ARE YOUNG GALAXIES VISIBLE?

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ABSTRACT

The purpose of this paper is to assess the general possibility of observing distant, newly formed galaxies. To this end a simple model of galaxy formation is introduced. According to the model galaxies should go through a phase of high luminosity in early stages of their evolution. The estimated luminosity for a galaxy resembling our own is $\sim 3 \times 10^{46}$ ergs/sec, roughly 700 times higher than the present luminosity. The bright phase would occur at an epoch of about 1.5×10^8 years, corresponding to a redshift between 10 and 30, depending on the cosmological model assumed.

The possibility of detecting individual young galaxies against the background of the night sky is discussed. Although the young galaxies would be numerous and would have sufficiently large angular diameters to be easily resolved, most of the radiation from the young galaxies would arrive at wavelengths of 1–3 μ where detection is difficult. However, it seems possible that the Lyman-α line might be detected if it is a strong feature of the spectra of young galaxies.

It is also shown how such an experiment might help us to distinguish between various cosmological models.

I. INTRODUCTION

The galaxies are thought to have formed from gaseous hydrogen originally distributed more or less uniformly throughout the expanding universe, but there is very little observational evidence on how or when the galaxies formed, or how they evolved. Some idea of the history of our own Galaxy is given by the details of its structure, but this is at best fragmentary, so it is important that some general information might be obtained from observations of very distant galaxies. The assumption here is that the universe is homogeneous, so that the light from very distant galaxies would reveal the universe as it was at earlier times. The observation might be a measurement of the integrated infrared background from distant galaxies (McVittie and Wyatt 1959; Whitrow and Yallop 1964), or it might be an attempt to pick out individual sources against the local background.

Our purpose in this paper is to discuss the general possibility of detecting individual galaxies in an early stage of evolution. The discussion is in two parts, the first being a description of how the galaxies may have formed and evolved. This necessarily involves a number of assumptions, most of which are conventional but nevertheless have little or no connection with available observations. However, we believe that we must present a fairly definite model, and in some detail, if we are to evaluate the possibility of future experiments and, indeed, if we are to understand how the experimental results might be interpreted. In the second part of this article we derive from the model the expected properties of the young galaxies as they would now be observed. Finally, we consider the possibility of detecting the light from these distant galaxies against the local background.

For convenience, we list below some of the important results of the model presented in the following sections.

1. Young galaxies go through a stage of high luminosity, commencing at an epoch of $\sim 1.5 \times 10^8$ years and lasting on the order of 3×10^7 years. The redshift of the young galaxies would be in the range $z + 1 = 10$–30.

2. The luminosity of a galaxy like our own during this phase is a few times 10^{46} ergs/sec.: for definiteness, we have assumed a value of 3×10^{46} ergs/sec. This luminosity corresponds to the conversion of 2 per cent of the hydrogen to heavier elements in 3×10^7 years.

3. The Lyman-α flux from these distant galaxies might conceivably be as much as 6–7 per cent of the total radiation. The line width is estimated to be $2(1 + z)$ Å ~ 20 Å.

The observations might be completely frustrated if there is an appreciable density of ionized intergalactic matter (Field 1959; Gunn and Peterson 1965). Using the formula given by Bahcall and Salpeter (1965) we find that, if the dominant mass density in the universe is ionized intergalactic hydrogen, the optical depth for scattering of the light from the young galaxies by free electrons would reach unity at a redshift $z + 1 = 8$ for an acceleration parameter $q_0 = \frac{1}{2}$, and at a redshift $z + 1 = 5$ for an acceleration parameter $q_0 = 1$. Therefore, it is also worthwhile to consider the possibility of a search above the atmosphere for the integrated light from the galaxies. This test would not be affected by the free electron scattering. We will discuss in a separate paper the expected background from the galaxies and the possible implications of this simpler experiment.

II. THE FORMATION AND EVOLUTION OF THE GALAXIES

a) Instability Model for the Formation of Galaxies

In this section we present a simple picture of galaxy formation, from which we can derive in the following sections two important parameters: the time of formation of the galaxies and the luminosity of the young galaxies. The discussion is not carried beyond this limited goal. We will be directing considerable attention to the formation of our own Galaxy, because we have some useful observational evidence here: evidently it is an additional and uncertain assumption that the Galaxy is typical. We shall adopt conventional general relativity, without the cosmological term, and we assume that the universe is homogeneous and isotropic when averaged over the clusters of galaxies. This is not adequately established by direct observations (galaxy counts and redshift–apparent-magnitude measurements) although the primordial fireball (Dicke, Peebles, Roll, and Wilkinson 1965), if it exists, provides a good measure of isotropy over the visible universe, and so excellent evidence of homogeneity in the large (Peebles 1965*b;* Wilkinson and Partridge 1966).

We assume that the galaxies formed due to the gravitational instability of the nearly homogeneous and uniformly expanding distribution of gas which filled the early universe. That is, in regions where the gas density happened to be somewhat higher than average, the gravitational attraction would have slowed the rate of expansion relative to surrounding regions, so that the perturbation would have grown more pronounced with time, until eventually the material stopped expanding and collapsed more or less radially to a bound system. This instability picture seems to be suggested by the existence of groups and clusters of galaxies (Oort 1958; Ambartsumian 1958), but it has been criticized on the grounds that the universe is in fact stable against the growth of perturbations which are so small that the geometry is not strongly perturbed (Lifshitz 1946). It has been shown also that thermal fluctuations in the mass distribution taken at a time when the mean density of matter in the universe was equal to nuclear density would not have had time to have grown into galaxies (Bonnor 1957; Layzer 1964). However, since galaxies do exist, these assumptions evidently are too restrictive, and we shall assume that in the early universe the necessary small, but not negligible, irregularities in the mass distribution did exist. This assumption should not be considered completely *ad hoc* in the sense that we have traded the problem of understanding why the galaxies formed for the problem of understanding why the initial conditions were just such as to lead to the formation of galaxies. The possible discovery of the primordial fireball (Dicke *et al.* 1965; Penzias and Wilson 1965; Roll and Wilkinson 1966) suggests an important reason why the galaxies formed when they did (Peebles 1965*a*). The primordial fireball would have prevented the formation of stable bound systems, whether stars or proto-galaxies, until the universe was about 10^5 years old. At this time the assumed primordial fireball radiation temperature drops to 4000° K, the plasma recombines and becomes decoupled from the radiation. At this epoch non-relativistic bound systems can start to form.

In this picture stars would not form until the development of systems so massive that the internal kinetic energy is equivalent to 10^4 ° K, so that hydrogen can be ionized. Since there would be no primordial heavy elements in the big-bang cosmology, ionized hydrogen is necessary for appreciable energy radiation. At the epoch where matter decouples from the radiation the mean density would be in the range 10^3–10^4 protons/cm^3, depending on the present value of the mean density. At this density the minimum mass necessary to ionize the hydrogen is about $10^7\ \mathfrak{M}_\odot$. As the universe expands the mean density decreases and this minimum mass for ionization increases.

It should be noted that the stars would tend to remain bound in the system in which they formed. It appears from this that stars could not have formed to any great extent in our Galaxy until the proto-galaxy formed as a distinct bound system. Otherwise, the Galaxy would have ended up as a group of smaller, more dense star systems. There is reason to believe that the globular clusters are not an exception to this. These clusters are strongly concentrated toward the center of the Galaxy, and apparently this was due to an appreciable drag on the material in the clusters by the general distribution of gas in the proto-galaxy. This is possible only if the globular clusters were gas clouds, rather than star systems, when the proto-galaxy formed (cf. Eggen, Lynden-Bell, and Sandage 1962, p. 761).

At the point of maximum expansion the newly formed proto-galaxy is not expected to have been particularly symmetrical, so it could not have collapsed by a very large factor in radius before the directed motion of collapse was translated into appreciable turbulent and thermal energy. The energy thus available would be equivalent to 3×10^6 ° K if the Galaxy collapsed from a radius of 20 kpc to a radius of 10 kpc. This means that in the early stages of the collapse turbulence could ionize the material and cause density irregularities sufficient to initiate endemic star formation. It has been suggested that the halo Population II stars are the remnants of this first generation of stars (Eggen *et al.* 1962; Oort 1964, 1965; cf. also Gamow 1948; Hoyle 1953; Mestel 1963; Field 1964).

This picture has two important consequences for our purposes which will be more fully discussed in the following sections. First, the halo stars preserve some memory of the maximum size of the proto-galaxy. Second, the onset of widespread star formation is a fairly well-defined event occurring at an early stage in the initial collapse of the proto-galaxy.

b) The Time of Formation of the Galaxies

In this section we will use a spherically symmetric model for the formation of the proto-galaxy. Although the proto-galaxy is not expected to have been very symmetrical, the model is adequate for a description of the expanding phase. Also, since the gravitational potential of the Galaxy is small ($G\mathfrak{M}/Rc^2 \sim 10^{-6}$), it is an excellent approximation to use Newtonian mechanics to describe the final development of the proto-galaxy (Callan, Dicke, and Peebles 1965).

Let $\mathfrak{M}$ be the mass contained within a sphere centered at the point of symmetry in the spherical model. At early times the gas is expanding. The sphere is fixed to the gas, so that the radius R increases with time. Here and throughout all lengths and times are proper quantities, as measured with ordinary rods and clocks. The equation of motion of the sphere is, in the Newtonian approximation,

$$\ddot{R}(t) = -G\mathfrak{M}/R^2 \ , \tag{1}$$

where $\mathfrak{M}$ is independent of time. The mass density of the black-body radiation would have been negligible compared to the matter density when the galaxies formed. The solution to equation (1) is the parametric form

$$R=\left(\frac{3\,\mathfrak{M}}{32\pi\rho_p}\right)^{1/3}(1-\cos\eta)\,,\qquad t=\left(\frac{3}{32\pi G\rho_p}\right)^{1/2}(\eta-\sin\eta)\,,\tag{2}$$

where η is the parameter. Here t is the proper world time of the usual homogeneous isotropic cosmological models. Since we envisage irregularities on a scale small compared with the size of the visible universe, no ambiguities arise in the definition of this time. The constants of integration in equations (2) are fixed by the two conditions that in the limit of very early time the density be uniform, and that at the moment the sphere stops expanding the mean mass density within the sphere be ρ_p. The material stops expanding at the world time

$$t_p = (3\pi/32G\rho_p)^{1/2}\,.\tag{3}$$

In evaluating equation (3) we assume that the mass of the proto-galaxy is (Perek 1962; Schmidt 1965; Innanen 1966*b*)

$$\mathfrak{M} = 1.2\times 10^{11}\,\mathfrak{M}_\odot\,.\tag{4}$$

A reasonable minimum value of the radius of the proto-galaxy when the expansion stops is

$$R_p = 20\text{ kpc.}\tag{5}$$

Then equation (3) yields a minimum value for the time of formation of the Galaxy,

$$t_p = 1.4\times 10^8\text{ years.}\tag{6}$$

The radius (5) is roughly the present outer boundary for the globular clusters (Arp 1965) and for the computed motions of the halo stars observed in the solar neighborhood (Eggen *et al.* 1962). The assumption here is that in the initial collapse of the proto-galaxy the outermost stars managed to form without suffering any appreciable radial gas drag. Individual stars may be found beyond the radius (5), but it does not follow that the value of R_p should be increased correspondingly, for it is possible that the turbulence during the collapse imparted kinetic energy to some of the material. If $R_p = 20$ kpc, then to explain the high circular velocity of the disk compared to the halo stars near the Sun, we must assume that the material in the outer part of the disk gained angular momentum from the more dense, more highly contracted inner part of the Galaxy. There is some evidence for the opposite assumption, that the Galaxy may have formed in such a way that the angular momentum of each mass element was conserved (Innanen 1966*a*). If this were the case it would indicate that the material in the disk near the solar orbit collapsed from an initial radius R_p larger than 20 kpc (Eggen *et al.* 1962), and this in turn would increase the time of formation of the Galaxy, the time varying as the $\frac{3}{2}$ power of R_p.

It appears unreasonable to suppose that the proto-galaxy could have collapsed from a radius very much greater than (5). This would require that the system be highly symmetric. However, at the time of maximum expansion the system surely was not very symmetric, and the irregularity would have grown as the system collapsed. The system could not have collapsed very far before appreciable turbulence developed, and we have seen that the turbulence energy would quickly have become sufficient to ionize the material collisionally, so that it could radiate and continue collapsing. However, once this happens we must also expect to find gravitationally bound subsystems which can radiate and so collapse all the way to star formation. That is, we believe that with a reasonably irregular initial proto-galaxy at the point of maximum expansion the system could not have collapsed by a very large factor in radius before the halo stars formed and defined the size of the proto-galaxy.

It is concluded that the time (6) is a minimum value for the time of formation of the galaxies: one could imagine increasing the time of formation by a factor of 2 or so, by assuming a somewhat larger value for the maximum radius, but it appears unreasonable to suppose that the time could be increased by a factor substantially larger than this.

c) *Luminosity of the Young Galaxies: Element Production*

There are two general reasons for believing that the young galaxies must have been much brighter than they are now: the first generation of stars would be expected to have included a large number of highly luminous stars of the O–B type; also a high luminosity is required to account for the rapid production of elements during the early stages of evolution of the Galaxy. We shall consider the general question of element production in the present section, and the expected luminosity of the first generation of stars in the next section.

From equation (2) we see that the time required for free fall of the galaxy from the point of maximum expansion to a highly contracted state is given by equation (3). Adopting the values (4) and (5), the minimum time at which the disk could have formed is therefore $2 \times t_p \gtrsim 3 \times 10^8$ years. This is consistent with estimates of the time required to form the disk: Eggen *et al.* (1962) suggested a time interval of a few times 10^8 years on the basis of the high eccentricities of the halo star orbits, and Spitzer (1966) argued for a similar time interval for the relaxation of the initial turbulence in the Galaxy.

During the initial collapse the gaseous hydrogen is polluted with material from the first generations of halo stars, and these stars must have been active enough to complete heavy-element production by the time the disk formed (van den Bergh 1961*b*; Dicke 1962). This condition yields a lower limit to the luminosity of the young galaxies. Let ΔX be the abundance by mass of all elements created due to hydrogen burning during the collapse. Then the luminosity of a young galaxy during this phase of rapid hydrogen burning is

$$\mathfrak{L}_y \simeq 0.007\, \mathfrak{M} c^2\, \Delta X/\Delta t \simeq 5 \times 10^{47}\, \Delta X/\Delta t_8 \text{ ergs/sec} , \tag{7}$$

where Δt_8 is the time interval, expressed in units of 10^8 years, during which the disk formed. Assuming that the mass-to-light ratio for the Galaxy now is 10 solar units, we find that the young Galaxy must have been more luminous than it is now by the factor

$$\mathfrak{L}_y/\mathfrak{L} \simeq 1 \times 10^4\, \Delta X/\Delta t_8 . \tag{8}$$

Since the production of heavy elements was well advanced by the time the disk formed, $\Delta X \gtrsim 0.02$. As mentioned above we may assume that $\Delta t_8 \lesssim 2$. In § II*d* we argue that a reasonable estimate is $\Delta t_8 \simeq 0.3$. Using this value, with $\Delta X \gtrsim 0.02$, we find

$$\mathfrak{L}_y \gtrsim 3 \times 10^{46} \text{ ergs/sec.} \tag{9}$$

The amount of helium produced is less certain because we do not have reliable measures of helium abundances in old stars. If the primordial helium abundance were small, we would suppose that the growth of the helium abundance paralleled the growth of the heavy elements during the very active initial phase of evolution of the Galaxy. Assuming that the helium abundance had grown to 20 per cent by mass when the disk formed, the luminosity $\mathfrak{L}_y$ would have been a factor of 10 larger than the value (9).

d) *The Luminosity of the Young Galaxies: Luminosity Function*

In this section we shall assume that the first generation of stars in young galaxies was formed with the same distribution of luminosities as the Population I stars of the solar neighborhood. The validity of this assumption will be considered briefly at the end of this section and will be discussed in greater detail in a subsequent paper.

Initial luminosity functions, which give the number of stars, $\psi(M_v)$, created in each (visual) magnitude interval, $M_v - \frac{1}{2}$ to $M_v + \frac{1}{2}$, have been computed by a number of authors, among them, Sandage (1957), Salpeter (1959), Schmidt (1959), Mathis (1959), and Limber (1960). In this paper we shall adopt the luminosity function of Limber and also his values for stellar masses and lifetimes as tabulated by Truran, Hansen, and Cameron (1965). This information may be found in columns (1), (2), (3), (5), and (7) of Table 1. We assume that nearly all the mass of the proto-galaxy was rapidly transformed into stars. This is quite different from the assumptions used in previous models

TABLE 1

LUMINOSITY FUNCTION

M_v (1)	M_{bol} (2)	$\psi(M_v)$ (Limber 1960) (3)	Equiv. No of $M_{bol}=0$ stars (4)	$\mathfrak{M}(M_v)$ (Solar Masses) (5)	$\psi\mathfrak{M}$ (6)	τ_{MS} in 10^6 years (Limber 1960) (7)	ϵ for $\Delta t_g = 3\times10^7$ years (8)
−6	−8 54	0 5	1300	~90	~45	~3 2	~0 107
−5	−7 62	2 5	2800	50	125	4 1	0 137
−4	−6 60	6 4	2800	27	173	6 8	0 226
−3	−5 54	11 2	1850	15 5	174	11 5	0 384
−2	−4 38	20 0	1130	9 4	188	20	0 667
−1	−2 97	34 5	531	6 0	207	37	1 00
0	−1 62	63 0	280	3 95	249	80	1 00
+1	−0 18	115	137	2 8	322	185	1 00
+2	+1 30	158	48	2 1	334	470	
+3	+2 72	179	15	1 65	296	1300	. .
+4	+3 98	222	6	1 25	278	3900	
+5	+5 00	256	3	1 00	256	12000	
+6	+6	337	1	0 86	290	30000	
+8	+8	470	0	0 61	570		
+10	+10	740		0 40	590		
+12	+12	1120		0 25	560		
+14	+14	1400		0 15	420		
				Add for low-mass stars:	~300		
Total			10900		5,400		

for the evolution of the Galaxy, and the justification for it will be discussed in the subsequent article. Normalization of the luminosity function may be achieved by setting the sum of $\psi(M_v)\ \mathfrak{M}(M_v)$ equal to $1.2 \times 10^{11}\ \mathfrak{M}_\odot$ (eq. [4]).

In column (4) the equivalent number of stars of bolometric magnitude zero is computed for each magnitude interval, using columns (2) and (3). The luminosity of a $M_{bol} = 0$ star is 3×10^{35} ergs/sec (Allen 1963). By summing column (4), we may calculate the luminosity of our Galaxy assuming that the gas was suddenly converted to stars according to $\psi(M_v)$. With this assumption, the value of the luminosity $\mathfrak{L}_y$ is 7.5×10^{46} ergs/sec. Field (1964) has used a somewhat similar approach, and finds that the initial luminosity of a galaxy with mass equal to $10^{11}\ \mathfrak{M}_\odot$ would be 7×10^{46} ergs/sec from main-sequence stars alone. Again this result holds if all the stars are formed at substantially the same time.

It is necessary, however, to consider the time scales involved in the formation and evolution of these stars. From columns (4) and (7) of Table 1, it is clear that the massive, short-lived stars contribute most of the luminosity. In fact, the few stars with lifetimes on the main sequence of less than 10^7 years are as luminous as all the fainter stars combined. It is therefore necessary to ask whether the first stars turned on in periods less than or comparable to 10^7 years.

The time taken for a massive star to reach the main sequence through Helmholtz contraction is much less than 10^7 y. For a star of 15 $\mathfrak{M}_\odot$, for instance, it is approximately 6×10^4 years (see Schwarzschild 1958, chap. iv).

An estimate of the time interval over which proto-stars *begin* their contraction to the main sequence is more difficult. First, we note that the model of collapse assumed in § II is independent of scale (cf. eq. [3] and Hunter 1962), so that the onset of stellar contraction and galactic contraction would have occurred at the same time.

Several other arguments also indicate that the interval over which stars began to contract was short compared to 10^8 years. First, we refer back to the model of stellar formation discussed in § II*a*, which implies a rapid onset of contraction. Also, von Hoerner (1960) has presented arguments for an early burst of star formation lasting perhaps a few times 10^8 years. Eggen *et al.* (1962) adduce dynamical arguments to show that presently observed dwarf stars with highly elliptical orbits must have formed in a period considerably less than 2×10^8 years, the galactic rotation period. If the onset of contraction of the proto-stars had been long delayed, gas drag on the nascent stars when they passed through the galactic plane would have been sufficient to slow them and distort their orbits.

Finally, the oldest known disk stars show considerable metal abundance. It is therefore necessary to suppose that a considerable number of the initial stars had completed their evolution and added heavy elements to the interstellar gas by the time the disk formed. Figure 1 indicates that very little mass could have been converted in times less than 5–8 $\times 10^7$ years. But the disk itself is thought to have formed about 10^8 years after the proto-galaxy began to contract.

These arguments suggest a time scale of a few times 10^7 years for the onset of stellar formation. It is interesting to note that stellar formation on a small scale, as in young galactic clusters, appears to take place in intervals of less than 10^7 years (Iben and Talbot 1966). For definiteness, we will take a time scale of 3×10^7 years, and use it in all subsequent calculations. We feel this value will not be in error by more than a factor of 2. The length of this interval has several implications.

1. Since it is longer than the lifetimes of the most luminous stars, it will determine the duration Δt of the initial bright period in the history of young galaxies. We assume

$$\Delta t = 3 \times 10^7 \text{ years.} \tag{10}$$

2. Consequently, in computing the luminosity $\mathfrak{L}_y$, it is necessary to note that only a fraction of the most luminous stars would be radiating at any time. For instance, for stars of $M_v = -5$, this fraction, ϵ, would be about 0.14 for $\Delta t = 3 \times 10^7$ years. Similar adjustments are needed for all stars with short lifetimes: the fractions appearing in column (8) of Table 1 are applicable for $\Delta t = 3 \times 10^7$ years. When these corrections are applied to column (4), the luminosity $\mathfrak{L}_y$ is reduced to

$$\mathfrak{L}_y \cong 2.5 \times 10^{46} \text{ ergs/sec.} \tag{11}$$

3. Another consequence is that massive stars formed early in the interval Δt will have completed their evolution quickly and returned some of their matter to the interstellar gas. This would allow for the creation of second-generation stars, tending to increase $\mathfrak{L}_y$. However, it may be seen from Table 1 or Figure 1 that the amount of material returned to the interstellar medium in 3×10^7 years is not large. In addition, assuming that the luminosity function $\psi(M_v)$ applied to these stars, most of the recycled matter would end up in small stars of negligible luminosity.

The value of $\mathfrak{L}_y$ calculated above agrees well with expression (9). The agreement suggests strongly that the model of star formation assumed in this section does not allow for the production of much helium. Considerable helium could have been produced only if greater numbers of bright stars than predicted by the luminosity function were formed initially in young galaxies. Evidence that this may have been the case in our own Galaxy has been put forward by Schmidt (1963).

Finally, we may look briefly at some of the assumptions made in this section. We have assumed throughout that the properties of the original stars of our Galaxy (and others) match those of the recently formed disk stars. In particular, we have ignored

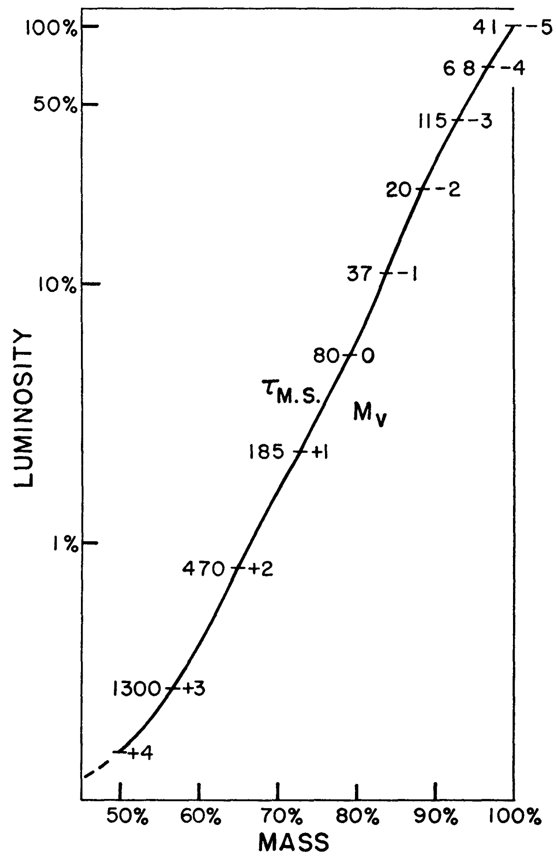

FIG. 1 —The amount of mass bound up in stars with absolute magnitudes $\geq M_v$ is plotted here as a function of the luminosity contributed by these stars. The values of M_v are indicated along the curve, together with $\tau_{M\ S}$, the lifetime on the main sequence expressed in millions of years. Note that 50 per cent of the luminosity is contributed by about 6 per cent of the mass.

changes in chemical composition. The assumed absence of heavy elements in the earliest stars might appreciably affect both the contraction time and the main-sequence lifetime of these stars. Its effect on $\psi(M_v)$, which depends essentially on the conditions in the interstellar medium, is less certain. Observational evidence for the universality of an initial luminosity function is fully discussed by Limber (1960). He notes that observations of the mass-to-light ratio of elliptical galaxies seem difficult to reconcile with the function of Table 1. Two possibilities are suggested:

1. That the number of small, faint stars formed early in the history of elliptical galaxies was greater than predicted by $\psi(M_v)$. This would tend to reduce $\mathfrak{L}_y$.

2. That much of the mass of ellipticals is present in the forms of white dwarfs. These

would presumably be the remnants of stars which have completed their evolution. Large numbers of white dwarfs argue for relative increase in the number of stars with main-sequence lifetimes $\lesssim 6 \times 10^9$ years, suggesting an *increase* in $\mathfrak{L}_y$ at *early* epochs.

The latter possibility is in accord with the findings of Schmidt (1963) for our own Galaxy, and with our calculations if we assume a low primordial helium abundance. However, there is not now sufficient observational evidence to allow us to choose between these two alternatives.

We have not yet considered the possible increase in $\mathfrak{L}_y$ due to supernovae in young galaxies. Making the extreme assumption that *all* massive, short-lived stars become supernovae at the end of their evolution, the data in Table 1 may be used to show that supernovae would occur at a rate of about 70 per year. Assuming that the supernovae each deliver on the order of 10^{50} ergs of ultraviolet and visible radiation, it may be seen that their effect on the luminosity of young galaxies is at most 1 per cent of the total (eq. [11]).

e) Spectrum

The qualitative nature of the spectra of young galaxies will be determined by the effective temperature of their most luminous stars. From Table 1 it is clear that stars with $M_v = -6$ to -2 produce most of the energy output of young galaxies. Main sequence stars in this range of magnitude are O and B types, with effective temperatures ranging from 20000° to 35000° K. As an average value, we will take 30000° K. This temperature determines a black-body spectrum having its maximum near 1000 Å. The actual spectrum of a young galaxy will be made flatter in the region of 1000–2000 Å by the presence of numerous dim stars.

Two important modifications of this simple picture are necessary: stellar opacities must be estimated, especially below the Lyman absorption edge at 912 Å, and interstellar absorption must be taken into account.

Models for hot, massive stars have been computed by Mihalas (1965). For stars with $T_e = 30000°$ K, his calculations suggest a Lyman decrement,

$$\frac{F_\lambda(912+)}{F_\lambda(912-)} \sim 18\,.$$

The decrement is shown in Figure 2, which is plotted on a frequency scale for enhanced clarity. The absorption coefficient shortward of 912 Å has been computed using a ν^{-3} law. This is appropriate when only hydrogen absorption is important. We have not attempted to include the stellar Lyman lines in Figure 2.

For a black body with a temperature of 30000° K, ~22 per cent of the flux is emitted with $\lambda < 912$ Å. We see from Figure 2 that about half of this (about 10 per cent of the total flux) escapes from the surface of the stars. If the galaxy is not optically thick to ionizing radiation, the fraction of the ionizing photons absorbed by atomic hydrogen is equal to the ratio of the total rate

$$\int \alpha n_e n_p dV$$

of production of hydrogen atoms to the total rate I of stellar generation of ionizing photons,

$$R = \int \alpha n_e n_p dV / I. \tag{12}$$

(The recombination coefficient, α, is given by Allen [1963], p. 91.) To estimate the value of equation (12) we assume that 10 per cent of the mass of the galaxy (4) remains as gaseous hydrogen, at a temperature of 10^4 ° K. If the gaseous hydrogen were distributed uniformly over a sphere of 15 kpc radius, the fraction (12) would amount to $R \sim 10^{-3}$. In this case most of the ionizing radiation would readily escape and the spectrum of the young galaxies would not differ greatly from the assumed stellar spectrum shown as a solid line in Figure 2. However, if the gaseous hydrogen were distributed in clouds, with a density within the clouds in the range of 10 to 100 protons/cm^3, an appreciable fraction of the ionizing radiation would be converted to Lyman photons (Menzel 1926; Zanstra 1927). As much as 10 per cent of the total luminous emission could be in the Lyman lines. In the extreme case that almost all the ionizing photons were converted to Lyman photons, the spectrum would resemble Figure 3, where we have shown only the Lyman-α

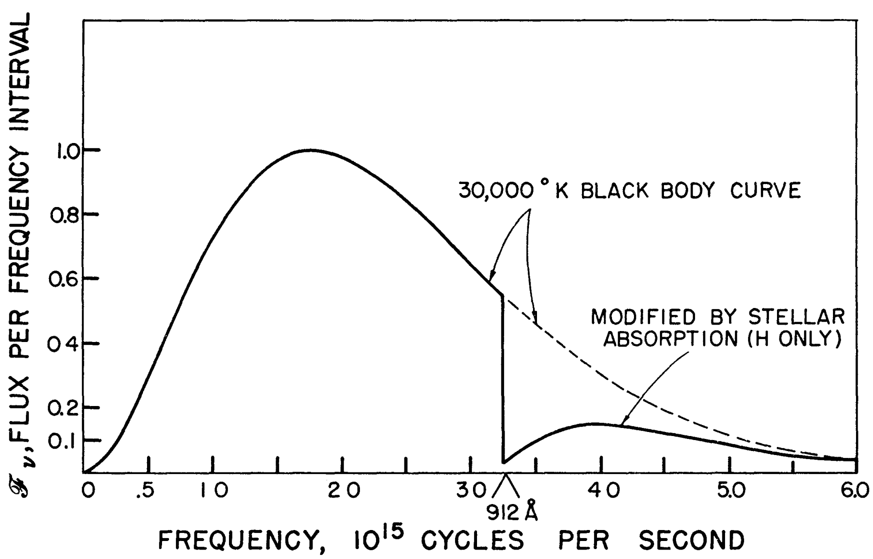

FIG. 2.—The spectrum is a black-body spectrum calculated for $T_e = 30000°$ K (the assumed effective temperature), modified below the Lyman limits by hydrogen absorption. No attempt has been made to calculate the details of the expected spectrum, such as the stellar Lyman lines.

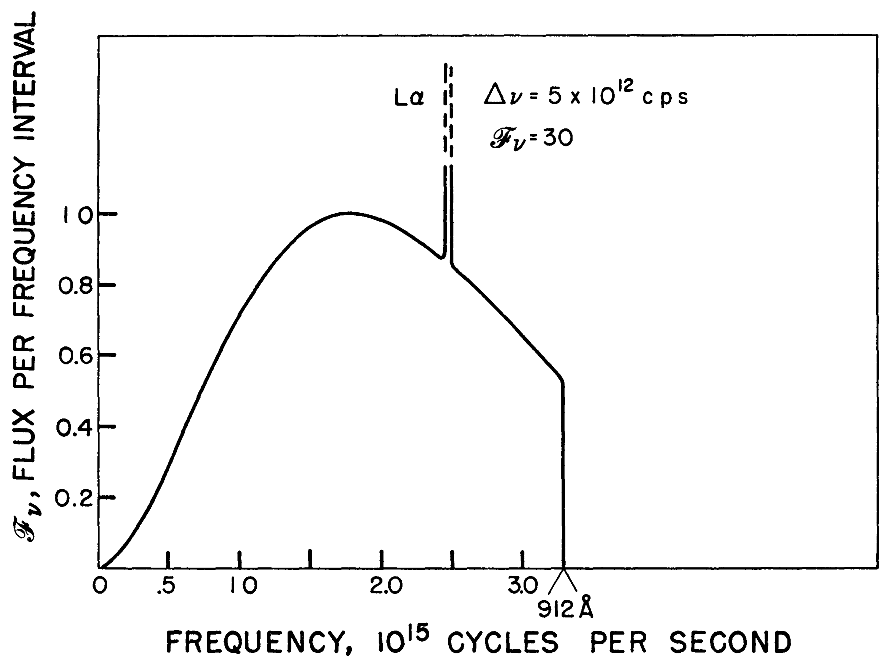

FIG. 3.—This curve represents semi-quantitatively the expected spectra of young galaxies in the extreme case that all photons of ionizing radiation have been converted to Lyman photons (§ IIe). The expected Lyman-α flux is calculated with $\Delta\nu = 0.002\ \nu$ for the line.

line of the series, which might alone contain 6 to 7 per cent of the total luminous emission of a young galaxy. Bahcall (1966) has remarked that, where the fraction of absorbed ionizing photons (12) is fairly high, the Lyman-β line (and to a lesser extent the higher Lyman lines) may also be prominent spectral features, and that this would be an important factor in interpreting the redshifted spectrum. The width of the lines may be estimated on the assumption that only Doppler broadening is important, and that this is due to the macroscopic motions of the stars and the gas in the young galaxies. For speeds of 300 km/sec, $\Delta\nu \approx 0.002\nu$. Thus the line width of the Lyman-α line would be 2.5 Å at the source, and this figure has been used in deriving $\mathfrak{F}_\nu$ for the line in Figure 3.

Two other possible spectral features may be mentioned. The first is the sharp cutoff in intensity at the Lyman limit caused primarily by atmospheric absorption in the bright stars of the young galaxies. In addition, if there is a substantial amount of interstellar

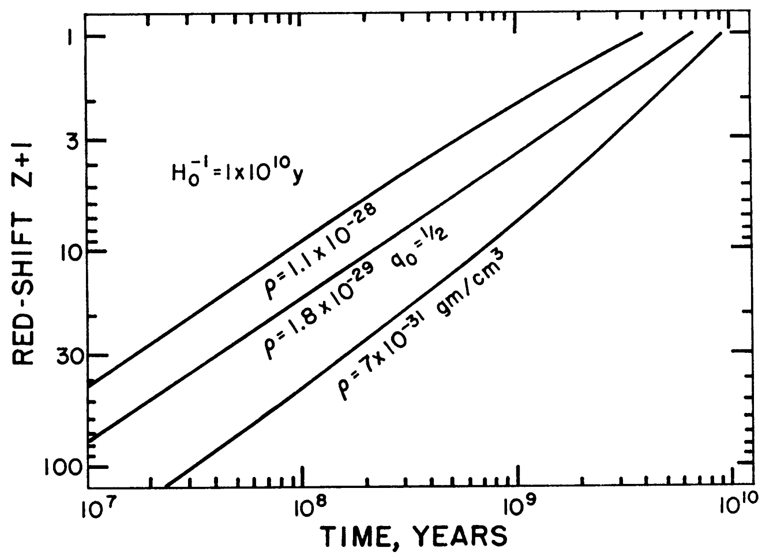

Fig. 4.—Redshift and epoch of formation of the galaxies. The connection between redshift and epoch (time since expansion of the model commenced from infinite density) is shown for three cosmological models, corresponding to three possible values of the mean density of the present universe.

helium present, either primordial or created by stellar processes, line emission at 584 Å might be expected. In the normalized units of Figure 3, $\mathfrak{F}_\nu$ for the He line would be $\lesssim 5$, assuming 20 per cent by mass of He in the interstellar medium.

III. OBSERVATIONAL PARAMETERS OF YOUNG GALAXIES

a) Redshift

Given a model for the development of the galaxies, the expected observational properties of the young galaxies depend on the cosmological model. Assuming a conventional isotropic homogeneous general relativistic model, with zero cosmological constant, the disposable parameters are the present values of the Hubble constant and the mean mass density. We assume that the energy density in radiation and neutrinos may be neglected. This is well established for electromagnetic radiation, but we do not have adequate limits on the energy density of neutrinos below a few Mev energy, or of gravitational radiation. If there were an appreciable density in such forms it would decrease the time scale for expansion and reduce the computed redshifts.

The relation between the redshift of the young galaxies and the epoch at which the galaxies formed is given in Figure 4 for three different cosmological models. These curves were obtained from the standard formulae given by Landau and Lifshitz (1962). In the standard notation z is the redshift, where $1 + z$ is the ratio of observed to emitted wavelength. We have adopted for the reciprocal of the Hubble constant $H_0^{-1} = 1 \times 10^{10}$ years. In the first model we have taken the mean density as $\rho_0 = 7 \times 10^{-31}$ gm/cm^3, the estimated mass density in galaxies (Oort 1958; Kiang 1961; van den Bergh 1961*a*). As a measure of the mass in ordinary galaxies this number should not be in error by a factor of more than 2. However, considerable amounts of matter could exist in other forms, and we have therefore included higher-density models. In the second model we have supposed that we are in the very early stages of expansion either of a closed or open universe, so that the acceleration parameter is $q_0 = \frac{1}{2}$. This requires a mass density 1.8×10^{-29} gm/cm^3. In the final model the mass density is six times larger than in the second. This requires an acceleration parameter $q_0 = 3$, which is consistent with the observations (Sandage 1961), but it will be noted that this model makes the age of the universe only 4×10^9 years, slightly less than the radioactive-decay age of the Earth (Patterson 1956).

From the point of view of the problem of time scale, the first two models appear to be more acceptable than the third, with the second ($\rho_0 = 1.8 \times 10^{-29}$ gm/cm^3) perhaps to be preferred on philosophical grounds (e.g., Dicke *et al.* 1965). We have argued that the galaxies would have been brightest at the epoch $t \sim 1.5 \times 10^8$ years. The redshift of the young galaxies therefore should be in the range 10–30. Since the radiation from the young galaxies is expected to be maximum in the ultraviolet, at wavelengths of the order of 10^3 Å, the observed radiation thus would be in the range 1–3 μ.

b) Surface Brightness of the Young Galaxies

As we shall show below, it is expected that young galaxies would be resolved by a telescope of moderate aperture, so that we are interested in the surface brightness of the galaxies. This is simply derived from formulae first obtained by Tolman. On dividing the equation for the luminosity of the source (Tolman 1934, eq. [178.8]) by the square of the angular diameter (eq. [180.3]) and inserting an extra factor of $(1 + z)$ in the denominator to yield the flux per unit wavelength interval we obtain directly the photon flux, per unit solid angle and wavelength,

$$B(\lambda) = N[\lambda/(1 + z)]/\pi^2 d^2 (1 + z)^4 \; . \qquad (13)$$

Here λ is the observed (redshifted) wavelength, and d is the proper diameter, which we take to be 30 kpc. Also $N(\lambda)$ is the luminosity of the young galaxies, expressed as the number of photons radiated per second and per unit wavelength.

For the assumed effective temperature, $T_e = 3 \times 10^4$ ° K, the wavelength of maximum photon flux is (Allen 1963, p. 103):

$$\lambda_m = 0.12(1 + z)\ \mu . \qquad (14)$$

At this wavelength, the surface brightness of a young galaxy with $\mathfrak{L}_y = 3 \times 10^{46}$ ergs/sec would be

$$B_\lambda = \frac{3.1}{(1+z)^2} \text{ protons sec}^{-1}\text{ cm}^{-2}\text{ (arc sec)}^{-2}\ \mu^{-1} . \qquad (15)$$

Although the surface brightness (15) represents a quite appreciable photon flux even for the maximum expected redshift, $z + 1 = 10$–30, the observational difficulty is that the radiation would be arriving at wavelengths in the range of 1–3 μ, where detector efficiency is falling and the night-sky emission rising. These problems are discussed below in § IV*a*.

c) Angular Diameters

Surface brightness is a useful quantity only if the young galaxies can be resolved: to show that this will be the case for the parameters we have assumed, we have plotted in Figure 5 the angular diameter θ of galaxies as a function of redshift. Equations (10) and (11) of McVittie (1965) were used to calculate the curves. Since the angular diameter will be greater than $5''$, it is clear that the young galaxies may be easily resolved.

The observed angular diameter of a highly redshifted source evidently depends strongly on the acceleration parameter q_0 (Sandage 1961; Hoyle 1959). Because of their large redshift, the determination of the angular diameter of young galaxies, combined with even a rough estimate of their physical diameter, would be very useful in helping to fix q_0.

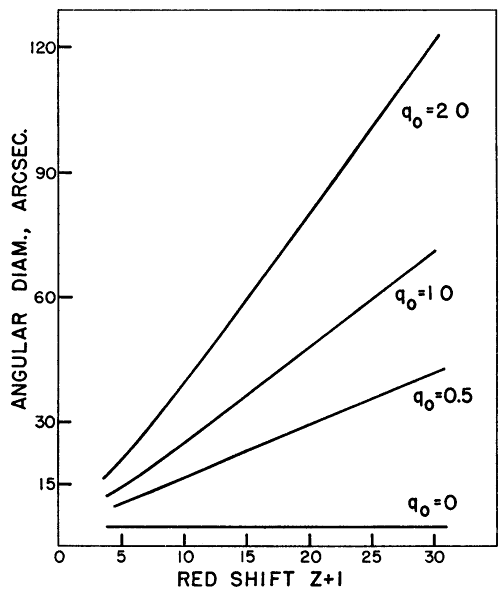

FIG. 5.—In computing the angular diameter of a young galaxy, a proper (local) diameter of 30 kpc has been assumed. For all cosmological models but the open one ($q_0 = 0$), the anguar diameter is an increasing function of $z + 1$ and hence of distance.

It should be emphasized at this point that we have assumed that the galaxies will be equally bright at every point on their surface, out to a radius of 15 kpc. Geometrical arguments, and the possibility of an increased rate of star formation at the center of collapsing proto-galaxies (Field 1964) suggest a possibly greater surface brightness for the central regions.

Providing the diameter of this central region is not much less than 30 kpc, so that the young galaxies may still be resolved, equation (13) may still be applied with both $N(\lambda)$ and d suitably altered. These brighter central regions would be easier to detect against the night-sky background (see § IV*a* below): however, we shall continue with the conservative assumption that the brightness distribution is uniform across the surface.

d) Numbers of Young Galaxies

If young galaxies can be detected at all, large numbers of them will be visible. An estimate of the number may be obtained from the following considerations: the probability that a random line of sight will pass through a highly luminous young galaxy is given by the expression

$$P=\sigma\mathfrak{N}c\int_{t_p}^{t_p+\Delta t}(1+z)^3dt. \tag{16}$$

Here σ is the geometrical cross-section of a galaxy, and $\mathfrak{N}$ is the spatial density of galaxies in the present universe. We have assumed that the galaxies turned on at time t_p (eq. [6]) and remained highly luminous for a time interval Δt thereafter (eq. [10]). For any reasonably possible cosmological model the expansion rate at the time the galaxies formed can be represented with reasonable accuracy as

$$1+z(t)=[1+z(t_p)](t_p/t)^{2/3}\ . \tag{17}$$

Combining this formula with equation (16) we obtain

$$P=\sigma\mathfrak{N}c[1+z(t_p)]^3t_p\Delta t/(t_p+\Delta t)\ . \tag{18}$$

The spatial density of galaxies brighter than our own Galaxy (assumed to have absolute magnitude $M_{pg}=-19.7$) is (Kiang 1961)

$$\mathfrak{N}\cong 8\times 10^{-3}/\text{Mpc}^3\ .$$

If we adopt an effective radius of 15 kpc for the young galaxies, the expression for the probability for intersecting a galaxy along a randomly chosen line of sight becomes

$$P\cong 5\times 10^{-5}(1+z)^3\ , \tag{19}$$

where we have employed the values (6) and (10).

With a redshift $z+1=10$–30 the probability (19) is at least 5 per cent. Thus, if young galaxies could be detected at all, it would not be a serious search problem to locate them: an appreciable part of the sky would be covered by the young galaxies.

IV. IS A SEARCH FOR YOUNG GALAXIES POSSIBLE?

a) Observational Assessment

The primary obstacle to a ground-based search for young galaxies is the night glow of the terrestrial atmosphere. The restriction imposed by the night glow is particularly severe in the near-infrared, the spectral region of present interest. In the following, we shall ignore the night-sky brightness contributed by zodiacal light and faint stars in the Galaxy. These effects are smaller than the light from the night sky if one avoids the ecliptic and the galactic equator.

The brightness of the night sky is shown in Figure 6. In preparing this figure we used four investigations in separate but overlapping parts of the spectrum. In the visible part of the spectrum, the emission of the night sky is more or less continuous, interspersed with a few strong nitrogen and oxygen lines (see the atlas of the airglow spectrum of Krassovsky, Shefov, and Yarin 1962). For the zenith brightness of the visible continuum, we adopt the figures compiled by Chamberlain (1961, p. 504), with the units suitably altered. Absolute measurements of the brightness of the night sky are quite uncertain: there is, for instance, considerable variation with latitude and season (Dufay and Dufay 1951). For this reason, the values we have plotted in Figure 6 may be in error by 50–100 per cent. In general, we have tried to err on the side of overestimating the brightness of the night sky.

In the near-infrared, from 0.7 to 1.6 μ, the situation is dominated by OH band emission and water-vapor absorption. The OH bands appear to originate at a height of 50–

100 km, well above most of the atmospheric water vapor. Thus both the OH bands and the infrared flux from young galaxies will suffer the same absorption. It is therefore proper to compare observed night-sky brightness with the surface brightness of young galaxies corrected for atmospheric absorption. Allen's (1963) tables have been used to make this correction, which is negligible below 8500 Å wavelength.

The 1.0–1.6-μ region was investigated by Harrison and Vallance Jones (1957): their measurements of absolute brightness at the zenith have also been included in Figure 6. The majority of the flux at these wavelengths is contributed by OH.

To fill in the intermediate region of the spectrum, we employ the paper of Chamberlain and Smith (1959) on the OH bands. It includes a summary of observational measures of the relative intensity of the various bands. The tabulated intensities for longer wavelength bands may be compared to the findings of Harrison and Vallance Jones to provide a normalization factor which is then employed for $\lambda \leq 1.0\ \mu$. Since in this region the OH emission is of the same order as the continuum emission, the brightness of the continuum at 6500 Å ($\sim 4 \times 10^{-2}$ photons sec^{-1}cm^{-2} [arc sec]$^{-2}$ μ^{-1}) has been added to the values we obtained for the OH emission alone.

A recent paper by Ford and Rubin (1965) presents a spectrum of the night sky from 4000 to 8000 Å. The results of this paper have also been included in Figure 6. Their

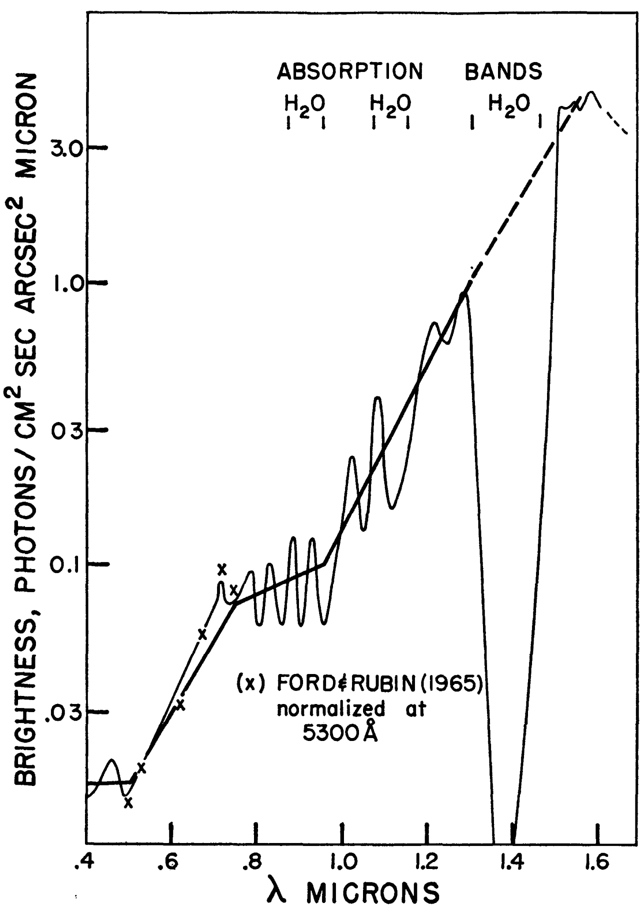

FIG. 6.—The emission of the night sky. The thin line is a detailed spectrum based on the sources referred to in § IV*a*. For wavelengths greater than 0.7 μ, the spectrum is dominated by OH band emission. New values taken from a recent paper by Ford and Rubin (1965) have been included as (*x*). We have added (*heavy line*) a smoothed spectrum for the night sky, and have also indicated water-vapor absorption bands.

spectrum was corrected approximately for the response of the photomultiplier (S20) and normalized at 5300 Å. The agreement with our estimates in the region 5000–7500 Å is good.

In Figure 6 we have also included a smoothed curve which reproduces the qualitative features of night-sky emission but ignores the line structure. Note the sharp increases between 0.5 and 0.75 μ and again beyond 1.0 μ.

The smoothed curve is compared in Figure 7 with the surface brightness of a young galaxy calculated for the wavelength of maximum photon flux (eq. [14]). We have used equation (15) to evaluate the surface brightness and have corrected for water-vapor absorption. The results are discouraging in the sense that the surface brightness of

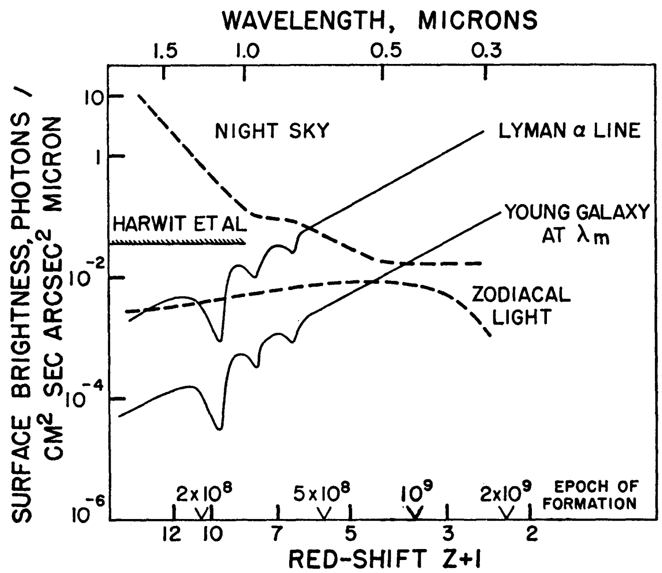

FIG. 7.—Local background and the brightness of young galaxies. The lower solid line is the computed surface brightness of a young galaxy at the wavelength of maximum brightness, and the upper solid line is the maximum possible brightness in the redshifted Lyman-α line. Both curves have been corrected for absorption in the atmosphere. The brightnesses are plotted as functions of the assumed redshift $(z + 1)$ of the object. The indicated connection between redshift and epoch of formation corresponds to a cosmological model with acceleration parameter $q_0 = \frac{1}{2}$. The connection between redshift and wavelength of maximum brightess is given by eq. (14). This is also nearly equal to the redshifted Lyman-α line. The brightness of the night sky and the brightness of the zodiacal light well away from the ecliptic are shown as a function of the wavelength of maximum observed brightness. The upper limit to the brightness of the sky above the atmosphere obtained by Harwit *et al.* (1966) is also indicated.

young galaxies falls below the emission of the night sky if λ_m is greater than 5500 Å, that is, if $(1 + z) > 4.5$. A young galaxy with a redshift $(1 + z) = 4.5$ would have a surface brightness equivalent to $M_{\text{vis}} = 23$ (arc sec)$^{-2}$ (eq. [15]). This is substantially brighter than the limit of photographic plates. Thus if the bright phase in the history of galaxies occurred at a somewhat later epoch than we have suggested, say, at 7×10^8 years (for $q_0 = \frac{1}{2}$), they should be readily detectable by ground-based instruments.

Young galaxies may appear very bright in the redshifted Lyman-α line. The upper line of Figure 7 was calculated assuming 6 per cent of the flux of a young galaxy lay in the line (see § II*e*). Lyman-α radiation of this amount should be easily detectable for redshifts $1 + z$ less than 6. Beyond this value, for $\lambda_m > 7500$ Å, not only the night-sky brightness but also detector sensitivities become major problems. Conceivably, an S-1 response image tube might be used down to 1.0 μ, where its quantum efficiency is $\sim$0.1 per cent.

To determine whether the detection of young galaxies at a wavelength of 1.0 μ $(z +$

1 = 8) is feasible, we estimate the signal-to-noise ratio that might be obtained with a telescope of moderate aperture. We will assume a 36-inch telescope and a band width of 20 Å. For a $q_0 = \frac{1}{2}$ cosmological model, a young galaxy with a redshift $(1 + z) = 8$ would subtend about 160 arc sec². From Figure 7, the night-sky background at 1.0 μ is $\sim$275 photons/sec from an area of 160 arc sec². Hence the expected noise is about 0.3 photoelectrons per second. The photon flux for a young galaxy is 33 photons/sec. Thus to distinguish a young galaxy from the night-sky background, an integration time of about 5 min would be required.

The problem of detecting young galaxies would be greatly eased if observations could be made above the atmosphere. With the emission of the night sky absent, the most important source of local background radiation would be the zodiacal light. In Figure 7 we have plotted the brightness of the zodiacal light well away from the ecliptic (Roach 1964) assuming that the zodiacal light has a solar spectrum. This is an upper limit: in the infrared the zodiacal light should be substantially dimmer than this if the scattering dust has a characteristic radius less than a few microns (see subsequent paper for fuller discussion). The zodiacal light shown on Figure 7 is less than the maximum expected brightness of the young galaxies in the Lyman-α line. Outside this band the surface brightness of the young galaxies is smaller than the brightness of the zodiacal light by a factor of as much as 10.

It is concluded that, on the basis of the model presented here, to search for young galaxies would require a technique capable of detecting signals as small as 10 per cent of the noise. Also, since the photon flux from a single galaxy would be small (of the order of 0.1 photons per sec cm² of collecting area), a detector with appreciable quantum efficiency would be required (of the order of 0.1 per cent). Either a photomultiplier or a photographic plate might satisfy these conditions for wavelengths as long as 1.2 μ. For a search at longer wavelengths, some form of bolometer would be necessary. We have plotted in Figure 7 an upper limit to the brightness of the sky above the atmosphere obtained by Harwit, McNutt, Shivanandan, and Zajac (1966). If their instrument could be made 30–100 times more sensitive, perhaps by increasing the integrating time used, it would be sufficient to detect individual young galaxies. Finally, we remark that such an experiment would not make stringent demands on pointing accuracy, since young galaxies are expected to have angular diameters greater than 5 arc sec.

b) Summary

Our purpose has been to discuss the possibility of obtaining observational evidence on the formation of galaxies. The experimental problem is to detect faint, extended objects at wavelengths of 1–3 μ against the local background. With the assumed model, the surface brightness of the objects would be at least 10 per cent of the local background above the atmosphere. If we assume there was no primordial helium, the surface brightness would be roughly comparable to the local background above the atmosphere. With available experimental techniques the experiment appears to merit consideration as a means of improving our knowledge of how galaxies formed. According to the instability model the young galaxies probably could not be detected from the ground. It should be noted, however, that a ground-based experiment could set useful upper limits on the luminosity and epoch of formation of the galaxies. According to our model an experiment above the atmosphere would appear to be capable of detecting the young galaxies.

If discovered, the interesting parameters of such objects would be their angular size, surface brightness, abundance and distribution, and possibly the redshift if the Lyman lines were prominent. From the redshift and angular size, and a fairly rough estimate of the proper (local) diameter, it would be possible to find the acceleration parameter q_0 (Fig. 5), and so find the epoch of formation of the galaxies (Fig. 4). This would serve as a test of the instability theory in § II. Also, it would be possible to gain some information on the absolute luminosity (eq. [13]) and the spectrum of young galaxies, quantities of considerable interest for any more detailed discussion of the earliest star population of galaxies.

Even the detection of young galaxies without a determination of their redshift would be of value. It is clear that they could not be detected at all unless the luminosity of galaxies was considerably higher at early epochs than it is now. Thus the detection of faint, extended objects in the infrared, which might reasonably be identified with young galaxies, would argue strongly for an initial bright phase in galactic evolution.

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