CHAPTERS 23 AND 24 NOTES

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Contents

23 Electric Potential

When a charged particle moves in an electric field, the field exerts a force that can do work on the particle. This work can be expressed in terms of electric potential energy. Just as gravitational potential energy depends on the height of a mass above the earth's surface, electric potential energy depends on the position of the charged particle in the electric field. In circuits, a difference in potential from one point to another is often called voltage.

23.1 Electric Potential Energy.

Definition 1. Work done by a Force: In non-electrical circumstances, the work done by a pushing force can be summarized as the exchange in energy by the force, and can be expressed by

$$W_{a\to b} = \int_a^b \vec{f} \cdot d\vec{l} = \int_a^b F cos\phi dl$$

where the force is positive if in same direction as movement.

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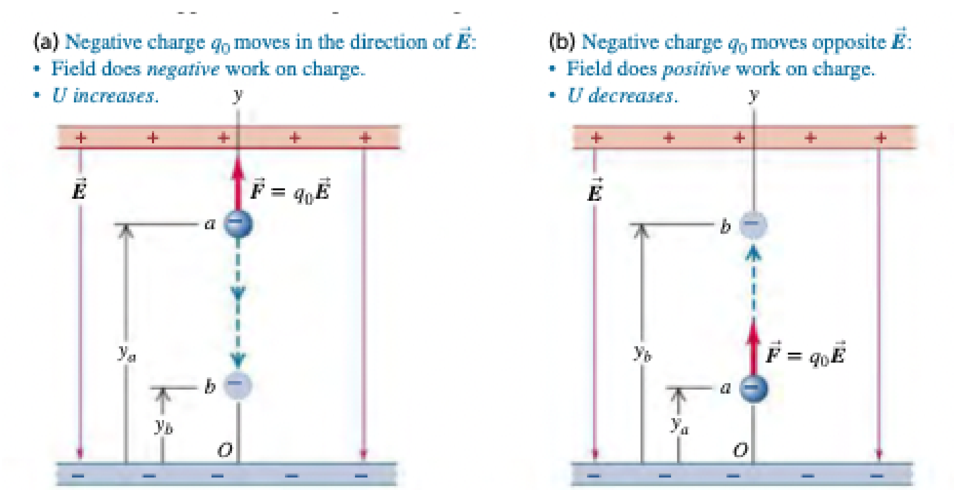

FIGURE 1. Electric Field doing work on negative charge

Theorem 1. Work-Energy Theorem: Work is defined as the negative of the change in *potential energy*, in that a decrease in potential means an increase in energy through positive work. This can be expressed mathematically as

$$W_{a\to b} = -\Delta U$$

for the specific case the work is conservative. This theorem also makes so *Total Mechanical Energy* is conserved.

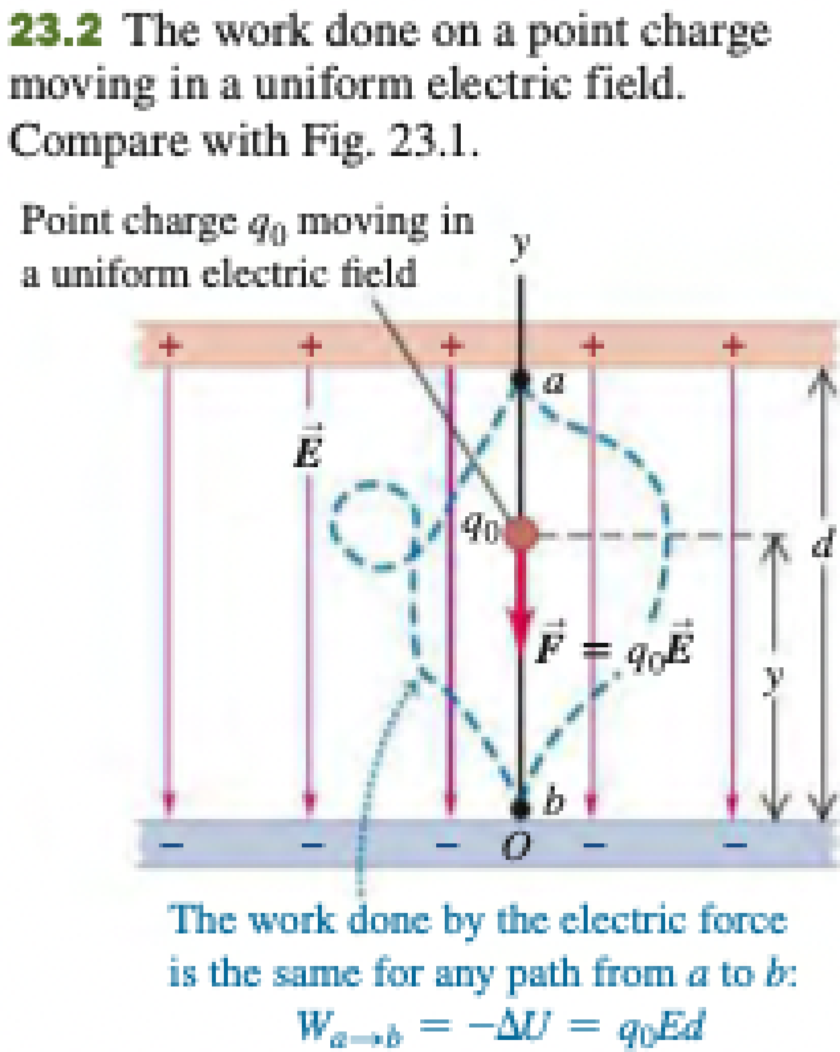

FIGURE 2. Electric Field as Conservative Force

23.1.1 Electric Potential Energy in a Uniform Field.

Definition 2. Electric Potential Energy in a Uniform Field: In the case of a uniform field, the value of the potential energy can be described as

$$U = -q_0 E y$$

where the value of q depends on the charge and the sign of the potential energy depends on direction of travel, either along the electric field or opposite.

23.1.2 Electric Potential Energy of Point Charges.

Definition 3. Electric Potential Energy of 2 point charges: Expressed mathematicall as

$$U = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r}$$

Because potential energy is a shared property the relation is 1/r, also noting that the sign of charges are kept as If q and q_0 have the same sign, the interaction is repulsive, this work is positive, and U is positive at any finite separation. If the charges have opposite signs the interaction is attractive and the work done is negative so U is negative.

Definition 4. Electric Potential Energy of Several Point Charges: Because the Electric Potential of 2 points is a vector sum we can simply sum all the point charges together as path of travel does not matter for conservative forces. Thus our equation becomes

$$U = \frac{q_0}{4\pi\epsilon_0}\Sigma_1\frac{q_i}{r_i}$$

Remark. We can represent any charge distribution as a collection of point charges, so it follows that for every electric field due to a static charge distribution, the force exerted by that field is conservative.

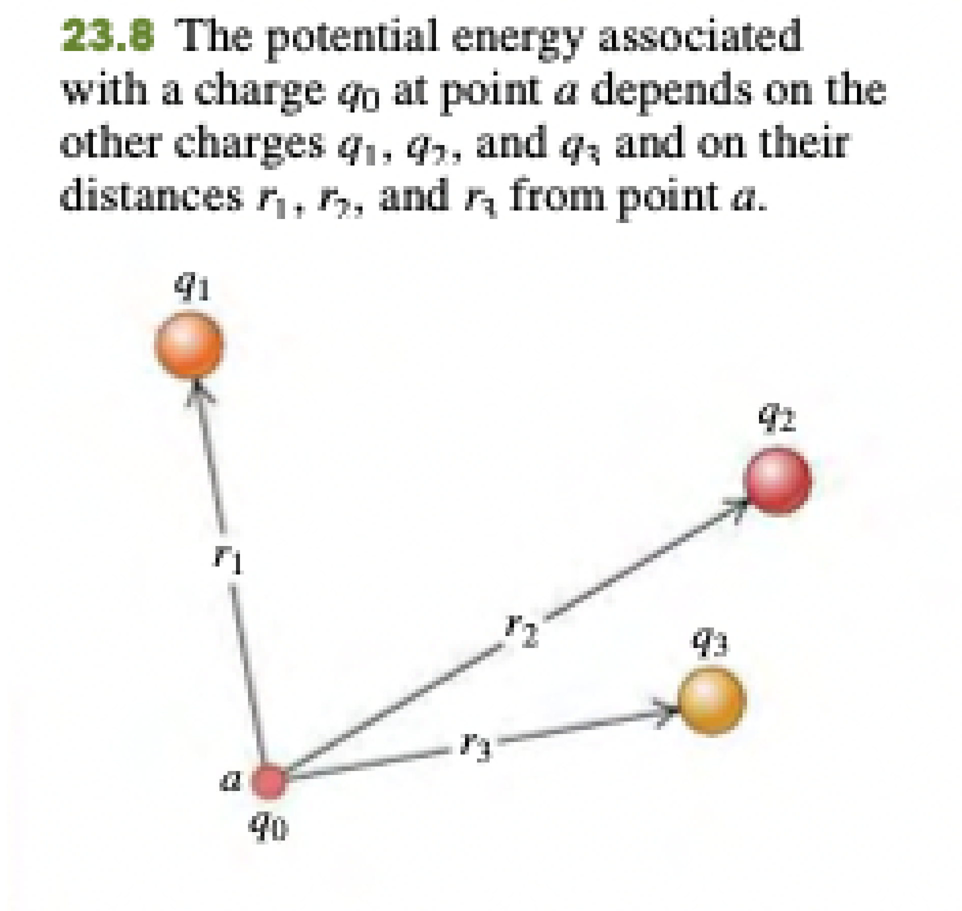

FIGURE 3. Potential Energy of Charges

23.2 Electric Potential.

Definition 5. Electric Potential: Otherwise called just *Potential*, this concept aids in determining involved energies of charged particles, and is defined as potential energy per unit charge, and written mathematically as

$$V = \frac{U}{q_0} \Leftrightarrow U = q_0 V$$

and has the SI unit of volt (V) or as joules/coulomb(J/C). When U is expanded the equation becomes:

(1) for a single point charge

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

(2) for multiple point charges

$$V = \frac{1}{4\pi\epsilon_0}\Sigma_i\frac{q_i}{r_i}$$

(3) and for a continuous distribution of charge.

$$V = \frac{1}{4\pi\epsilon_0}\int\frac{dq}{r}$$

Remark. Potential Energy and Charge are both scalars, so Electric Potential is also a scalar.

Definition 6. Voltage: Voltage V_{ab} is the potential (in Volts) of a with respect to b, and equals the work (in Joules) done by the electric force when a UNIT (1-C) charge moves from a to b.

23.2.1 Electric Potential from Electric Field.

Definition 7. Electric Potential from Electric Field: Because there is an easy conversion from force to Electric field in $F = \vec{E}q$ we can simply substitute in $\vec{E}$ such that

$$V_a - V_b = -\int_b^a \vec{E}\cdot d\vec{l}$$

This also makes sense because the units match: 1 V/m = 1 volt/meter = 1 N/C = 1 newton/coulomb

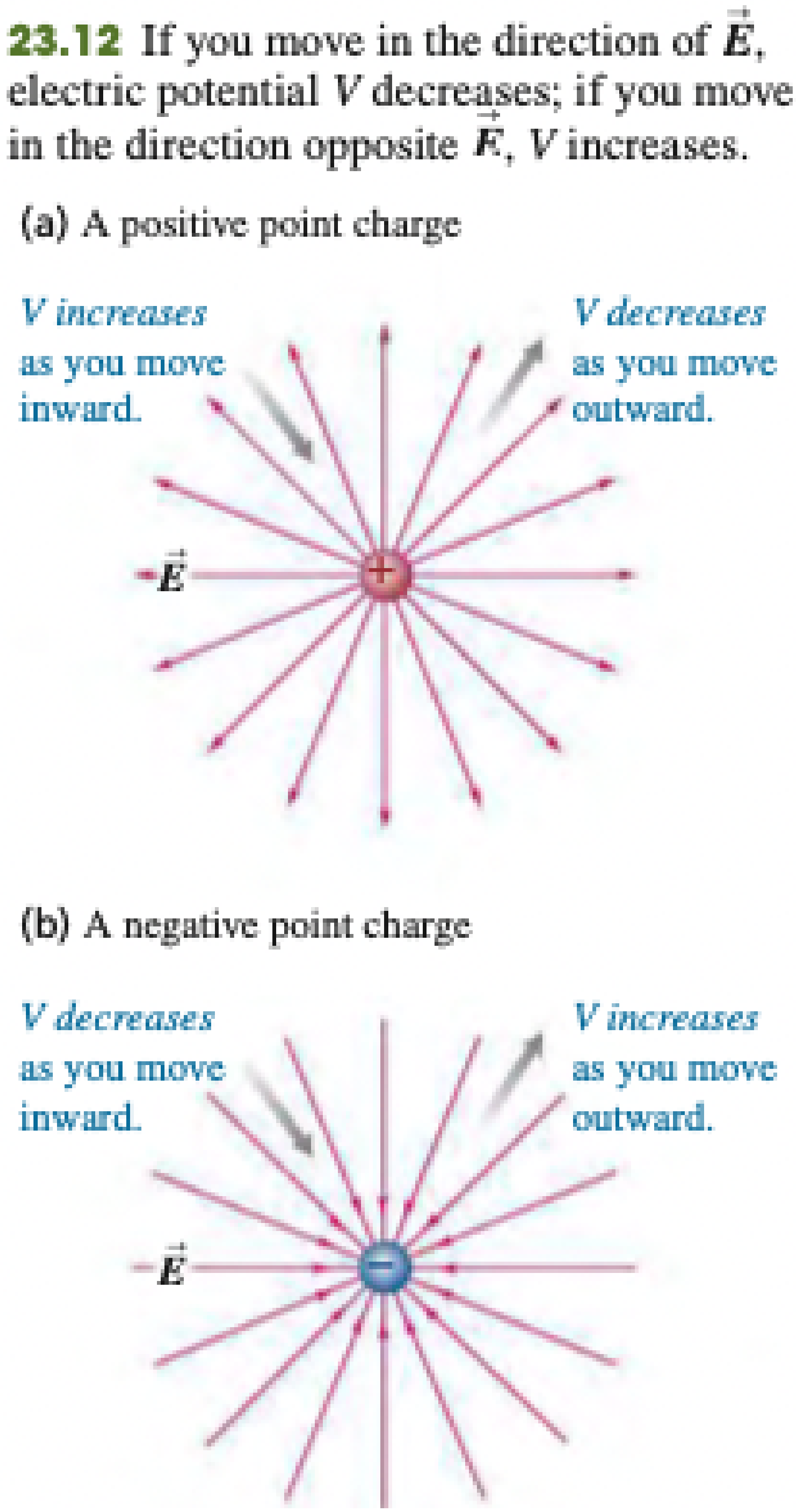

FIGURE 4. Electric Potential and Electric Field

23.2.2 Electron Volt.

Definition 8. Electron Volt: The quantity of energy equivalent of a single electron with a potential difference of 1 V. 1eV $= 1.602 * 10^{-19} J$

23.4 Equipotential Surfaces.

Definition 9. Equipotential Surfaces: In a similar manner to field lines, these surfaces are countout lines for equal electric potential and emit radially from a charge. Potential along lines are equal and thus no work to go along. Lines close together indicate the field doing large amounts of work in small displacement and $\vec{E}$ is large.

Remark. The electric field must always be perpendicular to the Equipotential surface except in a uniform field where equipotential surfaces are parallel planes perpendicular to field lines

Remark. When all charges are at rest, the surface of a conductor is always an equipotential surface and the entire volume of a conductor is at equal potential

23.5 Potential Gradient.

Definition 10. Potential Gradient: If the potential V is known as a function of the coordinates x, y, and z, the components of electric field E at any point are given by partial derivatives of V:

$$E_x = -\frac{\partial V}{\partial x}, E_y = -\frac{\partial V}{\partial y}, E_z = -\frac{\partial V}{\partial z}$$

which in vector form produces

$$\vec{E} = -(\hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z})$$