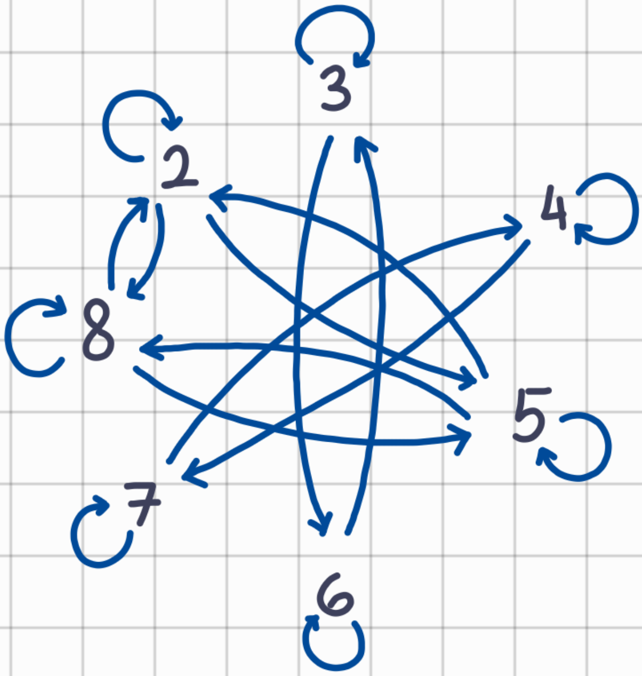
RELATION

1. $R = \{(2,2), (2,5), (2,8), (3,3), (3,6), (4,4), (4,7), (5,2), (5,5), (5,8), (6,3), (6,6), (7,4), (7,7), (8,2), (8,5), (8,8)\}$

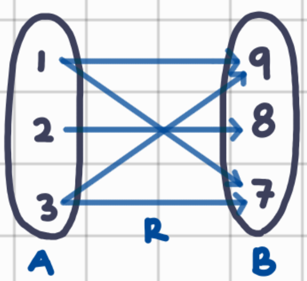


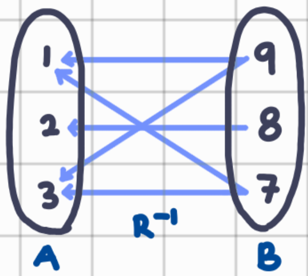
R is reflexive, symmetric and transitive.
R is equivalence relation on A.

2. a. $R = \{(1,9), (1,7), (2,8), (3,9), (3,7)\}$

$R^{-1} = \{(9,1), (9,3), (8,2), (7,1), (7,3)\}$

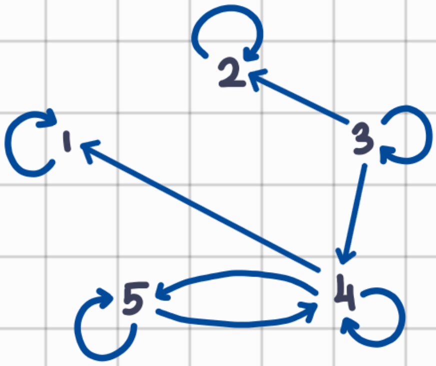
b.





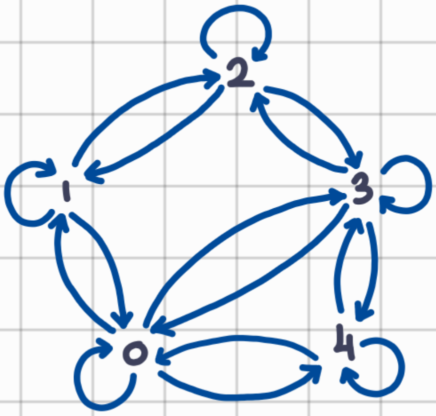
c. R^{-1} is the opposite of R.
If R : A to B , then R^{-1} : B to A.

3.



	1	2	3	4	5
In-degree	2	2	1	3	2
Out-degree	1	1	3	3	2

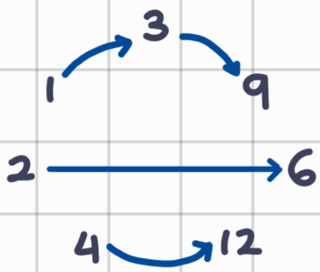
4.



∴ R is reflexive, symmetric and not transitive.

$1 R 0$, $0 R 3$, $1 \not R 3$

5. $R = \{(1,3), (2,6), (3,9), (4,12)\}$



a. R is irreflexive because all $x \not R x$. There is no loop. $(1,1) \notin R$ $(2,2) \notin R$

b. R is antisymmetric. $(2,6) \in R$
R is asymmetric. $(6,2) \notin R$

c. R is not transitive. $(1,3)$ and $(3,9) \in R$, but $(1,9) \notin R$.

6. a. $\begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

FUNCTION

7. Function : all xRy & $(x,y),(x,y') \in f$, $y = y'$ (only one output)

Relation : some xRy & $(x,y),(x,y') \in R$, $y \neq y'$ (can have multiple output)

8. (i) Function. The domain R, $\{2,3,4,5\}$ is equal to A.

(ii) Function. The domain R, $\{2,3,4,5\}$ is equal to A.

(iii) Not function. The domain R, $\{2,5\}$ is not equal to A.

$(2,3), (2,4) \in R$, $3 \neq 4$

(iv) Not function. The domain R, $\{2,3,4\}$ is not equal to A.

(v) Not function. The domain R, $\{2,4\}$ is not equal to A.

$(2,2), (2,3) \in R$, $2 \neq 3$ & $(4,4), (4,5) \in R$, $4 \neq 5$

9. $R = \{(1,6), (2,7), (3,8), (4,9), (5,10)\}$

Domain : $\{1,2,3,4,5\}$

Range : $\{6,7,8,9,10\}$

10. v) $f(x_1) = f(x_2)$

$1 - 2x_1 = 1 - 2x_2$ (-1)

$-2x_1 = -2x_2$ $(\div -2)$

$x_1 = x_2$

$\therefore$ f(x) is one-to-one and onto. f(x) is bijective

$y = 1 - 2x$

$x = -\dfrac{y-1}{2}$

$f(x) = 1 - 2\left(-\dfrac{y-1}{2}\right)$

$y = 1 + y - 1$

$y = y$

vi) $f(x_1) = f(x_2)$

$5x_1^2 - 1 = 5x_2^2 - 1$ $(+1)$

$5x_1^2 = 5x_2^2$ $(\div 5)$

$x_1^2 = x_2^2$

$x_1 = x_2$ or $x_1 = -x_2$

$\therefore$ f(x) is not one-one.

f(x) is onto.

$y = 5x^2 - 1$

$x = \sqrt{\dfrac{y+1}{5}}$

$f(x) = 5\left(\sqrt{\dfrac{y+1}{5}}\right)^2 - 1$

$y = y + 1 - 1$

$y = y$

vii) $f(x_1) = f(x_2)$

$x_1^4 = x_2^4$

$x_1 = x_2$ or $x_1 = -x_2$

$\therefore$ f(x) is not one-one.

f(x) is onto

$y = x^4$

$x = \sqrt[4]{y}$

$f(x) = (\sqrt[4]{y})^4$

$y = y$

viii) $f(x_1) = f(x_2)$

$$\frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$$

$$(x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$$

$$x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 3x_2 - 2x_1 + 6 \qquad (-x_1x_2 - 6)$$

$$-3x_1 - 2x_2 = -3x_2 - 2x_1$$

$$-3x_1 + 2x_1 = -3x_2 + 2x_2$$

$$x_1 = x_2$$

$$y = \frac{x-2}{x-3}$$

$\therefore f(x)$ is one-one and onto. $f(x)$ is bijective

$$y(x-3) = x - 2$$

$$xy - x = -2 + 3y$$

$$x(y-1) = 3y - 2$$

$$x = \frac{3y-2}{y-1}$$

$$f(x) = \frac{\left(\frac{3y-2}{y-1}\right) - 2}{\left(\frac{3y-2}{y-1}\right) - 3}$$

$$y = \frac{3y-2-2(y-1)}{\cancel{y-1}} \times \frac{\cancel{y-1}}{3y-2-3(y-1)}$$

$$y = \frac{3y-2-2y+2}{3y-2-3y+3}$$

$$y = \frac{y}{1}$$

$$y = y$$

11 ix) $f(g(x)) = f(x^2-1)$
$= 3(x^2-1)-1$
$= 3x^2-4$

$x=0$
$= 3(0)^2-4$
$= -4$

$x=1$
$= 3(1)^2-4$
$= -1$

$x=2$
$= 3(2)^2-4$
$= 8$

$x=3$
$= 3(3)^2-4$
$= 23$

x) $f(g(x)) = f(5x-6)$
$= (5x-6)^2$
$= 25x^2-60x+36$

$x=0$
$= 25(0)^2-60(0)+36$
$= 36$

$x=1$
$= 25(1)^2-60(1)+36$
$= 1$

$x=2$
$= 25(2)^2-60(2)+36$
$= 16$

$x=3$
$= 25(3)^2-60(3)+36$
$= 81$

xi) $f(g(x)) = f(x^3+1)$
$= (x^3+1)-1$
$= x^3$

$x=0$
$= (0)^3$
$= 0$

$x=1$
$= (1)^3$
$= 1$

$x=2$
$= (2)^3$
$= 8$

$x=3$
$= (3)^3$
$= 27$

RECURRENCE RELATION

12. xii) $a_0 = 1$

$a_1 = 6$

$a_2 = 6(a_1) - 9(a_0) = 6(6) - 9(1) = 27$

$a_3 = 6(a_2) - 9(a_1) = 6(27) - 9(6) = 108$

$a_4 = 6(a_3) - 9(a_2) = 6(108) - 9(27) = 405$

$a_5 = 6(a_4) - 9(a_3) = 6(405) - 9(108) = 1458$

1, 6, 27, 108, 405, 1458, ...

xiii) $a_0 = 2$

$a_1 = 5$

$a_2 = 15$

$a_3 = 6(a_2) - 11(a_1) + 6(a_0) = 6(15) - 11(5) + 6(2) = 47$

$a_4 = 6(a_3) - 11(a_2) + 6(a_1) = 6(47) - 11(15) + 6(5) = 147$

$a_5 = 6(a_4) - 11(a_3) + 6(a_2) = 6(147) - 11(47) + 6(15) = 455$

$a_6 = 6(a_5) - 11(a_4) + 6(a_3) = 6(455) - 11(147) + 6(47) = 1395$

2, 5, 15, 47, 147, 455, 1395, ...

xiv) $a_0 = 1$

$a_1 = -2$

$a_2 = -1$

$a_3 = -3(a_2) - 3(a_1) + a_0 = -3(-1) - 3(-2) + 1 = 10$

$a_4 = -3(a_3) - 3(a_2) + a_1 = -3(10) - 3(-1) - 2 = -29$

$a_5 = -3(a_4) - 3(a_3) + a_2 = -3(-29) - 3(10) - 1 = 56$

$a_6 = -3(a_5) - 3(a_4) + a_3 = -3(56) - 3(-29) + 10 = -71$

1, -2, -1, 10, -29, 56, -71, ...

13. $a_1 = k$

$a_2 = 5(a_1) - 3 \quad = 5k - 3$

$a_3 = 5(a_2) - 3 \quad = 5(5k-3) - 3 \quad = 25k - 18$

$a_4 = 5(a_3) - 3 \quad = 5(25k-18) - 3 \quad = 125k - 93$

$k,\ 5k-3,\ 25k-18,\ 125k-93,\ \ldots$

i) $a_4 = 125k - 93$

ii) $a_4 = 7$

$125k - 93 = 7$

$125k = 100$

$k = \dfrac{100}{125}$

$k = 0.8$