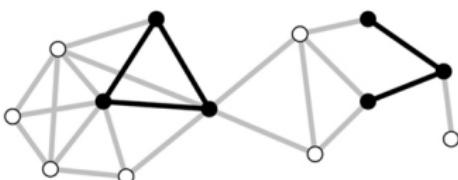


Modelling the nonrandom and dynamic structure of local cortical circuits

Felix Z. Hoffmann

Slides: bit.ly/bx18s



Bernstein Conference 2018 Satellite Workshop:
Adaptivity and Inhomogeneity in Neuronal Networks

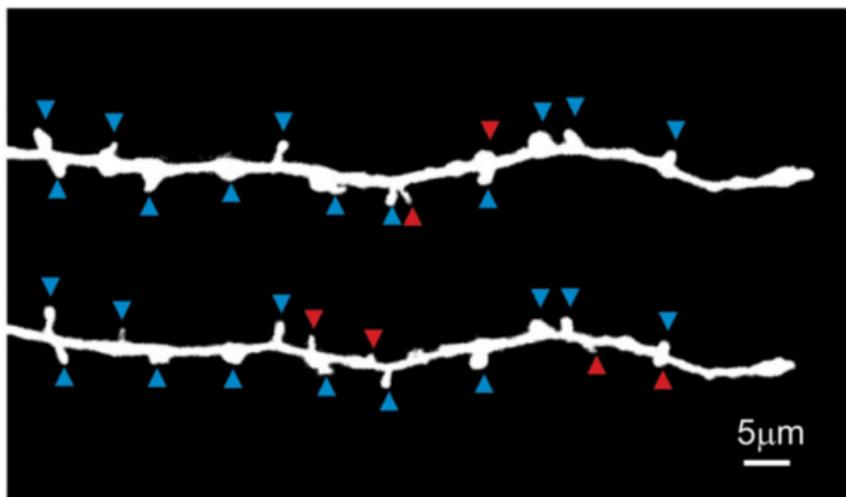


FIAS Frankfurt Institute
for Advanced Studies



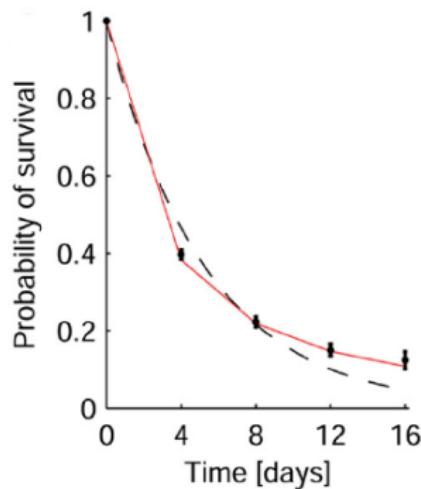
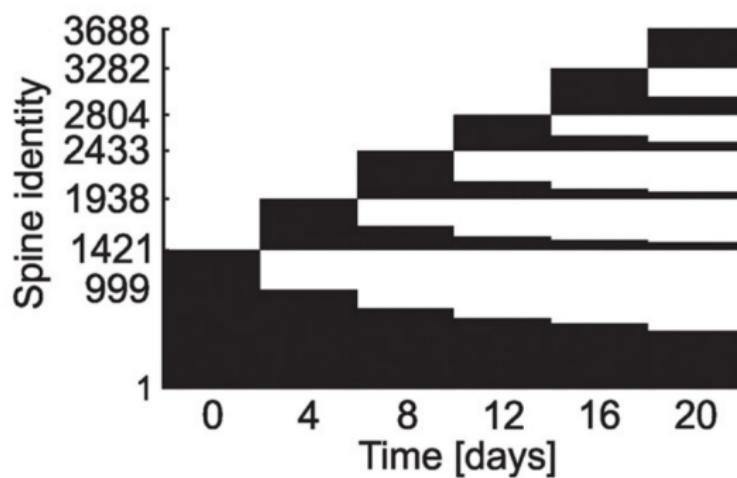
The dynamic connectome

At the synapse level, brain circuitry is constantly making new connections and abolishing old ones



The dynamic connectome

At the synapse level, brain circuitry is constantly making new connections and abolishing old ones



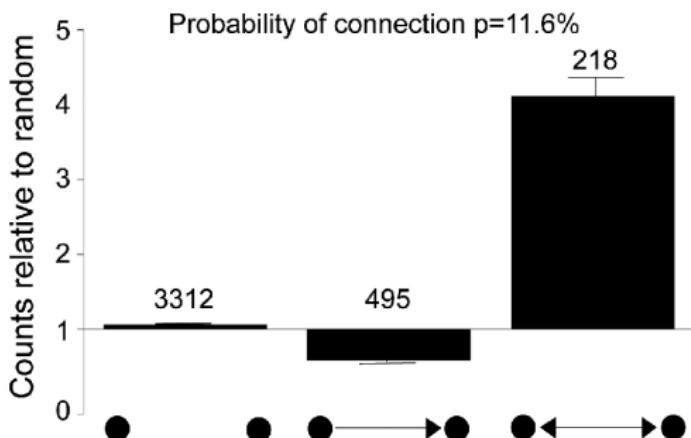
Robust nonrandom connectivity patterns

A

Null hypothesis assumes independent connection probabilities

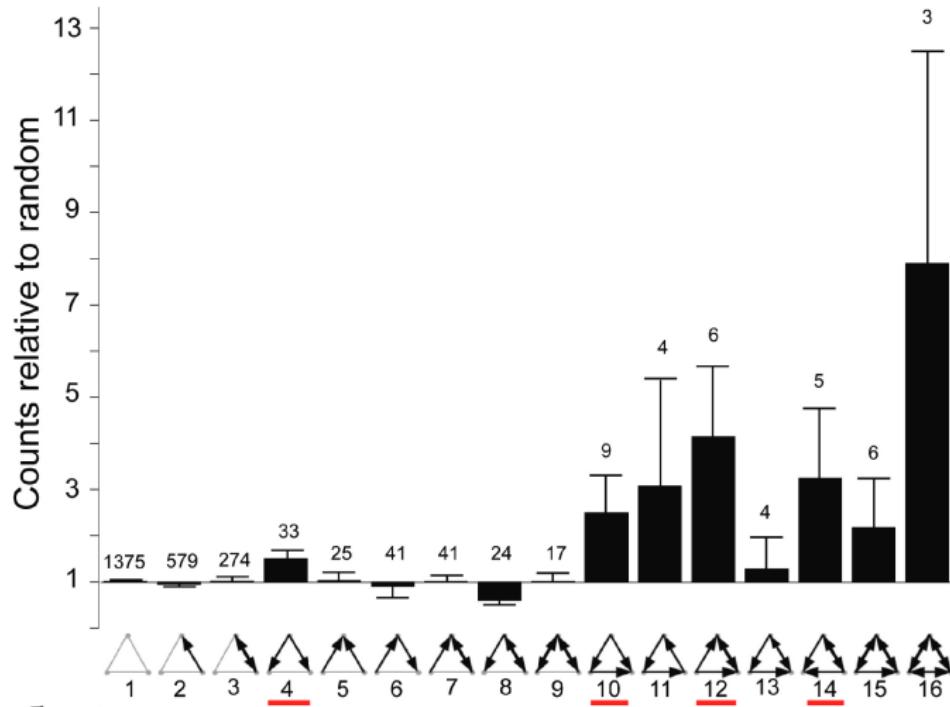


B



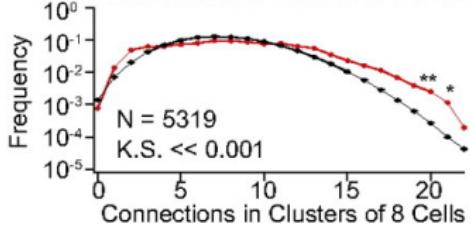
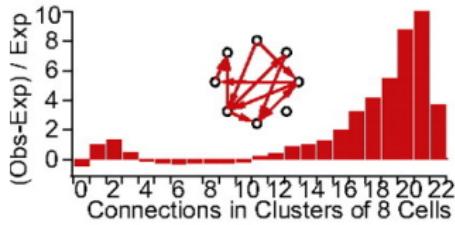
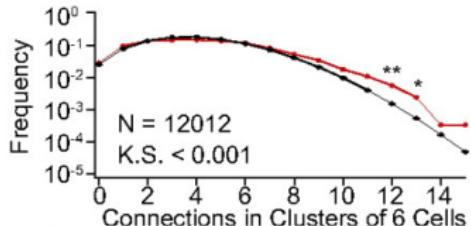
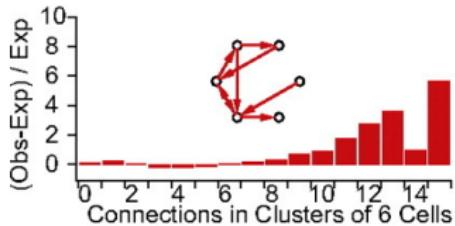
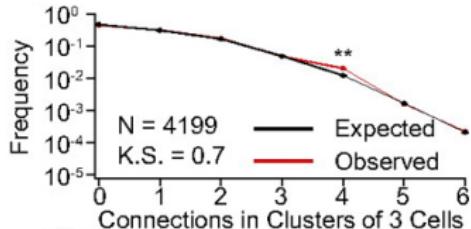
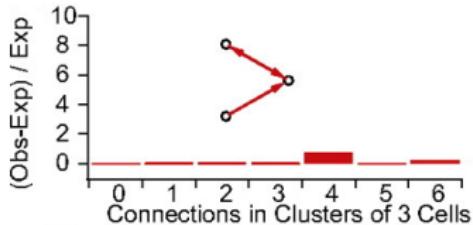
1. Overrepresentation of reciprocal connections

Robust nonrandom connectivity patterns



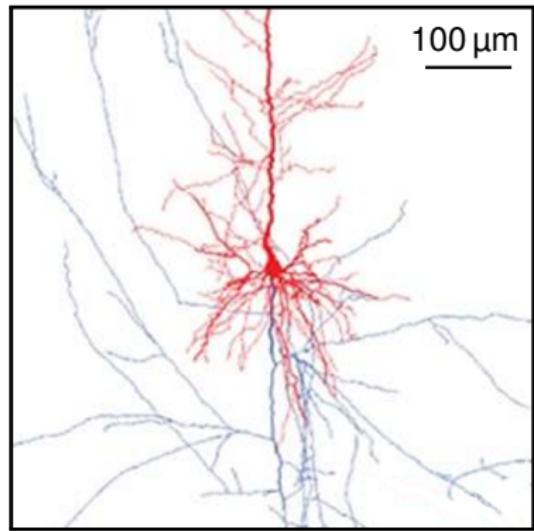
2. Characteristic occurrence of triplet motifs

Robust nonrandom connectivity patterns



3. High degree of connectivity in clusters occurs frequently

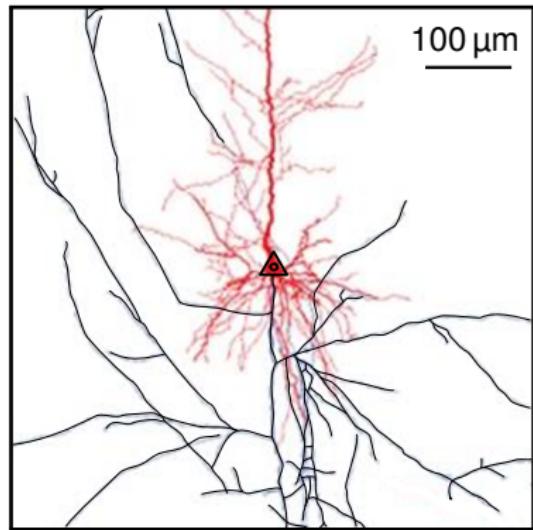
Morphological anisotropy in spatial connectivity



axon

morph. data from Romand et al. 2011

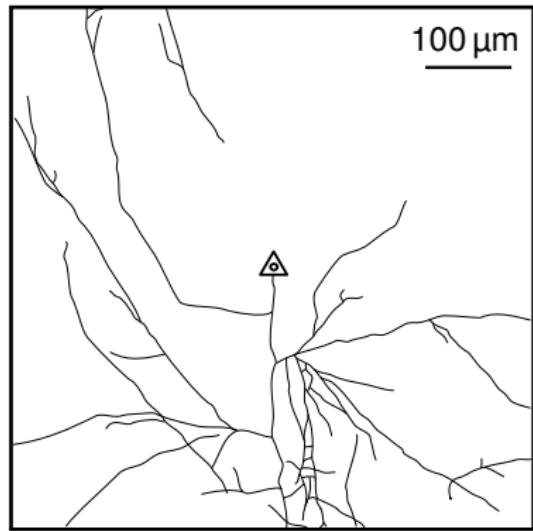
Morphological anisotropy in spatial connectivity



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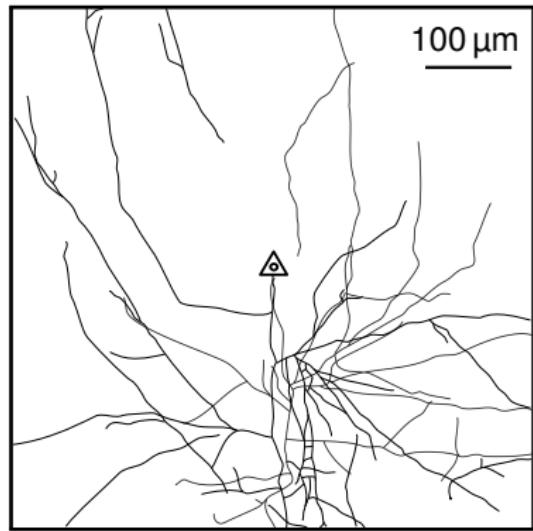
Morphological anisotropy in spatial connectivity



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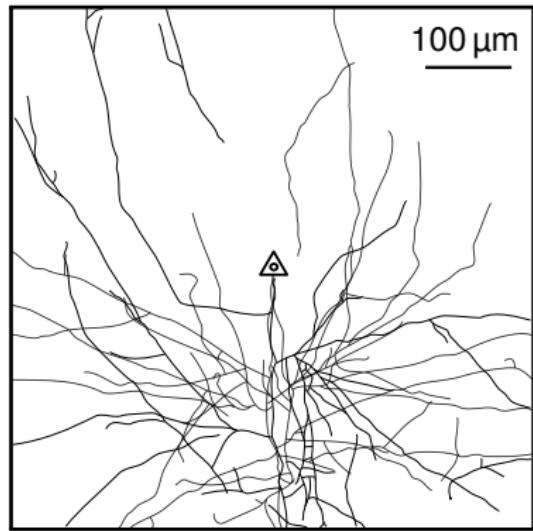
Morphological anisotropy in spatial connectivity



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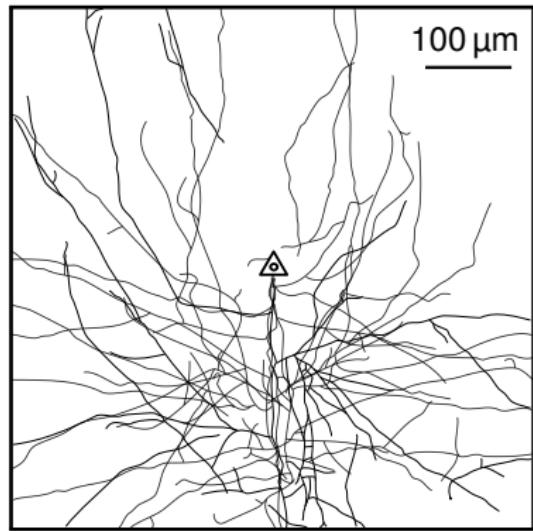
Morphological anisotropy in spatial connectivity



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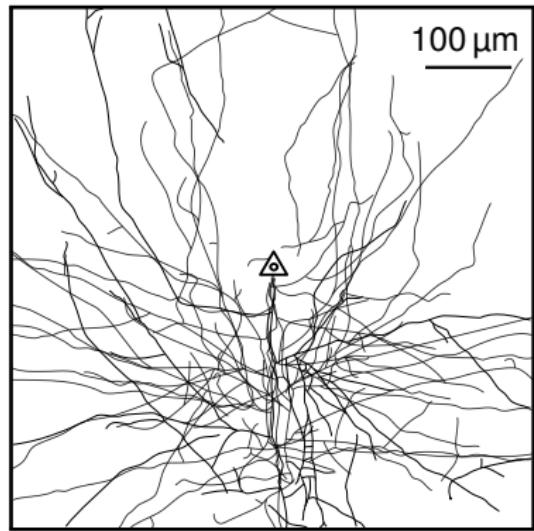
Morphological anisotropy in spatial connectivity



axon

morph. data from Romand et al. 2011

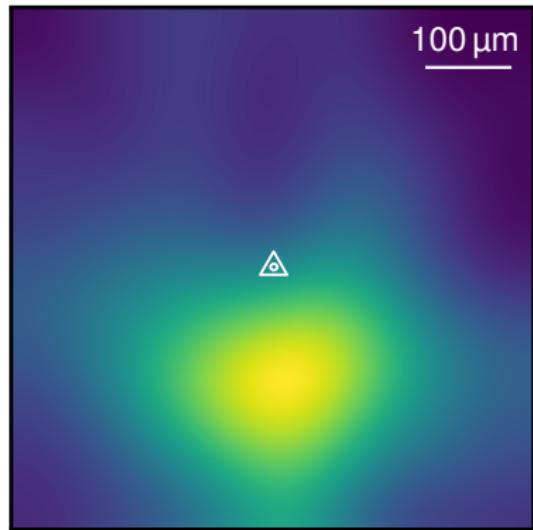
Morphological anisotropy in spatial connectivity



axon

morph. data from Romand et al. 2011

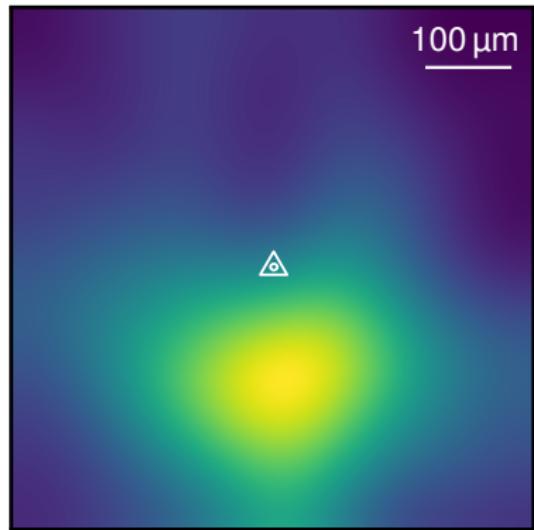
Morphological anisotropy in spatial connectivity



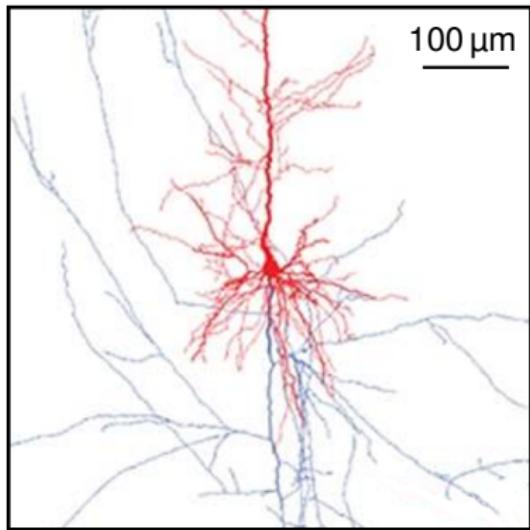
axon

morph. data from Romand et al. 2011

Morphological anisotropy in spatial connectivity



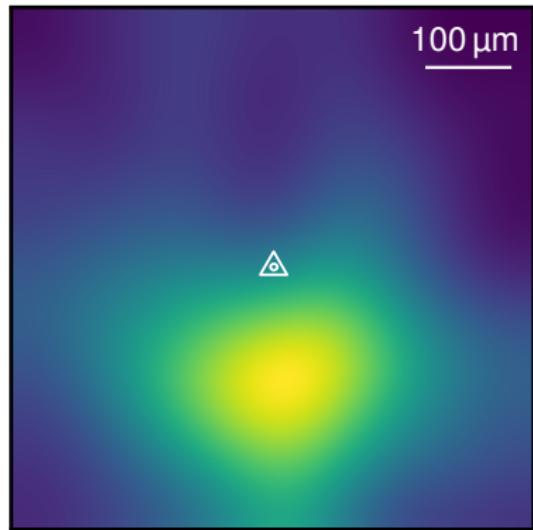
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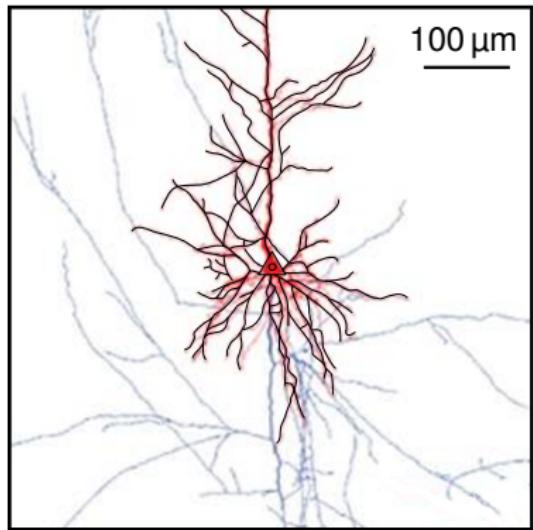
dendrite

morph. data from Romand et al. 2011

Morphological anisotropy in spatial connectivity



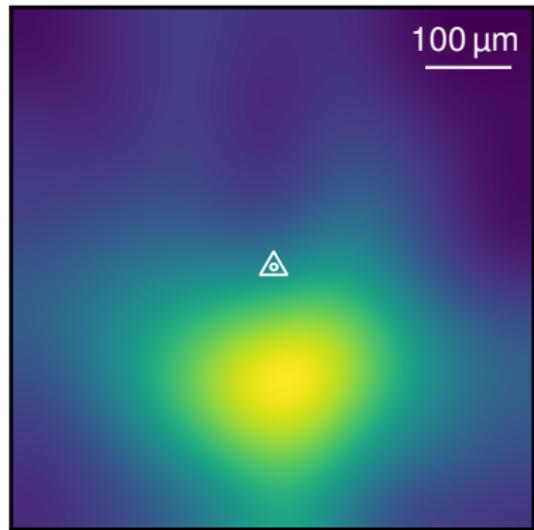
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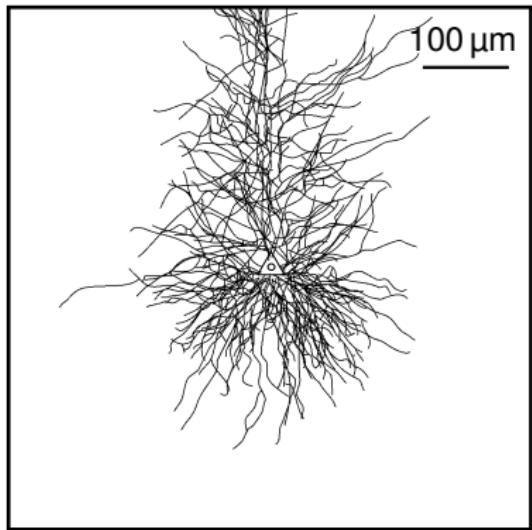
dendrite

morph. data from Romand et al. 2011

Morphological anisotropy in spatial connectivity



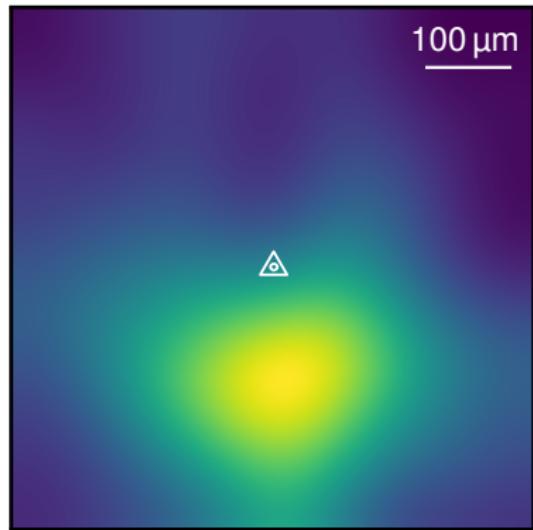
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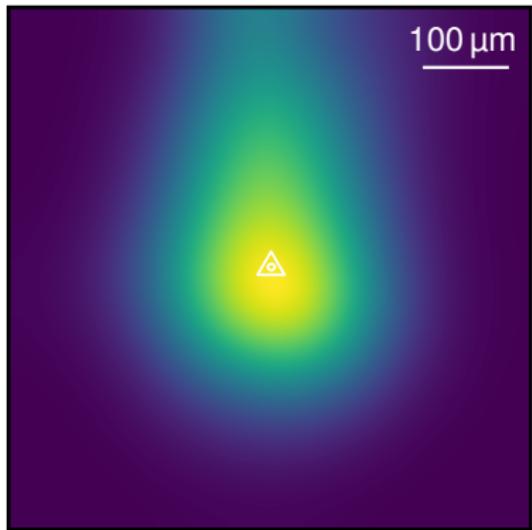
dendrite

morph. data from Romand et al. 2011

Morphological anisotropy in spatial connectivity



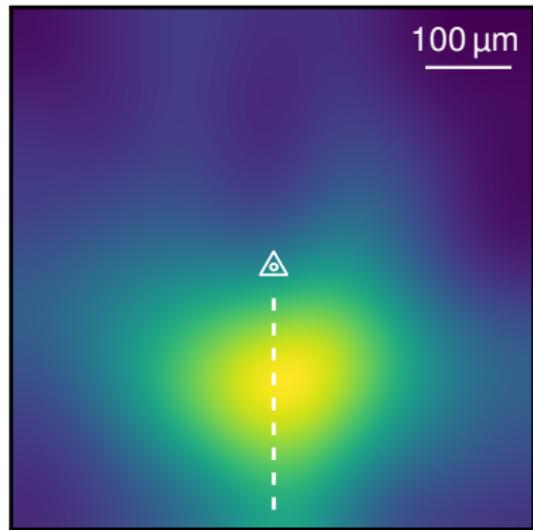
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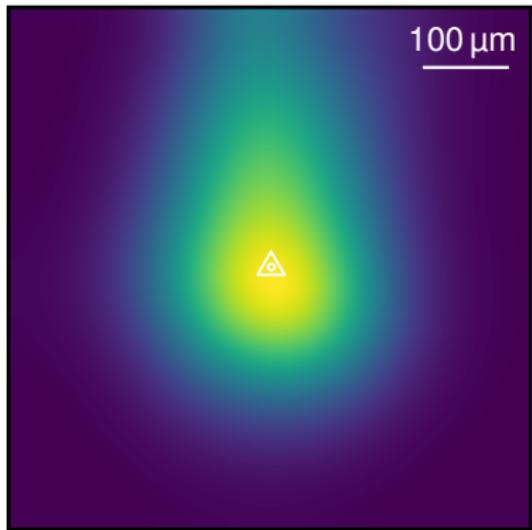
dendrite

morph. data from Romand et al. 2011

Morphological anisotropy in spatial connectivity



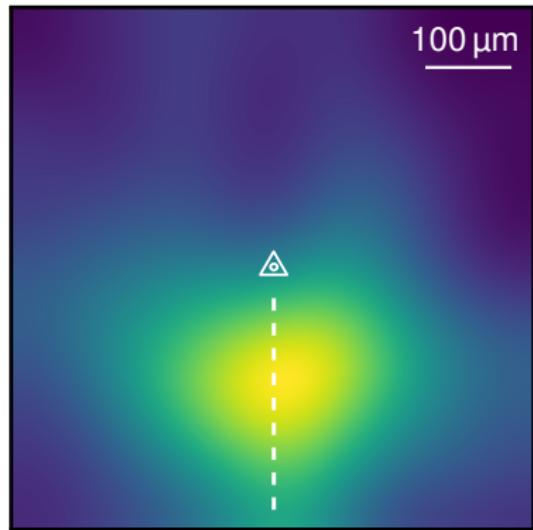
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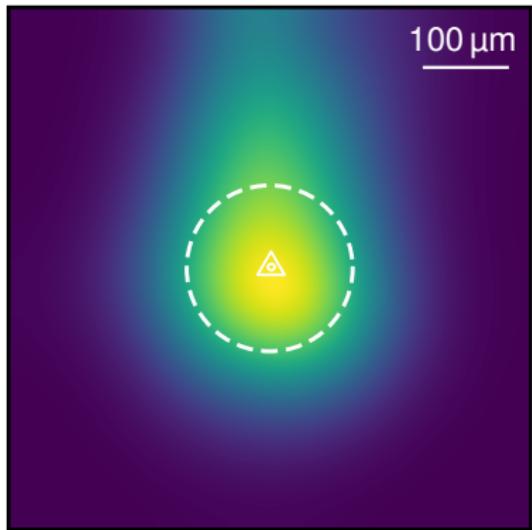
dendrite

morph. data from Romand et al. 2011

Morphological anisotropy in spatial connectivity



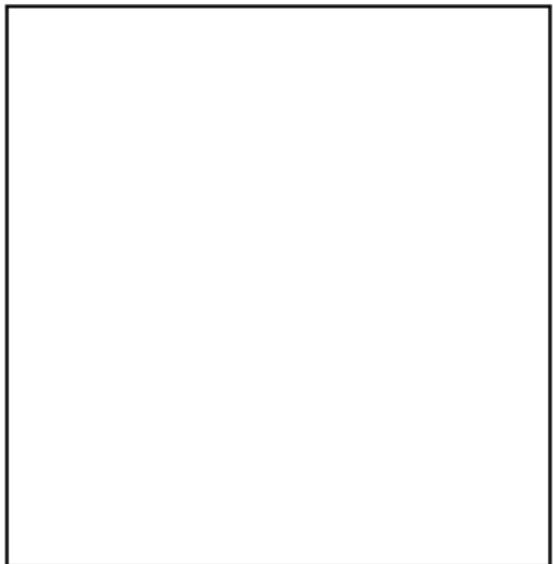
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dendrite

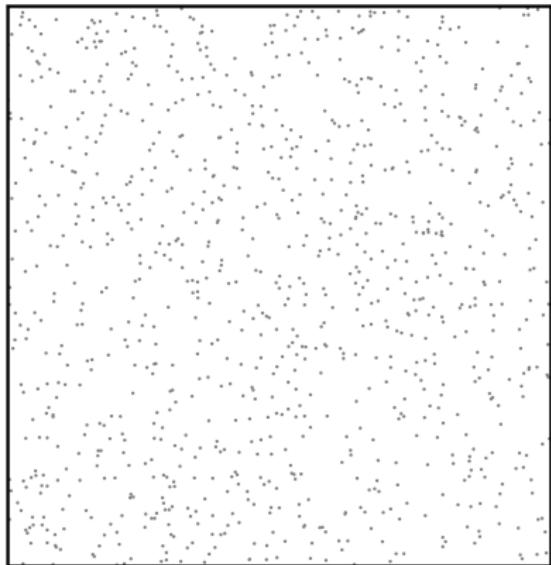
morph. data from Romand et al. 2011

Anisotropic network model



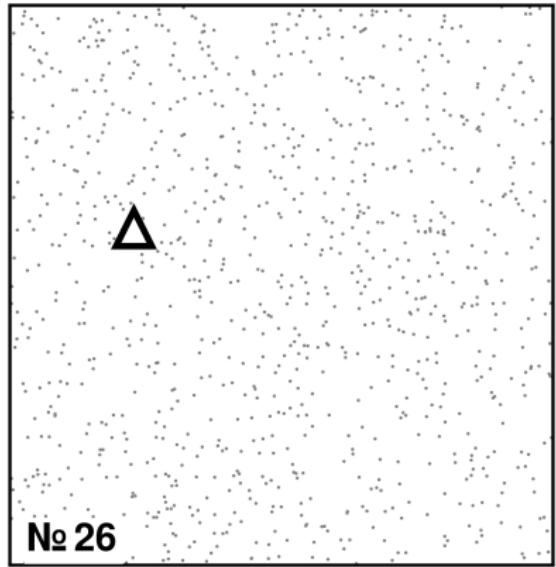
Distribute $N = 1000$ neurons randomly on a square surface

Anisotropic network model



Distribute $N = 1000$ neurons randomly on a square surface

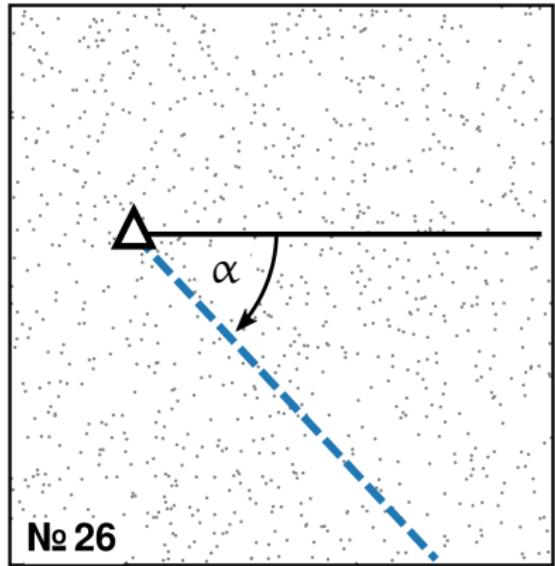
Anisotropic network model



Distribute $N = 1000$ neurons randomly on a square surface

For each neuron

Anisotropic network model

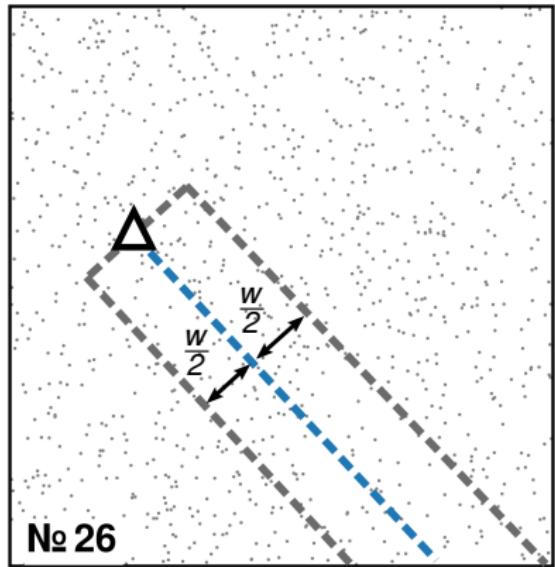


Distribute $N = 1000$ neurons randomly on a square surface

For each neuron

- assign a random angle α

Anisotropic network model

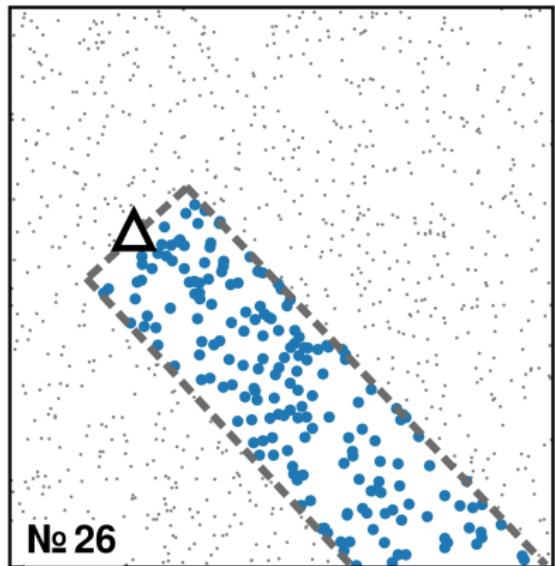


Distribute $N = 1000$ neurons randomly on a square surface

For each neuron

- assign a random angle α
- find targets within distance $w/2$ of α -projection

Anisotropic network model

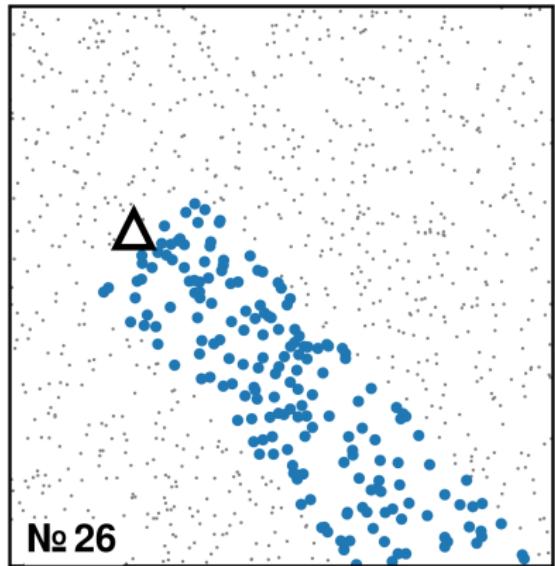


Distribute $N = 1000$ neurons randomly on a square surface

For each neuron

- assign a random angle α
- find targets within distance $w/2$ of α -projection
- connect to targets

Anisotropic network model

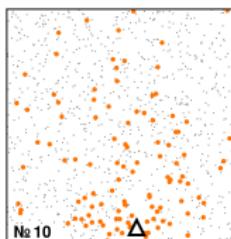
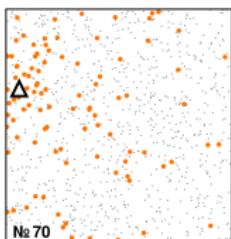
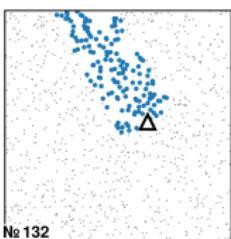
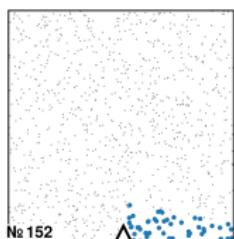
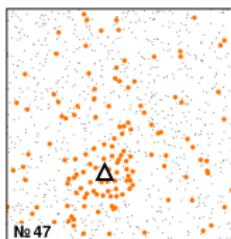
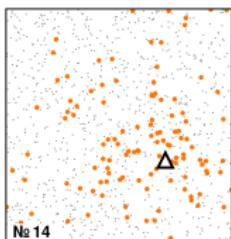
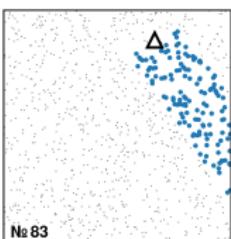
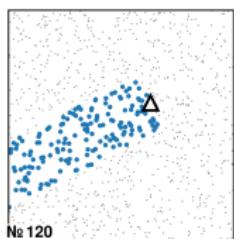
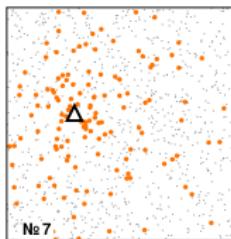
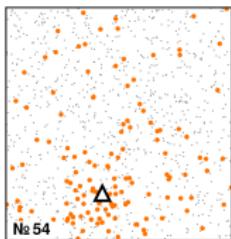
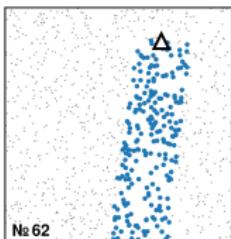
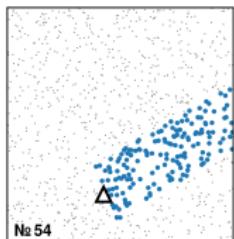


Distribute $N = 1000$ neurons randomly on a square surface

For each neuron

- assign a random angle α
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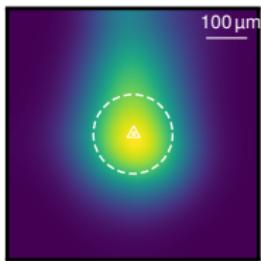
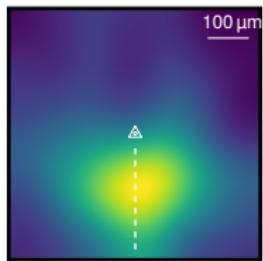
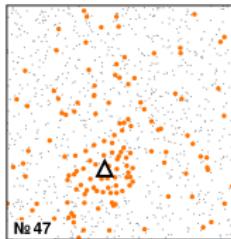
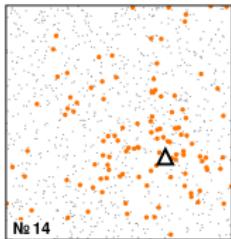
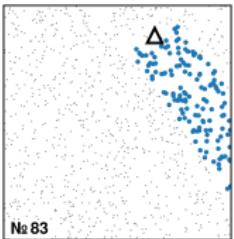
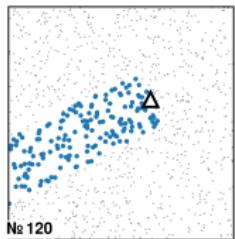
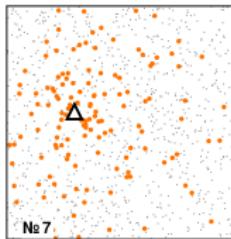
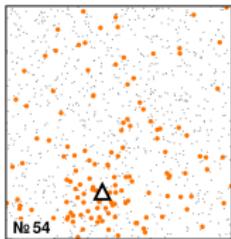
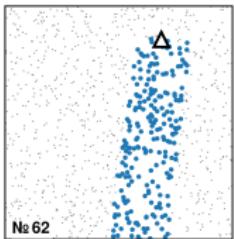
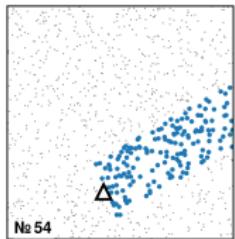
Anisotropic network model



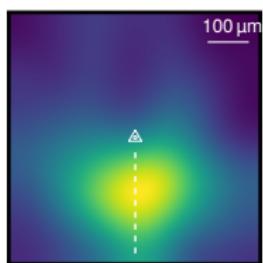
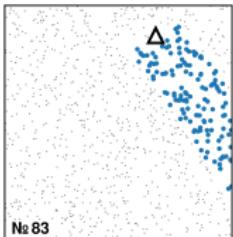
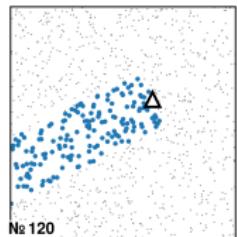
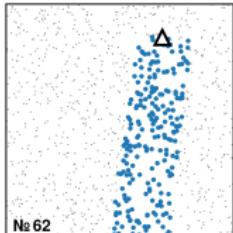
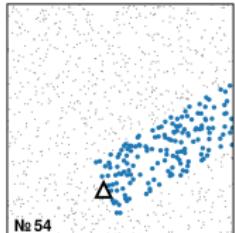
targets

inputs

Anisotropic network model



Anisotropic network model



Model parameters

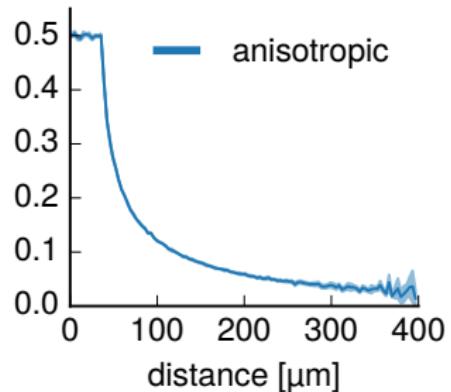
Number of nodes $N = 1000$

edge length of surface $l = 296 \mu\text{m}$

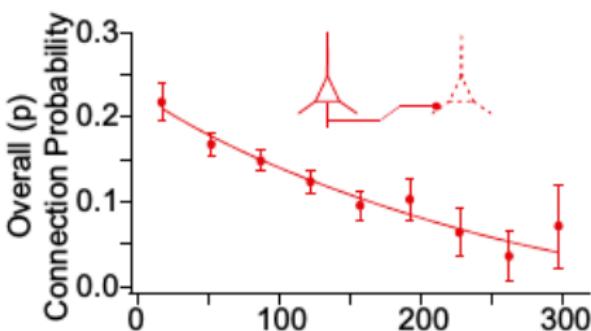
width $w = 74.6 \mu\text{m}$

l and w tuned to give connection density of $p = 0.116$

Distance-dependent connectivity in anisotropic networks

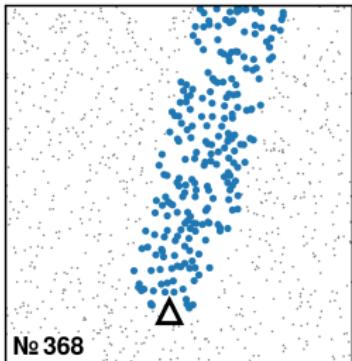


anisotropic network

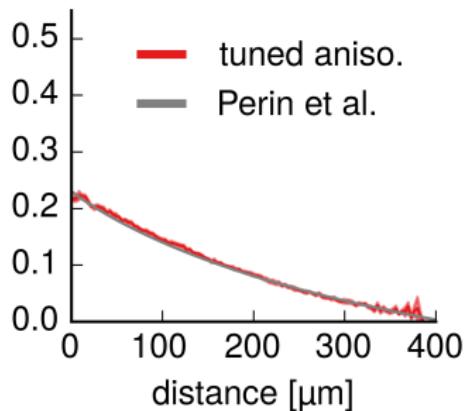
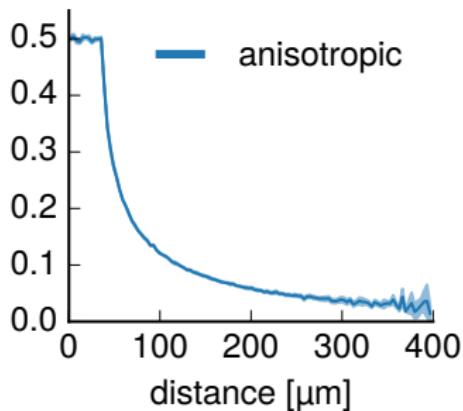
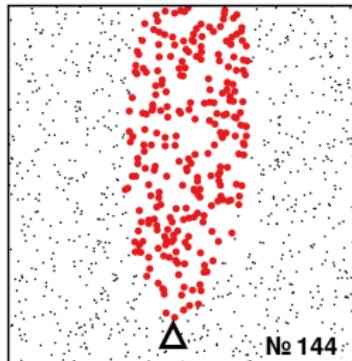


somatosensory cortex
Perin et al. (2011)

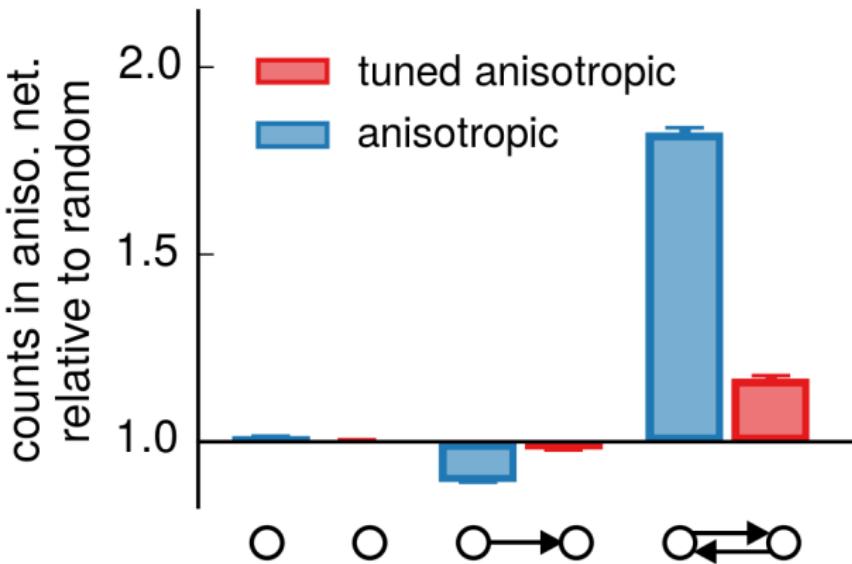
Anisotropic network



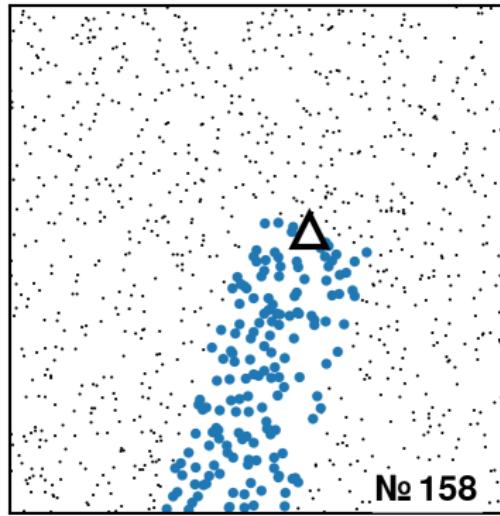
Tuned anisotropic network



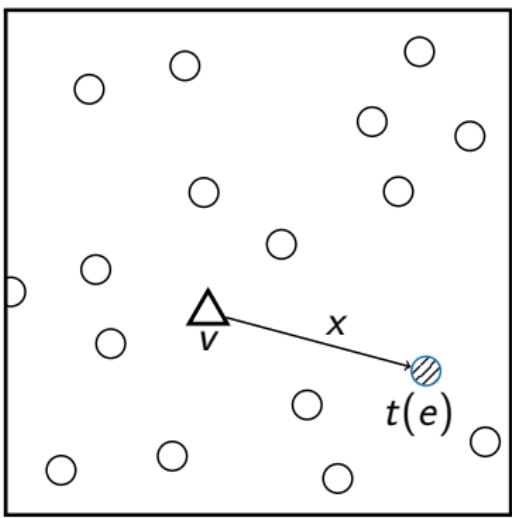
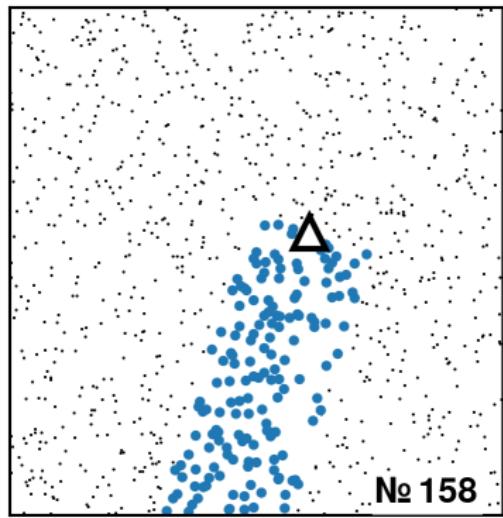
Results – Overrepresentation of reciprocal connections



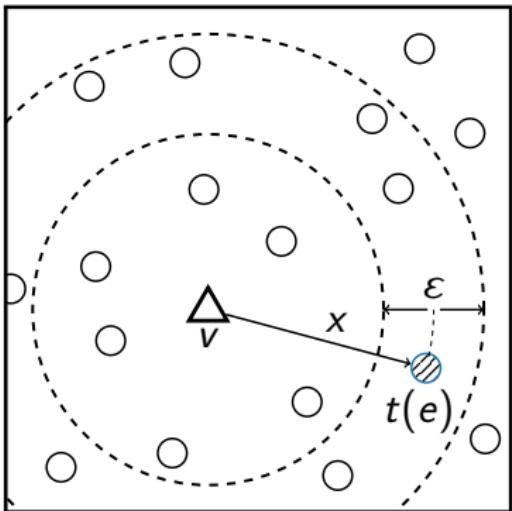
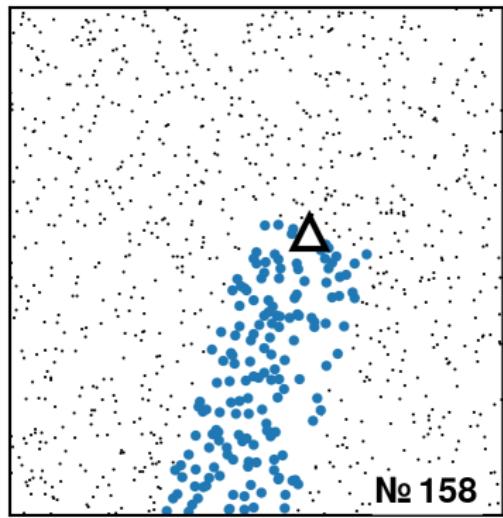
Rewiring anisotropic networks



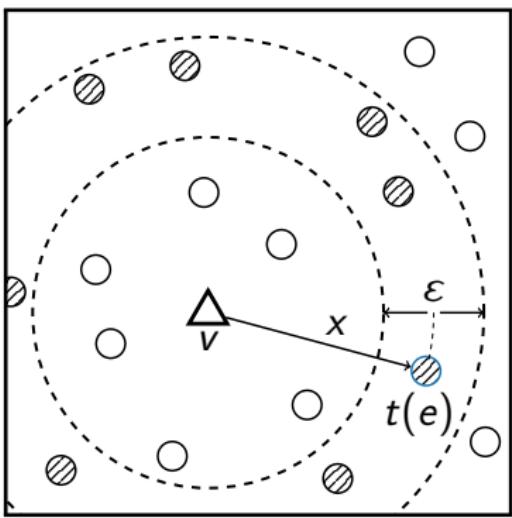
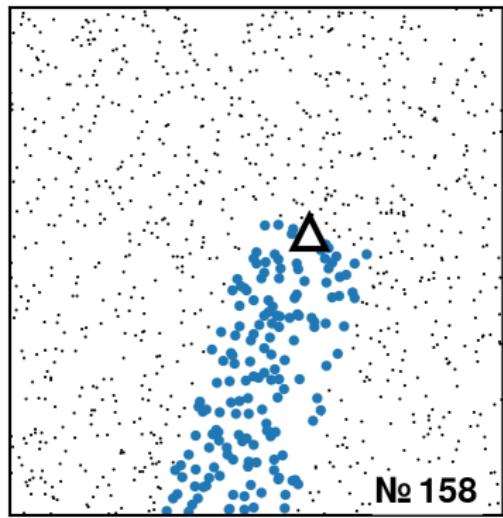
Rewiring anisotropic networks



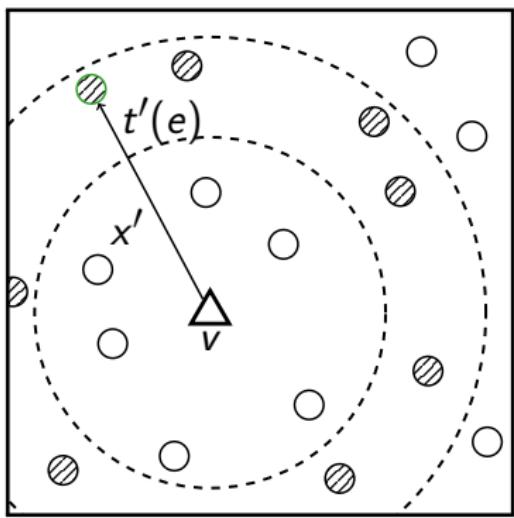
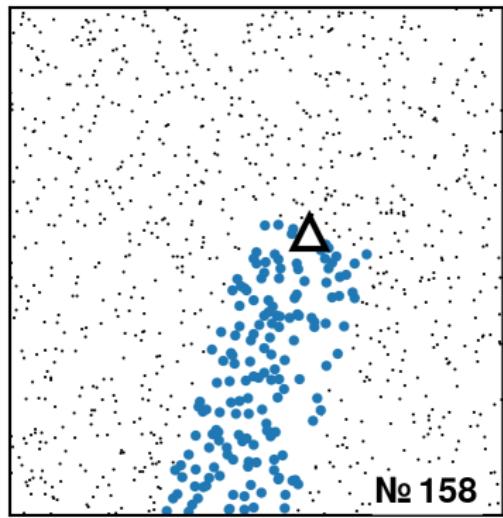
Rewiring anisotropic networks



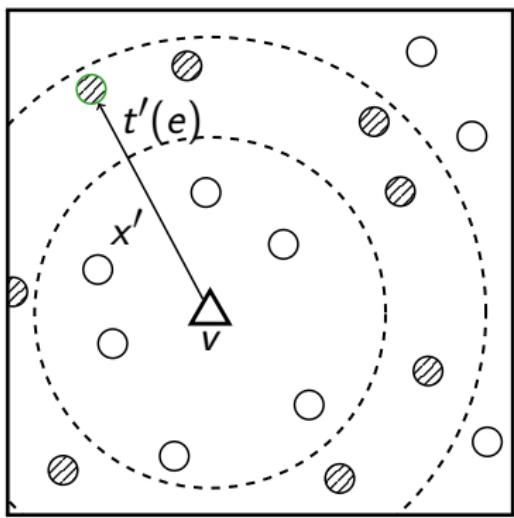
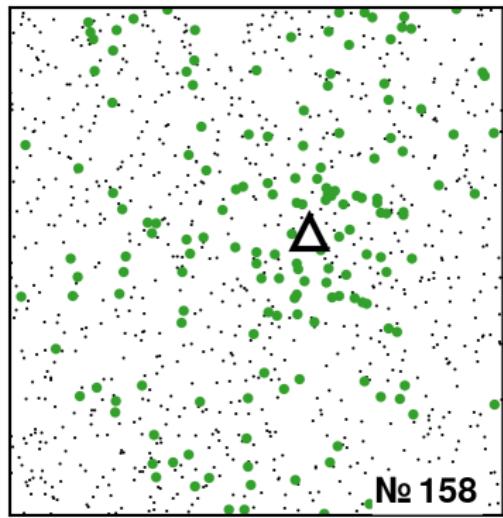
Rewiring anisotropic networks



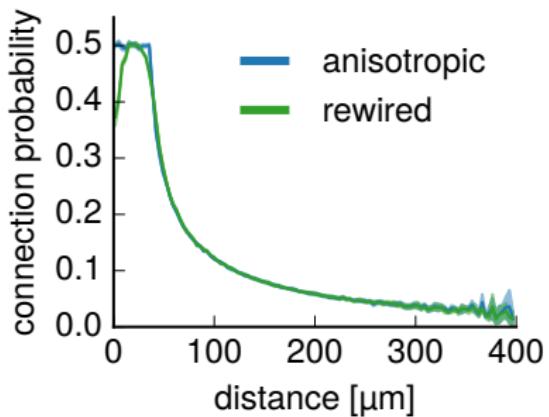
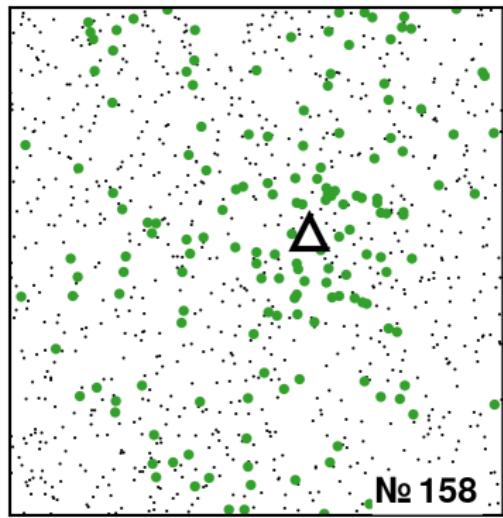
Rewiring anisotropic networks



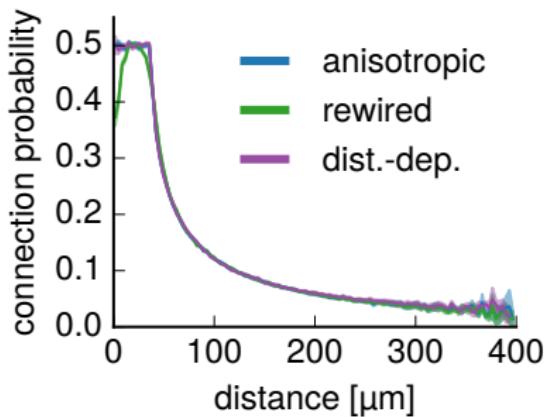
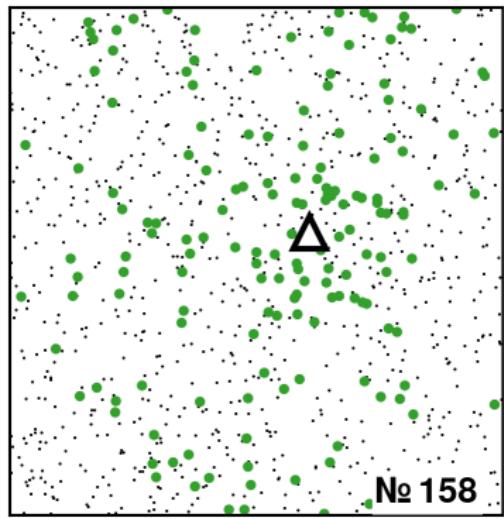
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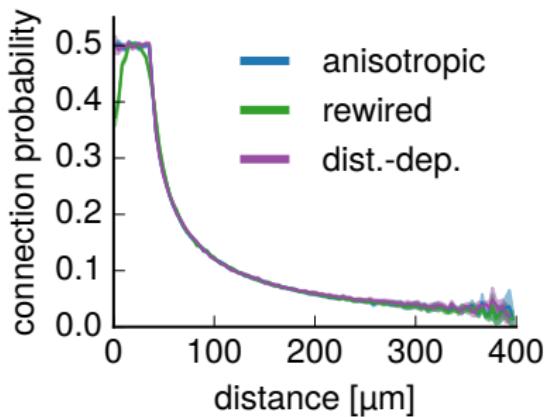
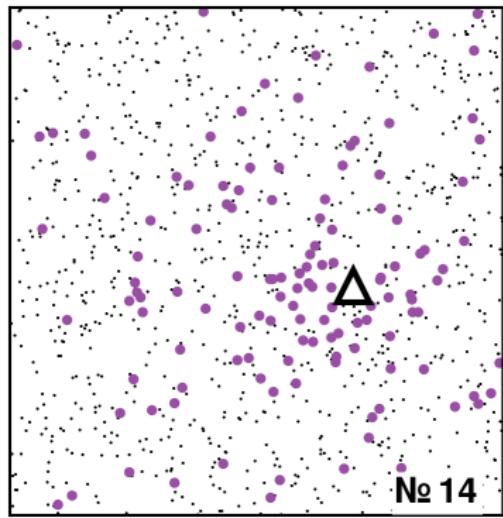
Rewiring anisotropic networks



Rewiring anisotropic networks

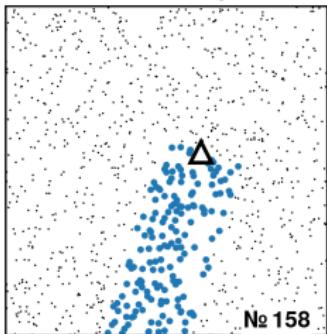


Distance-dependent network



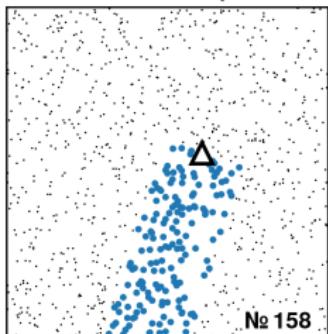
Network model overview

anisotropic

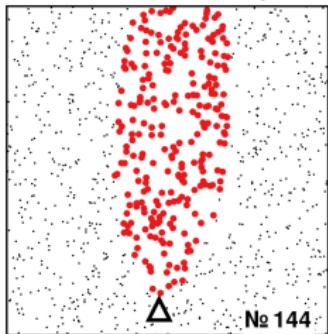


Network model overview

anisotropic

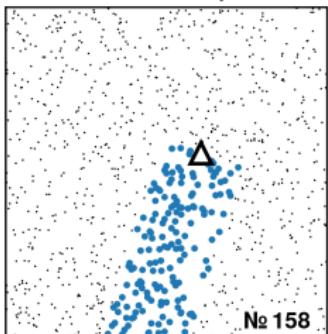


tuned anisotropic

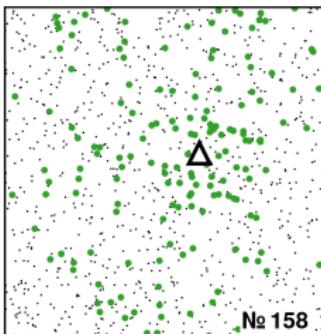


Network model overview

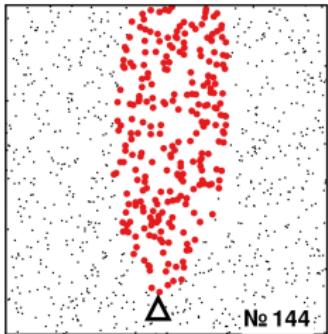
anisotropic



rewired

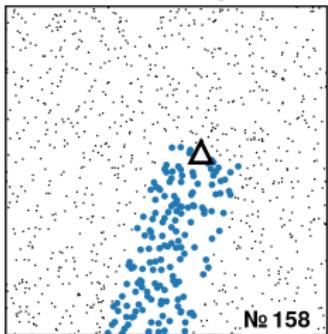


tuned anisotropic

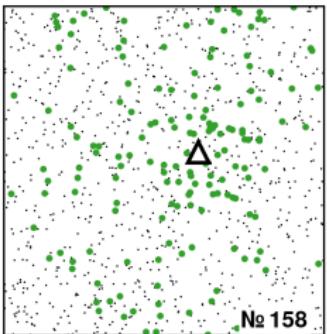


Network model overview

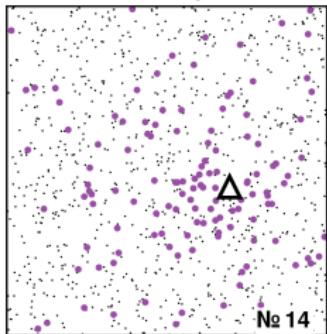
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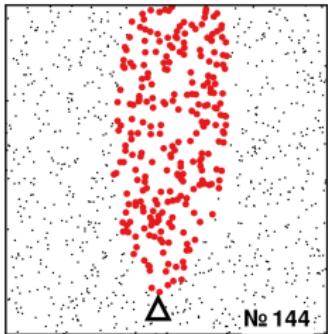
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distance-dependent

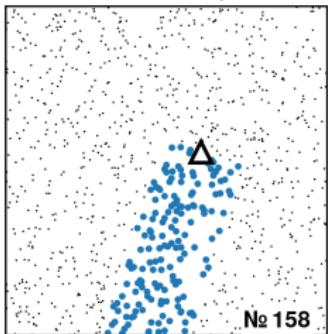


tuned anisotropic

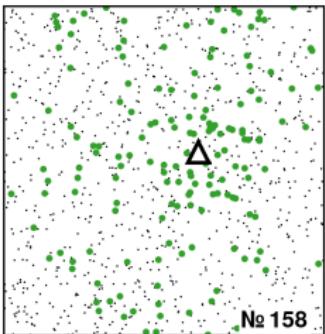


Network model overview

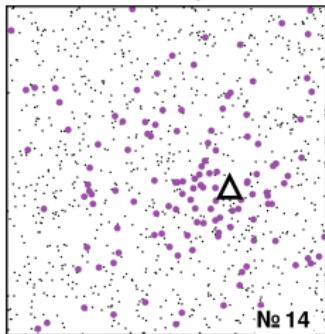
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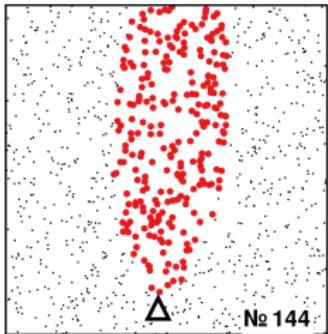
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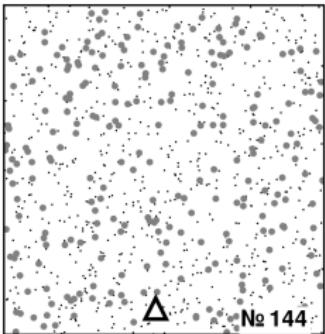
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tuned anisotropic

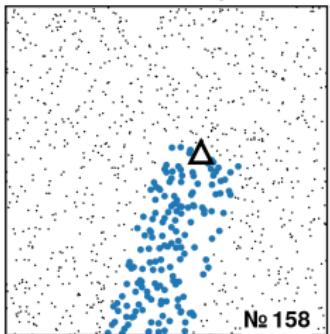


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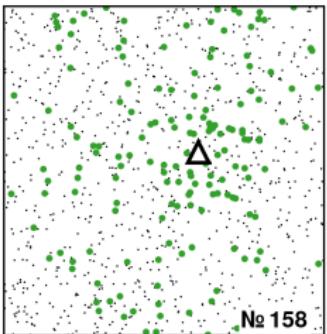


Network model overview

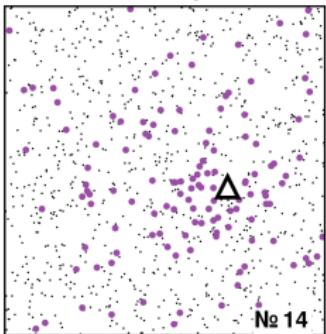
anisotropic



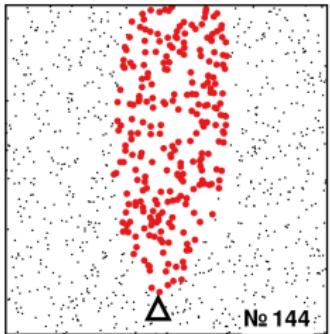
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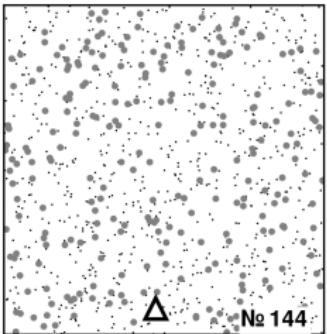
distance-dependent



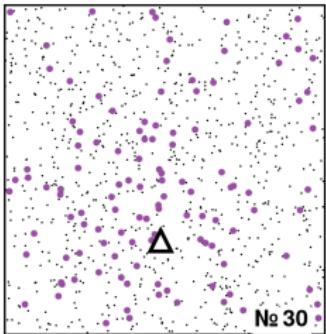
tuned anisotropic



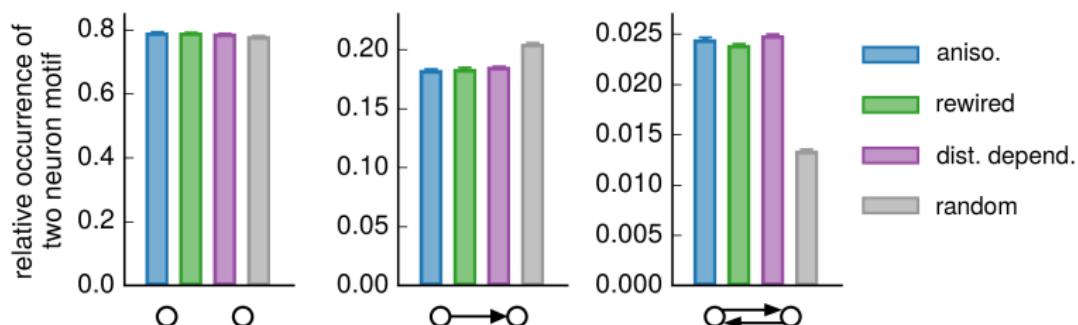
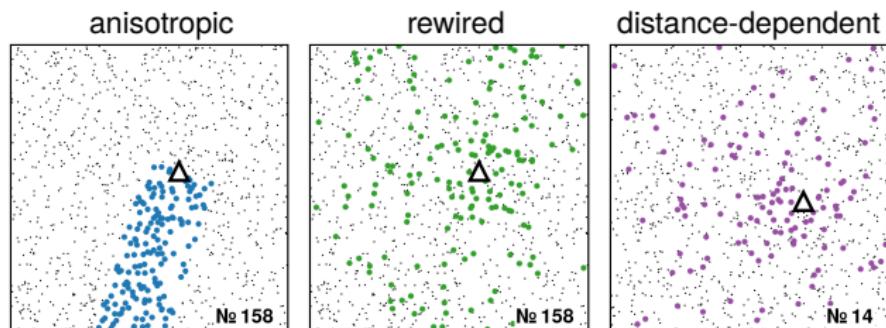
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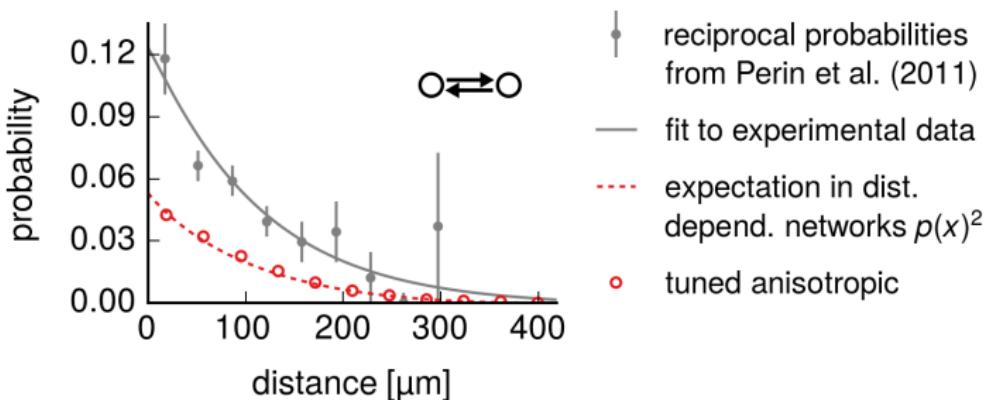
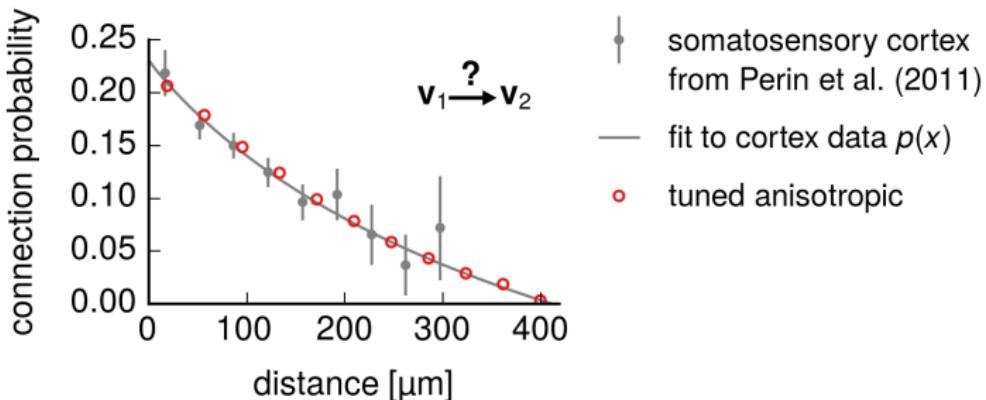
distance-dependent



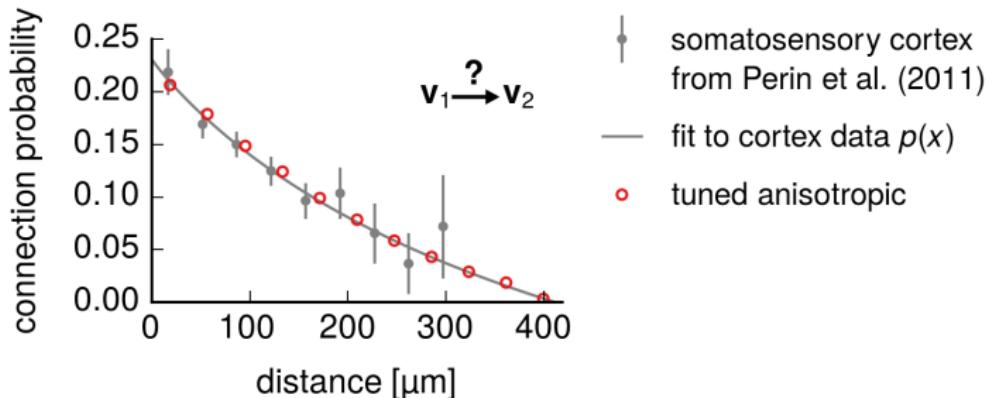
Results – Overrepresentation of reciprocal connections



Results – Overrepresentation of reciprocal connections

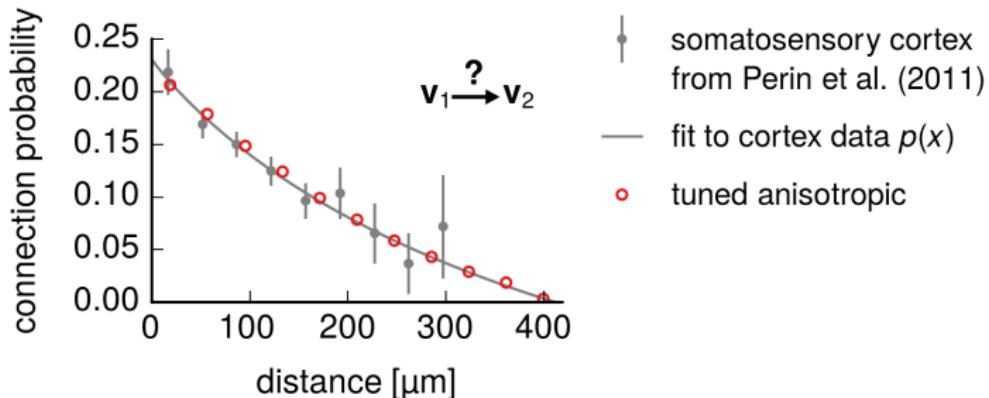


Results – Overrepresentation of reciprocal connections



Other sources for the overrepresentation of bidirectional connections?

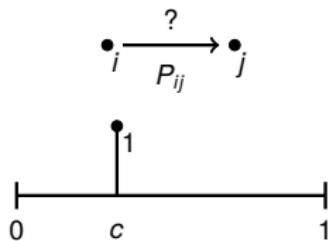
Results – Overrepresentation of reciprocal connections



Other sources for the overrepresentation of bidirectional connections?

⇒ Hoffmann, FZ and Triesch, J (2017). Nonrandom Network Connectivity Comes in Pairs. *Network Neuroscience*

Standard random network model



Probability of connection a constant P_{ij} ,

$$P_{ij} = c$$

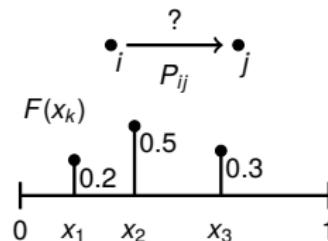
Overall connection probability

$$\mu = P_{ij} = c$$

Bidirectional connection

$$P_{\text{bidir}} = P_{ij}P_{ji} = c^2$$

Varying connection probabilities



Probability of connection a random variable P_{ij} ,

$$\mathbf{Prob}(P_{ij} = x_k) = F(x_k)$$

Overall connection probability

$$\mu = \sum_{k=1}^m F(x_k)x_k$$

Bidirectional connection

$$P_{\text{bidir}} = \sum_{k=1}^m \sum_{l=1}^m F(x_k)x_k F(x_l|x_k)x_l$$

Assume that connection probabilities within pairs are identical,

$$F(x_l|x_k) = \begin{cases} 1 & \text{if } l = k \\ 0 & \text{otherwise.} \end{cases}$$

Then

$$P_{\text{bidir}} = \sum_{k=1}^m \sum_{l=1}^m F(x_k)x_k F(x_l|x_k)x_l = \sum_{k=1}^m F(x_k)x_k^2.$$

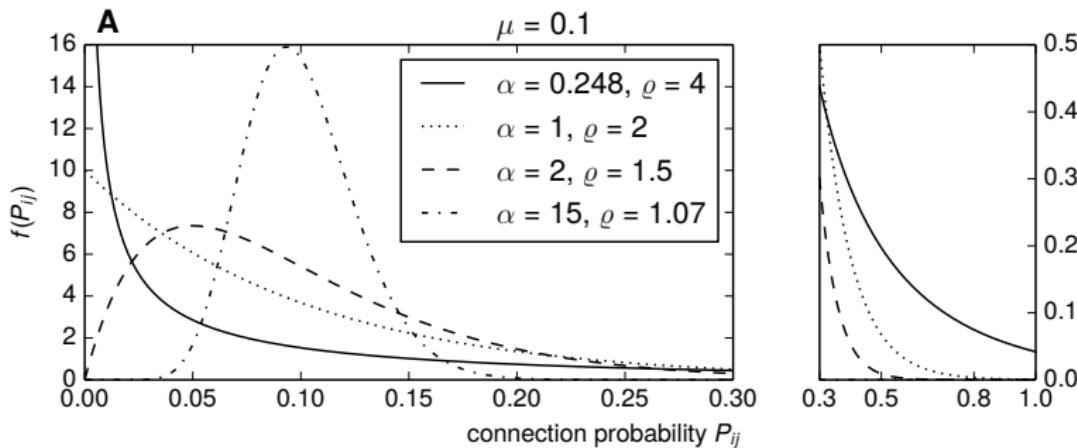
Relative overrepresentation ρ is the fraction

$$\rho = \frac{P_{\text{bidir}}}{\mu^2} = \frac{\sum_{k=1}^m F(x_k)x_k^2}{\left(\sum_{k=1}^m F(x_k)x_k\right)^2}.$$

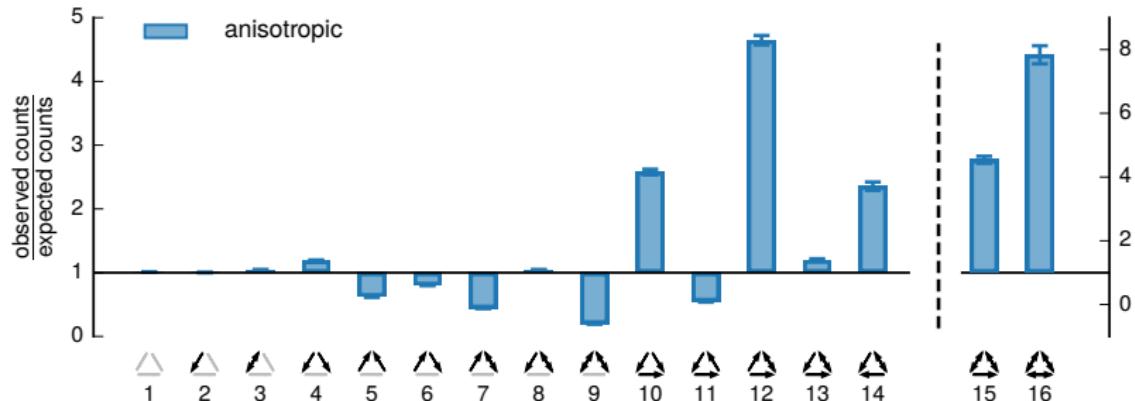
By Jensen's inequality,

$$\left(\sum_{k=1}^m F(x_k)x_k\right)^2 \leq \sum_{k=1}^m F(x_k)x_k^2 \quad \text{and thus} \quad \rho \geq 1.$$

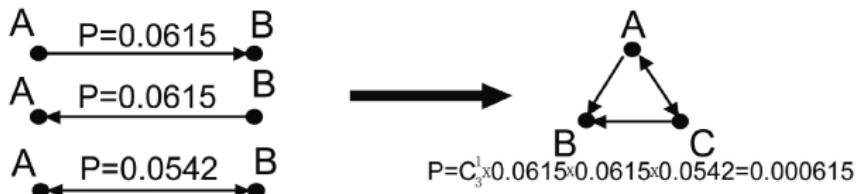
- multiple neuron properties may compound to give rise to broad connection probability distribution
- for example, higher connection probability in functionally related cells (Lee et al. 2016)



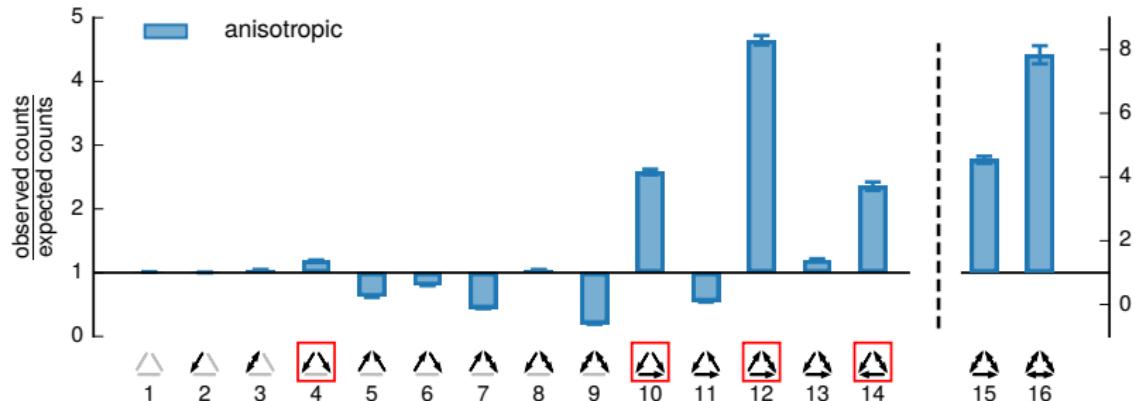
Three neuron motifs



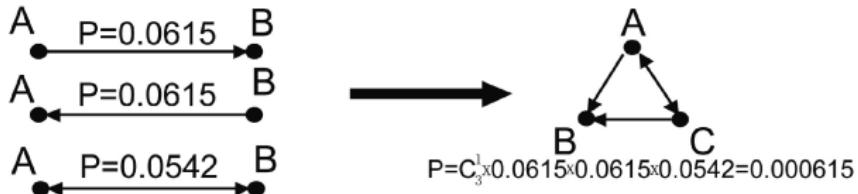
Null hypothesis assumes independent combination of pair connection probabilities



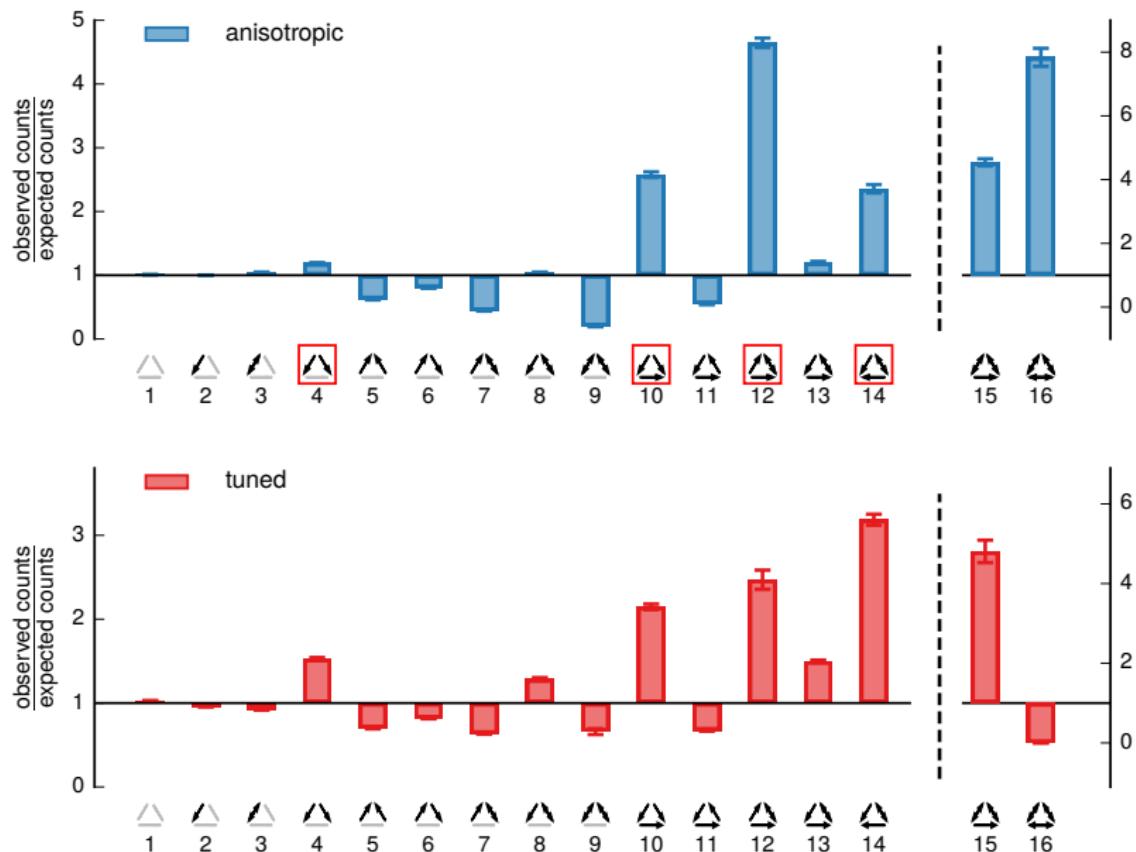
Three neuron motifs



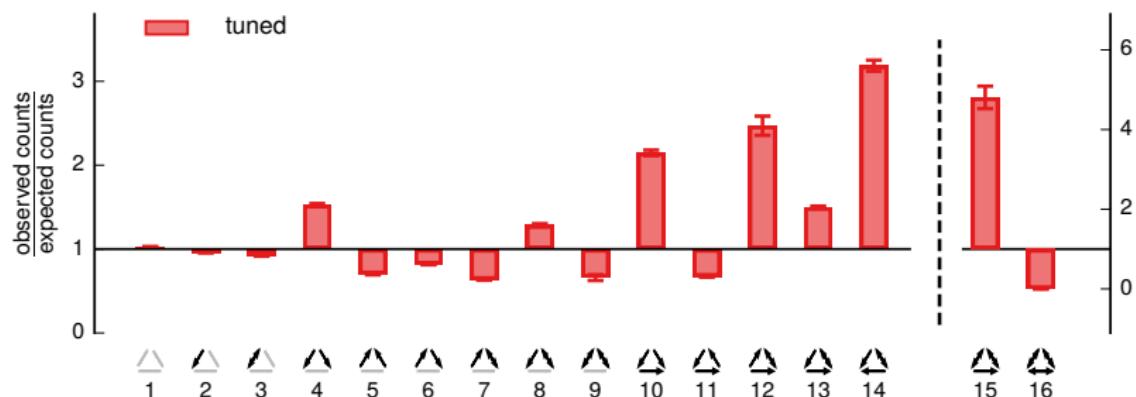
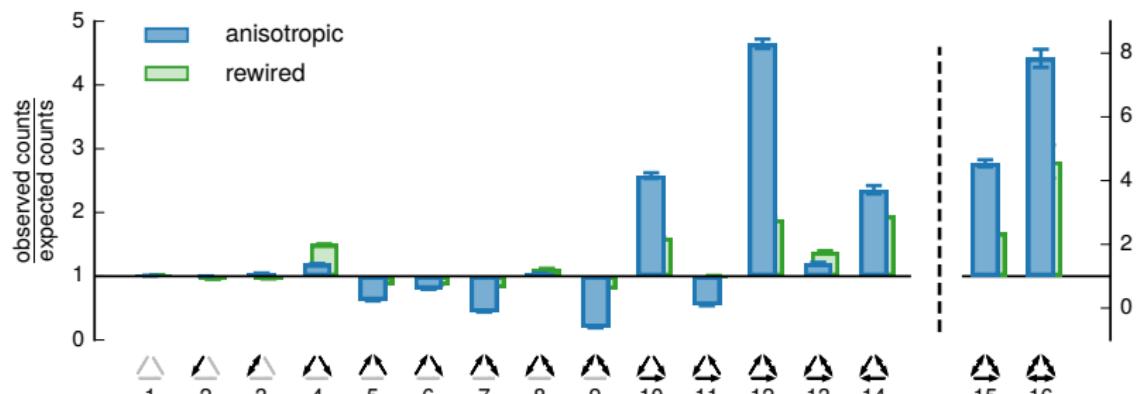
Null hypothesis assumes independent combination of pair connection probabilities



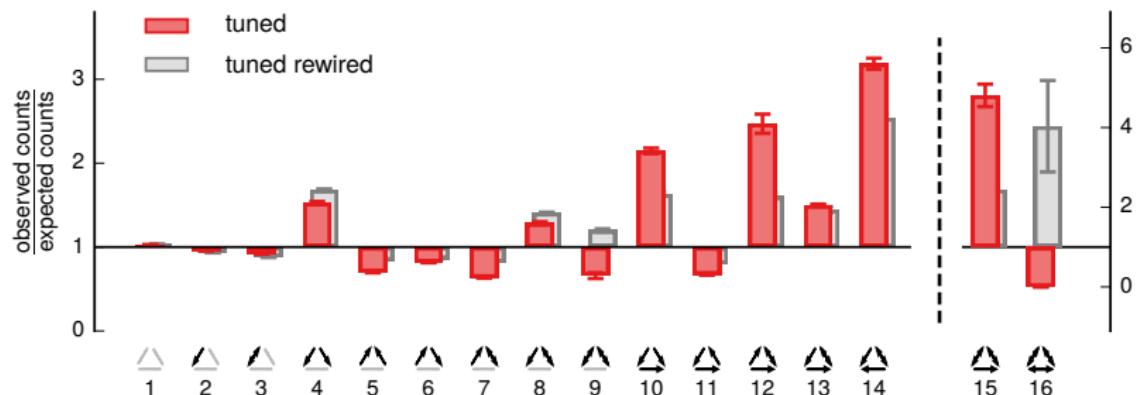
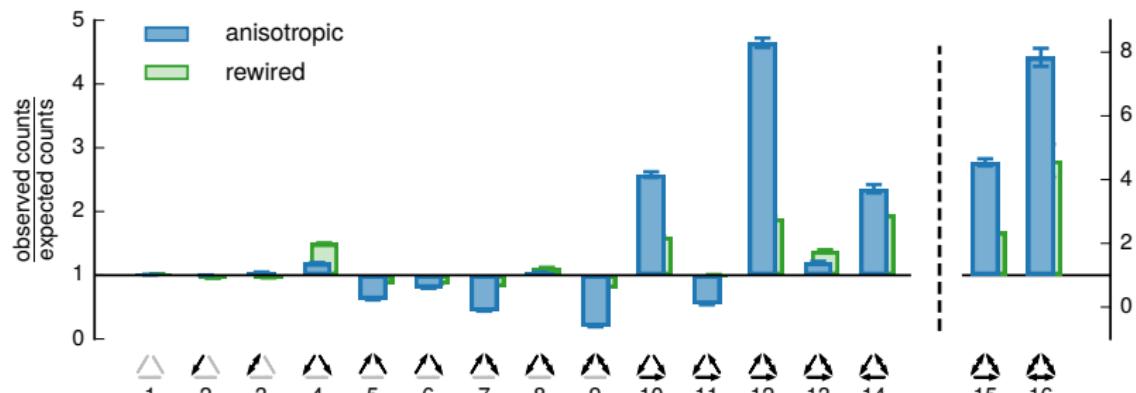
Three neuron motifs



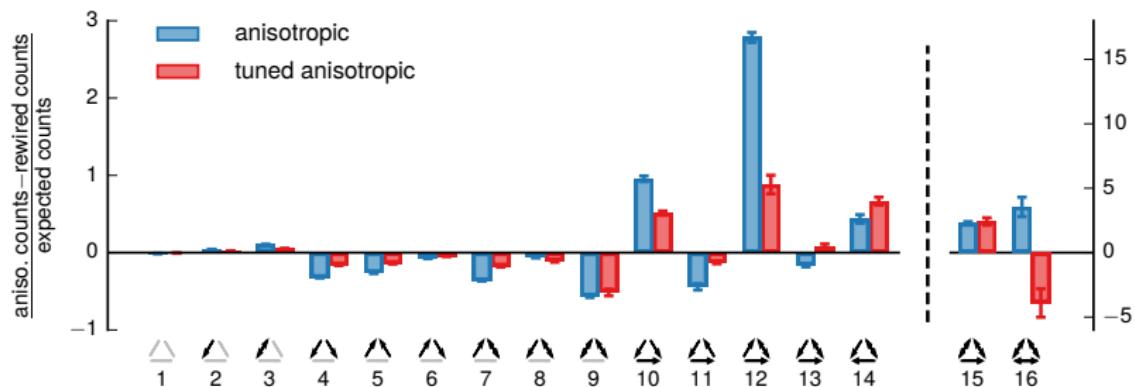
Three neuron motifs



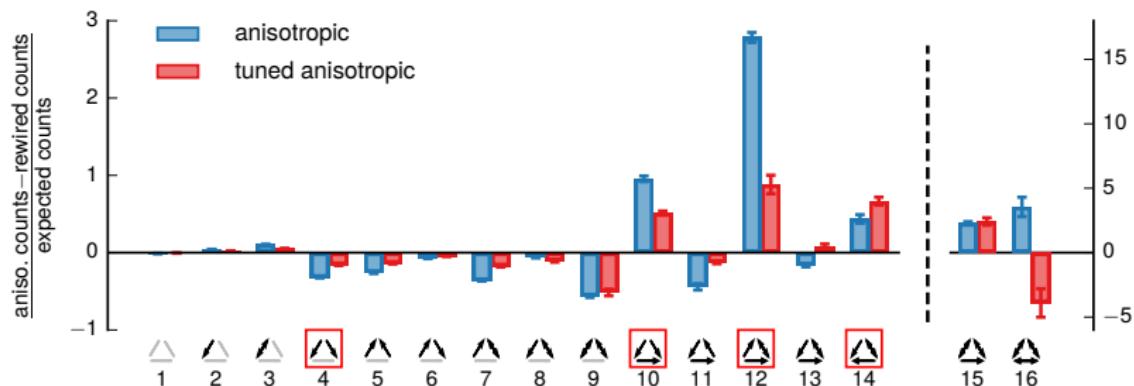
Three neuron motifs



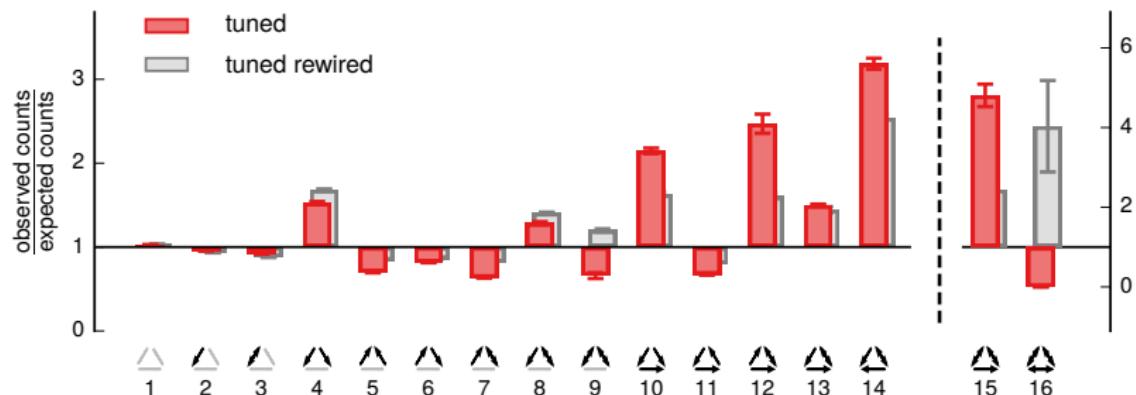
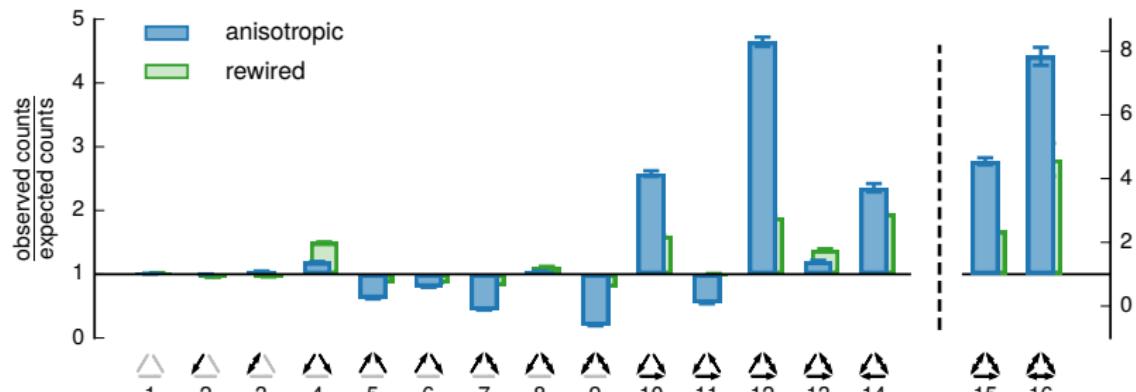
Three neuron motifs



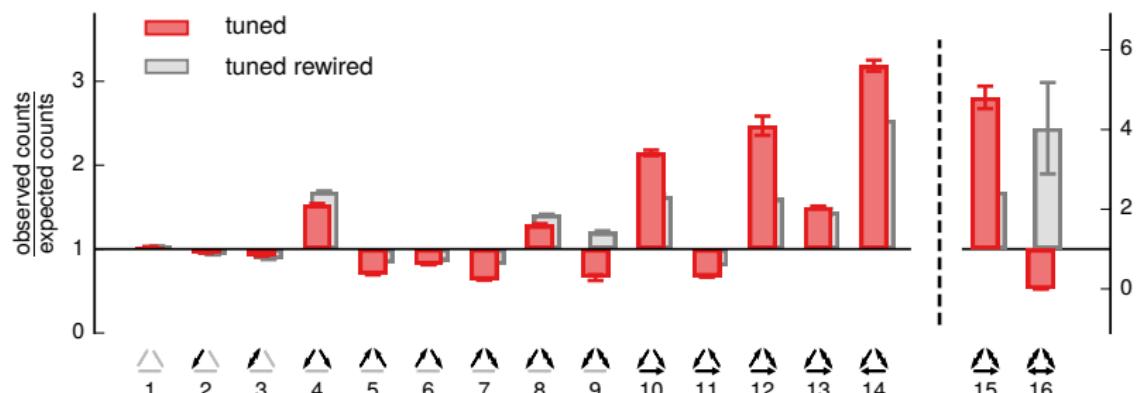
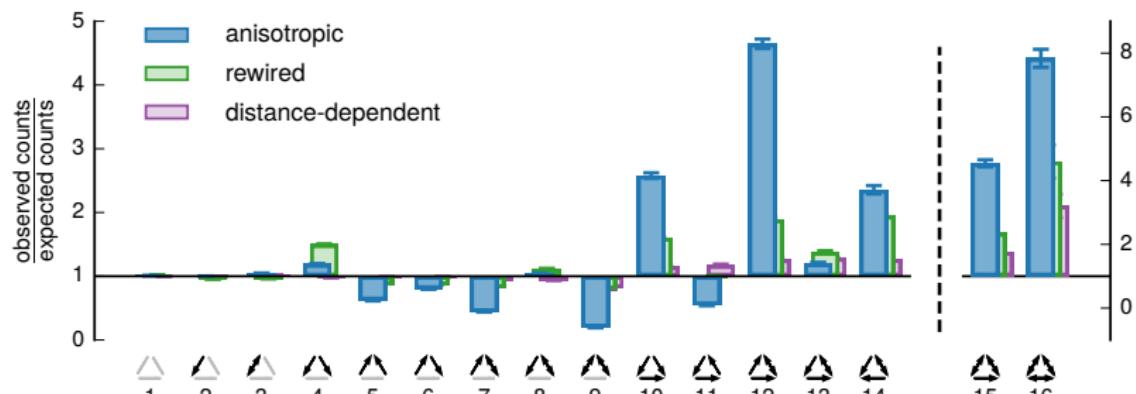
Three neuron motifs



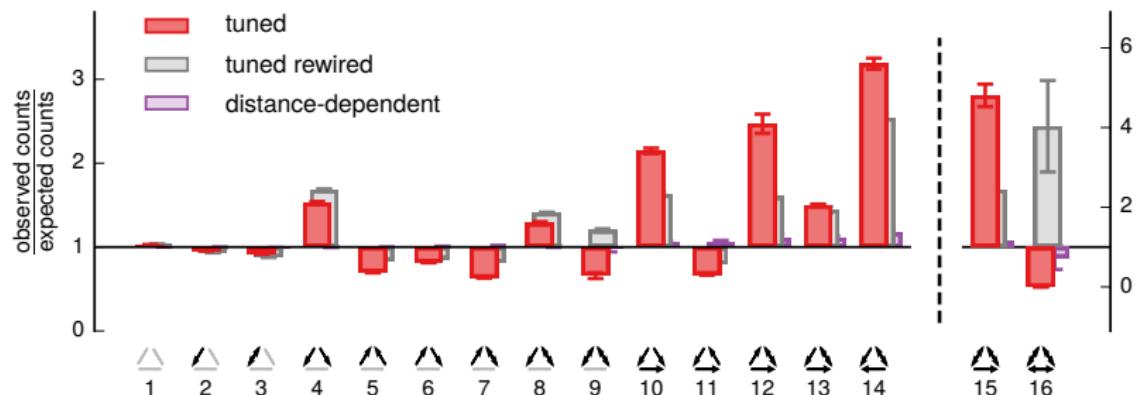
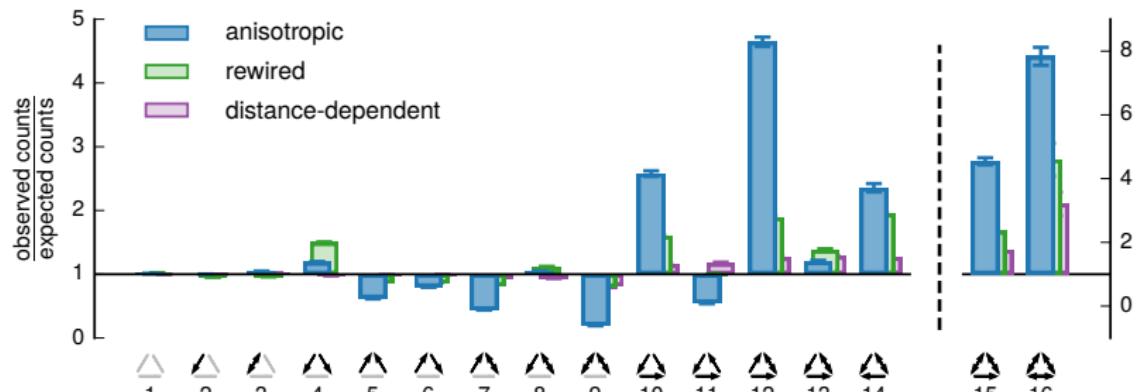
Three neuron motifs



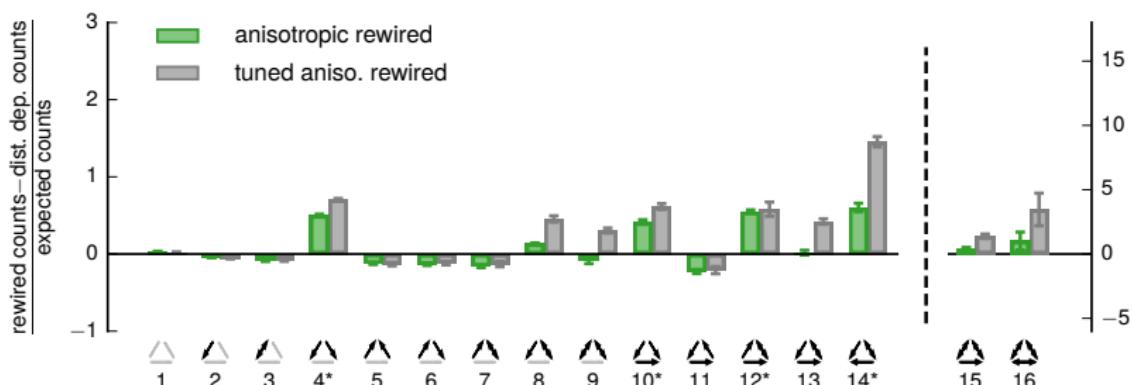
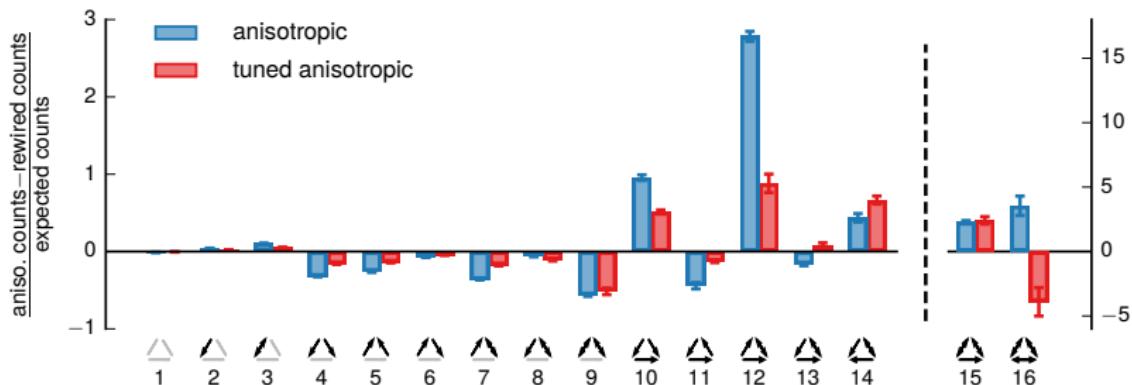
Three neuron motifs



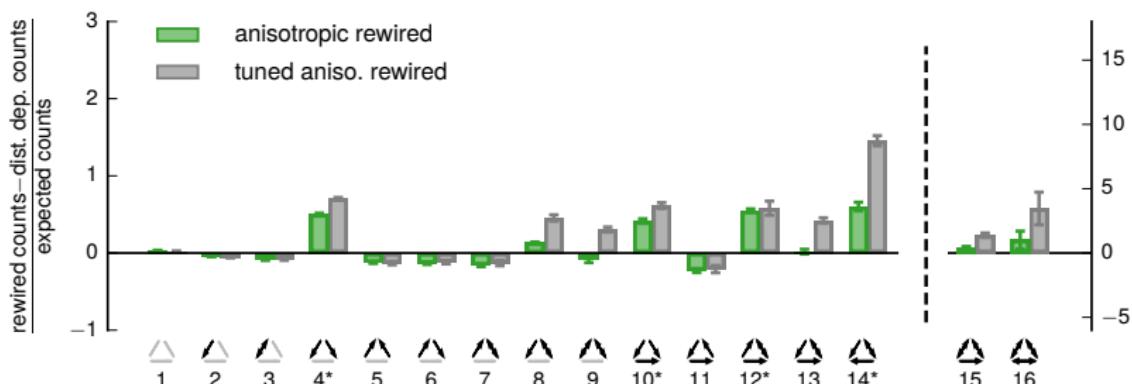
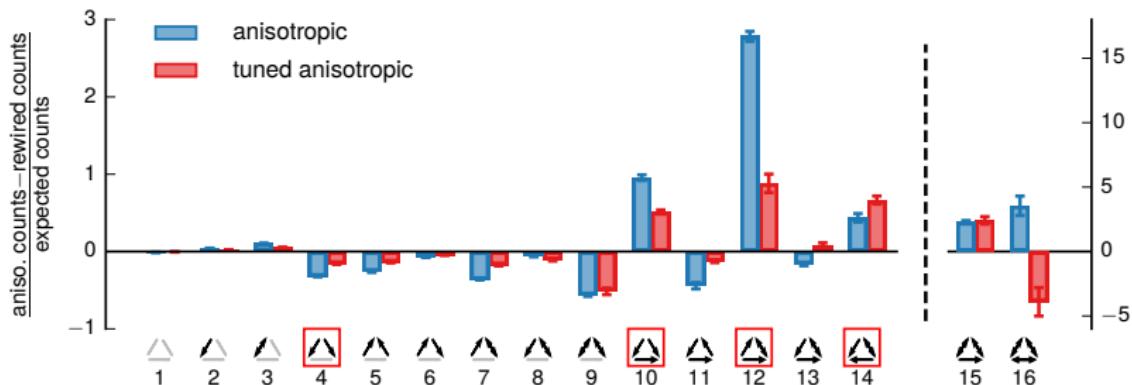
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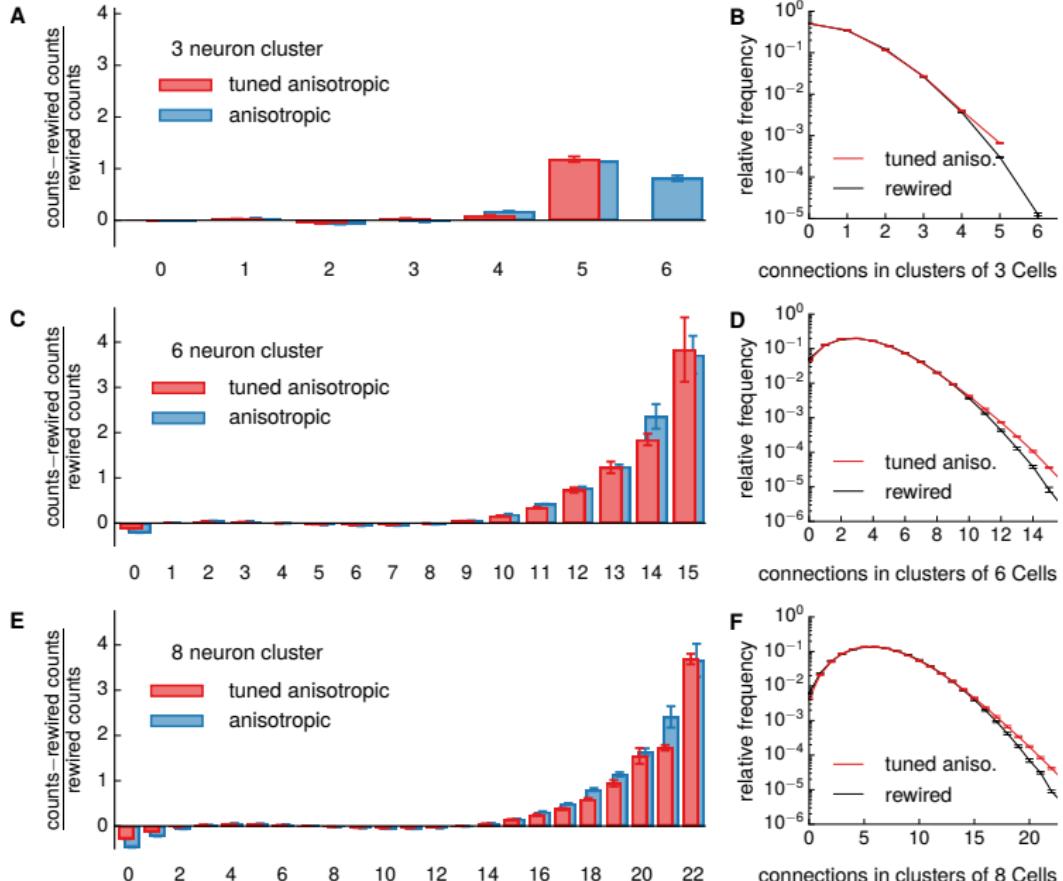
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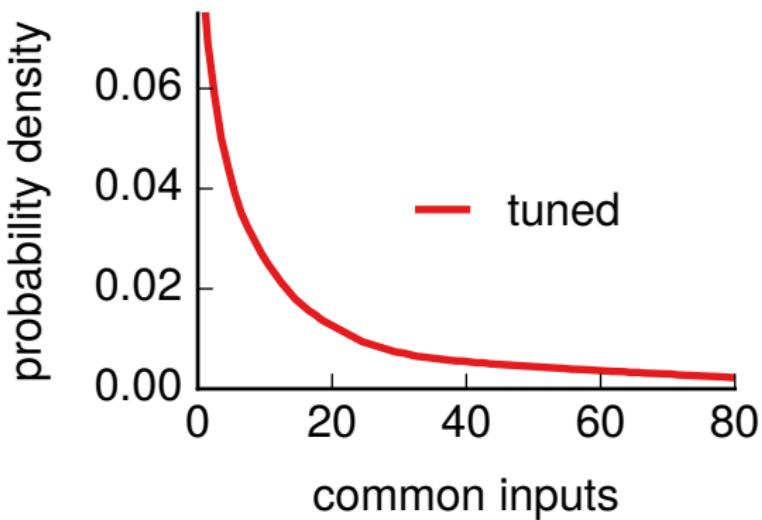
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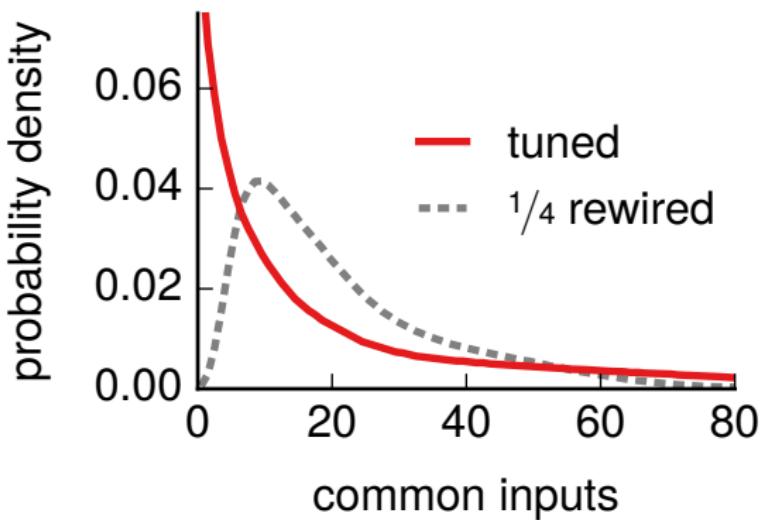
Connection density in clusters



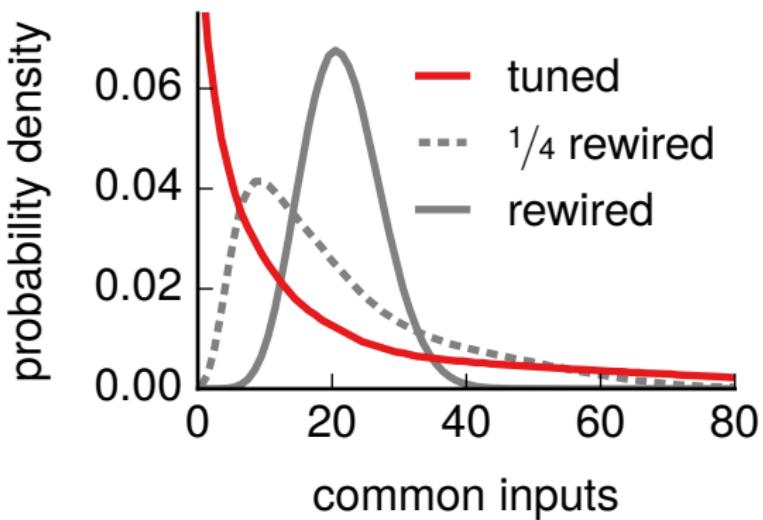
Common inputs statistics



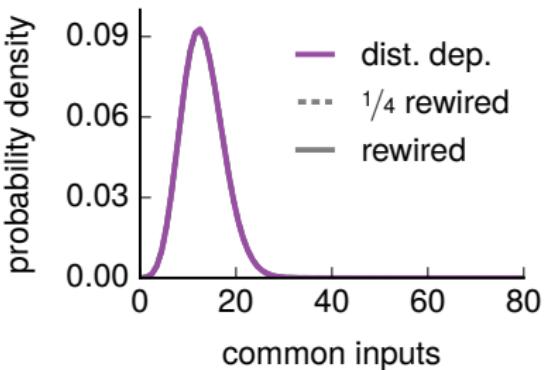
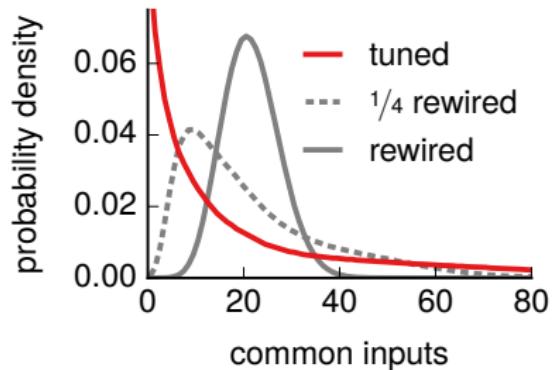
Common inputs statistics



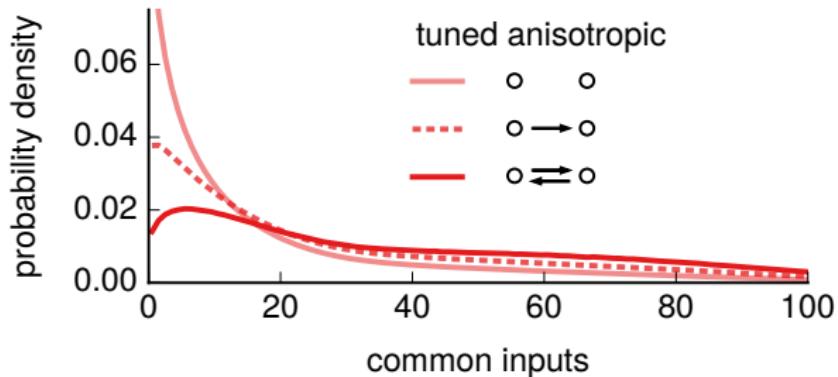
Common inputs statistics



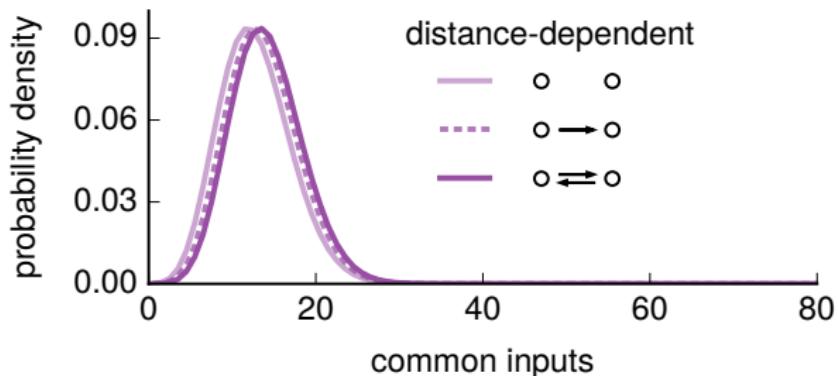
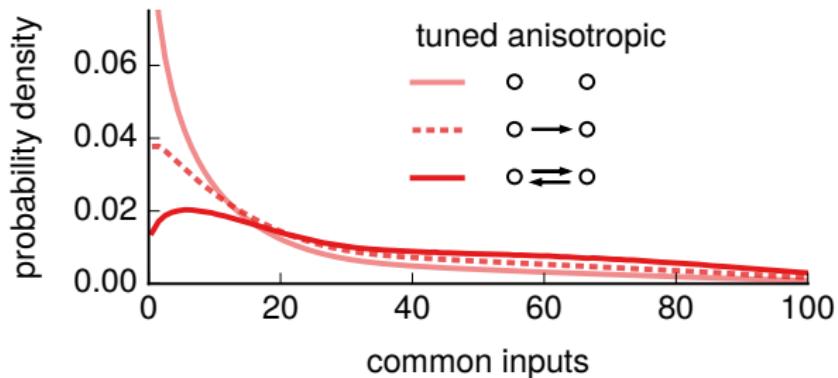
Common inputs statistics



Common inputs statistics



Common inputs statistics



Summary 1-1

Model

- anisotropy in spatial connectivity as a result of stereotypical axon and dendrite morphology
- anisotropic network as a simple model to test how anisotropy in spatial connectivity impacts network connectivity

Results

- reciprocal connections are overrepresented but only due induced distance-dependency
- various neuron properties (such as functional similarity) may compound with distance-dependency to induce observed reciprocity in cortical circuits

Summary 1-2

Results

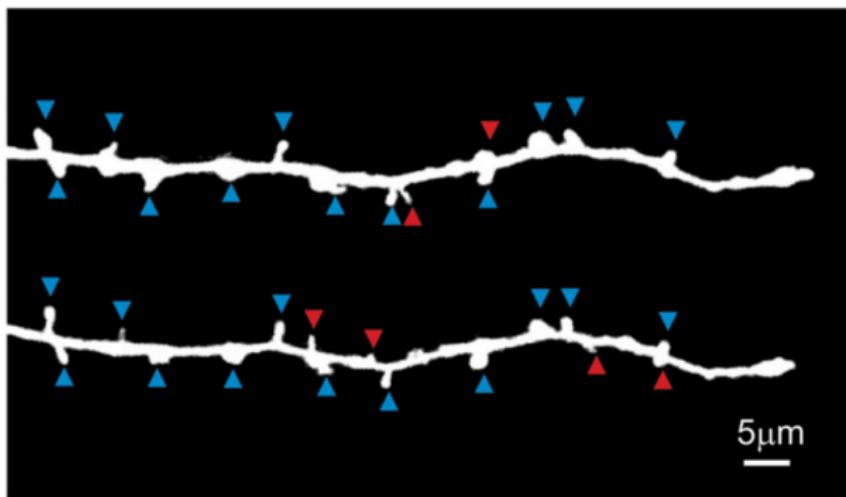
- specific three neuron motifs occur over- and underrepresented due to anisotropy matching data from cortical circuits
- observed frequent occurrence of high connection counts in neuron groups as a direct result of anisotropy in spatial connectivity

Predictions

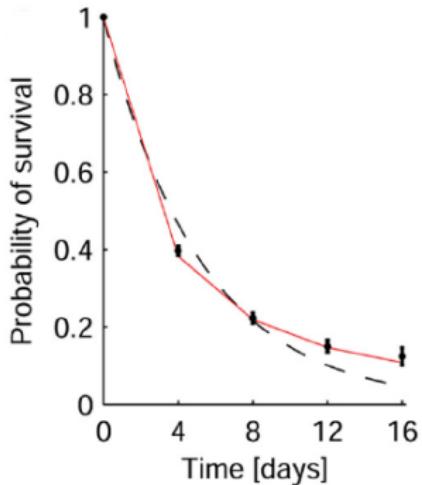
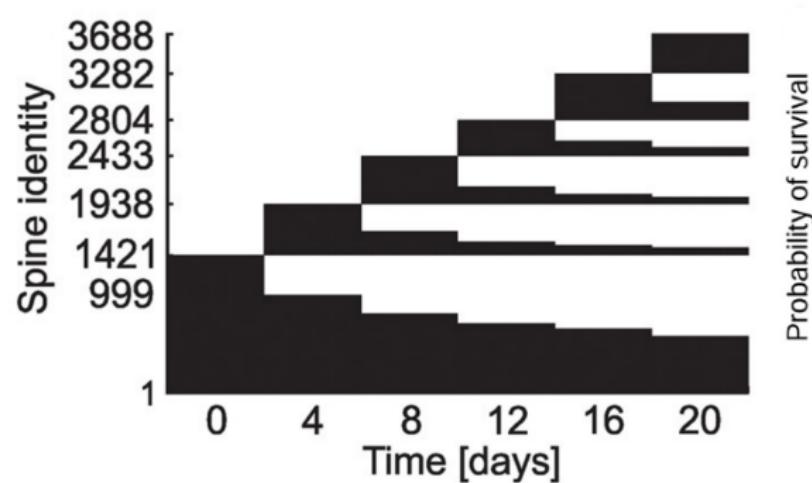
- anisotropic network model predicts broad distributions of common inputs for neuron pairs
- anisotropic network model predicts stronger sensitivity to connection type for common input distributions

The dynamic connectome

At the synapse level, brain circuitry is constantly making new connections and abolishing old ones



The dynamic connectome

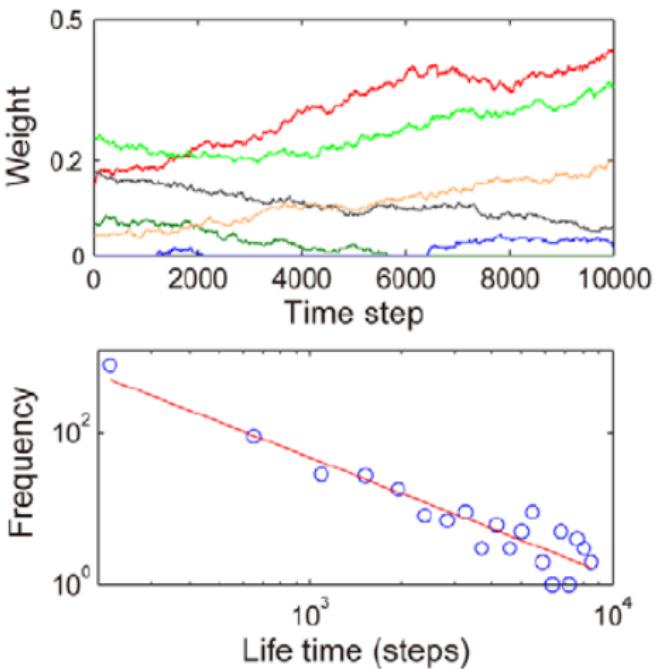


Survival probability of $\rho(t) = (t + 1)^{-\gamma}$ with $\gamma \approx 1.4$,

equivalently lifetime distribution of $f(t) = \gamma(t + 1)^{-(\gamma+1)}$

Synapse dynamics in self-organizing recurrent networks

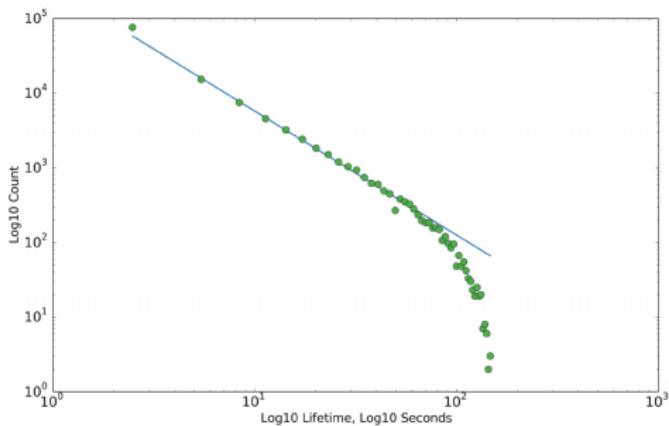
Power law synaptic lifetime distribution – Zheng et al. 2013



Synapse dynamics in self-organizing recurrent networks

Power law synaptic lifetime distribution – Zheng et al. 2013

Extension to leaky integrate-and-fire neuron – Miner and Triesch 2016

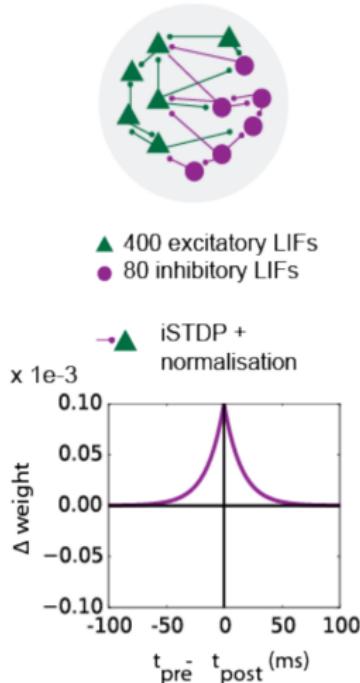


Synapse dynamics in self-organizing recurrent networks

Power law synaptic lifetime distribution – Zheng et al. 2013

Extension to leaky integrate-and-fire neuron – Miner and Triesch 2016

Synapse dynamics under inclusion of iSTDP – Kleberg and Triesch 2018



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FIAS Frankfurt Institute
for Advanced Studies



References

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