T3 :-

1) $K_v = 4 \text{ sec}^{-1}$ Phase margin $= 50^\circ$ Gain margin $\geq 8dB$

$$\frac{(2s+0.1)}{s(s^2+0.1s+4)} = G$$

We can design a PID controller.

$$G_c = \frac{(as+1)(bs+1)}{s}$$

T = overall transfer function $= G_c \times G \times K$

$$= \frac{K(as+1)(bs+1)(2s+0.1)}{s^2(s^2+0.1s+4)}$$

$$K_v = \lim_{s \to 0} sT(s) = 4$$

$$= \lim_{s \to 0} \frac{K(as+1)(bs+1)(2s+0.1)}{s(s^2+0.1s+4)}$$

Not defined.

We cannot use PID.

~~Let us try Lead lag com~~

Let us use ~~PD for phase margin and Lag compensator for static velocity error.~~

Let us use a PD controller $(as+1)$.

$$\Rightarrow T = K G_c G = \frac{K(as+1)(2s+0.1)}{s(s^2+0.1s+4)}$$

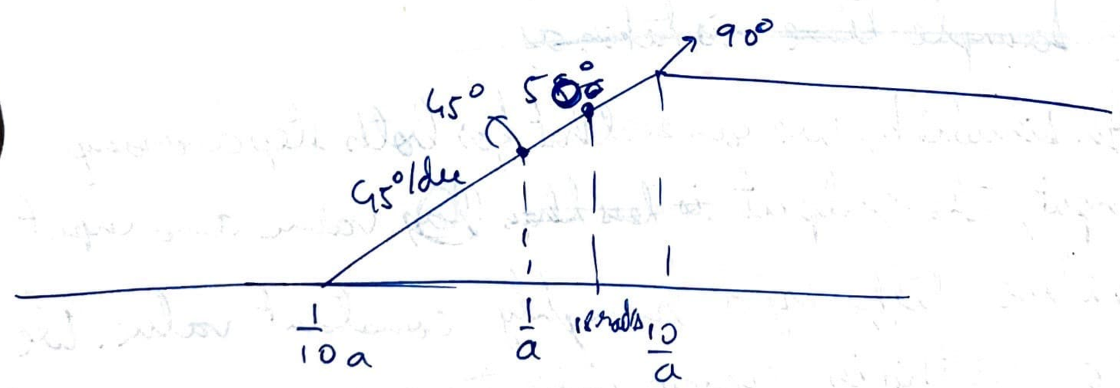
$$K_v = 4$$

$$\lim_{s\to 0} sT(s) = 0 \Rightarrow \frac{K \times 0.1}{4} = 4$$

$$K = 160$$

$$\Rightarrow T = \frac{160(as+1)(2s+0.1)}{s(s^2+0.1s+4)}$$

Bode plot of 160 G gives us ∞ the gain margin and phase margin of $\approx 0°$ at 18 rad/s. We need to pull this up by about 50° ~~say 55 to be slightly conservative~~.



$$\theta - 0 = 45 \times \left(\log \omega - \log \frac{1}{10a}\right)$$

$\theta = 50°$ at $\omega = 18$ rad/s

$$\frac{50}{45} = \log 18 + \log 10 + \log a \Rightarrow a = 10^{\frac{50}{45} - \log 180}$$

Now we have 62.1° phase margin at 26.2 rad/s and ∞ gain margin.

Thus, our compensator is $(as+1)$ where

$a = 10^{\frac{50}{45} - \log_{10} 180} = 0.07175$

Note:- Reason for PD is explained below.

Say, $G_c = \dfrac{(as+1)(bs+1) \cdots (zs+1) \cdots}{(a_1s+1)(b_1s+1) \cdots (z_1s+1) \cdots}$

No matter what we take as G_c, $K = 160$ as $\lim_{s \to 0} G_c = 1$

In the end, we see that increase in angle is less than 90°, so one zero should suffice.

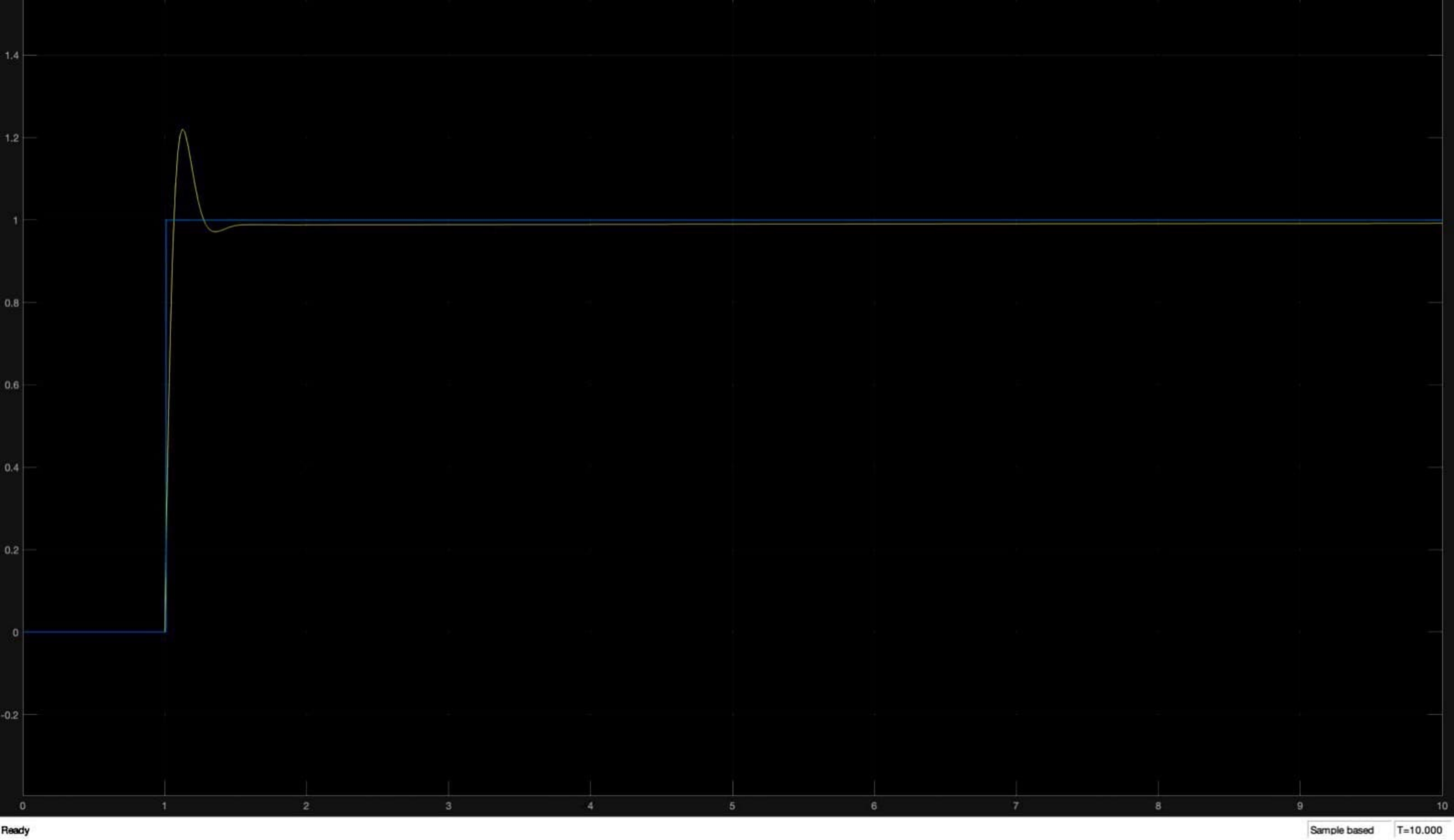
Thus, we use PD.

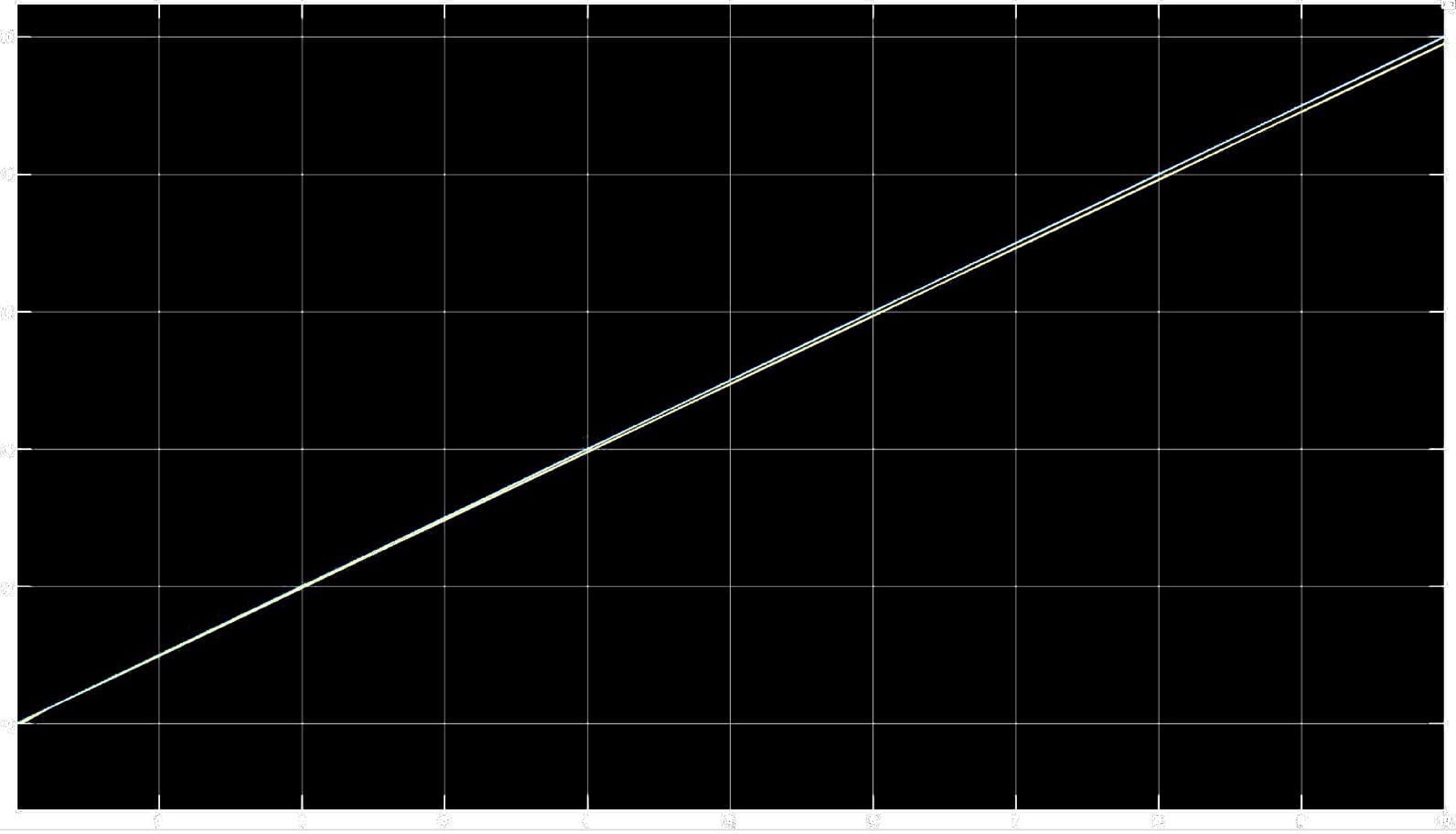
~~In simulink, nearly all output~~

~~In the end, we get~~

~~⇒ Sample time is taken as~~

In simulink, we can see that for both step and ramp input, the output ~~is less than~~ value and input value differs by a roughly constant value. We can see this in ramp, indicating our success in setting K_v.


1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
1
2
3
4
5
6
7
8
9
10
Ready
Sample based
T=10.000



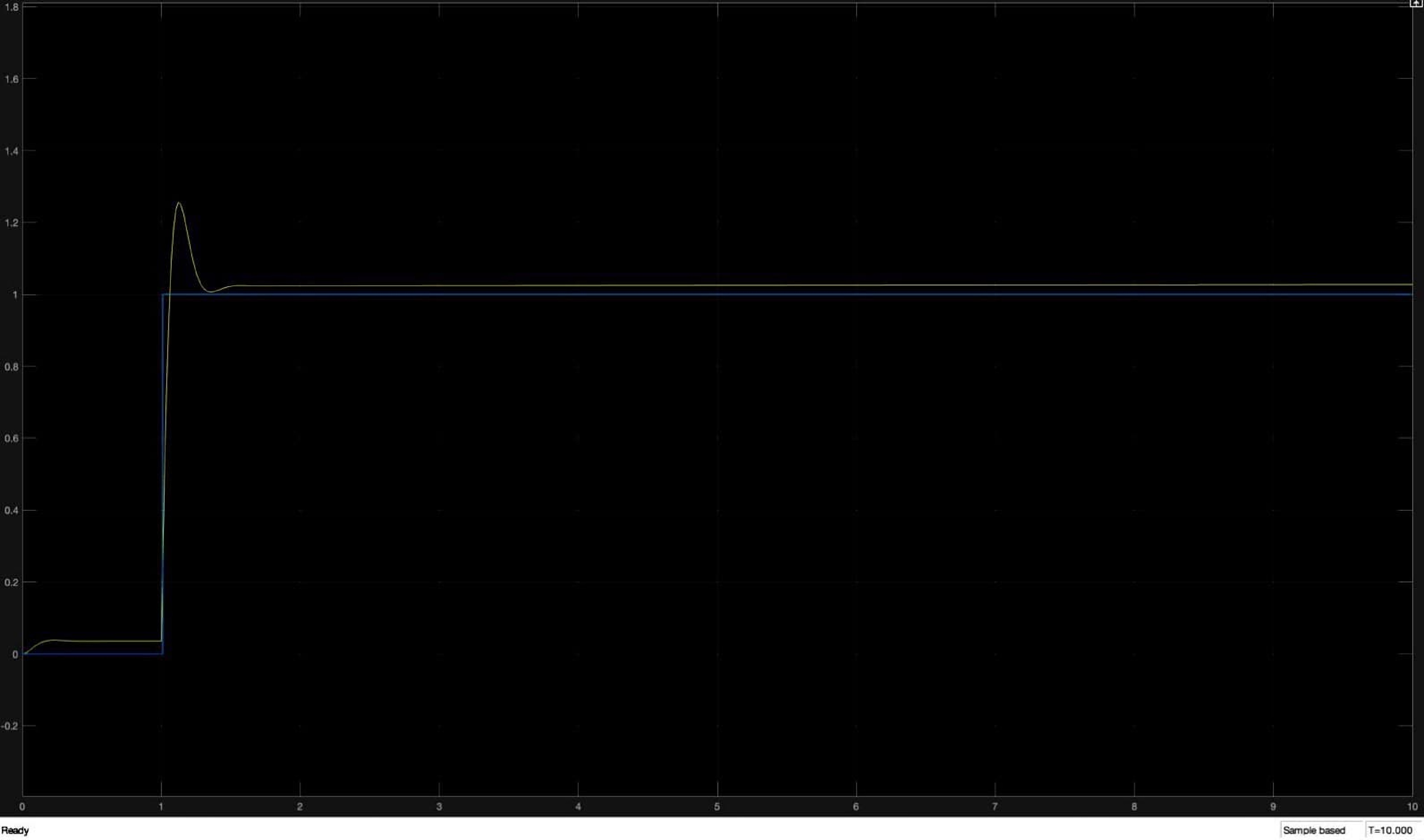
For PID block, N is very large so we can set ~~prope~~ improper transfer functions.

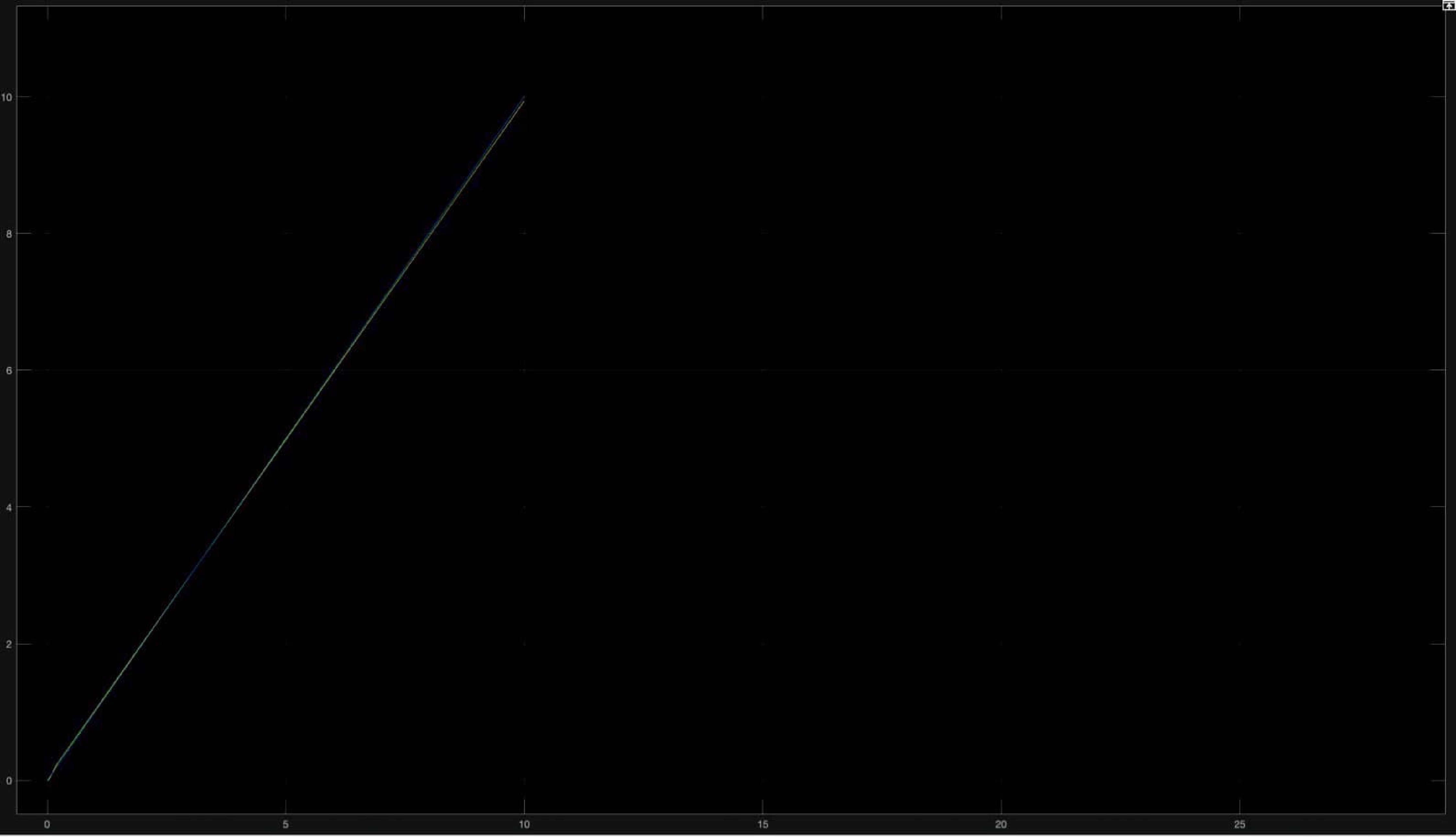
2) Sample time is set as 10 secs, ~~as anything less go~~
The lesser it is, the more erratic the output is.

For this sample time, there is not much appreciable change from the output without noise.

For step, the max value is greater and the difference b/w output and input is more pronounced.

For ramp, there is not much change other than the fact that the error b/w the input and output changes sign once through the 10 sec time.


1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
1
2
3
4
5
6
7
8
9
10
Ready
Sample based
T=10.000



Ready

Sample based

T=10.000

3) As in q1, initial phase margin $\simeq 0°$ at 18 rad/s.

We want to design lag compensator such that it barely affects the overall system in terms of phase and gain margin.

This can work if we give $\omega = 0.18$ a lag of $5°$ (result obtained by trial and error)

say lag comp is $\dfrac{rs+1}{ts+1}$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{\omega}{\frac{1}{r}}\right) - \tan^{-1}\left(\frac{\omega}{\frac{1}{t}}\right)$$

choose $\omega = 0.18$, $\frac{1}{r} = 0.1 \Rightarrow r = 10$

$$\Rightarrow -5 = \tan^{-1}\left(\frac{0.18}{0.1}\right) - \tan^{-1}\left(\frac{0.18}{\frac{1}{t}}\right)$$

$$0.18t = \tan\left(5 + \tan^{-1}\left(\frac{0.18}{0.1}\right)\right)$$

$$t = 12.446 \simeq 12.5$$

$$\frac{1}{t} = 0.08$$

$$\therefore \frac{rs+1}{ts+1} = \frac{10s+1}{12.5s+1}$$

On applying this, we get phase margin $\simeq 0°$ at $\omega = 16.1$ rad/s and ∞ to gain margin.

Now we create our lead compensator,

Let us say our ~~lag~~ lead compensator is $\left(\frac{ps+1}{qs+1}\right)$

We can say that $ps+1$ can contribute 90° and $qs+1$ can remove 40°.

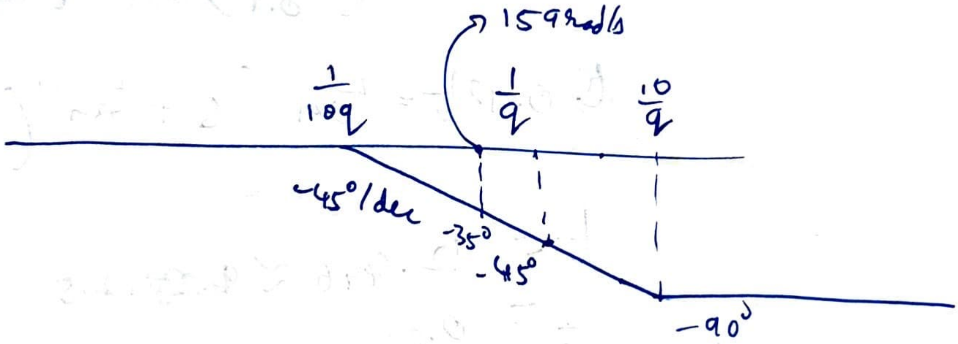
$ps+1 \rightarrow 90^\circ \Rightarrow \frac{1}{p} = \frac{16.1}{10} \Rightarrow p = \frac{10}{16.1}$

$p \simeq 0.62$

This gives us 89.4° phase margin at 159 rad/s and ∞ the gain margin.

Now, let us say $qs+1$ removes 39°.

$\Rightarrow$



$$\theta = -45\left(\log \omega - \log \frac{1}{10q}\right)$$

$$-39 = -45\left(\log 159 + \log 10 + \log q\right)$$

$$q = 10^{\left(\frac{39}{45} - \log 1590\right)} = 0.0046$$

This finally gives us 57.4° phase margin at 135 rad/s and ∞ the gain margin.

$$\therefore G_c = \left(\frac{10s+1}{12.5s+1}\right)\left(\frac{0.62s+1}{0.0046s+1}\right)$$

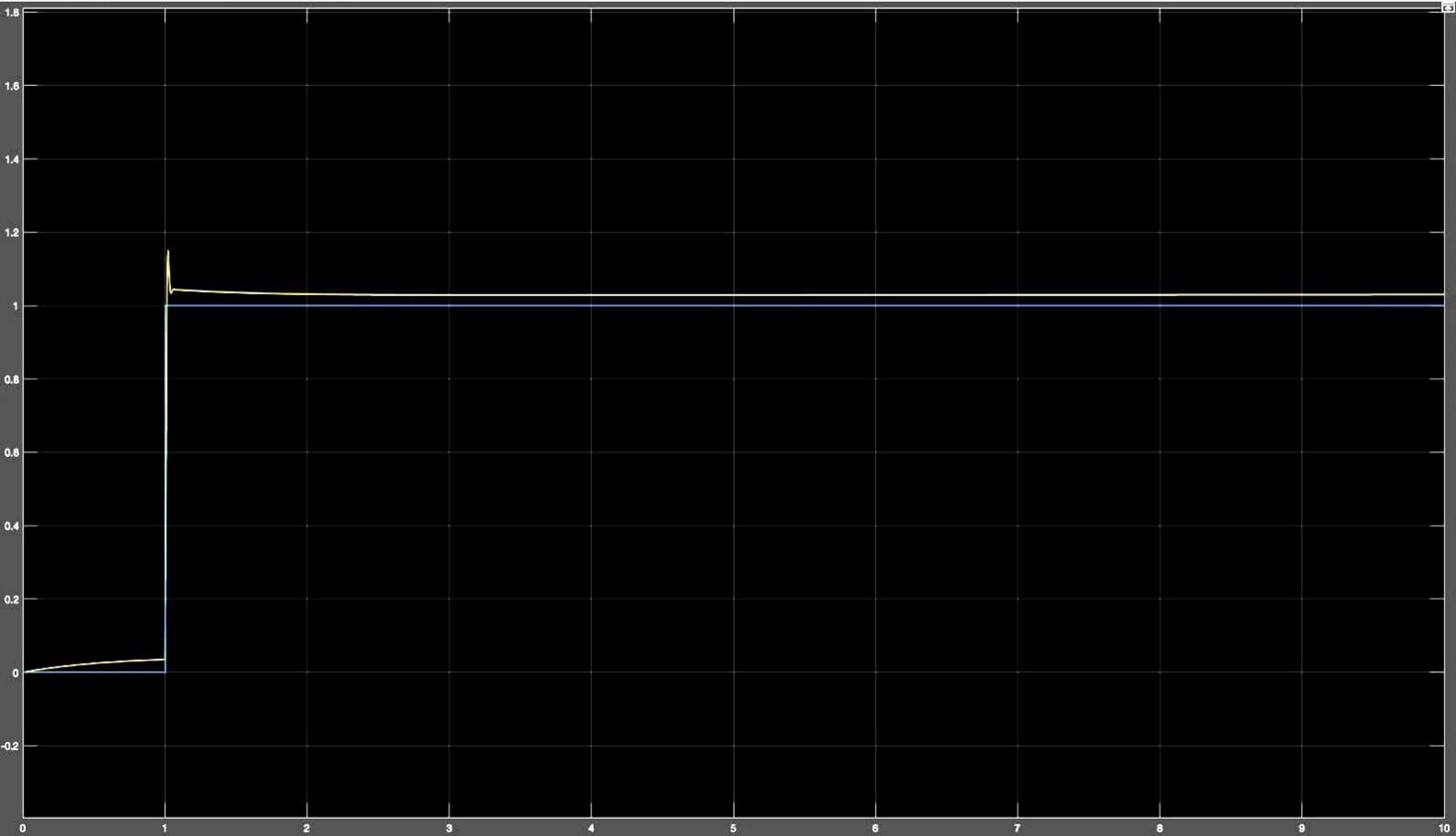
~~In simulink, we see that the output is close to the input for both step and ramp~~

③ The simulink results are similar to that of the second question.

For step, the peak value is less than 1.2

Overall, all of the above controllers give satisfactory results.

2) F



1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
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-0.2
0
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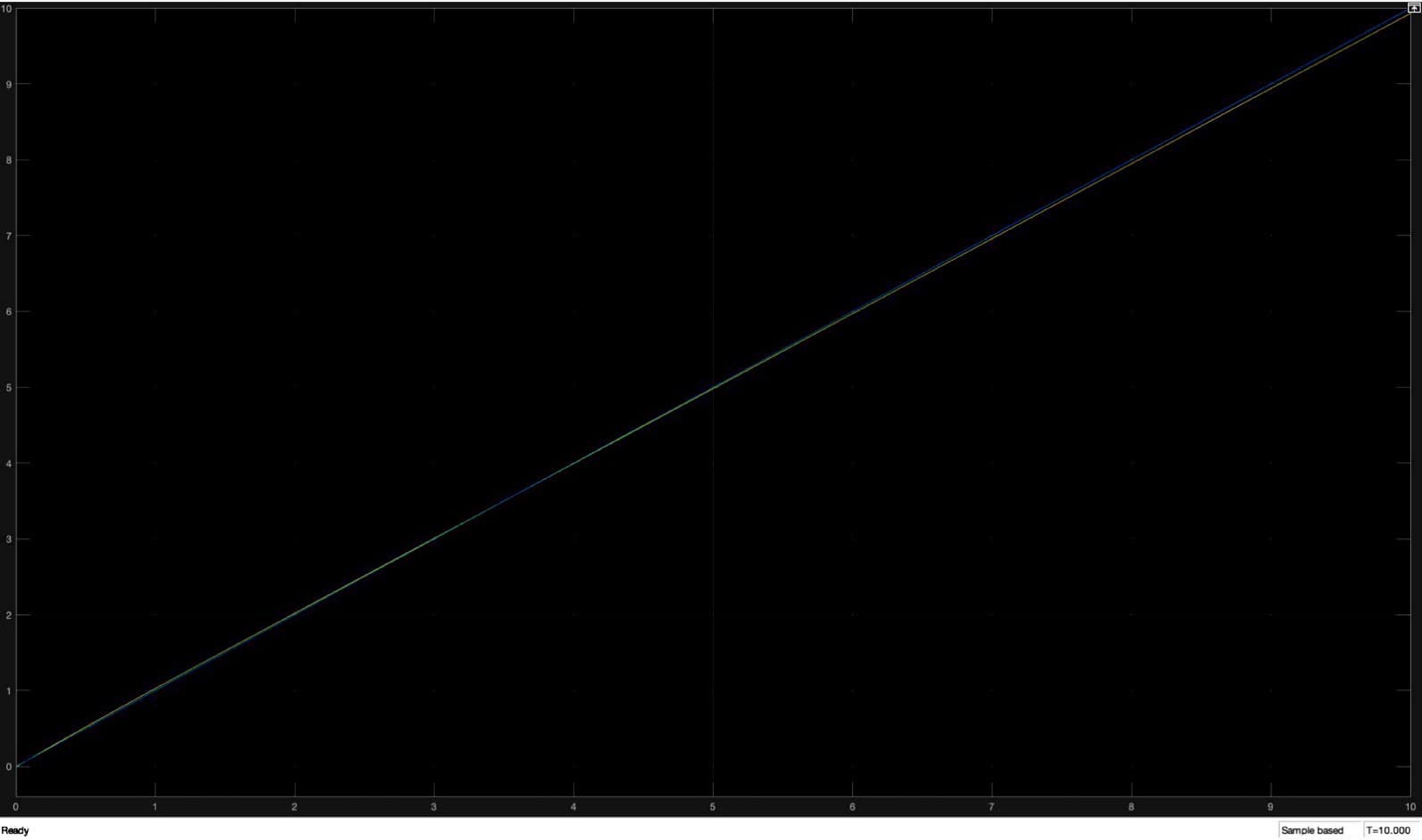

10
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6
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8
9
10
Ready
Sample based
T=10.000

Note :- the last two images are the noisy lead-lag compensated ~~input vs~~ output ~~vs~~ vs input.