

CHAPTERS 25 AND 26 NOTES

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25 CURRENT, RESISTANCE, AND ELECTROMOTIVE FORCE

This chapter involves electric charges *in motion* rather than static as before. An *electric current* consists of charges in motion from one region to another. If the charges follow a conducting path that forms a closed loop, the path is called an *electric circuit*.

25.1 Current.

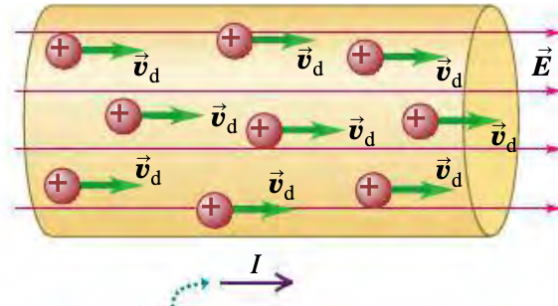
Definition 1. Current (I): Any motion of charge from one region to another and is induced by the ability of electrons to freely move. Current is zero everywhere in electrostatics. Current can be defined as

$$I = \frac{dQ}{dt}$$

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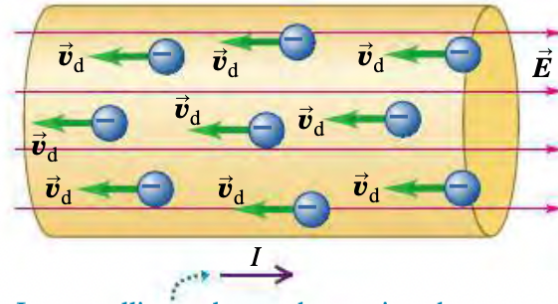
where Q is Coulombs and its unit is Amperes ($1 \text{ A} = 1 \text{ C/s}$). Because Current is a scalar it must be accompanied by a statement of direction: "25 Amps in the clockwise direction"

Remark. We define the current, denoted by I , to be in the direction in which there is a flow of positive charge and describe currents as though they consisted entirely of positive charge flow, even in cases in which we know that the actual current is due to electrons.



A **conventional current** is treated as a flow of positive charges, regardless of whether the free charges in the conductor are positive, negative, or both.

(b)



In a metallic conductor, the moving charges are electrons — but the *current* still points in the direction positive charges would flow.

FIGURE 1. Current Flow Diagram

Definition 2. Drift Velocity (v_d): The average velocity of charged particles moving in the direction of the electric force $\vec{F} = q\vec{E}$, though individual charge path is random. This value gives us an alternate calculation of I of

$$I = nqv_dA$$

where n is concentration of particles, q is unit charge and A is area.

25.1.1 Direction of Current Flow.

Remark. Different current-carrying materials may have differently charged moving particles. In metals the moving charges are always electrons, while in an ionized gas (plasma) or an ionic solution the moving charges may include both electrons and positively charged ions. In a semiconductor conduction is partly by electrons and partly by motion of *vacancies*, also known as holes; these are sites of missing electrons and act like positive charges.

25.1.2 Current Density.

Definition 3. Current Density (J):

The current per unit cross-section area or

$$J = I/A = nqv_d$$

where n is charge concentration, q is the charge per particle and v_d is the drift velocity.

25.2 Resistivity.

Theorem 1. Ohm's Law: A relationship in an idealized model that states that the ratio of the electric field and the current density is constant in metals at a given temperature, and this is known as Resistivity.

Definition 4. Resistivity (ρ): The permittivity of electrons to move freely in a material, linked with resistance. Resistivity is defined as

$$\rho = \frac{E}{J}$$

where E is the magnitude of the electric field and J is the current density. unit is ohm-meters ($1 \Omega \cdot m = V \cdot m/A$) Good insulators have high resistivity and conductors have low Resistivity.

Resistivity can also be calculated as a function of Temperature:

$$\rho(T) = \rho_0[1 + \alpha(T - T_0)]$$

where α is a temperature coefficient of resistivity and ρ_0 being the Resistivity at a reference temperature T_0

Definition 5. Conductivity: The reciprocal of resistivity whose units are $(\Omega \cdot m)^{-1}$ Good conductors obviously have high Conductivity.

Definition 6. Semiconductor: Materials with properties intermediate of metals and insulators, whose resistivity is likewise between these two groups.

Remark. A material that obeys Ohm's law reasonably well is called an *ohmic conductor* or a *linear conductor*, those that don't are *nonohmic*, or *nonlinear*. In the latter materials, J depends on E in a more complicated manner.

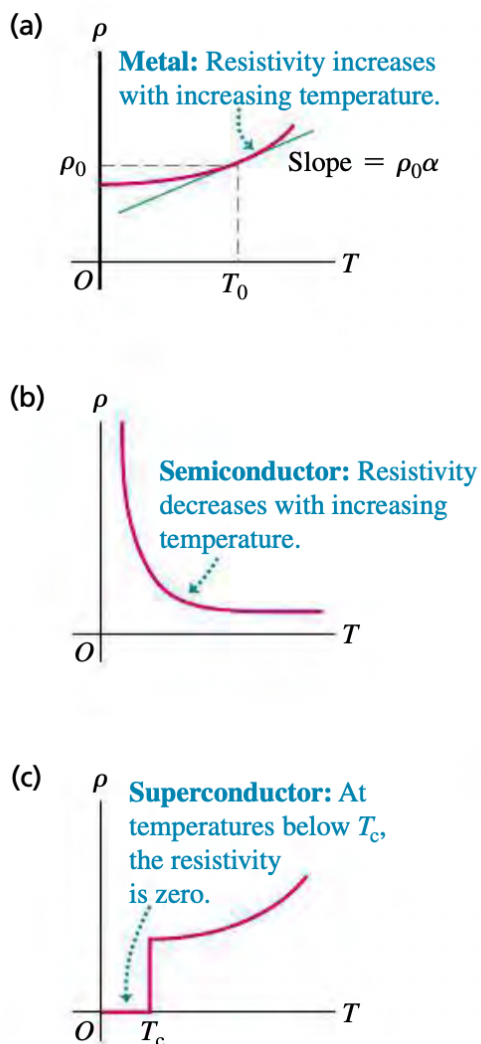


FIGURE 2. Resistivity Across Material Types

25.3 Resistance.

Definition 7. Resistance (R): The ratio of Potential Difference V to Current I for a particular conductor. Resistance measures the opposition to current flow in an electrical circuit. The resistance of a

conductor can be calculated by the equation

$$R = \frac{\rho L}{A} = \frac{V}{I}$$

and its unit is Ohm ($1 \Omega = 1V/A$) and similar to resistivity can be a function of Temperature:

$$R(T) = R_0[1 + \alpha(T - T_0)]$$

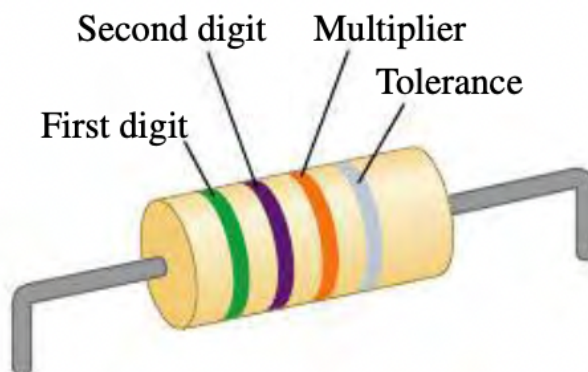


FIGURE 3. Labeling Guide for Resistors

25.4 Electromotive Force and Circuits.

Definition 8. Electromotive Force (\mathcal{E}): In an electric circuit there must be a device somewhere in the loop that acts like the water pump in a water fountain. This pumping action is called Electromotive Force or emf and the device is called a source of emf. This is not actually a force but a energy-per-unit-charge and thus has the unit Volt ($1 \text{ V} = 1 \text{ J/C}$).

Emf can be calculated in multiple ways:

$$\mathcal{E} = V_{ab} = IR$$

for ideal sources and where V_{ab} is the Terminal Voltage or

$$\mathcal{E} = V_{ab} + Ir$$

for when there is internal resistance (r).

TABLE 25.4 Symbols for Circuit Diagrams



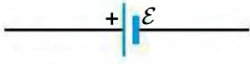

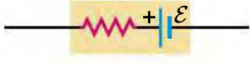
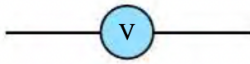
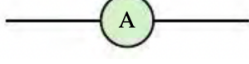
	Conductor with negligible resistance
	Resistor
	Source of emf (longer vertical line always represents the positive terminal, usually the terminal with higher potential)
	Source of emf with internal resistance r (r can be placed on either side)
or 	
	Voltmeter (measures potential difference between its terminals)
	Ammeter (measures current through it)

FIGURE 4. Table of Circuit Labels

25.5 Energy and Power in Electric Circuits.

Definition 9. Power(P): the rate, per unit time, at which electrical energy is transferred in or out of an electric element. Power can be found by the following equations:

$$P = V_{ab}I = I^2R = \frac{V_{ab}^2}{R} = \mathcal{E}I - I^2r$$

The SI unit of power is the watt($1 \text{ W} = 1 \text{ J/s}$)

Remark. The moving charges in flowing current collide with atoms in the resistor and transfer some of their energy, increasing the *internal energy* of the material. Either the temperature of the resistor increases or there is a flow of heat out of it, or both. Every resistor has a power rating, the maximum power the device can dissipate without becoming overheated and damaged.

26 DIRECT-CURRENT CIRCUITS

Our principal concern in this chapter is with *direct-current* (*dc*) circuits, in which the direction of the current does not change with time. Larger appliances and household equipment use *alternating current* (*ac*), in which the current oscillates back and forth.

26.1 Resistors in Series and Parallel.

Definition 10. Series Circuit: When circuit elements are connected in sequence with a single current path between points. In a Series Circuit Capacitors all have the same *Capacitance*

Definition 11. Parallel Circuit: An arrangement where each resistor provides an alternate path between the points. For elements connected in parallel the potential differences is the same across each element.

Definition 12. Equivalent Resistance: The total resistance of a circuit equal to the sum of its parts. Equivalent Resistance differs depending on configuration.

- (1) For Resistors in Series: $R_{eq} = R_1 + R_2 + ..$
- (2) For Resistors in Parallel: $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + ..$

Remark. The equivalent resistance of a series combination equals the sum of the individual resistances. The equivalent resistance is greater than any individual resistance. The reciprocal of the equivalent resistance of a parallel combination equals the sum of the reciprocals of the individual resistances. The equivalent resistance is always less than any individual resistance.

26.2 Kirschhoff's Rules.

Definition 13. Junction: A junction in a circuit is a point where three or more conductors meet.

Definition 14. Loop: A loop is any closed conducting path in a circuit, including the circuit itself.

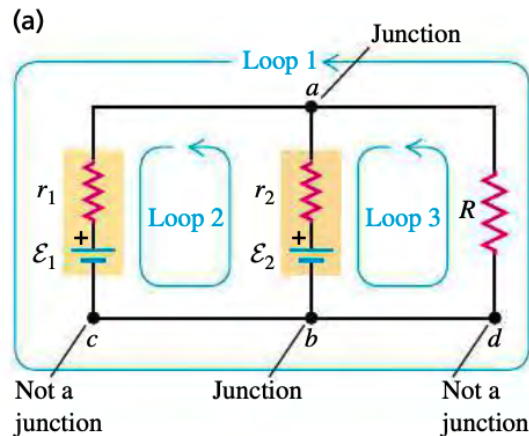


FIGURE 5. Example of Junctions and Loops

Definition 15. Kirschoff's Junction Rule: The sum of the currents into any junction is 0, or in other words the current that flows in a junction must equal the current flowing out the junction.

$$\Sigma I = 0$$

This is valid at any junction.

Definition 16. Kirschoff's Loop Rule: The sum of the potential differences around any loop must equal 0, or in other words the sum voltage at the end of a circuit or loop must equal 0.

$$\Sigma V = 0$$

This is valid at any closed loop.

26.8 Use these sign conventions when you apply Kirchhoff's loop rule. In each part of the figure "Travel" is the direction that we imagine going around the loop, which is not necessarily the direction of the current.

(a) Sign conventions for emfs

(b) Sign conventions for resistors

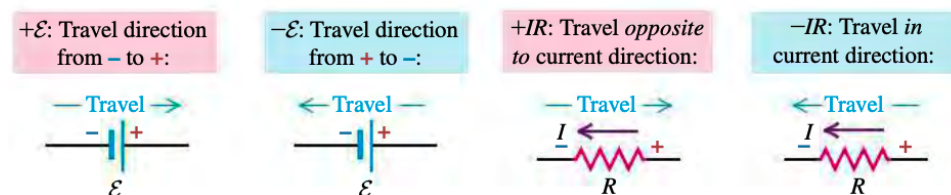


FIGURE 6. Picking a sign for loops

26.3 Electrical Measuring Instruments.

Definition 17. d'Arsonval galvanometer: Tool used to measure potential difference, current, or resistance by evaluating the torque caused by the magnetic field on a coil by current. For a larger current range, a shunt resistor is added, so some of the current bypasses the meter coil, and this makes an ammeter. If materials are ohmic the ammeter can be converted to a voltmeter.

Remark. Ammeters should be placed in series with resistors to measure current due to having little resistance and thus drawing current. Voltmeters should be attached in parallel as they have very high resistance and thus can block current flow if attached in series.

26.14 A d'Arsonval galvanometer, showing a pivoted coil with attached pointer, a permanent magnet supplying a magnetic field that is uniform in magnitude, and a spring to provide restoring torque, which opposes magnetic-field torque.

Magnetic-field torque tends to push pointer away from zero.

Spring torque tends to push pointer toward zero.

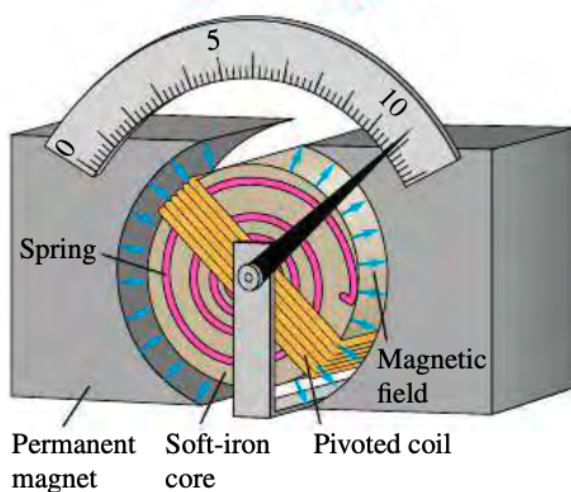


FIGURE 7. Example of d'Arsonval galvanometer

26.4 R-C Circuits.

Definition 18. R-C Circuit: A circuit containing resistors and capacitors in series with an ideal battery and ideal wires. In these circuits we take into consideration time to charge a capacitor, as time is not constant. For these situations use the following equations:

(1) For Charge:

$$q = C\mathcal{E}(1 - e^{-t/RC}) = Q_f(1 - e^{-t/RC})$$

Where t is time since switch closed and Q_f is final capacitor charge $= C\mathcal{E}$

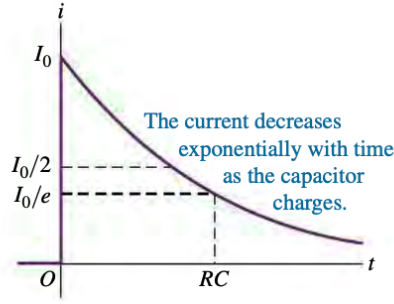
(2) For Current:

$$i = \frac{dq}{dt} = \frac{\mathcal{E}}{R}e^{-t/RC} = I_0e^{-t/RC}$$

Where t is time since switch closed and I_0 is initial current charge $=\mathcal{E}/R$

Remark. Variables are in lower case to distinguish they are time-varying quantities and separate from their constant capitalized counterparts.

(a) Graph of current versus time for a charging capacitor



(b) Graph of capacitor charge versus time for a charging capacitor

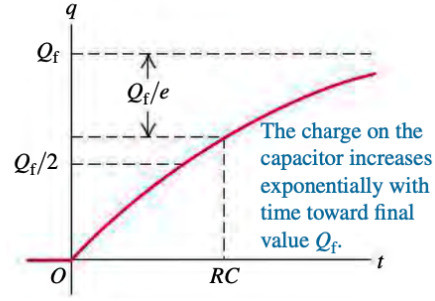


FIGURE 8. Capacitor Charging Curve

26.4.1 Time Constant.

Definition 19. Time Constant (τ): A time constant is the time equal to RC or the time at which the current has decreased to $1/e$ its original value and the capacitor has reached $(1-1/e)$ its final value $Q_f = C\mathcal{E}$. It is thus a measure of how quickly a capacitor charges. It can be calculated with

$$\tau = RC$$

The unit for τ is seconds ($1s = 1F \cdot C$)

Remark. When τ is small, the capacitor charges quickly; when it is larger, the charging takes more time.

26.4.2 Discharging a Capacitor.

Remark. The functions for discharging a Capacitor are the same as for charging except that the sign of current is opposite as current is flowing out

For Charge:

$$q = Q_f(1 - e^{-t/RC})$$

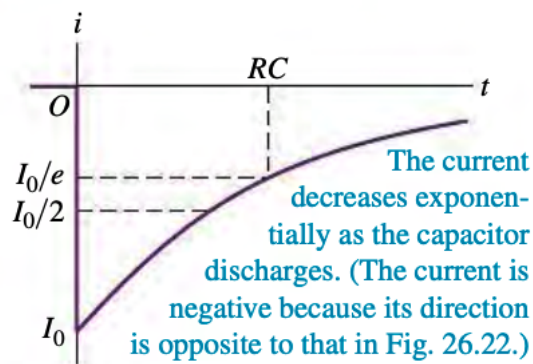
Where t is time since switch closed and Q_f is initial capacitor charge $=C\mathcal{E}$

For Current:

$$i = \frac{dq}{dt} = -\frac{Q_0}{RC}e^{-t/RC} = I_0e^{-t/RC}$$

Where t is time since switch closed and I_0 is initial current charge $= -Q_0/RC$

(a) Graph of current versus time for a discharging capacitor



(b) Graph of capacitor charge versus time for a discharging capacitor

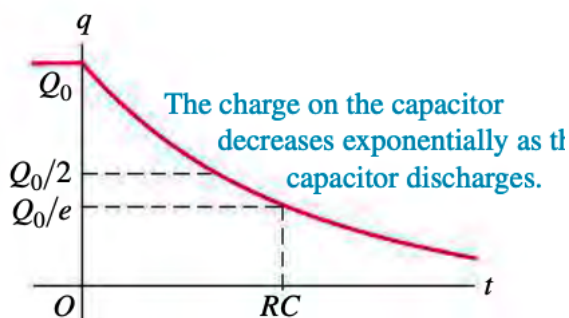


FIGURE 9. Capacitor Discharging Curve