LECTURE 7 AND 8

ANISH SUNDARAM

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7 Jointly Distributed Random Variables

7.1 JDRV Discrete Case.

Definition 1. JDRV: Can be described as the computation

$$\mathcal{P}(X, Y) \in B$$

for all possible sets.

Remark. Summing across all values x and y will result in 1 and the probability of p(x,y) must at least be 0.

0.1. 7.2 Marginal Distributions.

Definition 2. Marginal Distributions: The probability distribution of the variables contained in the subset. Can be found using the equation

$$p_Y(y) = \sum{}_x p_{X,Y}(x, y)$$

. Suppose we roll a pair of dice and let X be the minimum and Y the maximum of the two scores: The conditional PMF of Y given $X = x_0$

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is defined as:

$$p_{Y|X} := P[Y = y|X = x_0] = \frac{p_{X,Y}(x_0, y)}{p_x(x_o)}$$

7.2 Conditional Distributions.

Definition 3. Conditional Distribution: A probability distribution for a sub-population. In other words, it shows the probability that a randomly selected item in a sub-population has a characteristic you're interested in.

Solution: Let's complete the table of the joint PMF:

$X \backslash Y$	1	2	3	4	5	6	p_X
1	1/36	2/36	2/36	2/36	2/36	2/36	11/36
2	0	1/36	2/36	2/36	2/36	2/36	9/36
3	0	0	1/36	2/36	2/36	2/36	7/36
4	0	0	0	1/36	2/36	2/36	5/36
5	0	0	0	0	1/36	2/36	3/36
6	0	0	0	0	0	1/36	1/36
p_Y	1/36	3/36	5/36	7/36	9/36	11/36	1

FIGURE 1. Example of a Joint Distribution

8 JOINTLY DISTRIBUTED RANDVARS

8.1 Covariance.

Definition 4. Covariance: Defined as the measure of joint variability of two random variables. Do they trend together or inversely?At all? Covariance of 2 random variables X and Y is defined as

$$Cov(X, Y) = E[(X - \mu_x)(Y - \mu_y)]$$

where

$$\mu_x = E[X]$$

and

$$\mu_y = E[Y]$$

are the means of X and Y respectively and can also be seen as

$$\frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X_n})(Y_i - \bar{Y_n})$$

for population and

$$\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X_n})(Y_i - \bar{Y_n})$$

for samples

Definition 5. Properites of Covariance: Covariance is sensetive to scale, and has various properties:

(1) Symmetry: $Cov(X, Y) = Cov(Y, X)$
(2) Linearity:
(3) If X and Y are independent then $Cov(X, Y) = 0$

8.2 Correlation Coefficient.

Definition 6. Correlation Coefficient(ρ): A unitless statistical measure of the strength of the relationship between the relative movements of two random variables. Defined mathematically as:

$$Corr(X, Y) = \frac{Cov(X, Y)}{\sigma_X \sigma_y} = \frac{\sum(X_i - \bar{X_n})(Y_i - \bar{Y_n})}{\sqrt{\sum x_i - \bar{x})^2 \sum(y_i - \bar{y})^2)}}$$

Theorem 1. *Cauchy-Schwarts*:

8.3 Density of Transformation.

8.3 Transformations of RVs.

Definition 7. linear

Definition 8. nonlinear