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grupa 3.1

The spring elastic constant evaluation

1. Purpose

The objective of the experiment is to determine the spring constant of a spiral spring using Hooke's law and the period of oscillatory motion in response to a weight.
Apparatus: A spiral spring, a set of weights, a weight hanger, a stop watch, and a lab scale.

2. Theory

A. Static method

We use a spiral spring with elastic constant k and undeformed length l_o and bodies with different mass.

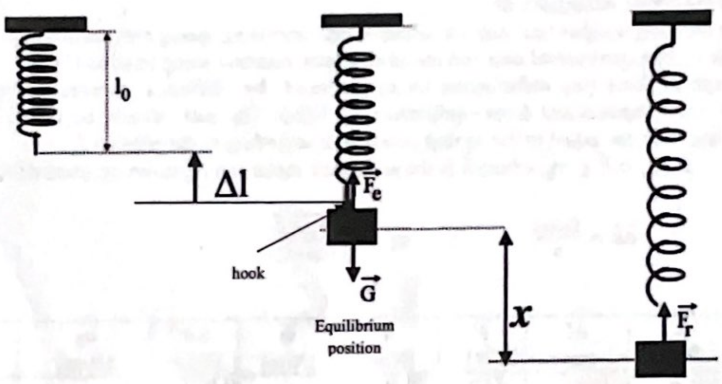

Fig. 1.

When a spring is stretched, according to Hooke's law, a restoring force F proportional to its elongation, x (or $\Delta l = l - l_o$) appears. Every spring obeys the Hooke's law if the deformation is not too great.

$$\vec{F}_e = -k\overrightarrow{\Delta l} \quad (1)$$

For the equilibrium

$$\Rightarrow mg = k\Delta l \quad (2)$$

$$k = \frac{mg}{\Delta l} \quad (3)$$

B. Dynamic method

When we move the body connected to a spring from its equilibrium position it starts to oscillate around the equilibrium position under the action of restoring (elastic) force. With x the distance from equilibrium position the Newton's second law is:

$$\mathrm{ma} = -\mathrm{kx} \quad (4)$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \quad (5)$$

We note $\omega = \sqrt{\frac{k}{m}}$ and we call it the natural angular frequency. The second Newton law for the spring (5) is a differential equation having the solution

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CTI engleză
An I
grupa 3.1.

$$x(t) = A\sin(\omega t + \varphi) \tag{7}$$

The period for the harmonic oscillation is connected to the natural angular frequency throught:

$$\omega = \frac{2\pi}{T} \tag{8}$$

$$k = m\omega^2 = 4\pi^2 \frac{m}{T^2} \tag{9}$$

3. What to do

Static method

1) We measure the undeformed spring length l_o.
2) We hang the weight hanger with mass m_l on the spring and we measure the deformed spring length l_l. We calculate the spring elongation Δl_1.
3) Successively we hang weights (m_i) and we calculate corresponding spring elongations Δl_i.
4) Use the Table 1. for experimental data and calculate elastic constant using relation (3).
5) Plot the graph of force (the deformative force) produced by different masses ($F=m\cdot g$) as a function of the displacement from equilibrium Δl: **F(Δl)**. The data should be linear. Hence, the slope of the line will be equal to the spring constant k according to the relation 2.
6) Final result: $k_{true} = \bar{k} \pm \sigma_{\bar{k}}$, where $\bar{k}$ is the arithmetic mean and $\sigma_{\bar{k}}$ is the standard deviation of the mean.

$\Delta k_i = k_i - \bar{k}$ $\qquad \overline{\Delta k} = \frac{\sum|\Delta k_i|}{n}$ $\qquad \sigma_{\bar{k}} = \sqrt{\frac{\sum_1^n (\Delta k_i)^2}{n(n-1)}}$

Table 1.

$l_0 =$ 44 cm

Nr. crt	m [kg]	l [cm]	Δl [m]	F [N]	k [N/m]	$\bar{k}$ [N/m]	Δk [N/m]	$\sigma_{\bar{k}}$ [N/m]	k_{true} [N/m]
1	0.05	50.5	0.065	0.49	7.538	7,1283	0.4097	0,0686	K_{true_1} = 7,1969 K_{true_2} = 7,0597
2	0.066	53.5	0.095	0.64	6.736		-0.3923		
3	0.083	55	0.11	0.81	7.36		0.2317		
4	0.091	56.5	0.125	0.89	7.134		0.0057		
5	0.099	57.5	0.135	0.97	7.186		0.0577		
6	0.105	58.5	0.145	1.029	7.096		-0.032		
7	0.134	62.5	0.185	1.313	7.0983		-0.03		
8	0.178	68.5	0.245	1.744	7.12		-0.083		
9	0.207	73	0.29	2.028	6.995		-0.133		
10	0.24	77.5	0.335	2.352	7.02		-0.1083		

Dynamic method

1) Hang the weight hanger with several weights (mass m_l) on the spring and set the equilibrium position of the system.
2) Pull the system out of its equilibrium position to make oscillations with 1-2cm amplitude.
3) Record the time for n=20 oscilations and find the period: $T = t/n$.
4) Repeat 1), 2) and 3) for different masses.
5) Complete the Table 2 using relation (9) for elastic constant.
6) Plot the graph of $T^2(s^2)$ as function of m(kg). The data should be linear. Find elastic constant from the slope (i.e. $T^2 = \frac{4\pi^2}{k} m \Leftrightarrow y = \text{slope} \cdot x$).
7) Final result: $k_{true} = \bar{k} \pm \sigma_{\bar{k}}$, where $\bar{k}$ is the arithmetic mean and $\sigma_{\bar{k}}$ is the standard deviation of the mean.

$\Delta k_i = k_i - \bar{k}$ $\qquad \overline{\Delta k} = \frac{\sum|\Delta k_i|}{n}$ $\qquad \sigma_{\bar{k}} = \sqrt{\frac{\sum_1^n (\Delta k_i)^2}{n(n-1)}}$

Pîrvulescu Antonia

CTI engleză

An I

grupa 3 1.

Table 2.

Nr. crt.	m [kg]	t [s]	n	T [s]	T^2 (s^2)	k [N/m]	$\bar{k}$ [N/m]	Δk [N/m]	$\sigma_{\bar{k}}$ [N/m]	k_{true} [N/m]
1	0.06	6.834	12	0.569	0.323	7.326	7,104	0,222	0,0531	k_{true_1} = 7,1571
2	0.09	8.602	12	0.716	0.512	6.932		-0,172		k_{true_2} = 7,0509
3	0.12	9.717	12	0.809	0.654	7.236		0,132		
4	0.15	11.1	12	0.925	0.855	6.919		-0,185		
5	0.18	11.934	12	0.994	0.988	7.185		0,081		
6	0.21	12.885	12	1.073	1.151	7.195		0,091		
7	0.24	13.952	12	1.162	1.35	7.011		-0,093		
8	0.27	14.77	12	1.231	1.515	7.028		-0,076		

Compare the results obtained by the 2 methods, respectively by arithmetic and graphic mediation !!

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Static method – calculus

1) $\Delta l_1 = l_1 - l_0 = 50.5 - 44 = 6{,}5\,cm$
$= 0{,}065\,m$

2) $F_1 = m_1 \cdot g = 0{,}05 \cdot 9{,}8 = 0{,}49\,N$

3) $k_1 = \frac{m_1 \cdot g}{\Delta l_1} = \frac{F_1}{\Delta l_1} = \frac{0{,}49}{0{,}065} = 7{,}538\,N/m$

4) $\bar{k} = \frac{k_1 + k_2 + \dots + k_{10}}{10} = \frac{(7{,}538 + 6{,}736 + 7{,}36 + 7{,}134)}{10}$

$+ \frac{7{,}186 + 7{,}096 + 7{,}0983 + 7{,}12 + 6{,}995 + 7{,}02}{10} =$

$= \frac{71{,}283}{10} = 7{,}1283\ N/m$

5) $\Delta k_1 = k_1 - \bar{k} = 7{,}538 - 7{,}1283 = 0{,}4097\ N/m$

6) $\sigma_{\bar{k}} = \sqrt{\frac{\sum_1^n (\Delta k_i)^2}{n(n-1)}}$

$\sum_1^n (\Delta k_i)^2 = 0{,}1678 + 0{,}1538 + 0{,}053 + 0{,}000032 + 0{,}00332$
$+ 0{,}001 + 0{,}009 + 0{,}0068 + 0{,}0176 + 0{,}0117 = 0{,}424$

$\Rightarrow \sigma_{\bar{k}} = \sqrt{\frac{0{,}424}{9 \cdot 10}} = \sqrt{0{,}0047} = 0{,}0686\ N/m$

7) $k_{true} = \bar{k} \pm \sigma_{\bar{k}}$

$\Rightarrow k_{true_1} = 7{,}1283 + 0{,}0686 = 7{,}1969\ N/m$

$k_{true_2} = 7{,}1283 - 0{,}0686 = 7{,}0597\ N/m$

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The dynamic method – calculus.

1) $T_1 = \frac{t_1}{n} = \frac{6,834}{12} = 0,569\ s$

2) $T_1^2 = 0,569 \cdot 0,569 = 0,3235^2$

3) $\omega_0^2 = \frac{k_e}{m}$, $\omega_0 = \frac{2\pi}{T}$

For $m_1 = 0,06$ $\Rightarrow$ $k_1 = \frac{4\pi^2 m_1}{T_1^2} = \frac{4 \cdot 3,14 \cdot 3,14 \cdot 0,06}{0,323} =$

$= \frac{2,366}{0,323} = 7,326\ N/m$

4) $\bar{k} = \frac{7,326 + 6,932 + 7,236 + 6,919 + 7,185 + 7,195 + 7,011 + 7,028}{8}$

$= 7,104\ N/m$

5) $\Delta k_1 = k_1 - \bar{k} = 7,326 - 7,104 = 0,222\ N/m$

6) $\sigma_{\bar{k}} = \sqrt{\frac{\sum_1^n (\Delta k_i)^2}{n(n-1)}}$

$\sum_1^n (\Delta k_i)^2 = \frac{0,049 + 0,029 + 0,0174 + 0,034 + 0,0065 + 0,0082 + 0,0086}{7 \cdot 8}$

$+ \frac{0,0057}{7 \cdot 8} = \frac{0,1584}{7 \cdot 8} = 0,00282\ N/m$

$\Rightarrow \sigma_{\bar{k}} = \sqrt{0,00282} = 0,0531\ N/m$

7) $k_{true} = \bar{k} \pm \sigma_{\bar{k}}$

$k_{true_1} = \bar{k} + \sigma_{\bar{k}} = 7,104 + 0,0531 = 7,1571\ N/m$

$k_{true_2} = \bar{k} - \sigma_{\bar{k}} = 7,104 - 0,0531 = 7,0509\ N/m$

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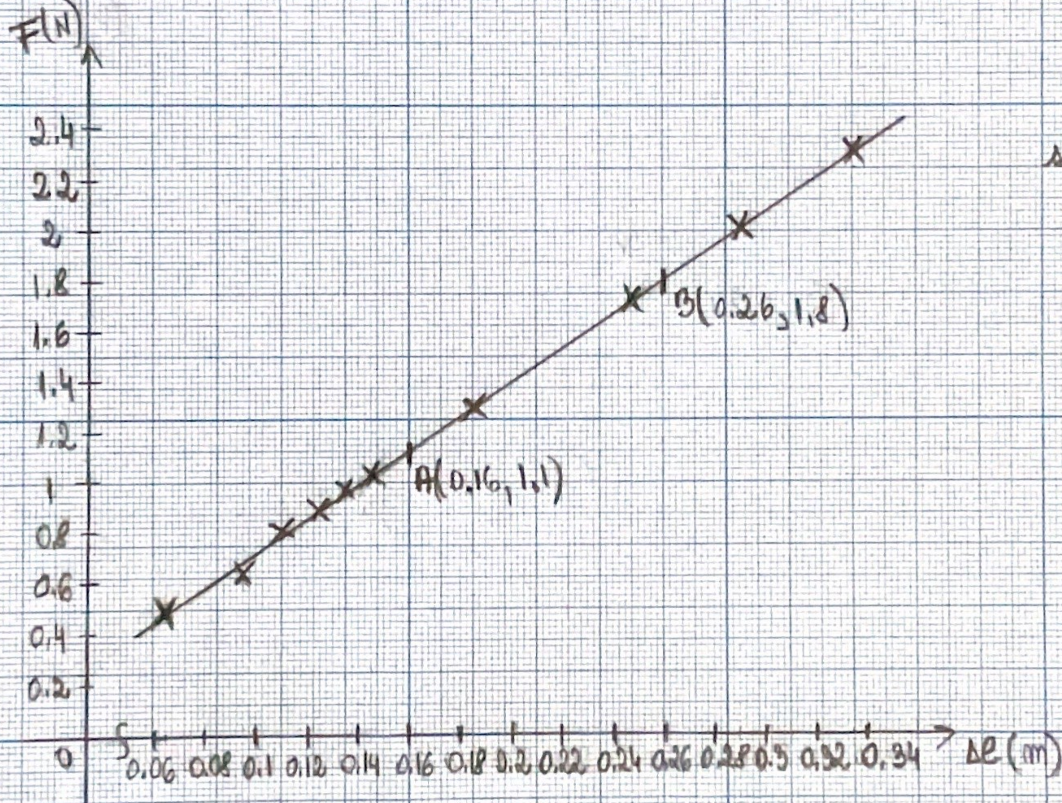
— static method —



$$\text{slope} = \frac{F_B - F_A}{\Delta l_B - \Delta l_A} = \frac{1.8 - 1.1}{0.26 - 0.16} = \frac{0.7}{0.1} = 7$$

$$\Rightarrow k_e = 7 \ N/m$$

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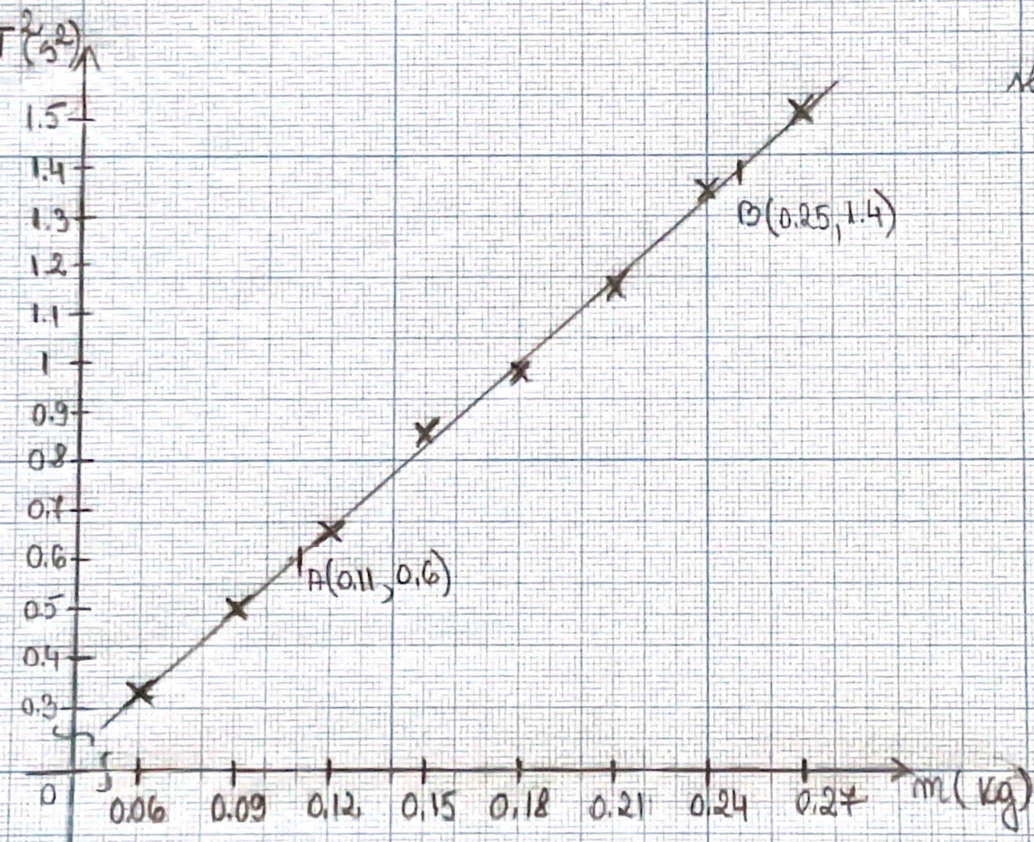
-dynamic method-



$$\text{slope} = \frac{T_B^2 - T_A^2}{m_B - m_A} = \frac{1.4 - 0.6}{0.25 - 0.11} = \frac{0.8}{0.14} = 5.71428 = \frac{4\pi^2}{k}$$

$$\Rightarrow k = \frac{4\pi^2}{\text{slope}} = 6.90172 \text{ N/m}$$