LECTURE 5 AND 6

ANISH SUNDARAM

Contents

5 Random Variables

5.1 Random Variables.

Definition 1. Random Variable(X): A variable whose value is the numerical output of a random event. There are many forms of Randvars but each is the output of some probability function.

5.2 Discrete Random Variables.

Definition 2. Discrete Random Variable: A random variable X is said to be discrete if it takes either finitely many or a countable many values.

Remark. Examples:

(1) Experiment: Toss a fair coin n times and let X be the number ofHeads.
(2) Experiment: The number X of accidents on I-95 during next week.

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(3) Experiment: Let X be the number of coin-tosses until the first Head comes up

Definition 3. Continous Random Variable: A random variable X is said to be discrete if it can take any number within a variable.

Remark. Examples include the exact time of an event happening or measurements.

5.2 Probability Mass Function.

5.3 The Bernoulli Distributions.

Definition 4. Bernoulli Trial: A defined single trial that is classified as:

(1) Binary Outcomes, ie Success or Failure
(2) Independent observations
(3) Fixed probability

Remark. Some times Bernoulli Randvars are called **Indicator Variables**

Definition 5. Probability Mass Function: An equation which defines the probability the X can take on a value, used for Bernoulli Trials:

$$P(X = x) = p^x(1 - p)^{1-x}$$

where X is the random variable and x is the specific value(1,0)

Definition 6. Bernoulli Distribution: A probabilistic deescription of the successes or failures of a single Bernoulli experiment/trial. Probability of Success(1) in a Bernouli trial is seen as p whereas probability of failure(0) is seen as $1 - p$

Definition 7. Mean and Variance of a Bernouli Distribution: Because the values a Bernoulli trial can have(1,0), we can see that the mean μ can be described as

$$E(X) = p$$

and the variance as

$$Var[X] = pq = p(1 - p)$$

5.4 Binomial Distributions.

Definition 8. Binomial Distribution: Distribution of the number of successes of a fixed amount(n) Bernoulli trials. For a binomial distribution the probability of individual events can be calculated using the formula:

$$P(X = x) =_n C_x p^x q^{n-x}$$

Definition 9. Mean and Variance of a Binomial Distribution: Because a Binomial distribution is multiple Bernoulli trials the mean can be found by

$$\mu = np$$

and the variance can be found by

$$\sigma^2 = np(1-p)$$

6 Discrete and Continous Random Variables

6.1 Expectations of Discrete Randvars.

Definition 10. Expected Value of Binomial Distribution: For a Binomial Distribution, the mean ends up being the expected value in that

$$E(X) = \mu = X \cdot p_x$$

across the differe'nt values resulting in:

$$E(x) = \Sigma x \cdot p(x)$$

6.2 Variance of Expectation.

Definition 11. Variance of Expected Value: The variance of the expected value depends on the variance of the distribution, which means that we get the relation

$$Var(X) = E[(X-\mu)^2] = E(X^2) - (E(X))^2 = E(X^2) - \mu^2$$

which is all derived from

$$\sigma^2 = \Sigma(x-\mu)^2 \cdot p(x)$$

from which we can obviously get

$$\sigma = \sqrt{Ver[x]}$$

6.3 Expectation of sums of Randvars.

Definition 12. Expectation of sums of Randvars: We have seen that

$$E[aX + b] = aE[x] + b$$

. From this we can get the result that

$$E[X+Y] = E[X] + E[Y]$$

and can be generally described as

$$E[X_1 + \ldots X_n] = E[X_1] + \ldots + EX_n$$

6.4 Cumulative Distribution Functions(CDF).

Definition 13. Cumulative Distribution Functions(CDF): Functions used to calulate the area under the curve left of an input, where area under the curve is equal to the probability in a distribution.

6.5 Probability Density Function(PDF).

Definition 14. Probability Density Function(PDF): Describes the shape of the distribution. Also tells us what the probability is at a specific point on the curve. The PDF must integrate from one such that

$$1 = \int_{-\inf}^{\inf} f_x(z_d z)$$

of which the limit bounds may change