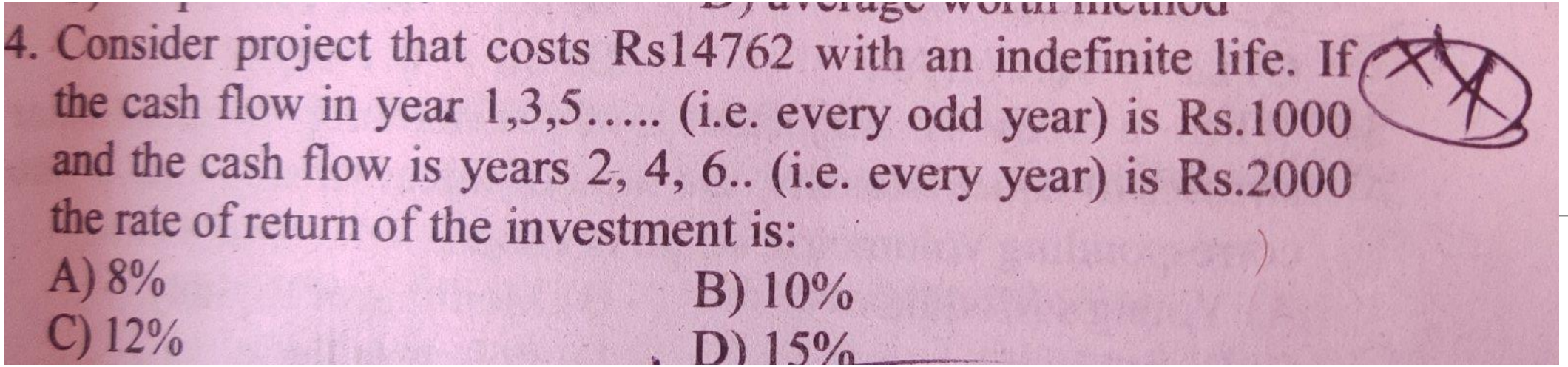
c) Maintenance cost d) none

3. Find the effective interest rate if 10% is compounded 3 times per year

A) 10.11 b) 10.22 c)10.33 d)10.44

4. Find the effective interest rate if 12% is compounded quarterly

A) 12.22 b) 12.33 c)12.44 d)12.55


4. Consider project that costs Rs14762 with an indefinite life. If the cash flow in year 1,3,5….. (i.e. every odd year) is Rs.1000 and the cash flow is years 2, 4, 6.. (i.e. every year) is Rs.2000 the rate of return of the investment is:

A) 8%
B) 10%
C) 12%
D) 15%

Compound rate: Rate used to calculate Future value of Present cash flow.

Discount rate: Rate used to calculate present value of series of future cash flow. Inverse of compound rate

Accounting Rate of return

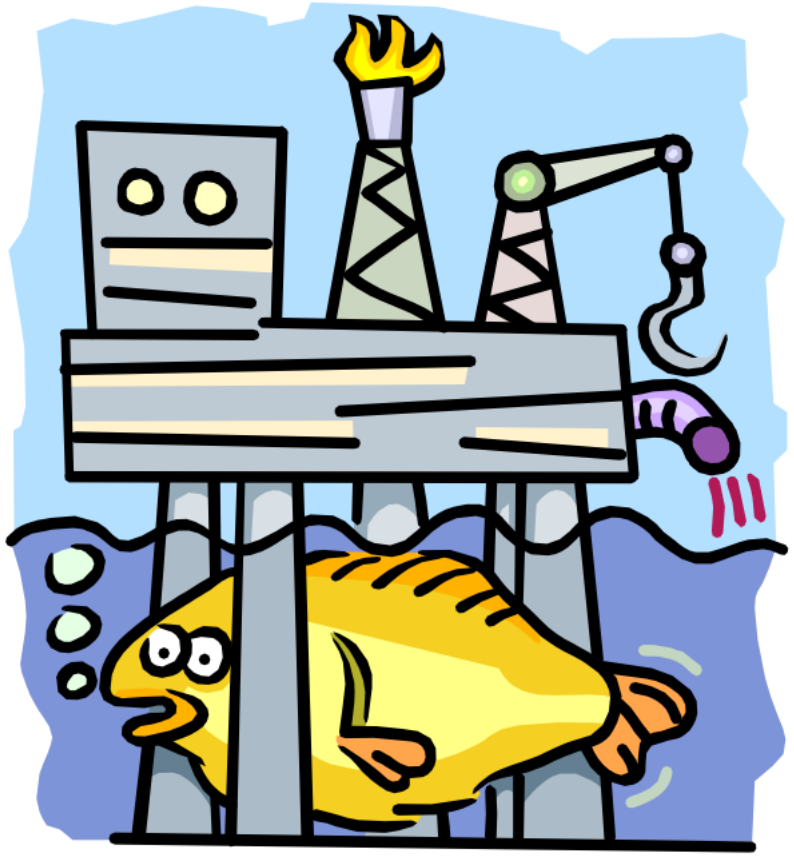
(does not consider time value of money)

ARR = Average Income/ Average Investment

ARR> MARR

Origins of Project Risk

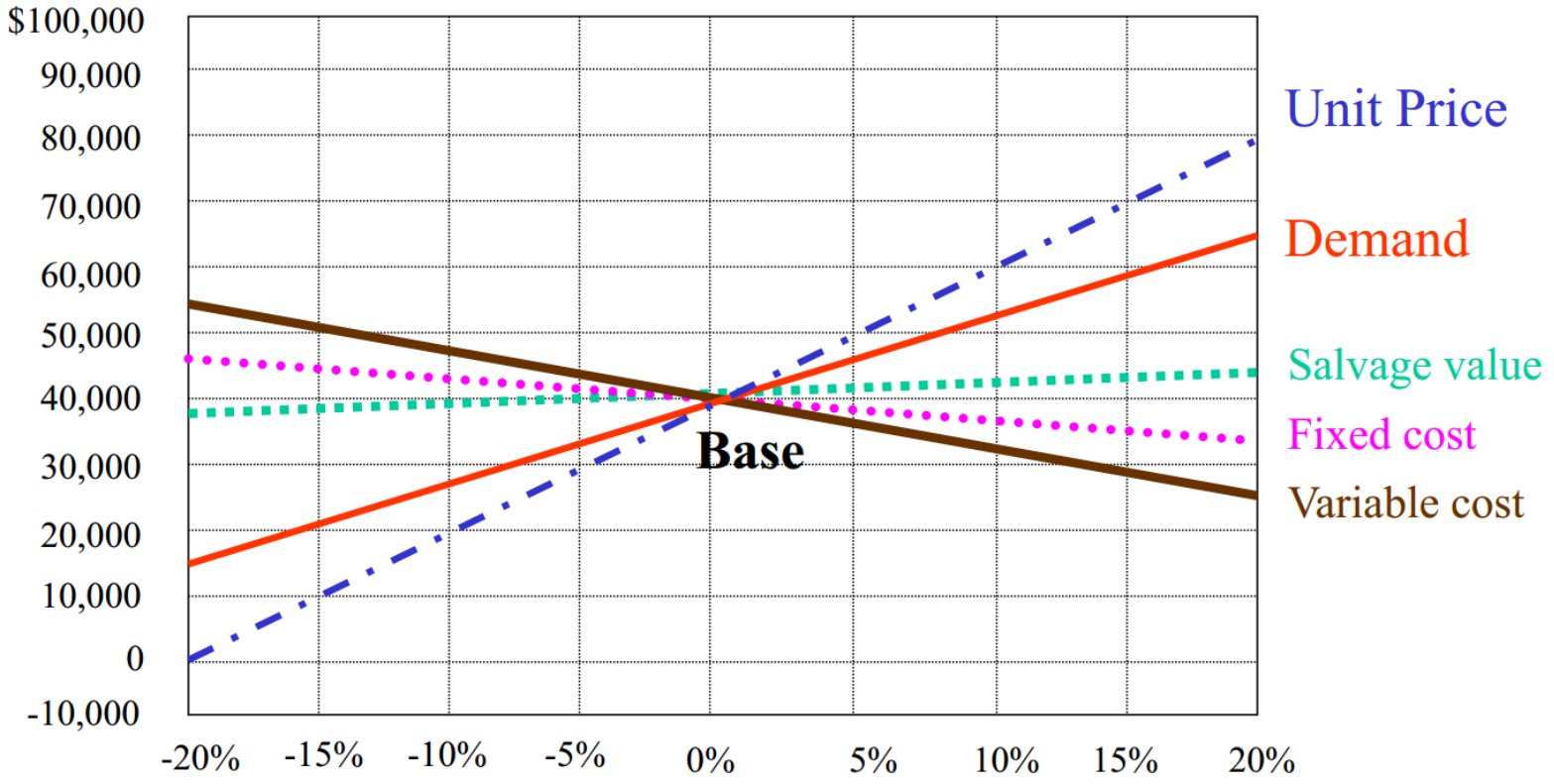
- Risk: the potential for loss
- Project Risk: variability in a project's NPW
- Risk Analysis: The assignment of probabilities to the various outcomes of an investment project



Methods of Describing Project Risk

- Sensitivity Analysis: a means of identifying the project variables which, when varied, have the greatest effect on project acceptability.
- Break-Even Analysis: a means of identifying the value of a particular project variable that causes the project to exactly break even.
- Scenario Analysis: a means of comparing a "base case" to one or more additional scenarios, such as best and worst case, to identify the extreme and most likely project outcomes.

Sensitivity Analysis



- Determining how sensitive our NPV analysis is.
- Can be called Spider plot
- If used for infinite period, it is called Monte Carlo Simulation

Some Definitions

Cost Terminologies

Fixed cost:

- Cost which are associated with fixed factors of production like land, plant, building, equipment etc.
- It remains constant in the production process for certain level of output capacity.

Variable cost:

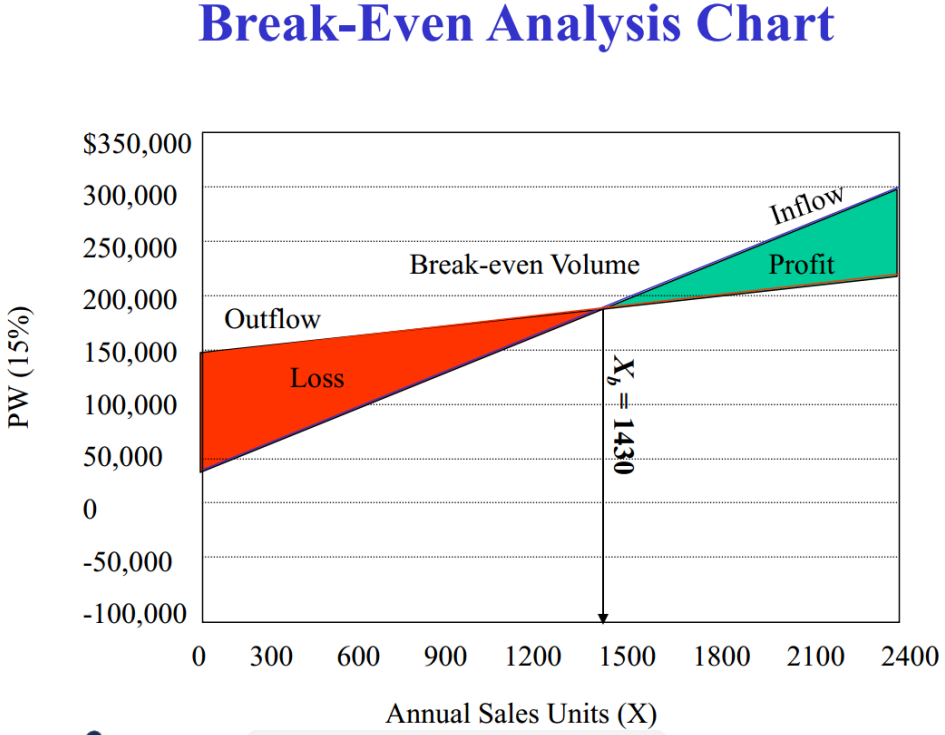
- Cost which are associated with variable factors of production and vary with change in level of output like labour, raw materials, fuel etc.

Total cost/ Sum cost: Sum of fixed and variable cost

Sunk cost:

- All the past cost which cannot be recovered when a firm leave from an industry.
- Cost of engineering design, wages, decoration, depreciation, registration, license etc.

Breakeven Analysis



- Break even quantity
= Fixed costs / (Sales price per unit – Variable cost per unit)

- Total Revenue = Total cost
- Revenue sufficient to cover all expense
- Organization is neither in gain nor in loss.

Other: Export = Import; Supply = Demand;
Cost of two products being equal

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- **Profit** when **Revenue > Total Variable cost + Total Fixed cost**
- **Break-even point** when **Revenue = Total Variable cost + Total Fixed cost**
- **Loss** when **Revenue < Total Variable cost + Total Fixed cost**

Sensitivity Analysis

Break even analysis is often a component of sensitivity analysis and scenario analysis performed in financial modeling. Using Goal Seek in Excel, an analyst can backsolve how many units need to be sold, at what price, and at what cost to break even.

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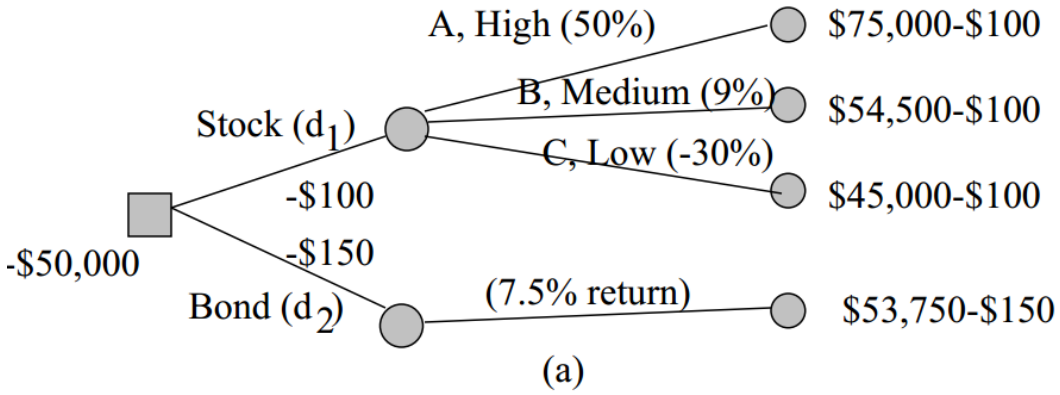

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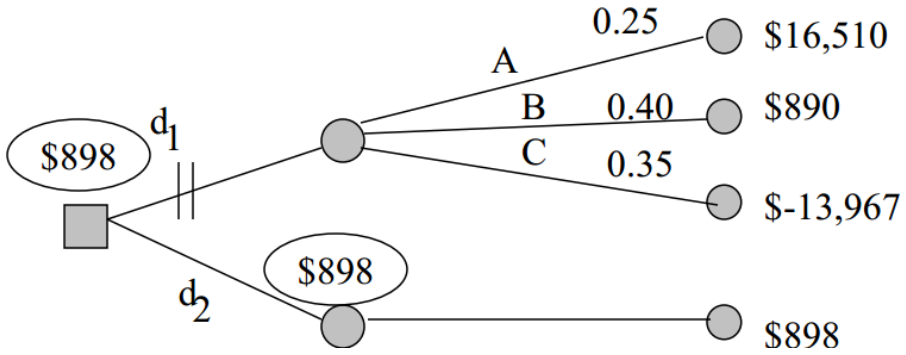


(b) Net present worth for Each decision path

COST

Q. The benefit that is forgone by engaging the resource in a chosen activity instead of engaging the same resource in forgone activity is known as:

A)Marginal cost, B)Opportunity cost,

C) Incremental cost D) Life cycle cost

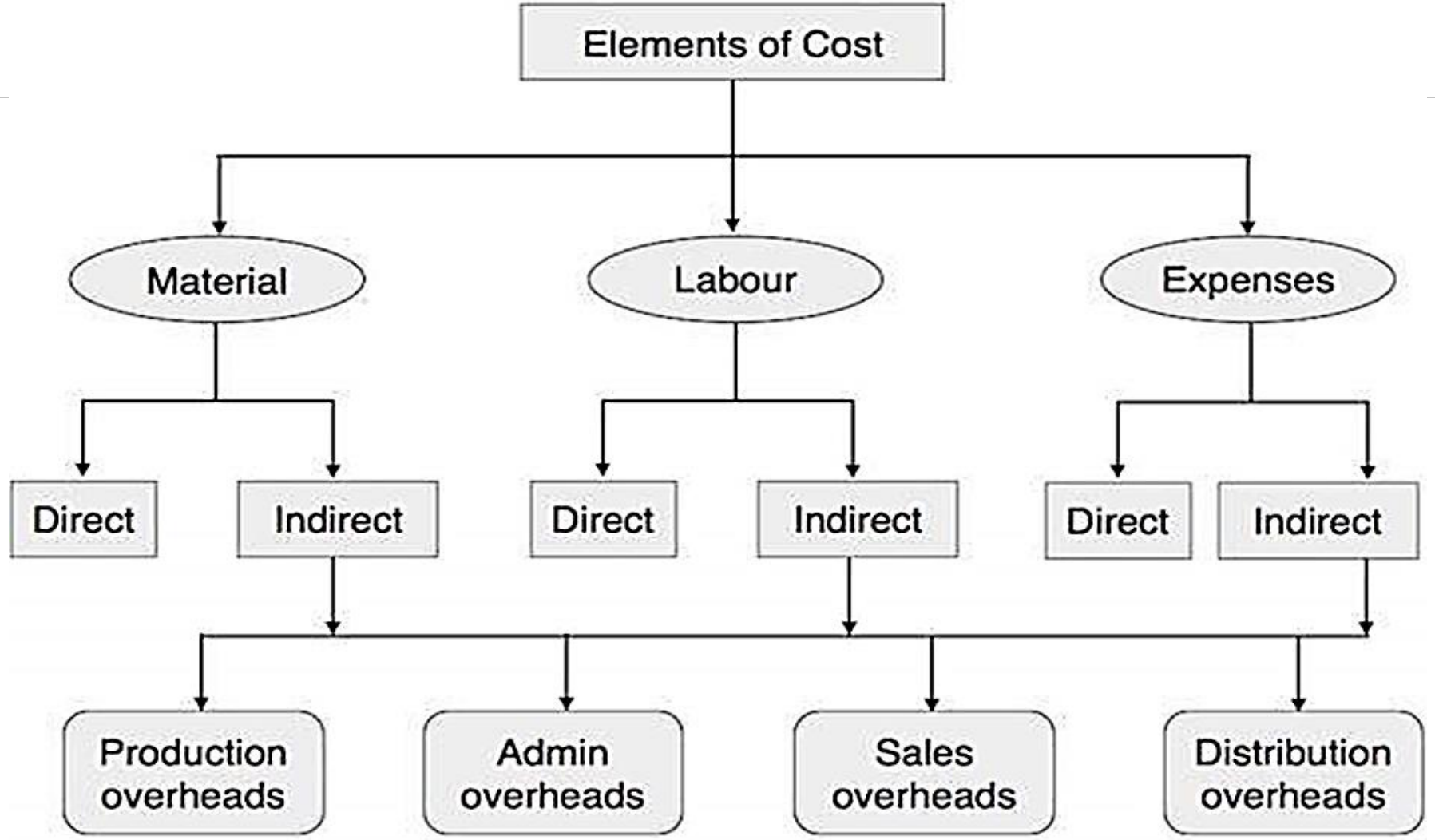
Marginal Cost: The cost of producing one more unit of good

Opportunity cost: Opportunity costs represent the potential benefits an individual, investor, or business misses out on when choosing one alternative over another

projects that are undertaken usually have an expected return at least as great as the return on another alternative that is not funded. The expected rate of return on the unfunded project is called the ***opportunity cost.***

The opportunity cost is the rate of return of a forgone opportunity caused by the inability to pursue a project. Numerically, it is the largest rate of return of all the projects not accepted (forgone) due to the lack of capital funds or other resources. When no specific MARR is established, the de facto MARR is the opportunity cost, i.e., the ROR of the first project not undertaken due to unavailability of capital funds.

Element of cost



Cont…

- Material; the substance from which a product is made is known as material. It may be in a raw or a manufacture state.

- Labour; For conversion of material into finish goods, human effort is needed and such human effort is called labour.

- Expenses; these include cost of special design, drawing, layout, cost of purchase or hire of tools and plants for a particular job and maintenance of such tools and equipment etc.

- Types of cost
  - **Direct cost**; Direct costs are those costs that can be identified specifically with a particular project, or that can be directly assigned to such activities relatively easily with a high degree of accuracy.
  - **Indirect cost;** Indirect Costs are those that are incurred for common or joint objectives and therefore cannot be identified readily and specifically with a particular project.

Classification of cost

- **Total Cost= Total prime cost+ total overhead cost**
- Prime cost: Prime cost is those costs that can be reasonably measured and allocated to specific output or work activity.
- Total prime cost= Direct material cost+ Direct Labor cost+ Direct Expenses.
- Overhead cost: Overhead costs are those that occur but cannot be reasonably measured and allocated to a specific output or activity.
- Total Overhead cost= Indirect material cost+ Indirect Labor cost+ Indirect Expenses.

Some Definitions

Direct cost/Prime cost:

- Cost which can be reasonably measured and allocated with specific output or work activity.
- Direct Material: The cost of material that can be allocable to production.
  Example: Raw material consumed during production of the unit.
- Direct Labor: Wages to the laborers that can be identified with a cost object.
  Example: The term wages include bonus, gratuity, provident fund, incentives, etc.
- Direct Expenses: It includes all the other expenses that are directly linked to the production of a product.
  Example: Job processing charges, hire charges for tools and equipment, subcontracting expenses.

Some Definitions

Indirect and Overhead cost:

- All cost other than direct cost.
- Cost that cannot be reasonably measured and allocated to a specific output or work activity.
- Indirect Material: Material Cost which cannot be identified with a particular product or project.
  Example: Fuel, lubricants, small tools etc.
- Indirect Labor: Salary to the employees that cannot be allocable to a particular cost object.
  Example: Overtime pay, holiday pay, maintenance charge, material handlers, supervisor etc.
- Indirect Expenses: All the expenses other than indirect material and labor are included in this category.
  Example: Interest, Rent, Tax, Duty, etc.

Cost variance analysis

- When actual performance are recorded and compared with a standard set, some deviations are observed. These deviations are popularly called variance.
- Variance(V)= Actual cost(AC) – Estimated Cost or Standard cost(SC)
- If AC< SC, Favorable condition and
- If AC> SC, Adverse condition.

A. Direct material variances: 1+2

1. Direct material price variance(DMPV)

DMPV= (AQ*AR)- (AQ*SR)= AQ(AR-SR)

2. Direct material usage variance(DMUV)

DMUV= AQ*SR- SQ*SR= SR(AQ-SQ)

where, AQ=Actual Quantity, SQ= Standard Quantity for actual production,

AR= Actual Rate, SR= Standard Rate

Cont…

B. Direct wages variance (Labour variance): 1+2

1. Direct wages(labour) rate variance(DWRV)

DWRV= AH(AR-SR)

2. Direct labour efficiency variance(DLEV)

DLEV= SR(AH-SH)

C. Variable overhead variance: 1+2

1. Variable overhead expenditure variance(VOEV)

VOEV= AH(AR-SR)

2. Variable overhead efficiency variance(VOEV1)

VOEV1= SR(AH-SH)

Where, AH=Actual Hour, SH= Standard Hour,

AR= Actual Rate, SR= Standard Rate.

Cont..

D. Fixed overhead variance:

1. Fixed overhead expenditure variance(FOEV)

FOEV= Actual fixed overhead(AH*AR)-

budgeted fixed overhead

2. Fixed overhead capacity variance(FOCV)

FOCV= Budgeted fixed overhead- (AH*SR)

3. Fixed overhead efficiency variance(FOEV)

FOEV= (AH*SR)-(SH*SR) = SR*(AH-SH)

4. Fixed overhead volume varience(FOVV)

FOVV= Capacity variance(FOCV)+ Efficiency variance(FOEV)

Where, AH=Actual hour, SH= Standard hour,

AR= Actual rate, SR= Standard Rate.

Some Definitions

Inflation

- Loss in purchasing power of money over time.
- If the supply of money were to grow faster than demand for it.
- When demand exceeds supply, because of less supply, the net prices of the services or goods increases and this kind of situation is known as inflation.
- Because of inflation rupees in one time are not equivalent to rupees in another time.

A) Cost Push Inflation B) Demand Pull Inflation

Deflation:

- Opposite of inflation.
- Increase in value of currency.
- Deflation encourages individuals and corporations to hoard their money.
- Deflation is cause of recession, that slowdown the economic activities

HYPERINFLATION !!

STAGFLATION !!

Some Definitions

Depreciation:

- Decline in value of capital assets due to wear and tear, passage of time, obsolescence.
- Allocation of the cost of fixed asset over its recovery period.

Depletion:

It is defined as the reduction of the value of certain natural resources such as mines, oil, timber, quarries etc. due to the gradual extraction of its contents.

Q. What is defined as the reduction or fall of the value of an asset due to constant use and passage of time? Depletion, Inflation, Depreciation, Deflation

Some Definitions

Any assets will have following cost components:

i) Capital recovery cost – Earning back of the initial funds.

ii) Average operating and maintenance cost

iii) Total cost

The point where the total cost is minimum is called economic life of machine.

Economic life is the period of time that results in the minimum equivalent uniform annual cost of owning and operating an asset.

Life cycle cost: Sum of all expenditure associated with the project during its entire service life.

i) First cost: Land, Design and development, Building, Machine and installation, Training and special tooling, Supporting equipment, computer and transportation, Manufacturing and fabrication cost etc.
ii) Operating and Maintenance cost: Labour, Material, Overhead, Transportation etc.
iii) Disposal cost: Labour for removal, Material, Transportation for disposal.

Marginal cost/ Incremental cost: **Marginal cost** is the change in the total cost that arises when the quantity produced is incremented by one unit; that is, it is the cost of producing one more unit of a good.

Marginal revenue: The change in total revenue due to one extra unit of quantity sold.

Some Definitions

Salvage Value:

It is the estimated value of the property at the end of its useful life or recovery period. It is the expected selling price of a property when the asset can no longer be used productively by its owner.

Market value:

The amount that will be paid by a willing buyer to a willing seller for a property where each has equal advantage and it is under no compulsion to buy or sell.

Scrap value/Residual value/Break-up value:

Scrap value is the worth of a physical asset's individual components when the asset itself is deemed no longer usable.

The main difference between **scrap** and **salvage** is that a **salvage** vehicle has the potential to be repaired and then returned to the road. Whereas a **scrap** vehicle cannot be returned to the road and will be crushed and sold for materials.

Cash cost and Book cost: Cost that involves payment of cash from is called cash cost. Cost that doesn't involve cash transaction but is reflected in the accounting system is called book cost or non-cash cost. Eg. Depreciation charge of equipment.

Job (Specific order) Costing:Job costing refers to calculating the cost of a special contract, work order where work is performed as per client's or customer's instruction or specification.

- Eg. Building construction, Furniture etc.

Process costing: It refers to costing of one or more processes involved while converting a raw material to finished output.

- This method is used where it is not possible to trace the items of prime cost to a particular order, because its identity is lost.
- Eg. Chemical plant, Oil refining, dairy etc.

- *Uniqueness of product*. Job costing is used for unique products, and process costing is used for standardized products.
- *Size of job*. Job costing is used for very small production runs, and process costing is used for large production runs.

Cont..

Job costing	Process costing
1. **Production is made by specific orders.** 2. **The different jobs may be independent of each other** 3. **Each job is allotted a number and costs are collected against the same job number.** 4. **Job costing is computed when the job is completed.**	1. Uniform production in continuous flow. 2. Being manufactured in a continuous flow, product loses their individual identity. 3. The unit cost of process which is computed by dividing the total costs for the period by the total output, is an avg. cost of the period. 4. Process cost is calculated at the end of the cost period.

Assets, Capital, Liabilities

Asset:

- An economic resource of an entity.
- Real asset: Land, building, plant, machinery, equipment etc.
- Financial asset: cash, shares, securities etc.

Capital:

- Factors of production
- Term for describing wealth.
- Man-made factor that contribute to the production of goods and services.
- Non-human ingredients that contribute to the production of goods and services including land, buildings, raw and semi-finished materials, finished product, inventories etc.

What is the difference between Capital and Asset?

• Capital is the net worth of a company or the money that is required to produce goods

• Assets are things that have a value and can be sold in the market for a monetary value

• As such capital is a type of asset

• All capital is asset, but not all assets are capital as there are intangible assets that cannot be sold to make money

Assets = **Liabilities** + Owner's Equity (Capital)

When a business is put up, its resources (assets) come from two sources: contributions by owners (capital) and those acquired from creditors or lenders (liabilities).

The fund invested by the owner in the business or the net amount claimable by the owner from the business is known as the Capital or Owner's Equity or Net Worth

Some Definitions

Asset: Tangible assets vs Non tangible assets

Tangible asset

Fixed assets: Physical or infrastructure assets which are not intended for resale in the normal course of business and provide benefit over a long period of time to the business. Ex. Land and Building, Equipment, Furniture, Vehicles etc.

Current asset: The assets which are in the form of cash or can be converted into cash within a year. Ex. Cash in hand, cash in bank, bill receivable, inventories, marketable securities, accrued income etc.

Non tangible : Goodwill, Trademark, copyrights

Liabilities: Liabilities are borrowed debts and payable obligation of organization.

- Long term Liabilities: Raised for more than one year. Bond/Debenture, Bank loan etc.
- Current (Short term) Liabilities: To be repaid within one year. Bill payable, advance income etc. A liability is considered current if it is due within 12 months after the end of the balance sheet date

Total debt = Long term debt + Current liability

Using Financial Ratios

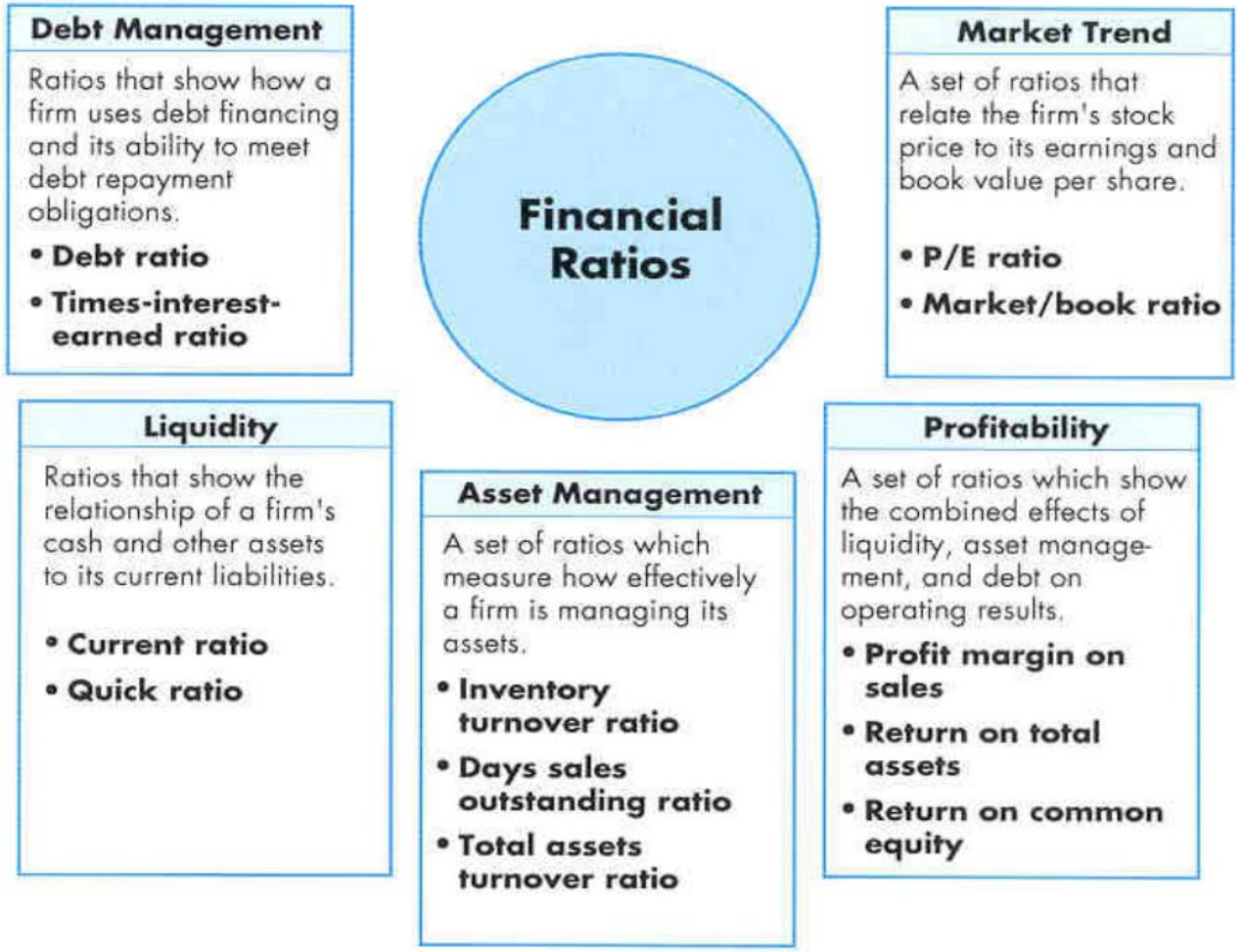


Figure 2.4 Types of ratios used in evaluating a firm's financial health

Debt Ratio

- What It Measures: The extent to which a form uses debt financing
- How You Compute: The ratio of total debt to total assets

$$\text{Debt ratio} = \frac{\text{Total debt}}{\text{Total assets}} = \frac{\$6{,}163}{\$11{,}471} = 53.73\%$$

Debt ratio (Solvency/Leverage/Capital structure ratio):

Debt ratio : debt ratio divides total debt by total assets.

Debt to equity ratio(DER): Total debt/ Shareholder's equity : Measures long term solvency.

- Lower ratio indicates more security to the creditors.
- Higher ratio represents more risk to creditors and owners.

Time-Interest-Earned Ratio

- What It Measures: The ability of the firm to meet its annual interest payments
- How You Compute: The ratio of earnings before interest and taxes (EBIT) to interest charges

$$\text{Time - interest - earned ratio} = \frac{\text{EBIT}}{\text{Interest expense}} = \frac{(\$2{,}451 + \$34)}{\$34} = 73 \text{ times}$$

Also called Interest coverage ratio

Interest coverage ratio = Operating income / Interest expenses

Current Ratio

- What It Measures: The extent to which the claims of short-term creditors are covered by assets
- How You Compute: The ratio computed by dividing current assets by current liabilities

$$\text{Current ratio} = \frac{\text{Current assets}}{\text{Current liabilities}} = \frac{\$7{,}681}{\$5{,}192} = 1.48 \text{ times}$$

Current ratio measures company ability to pay short term obligations

Quick (Acid Test) Ratio

- What It Measures: The firm's ability to pay off short-term obligations without relying on the sale of inventories.
- How You Compute: This ratio is computed by deducting inventories from current assets and dividing the remainder by current liabilities.

$$\text{Quick ratio} = \frac{\text{Current assets - Inventories}}{\text{Current liabilities}} = \frac{\$7{,}681 - \$391}{\$5{,}192} = 1.40 \text{ times}$$

Ratio

Current ratio: Current asset/Current Liabilities ; cash asset ratio

Represents ability of firm to pay its short-term debt.

Current ratio of 2:1 is regarded as an acceptable standard.

Less than 2:1 represents firm has not paid its urgent debt on time, which shows bad business performance.

Quick ratio or Acid test ratio or Liquid ratio:

Quick asset/Current liabilities

Quick asset = Current asset – stock – prepaid expenses

Higher the quick ratio, better the liquidity position. Ideal quick ratio is 1:1

Economic system

Economical System

a)Capitalistic Economic sytem / Free Market Economy

b) Socilastic System/ Controlled Economy

c) Mixed System

Utility : Satisfaction that the consumer derived from the good and service consumed. Measure of satisfaction

Sunk cost: Capital already invested that for some reason cannot be retrieved. Expense which has happened in the past. No relevance to alternatives being considered.

Law of Diminishing..

Law of Diminishing Marginal Utility:

According to Marshall: The additional benefit which a person derives from an increase of his stock of a thing diminishes with the growth of the stock that he already has.

Law of Diminishing Returns:

It states that when more and more units of variable input are applied to a given quantity of fixed inputs, the total output may initially increase and then will be constant and it will eventually decrease.

Market and Law of Demand

A market is a group of buyers and sellers of a particular good or service. The buyers as a group determine the demand for the product, and the sellers as a group determine the supply of the product.

The quantity demanded of any good is the amount of the good that buyers are willing and able to purchase.

Law of demand:

Other things being equal, when the price of a good rises, the quantity demanded of the good falls, and when the price falls, the quantity demanded rises.

demand schedule : a table that shows the relationship between the price of a good and the quantity demanded

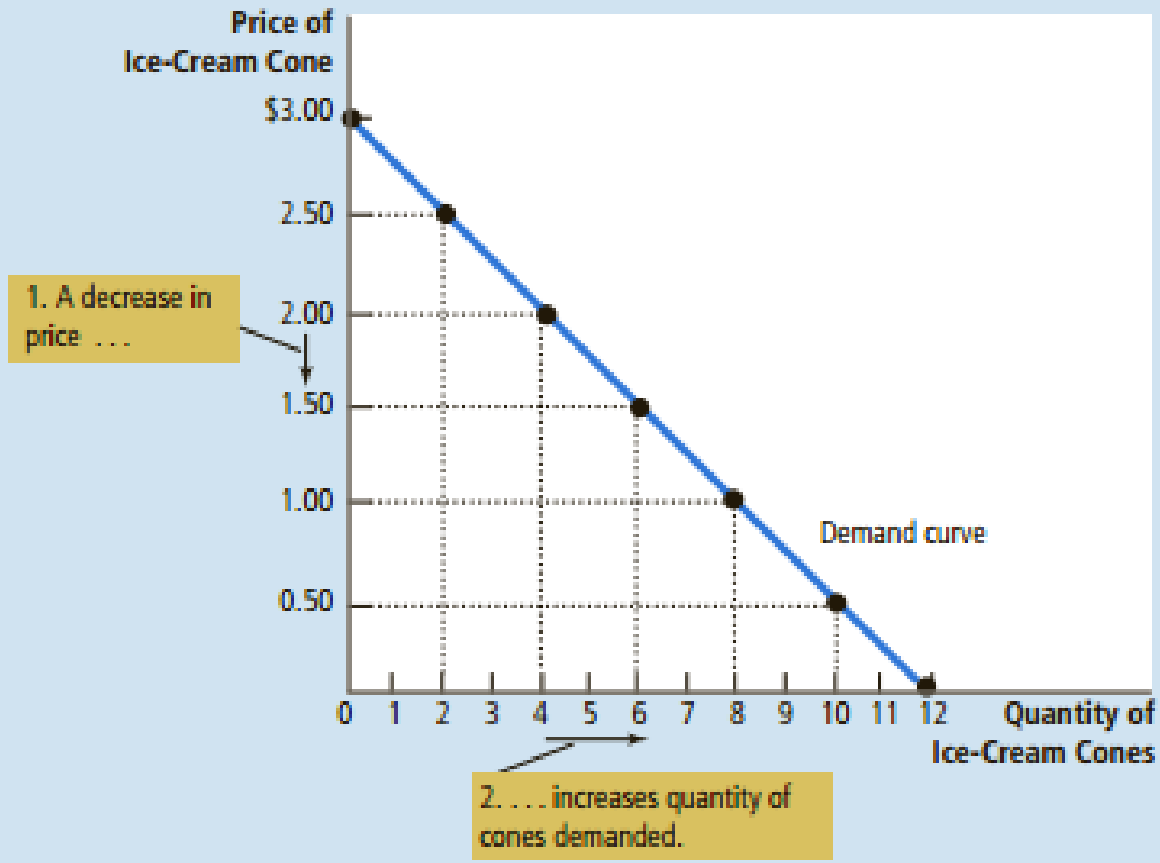
Demand curve & Schedule

FIGURE 1

Catherine's Demand Schedule and Demand Curve

The demand schedule is a table that shows the quantity demanded at each price. The demand curve, which graphs the demand schedule, illustrates how the quantity demanded of the good changes as its price varies. Because a lower price increases the quantity demanded, the demand curve slopes downward.

Price of Ice-Cream Cone	Quantity of Cones Demanded
$0.00	12 cones
0.50	10
1.00	8
1.50	6
2.00	4
2.50	2
3.00	0



Law of Supply

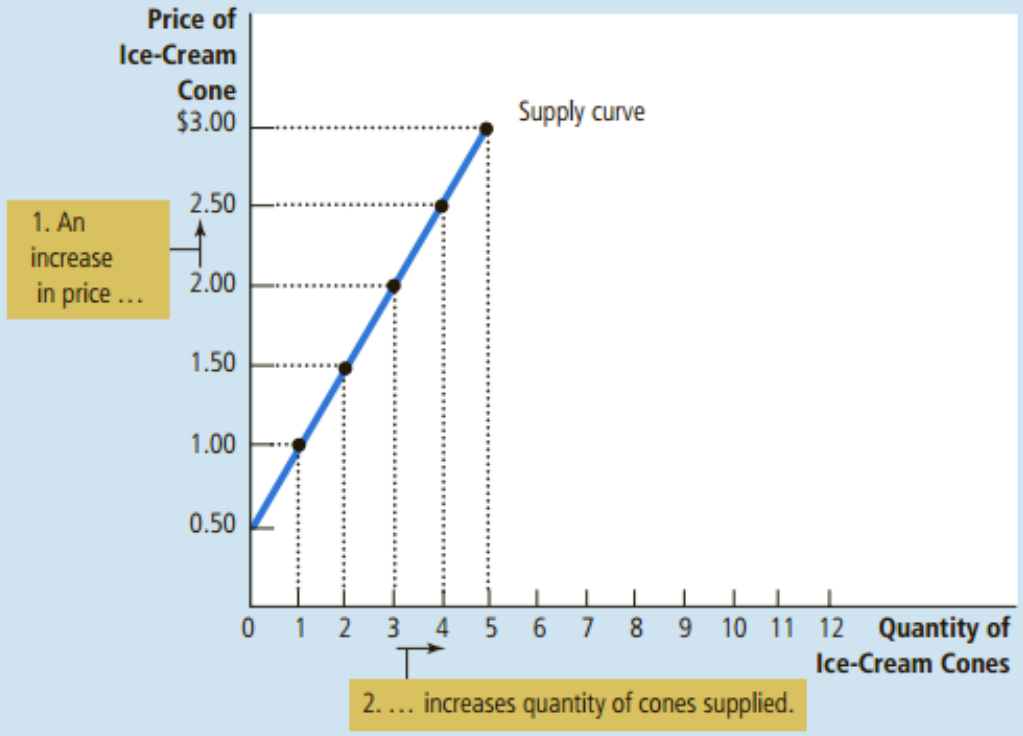
Other things being equal, when the price of a good rises, the quantity supplied of the good also rises, and when the price falls, the quantity supplied falls as well.

The supply schedule is a table that shows the quantity supplied at each price. This supply curve, which graphs the supply schedule, illustrates how the quantity supplied of the good changes as its price varies. Because a higher price increases the quantity supplied, the supply curve slopes upward.

FIGURE 5

Ben's Supply Schedule and Supply Curve

Price of Ice-Cream Cone	Quantity of Cones Demanded
$0.00	0 cones
0.50	0
1.00	1
1.50	2
2.00	3
2.50	4
3.00	5



Equilibrium of Supply and Demand

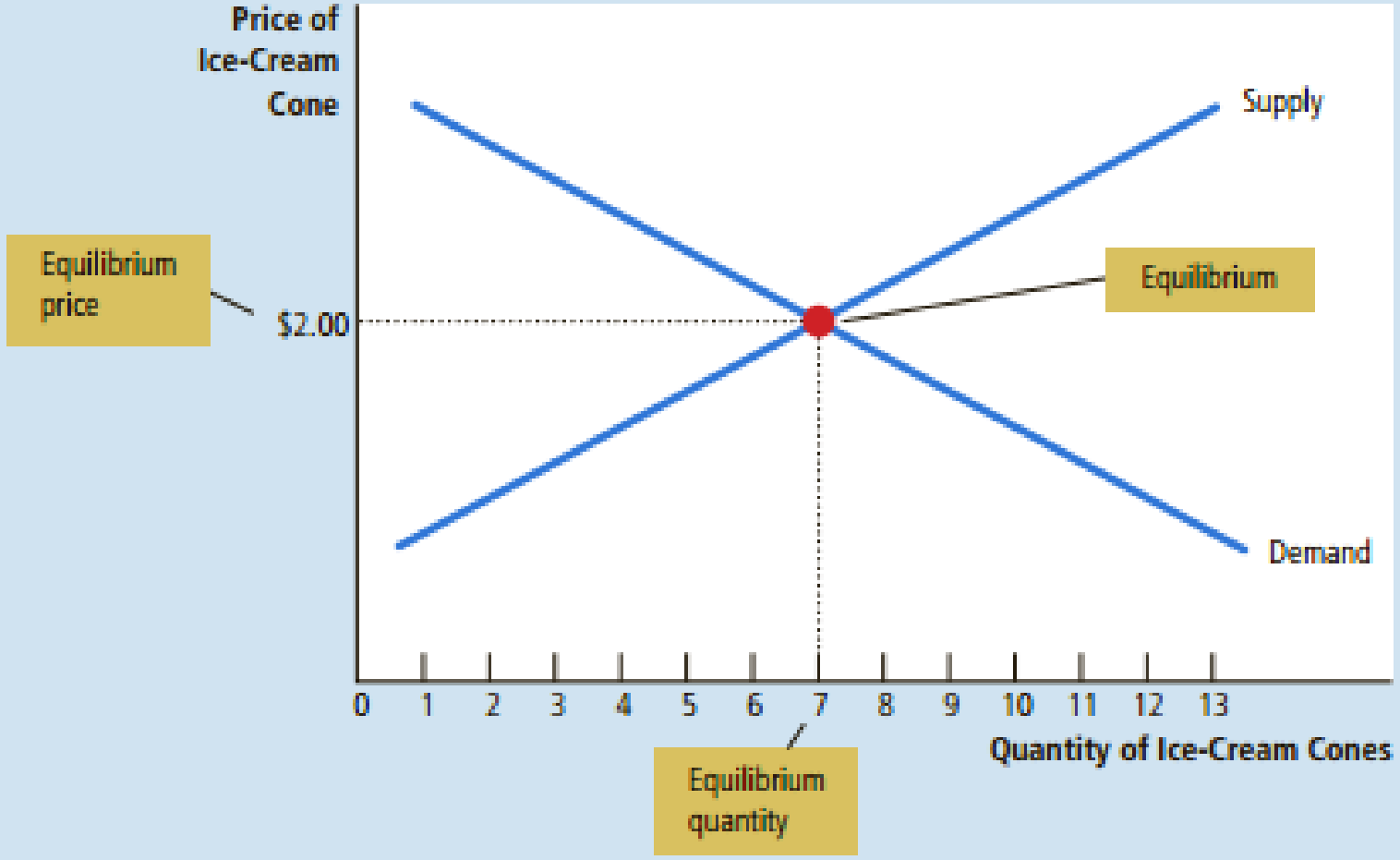


FIGURE 8

The Equilibrium of Supply and Demand

The equilibrium is found where the supply and demand curves intersect. At the equilibrium price, the quantity supplied equals the quantity demanded. Here the equilibrium price is $2.00: At this price, 7 ice-cream cones are supplied and 7 ice-cream cones are demanded.

Shifts in Demand/ Factors affecting Qd

Price of commodity

Income of Consumer

Price of related goods

Change in Taste

Weather

Future Expectations

Nos of Buyers

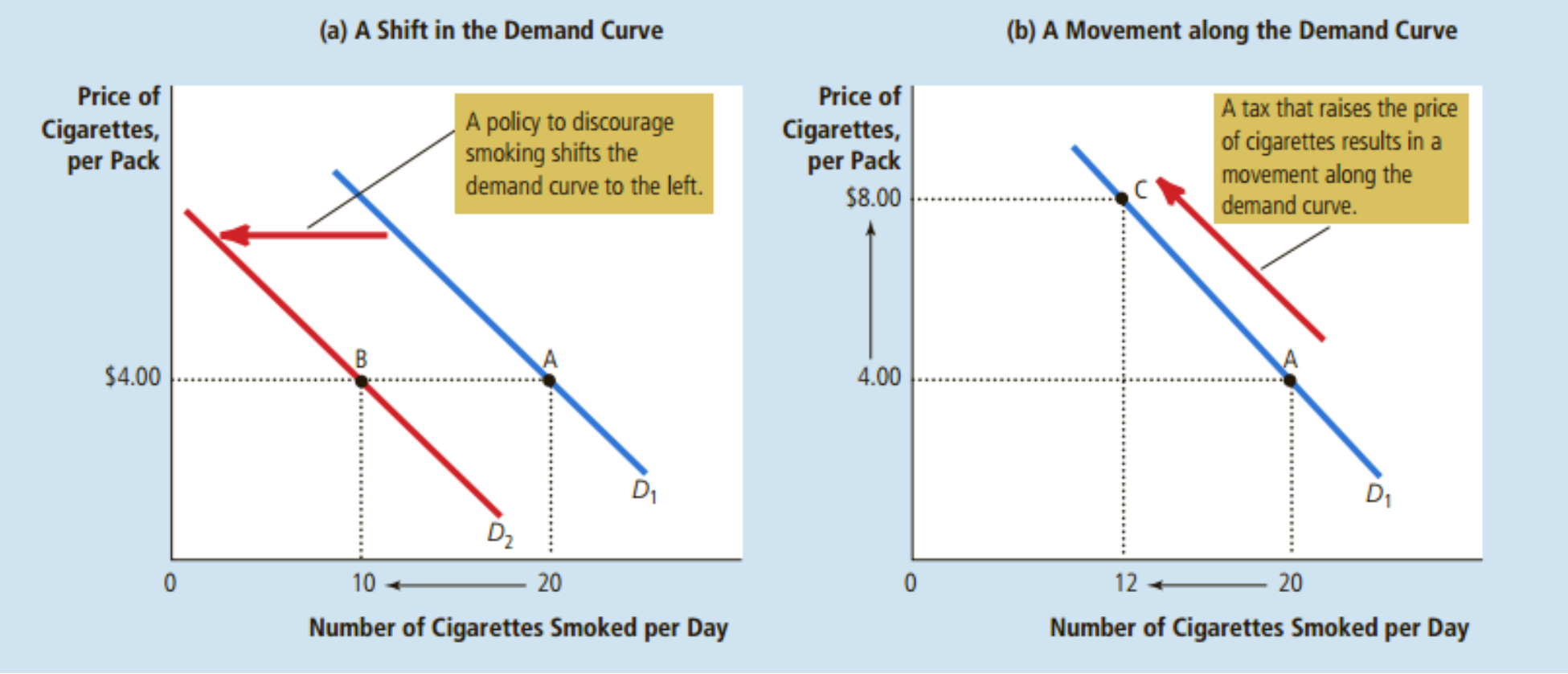
Variable	A Change in This Variable . . .
Price of the good itself	Represents a movement along the demand curve
Income	Shifts the demand curve
Prices of related goods	Shifts the demand curve
Tastes	Shifts the demand curve
Expectations	Shifts the demand curve
Number of buyers	Shifts the demand curve

TABLE 1

Variables That Influence Buyers

This table lists the variables that affect how much of any good consumers choose to buy. Notice the special role that the price of the good plays: A change in the good's price represents a movement along the demand curve, whereas a change in one of the other variables shifts the demand curve.

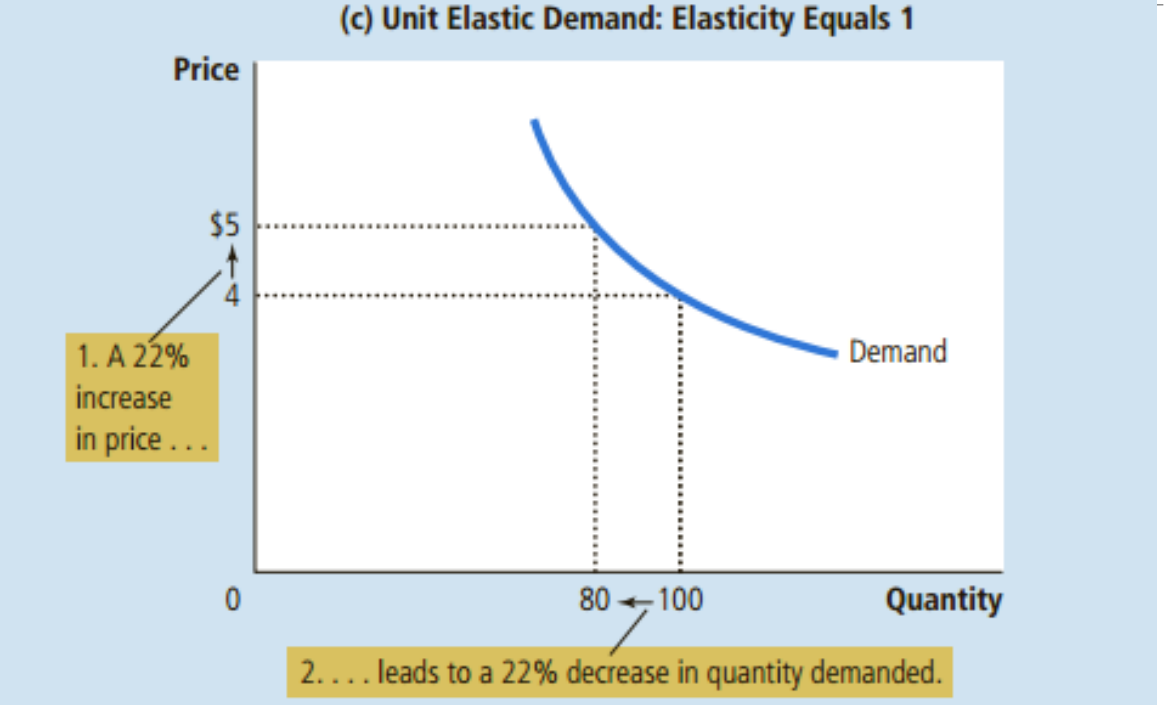
Shifts vs movement



Elasticity

Elasticity is a measure of how much buyers and sellers respond to changes in market conditions

a measure of how much the quantity demanded of a good responds to a change in the price of that good, computed as the percentage change in quantity demanded divided by the percentage change in price.

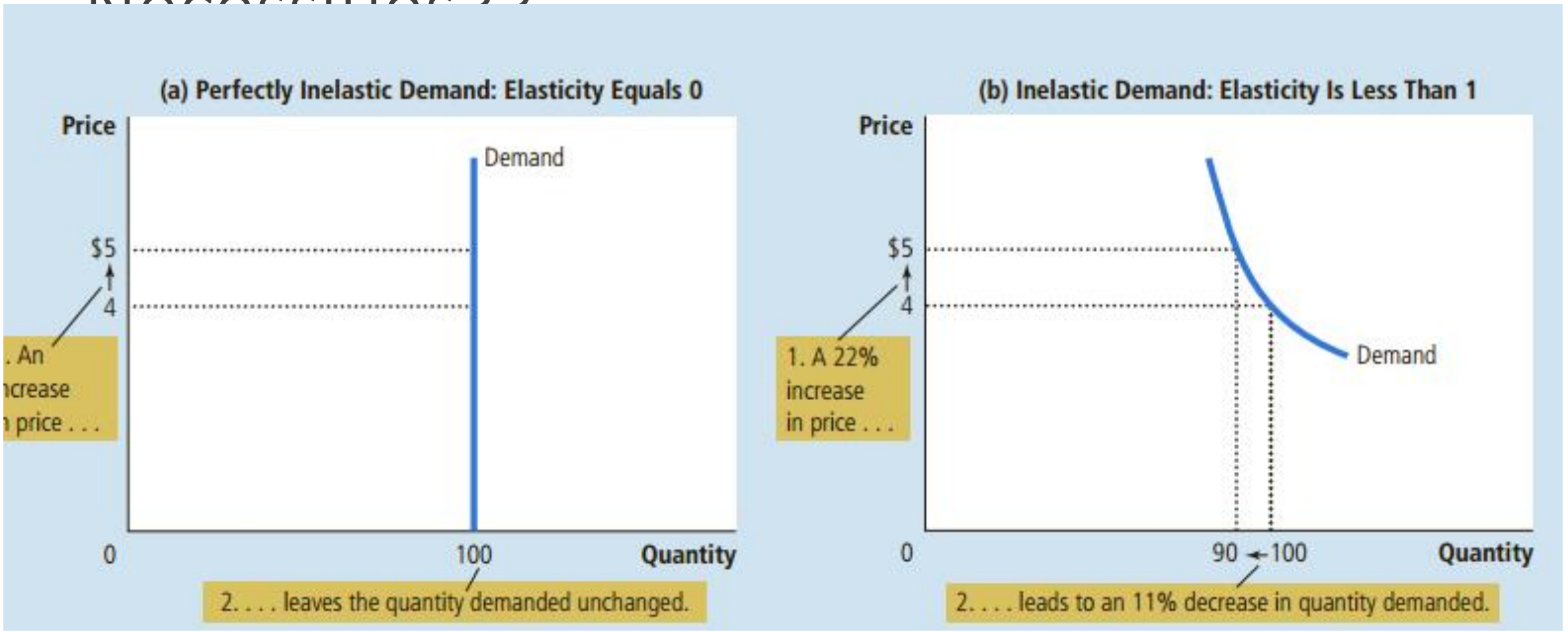


$$\text{price elasticity of demand} = \frac{\text{percentage change in quantity demanded}}{\text{percentage change in price}}$$

$$\text{Symbolically, } E_P = \frac{\frac{\Delta Q}{Q}}{\frac{\Delta P}{P}} = \frac{\Delta Q}{Q} \times \frac{P}{\Delta P} = \frac{\Delta Q}{\Delta P} \times \frac{P}{Q}$$

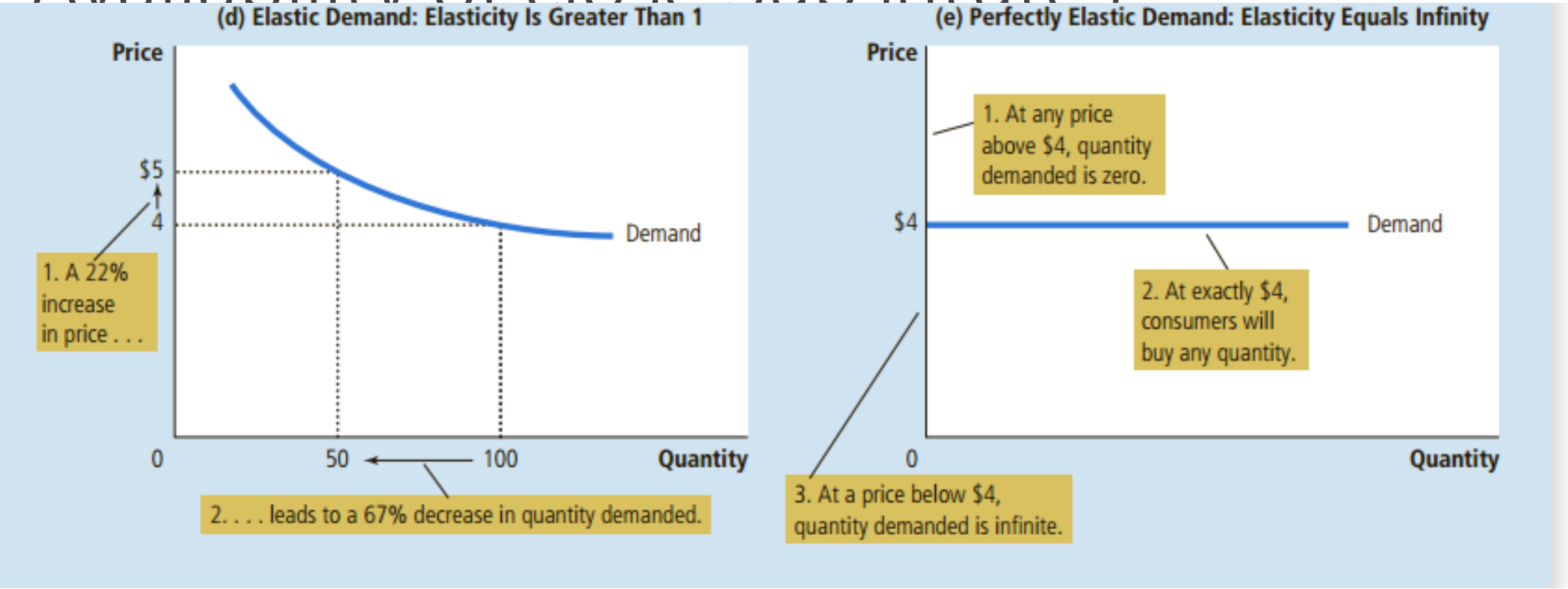
Where, E_P = Price elasticity

Necessities??



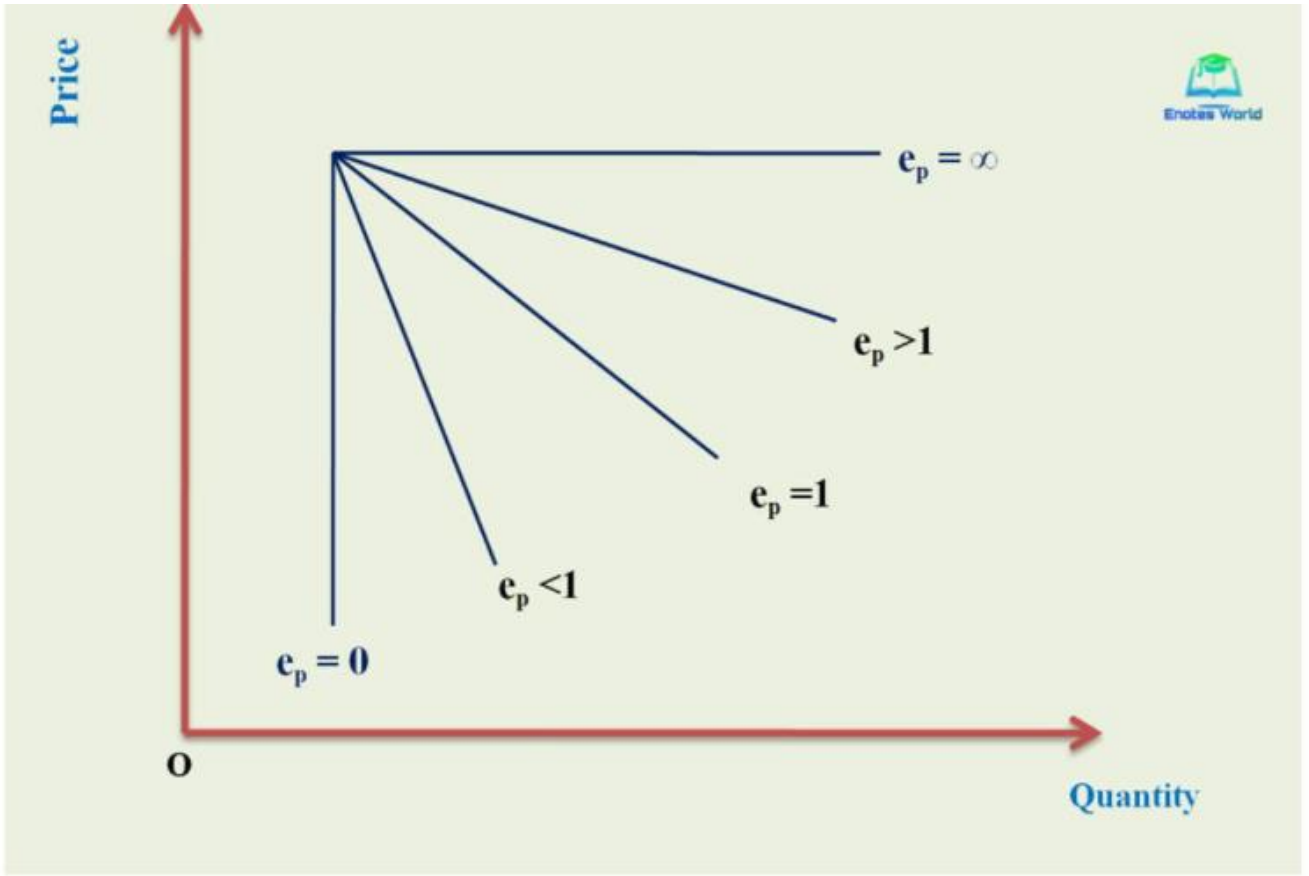
Also Called Relatively Inelastic Demand

Availability of close Substitutes

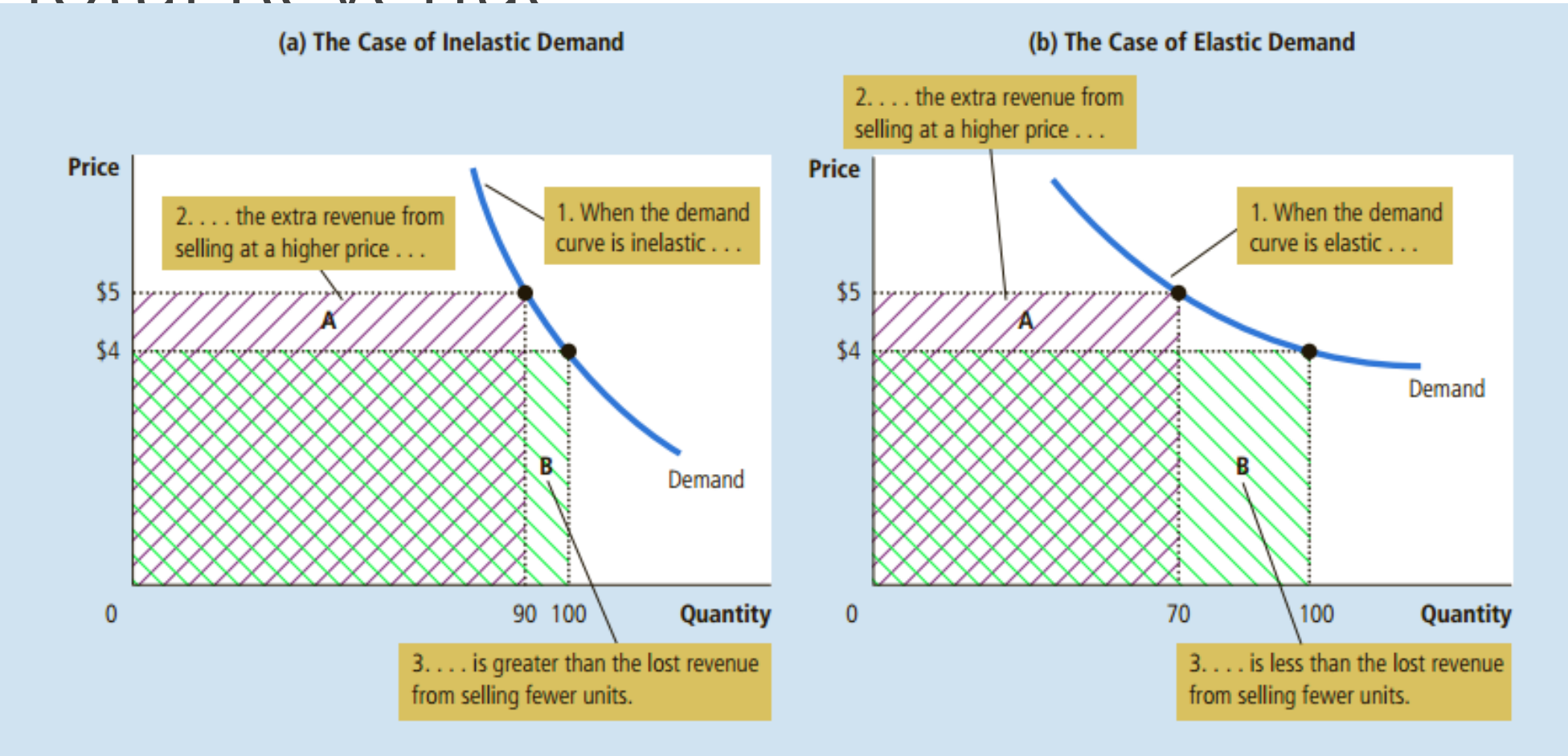


Also Called Relatively elastic Demand

Types of Price Elasticity of Demand: A Summary Diagram



Total Revenue



Other Elasticities

The Income Elasticity of Demand The **income elasticity of demand** measures how the quantity demanded changes as consumer income changes. It is calculated as the percentage change in quantity demanded divided by the percentage change in income. That is,

$$\text{income elasticity of demand} = \frac{\text{percentage change in quantity demanded}}{\text{percentage change in income}}$$

The Cross-Price Elasticity of Demand The **cross-price elasticity of demand** measures how the quantity demanded of one good responds to a change in the price of another good. It is calculated as the percentage change in quantity demanded of good 1 divided by the percentage change in the price of good 2. That is,

$$\text{cross-price elasticity of demand} = \frac{\text{percentage change in quantity demanded of good 1}}{\text{percentage change in the price of good 2}}$$

Income elasticity of demand (E_y) = $\frac{\text{Proportionate change in quantity demanded}}{\text{Proportionate change in income}}$

Symbolically, $E_y = \frac{\frac{\Delta Q}{Q}}{\frac{\Delta Y}{Y}} = \frac{\Delta Q}{Q} \times \frac{Y}{\Delta Y} = \frac{\Delta Q}{\Delta Y} \times \frac{Y}{Q}$

where, E_y = *Income elasticity*

ΔQ = *Change in quantity demanded*

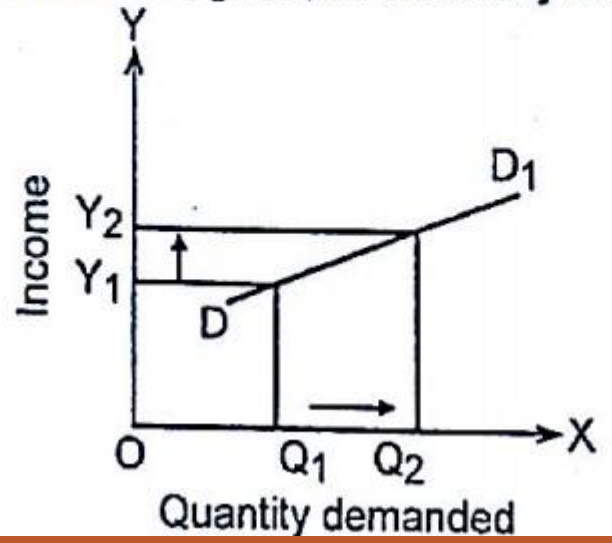
Q = *Initial demand*

ΔY = *Change in income*

Y = *Initial income*

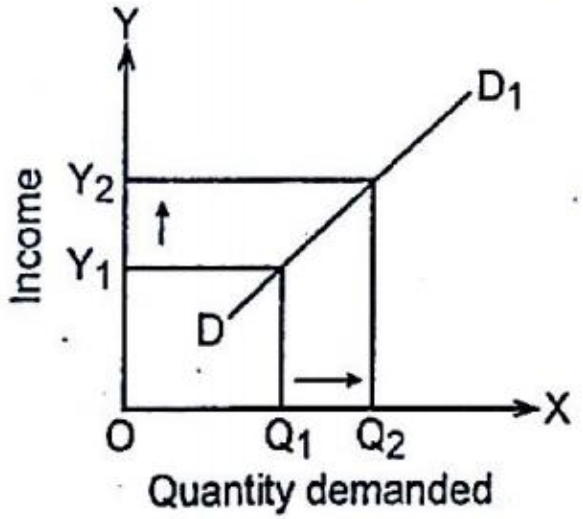
(i) Greater than unity income elasticity ($E_y > 1$)

If percentage change in quantity demanded is greater than percentage change in income, then it is said to be greater than unity income elasticity of demand. In the case of luxurious goods, such elasticity can be found.



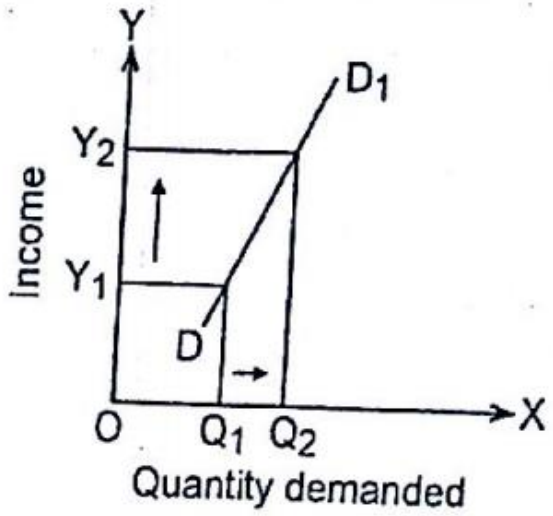
(ii) Equal to unity income elasticity ($E_y = 1$)

If the change in income brings the equal change in quantity demanded, then such elasticity is said to be equal to unity. In another words, it is the situation of change in demand and income by equal proportion.



(iii) Less than unity income elasticity ($E_y < 1$)

If the percentage change in quantity demanded is less than percentage change in income, then it is said to be less than unity income elasticity. In case of normal goods, such elasticity can be realized.

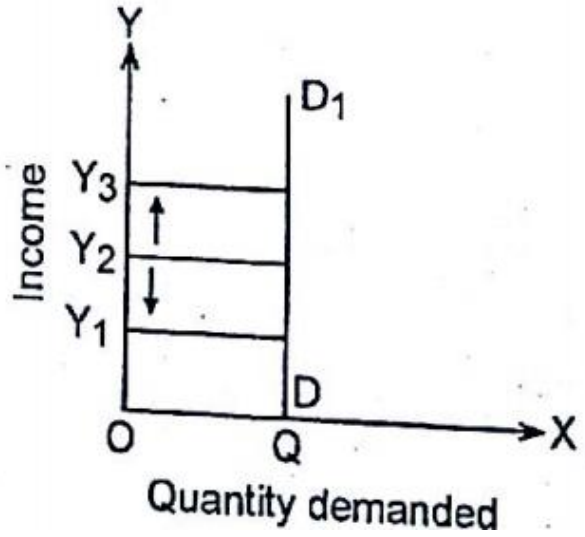


So If Qd increases when Price decrease, Good is NORMAL

So If Qd increases when Income increase, Good is NORMAL

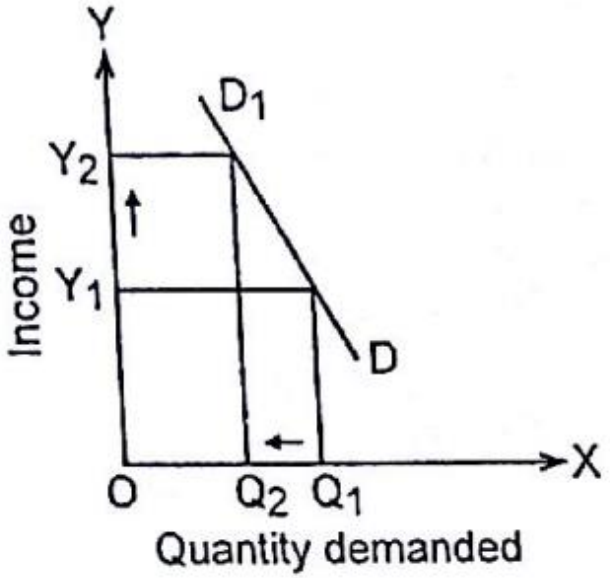
(iv) Zero income elasticity ($E_y = 0$)

If demand for the commodity does not change with change in income of the consumer, then it is called zero income elasticity. It is the situation where no any response in quantity demanded due to any change in income of the consumer. This happens in case of neutral (more necessary) goods like salt, matches etc.



(v) Negative income elasticity ($E_y < 0$)

If the demand decreases with increase in income, then it is called negative income elasticity. If goods are giffen or inferior, then the demand varies inversely with income because higher income inspires people to consume superior goods.



Giffen goods are rare forms of **inferior** goods that have no ready substitute or alternative such as bread, rice, and potatoes. The only difference from traditional **inferior** goods is that demand increases even when their price rises, regardless of a consumer's income.

If Qd decreases when Income increase, Good is INFERIOR

If Qd increases when Price Increase, Good is Giffen

(Irrespective of Change in Income)

Cross elasticity of demand;

$$(E_C) = \frac{\text{Proportionate change in quantity demanded of X}}{\text{Proportionate change in price of Y}}$$

Symbolically, $E_C = \dfrac{\frac{\Delta Q_X}{Q_X}}{\frac{\Delta P_Y}{P_Y}} = \dfrac{\Delta Q_X}{Q_X} \times \dfrac{P_Y}{\Delta P_Y} = \dfrac{\Delta Q_X}{\Delta P_Y} \times \dfrac{P_Y}{Q_X}$

where, ΔQ_X = *Change in quantity demanded for X commodity*

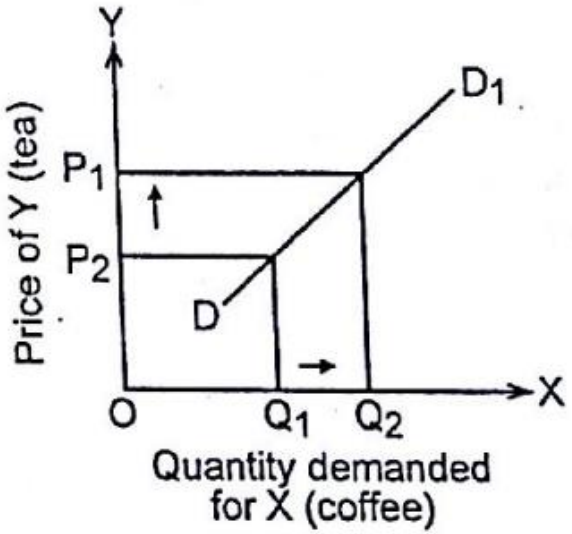
Q_X = *Initial demand for X commodity*

ΔP_Y = *Change in price of Y commodity*

P_Y = *Initial price of Y commodity*

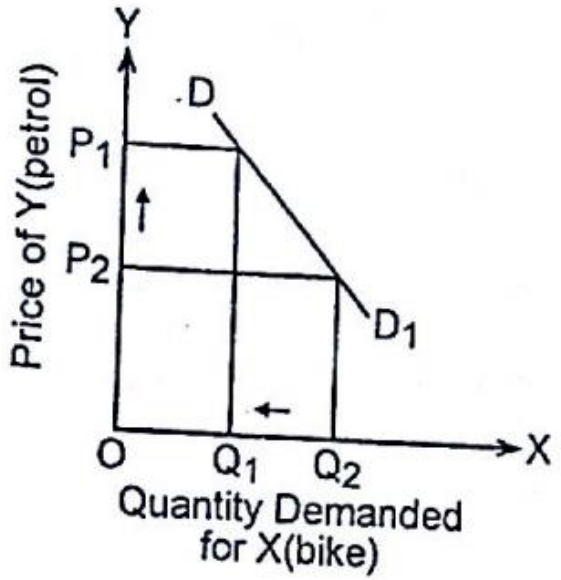
(i) **Positive cross elasticity (E_C = +ve)**

If the quantity demanded for one commodity (say X) varies positively with the change in price of another commodity (say Y), the cross elasticity will be positive. It is connected with substitute goods like tea and coffee.



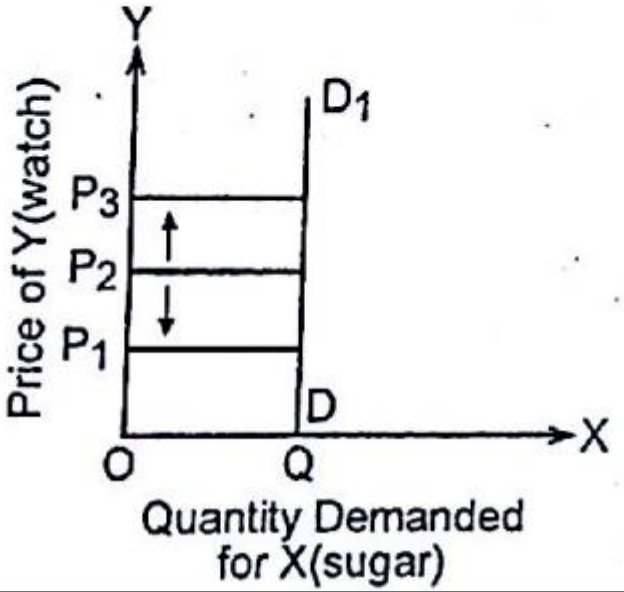
(ii) Negative cross elasticity ($E_C = -ve$)

If the quantity demanded for one commodity (say X) varies inversely with the price of another commodity (say Y), the cross elasticity will be negative. It is connected with complementary goods like petrol and motorbike.



(iii) Zero cross elasticity ($E_C = 0$)

If there is no any response in quantity demanded for one commodity (say X) due to the change in price of another commodity (say Y), the cross elasticity will be zero. It is connected unit perfectly non–related goods like watch and sugar.



questions

Q. If the percentage fall in quantity demanded is greater than the percentage rise in price charged, the revenue will…….

Rise, fall , not related, none of the above

Q. If the change in quantity demanded is LESSER than change in price, it is said to be

a) Perfectly elastic demand

b) Unitary elastic demand

c) Relatively inelastic demand

d) Relatively elastic demand

GDP & GNP

In economics, Gross Domestic Product (GDP) is used to calculate the total value of the **goods and services produced within a country's borders,**

while Gross National Product (GNP) is used to calculate the total value of the goods and services produced **by the residents of a country, no matter their location.**

Essentially, **GDP** looks for the amount of economic activity within a **nation's economy**, while **GNP** looks at the value of the economic activity generated by the **nation's people.**

GDP Nepal ko, GNP kasko ?? Nepali ko !!

Approach for GDP Calculation

- **Expenditure approach**
- **Income approach**
- **Product approach**
  1. **Final product approach**
  2. **Value added approach**

Methods of GDP Calculation

The expenditure approach takes into account adding up all the amount spent on goods and services during the period.

GDP = C + I + G + (X – M)

Where,

C = Consumption spending

I = Investments

G = Government purchases

X = Exports

M = Imports

X-M = Net Exports

GNP

GNP is known as gross national product and represents the total value of goods and services produced by the residents of a country during a financial year.

It takes the income earned by the citizens of the country present within or outside the country into consideration. It excludes the income generated by the foreign nationals who are residing in the country. It can be calculated as:

GNP = GDP + NR – NP

Where,

GDP = Gross domestic product

NR = Net income receipts

NP = Net outflow to foreign assets

Engineering Economics MCQ

Collection by Sushil Rijal

questions

- **In a cash-flow diagram:**

(A) Time 0 is considered to be the present

(B) Time 1 is considered to be the end of time period 1

(C) A vertical arrow pointing up indicates a positive cash flow

(D) All of these

Cost that are not borne by the parties to the economic….. Are called externalities..

a) Gain b)transaction c)transfer d)loss

- In the process of selecting public project the criterial usually adopted are

a) Economic evaluation b) financial evaluation c) both

- The benefit cost ratio is

A. Directly proportional to discount rate
B. Inversely proportional to discount rate
C. No any relation with discount rate
D. All of the above

- Q. If one has deposited Rs. 10,000 in bank and pay Rs 11025 after one year, the interest rate could be…

A10% compounded semiannually
B 10.25% annually
C Both of above
D None of above

Q. Which of the following is not direct cost for the project

a) Wages of labor b) cost of materials c) insurance costs d) sub contract cost

- Q. The numerical value of MARR is

A. Equal to IRR
B. Less than IRR
C. Greater than IRR
D. All of the above

- Q. IRR should be…

A. Less than borrowing rate
B. More than borrowing rate
C. Equal to borrowing rate
D. All of the above

(1-1) The concept that different sums of money at different points in time can be said to be equal to each other is known as:

a) Evaluation criterion
b) Equivalence
c) Cash flow
d) Intangible factors

(1-2) The evaluation criterion that is usually used in an economic analysis is:

a) Time to completion
b) Technical feasibility
c) Sustainability
d) Financial units (dollars or other currency)

(1-3) All of the following are examples of cash outflows, except:

a) Asset salvage value
b) Income taxes
c) Operating cost of asset
d) First cost of asset

Question	***answer***
(1-1)	b
(1-2)	d
(1-3)	a

(1-4) In most engineering economy studies, the best alternative is the one that:

a) Will last the longest time
b) Is most politically correct
c) Is easiest to implement
d) Has the lowest cost

(1-5) The following annual maintenance and operation (M&O) costs for a piece of equipment were collected over a 5-year period: $12,300, $8900, $9200, $11,000, and $12,100. The average is $10,700. In conducting a sensitivity analysis, the most reasonable range of costs to use (i.e., percent from the average) is:

a) ±5%
b) ±11%
c) ±17%
d) ±25%

(1-6) At an interest rate of 10% per year, the equivalent amount of $10,000 one year ago is closest to:

a) $8264
b) $9091
c) $11,000
d) $12,000

(1-7) Assume that you and your best friend each have $1000 to invest. You invest your money in a fund that pays 10% per year compound interest. Your friend invests her money at a bank that pays 10% per year simple interest. At the end of 1 year, the difference in the total amount for each of you is:

a) You have $10 more than she does
b) You have $100 more than she does
c) You both have the same amount of money
d) She has $10 more than you do

(1-8) The time it would take for a given sum of money to double at 4% per year simple interest is closest to:

a) 30 years
b) 25 years
c) 20 years
d) 10 years

Question	***answer***
(1-1)	b
(1-2)	d
(1-3)	a
(1-4)	d
(1-5)	c
(1-6)	b
(1-7)	c
(1-8)	b

Qn. In case of construction industry, the cost of material is normally

- A) more than cost of labour
- B) equal to
- C) less than

2.The difference between actual cost of the project with standard/targeted cost is called…..

3. For selection of any project the NPV should be…

Thank you