Introductory physics: The new scholasticism

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Most introductory physics textbooks neglect air resistance in situations where an astute student can observe that it dominates the dynamics. We give examples from many books. Using dimensional analysis we discuss how to estimate the relative importance of air resistance and gravity. The discussion can be used to mitigate the baleful influence of these textbooks. Incorrectly neglecting air resistance is one of their many unphysical teachings. Shouldn't a physics textbook teach correct physics?

I. THE PROBLEM

You teach introductory physics, say Newtonian mechanics. You want students to use everyday experience to develop physical intuition. On occasion you get a student who *does* see her or his experiences as a physics laboratory. She avidly observes sporting events, street life, and kitchen appliances for insight into how macroscopic objects behave. Then she encounters problem 4–34 from Halliday, Resnick & Walker (see the list in Table I): to analyze a golf drive ignoring air resistance; *i.e.,* with the ball traveling on a parabolic trajectory.

The observant student knows—from exploring the world—that golf drives rise quickly and almost straight and drop parabolically only near the end. This student learns that the world described by her physics textbook is not the *real* world and that careful observation is irrelevant to physics.

Contrast the experience of the curious student with that of a student who parrots equations and regurgitates textbook paragraphs. This student is untroubled by the golf problem or its variants listed in Table I because he knows the easily memorized 'fact' that all trajectories are parabolae. The parrot student correctly answers this problem, confirming that physics means memorizing disembodied mathematical facts. Observant student: 0; parrot student: 1.

Our objection is not only to the treatment of air resistance. Most textbooks leave many assumptions unjustified and unexamined: be they massless, inextensible strings, frictionless pulleys, or pointlike particles. Air resistance, or its absence, is merely one glaring example, which we now discuss in detail.

II. DIMENSIONAL ANALYSIS

You are a physicist. You begin any solution by asking not what equation matches but what physics is relevant and what approximations to use. Studying the flight of a golf ball, *as a start* you might neglect air resistance; then from dimensions you could argue that the velocity of a golf ball hit a distance L (say, $250\,\mathrm{m}$) is roughly

$$v \sim \sqrt{gL} \sim (10\,\mathrm{m\,s^{-2}} \times 250\,\mathrm{m})^{1/2} \sim 50\,\mathrm{m\,s^{-1}}. \tag{1}$$

You check this assumption by comparing the gravitational force, about $0.5\,\mathrm{N}$, to the drag force f (from air resistance). By dimensional analysis again,

$$f \sim \rho A v^2, \tag{2}$$

where ρ is the density of air and A is the cross-sectional area of the golf ball. The drag is due to the power fed into the turbulent eddies that eventually become small eddies gobbled by viscosity. The analysis of the turbulence is difficult, if not impossible, but dimensional analysis gives the correct form of the drag force.

With this result and reasonable values $\rho \sim\sim 1\,\mathrm{kg\,m^{-3}}$ and $A \sim 10^{-3}\,\mathrm{m^2}$, the drag force is

$$f \sim 1\,\mathrm{kg\,m^{-3}} \times 10^{-3}\,\mathrm{m^2} \times (50\,\mathrm{m\,s^{-1}})^2 \sim 2.5\,\mathrm{N}. \tag{3}$$

TABLE I: Textbook prices, masses, and problematic air-resistance problems.

Author/Title	Edition	Mass	Price[a]	Problematic problems
ONE OF SEVERAL VOLUMES				
D. Halliday, R. Resnick & J. Walker, *Fundamentals of Physics*[b]	7th ed., volume 1, John Wiley, 2005, ISBN 0-471-42959-7	3.4 lb	US$93.95	4–34, 4–26 (tennis), 4–28 (soccer), and 4–37 (baseball)
R. Resnick, D. Halliday & K. S. Krane, *Physics*[c]	5th ed. volume 1, John Wiley, 2002, ISBN 0-471-32057-9	3.4	93.95	
D. C. Giancoli, *Physics for Scientists & Engineers*	3rd ed., volume 1, Prentice Hall, 2000, ISBN 0-13-021518-X	3.1	102.67	3–22 (skiing), 3–31 (football), and 3–82 (baseball)
A. F. Rex & M. Jackson, *Integrated Physics and Calculus*	volume 1, Addison Wesley, 2000, ISBN 0-201-47396-8	2.6	73.00	4–21 through 4–24
P. A. Tipler, *Physics for Scientists and Engineers*	4th ed., volume 1, W. H. Freeman, 1999, ISBN 1-57259-491-8	3.6	84.95	3–88 (baseball)
COMPLETE IN ONE VOLUME				
K. Cummings, P. W. Laws, E. F. Redish & P. J. Cooney, *Understanding Physics*[d]	Wiley, 2004, ISBN 0-471-37099-1	6.4	138.95	5–8 (golf), 5–15 (football), 5–18 (soccer), and 5–23 (baseball)
P. M. Fishbane, S. G. Gasiorowicz & S. T. Thornton, *Physics for Scientists and Engineers with Modern Physics*	3rd ed., Pearson Prentice Hall, 2005, ISBN 0-13-035299-3	6.7	160.00	3–44 (football) and 3–71 (golf)
D. C. Giancoli, *Physics: Principles with Applications*	6th ed., Prentice Hall, 2005, ISBN 0-13-060620-0	5.2	146.67	3–22 (football), 3–29 (football), 3–62 (baseball), and 3–68 (skiing)
R. D. Knight, *Physics for Scientists and Engineers with Modern Physics: A Strategic Approach*	Addison–Wesley, 2004, ISBN 0-8053-8685-8	9.9	160.00	6–29 (tennis) and 6–38 (football)
R. A. Serway & J. W. Jewett Jr., *Principles of Physics: A Calculus-Based Text*	3rd ed., Brooks/Cole, 2002, ISBN 0-03-027157-6	6.0	148.95[e]	3–15 (football) and 3–45 (baseball)
J. S. Walker, *Physics*	2nd ed., Pearson Prentice Hall, 2004, ISBN 0-13-101416-1	5.7	146.67	4–26 (soccer), 4–31 (golf), 4–33 (football), 4–42 (golf), 4–46 (soccer), and 4–48 (golf)
H. D. Young & R. A. Freedman, *University Physics with Modern Physics*	11th ed., Addison Wesley, 2004, ISBN 0-8053-8684-X	7.4	160.00	3.15 (football), 3.17 (flare gun), 3.19 (baseball), and 3.85 (soccer)

[a]Mass and price from `amazon.com` on 2006-03-28 using the `isbn2info.py` script in the physics/0412107 source.

[b]At least Halliday *et al.* mention v^2 drag.

[c]describes the drag force on a basketball or a skydiver as proportional to v rather than v^2 (p. 72)!

[d]discusses air resistance but not in relation to the relevant problems, where it merely mentions that 'In some of the problems, exclusion of the effects of the air is unwarranted but helps simplify the calculations' (p. 131).

[e]from price sticker on the back (no Amazon list price).

Since the gravitational force is roughly $0.5\,\mathrm{N}$, the drag force is *5 times the gravitational force!* So you reject the original assumption of no drag and numerically integrate to approximate the trajectory (easy with spreadsheets used by students in other contexts). An early conclusion of your approach—the physical approach—is that the golf ball must be hit much faster than $50\,\mathrm{m\,s^{-1}}$, which is the speed it would have to be hit if there were no drag. Therefore the drag is even larger than the estimate in (3).

III.   DIMENSIONLESS RATIOS

As a physicist, you also know about the physical insight provided by dimensionless ratios. The interesting ratio here is of drag force to gravitational force:

$$\frac{\text{drag force}}{\text{gravitational force}} \sim \frac{\rho_{\text{air}}Av^2}{mg}. \tag{4}$$

If d is the diameter of the golf ball, then its cross-sectional area is $A \sim d^2$ and its mass is $m \sim \rho_{\text{ball}}d^3$. Furthermore the v^2 in the drag force is roughly gL, from the estimate (1). So, after dividing out the common factor of g,

$$\frac{\text{drag force}}{\text{gravitational force}} \sim \frac{\rho_{\text{air}}Ld^2}{\rho_{\text{ball}}d^3}. \tag{5}$$

The denominator is mass of the golf ball. What about the numerator? Since Ld^2 is roughly the volume of air that the golf ball sweeps out, the numerator is

$$\rho_{\text{air}}Ld^2 \sim \rho_{\text{air}} \times \text{volume swept out}, \tag{6}$$

which is the mass of air that the golf ball sweeps out. So

$$\frac{\text{drag force}}{\text{gravitational force}} \sim \frac{\text{mass of air golf ball sweeps out}}{\text{mass of golf ball}}, \tag{7}$$

or in English:

Air resistance is significant if the golf ball sweeps out a mass comparable to itself.

In the ratio (5), two powers of the diameter d divide out:

$$\frac{\text{drag force}}{\text{gravitational force}} \sim \frac{\rho_{\text{air}}}{\rho_{\text{ball}}} \times \frac{L}{d}. \tag{8}$$

The density ratio is $\sim 10^{-3}$: Golf balls roughly float on water so $\rho_{\text{ball}} \sim 10^3\,\text{kg}\,\text{m}^{-3}$. The length ratio is roughly

$$\frac{L}{d} \sim \frac{250\,\text{m}}{4\,\text{cm}} \sim 6000, \tag{9}$$

where $d \sim 4\,\text{cm}$ is a typical golf-ball diameter and $L \sim 250\,\text{m}$ is the range used in (1). The product of the density and length ratios is 6, which echoes the ratio of 5 between drag and gravity estimated after (3).

This argument—a quantitative argument about the relative importance of gravity and air resistance—uses neither advanced mathematics nor abstruse concepts. Yet it is sophisticated and leads to a general insight connecting the importance of air resistance and the ratio of displaced masses. It is sophisticated because it uses the physicist's special tools: estimation and dimensional analysis. This argument appears in *no* introductory text. Why not? Not because students find it difficult, for it applies simple principles and mathematics to familiar objects. Not because such arguments have failed, for they have not even been tried. Perhaps it does not appear in introductory textbooks because it refutes so many standard problems.

Of the textbooks in Table I that come in a single volume (second portion of the table), the average price is \$152 and the average weight is 6.8 pounds. We should not teach from, take problems from, or ask students to buy obese, expensive books filled with bogus physical analyses of easily observed phenomena. If you agree with this principle, then you will not voluntarily assign or use the textbooks in Table I nor others that incorrectly analyze golf shots, baseball hits, tennis serves, downhill skiing, or any phenomena where air resistance is important. *The requirement that physics textbooks teach correct physics excludes most books on the market.*

One book treats air resistance correctly: *Matter & Interactions I: Modern Mechanics*[1] (pp. 180–188). In problem 5.3, students numerically integrate the trajectory of a baseball with and without air resistance. The authors point out that 'the effect is surprisingly large—about a factor of 2!' The problem even compares the effect of air resistance at high altitude (Denver) with its effect at sea level. This book and the companion volume on electricity and magnetism, which have been reviewed by Titus,[2] are highly recommended to teachers who want to return physics to introductory physics.

IV.   CALCULUS TEXTBOOKS

Physics textbooks often ignore air resistance. Articles on teaching often perpetuate an alternative misconception: that drag is linear in v. For example, Warburton and Wang[3] analyze the large-velocity limit saying: 'For simplicity, we will assume linear air resistance...'! The mathematics textbooks, perhaps where many of us first saw analyses of air resistance, are hardly role models. One author (SM) was mistreated by the following argument in high-school calculus. The author wrote the drag force as a Taylor series in v:

$$f = c_1 v + c_2 v^2 + c_3 v^3 + \cdots. \tag{10}$$

Following the usual practice with $\sin x \approx x - x^3/3 + \cdots$, he argued that the term with the lowest power of v, in other words $c_1 v$, is the dominant term. Therefore

$$f \propto v \qquad \text{(approximately)}. \tag{11}$$

The gentle reader will enjoy finding the flaw in this argument.

In a current calculus textbook,[4] the introduction offers heartening news (p. viii) '...we have not compromised our belief that the fundamental goal of a calculus book is to prepare students to enter the scientific community.' Sadly, the section on First Order Differential Equations (the usual location of this unphysical result) contains the ominous subhead 'Resistance proportional to velocity' (p. 534). Therein the reader learns:

> In some cases it makes sense to assume that, other forces being absent, the resistance encountered by a moving object, like a car coasting to a stop, is proportional to the object's velocity. The slower the object moves, the less its forward progress is resisted by the air through which it passes.

A car coasting to a stop! Its Reynolds number is

$$\text{Re} \sim \frac{vl}{\nu}, \tag{12}$$

where $\nu 1.5 \times 10^{-5}\,\text{m}^2\,\text{s}^{-1}$ is the kinematic viscosity of air, and l is a typical length scale for a car, perhaps $2\,\text{m}$. At a typical speed of $v \sim 20\,\text{m s}^{-1}$, the Reynolds number is

$$\text{Re} \sim \frac{20\,\text{m s}^{-1} \times 2\,\text{m}}{1.5 \times 10^{-5}\,\text{m}^2\,\text{s}^{-1}} \sim 3 \times 10^6. \tag{13}$$

Drag switches from v^2 (turbulent) to v (Stokes) drag at a Reynolds number of say $\text{Re} \sim 3$ (see Tritton's excellent textbook[5] (pp. 32–34)), which occurs when v falls by a factor of 10^6 so when $v \sim 2 \times 10^{-5}\,\text{m s}^{-1}$. The car has certainly coasted to a stop!

Digging the hole deeper, the section offers an example (p. 536):

> Example 8. For a 192-lb ice skater, the k in $[F_{\text{drag}} = kv]$ is about 1/3 slug/sec and $m = 192/32 = 6$ slugs. How long will it take the skater to coast from 11 ft/sec (7.5 mph) to 1 ft/sec? How far will the skater coast before coming to a complete stop?

Leaving aside the obscure units, the authors take formulae derived from bogus physics and ask students to insert alleged constants into the resulting bogus formulae!

V.   THE DESERT

We live and teach in the desert of the real[6] (p. 1). Our betters may force us to assign a book in Table I. We can mitigate its harm by teaching students how to make quantitative checks on the approximations and assumptions and by class discussion on dubious problems.

Great value still lies in explaining and analyzing trajectories neglecting air resistance, such as the trajectory of a baseball or golf ball when tossed gently from hand to hand or of a solid, dense metal ball falling a few meters. The golf drive can be analyzed neglecting air resistance so long as the problem's unreality is emphasized and physically reasonable approaches follow. Our recommendation need not make introductory physics more difficult. We just want it to become more *physical*.

If a student can tell you that an object with constant acceleration moves on a parabolic trajectory, then the student has learned or memorized a mathematical fact. If a student can tell you when and why constant acceleration usefully approximates a real system, then the student has learned physics and understands important patterns in our Universe. Physics studies the world, not just mathematical relationships. By that standard, introductory 'physics' has become as remote from students' reality as have medieval disputations about the size of angels.

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[1] R. Chabay and B. Sherwood, *Matter & Interactions I: Modern Mechanics* (John Wiley, 2002), `http://www4.ncsu.edu/~rwchabay/mi/`.

[2] A. Titus, American Journal of Physics **74**, 362 (2006).

[3] R. D. H. Warburton and J. Wang, American Journal of Physics **72**, 1404 (2004).

[4] J. George B. Thomas and R. L. Finney, *Calculus and analytic geometry* (Addison Wesley, 2003), alternate ed.

[5] D. J. Tritton, *Physical fluid dynamics* (Oxford, 1988), 2nd ed.

[6] J. Baudrillard, *Simulacra and Simulation* (University of Michigan Press, 1995), translated by Sheila Faria Glaser.