Birla Institute of Technology & Science - Pilani, Hyderabad Campus

Second Semester 2013-2014 Test – 1 Date: 24.02.2014

CS F211 / IS F211: Data Structures & Algorithms

Type: Closed Book Time: 60mins Marks: 60

All parts of the same question should be answered together.

1.a. Derive the time complexity of the below algorithms, if possible, in O-notation.

Algorithm 1: [2 Marks]

```
i = 2048
while (i > 1) do begin
      print i
      i = floor(i/2)      /* comment: floor truncates to nearest integer */
end                         /* less than or equal to the value */
```

Algorithm 2: [3 Marks]

```
i = 1;
while (i < n) do begin
      print i
      i = i*i
end
```

Algorithm 3: [5 Marks]

Assume, procedure mit and procedure cmu take O(1) and O(1/n) unit of time, respectively.

```
algorithm what(n)
      begin
            if n = 1 then call mit()
            else begin
                  what(n-1)
                  call cmu(n)
            end
      end.
```

1.b. Algorithms A and B spend exactly TA(n) = $0.1n^2 \log_{10}n$ and TB(n) = $2.5n^2$ microseconds, respectively, for a problem of size n. Choose the algorithm, which is better in the Big-Oh sense, and find out a problem size n_0 such that for any large size $n > n_0$ the chosen algorithm outperforms the other. If your problems are of size $n \le 10^9$, which algorithm will you recommend to use? [4 Marks]

1.c. Devise an algorithm of $o(n^{0.66})$ complexity to find whether the given integer 'n' is prime or not.
Note: o in o(n) refers to small o but not big O. [5 Marks]

2.a. Suppose you are given an array A[1 .. n] of distinct integers. A pair (i, j) is said to be an inversion of A if i < j and A[i] > A[j]. Each permutation of A[1 .. n] will have k (≥ 0) inversions. Devise an algorithm of O(n logn) complexity to find the permutation of A[1 .. n] that has maximum number of inversions. [8 Marks]

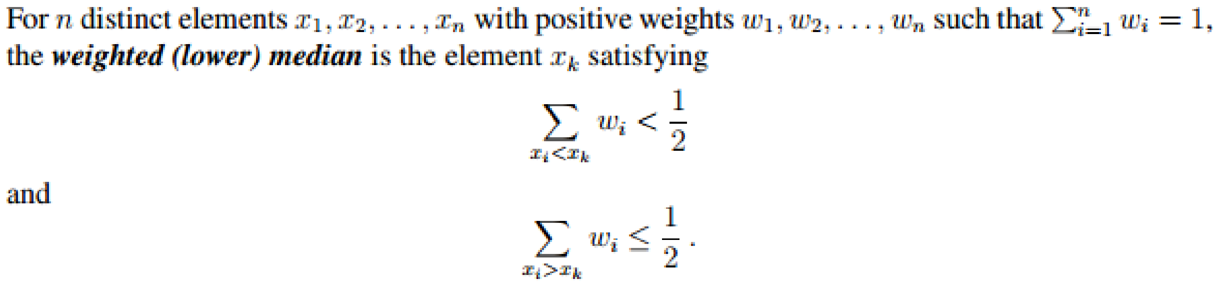
2.b. Suppose you are given an array A[1 .. n] of ‘n’ real numbers between 0 and 1. Design Bucket Sort algorithm to get ascending order of the given elements. And derive best, worst and average case of this algorithm. [8 Marks]

2.c. Suppose we are given sequence of n elements, each of which is an integer in the range [-3n/2, 5n/2]. Can there be an algorithm with o(n log n) complexity to sort given sequence of n elements in an non-increasing order. [5 Marks]
Note: o in o(n) refers to small o but not big O.

3.a. An array A contain n-1 unique integers in the range [0,n-1], that is, there is one number from this range that is not in A. Design an O(n)–time algorithm for finding that number. You are allowed to use only O(1) additional space besides the array A itself. [4 Marks]

3.b You are given ‘n’ bags of gold coins where in ‘n -1’ bags contain coins that each weigh 10 grams and the remaining one bag contains all false coins that weigh one gram less. Provide an algorithm by which you must identify this bag in just one weighing. You have a digital balance that reports the weight of what is placed on it. [8 Marks]
Note: You can assume that there are more than ‘n’ gold coins in each bag.

3.b. [8 Marks]

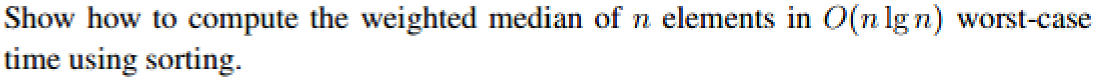


For n distinct elements $x_1, x_2, \ldots, x_n$ with positive weights $w_1, w_2, \ldots, w_n$ such that $\sum_{i=1}^{n} w_i = 1$, the ***weighted (lower) median*** is the element x_k satisfying

$$\sum_{x_i < x_k} w_i < \frac{1}{2}$$

and

$$\sum_{x_i > x_k} w_i \le \frac{1}{2} .$$



Show how to compute the weighted median of n elements in $O(n \lg n)$ worst-case time using sorting.