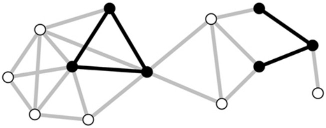
Modelling the nonrandom and dynamic structure of local cortical circuits

Felix Z. Hoffmann

Slides: bit.ly/xbcn18



Bernstein Conference 2018 Satellite Workshop:
Adaptivity and Inhomogeneity in Neuronal Networks

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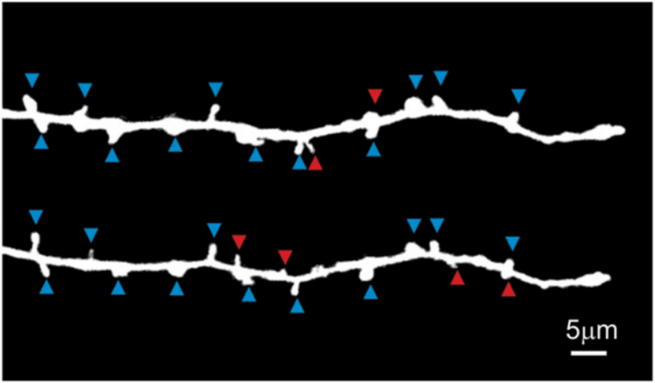
Wissenschaft offen gestalten
FIAS Frankfurt Institute for Advanced Studies
SPP 2041 Computational Connectomics

The dynamic connectome

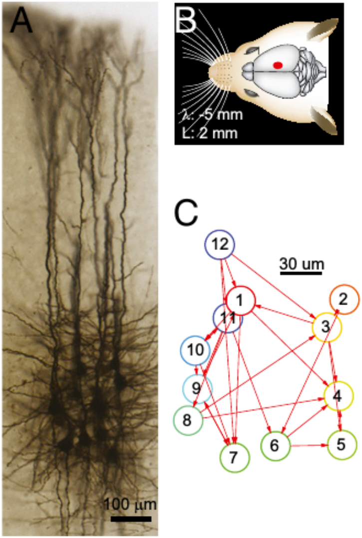


Day 0

Day 4

Loewenstein et al. 2015

Robust nonrandom connectivity patterns



Perin et al. 2011; Song et al. 2005; Markram et al. 1997; Miner and Triesch 2016; Gal et al. 2017; Vegué et al. 2017

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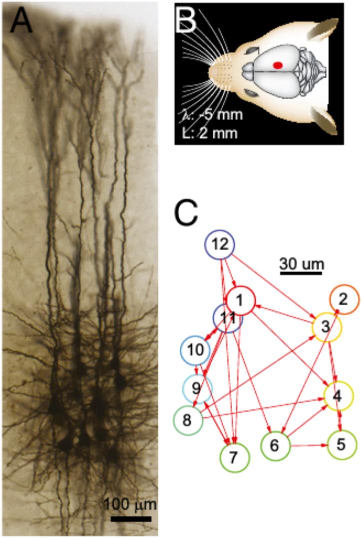

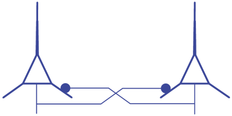

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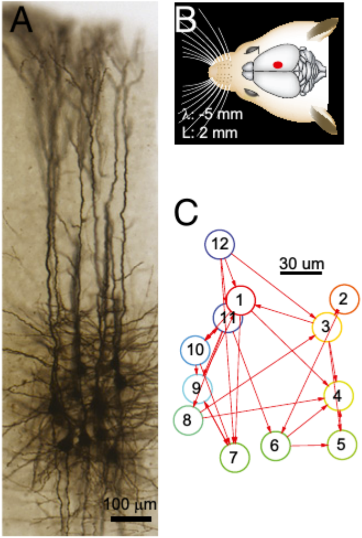

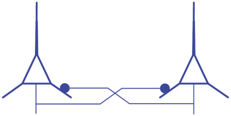

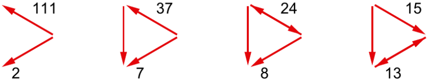

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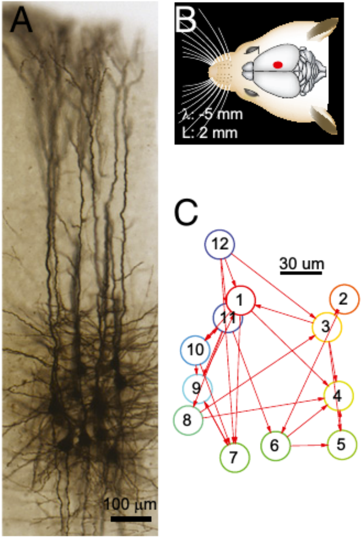

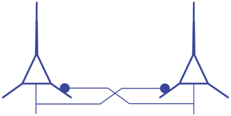

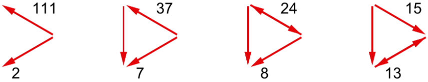

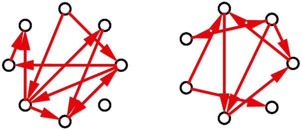

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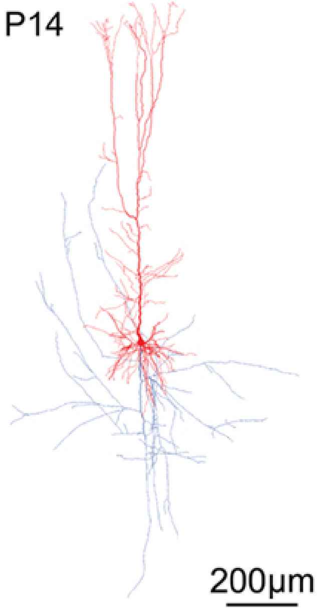
Anisotropy in stereotypical axon morphology



Romand et al. 2011

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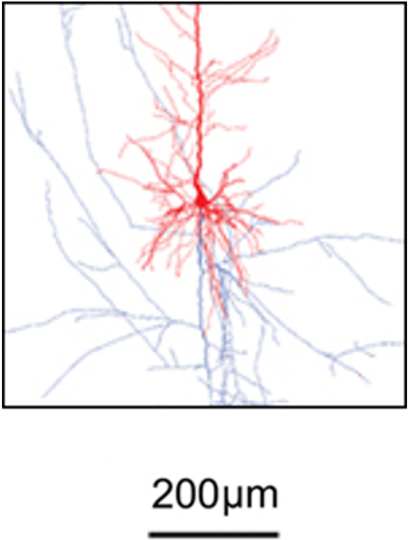
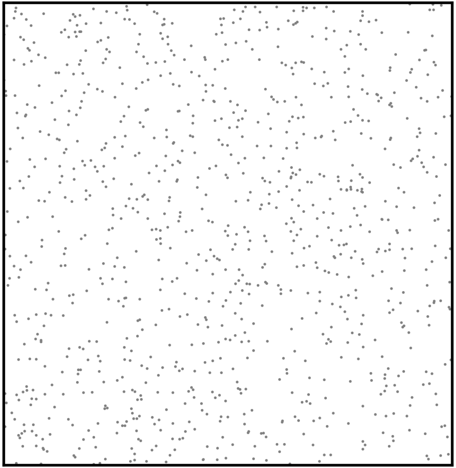


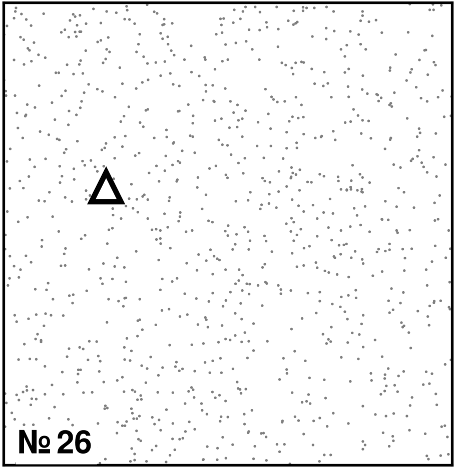
Romand et al. 2011; Stepanyants and Chklovskii 2005

Anisotropic network model



N = 1000

Anisotropic network model



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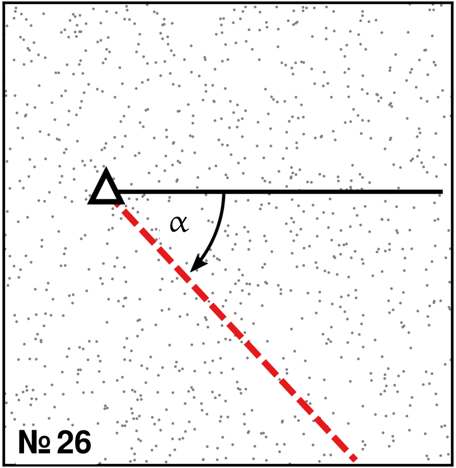

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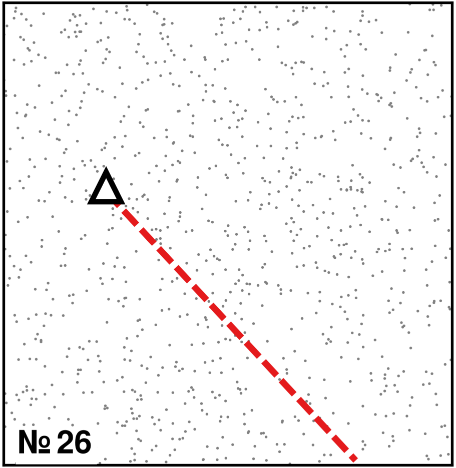

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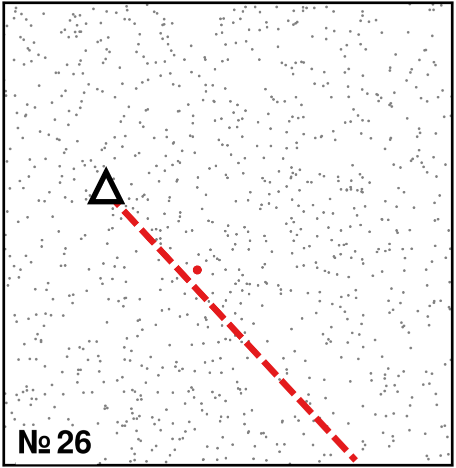

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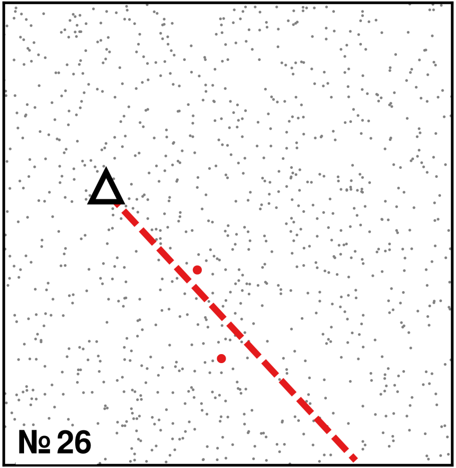

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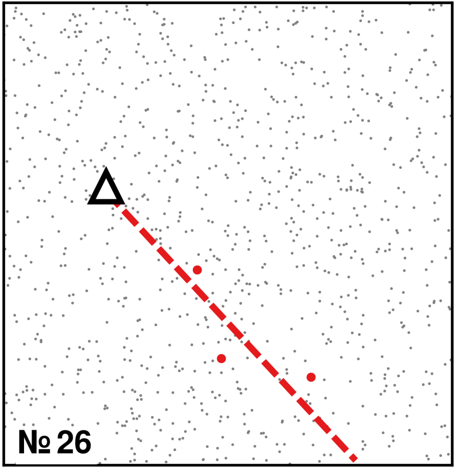

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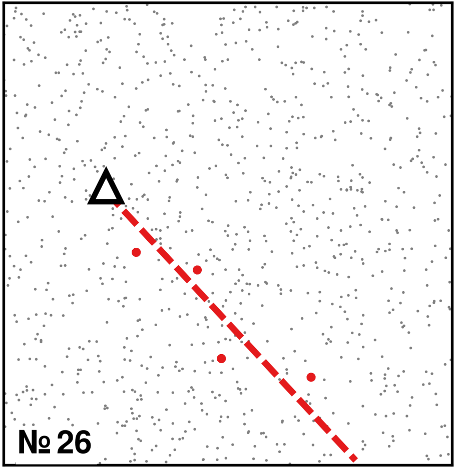

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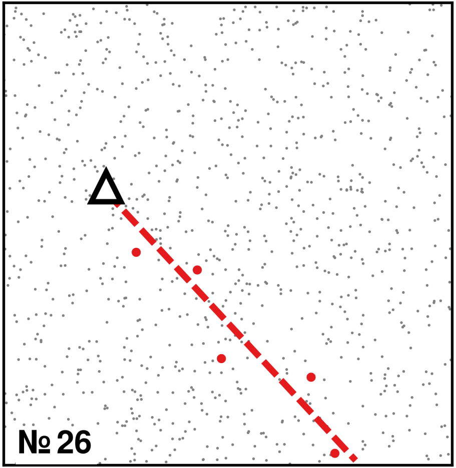
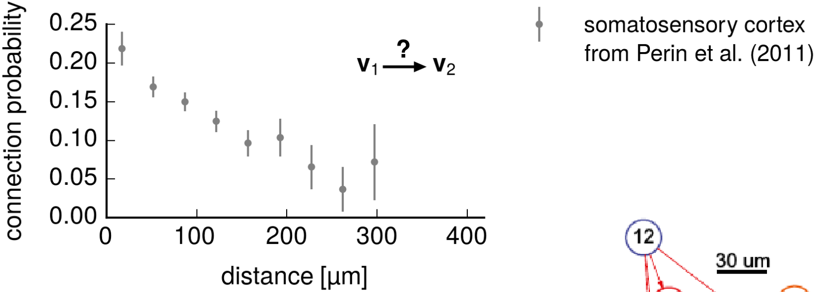
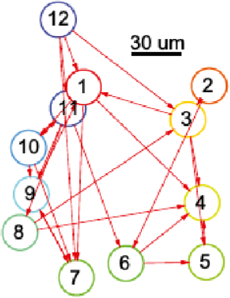


N = 1000

Anisotropic network model – Distance-dependency





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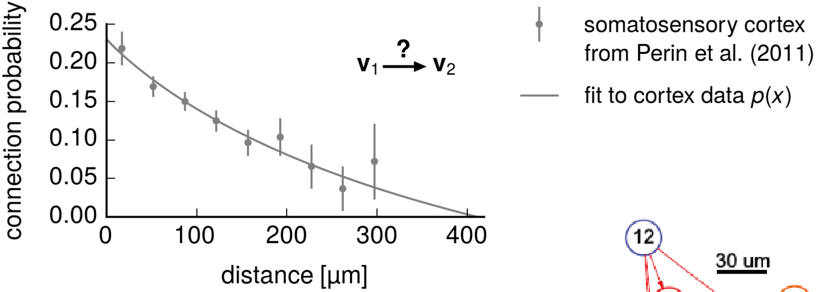

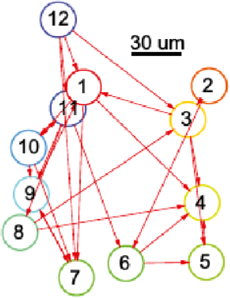

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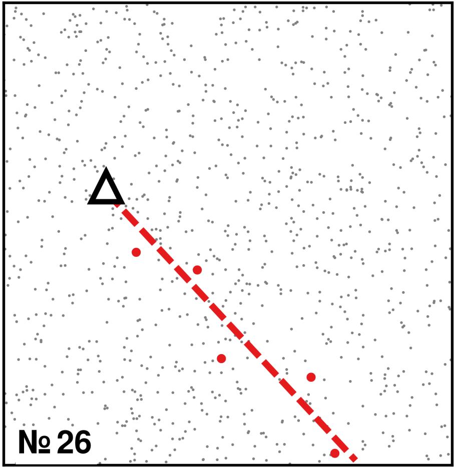

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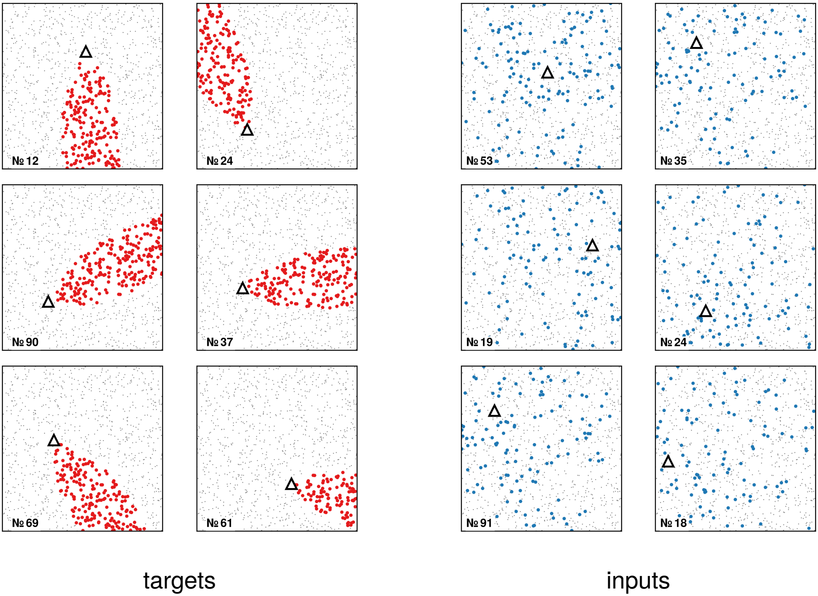
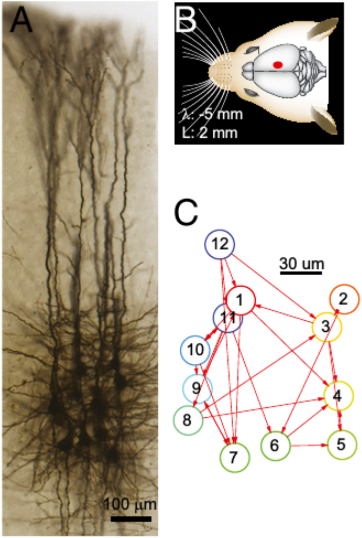
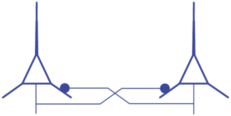
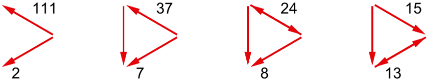
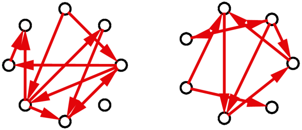


Robust nonrandom connectivity patterns



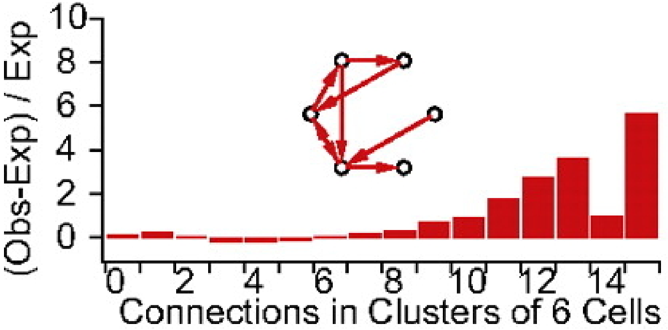






Perin et al. 2011; Song et al. 2005; Markram et al. 1997; Miner and Triesch 2016; Gal et al. 2017; Vegué et al. 2017

Connection counts in neuron clusters



Perin et al. 2011

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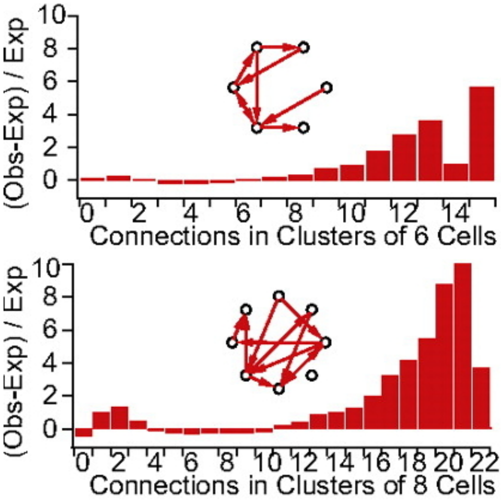

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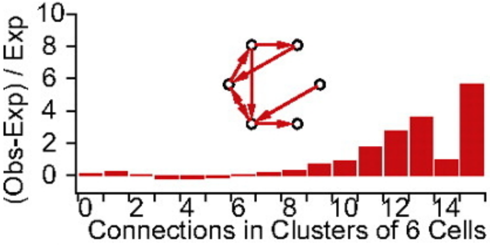

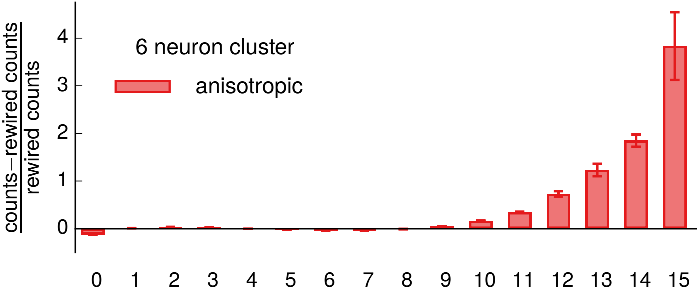

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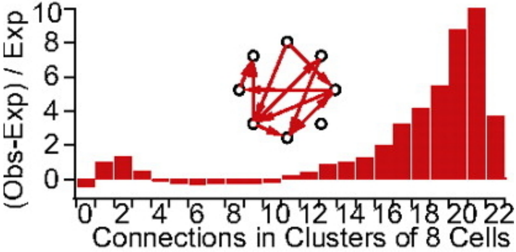
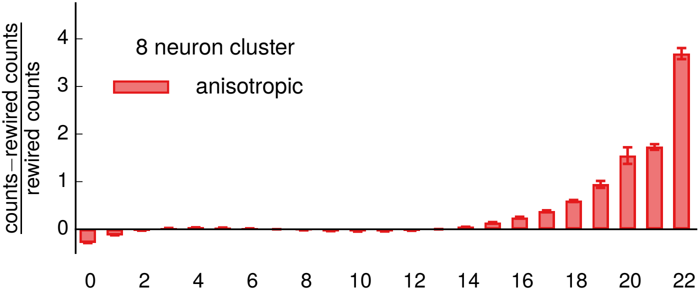
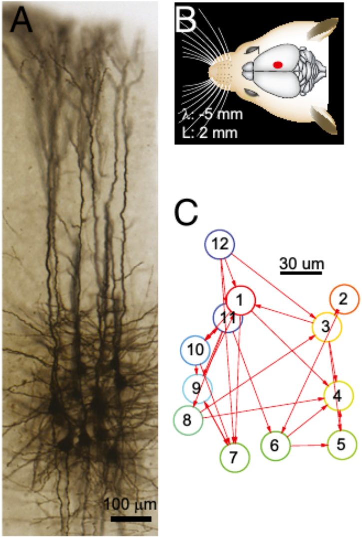
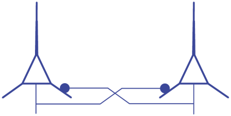


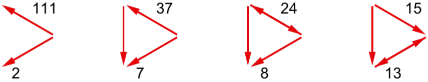


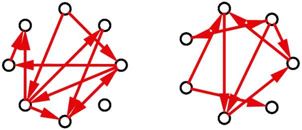
Results

Robust nonrandom connectivity patterns









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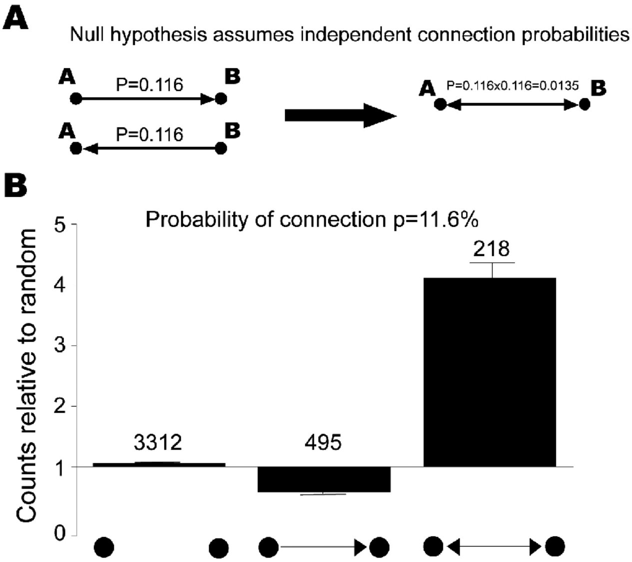
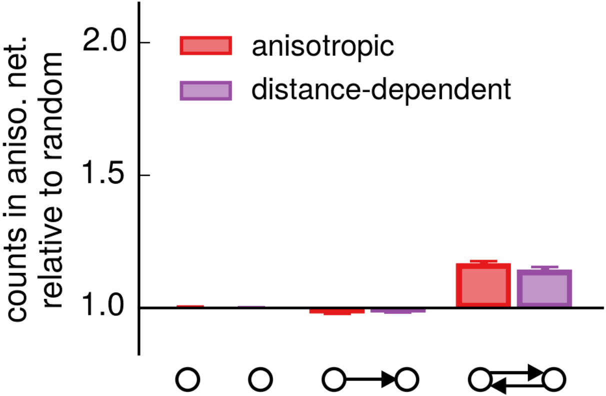
Overrepresentation of reciprocal connections

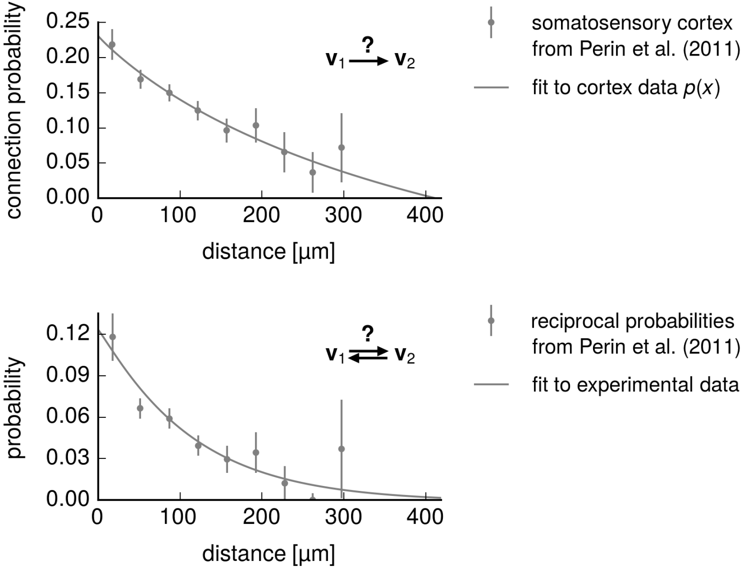


Song et al. 2005

Overrepresentation of reciprocal connections



Reciprocity – Distance-dependent?



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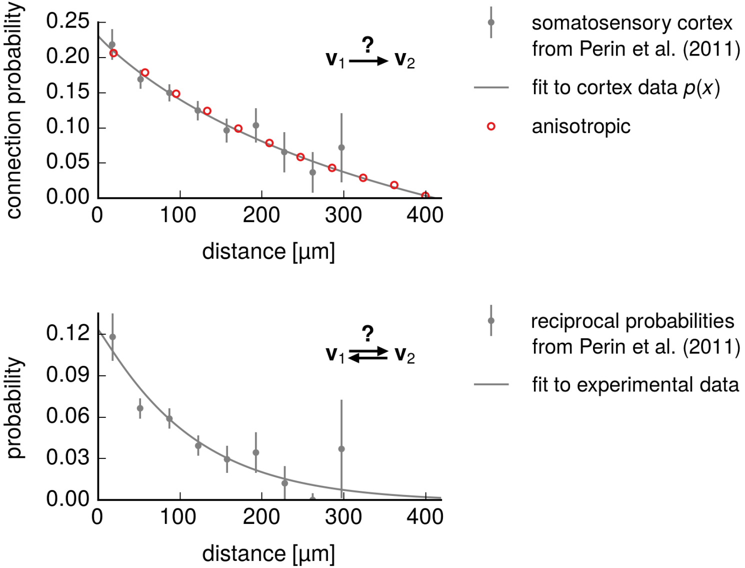

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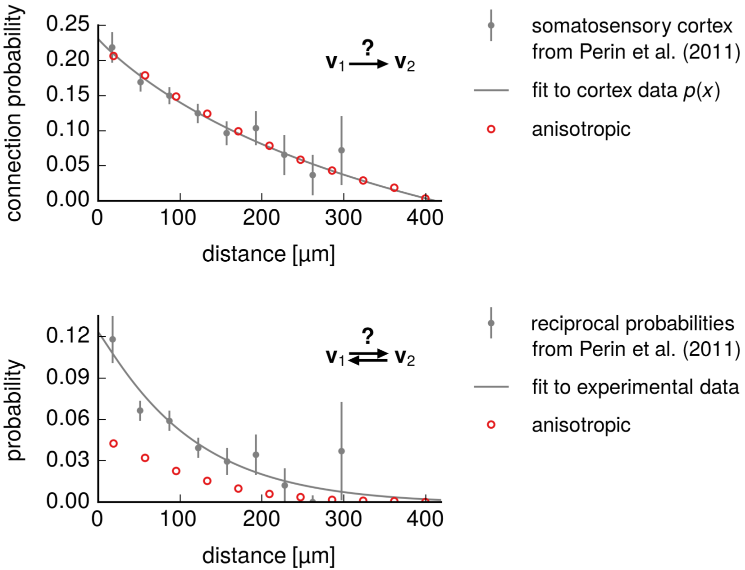

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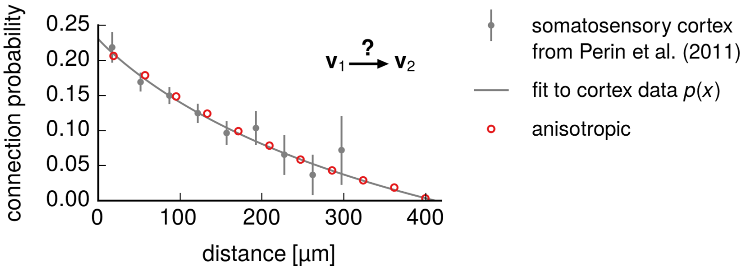

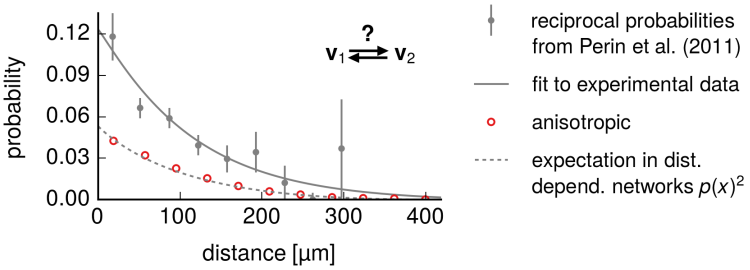

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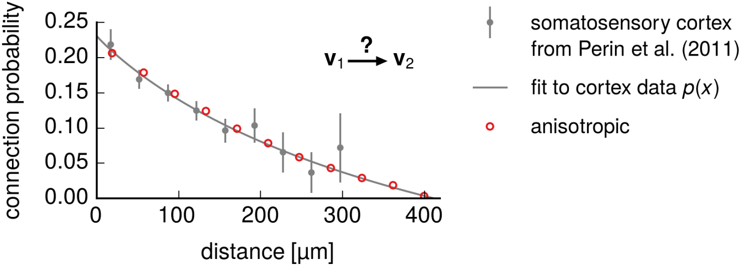
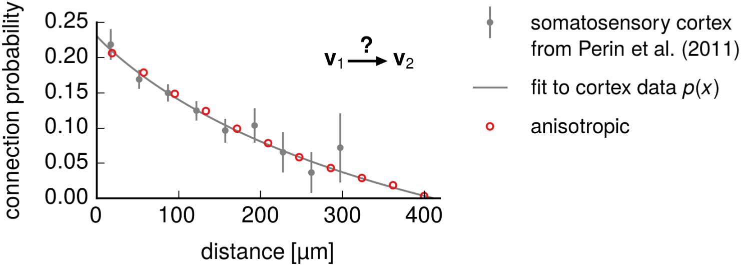


Other sources for the overrepresentation of bidirectional connections?

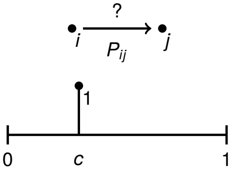
Reciprocity – Distance-dependent?



Other sources for the overrepresentation of bidirectional connections?

⇒ Hoffmann, FZ and Triesch, J (2017). Nonrandom Network Connectivity Comes in Pairs. *Network Neuroscience*

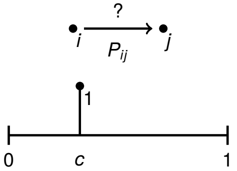
Standard random network model



Probability of connection a constant P_{ij},

$$P_{ij} = c$$

Standard random network model



Probability of connection a <u>constant</u> P_{ij},

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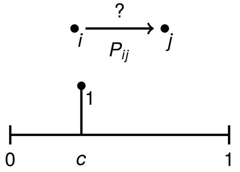
Overall connection probability

$$\mu = P_{ij} = c$$

Standard random network model



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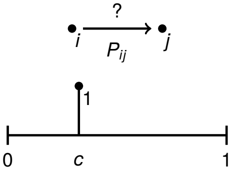
Bidirectional connection

$$P_{\text{bidir}} = P_{ij} P_{ji} = c^2$$

Standard random network model



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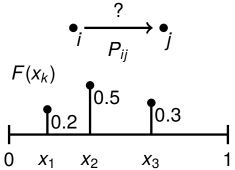
Varying connection probabilities



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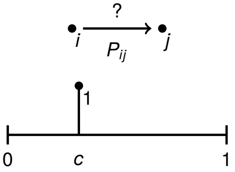

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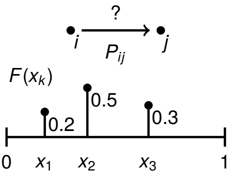
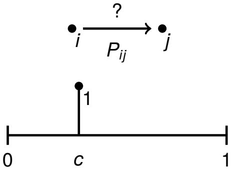


Probability of connection a <u>random variable</u> P_{ij},

$$\textbf{Prob}(P_{ij} = x_k) = F(x_k)$$

Standard random network model



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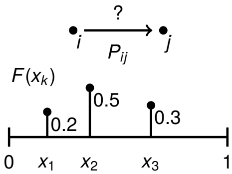

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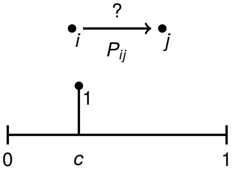
Overall connection probability

$$\mu = \sum_{k=1}^{m} F(x_k)x_k$$

Standard random network model



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Bidirectional connection

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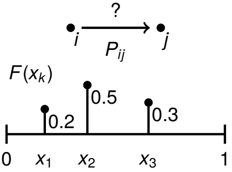
Varying connection probabilities



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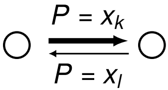
Bidirectional connection

$$P_{\mathrm{bidir}} = ?$$



Probability of connection a constant P_{ij},

$$P_{ij} = c$$

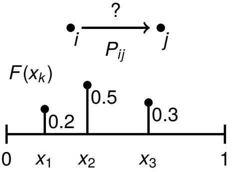
Overall connection probability

$$\mu = P_{ij} = c$$

Bidirectional connection

$$P_{\text{bidir}} = P_{ij} P_{ji} = c^2$$

Varying connection probabilities



Probability of connection a random variable P_{ij},

$$\mathbf{Prob}(P_{ij} = x_k) = F(x_k)$$

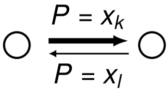
Overall connection probability

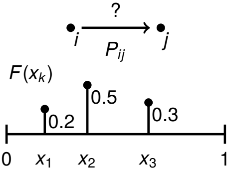
$$\mu = \sum_{k=1}^{m} F(x_k) x_k$$

Bidirectional connection

$$P_{\text{bidir}} = ?$$

Varying connection probabilities





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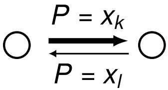

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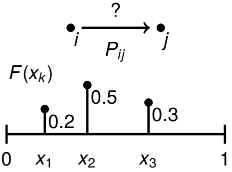

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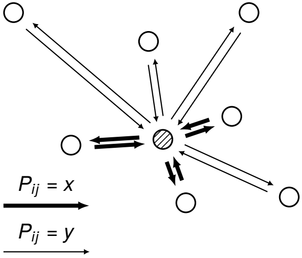
$$P_{\text{bidir}} = \sum_{k=1}^{m}\sum_{l=1}^{m} F(x_k)x_k\underline{F(x_l|x_k)}x_l$$

$$F(x_l|x_k) = \begin{cases} 1 & \text{if } l = k \\ 0 & \text{otherwise.} \end{cases}$$



Hoffmann and Triesch 2017

$$P_{\text{bidir}} = \sum_{k=1}^{m} \sum_{l=1}^{m} F(x_k) x_k F(x_l | x_k) x_l$$
$$= \sum_{k=1}^{m} F(x_k) x_k^2 .$$

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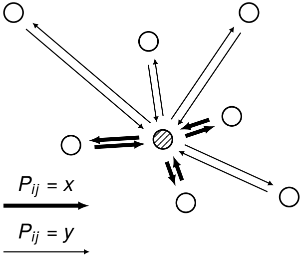

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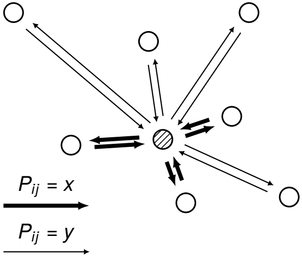
Relative overrepresentation ρ is the fraction

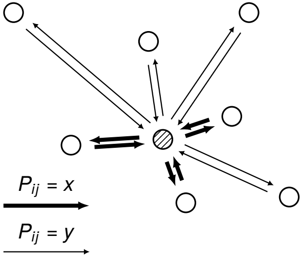
$$\rho = \frac{P_{\text{bidir}}}{\mu^2} = \frac{\sum_{k=1}^{m} F(x_k) x_k^2}{\left(\sum_{k=1}^{m} F(x_k) x_k\right)^2} .$$



Hoffmann and Triesch 2017

$$P_{\text{bidir}} = \sum_{k=1}^{m} \sum_{l=1}^{m} F(x_k) x_k F(x_l|x_k) x_l \qquad F(x_l|x_k) = \begin{cases} 1 & \text{if } l = k \\ 0 & \text{otherwise.} \end{cases}$$

$$= \sum_{k=1}^{m} F(x_k) x_k^2 .$$



Relative overrepresentation ρ is the fraction

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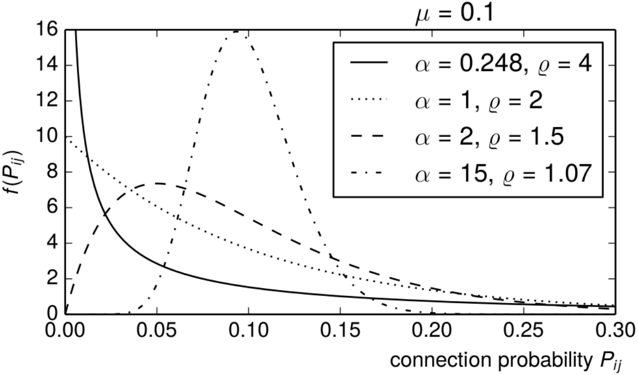
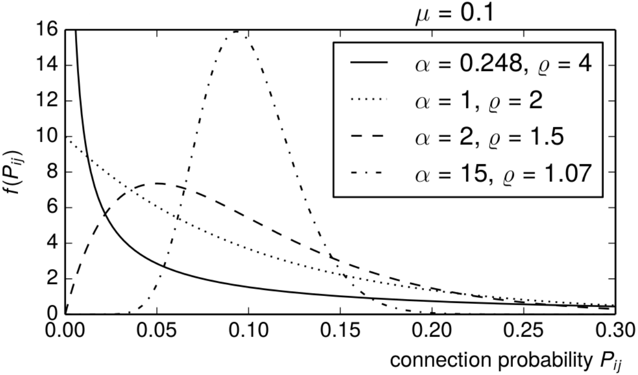
By Jensen's inequality,

$$\left(\sum_{k=1}^{m} F(x_k) x_k \right)^2 \leq \sum_{k=1}^{m} F(x_k) x_k^2 \quad \text{and thus} \quad \rho \geq 1.$$

Hoffmann and Triesch 2017

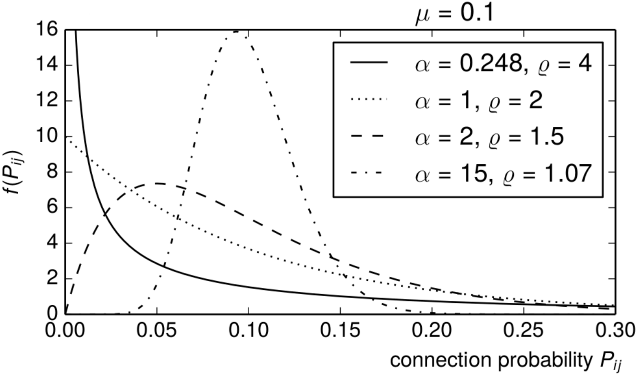


Hoffmann and Triesch 2017



– multiple neuron properties together can cause strong overrepresentation

Hoffmann and Triesch 2017

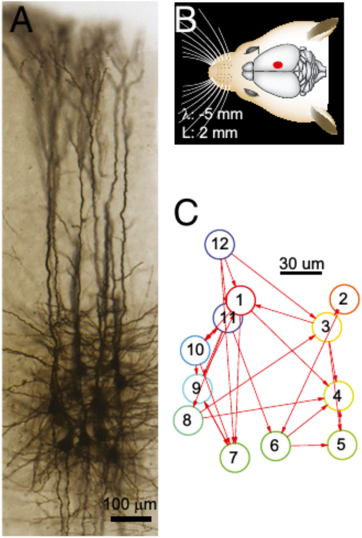


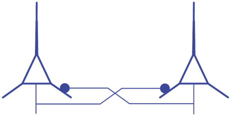
– multiple neuron properties together can cause strong overrepresentation

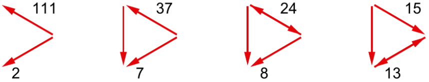
– example: higher connection probability in functionally related cells (Lee et al. 2016)

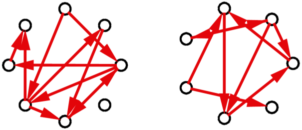
Hoffmann and Triesch 2017

Robust nonrandom connectivity patterns



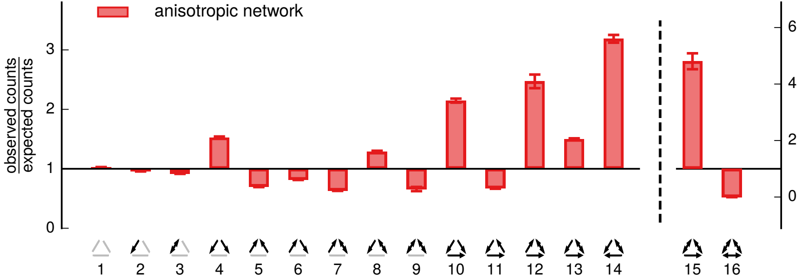






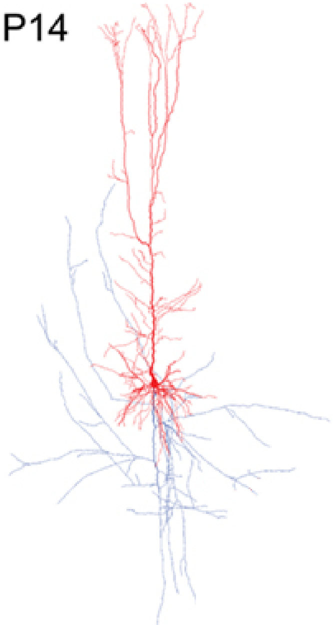
Perin et al. 2011; Song et al. 2005; Markram et al. 1997; Miner and Triesch 2016; Gal et al. 2017; Vegué et al. 2017

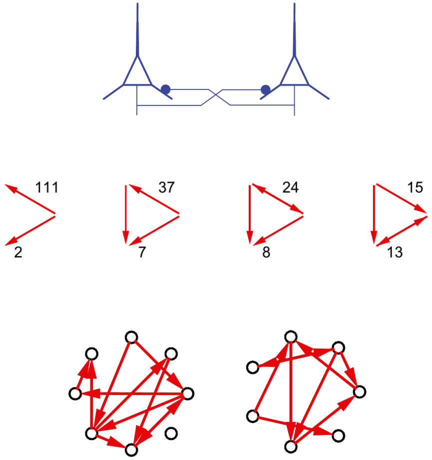
Triplet motif statistics in anisotropic networks



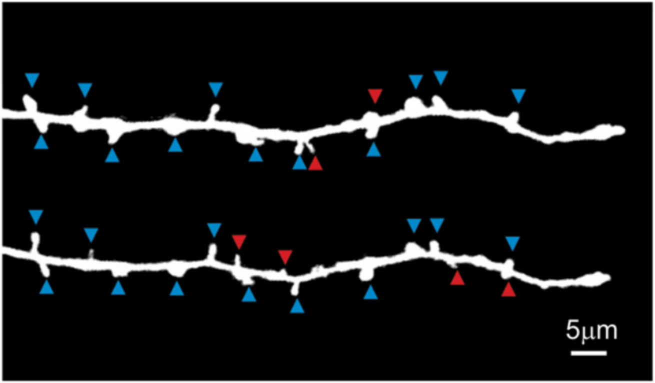
$\Rightarrow$ More aboout this: Poster W77, Wednesday!

Nonrandom connectivity from anisotropy





The dynamic connectome

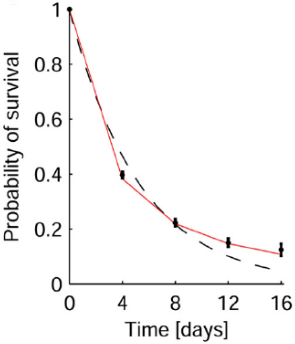


Day 0

Day 4

Loewenstein et al. 2015

The dynamic connectome

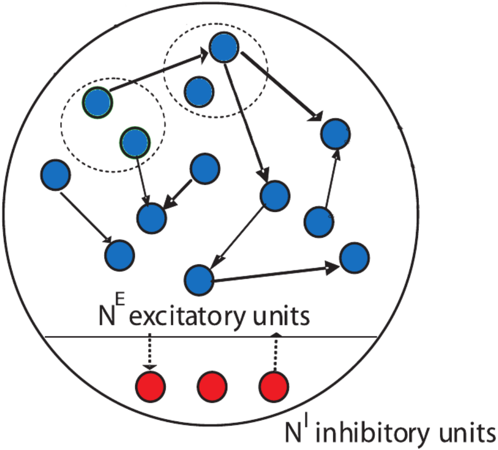


Survival probability of $\rho(t) = (t + 1)^{-\gamma}$ with $\gamma \approx 1.4$,

equivalently lifetime distribution of $f(t) = \gamma(t + 1)^{-(\gamma+1)}$

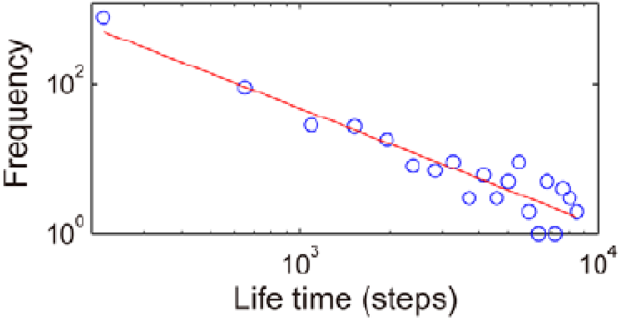
Loewenstein et al. 2015

Synapse dynamics in self-organizing recurrent networks



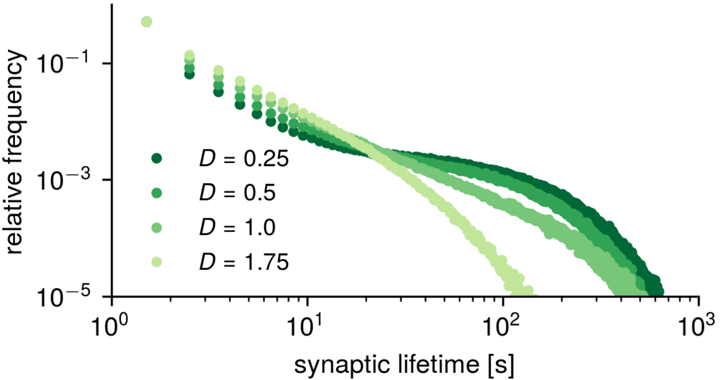
Zheng et al. 2013; Miner and Triesch 2016; Kleberg 2018

Synapse dynamics in self-organizing recurrent networks



Zheng et al. 2013; Miner and Triesch 2016; Kleberg 2018

Lifetimes in the self-organizing recurrent network



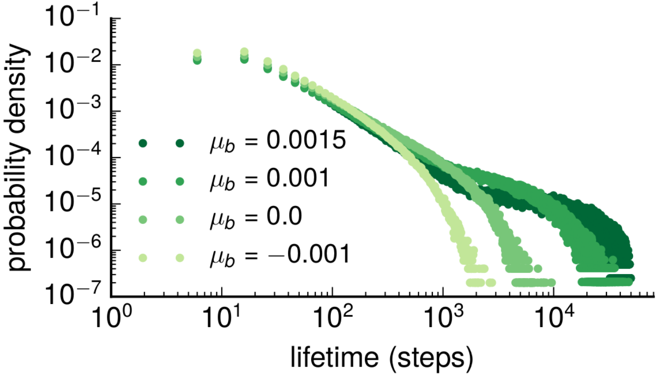
D = LTD–LTP balance

Lifetimes modelled by a stochastic process

Kesten process (Kesten 1973; Statman et al. 2014)

$$X_{n+1} = a_n X_n + b_n$$

as a model for synapse dynamics



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Triesch lab

bcf

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SPP 2041
Computational Connectomics

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