

# CHAPTERS 23 AND 24 NOTES

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## 23 ELECTRIC POTENTIAL

When a charged particle moves in an electric field, the field exerts a force that can do work on the particle. This work can be expressed in terms of electric potential energy. Just as gravitational potential energy depends on the height of a mass above the earth's surface, electric potential energy depends on the position of the charged particle in the electric field. In circuits, a difference in potential from one point to another is often called voltage.

### 23.1 Electric Potential Energy.

**Definition 1. Work done by a Force:** In non-electrical circumstances, the work done by a pushing force can be summarized as the exchange in energy by the force, and can be expressed by

$$W_{a \rightarrow b} = \int_a^b \vec{f} \cdot d\vec{l} = \int_a^b F \cos \phi dl$$

where the force is positive if in same direction as movement.

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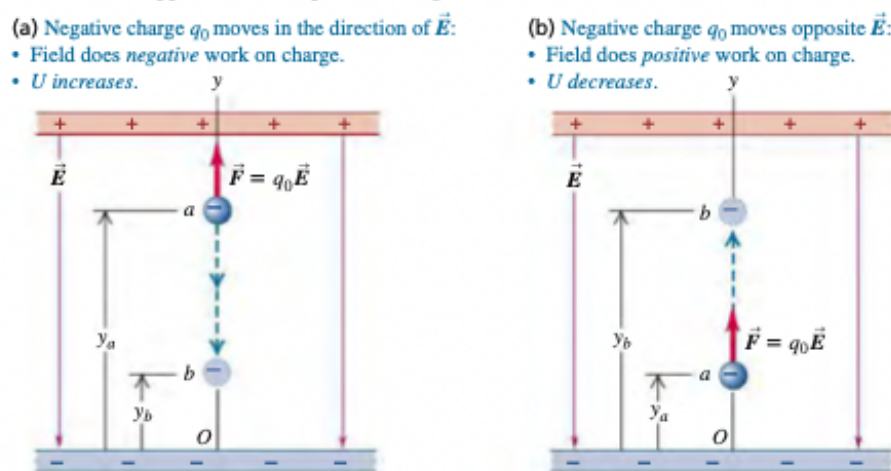


FIGURE 1. Electric Field doing work on negative charge

**Theorem 1.** Work-Energy Theorem: Work is defined as the negative of the change in *potential energy*, in that a decrease in potential means an increase in energy through positive work. This can be expressed mathematically as

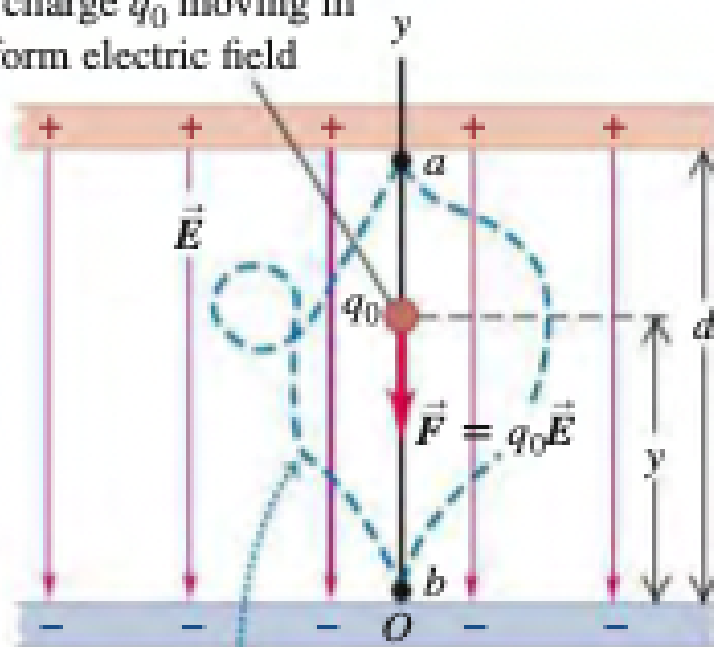
$$W_{a \rightarrow b} = -\Delta U$$

for the specific case the work is conservative. This theorem also makes so *Total Mechanical Energy* is conserved.

## 23.2 The work done on a point charge moving in a uniform electric field.

Compare with Fig. 23.1.

Point charge  $q_0$  moving in a uniform electric field



The work done by the electric force is the same for any path from  $a$  to  $b$ :

$$W_{a \rightarrow b} = -\Delta U = q_0 E d$$

FIGURE 2. Electric Field as Conservative Force

### 23.1.1 Electric Potential Energy in a Uniform Field.

**Definition 2. Electric Potential Energy in a Uniform Field:** In the case of a uniform field, the value of the potential energy can be described as

$$U = q_0 E y$$

where the value of  $q$  depends on the charge and the sign of the potential energy depends on direction of travel, either along the electric field or opposite.

### 23.1.2 Electric Potential Energy of Point Charges.

**Definition 3. Electric Potential Energy of 2 point charges:** Expressed mathematicall as

$$U = \frac{1}{4\pi\epsilon_0} \frac{qq_0}{r}$$

Because potential energy is a shared property the relation is  $1/r$ , also noting that the sign of charges are kept as If  $q$  and  $q_0$  have the same sign, the interaction is repulsive, this work is positive, and  $U$  is positive at any finite separation. If the charges have opposite signs the interaction is attractive and the work done is negative so  $U$  is negative.

**Definition 4. Electric Potential Energy of Several Point Charges:**

Because the Electric Potential of 2 points is a vector sum we can simply sum all the point charges together as path of travel does not matter for conservative forces. Thus our equation becomes

$$U = \frac{q_0}{4\pi\epsilon_0} \sum_1 \frac{q_i}{r_i}$$

**Remark.** We can represent any charge distribution as a collection of point charges, so it follows that for every electric field due to a static charge distribution, the force exerted by that field is conservative.

**23.8** The potential energy associated with a charge  $q_0$  at point  $a$  depends on the other charges  $q_1$ ,  $q_2$ , and  $q_3$  and on their distances  $r_1$ ,  $r_2$ , and  $r_3$  from point  $a$ .

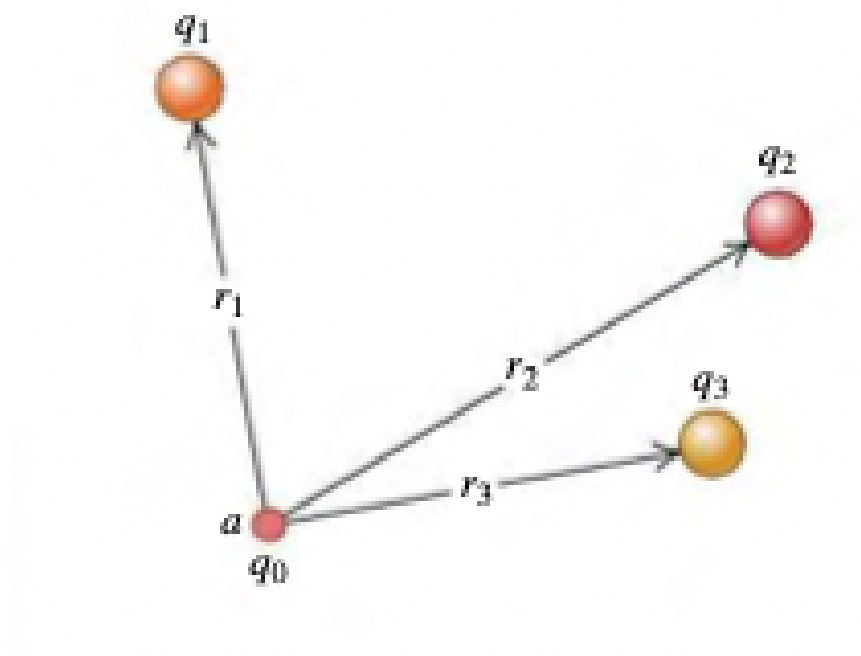


FIGURE 3. Potential Energy of Charges

### 23.2 Electric Potential.

**Definition 5. Electric Potential:** Otherwise called just *Potential*, this concept aids in determining involved energies of charged particles, and is defined as potential energy per unit charge, and written mathematically as

$$V = \frac{U}{q_0} \Leftrightarrow U = q_0 V$$

and has the SI unit of volt (V) or as joules/coulomb(J/C). When  $U$  is expanded the equation becomes:

(1) for a single point charge

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

(2) for multiple point charges

$$V = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i}$$

(3) and for a continuous distribution of charge.

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}$$

**Remark.** Potential Energy and Charge are both scalars, so Electric Potential is also a scalar.

**Definition 6. Voltage:** Voltage  $V_{ab}$  is the potential (in Volts) of  $a$  with respect to  $b$ , and equals the work (in Joules) done by the electric force when a UNIT (1-C) charge moves from  $a$  to  $b$ .

### *23.2.1 Electric Potential from Electric Field.*

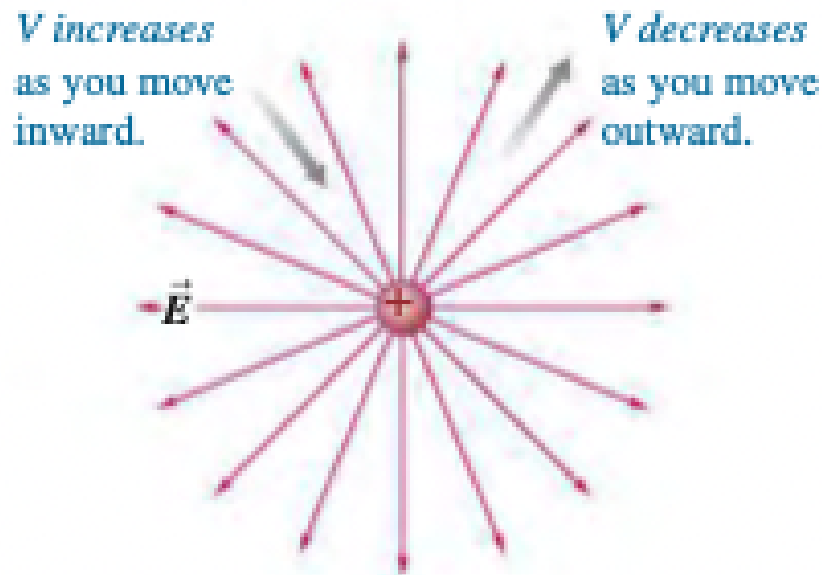
**Definition 7. Electric Potential from Electric Field:** Because there is an easy conversion from force to Electric field in  $F = \vec{E}q$  we can simply substitute in  $\vec{E}$  such that

$$V_a - V_b = - \int_b^a \vec{E} \cdot d\vec{l}$$

This also makes sense because the units match:  $1 \text{ V/m} = 1 \text{ volt/meter} = 1 \text{ N/C} = 1 \text{ newton/coulomb}$

**23.12** If you move in the direction of  $\vec{E}$ , electric potential  $V$  decreases; if you move in the direction opposite  $\vec{E}$ ,  $V$  increases.

(a) A positive point charge



(b) A negative point charge

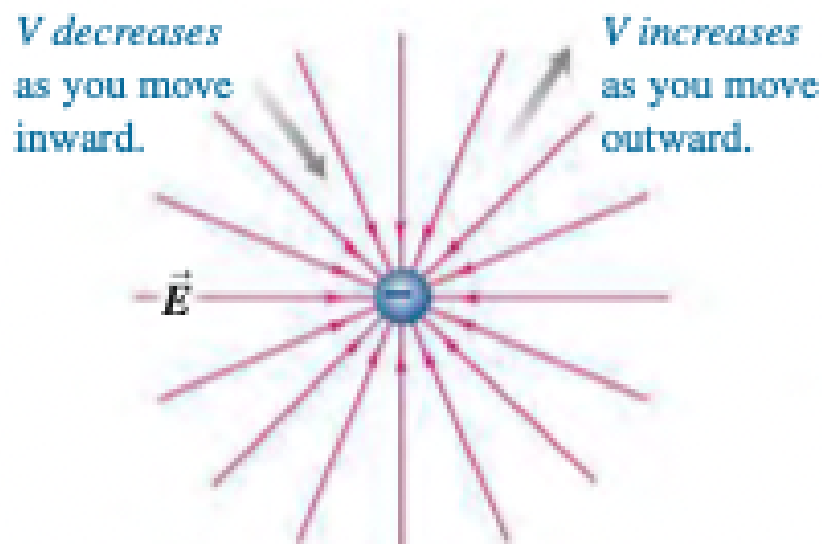


FIGURE 4. Electric Potential and Electric Field

### 23.2.2 Electron Volt.

**Definition 8. Electron Volt:** The quantity of energy equivalent of a single electron with a potential difference of 1 V.  $1\text{eV} = 1.602 * 10^{-19} J$

## 23.4 Equipotential Surfaces.

**Definition 9. Equipotential Surfaces:** In a similar manner to field lines, these surfaces are countout lines for equal electric potential and emit radially from a charge. Potential along lines are equal and thus no work to go along. Lines close together indicate the field doing large amounts of work in small displacement and  $\vec{E}$  is large.

**Remark.** The electric field must always be perpendicular to the Equipotential surface except in a uniform field where equipotential surfaces are parallel planes perpendicular to field lines

**Remark.** When all charges are at rest, the surface of a conductor is always an equipotential surface and the entire volume of a conductor is at equal potential

## 23.5 Potential Gradient.

**Definition 10. Potential Gradient:** If the potential V is known as a function of the coordinates x, y, and z, the components of electric field E at any point are given by partial derivatives of V:

$$E_x = -\frac{\partial V}{\partial x}, E_y = -\frac{\partial V}{\partial y}, E_z = -\frac{\partial V}{\partial z}$$

which in vector form produces

$$\vec{E} = -(\hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z})$$



IMPORTANT EQUATIONS