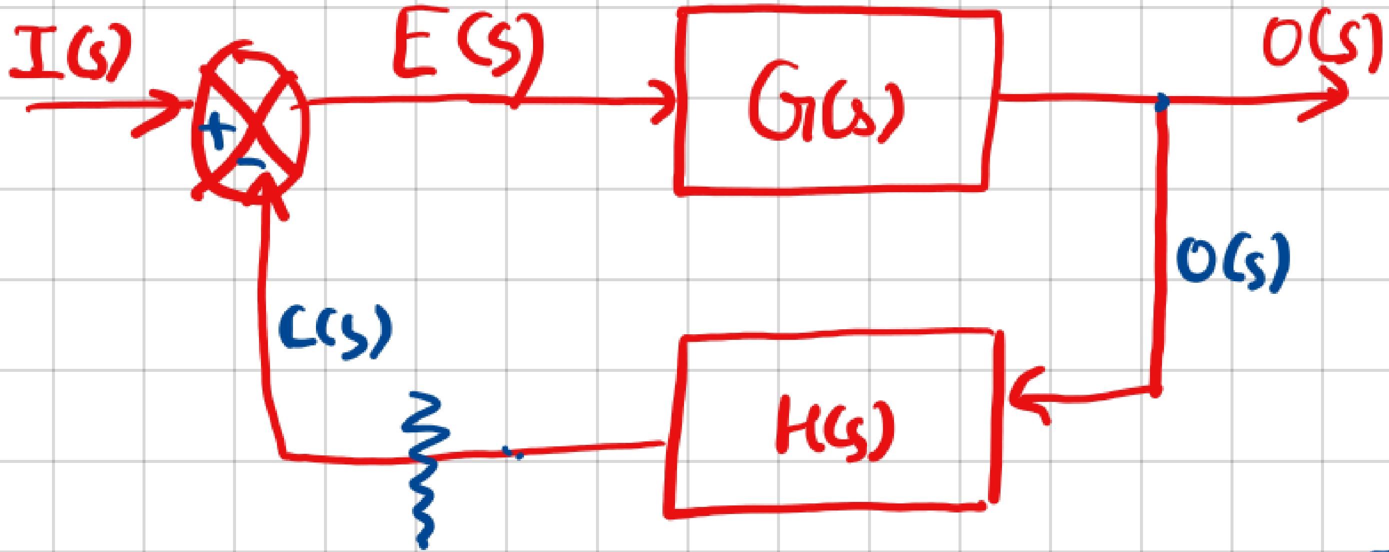
ROOT Locus TECHNIQUES



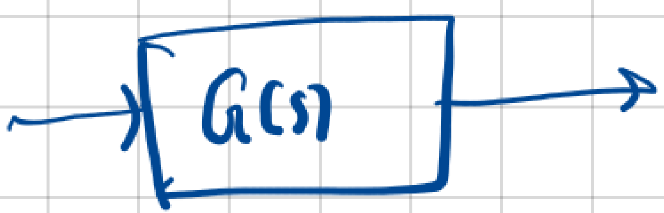
$$E(s)\cdot G(s) = O(s)$$

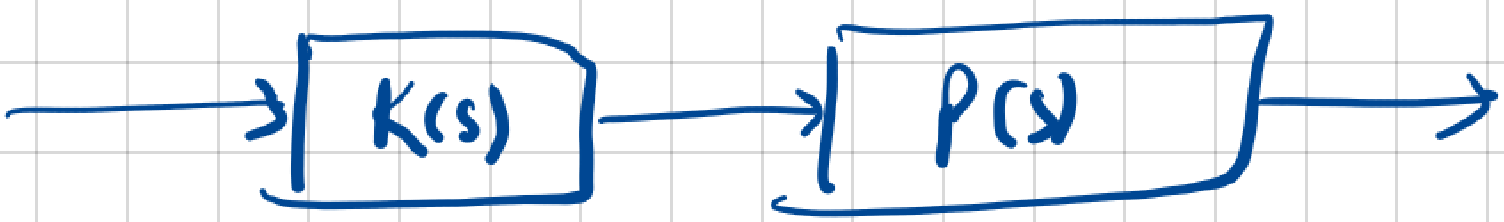
$$E(s) = I(s) - C(s)$$
$$= I(s) - O(s)\cdot H(s)$$

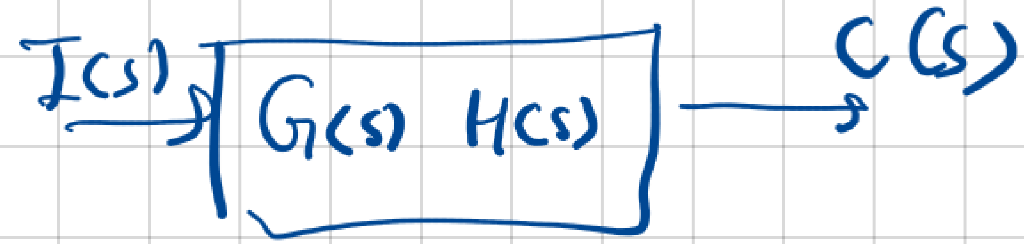
$$[I(s) - O(s)\,H(s)]\cdot G(s) = O(s)$$

$$I(s)\cdot G(s) - O(s)\,H(s)\,G(s) = O(s)$$

$$Q(s) = \frac{O(s)}{I(s)} = \frac{G(s)}{1 + G(s)\,H(s)}$$



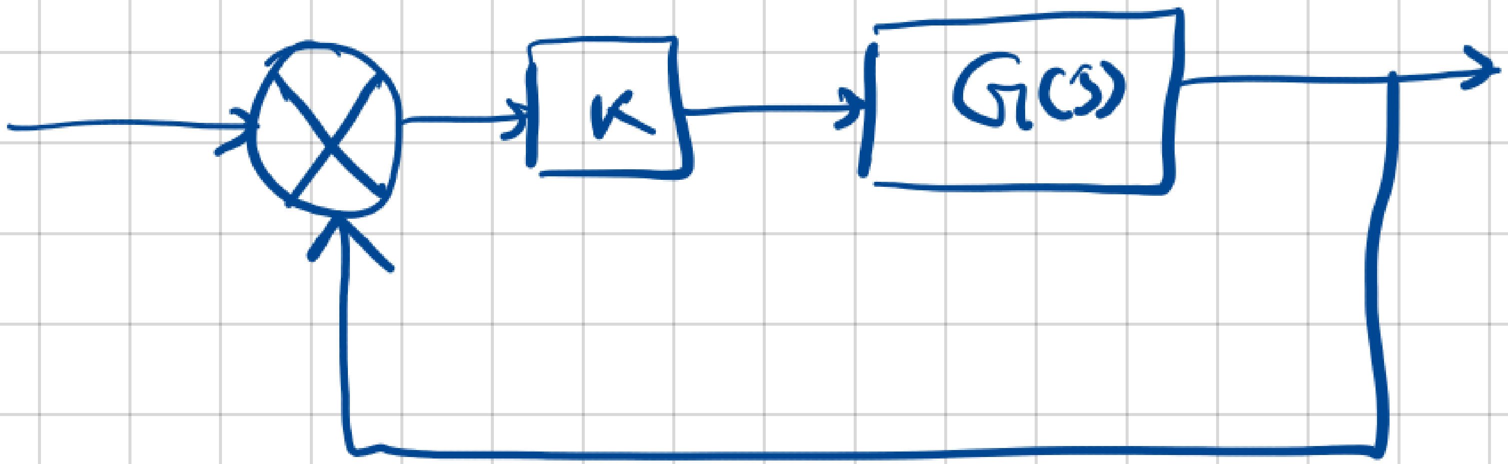




↳ open loop TF.

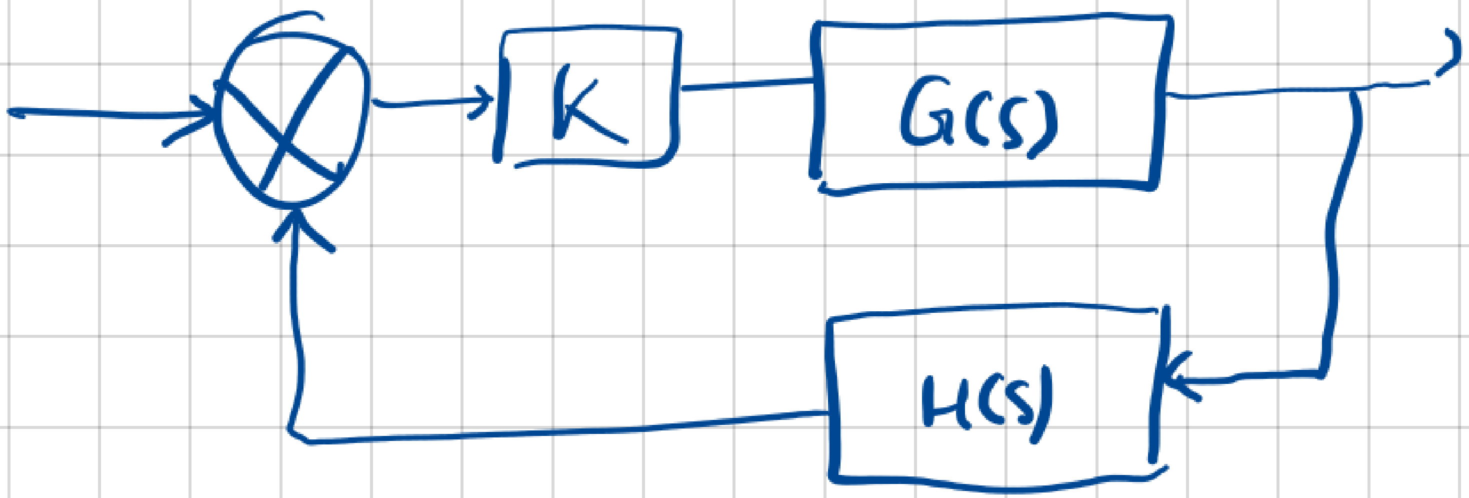
$$G(s) = \frac{(s+a)}{(s+b)(s+c)(s+q)(s+r)}$$

$$Q(s) = \frac{KG(s)}{1+KG(s)}$$



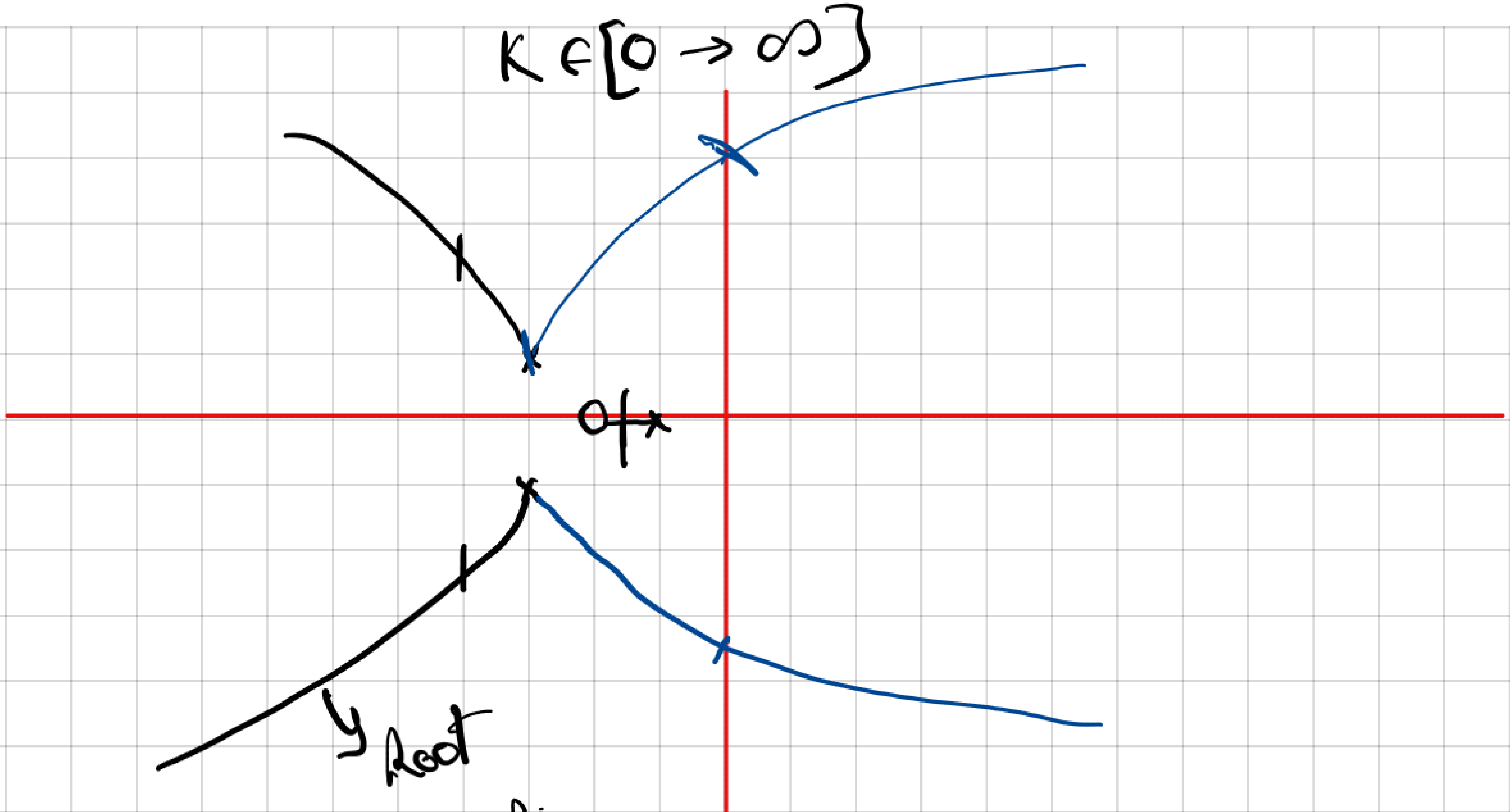
$$G(s) = \frac{N_G}{D_G}$$

$$H(s) = \frac{N_H}{D_H}$$



$$Q(s) = \frac{N_G / D_G}{1 + K \frac{N_G}{D_G} \frac{N_H}{D_H}}$$

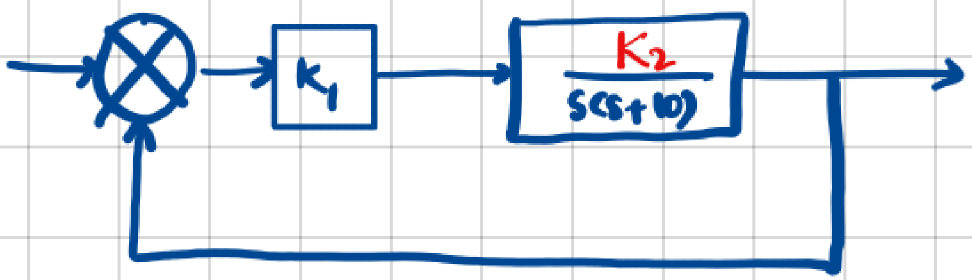
$$\boxed{Q(s) = \frac{N_G \cdot D_H}{D_H D_G + K N_G N_H}}$$

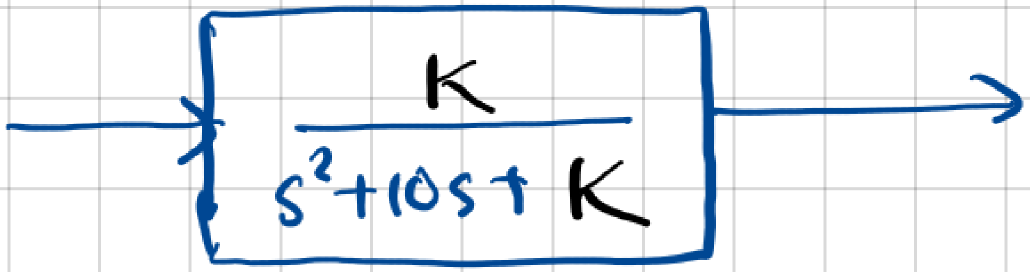


Root locus.

→ Path taken by a root as $K \uparrow$

Defining a R.L.





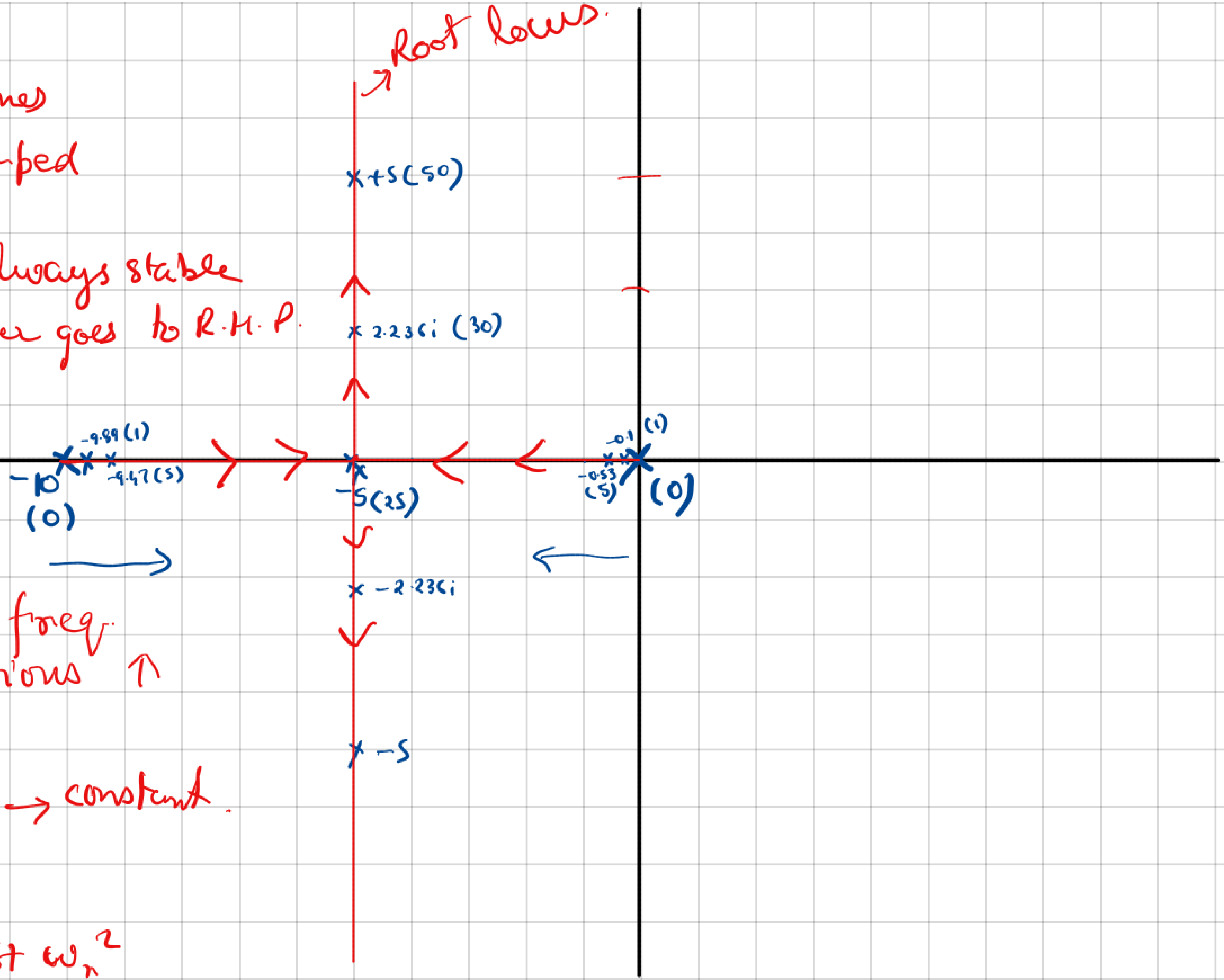
$K_1 K_2 = K$

verify if at $K=5$, -9.4721 is a pole of the closed loop system

$$\frac{5}{(-9.4721)(-9.4721+10)} = -0.9999.... \simeq -1$$

K	Pole 1	Pole 2
0	-10	0
1	-9.899	-0.1010
5	-9.4721	-0.5279
25	-5	-5
36	$-5+2.361i$	$-5-2.361i$
35	$-5+3.1623i$	$-5-3.1623i$
50	$-5+5i$	$-5-5i$

$K > 25 \rightarrow$ Sys becomes underdamped



→ Sys is always stable as R.L. never goes to R.H.P.

→ Damped freq. of oscillations ↑

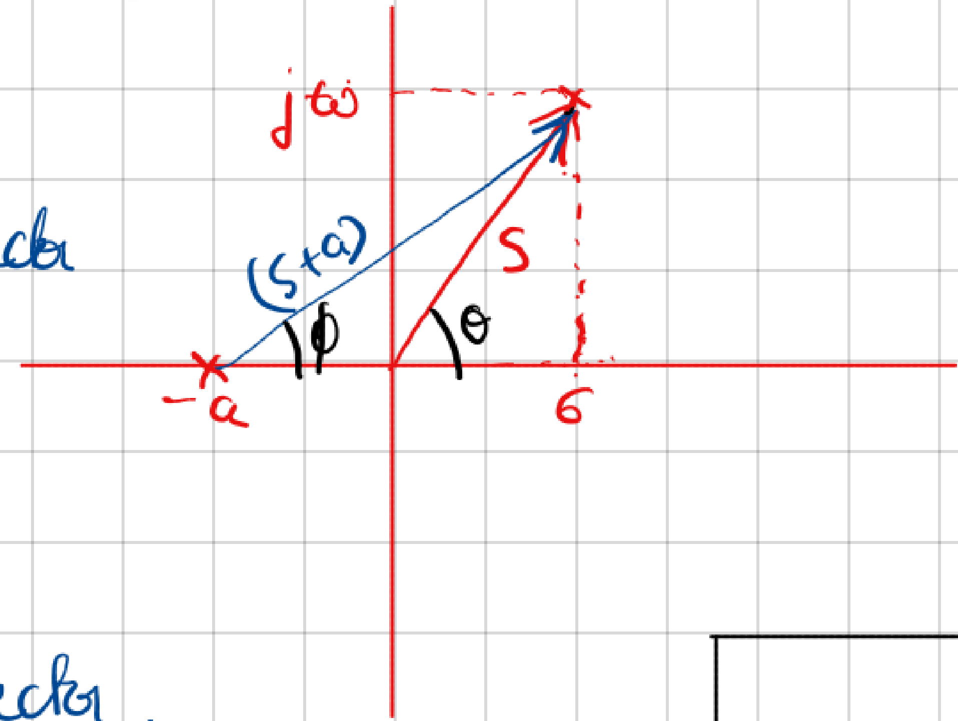
98% $T_s \rightarrow \frac{4}{\xi \omega_n} \rightarrow$ constant.

$s^2 + 2\xi\omega_n s + \omega_n^2$

Vector Representation of Complex Nos.

$s = \sigma + j\omega$

can be represented as a vector



$(s+a)$

vector representation is as shown

$s = -a$ is the tail of the vector.

$(s+a)$

$|s+a| \angle (s+a)$

Assume a function

$$F(s) = \frac{\prod_{i=1}^{m} (s+z_i)}{\prod_{j=1}^{n} (s+p_j)}$$

\# poles = n

\# zeros = m

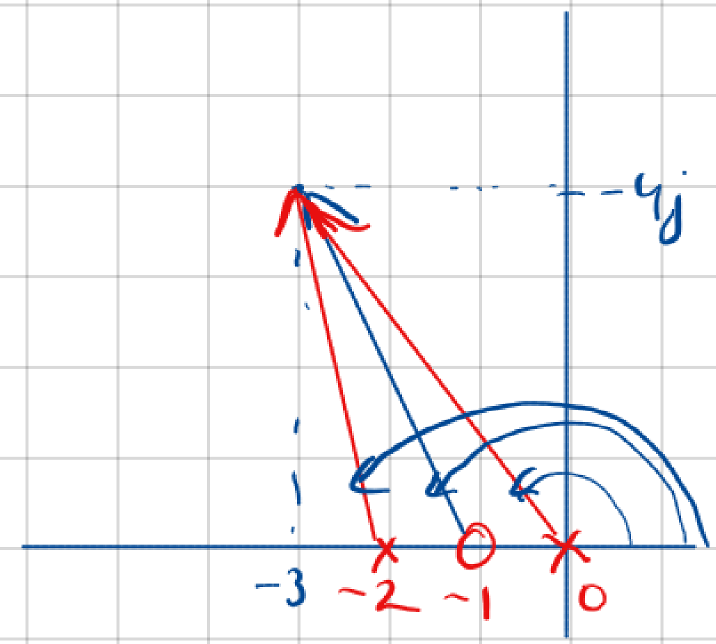
magnitude of $F(s)$ at a given value of s

$$\boxed{M = \frac{\prod_{i=1}^{m} |s+z_i|}{\prod_{j=1}^{n} |s+p_j|}}$$

$$\angle F(s) = \sum_{1}^{m} \text{Zero } \angle\text{'s} - \sum_{1}^{n} \text{Pole angles}$$

$$= \sum_{1}^{m} \angle (s+z_i) - \sum_{1}^{n} \angle (s+p_j)$$

$$F(s) = \frac{(s+1)}{s(s+2)} \qquad @\, s = -3+4j$$



$$(s+1) \rightarrow \sqrt{20}\angle 116.56^\circ$$

$$s \rightarrow 5\angle 126.9^\circ$$

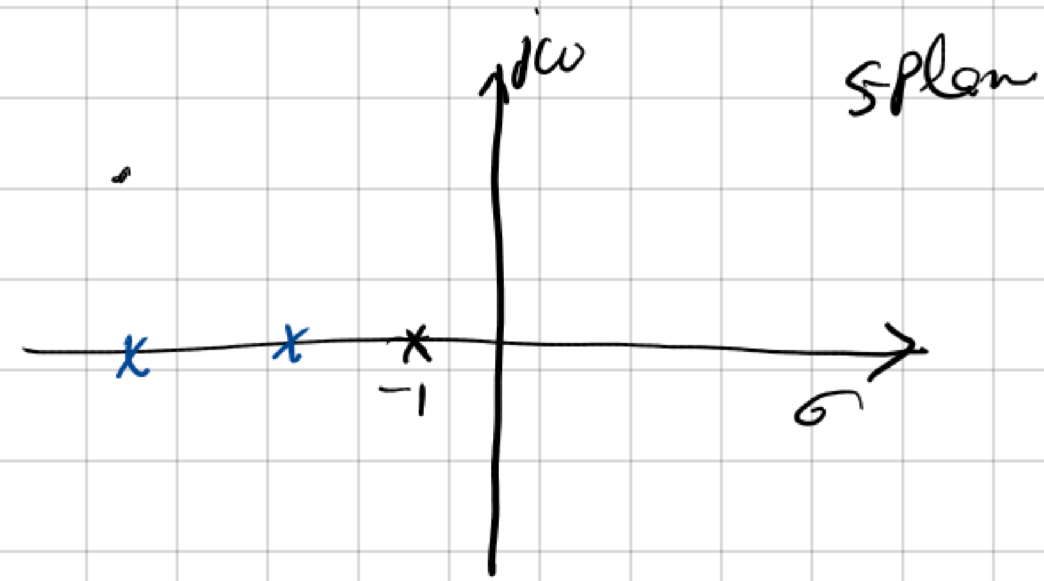
$$(s+2) \rightarrow \sqrt{17}\angle 104$$

$$F(s) = \frac{\sqrt{20}}{5\sqrt{17}}\angle(116.56 - 126.9 - 104)$$

$$= 0.217\angle -114.35^\circ$$

Properties of Root locus

$$F(s) = \frac{K\,G(s)}{1 + K\,G(s)\,H(s)}$$



For S to lie on R.L. $(1 + K\,G(s)\,H(s))_s = 0$

$$K\,G(s)\,H(s) = -1$$

$|K\,G(s)\,H(s)| = 1$ & $\angle K\,H(s)\,G(s) = 180^\circ \Rightarrow$ $\boxed{\angle H(s)\,G(s) = (2p+1)\,180^\circ}$

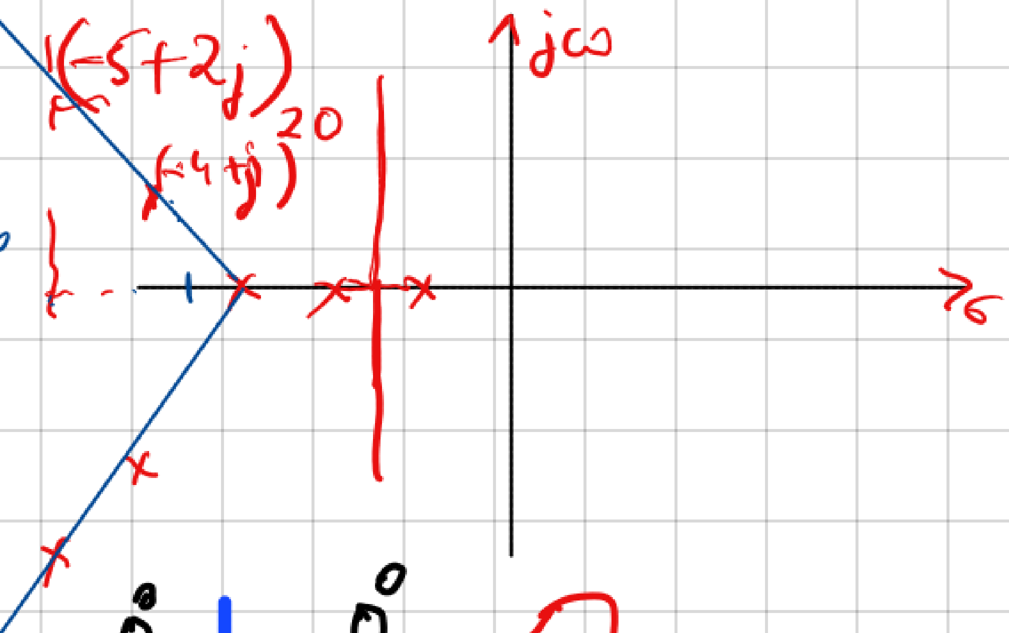
K is positive $\Rightarrow$ $\boxed{K = \frac{1}{|G(s)\,H(s)|}}$

Pole causes an angle of 180° for $K\,G(s)\,H(s)$

Sketch a Root Locus:

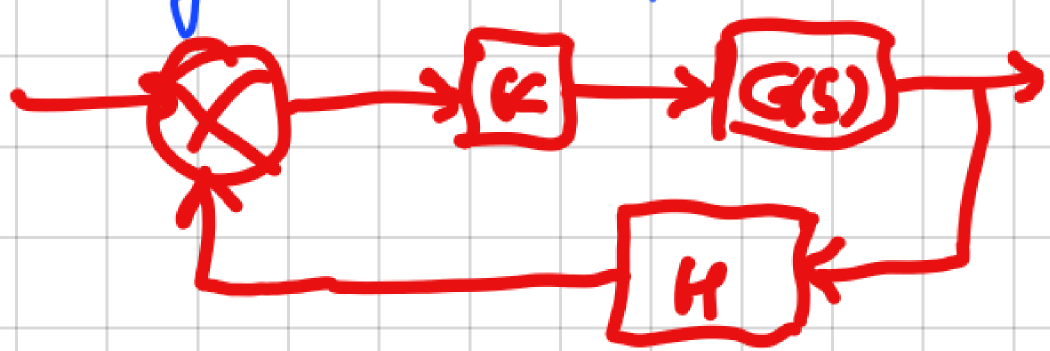
Rule 1: No. of branches = No. of poles of the closed loop system.

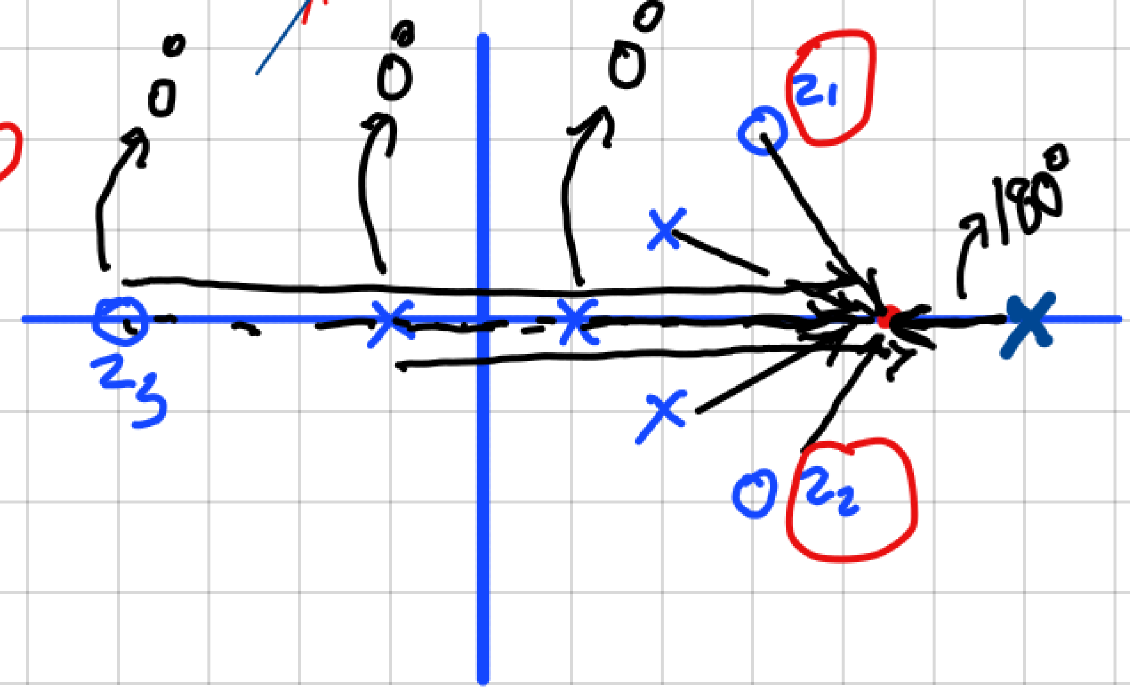
Rule 2: Root Locus is symmetric about the real axis since complex roots can only exist as complex conjugates



Rule 3: Segments of Real Axis

say - Poles(x) zeros(o) → of $K\,G(s)\,H(s)$





Angle contribution of complex poles & zeros TO THE REAL AXIS segme

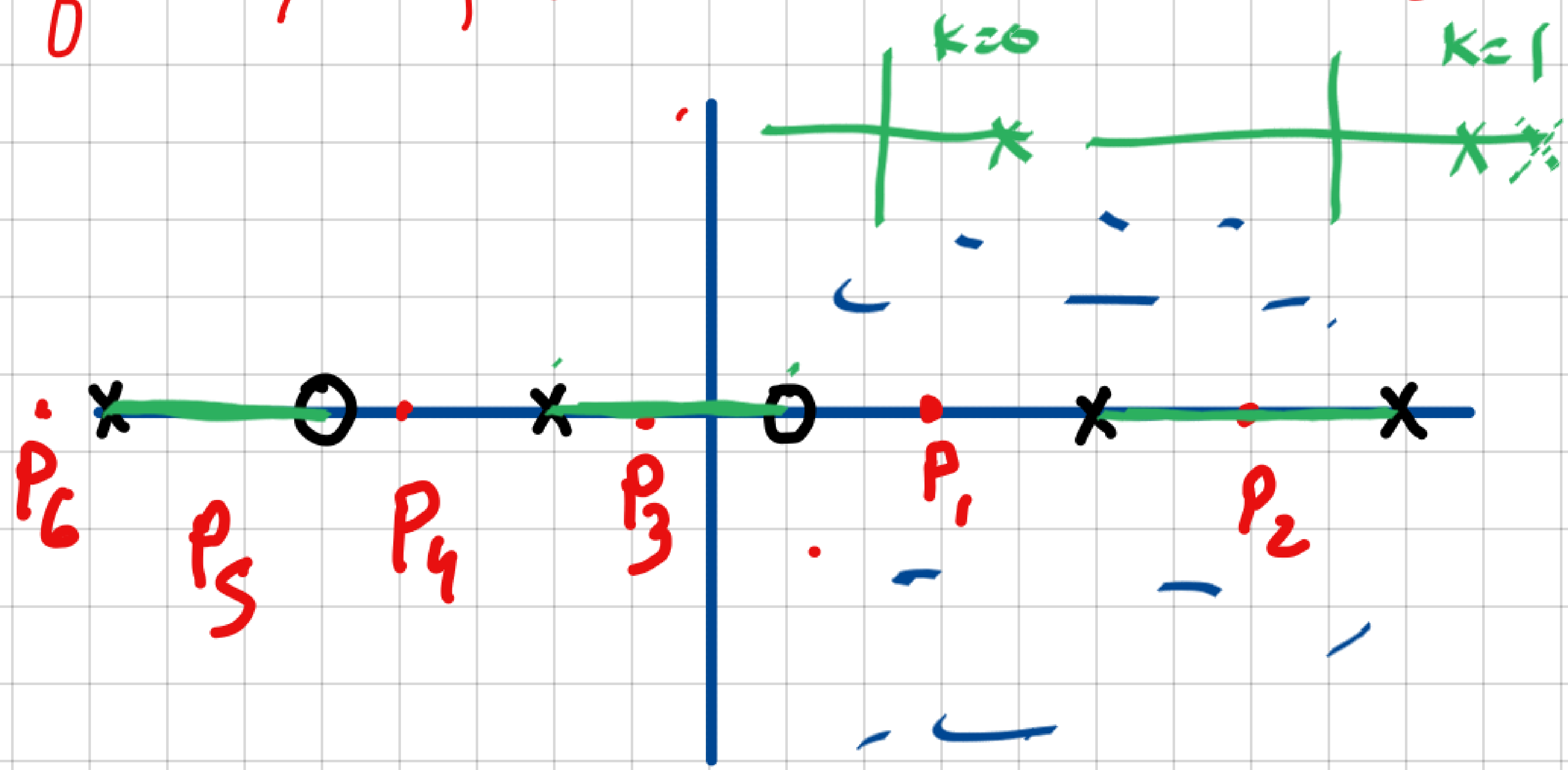
is = 0

Part of R.L.

P_2

P_3

P_5



All points on the LEFT of ODD # poles & zeros lie on the R·L real axis

Start & end of Brances of R. L.

Extend the R.L. beyond the real axis

$$T(s) = \frac{K\ N_G D_H}{\underline{D_G D_H} + \underline{K N_G N_H}}$$

$K \to \underset{\varepsilon}{0}$

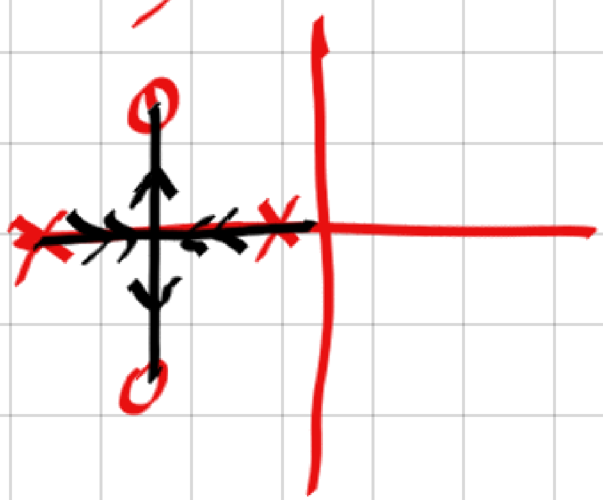
$D_T = D_G D_H + \underset{\to 0}{\varepsilon} \Rightarrow$ poles at $K = 0$ = poles of open loop system

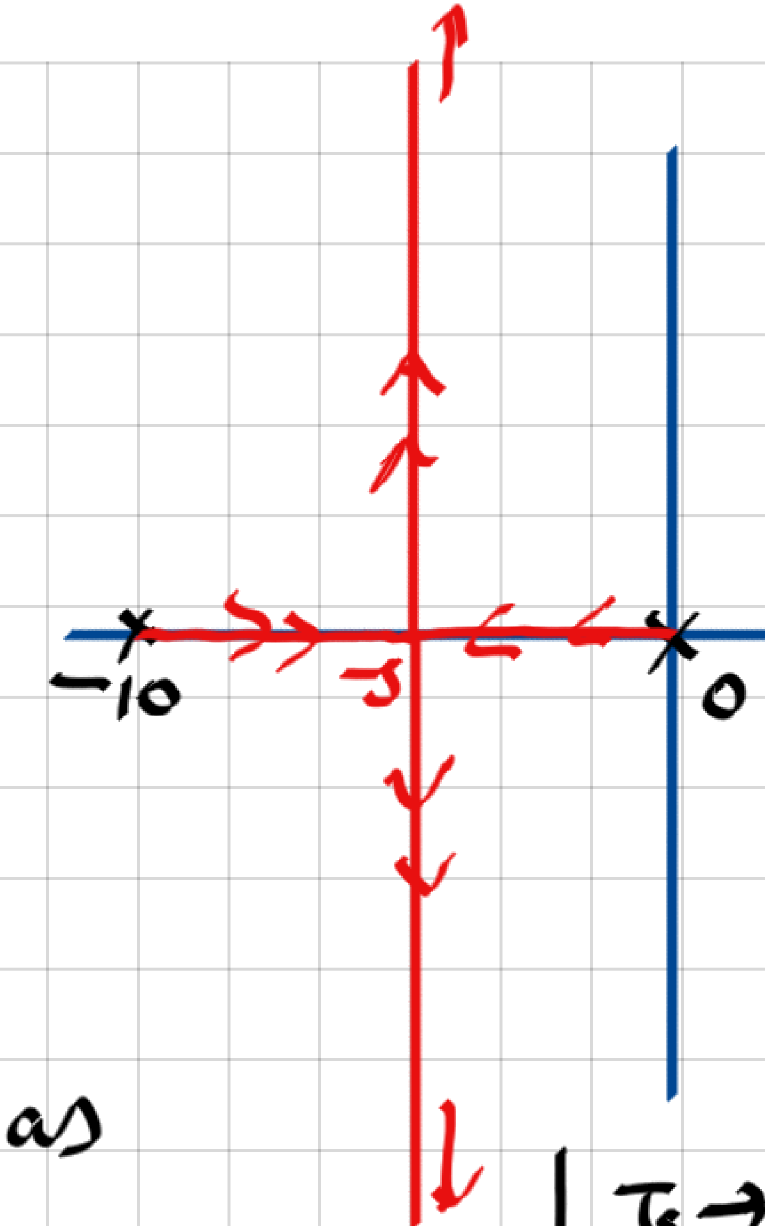
$K \to \infty$

$D_T = \dfrac{K N_G D_H}{\varepsilon + K N_G N_H} \Rightarrow$ poles at $K = \infty$ = zeros of open loop system.

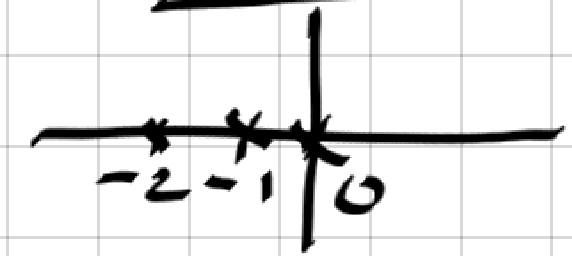
Rule 5 Behavior at ∞

$$\frac{(s+a)(s+b)}{(s+c)(s+d)}$$





$$\frac{K}{s(s+1)(s+2)}$$



$$T = \frac{N \to \infty}{D \to 0}$$

There are as many zeros as poles when ∞ is considered as the poles & zeros of the system.

$T_s \to \infty$ $\infty \to$ pole of the system

$\boxed{\text{as } K \to \infty}$

$T_s \to 0$ $\infty \to$ zero of the system

$\boxed{K \to \infty}$

$$\frac{K}{s(s+1)(s+2)}$$

$s \to \infty \quad \approx \frac{K}{s\,s\,s} \Rightarrow$ system has 3 zeros at ∞

$$\frac{K}{s(s+1)(s+2)}$$