Predicate logic

Why

- Propositional logic cannot adequately express the meaning of all statements in mathematics and in natural language.

Example:

- Every computer connected to the university network is functioning properly.
- There is a computer on the university network that is under attack by an intruder.

Predicate

- A predicate is an expression of one or more variables defined on some specific domain.

Example: "$x > 3$"

The statement "x is greater than 3" has two parts. The first part, the variable x, is the subject of the statement. The second part—the **predicate**, "is greater than 3"—refers to a property that the subject of the statement.

- We can denote the statement "x is greater than 3" by $P(x)$,
- where P denotes the predicate "is greater than 3" and x is the variable.

Let $P(x)$ denote the statement "$x > 3$." What are the truth values of $P(4)$ and $P(2)$?

Solution: We obtain the statement $P(4)$ by setting $x = 4$ in the statement "$x > 3$." Hence, $P(4)$, which is the statement "$4 > 3$," is true. However, $P(2)$, which is the statement "$2 > 3$," is false.

Example 2

Let $A(x)$ denote the statement "Computer x is under attack by an intruder." Suppose that of the computers on campus, only CS2 and MATH1 are currently under attack by intruders. What are truth values of $A(\text{CS1})$, $A(\text{CS2})$, and $A(\text{MATH1})$?

Solution: We obtain the statement $A(\text{CS1})$ by setting $x = \text{CS1}$ in the statement "Computer x is under attack by an intruder." Because CS1 is not on the list of computers currently under attack, we conclude that $A(\text{CS1})$ is false. Similarly, because CS2 and MATH1 are on the list of computers under attack, we know that $A(\text{CS2})$ and $A(\text{MATH1})$ are true. ◀

Example 3

Let $Q(x, y)$ denote the statement "$x = y + 3$." What are the truth values of the propositions $Q(1, 2)$ and $Q(3, 0)$?

Solution: To obtain $Q(1, 2)$, set $x = 1$ and $y = 2$ in the statement $Q(x, y)$. Hence, $Q(1, 2)$ is the statement "$1 = 2 + 3$," which is false. The statement $Q(3, 0)$ is the proposition "$3 = 0 + 3$," which is true. ◀

Quantifiers

- The variable of predicates is quantified by quantifiers. There are two types of quantifier in predicate logic − Universal Quantifier and Existential Quantifier.

Universal Quantifier

- Universal quantifier states that the statements within its scope are true for every value of the specific variable. It is denoted by the symbol ∀.
- $\forall x P(x)$ is read as for every value of x, P(x) is true.
- **Example** − "Man is mortal" can be transformed into the propositional form ∀xP(x) where P(x) is the predicate which denotes x is mortal and the universe of discourse is all men.

Existential Quantifier

- Existential quantifier states that the statements within its scope are true for some values of the specific variable. It is denoted by the symbol ∃.
- ∃xP(x) is read as for some values of x, P(x) is true.
- Example – "Some people are dishonest" can be transformed into the propositional form ∃xP(x) where P(x) is the predicate which denotes x is dishonest and the universe of discourse is some people.

Nested Quantifiers

- If we use a quantifier that appears within the scope of another quantifier, it is called nested quantifier.
- **Example**
- $\forall$ a$\exists$bP(x,y) where P(a,b) denotes a+b=0