Birla Institute of Technology & Science – Pilani, Hyderabad Campus
Second Semester 2015-2016
CS F211: Data Structures & Algorithms
Comprehensive Examination

Type: Closed **Time: 60 mins** **Max Marks: 50** **Date: 26.02.2016**

1.a. Prove or disprove the following statements. Give a counter example if you are disproving. In other case make use of definitions of asymptotic notations only to prove the statement but not using any theorems derived in class:

(i). If $f(n) = O(g(n))$ and $g(n) = O(h(n))$, then $h(n) = \Omega(f(n))$. [2 Marks]

(ii). If $f(n) = O(g(n))$ and $g(n) = O(f(n))$ then $f(n) = g(n)$. [2 Marks]

1.b. State whether the following statements are True or False. You have to give mathematical / algorithmic reasoning in support of your answer.

(i). We can sort 7 numbers with 10 comparisons. You may use proof methodology of lower bound theorem for comparison based sorting algorithms but not the statement of the theorem to prove or disprove this statement. [1 + 2 Marks]

(ii) Merge sort can be implemented to be stable. [1 + 2 Marks]

1.c. Give the best lower bound that you can for the following code fragment, as a function of the initial value of n.

```
while (n > 1)
    if (ODD(n))
        n = 3 * n + 1;
    else
        n = n / 2;
```

Do you think that the upper bound is likely to be the same as the answer you gave for the lower bound? Note: In this problem best lower bound can be understood as it is lower bound for all inputs and for some input the algorithm attains the lower bound. [6 Marks]

2.a. Is it possible to devise a comparison based sorting algorithm on a sequence of $n^{0.99}$ elements that takes O(n) time in worst case. If so provide algorithm and prove its complexity in worst case. [6 Marks]

2.b. Modify the binary search routine to support search in an array of infinite size. In particular, you are given as input a sorted array and a key value K to search for. Call n the position of the smallest value in the array that is equal to or larger than K. Provide an algorithm that can determine n in O(log n) comparisons in the worst case. Explain why your algorithm meets the required time bound. [6 Marks]

2.c. Consider a recursive Mergesort implementation that calls Insertion Sort on sublists smaller than some threshold. If there are n calls to Mergesort, how many calls will there be to Insertion Sort? Why? [6 Marks]

3.a. Let Q be a non-empty queue, and let S be an empty stack. Using only the stack and queue ADT functions and a single element variable X, write an algorithm to reverse the order of the elements in Q. [2 Marks]

3.b. What is the average complexity of Bucket Sort and derive it completely. [8 Marks]

3.c. A program, Prog1, written by one of the programmer in an IT organization uses an implementation of the sequence ADT (data structure like an array, linked list) as its main component. It performs atRank, insertAtRank and remove operations in some unspecified order. It is known that Prog1 performs n^2 atRank operations, 2n insertAtRank operations, and n remove operations. Which implementation of the sequence ADT should the programmer use in the interest of efficiency: the array-based one or the one that uses a doubly-linked list? Support your answer with the help of asymptotic notation and analysis. [6 Marks]