CHAPTERS 27 AND 28 NOTES

ANISH SUNDARAM

Contents

27 Magnetic Field and Magnetic Forces

But the *fundamental* nature of magnetism is the interaction of moving electric charges. Unlike electric forces, which act on electric charges

Date: July 10, 2021.

whether they are moving or not, magnetic forces act only on *moving charges*.

27.1 Magnetism.

Remark. The earth itself is a magnet. Its north geographic pole is close to a magnetic south pole, which is why the north pole of a compass needle points north. The earth's magnetic axis is not quite parallel to its geographic axis (the axis of ro- tation), so a compass reading deviates somewhat from geographic north. This deviation, which varies with location, is called *magnetic declination or magnetic variation.*

Remark. While isolated positive and negative charges exist, there is no experimental evidence that one isolated magnetic pole exists; poles always appear in pairs. If a bar magnet is broken in two, each broken end becomes a pole

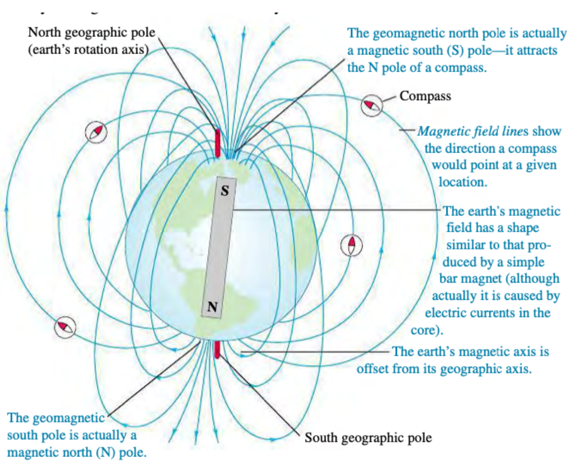

FIGURE 1. Depiction of Magnetic Field

27.2 Magnetic Field.

Definition 1. Summary of Electric interactions:

(1) A distribution of electric charge creates an electric field $\vec{E}$ in the surrounding space.
(2) The electric field exerts a force $\vec{F} = q\vec{E}$ on any other charge q that is present in the field.

Definition 2. Summary of Magnetic interactions:

(1) A moving charge or a current creates a **magnetic field (B)** in the surrounding space (in addition to its *electric* field).

(2) The magnetic field exerts a force $\vec{F}$ on any other moving charge or current that is present in the field.

Magnetic Field strength can be found by the equation

$$B = \frac{\mu_0 I}{2\pi r}$$

where μ_0 is the permeability of free space($4\pi \cdot 10^{-7}\ T \cdot m/A$) and r is the seperation.

Remark. Like electric field, magnetic field is a vector field–that is, a vector quantity associated with each point in space. We will use the symbol $\vec{B}$ for magnetic field.

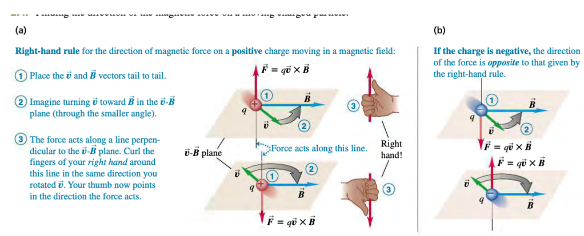

FIGURE 2. Right-Hand Rule

27.2.1 Magnetic Forces on Moving Charges.

Definition 3. Key Characteristics of Magnetic Force on Moving Charges:

(1) Force Magnitude is proportional to the magnitude of the charge.
(2) The magnitude of the force is also propor- tional to the magnitude, or "strength," of the field.
(3) The magnetic force depends on the particle's velocity.
(4) The magnetic force $\vec{F}$ does not have the same direction as the magnetic field $\vec{B}$ but instead is always perpendicular to both $\vec{B}$ and the velocity $\vec{v}$.

The Magnetic force can be found by the equation

$$F = |q|v \times B = |q|vBsin\phi$$

where ϕ is the angle between the direction of velocity and direction of the magnetic field and the magnitude is measured in teslas ($1T = 1N/A \cdot m$) or in Gauss(1G = 10^{-4}T)

Remark. The magnetic field of the earth is 10^{-4}T or 1 Gauss.

Remark. When a charged particle moves through a region of space where both electric and magnetic fields are present, *both fields exert forces on the particle.* The total force $\vec{F}$ is the vector sum of the electric and magnetic forces:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

27.3 Magnetic Field Lines and Magnetic Flux.

Definition 4. Magnetic Field Lines: Just like an electric field, a magnetic field can be represented through field lines. The more lines there are in a given area the stronger the field , and they never intersect. Magnetic Field Lines are not lines of force, just direction.

Definition 5. Magnetic Flux(Φ_B): A measurement of the total magnetic field which passes through a given area. The total magnetic flux through the surface is the sum of the contributions from the individual area elements. Can be found using the following equation:

$$\Phi_B = \int Bcos\phi dA = \int \vec{B} \cdot d\vec{A}$$

where ϕ is the angle between the two planes from the vertical. Magnetic Flux is scalar and its units are Weber(1 Wb = 1 $T \cdot m^2$ = 1 $N \cdot m/A$)

27.4 Motion of Charged Particles in a Magnetic Field.

Remark. Motion of a charged particle under the action of a magnetic field alone is always motion with constant speed.

Remark. When velocity and magnetic field are perpendicular, the particle exhibits circular motion. This relation is shown by

$$F = |q|vb = m\frac{v^2}{R}$$

whose radius can be found by

$$R = \frac{mv}{|q|B}$$

and whose angular speed can be found by

$$\omega = \frac{v}{r} = \frac{|q|B}{m}$$

27.5 Applications of Motion of Charged Particles.

27.5.1 Velocity Selector.

Definition 6. Velocity Selector: An arrangement of electric and magnetic fields that can be used to select only particles of a specific speed. Only particles with speed equal to E/B can pass through without being deflected, and this process works for electrons and protons.

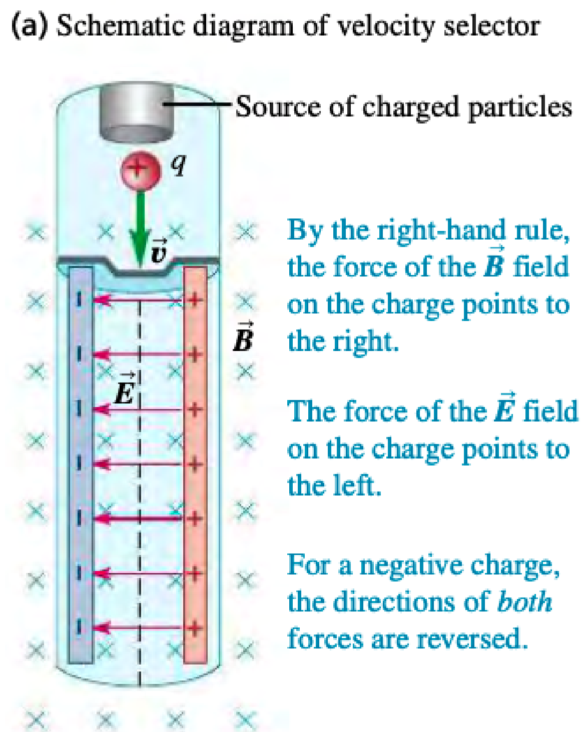

FIGURE 3. Model of Velocity Selector

27.6 Magnetic Force on a current-carrying Conductor.

Remark. Magnetic fields are caused by moving charges, and thus exist around a current carrying wire. We can manipulate the equation for single charges for currents,$\vec{F} = |q|vBsin\phi$, by factoring that the total force would be the force per charge multiplied by the amount of charges in the wire:

$$\vec{F} = |q|vBsin\phi = (nAl)(qvBsin\phi) = IlBsin\phi$$

and for infinitesmal wire segments:

$$d\vec{F} = Id\vec{l} \times \vec{B}$$

27.27 Magnetic field $\vec{B}$, length $\vec{l}$, and force $\vec{F}$ vectors for a straight wire carrying a current I.

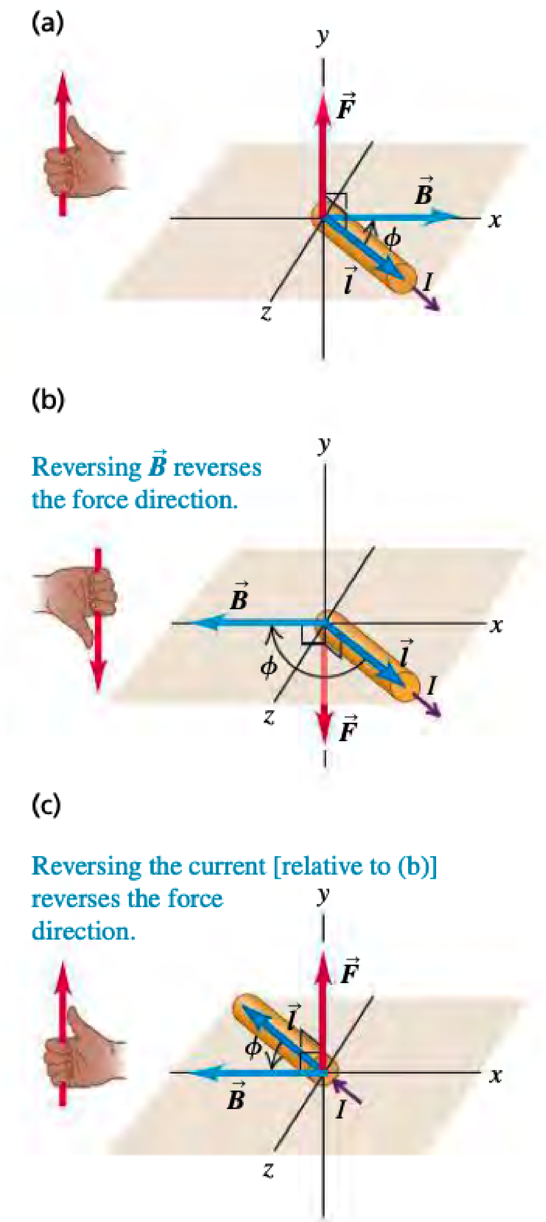

FIGURE 4. Right-Hand Rule for a Current-Carrying Wire

27.7 Force and Torque on a Current Loop.

Remark. The net force on a current loop in a uniform magnetic field is zero. However, the net torque is not in general equal to zero.

Definition 7. Torque on a Current Loop (τ): In a uniform field the total force is zero but torque is not necessary, as in a dipole. The magnitude of the magnetic torque can be found using the equation

$$\tau = IBAsin\phi$$

where A is the area of the loop within the field and ϕ is the area normal to loop plane and field direction.

Definition 8. Magnetic Dipole Moment(μ): Also called the **Magnetic Moment**, this is the product IA and is analogous to the electric dipole moment by having a north and south poles. A current loop or any body that experiences a magnetic torque is called a **Magnetic Dipole**.

27.7.1 Potential Energy for a Magnetic Dipole.

Definition 9. Potential Energy for a Magnetic Dipole(U): When a dipole changes orientation the field does work on it. Because potential energy is the negative of total work, the equation for finding U is the parallel of the potential energy in an electric field:

$$U = -\vec{\mu} \cdot \vec{B} = -\mu B cos\phi$$

where ϕ is the angle between μ and B.

27.7.2 Magnetic torque: Loops and Coils.

Definition 10. Solenoid: A helical winding of wire, such as a coil wound on a circular cylinder. The total torque on a solenoid in a megnetic field is the sum of the torques of the individual turns, or

$$\tau = NIABsin\phi$$

where ϕ is the angle between the axis of the solenoid and the direction of the field.

27.8 Direct-Current Motor.

Definition 11. Parts of Motor: The moving part of the motor is the *rotor*, a length of wire formed into an open-ended loop and free to rotate about an axis. The ends of the rotor wires are attached to circular conducting segments that form a *commutator*.

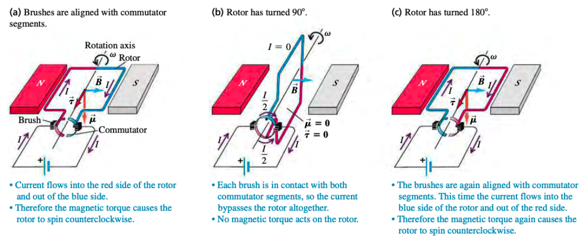

FIGURE 5. Model of a Motor

Remark. In a dc motor a magnetic field exerts a torque on a current in the rotor. Motion of the rotor through the magnetic field causes an induced emf called a back emf. For a series motor, in which the rotor coil is in series with coils that produce the magnetic field, the terminal voltage is the sum of the back emf and the drop Ir across the internal resistance.

27.9 The Hall Effect.

Definition 12. The Hall Effect: A potential difference perpendicular to the direction of current in a conductor, when the conductor is placed in a magnetic field. The Hall potential is determined by the requirement that the associated electric field must just balance the magnetic force on a moving charge. Hall-effect measurements can be used to determine the sign of charge carriers and their concentration n.

$$nq = \frac{-J_x B_y}{E_z}$$

where J_x is the current density or I/A

28 SOURCES OF MAGNETIC FIELD

We've learned that a charge creates an electric field and that an electric field exerts a force on a charge. But a magnetic field exerts a force on only a moving charge. Similarly, we'll see that only moving charges create magnetic fields. We'll begin our analysis with the magnetic field created by a single moving point charge. We can use this analysis to determine the field created by a small segment of a current-carrying conductor. Once we can do that, we can in principle find the magnetic field produced by any shape of conductor.

28.1 Magnetic Field of a Moving Charge.

Definition 13. Magnetic Field due to moving point charge: Using the equations we derived for electric fields we can create a formula to find $\vec{B}$ by using a vector product of magnitude and direction:

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v} \times \hat{r}}{r^2}$$

where μ_0 is the magnetic constant and $\hat{r}$ is the unit vector from the point charge towards the measure point.

28.2 Magnetic Field of a Current Element.

Theorem 1. Principle of Superposition of Magnetic Fields: The total magnetic field caused by several moving charges is the vector sum of the fields caused by the individual charges.

Definition 14. Law of Biot-Savart: A method of finding the total magnetic field $\vec{B}$ at any point in space due to the current in a complete circuit. We can find the magnetic field in an infinitesmal current element using this equation:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l} \times \hat{r}}{r^2}$$

and then integrate this equation over all segments $d\vec{l}$ for a complete circuit, resulting in:

$$d\vec{B} = \frac{\mu_0}{4\pi}\int \frac{Id\vec{l} \times \hat{r}}{r^2}$$

28.3 Magnetic Field of a Straight Current-Carrying Conductor.

Definition 15. Magnetic Field of a current-carrying wire: Integrating over a straight line means we can simplify our previous equation into a simpler form of

$$B = \frac{\mu_0 I}{2\pi r}$$

28.4 Force between Parallel Conductors.

Remark. Current-carrying wires each produce their own magnetic field, and these fields can interact with other nearby wires. Each conductor lies in the magnetic field set up by the other, so each experiences a force.

Definition 16. Force per unit length between 2 wires: Using our knowledge that $F_B = IlB$ and the equation for a magnetic field around a wire $B = \frac{\mu_0 I}{2\pi r}$, we can substitute for a new equation of the force between 2 wires:

$$\frac{F}{L} = \frac{\mu_0 I I^{'}}{2\pi r}$$

where I is the first current, $I^{'}$ is the second current and r is the distance between the 2 wires.

Definition 17. Ampere(A): One ampere is that unvarying current that, if present in each of two parallel conductors of infinite length and one meter apart in empty space, causes each conductor to experience a force of exactly 2×10^{-7} newtons per meter of length.

28.5 Magnetic Field of a Circular Current Loop.

Definition 18. Magnetic field on axid of a circular wire: Similar to the interactions discussed about electric fields, the strength of the field a certain point on axis from a circular wire can b expressed as the integration of the horizontal force at all points across the circular wire as the vertical components cancel out. Thus we get

$$B_x = \int \frac{\mu_0 I}{4\pi} \frac{adl}{(x^2 + a^2)^{\frac{3}{2}}} = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{\frac{3}{2}}}$$

where x is the distance along the axis from the loop's center to the field point and a is the radius of the loop.

28.5.1 Magnetic Field on the Axis of a Coil.

Definition 19. Magnetic field at center of N wire loops: Imagining that instead of a single loop there is a coil, we get an N amount of wire loops which generate their own field with only an x-component, leaving the result of

$$B_x = \frac{\mu_0 N I}{2a}$$

where N is the number of loops and r is the radius. If we know the magnetic total magnetic moment $\mu = N \cdot I\pi a^2$ we can substitute into the previous equation to get

$$B_x = \frac{\mu_0 \mu}{2\pi (x^2 + a^2)^{\frac{3}{2}}}$$

28.6 Ampere's Law. : For any closed loop path, the sum of the length elements times the magnetic field in the direction of the length element is equal to the permeability times the electric current enclosed in the loop. Put into an equation as

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

where I_{encl} is the net current cnlosed by path.

Remark. if $\oint \vec{B} \cdot d\vec{l} = 0$ it does not mean that $\vec{B} = 0$ everywhere along the path,only that the total current through an area bounded by the path is zero.

28.7 Applications of Ampere's Law.

Definition 20. Magnetic Fields due to Current Distributions:

Current Distribution	Point in Magnetic Field	Magnetic-Field Magnitude
Long, straight conductor	Distance r from conductor	$B = \frac{\mu_0 I}{2\pi r}$
Circular loop of radius a	On axis of loop	$B = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}}$
	At center of loop	$B = \frac{\mu_0 I}{2a}$ (for N loops, multiply these expressions by N)
Long cylindrical conductor of radius R	Inside conductor, $r < R$	$B = \frac{\mu_0 I}{2\pi} \frac{r}{R^2}$
	Outside conductor, $r > R$	$B = \frac{\mu_0 I}{2\pi r}$
Long, closely wound solenoid with n turns per unit length, near its midpoint	Inside solenoid, near center	$B = \mu_0 n I$
	Outside solenoid	$B \approx 0$
Tightly wound toroidal solenoid (toroid) with N turns	Within the space enclosed by the windings, distance r from symmetry axis	$B = \frac{\mu_0 N I}{2\pi r}$
	Outside the space enclosed by the windings	$B \approx 0$

FIGURE 6. Applications of Ampere's Law

28.8 Magnetic Materials.