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Market informed portfolio optimization methods with hybrid quantum computing

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Abstract

This document presents a portfolio optimization framework that employs a hybrid quantum computing algorithm and a futures market sentiment indicator—The Market Sentiment Meter (MSM) variable, developed jointly by CME Group and 1QBit. The methodology used was the Variational Quantum Eigensolver (VQE). The work presented here is divided into four portfolio optimization problem formulations, of binary and continuous variable formulations, determining which assets to pick their weights. This work demonstrates that adding the MSM variable can improve the performance of hybrid quantum solutions, by informing the asset selection problem with market environment information through the four MSM states.

KEYWORDS

market sentiment meter, portfolio optimization, quantum computing, variational quantum eigensolver

JEL CLASSIFICATION

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1 | INTRODUCTION

1.1 | Research overview and contributions

This work aims to bridge the gap between theoretical quantum computing models and practical financial applications, paving the way for future research and development in the field of quantum finance. By demonstrating the feasibility and advantages of this approach, we contribute to the growing body of knowledge on how quantum computing can revolutionize financial optimization and risk management.

In this work, several portfolio optimization frameworks that employ hybrid quantum computing algorithms in conjunction with a futures and options markets risk sentiment indicator. The primary methodology utilized is the Variational Quantum Eigensolver (VQE), integrated into a risk-modified Markowitz mean–variance portfolio optimization approach. This novel approach has the potential to leverage the strengths of quantum computing and classical methods.

Traditional portfolio optimization often relies on mean–variance analysis to balance expected returns against risk. However, these methods typically assume static market conditions and may not fully capture the dynamic nature of financial markets. By incorporating a risk sentiment indicator derived from futures and options markets, our framework adapts to changing market conditions, potentially leading to more resilient investment strategies.

Giancarlo Salirrosas Martínez and Jinglun Gao were equal first authors.

Further, by incorporating risk sentiment indicators, we enhance the traditional mean–variance optimization model, making it more responsive to market conditions. This integration helps in better capturing the risk dynamics and improving asset allocation decisions. The introduction of the risk sentiment variable within the hybrid quantum framework demonstrates improved performance in asset allocation, providing evidence that informed asset selection can lead to better portfolio outcomes.

1.2 | Literature review and interrelationship between physics and finance

Physics and finance have a long, intertwined history, going back as far as 1900 when mathematician Louis Bachelier, now considered the "Father of Modern Financial Mathematics", wrote his famous thesis "The Theory of Speculation" (Bachelier, 1900). In this paper he applied the concept of Brownian motion and used it to analyze the behavior of financial markets, at the time marking a new approach to finance. The relationship between the two fields may seem unlikely at first glance. However, at their cores, both sciences seek to understand and model the behavior of complex systems, whether it be the movement of particles in the physical world or the fluctuations in financial markets. In recent years, the use of advanced physics and mathematical techniques, such as chaos theory and complex systems analysis, have allowed for the creation of increasingly sophisticated financial models. These models have been used to better understand and predict market behavior, design advanced algorithmic trading strategies, and evaluate the risk of financial instruments.

The adoption of these physics and math-based approaches to tackle problems in the financial industry, continuous technological developments, and increasing amounts of data generated everyday by the markets gave birth to what we know today as quantitative finance, financial engineering, computational finance, and mathematical finance. Quantitative finance uses mathematical, computational, and statistical methods to model and analyze financial data for a set of many purposes such as pricing, time series forecasting, credit scoring and portfolio optimization.

It is important to notice that many of the mathematical models used in quantitative finance are based on the principles of physics, such as the Black-Scholes model for pricing options, which is based both on the idea of a "random walk" derived from the theory of Brownian motion and the heat transfer equation from traditional physics. One of the most well-known areas of overlap between physics and finance is the use of statistical mechanics in financial modeling. Statistical mechanics is the branch of physics that deals with the behavior of large systems composed of many interacting components. These concepts have been successfully applied to a variety of financial systems, including stock markets, foreign exchange markets, and agent-based modeling. Moreover, this intersection of physics and finance can be grouped in a new field known as econophysics, which in summary applies the principles and techniques of physics to the study of economic systems and financial markets. Econophysics has contributed to a deeper understanding of the behavior of financial markets and has led to the development of new modeling techniques and financial products.

In recent years, the principles of quantum mechanics are being applied to computer science, giving rise to a new paradigm of computation—quantum computing. Currently, research on quantum computing applications to the financial industry (Elsokkary et al., 2017; Farhi, 2014; Hen & Spedalieri, 2016; Marzec, 2016; QC Ware Corp, 2018; et al., 2018) has brought a considerable attention of both academic and industry practitioners, opening the door to innovative approaches by revisiting old problems with quantum computing techniques or by exploring how existing methodologies can potentially be improved through applied quantum algorithms.

The motivation of the present study is to explore whether current quantum computing algorithms can be used in realistic problems in the financial industry, focusing on the portfolio optimization problem. The broad question that this paper contributes to addressing is: How can a hybrid-quantum algorithm for portfolio optimization be improved to give better returns?

There are secondary hypotheses addressed in this research: (a) the direct relationship between risk and return, that is, a higher level of risk will produce higher returns does not hold for the case of futures contracts; (b) the inclusion of the CME Market Sentiment Meter (MSM) variable gives better results in terms of the performance of the portfolio optimization problem in both different approaches: classical and hybrid quantum algorithms.

Even though the field of quantum computing and its applications for the financial industry are in their early stages of research, we can reference some authors that have approached the portfolio optimization problem with quantum computing algorithms. Within this group of researchers, the algorithms employed by the authors fall into two different categories: quantum annealer and gate-based algorithms.

Certo et al. (2022) formulated real-world constraints for a portfolio of financial assets to use with a quantum annealer using D-Wave's Quantum Processor and showed how to add fundamental analysis to the static portfolio optimization problem (Certo, 2022). Fernandez-Lorenzo et al. (2021) formulated a hybrid quantum-classical optimization algorithm to undertake a quadratic optimization with cardinality and linear constraints applied to the financial industry (Fernandez-Lorenzo, 2021). Rebentrost & Lloyd, (2018) developed a quantum algorithm for portfolio optimization (Rebentrost & Lloyd, 2018). Mugel et al. (2020) implemented quantum and quantum-inspired algorithms to tackle the problem of dynamic portfolio optimization using D-Wave Hybrid quantum annealing and VQEs on IBM-Q (Mugel, 2020). Barkoutsos et al. (2020) showed that hybrid quantum—Classical variational algorithms can be implemented on quantum computers and can be used to find solutions for combinatorial optimization problems (Barkoutsos, 2020). Rosenberg et al. (2016) implemented a multi-period portfolio optimization problem which considered transaction costs including permanent and temporary market impact costs using quantum annealer (Rosenberg et al., 2016). Lopez de Prado (2015) demonstrated how the generalized dynamic portfolio optimization problem can be reformulated as an integer optimization problem in order to be solved by a quantum computer (López de Prado, 2015).

In Modern Portfolio Theory (MPT), Markowitz (1952) explained the risk (volatility) and expected return of a financial security can be denoted as the mean and standard deviation of a normal distribution. This is the cornerstone of quantitative analysis of finance. In traditional portfolio optimization, that is, the mean–variance optimization, volatility is the only risk factor at play (Markowitz, 1952). However, many have argued that volatility alone does not equal risk, including Putnam (2020) who introduced a new method to 'calculate' market sentiment through several market sentiment states, as an enhanced probe of volatility (Putnam, 2020).

It has been identified from the above literature review that most of the existing quantum portfolio research developed to-date lacks a key component in market sentiment, which this work aims to address.

2 | MATERIALS AND METHODS

2.1 | Hybrid quantum algorithms and the VQE

A hybrid quantum-classical algorithm is class of algorithms that utilize both quantum and classical aspects to perform a specific task. These algorithms take advantage of the strengths of current quantum and classical computing, that can operate within the limitations of current quantum hardware by making use of some classical methods.

In general, hybrid quantum-classical algorithms can be divided into two categories: those that use a quantum computer as a subroutine in a classical algorithm, and those that use a classical computer to control a quantum algorithm. In the first case, the quantum computer is used to perform a specific subroutine, such as factoring a large number, that is too difficult for a classical computer to perform efficiently. In the second case, the classical computer is used to control the execution of a quantum algorithm, such as a quantum simulation or optimization algorithm.

Hybrid quantum-classical algorithms have been proposed for a wide range of applications, including machine learning, optimization, and quantum chemistry. For example, in the field of machine learning, a hybrid quantum-classical algorithm can be used to perform gradient descent on a quantum computer, while the classical computer is used to perform the updates to the quantum circuit. In the field of optimization, a hybrid quantum-classical algorithm can be used to find the global minimum of a high-dimensional function.

There are several challenges in implementing hybrid quantum-classical algorithms, including the difficulty of interfacing between the quantum and classical systems, the need for efficient classical algorithms to control the quantum computation, and the need for efficient quantum algorithms to perform the desired computation. Presently, they serve as a mode to begin defining frameworks that will improve as the quantum computing hardware develops, while also serving as a benchmark to evaluate the utility of quantum computing for various applications.

In the context of optimization two of the most popular hybrid quantum-classical algorithms are the Quantum Approximate Optimization Algorithm (QAOA) and the VQE. QAOA is a hybrid quantum-classical algorithm that uses a quantum computer to approximate the ground state of a cost Hamiltonian and a classical computer to optimize the parameters of the quantum circuit that creates the approximation. This algorithm can be used to find approximate solutions to combinatorial optimization problems, such as the Maximum Cut problem, that are NP-hard and thus intractable for classical algorithms.

Another example of hybrid quantum-classical algorithms is the closely related VQE. VQE is a hybrid quantum-classical algorithm that uses a quantum computer to prepare a quantum state that encodes the solution to an optimization problem and a classical computer to optimize the parameters of the quantum state. VQE can be applied to a wide range of optimization problems, including the solution of the electronic structure of molecules, the simulation of quantum systems, portfolio optimization, and more.

VQE was first developed by Peruzzo et al. (Peruzzo et al., 2014), and in recent years has received significant attention from the research community specially oriented in the field of quantum computing for optimization purposes. This methodology computes the ground state energy of a Hamiltonian, a problem that is central to quantum physics, and through quantum computing can be applied to problems in other areas, like portfolio optimization.

Let $|\psi_i\rangle$ be an eigenvector of some matrix M that is invariant under transformation by M up to a scalar multiplicative constant, such that:

$$M|\psi_{\mathrm{i}}\rangle = \lambda_i \ |\psi_i\rangle$$

Where λ_i is the eigenvalue. Next, the Hamiltonian which describes the energetics of a quantum system is denoted as H. This is a Hermitian operator, meaning that it is equal to its own conjugate transpose, i.e.

$$\mathrm{H} = \mathrm{H}^{\dagger}$$

Following, the eigenvalues of this operator are real, such that $\lambda_i \in R$. By spectral theory, the operator may be expressed as

$$H = \sum\nolimits_{i=1}^{N} \lambda_i |\psi_i\rangle\langle\psi_i|$$

Moreover, the expectation of the operator H, on any arbitrary quantum state $|\psi\rangle$ gives

$$\langle H\rangle_{\psi} \equiv \langle\psi|H|\psi\rangle$$

The previous formulation can be expressed as a weighted sum of the eigenvectors of the Hermitian operator:

$$\langle H\rangle = \sum\nolimits_{i=1}^{N} \lambda_i |\langle\psi_i|\psi\rangle|^2.$$

Thus, it has been demonstrated that the average value of an observable in any state can be represented as a linear combination of the eigenvalues using H as the weights. This leads to

$$\lambda_{min} \leq \langle H\rangle_{\psi} = \langle\psi|H|\psi\rangle = \sum\nolimits_{i=1}^{N} \lambda_i \ |\langle\psi_i| \ \psi\rangle|\hat{}2.$$

From this expression, it is deducible that the expectation value of the wave function is the minimum eigenvalue associated with the Hamiltonian:

$$\langle\psi_{min}|H|\psi_{min}\rangle = \left\langle\psi_{min}|\lambda_{min}|\psi_{min}\right\rangle = \lambda_{min}$$

In other words, the objective of the VQE is to find a parameterization of $|\psi\rangle$, such that the expectation value of the Hamiltonian is minimized.

2.2 | Simultaneous perturbation stochastic approximation (SPSA)

The VQE algorithm can be implemented using a variety of optimization algorithms, such as gradient-based methods or stochastic optimization. The optimization problem is typically non-convex and may have multiple local minima, so the choice of optimization algorithm and the initialization of the classical variables can have a significant impact on the performance of the algorithm.

There are a wide range of classical optimization methods that can be implemented to work with VQE. However, Pellow-Jarman et al. (2021) compared the performance of 27 classical optimization methods, and demonstrated that the SPSA method appeared to perform the best under realistic noise levels. They also showed that other optimizers like COBYLA and Nelder–Mead unperformed notoriously in noisy environments. As such, SPSA was chosen in this work.

The idea behind SPSA is to use a sequence of random perturbations to the parameters of the function, and then use the resulting changes in the function value to estimate the gradient of the function (Spall, 1992). The goal is to find the parameters that minimize the function, which corresponds to the lowest eigenvalue of the Hamiltonian.

In order to implement the VQE-SPSA algorithm, one starts by preparing a trial state, which can be a parameterized quantum state, the parameters of which are then optimized using the SPSA algorithm. The expectation value of the Hamiltonian with respect to the trial state is then measured, and the results are used to update the parameters of the trial state. This process is repeated until the expectation value reaches a minimum, which corresponds to the lowest eigenvalue of the Hamiltonian. The methodology of SPSA is as follows:

Consider the following optimization with no constraints:

$$\min_{v \in R^n} S(v)$$

Where $S(v)$: $R^n \to R$ is differentiable with noisy measurements. Subsequently, an iterative process is computed as

$$v_{k+1} = v_k - a_k \widehat{p_k},$$

where $\widehat{p_k}$ is the gradient estimate of the objective function, $a_k \in R$ is a step size, and k accounts for the k^{th} iteration. Finally, the SPSA method is defined as follows:

$$\widehat{p_k} = \frac{S\left(v_k + u_k \zeta_k\right) - S\left(v_k - u_k \zeta_k\right)}{2u_k\left(\zeta_k\right)_i}$$

where ζ_k is a random perturbation vector, and u_k is a small positive scalar.

2.3 | Ansatz

In the context of quantum computing, the ansatz refers to the specific form of the quantum circuit to form the trial state in the quantum algorithm. There are several important groups of ansatz in quantum computing. The Unitary Coupled Cluster ansatz is based on the unitary coupled cluster method used in quantum chemistry, and it combines single-qubit rotations and multi-qubit gates to approximate the unitary coupled cluster wave function. This ansatz has been shown to provide high-fidelity solutions for quantum chemistry problems. The other type of ansatz is the Hardware Efficient ansatz which is designed to minimize the number of gates used in the circuit, making it more suitable for near-term quantum devices with limited resources. Meanwhile, this ansatz may have a lower representation power compared to other types of ansatz. Another type of ansatz is the Deep ansatz which represents a neural network, where the parameters of the ansatz are learned using the backpropagation algorithm. This type of ansatz can be highly expressive and provide good solutions for certain types of problems. However, it requires large amounts of quantum data for training and may be challenging to optimize.

The two-local circuit ansatz is composed of single-qubit rotations and two-qubit gates. Two-local circuits can be easily modified and adapted to different problems, making them a versatile choice for different applications. These circuits are more scalable than N-local circuits because they require fewer qubits and gates. This makes them more suitable for near-term quantum devices with limited resources. Further, the optimization problem of finding the best parameters for a two-local circuit ansatz is easier to solve than the optimization problem for an N-local circuit ansatz. As such, the two-local circuit ansatz was chosen for the hybrid quantum optimization algorithm (Figures 1 and 2).

2.4 | The CME MSM

Commonly, volatility is used as the proxy of risk. This is because of the normal distribution assumption in MPT. While in reality, risk and return don't always follow normal distributions. For example, when bearish news hits the market, it is likely to observe systematic risk that may skew the distribution or add fat tails to the distribution. A contrasting example is when an important event is scheduled on a known date and the outcome (usually binary) is unknown to the public, whereby opposite expectations and sentiments may grow in the market. This can lead to a bi-modal distribution in which

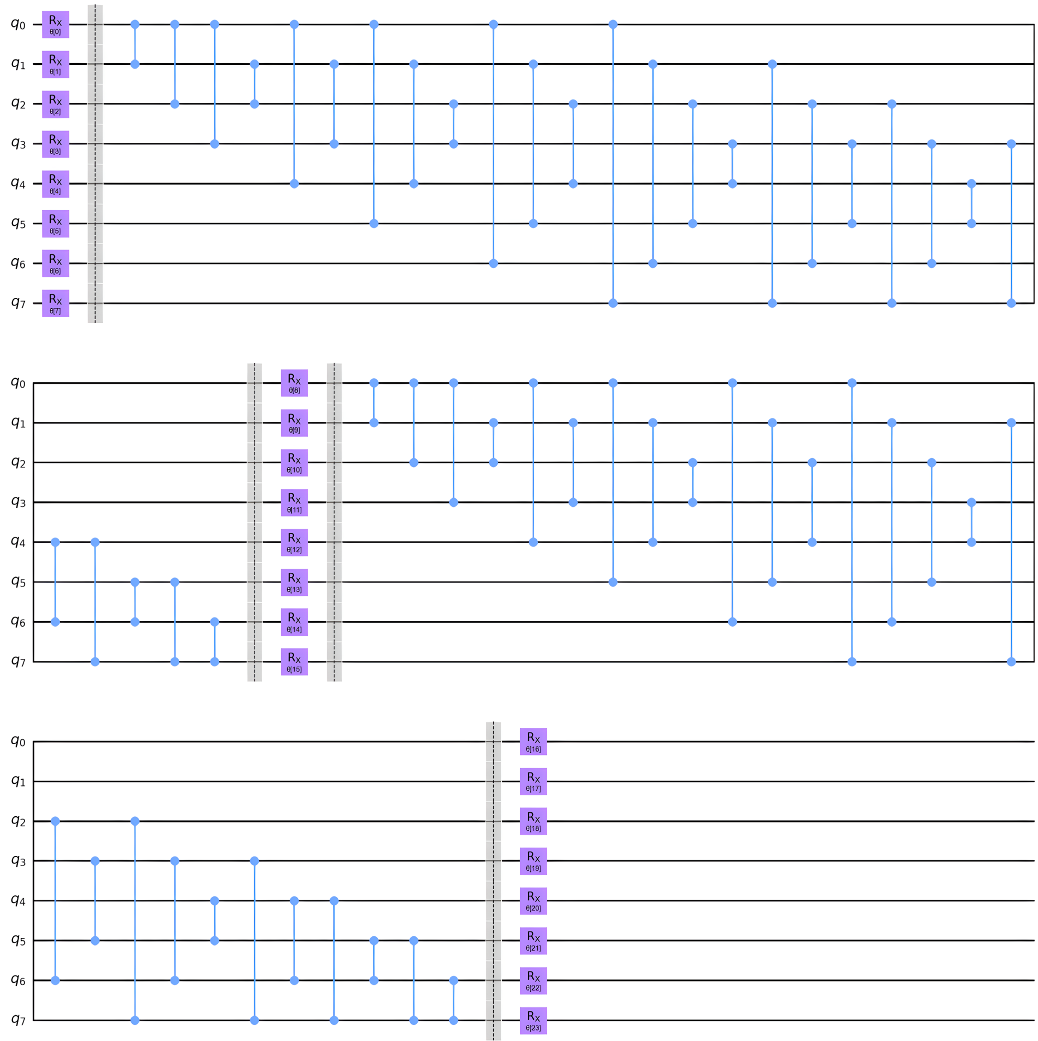

FIGURE 1 An illustration of the two-local circuit ansatz for 8 qubits used in this work, consisting of single-qubit rotations and two-qubit gates.

the two most possible expected outcomes gather around the peaks. In such circumstances, the assumption of normal distribution is not sophisticated enough.

The CME Group and 1QBit collaborated to develop the CME MSM to solve this problem. Based on the amount of money at risk in the open market, MSM aids in determining the historical market sentiment's size. The challenge is that the expected risk–return probability distributions cannot be directly observed. However, we can approximate some of these qualities by analyzing market behavior. A financial database serves as the main source of data for the MSM. Additionally, the MSM is created using CME End of Day (EOD) data. Blu Putnam (2020) found the key is to mix two normal distributions that each represent a different force in the market. One is the traditional distribution per MPT, while the other is enhanced by three vital factors that are particularly useful for understanding the shape of the probability distribution: (1) the changing relationship between put option trading volume and call option trading volume; (2) intraday market activity, particularly high/low spreads; and (3) indicated volatility from options pricing in comparison to historical volatility. The mixture distribution is then classified as one of the four sentiment states: Complacent, Balanced, Anxious and Conflicted. We can plot the hypothetical risk probability distribution of the MSM to visualize it.

The Complacent state is a narrow and tall distribution. It is rare and denotes a low level of market concern. The Balanced state is similar to the famous bell-shaped normal distribution, and is the most common market state, implying a typical amount of market anxiety. The Anxious state's distribution usually has fat tails or is highly skewed. It is uncommon and denotes a high level of market anxiety. The Conflicted state is a bi-modal distribution where

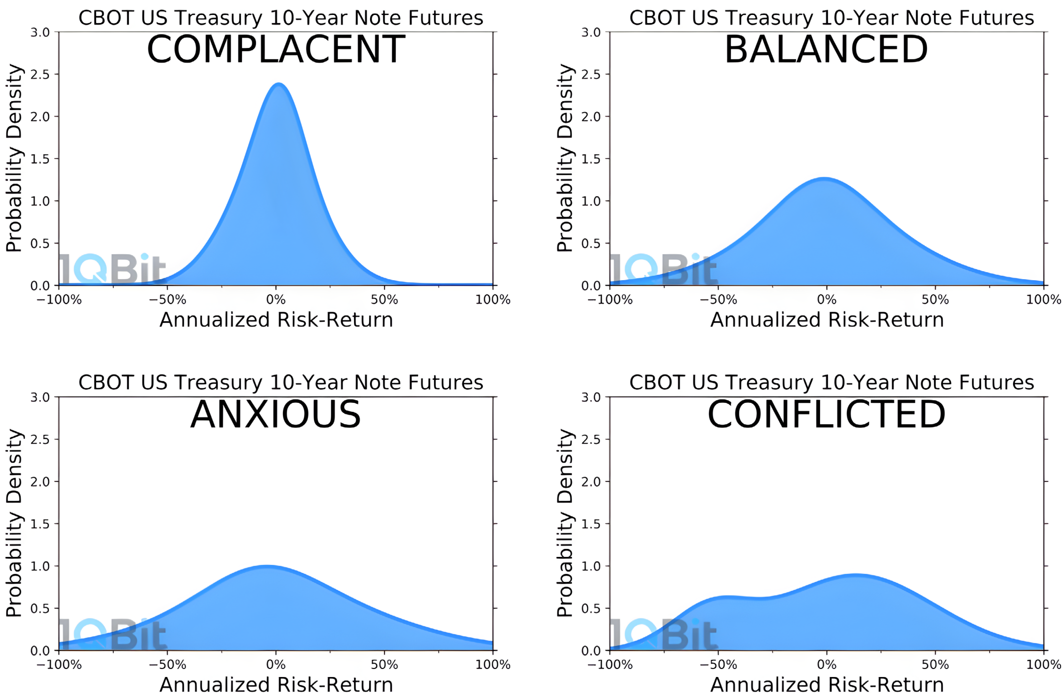

FIGURE 2 Illustrative plots of the 4 CME Market Sentiment Meter market sentiment states and their corresponding risk–return curves. The four states are labeled as Complacent, Balanced, Anxious, and Conflicted, where we note that the Conflicted state's distribution can be bimodal. Eight assets (E-mini S&P 500, Treasury 10-Year Note, Euro FX, Gold, WTI Crude Oil, Henry Hub Natural Gas, Corn, and Soybeans) have been monitored since 2012 by the MSM.

each mode is representative of different market opinions. It is extremely rare and signifies the presence of price gap anxiety.

The MSM has been proved to be effective in options trading. Kownatzki et al., (2021) implemented a trading strategy using strangles from E-Mini S&P 500 options. They used MSM state as their indicator to adjust trading directions (long or short). From Jan 2018 to Jan 2021, this strategy achieved over 200% capital gain with less than 50% maximum drawdown (Kownatzki et al., 2021).

2.5 | Portfolio optimization problem

The Quadratic Unconstrained Binary Optimization (QUBO) problem formulation has been key to many of the studies of applied quantum computing. Several works, such as Kochenberger, et al. (2014) and Anthony, et al. (2017), have shown that the QUBO problem can embrace an important variety of diverse combinatorial optimization problems found in industry. The following problem formulation can be solved with hybrid quantum algorithms to solve for the optimal portfolio, by selecting some subset from n total assets that can be chosen from, as:

$$\max_{x \in \{0,1\}^n} \left(\mu\hat{}T\, x - q\, x\hat{}T\, \sigma x \right)$$

$$\text{subject to: } 1^T x = B$$

In this optimization problem:

- $x \in \{0, 1\}^n$ is the vector of binary decision variables, which indicate which assets to pick ($x[i] = 1$) and which not to pick ($x[i] = 0$).
- $\mu \in R^n$ is the vector of means for the daily percent returns of each asset.
- $\sigma \in R^{n \times n}$ is the covariances matrix of the daily percent returns of the assets.

- $q > 0$ is the risk factor.
- B is the number of assets to be selected out of n total assets.

The equality constraint above, $1^T x = B$, is turned into a penalty by subtracting $\left(1^T x - B\right)^2$ scaled by a penalty parameter in the objective function.

2.6 | Optimization problem variations

Three different variations of the base portfolio optimization problem that will serve as the classical references of and bases of the hybrid quantum portfolio optimization problem.

2.6.1 | Portfolio optimization problem formulation 1

The first formulation modifies the QUBO by taking into account specific levels of risk, q, subject to the weights of the portfolio, which vary from 7% to 93%, sum equal to one. Note that the range of the weights have a broad range chosen, to allow the optimization algorithm to best explore a large space. In practice, the bounds can be changed based on insights into the assets and the needs of the portfolio, as one may not to overweigh a particular asset in their portfolio. The purpose of this problem formulation is to set a benchmark for the next results from the other problem formulations. The formulation goes as follows:

$$\max_{x \in \{0.07, 0.93\}^n} \left(\mu^T x - q x^T \sigma x\right)$$

$$\text{subject to: } 1^T x = 1$$

Where:

- $x \in \{0.07, 0.93\}^n$ is the vector of continuous decision variables, which indicates the weight of each asset.
- $\mu \in R^n$ is the vector of means for the daily percent returns of each asset.
- $\sigma \in R^{n \times n}$ is the covariance matrix of the daily percent returns of the assets.
- $q > 0$ is the risk factor.

2.6.2 | Portfolio optimization problem formulation 2

In this second formulation different levels of risk are implemented through q and we introduced the MSM variable, α. The weights of the portfolio vary from 7% to 93%, sum equal to one. Note that the range of the weights have a broad range chosen, to allow the optimization algorithm to best explore a large space. In practice, the bounds can be changed based on insights into the assets and the needs of the portfolio, as one may not to overweigh a particular asset in their portfolio. The goal of this problem formulation is to assess the impact of the MSM variable with different levels of risk on the portfolio balance. The formulation is shown as follows:

$$\max_{x \in \{0.07, 0.93\}^n} \left(\mu^T x - q x^T \sigma x - \alpha x^T x\right)$$

$$\text{subject to: } 1^T x = 1.$$

Where:

- $x \in \{0.07, 0.93\}^n$ is the vector of continuous decision variables, which indicates the weight of each asset.
- $\mu \in R^n$ is the vector of means for the daily percent returns of each asset.
- $\sigma \in R^{n \times n}$ is the covariance matrix of the daily percent returns of the assets.

- $q > 0$ is the risk factor.
- $\alpha > 0$ is the risk factor that equals the mean of the MSM states from the last period.
- MSM states to numerical values: Complacent: 0, Balanced: 0.1, Anxious: 0.4, Conflicted: 0.5.

2.6.3 | Portfolio optimization problem formulation 3

In the final formulation, we set the QUBO amenable to VQE, and we also include both risk factors, q as the risk appetite and α as the MSM variable. A vector of binary decision variables is employed. The constraint remains the same. The weights are set to be equal among the chosen assets. Continuous decision variables can in principle be used to calculate weights directly, but require significant reformulation, and are left for a future work. In practice, one can assign weights after the asset selection through various methods, like risk-parity analysis. The goal of this problem formulation is to assess if the hybrid quantum optimization algorithm with the MSM variable can perform better than the previous formulations. The mathematical representation of the problem is shown below:

$$\max_{x \in \{0,1\}^n} \left(\mu^T x - q x^T \sigma x - \alpha x^T x\right)$$

$$\text{subject to: } 1^T x = B.$$

Where:

- $x \in \{0,1\}^n$ is the vector of binary decision variables, which indicate which assets to pick ($x[i] = 1$) and which not to pick ($x[i] = 0$).
- $\mu \in R^n$ is the vector of means for the daily percent returns of each asset.
- $\sigma \in R^{n \times n}$ is the covariance matrix of the daily percent returns of the assets.
- $q > 0$ is the risk factor.
- $\alpha > 0$ is the risk factor that equals the mean of the MSM states from the last period.
- B is the number of assets to be selected out of n total assets.
- MSM states to numerical values: Complacent: 0, Balanced: 0.1, Anxious: 0.4, Conflicted: 0.5

2.6.4 | Data

The eight different assets evaluated were the CME E-mini S&P 500 e-mini futures, CBOT 10-Year Treasury Note futures, CME Euro FX EUR/USD futures, COMEX Gold futures, NYMEX WTI Crude Oil futures, NYMEX Henry Hub Natural Gas, CBOT Corn futures, and CBOT Soybeans futures over the period of January 4th 2012 to June 3th 2022, due to data availability of the MSM. To visualize the relationship between the asset price behaviors, the covariance of the assets over this period is plotted as a heatmap in Figure 3.

Trade Date, Most Active Futures Settlement Price, and Last Trade Date data were used for the above assets. The notional value of the futures contracts were calculated by multiplying the contract units with the settlement price of the assets, and it was established the level of 10% of the notional value as the margin of the contract. This can vary by asset, but this is a simplification taken for this work, to put focus onto the algorithms involved and the additions of the MSM variable into the objective functions. The contract units are as follows:

- Gold: 100 troy ounces
- Soybeans: 5000 bushels
- Treasuries: par on the basis of 100 points
- Nat-Gas: 10,000 MMBtu
- Oil: 1000 barrels
- Euro: 125,000 Euro (U.S. dollars and cents per Euro increment)
- Corn: 5000 bushel
- Equities: 50

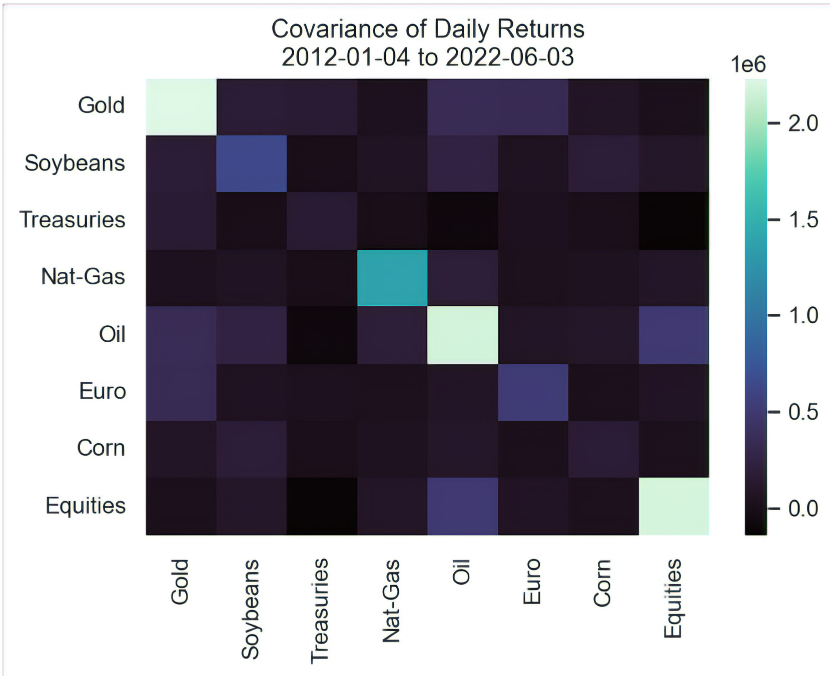

FIGURE 3 The covariance of daily returns for the 8 futures products from January 4 2012 to June 3 2022.

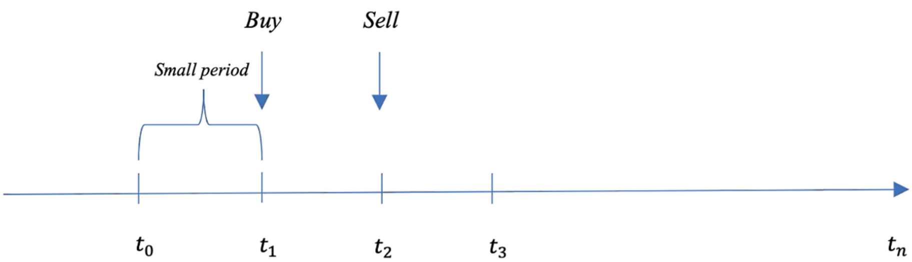

FIGURE 4 Illustration of the small time periods defined by the overlapping times in which the futures contracts were active. Over the period of 2012 to 2022, there are 342 small periods, where 193 small periods extend over a period greater than 7 days.

As each asset has a different last trade date, the whole time period has been divided into small time periods such that all the assets are active within the small period (Figure 4). There are 342 small periods, where 193 small periods have more than 7 days. The length of these small periods are determined by the expiry dates for each futures contract, as a way to get around effects of rolling futures contract expiries, as we are working with most active futures contract data.

3 | RESULTS

3.1 | Portfolio optimization formulation 1

The portfolio returns over the ten-year period for Portfolio Optimization Formulation 1 are plotted in Figure 5 for varying values of q.

3.2 | Portfolio optimization formulation 2

The portfolio returns over the ten-year period for Portfolio Optimization Formulation 2 are plotted in Figure 6 for varying values of q.

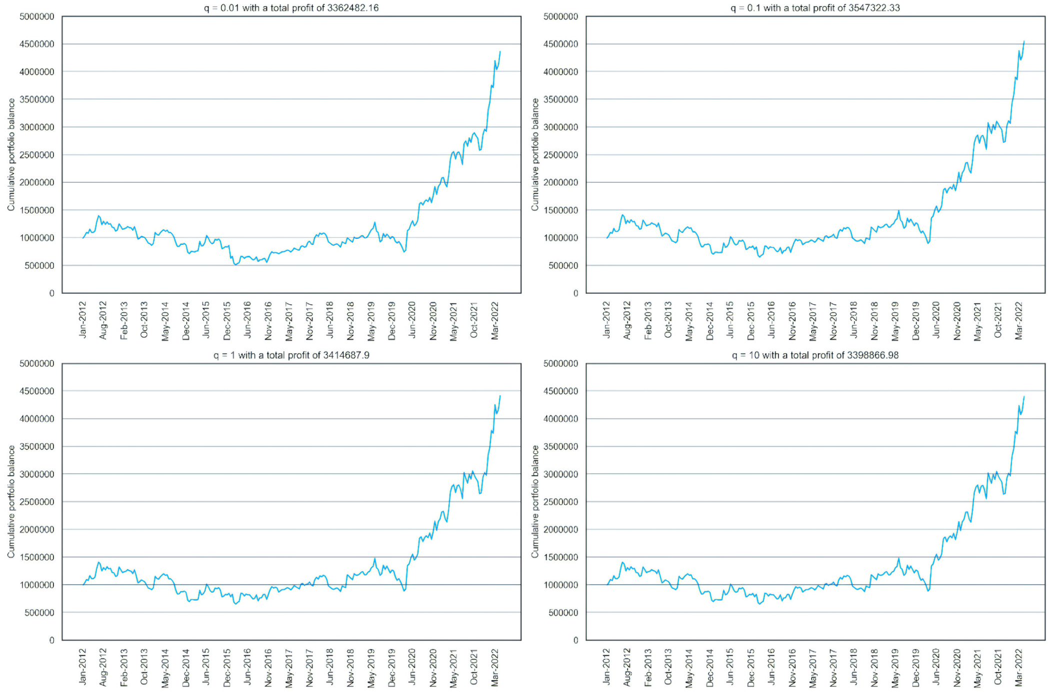

FIGURE 5 Returns of Portfolio optimization formulation 1, for varying values of q.

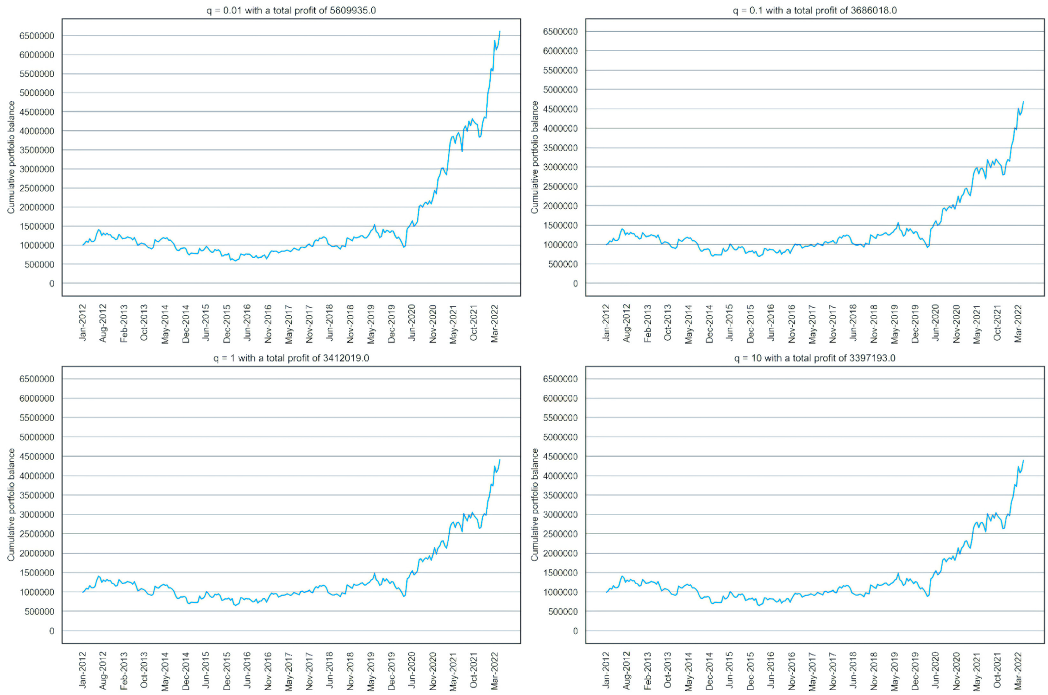

FIGURE 6 Returns of portfolio Optimization Formulation 2, for varying values of q.

3.3 | Portfolio optimization formulation 3

The portfolio returns over the ten-year period for Portfolio Optimization Formulation 3 are plotted in Figure 7 for varying values of q.

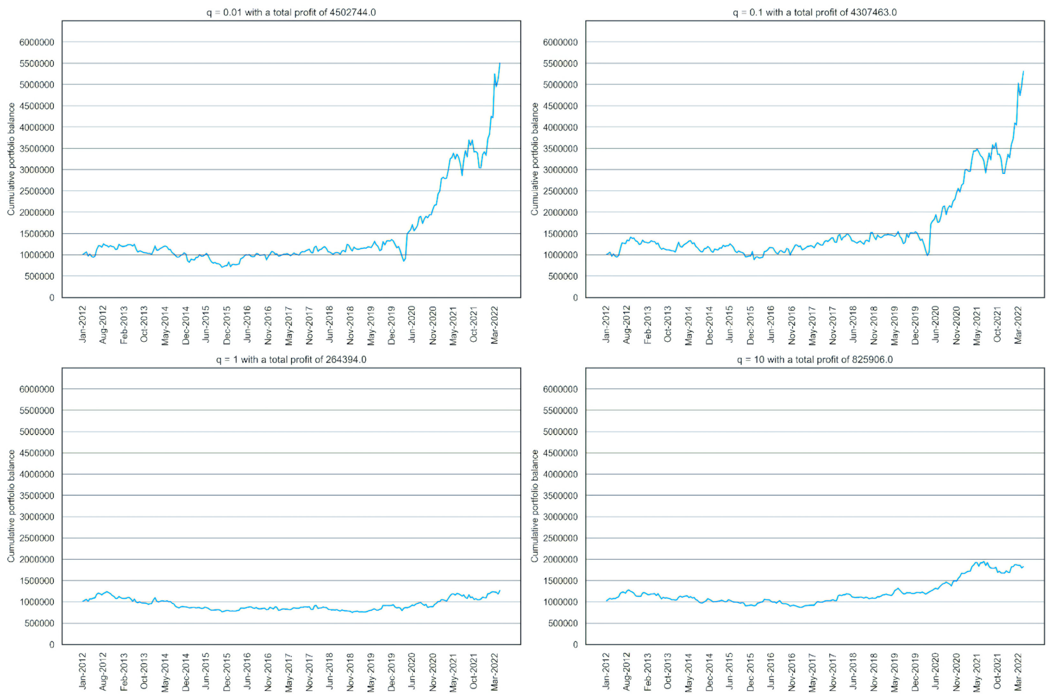

FIGURE 7 Returns of portfolio optimization formulation 3, for varying values of q.

4 | DISCUSSION

4.1 | Portfolio optimization formulation 1

In analyzing these results, we see the evolution of the portfolio over time in different scenarios. In order to understand the impact of a different level of risk tolerance, q, four values of q were evaluated (Figure 5).

For $q = .01$, the total cumulative portfolio balance reached a total profit of USD 3,362,482.16, or a return of 336.24%. The lowest level of this portfolio was between December 2015 and June 2016.

For $q = .1$, the total cumulative portfolio balance reached a total profit of USD 3,547,322.33, which represents a return of 354.73%. The lowest level of this portfolio was in the first half of 2016.

For $q = 1$, the total cumulative portfolio balance reached a total profit of USD 3,414,687.90, or a return of 341.47%. Paradoxically, a higher level of risk, q, did not result in a higher return of the portfolio. Meanwhile, as in the previous scenarios the lowest level of this portfolio was in the first half of 2016.

For $q = 10$, the total cumulative portfolio balance reached a total profit of USD 3,414,687.90, or a return of 341.47%. As in the previous case, an even higher level of q gave the second lowest return of the portfolio.

The two asset classes with more participation in the portfolio were Treasuries and Corn, and the least attractive assets in the portfolio were Gold and Oil. It is important to notice that during the years of the pandemic, December 2019 to October 2021, the algorithm did not choose almost anything from equities in the composition of the portfolio.

4.2 | Portfolio optimization formulation 2

For this formulation, we once more see the development of the portfolio over the aforementioned period of time for four levels of risk tolerance, q, in addition to the MSM information for the case of continuous variables.

For $q = .01$, the total cumulative portfolio balance reached a total profit of USD 5,609,935, or a total return of 560.99%. The lowest level of this portfolio was between December 2015 and June 2016.

For $q = .1$, the total cumulative portfolio balance reached a total profit of USD 3,686,018, or a total return of 368.60%. The lowest level of this portfolio was in the first half of 2016. Again, this result is paradoxical considering a higher level

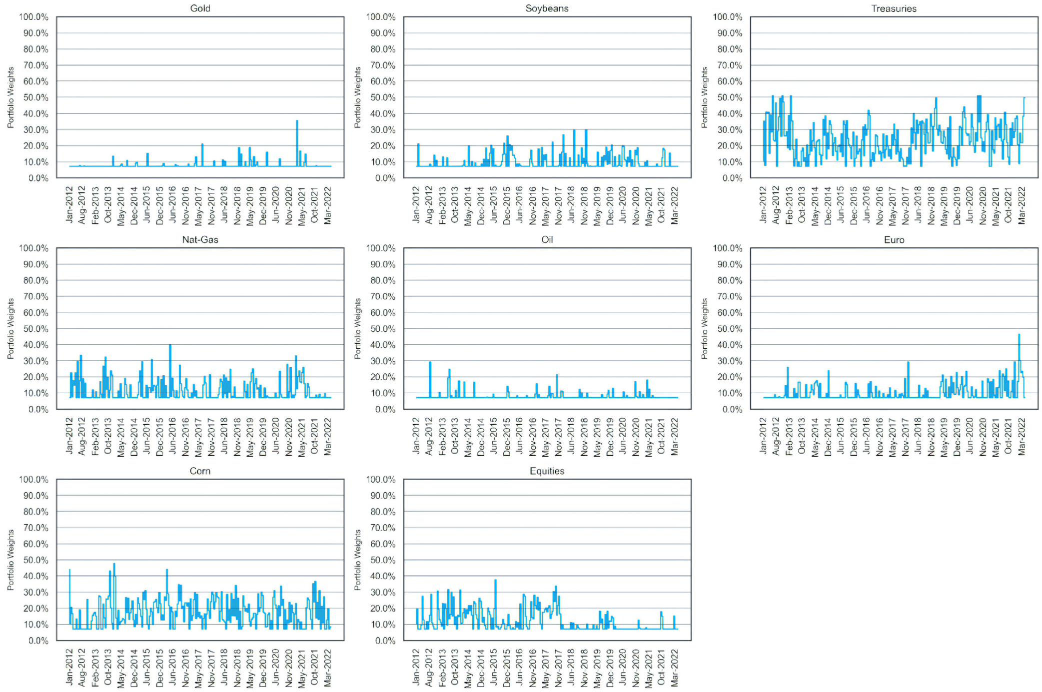

FIGURE 8 Portfolio Weights for each asset over time for Portfolio Optimization Formulation 2.

of risk did not result in a higher return in comparison with the previous scenario of $q = 0.1$. However, it is 13.87% higher than the same scenario of the previous section.

For $q = 1$, the total cumulative portfolio balance reached a total profit of USD 3,412,019, or a total return of 341.20%. This result is interesting because it is 0.27% lower than the same result shown in section 4.1 despite the inclusion of the MSM.

For $q = 10$, the total cumulative portfolio balance reached a total profit of USD 3,397,139, or a total return of 339.71%. This result is 0.18% lower than Portfolio Optimization Formulation 1.

In this section it has been shown that the inclusion of the MSM variable in the portfolio optimization makes a remarkable positive impact on the first two levels of risk, translated into much higher returns on the portfolio, and almost no change in the last two higher levels of risk.

Regarding the portfolio allocation, in this case the portfolio is slightly more balanced than the previous one. It shows more allocation in Gold, Soybeans and Oil. While on the other side, it shows a slightly less allocation on Treasuries and Corn (Figure 8).

4.3 | Portfolio optimization formulation 3

In this final formulation, the VQE-SPSA algorithm is applied. In order to make the results fairly comparable against classical optimization algorithms this analysis also was performed with the same levels of risk tolerance $q = .01, 0.10, 1, 10$ and in the same periods of time. Moreover, it is important to remember that this type of portfolio optimization outputs a binary decision variable, 0 or 1, where 0 means not selecting the asset and 1 choosing the asset. Thereby, taking into account the budget constraint, which in this case is restricted to five assets, the weights of the portfolio are estimated by equally dividing the selected assets by five, resulting in an equally weighted portfolio of 20%. Also, the hybrid quantum optimization algorithm incorporates the MSM variable. Here are the results:

For $q = .01$, the total cumulative portfolio balance reached a total profit of USD 4,502,744, or a total return of 450.27%. This is better than formulation 1 but not as good as formulation 2.

For $q = .1$, the total cumulative portfolio balance reached a total profit of USD 4,307,463, or a total return of 430.75%. Interestingly, this is better than both formulation 1 and 2.

For $q = 1$, the total cumulative portfolio balance reached a total profit of USD 264,394, or a total return of 26.44%.

For $q = 10$, the total cumulative portfolio balance reached a total profit of USD 825,906, or a total return of 82.59%.

In this formulation. The algorithm allocates marginally more capital into Equities and Euro; in the case of Natural Gas allocates the same but rather than allocating around June 2020 like the previous results it assigns the capital close to June 2018. On the other hand, this formulation has less participation of Oil in the portfolio in comparison with the previous result.

4.4 | Performance metrics for portfolio formulations

For completeness, tabulated below is the performance metrics for Portfolio Formulation 2 with $q = 0.01$, which is the best performing model overall. Generally, changes in the risk parameter result in expected behavior. Examining 2013 as an example, larger risk parameters can result in lower relative profit, compared to smaller values. This is because a larger risk parameter places a larger weight on the covariance in the optimization problem, as opposed to a lower risk parameter which would be geared towards maximizing profit as opposed to minimizing risk, using covariance as a probe for risk. Note that different formulations are all sensitive to changes in q, however, some can appear more or less sensitive, due to the addition of the market risk sentiment metric to the objective function. However, the overall behavior with q is common between all the formulations above, where Formulation 2 was chosen as an illustrative example, as per the tables below.

Formulation 3 with $q = 0.01$.

Year	Mean	Standard deviation	Maximum drawdown	Max profit	Relative profit
2012	1.52%	14.11%	24.91%	32.08%	16.02%
2013	−2.43%	11.20%	20.79%	25.36%	−27.81%
2014	−0.20%	12.72%	20.91%	40.92%	1.02%
2015	−1.02%	11.76%	25.03%	28.07%	−0.35%
2016	0.38%	12.13%	34.88%	25.95%	37.06%
2017	1.18%	6.17%	11.15%	14.59%	15.25%
2018	1.27%	14.68%	28.17%	46.56%	1.28%
2019	2.27%	16.72%	38.99%	37.41%	13.33%
2020	7.47%	29.10%	29.78%	90.39%	72.64%
2021	13.83%	46.99%	71.87%	116.78%	53.62%
2022	47.96%	66.04%	49.12%	160.58%	51.66%

4.5 | Case study—2022 market correction

2022 was an exceptional year, marked by a series of significant events that had a profound impact on the global market. The high inflation rates observed in both the US and Europe, along with the ongoing Russia-Ukraine conflict and the increasingly severe effects of climate change, all contributed to a highly uncertain economic landscape. As a result, managing portfolios became an exceptionally challenging task, requiring careful consideration of a range of complex factors.

In the first half of 2022 (the last period of our simulation), S&P 500 Index tumbled about 13%, Euro dropped 5%, gold went up 1.6%, and natural gas had a hefty return of 140%. Despite these varied performances, our portfolio realized a return of 153% with a volatility of less than 10%, which outperforms every single product. During times of heightened market stress, our key to success is to adjust the positions based on market sentiment.

The rally of natural gas in the first half of 2022 serves as an excellent example of our investment strategy's success. During the period from March 22 to March 28, 2022, our portfolio consisted of gold, corn, soybeans, equities, and treasuries, each with an equal weight allocation. On March 29, 2022, the sentiment surrounding natural gas shifted to a ‘complacent’, which indicated the lowest level of risk among the four states. This change in sentiment strongly signaled a positive market outlook for natural gas. Taking into account the mean and standard deviation as well, our system made the decision to drop soybeans and add natural gas to the portfolio.

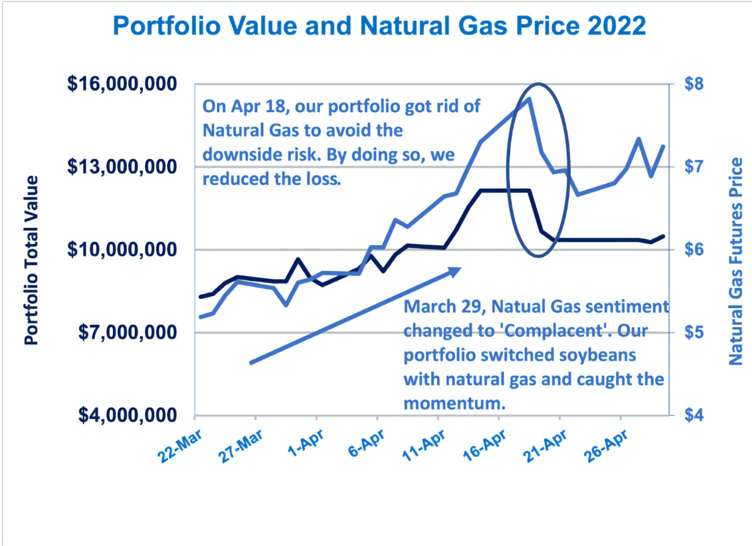

FIGURE 9 Portfolio behavior and natural gas futures price around the case study period.

From March 29 to April 18, 2022, natural gas experienced an impressive gain of 46.7%, while soybeans only increased by 4.4% during the same period. This 20-day period resulted in a remarkable 24% increase in the total value of our portfolio. However, on April 18, the sentiment surrounding natural gas turned 'anxious', indicating a higher level of risk associated with this asset. As a result, we promptly removed natural gas from our portfolio and reallocated our positions. Subsequently, from April 18 to April 27, 2022, natural gas witnessed a decline of 6.15% while our portfolio value only shrank by 5.7%. Once again, by reducing our exposure to risky assets, we successfully improved the performance of our portfolio.

Overall, these strategic adjustments based on market sentiment, mean, and standard deviation allowed us to capitalize on the positive market sentiment surrounding natural gas, resulting in significant gains for our portfolio. The reduced exposure to riskier assets further contributed to enhancing the overall performance of our investment strategy (Figure 9).

5 | CONCLUSION AND FUTURE DIRECTIONS

The present work evaluated the inclusion of market sentiment in hybrid quantum portfolio optimization solutions, by way of the CME MSM. It has been demonstrated that the inclusion of this information through a parameter that takes into account the market sentiment of eight different futures asset classes inside the portfolio optimization scenario improved the overall performance of the portfolio for different levels of risk tolerance. Thus, adding the MSM could allow for better performing results in diverse strategies for different market scenarios.

To this end, the VQE algorithm using the SPSA optimizer, selected five optimal stocks within the QUBO formulations, subject to the budget constraint and constructed a portfolio that outperformed a purely classical analogue formulation.

From this perspective, the addition of the MSM into the hybrid quantum portfolio optimization improved the results of the for varying levels of risk tolerance. Thus, the inclusions of a market sentiment metric through the MSM allowed portfolio optimization formulations 2 and 3 to outperform the formulations which did not include the MSM.

Now, a lower level of risk tolerance (and thus a higher risk) did not always translate to higher returns, however the inclusion of the MSM allows for one to capitalize on market sentiment information, to improve gains in the portfolio, as elaborated upon in the case study around natural gas futures. That is, the MSM allowed for reduced exposure to riskier assets, which further contributed to enhancing the overall performance of the investment strategy across the portfolio formulations that included the MSM in the classical and hybrid-quantum formulations.

Furthermore, this numerical experiment did not consider transaction costs, and therefore it is a matter of future study within the presented formulations. On the other hand, it is also a matter of further research to perform a quantum portfolio optimization with continuous decision variables. Additionally, another topic of further research is to include other assets besides Gold, Soybeans, Treasuries, Natural Gas, Oil, Euro, Corn and Equities on the MSM variable and assess possible impacts on the portfolio.

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