Determining the Decay Parameters of ^{22}Na-sourced Positronium in Air at Atmospheric Pressure

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Abstract

Our experiment used the β^+ decay of the unstable sodium isotope ^{22}Na as positronium source. Using thallium(Th)-doped sodium iodide(NaI) scintillators connected to a discriminator and time to amplitude converter, we were able to observe the delay between gamma rays associated with the creation and annihilation of the positron and positronium in the ^{22}Na decay, which we then used to observe the decay time of positronium events, which was plotted in a histogram. The histogram was fitted to the differential equation solution $N(t) = N_0 e^{-\lambda t}$ in order to determine the decay constant, λ, which we found to be $53 \pm 3\mu s^{-1}$ for positronium in air at 1 atm of pressure. This decay constant was then used to determine the half-life of positronium in air, which was shown to be 13 ± 1ns. The peak of the histogram was found to be at 45 ± 4ns, which corresponds to the most likely decay time for parapositronium.

Introduction

Positronium is an onium type exotic atom composed of an electron (e^-)bound with its antiparticle, the positron (e^+). This unique combination of provides an invaluable tool to study quantum electrodynamics (QED), as the pair of leptons allows the isolation of leptonic properties and photon fields without the much more massive nucleus.

The antielectron, or positron, was first conceptualised by Paul Dirac in 1927. His famed Dirac equation postulated that electrons were allowed both positive and negative energy as solutions to the equation. Experimentally, the positive energy solution was verified via the electron, but mathematically, the negative energy solution could be just as valid.[4] In 1931, building off of the controversy created by the negative energy solution to the Dirac equation, Dirac

theorised a new particle called an "anti-electron" with the equivalent, mass angular momentum, and wave properties as the electron, but with positive charge and thus negative energy.[5] In 1929, Dmitri Skobeltsyn first observed the positron while trying to detect gamma radiation in cosmic rays in a Wilson cloud chamber by noticing that there seemed to be an electron-like particle that curved the opposite way in an external magnetic field.[6] In 1932, Carl Anderson repeated this experiment with the intention of exploring this antielectron, the first evidence of antimatter, for which he won the 1936 Nobel Prize in Physics.[7,8]

Positronium, the bound state of the electron and positron, was first observed by Martin Deutsch at MIT in 1951. Based on his work, positronium has been used extensively in QED research.[1] Excited states of positronium are being used to further explore the electro-weak force and other QED interactions. Positronium has been shown to form bonds with larger molecules to form complex compounds, such as PsH, or positronium hydride. These interactions have been invaluable to radiation chemistry research. The study of positronium behaviour has been vital to research in charge conjugation, parity, and time-reversal (CPT) symmetry and invariance.

Our experiment used the β^+ decay of the unstable sodium isotope ^{22}Na as positronium source. Using thallium(Th)-doped sodium iodide(NaI) scintillators, we were able to observe the gamma rays associated with the creation and annihilation of the positron and positronium in the ^{22}Na decay, which we then used to observe the decay time of positronium events, which was plotted in a histogram.

Theoretical Background

β^+ Decay

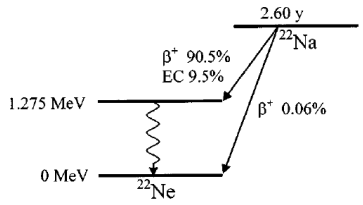

Figure 1: The β^+ decay of ^{22}Na.

The unstable isotope ^{22}Na undergoes what is known as β^+, or *positron emission*, decay, in which a proton decays into a neutron, releasing a positron and neutrino (Figure 1). ^{22}Na decays into a 1.275 MeV excited state of ^{22}Ne, a positron, and a neutrino. The ^{22}Ne quickly decays into its stable ground state,

emitting a "high energy" 1.275MeV gamma ray in the process. [3] As noted in the Experimental Procedure, this gamma photon detection is used as the start signal for our timer mechanism, alerting us that a free positron has been formed.

The binding energy of the daughter atom must exceed that of the original nucleus by at least $m_n c^2 - m_p c^2 + m_e c^2$, or 2.511 MeV, in order to observe β^+ decay. (Note that m_n is neutron mass, m_p is proton mass, and m_e is electron mass.) The binding energy of ^{22}Na, 174.15 MeV, subtracted from the binding energy of ^{22}Ne, 177.78 MeV, yields a binding energy difference of 3.63 MeV, which exceeds the minimum binding energy difference and allows for ^{22}Na β^+ decay.[2] Additionally, as a positron is emitted from the decay of the original nucleus, an electron must be shed from the outer orbital of the daughter atom in order to balance the charge. This means that atleast two electron masses are shed in the process of β^+ decay and the resulting daughter atom must be atleast two electron masses (1.022 MeV) lighter than the original atom.[9] Thus, these two energy-mass conditions must be met for β^+ decay to occur.

Positronium Formation

The positron resulting from the β^+ decay is quite energetic. According to *Ore Gap* theory, certain energy conditions must be fulfilled in order for an electron and positron to bind into positronium. In order to maintain a bound state with the electron and then undergo annihilation, the energy of positronium after formation, E_{Ps}, must be less than the dissociation energy of positronium, I_{Ps}. Thus, as $E_{Ps} < I_{Ps}$, the emitted positron kinetic energy, E_{e^+}, must be within limits in order for the positron to bind with an electron and produce positronium. The high energy positron thus undergoes inelastic collisions in the gaseous medium, in our case, air, until it slows to an energy sufficient for electron capture. Neglecting kinetic energy, the energy of the resulting positronium is then E_{Ps} such that $E_{Ps} = E_{e^+} - I_M + I_{Ps}$, where I_M is the ionization energy of the gas molecule. This relationship demonstrates that the energy of the free positron must be less than the ionization energy, i.e. $E_{e^+} < I_M$. However, the energy must be sufficient as to allow the positron to capture an atomic electron from its host, thus $E_{e^+} > I_M - I_{Ps}$ Positrons with energies below this threshold are unable to free an atomic electron in order to form a bound positronium atom and proceed to elastically interact with the medium until eventually annihilating with a free electron.[3]

Positronium Properties

There are two possible bound states for ground-state positronium after it has formed. These states are related to the relative spin states of the positron and electron in the positronium atom. *Orthopositronium* is a triplet state of positro-

nium characterised by parallel spin states between the positron and a total spin of $s = 1$. Conversely, *parapositronium* is a singlet state of positronium characterised by antiparallel spins between the positron and electron and a total spin of $s = 0$.[3] Significant overlap in the positron and electron wave functions lead the positron to eventually annihilate such that all of the positronium energy is converted into gamma rays. The observation of these gamma rays, as described in the experimental procedure, was our stop signal for the decay time measurements. The number of gamma rays emitted from annihilation corresponds to the total spin s via the charge conjugation selection rule:

$$(-1)^{l+s} = (-1)^n \tag{1}$$

where l is the relative orbital angular momentum, and n is the number of gamma rays emitted through the annihilation process. l must be zero in the ground state of positronium, therefore the number of gamma photons released depends thoroughly on the total spin number. Thus, orthopositronium must always decay into an odd number of photons while parapositronium must always decay into an even number of photons. As the release of a single gamma photon would violate conservation of momentum and energy, the most likely decay scenario for orthopositronium becomes three gamma photons, while parapositronium decays most probably into two gamma rays of energy 0.511 MeV each which travel in opposite directions, reflecting the mass-energy of the positron and electron pair. The orthopositronium decay also divides the total positronium energy among the the emitted photons; the photons emitted through orthopositronium decay exhibit a continuous energy distribution. The process of positronium decay, leading to multiple photon emission, reflects in the relatively long lifetimes of bound positron-electron pairs, with orthopositronium having a longer lifetime than parapositronium. Spin-exchange interactions with certain surrounding gaseous media with two available spin states (such as oxygen), can, however, cause spin changes in the positron or electron in the bound pair, thus converting orthopositronium into parapositronium in a process referred to as "pick-off". The resulting parapositronium decays quicker than the orthopositronium, reducing the observed lifetime of the orthopositronium.[1]

The random decay of positronium can be modeled with a differential equation relating the decay rate $-dN/dt$ as a function of the population of the positronium as a function of time. The differential equation can then be modeled as

$$\frac{-dN}{dt} = \lambda N(t) \tag{2}$$

where $N(t)$ is the population of the positronium as a function of time and λ is the decay constant of the positronium in the medium, given in units of time^{-1}.[2] The decay constant represents the rate of the exponential decay of the positronium, an intrinsic property of the positronium in the medium. The

solution to Equation 2 can then be given as:

$$N(t) = N_0 e^{-\lambda t} \tag{3}$$

where N_0 is the initial population of the positronium atoms in a sample.[2] This exponential function was later fitted on a histogram of measured decay times in order to solve for the decay constant. In our analysis, N_0 was arbitrary and inconsequential, as our ^{22}Na sample allowed us a steady rate of positronium creation. The half-life of the positronium $t_{1/2}$ can also be related to the decay constant as:

$$e^{-\lambda t_{1/2}} = 1/2 \tag{4}$$

[2]

Experimental Procedure

The decay of a small sample of ^{22}Na provided the constant source of positrons for our experiment. The sample was surrounded by two thallium-doped sodium iodide scintillators with photomultiplier tubes (PMTs), which were used to detect the gamma photons corresponding to the creation and destruction of the positronium. The scintillators were each powered by a respective 1.1 kV high voltage power supply. The less sensitive scintillator was used to detect the high energy gamma released from the decay of ^{22}Ne to its ground state. The high sensitivity scintillator was used to detect the lower energy photons from the annihilation of positronium. The lower sensitivity scintillator, was also connected to a fast amplifier in order to boost the low amplitude signal from the PMT. The scintillators were then both fed to a 300 MHz discriminator, with the respective channel thresholds set to represent the high energy and low energy gamma photons. This is reflected in the experimental block diagram shown in Figure 2. The discriminator outputted a pulse each time the input voltage exceeded the set threshold, allowing us to count the number of events that exceed the threshold using a timer/counter unit. Using the counter display, we were able to effectively set the voltage thresholds by looking for the peaks in the count for the high energy gamma and then the low energy, 0.511 MeV gamma. The discriminator channels were then connected respectively to the start and stop inputs on the Time to Amplitude Converter (TAC). The TAC served to create a voltage pulse with a magnitude reflecting the time delay between the start and stop signals. The TAC output was then sent through an Analogue to Digital Converter (ADC) and then in through an interface to the GammaVision software on our computer, which was set to record a histogram of counts versus "bins" corresponding to delay times. The TAC signal, and therefore the bin size in the GammaVision software were calibrated by attaching a time calibrator to the TAC temporarily. The time calibrator would create delayed signals with a

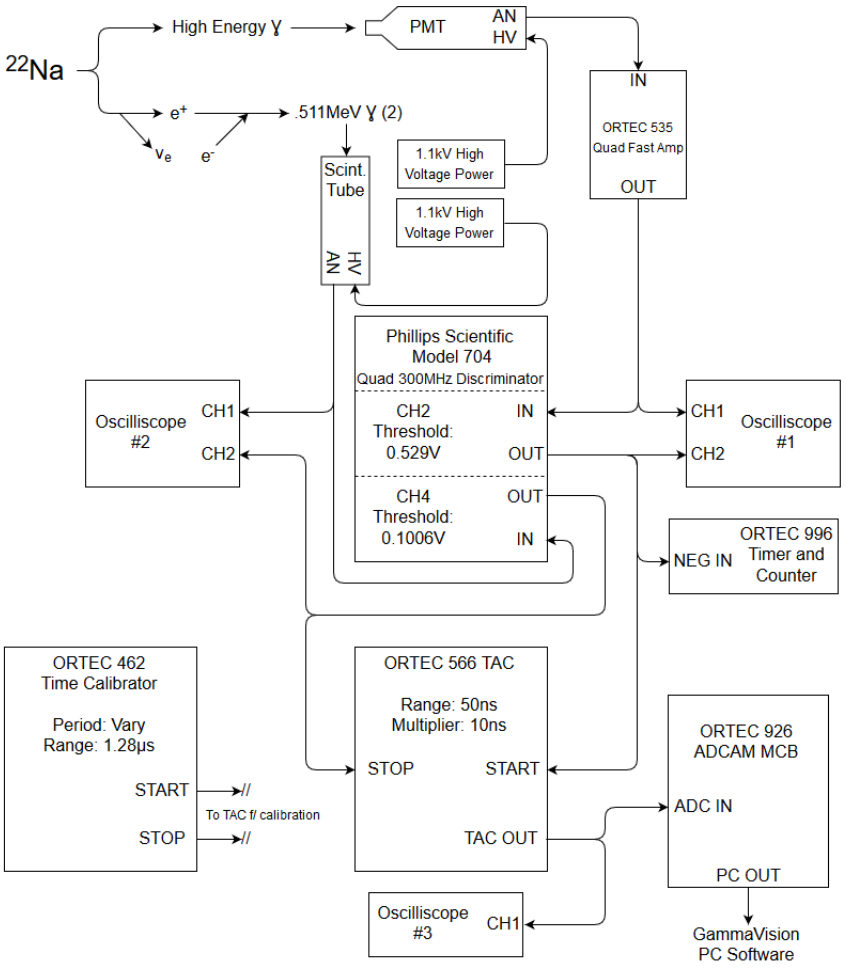

Figure 2: A block diagram of our experimental setup, including threshold voltages for the discriminator.

known spacing, which appeared as spikes on our GammaVision software. The spacing between bins could then be compared to the known spacing in the calibrator signal, allowing us to convert the raw bin data into real delay times. As shown in Figure 2, we also placed oscilloscopes at various points along the experimental circuit in order to visually monitor the experimental process.

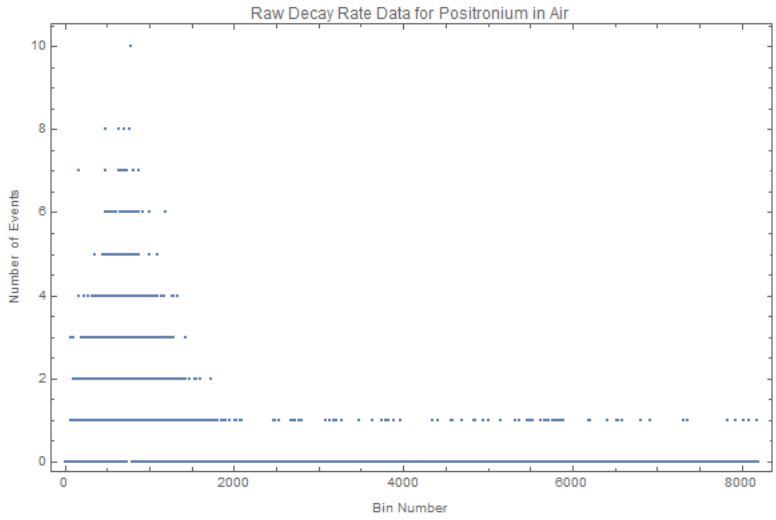

Figure 3: The raw data from the GammaVision software: a histogram of counts versus bins corresponding to delay times.

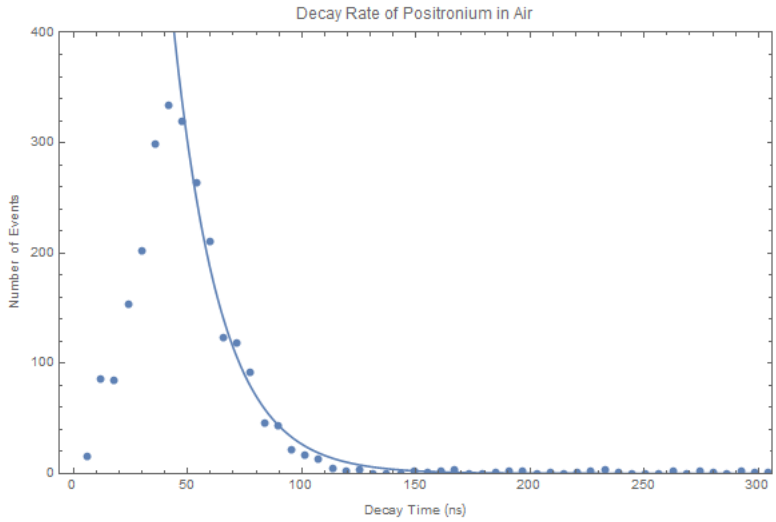

Figure 4: A histogram of number of events versus decay time. The peak represents the most likely parapositronium decay time. The fit of Equation 3 represents the decay of orthopositronium.

Data Analysis and Discussion

The calibration results, using various periods at a range of 1.28μs, demonstrated that 1μs was the equivalent of 16764.3 ± 0.1bins. Therefore the individual bins in GammaVision each corresponded to approximately 0.05965ns. The positron decay times of the events observed were plotted by the GammaVision software over the course of our two data runs. This data was in the form of a histogram of number of events per observed delay bin (Figure 3). We first consolidated these data points into bins of step size approximately 5.96 ± 0.06ns, which provided us with a far more consistent curve at the expense of a slight loss in precision, as shown in Figure 4. The curve qualitatively appeared to follow a Gaussian distribution with a peak around 45 ± 4ns, which we assumed to be the most likely decay time for parapositronium in air. However this was not conclusive and was not of concern for this experiment. Rather, we were concerned with the decay rate of the orthopositronium atom. According to Equation 3, the solution to the orthopositronium population differential equation, the falling slop of the data should fit an exponential curve with a decay constant λ. To fit the falling slope of our data, we omitted all data points before our peak of 45ns. Fitting the resulting curve to Equation 3, we obtained a decay constant(λ) value of $0.059 \pm 0.003\text{ns}^{-1}$ for our first, one-day data run, and $0.048 \pm 0.002\text{ns}^{-1}$ for our second, longer data run. Averaging the two, we obtained an estimated decay constant of $53 \pm 3\mu\text{s}^{-1}$, which we then input into Equation 4 in order to express the mean decay "half-life" quantity in a standardised unit of time. We found this quantity to be 13 ± 1ns, which we concluded is the half-life of orthopositronium in air. With approximately a 21% concentration of oxygen in air, we expected to see a large decay constant due to "pick off" from the oxygen, and this is reflected in our data quite well. Due to lack of positronium decay data in a homogeneous, non-oxygenated gas, we were unable to compare this decay constant in air quantitatively to a decay constant without "pick off" decay, however, the accepted value for the orthopositronium decay constant in 1 atm of pressure in a vacuum is approximately $7.038\mu\text{s}^{-1}$ ADD REFERENCE TO MANUAL, and our decay constant of $53\mu\text{s}^{-1}$ in air at approximately 1 atm of pressure is undoubtedly larger.

Error and Uncertainty

The largest source for error and uncertainty in this experiment was due to a quantitative lack of data points. Our rate of positronium events was quite small, with an event frequency of 1 event every 5 minutes or so. Even with our 24+ hour data runs, we encountered quite a bit of noise and uncertainty in our analysis. We attempted to counteract this by leaving the experiment running over the course of a weekend, however upon returning the following week, we discovered our fast amplifier had been turned of at some point over the weekend, leaving us with a similar data set to our first run. The bins created

by the GammaVision software in which to sort our data were quite precise, with each bin representing a step size of approximately .06ns. However, this was far too precise for our limited data, so we consolidated the bins into larger groupings of 100 small bins each, a step size of approximately 6ns, by totaling the counts for each individual bin in the range. We lost about three orders of magnitude of precision, however, our data was quite a bit more functional and manipulable, allowing us to fit and solve for the decay constant. The error in the constant was then simply the error in our fitted model for the orthopositronium decay. In spite of our limited data, we were able to quite conclusively derive the decay constant of orthopositronium in air to an precision of within 3 μs^{-1}. Our data, however, does not represent an accurate portrayal of pure orthopositronium decay, as we were unable to find a homogeneous, non-oxygenated, gas or a vacuum in which to conduct our experiment. The air, with a large oxygen concentration, led to "pick off" decay in which the orthopositronium decayed as parapositronium, thus skewing our decay constant to a larger value. We were unable to quantify this "pick off" decay to compensate as we had to other decay data recorded in order to compare.

Conclusion

Using thallium(Th)-doped sodium iodide(NaI) scintillators, we were able to observe the gamma rays associated with the creation and annihilation of the positron and positronium in the β^+ decay of the unstable sodium isotope ^{22}Na. The delay between the high energy gamma ray associated with ^{22}Ne decay that coincided with the creation of the positron and the low energy gamma ray associated with the annihilation of positronium was then plotted as a histogram of counts versus delay time. We then used this data to deduce the decay parameters of orthopositronium in air. We found the decay constant to be $53 \pm 3 \mu s^{-1}$ for positronium in air at 1 atm of pressure. The half life of positronium in air was then computed to be 13 ± 1ns. This experiment could have been improved through the use of a homogeneous gaseous medium in which to observe the orthopositronium decay, which could be compared to the decay in air to determine the "pick-off" rate of the orthopositronium. The homogeneous gas in a controlled container would also allows us to explore the effects of density and pressure on the positronium decay rate. Furthermore, the insufficient amount of data could be alleviated through the use of a larger ^{22}Na sample or other, faster, positron source coupled with longer data runs. However, our results still proved conclusive.

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