LECTURE 3 AND 4

ANISH SUNDARAM

Contents

3 Probability – intuition and fundamentals

3.1 Elements of Probability.

Definition 1. Experiment: A situation under which a specific treatment is applied to a group and one group is left untreated as a control group in order to compare effectiveness vs placebo. This is the basis of forming conclusions

Definition 2. Probability(P): The likelyhood of an event to occur, represented as a proportion of the times an event occurs over the total amount of events. Represented as

$$p_n = \frac{\text{times A occurs}}{n} \longrightarrow P(A)$$

with n approaching infinity

Definition 3. Sample Space($\Omega/S/U$): The total number of possible solutions to a roll, pull, Probabilistic event. it is the superset of all possible results.

Definition 4. Events: An statistic event is defined as an outcome or defined collection of outcomes of a random experiment. Every Event A is a subset of the Sample Space in that :

$$A \subset \Omega$$

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Terminology:
- $\emptyset$ is called the impossible event (it never occurs)
- Ω is the certain event (always occurs)
- If $A \cap B = \emptyset$, then A and B are called mutually exclusive – they can never occur together.

FIGURE 1. Additional terminology of sets

3.2 Operations with Events.

Definition 5. Operations with Events:

(1) **Intersection($\cap$)**: The elementary outcomes that are in BOTH A and B: $AB = A \cap B = \omega \in \Omega : \omega \in A\text{AND}\omega \in B$
(2) **Union($\cup$)**: The elementary outcomes that are in either A,B, or both inclusive: $AB = A \cup B = \omega \in \Omega : \omega \in A\text{OR}\omega \in B$
(3) **Complement(A^c)**: The elementary outcomes that are not in A: $A_c = \omega \in \Omega : \omega \notin A$
(4) **Symmetric Difference($A\Delta B$)**: The elementary outcomes that are in A or B BUT NOT IN BOTH: $A\Delta B = A_c B \cup AB_c = \omega \in \Omega : \omega \in A or \omega \in B$but not in both This is exclusive or "XOR" operation

Event	Logical statement of occurence
$A \cap B$	(A occurs) AND (B occurs)
$A \cup B$	(A occurs) OR (B occurs)
A^c	NOT (A occurs)
$A \Delta B$	(A occurs) XOR (B occurs)

FIGURE 2. Table of Set Operations

Definition 6. Set Properties: There are some identities to know when dealing with sets:

Properties of Unions of Sets	
Commutative Property	$A \cup B = B \cup A$
Associative Property	$(A \cup B) \cup C = A \cup (B \cup C)$
Identity Property	$A \cup \varnothing = \varnothing \cup A$
Distributive Property	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

FIGURE 3. Properties of sets

3.3 Probability.

Theorem 1. *Axioms of Probability*: The probability P is required to follow the following axioms:

(1) $P(\Omega) = 1$
(2) $P(A) \in [0, 1]$, for all events $A \in F$
(3) If we have a sequence of pair-wise Mutually exclusive events then: $P(U_i, A_i) = \Sigma_i P(A_i)$

Remark. The triplet (Ω, F, P) is called a **probability space**.

Definition 7. Mutually Exclusive Events: 2 Events that cannot happen at the same time, or in other words the intersection of set A and set B is the null set. If the probability of both is anything other than null it is **mutually inclusive**.

$$A = [1, 2, 3], B = [5, 6]; A \cap B = \emptyset \text{ and } P(A \cap B) = 0$$

Theorem 2. *Addition Rule of Probability*: A rule for breaking apart a set union which works regardess of inclusivity/exclusivity:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Theorem 3. *Inclusion-Exclusion Principle*: States that the addition rule of probability can be extended however many times a union is applied between two sets and can continue ad infinitum:

$$P(A\cup B\cup C) = P(A)+P(B)+P(C)-P(A\cap B)-P(A\cap C)-P(B\cap C)-P(B\cap C)+P(A\cap B\cap C)$$

3.4 Fundamental Principle of Counting.

Theorem 4. *Fundamental Principle of Counting*: If there are p ways to do one thing and q ways to do another thing, there are $p \cdot q$ ways to do both.

3.5 Permutations.

Definition 8. Permutations(P): Order of individual events does matter. Permutations are written as:

$${}_nP_R = \frac{n!}{(n-R)!}$$

3.6 Combinations.

Definition 9. Combinations(C): Order of events doesnt matter. Combinations are written as:

$${}_nC_R = \frac{n!}{(n-R)!R!}$$