Question 1

a) In order to know the habits of the FC students about social networks we have asked 150 students if they have an active account in some of the most famous social networks: Facebook, Instagram or Twitter. Considering the positive answers, we obtain that 25 people have only Facebook account, 30 have only Instagram account, and 20 have only Twitter account. 15 of them have Facebook and Instagram accounts, but not a Twitter one. Only 5 people have an account in the three social networks. After the experiment, we obtain 65 Facebook users, 55 Instagram users and 50 Twitter users.

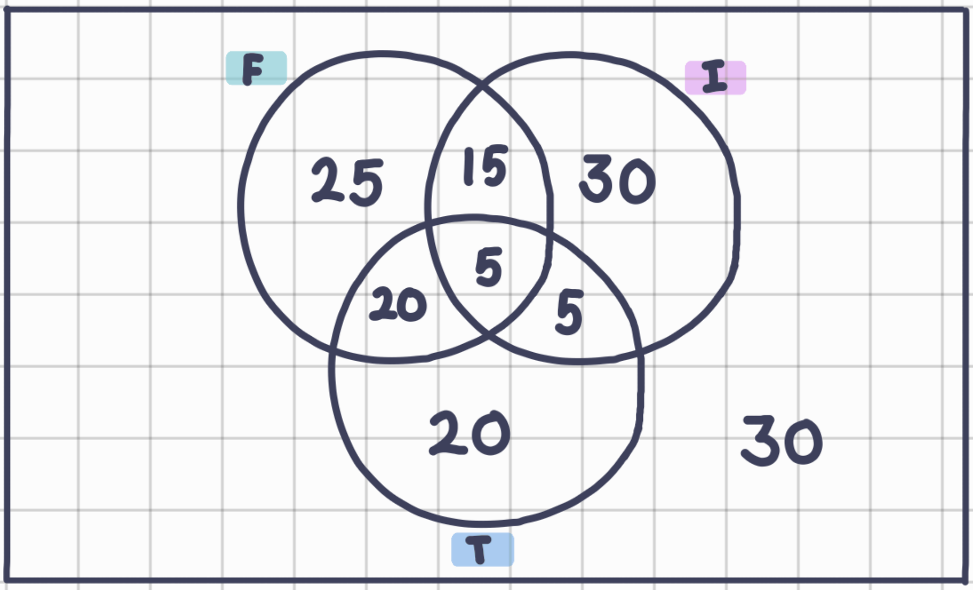
i) Draw a Venn diagram to represent to above problem. (2 marks)

ii) How many students do not have an account in any of the three social networks? (2 marks)

iii) How many students have exactly two social networks? (2 marks)

iv) How many students have social media account other than Facebook? (2 marks)

i) ξ



F : Facebook users
I : Instagram users
T : Twitter users
ξ : FC Students

ii) $(F \cup I \cup T)' = 30$ students

iii) $((F \cap I) \cup (F \cap T) \cup (I \cap T)) \cap (F \cap I \cap T)' = 40$ student

iv) $(F \cup I \cup T) - F = 55$ students

b) Suppose, $A = \{n \in N | n \ odd, 1 < n < 10\}$, where $N = \{natural\ number\}$
$B = \{n \in N | n \ is\ prime, 1 < n < 10\}$, $C = \{n \in N | n \ divisible\ by\ 3, 1 < x < 10\}$

i) Find $|A|$, $|B|$ and $|C|$, (3 marks)

ii) Find the number of proper subsets of A. (3 marks)

iii) Find $C \times B$ (2 marks)

$A = \{3, 5, 7, 9\}$ $B = \{2, 3, 5, 7\}$ $C = \{3, 6, 9\}$

i) $|A| = 4$, $|B| = 4$, $|C| = 3$

ii) $|P(A)| = 2^4$
$= 16$

No. of proper subsets of A $= |P(A)| - 1$
$= 16 - 1$
$= 15$

iii) $C \times B = \{(3,2), (3,3), (3,5), (3,7),$
$(6,2), (6,3), (6,5), (6,7),$
$(9,2), (9,3), (9,5), (9,7)\}$

Question 2

a) Verify $\sim(p \vee q) \vee (\sim p \wedge q) \equiv \sim p$, using both truth table and logic property law

(6 marks)

p	q	~(p∨q)	~p	(~p∧q)	~(p∨q)∨(~p∧q)
T	T	F	F	F	F
T	F	F	F	F	F
F	T	F	T	T	T
F	F	T	T	F	T

$$\begin{aligned} \sim(p \vee q) \vee (\sim p \wedge q) &\equiv (\sim p \wedge \sim q) \vee (\sim p \wedge q) && \text{De Morgan's Laws} \\ &\equiv \sim p \wedge (\sim q \vee q) && \text{Distributive Laws} \\ &\equiv \sim p && \text{Complement Laws} \end{aligned}$$

b) Write the following statement using *p* and *q* and logical connective

p: I go to the beach

q: it is a sunny summer day

r: it is Sunday

i) I go to the beach whenever it is Sunday and sunny summer day (2 marks)

ii) If it is not either Sunday or sunny summer day then I do not go to the beach. (2 marks)

iii) If I do not go to the beach then it is not either Sunday or sunny summer day. (2 marks)

i) $p \leftrightarrow (r \wedge q)$

ii) $(\sim r \vee \sim q) \rightarrow \sim p$

iii) $\sim p \rightarrow (\sim r \vee \sim q)$

c) Write the negation of $\forall x(x^2 + 2x - 3 = 0)$ and determine the resulting proposition is TRUE or FALSE with the domain of discourse is integer (5 marks)

$\exists x\,(x^2 + 2x - 3 \neq 0)$

Let $x = 2$: $2^2 + 2(2) - 3 = 5 \neq 0$

$\therefore$ The resulting proposition is true

d) Express the following statement using predicates, quantifier and logical connective with the domain of discourse consist of all students at your school (6 marks)

i) There is a student at your school who can speak Russian but does not know C++

ii) Every student at your school either can speak Russian or knows C++

iii) No student at your school can speak Russian or knows C++

Let:

$S(x)$ be "x is a student at your school"

$R(x)$ be "x can speak Russian"

$C(x)$ be "x knows C++"

i) There exist a student at your school who can speak Russian but does not know C++

$$\exists x\,(R(x) \wedge \sim C(x))$$

ii) For every student at your school either can speak Russian or knows C++

$$\forall x\,(R(x) \vee C(x))$$

iii) No student at your school can speak Russian or knows C++

$$\neg \exists x\,(R(x) \vee C(x))$$

Question 3

a) Prove the following theorem using indirect proof method. (5 marks)

For all integers, if $a^2 - 3b$ is even then a is even and b is even

Let :

$P(a,b)$: $a^2 - 3b$ is even

$A(a)$: a is even

$B(b)$: b is even

$$\forall a \forall b \, (P(a,b) \rightarrow (A(a) \wedge B(b)))$$

$$\equiv \neg(A(a) \wedge B(b)) \rightarrow \neg P(a,b) \quad \text{contrapositive}$$

$$\equiv \neg A(a) \vee \neg B(b) \rightarrow \neg P(a,b) \quad \text{De morgan's Laws}$$

- if a is odd or b is odd then $a^2 - 3b$ is odd

① · let a be odd and b even

- $a = 2n + 1$ and $b = 2m$
- $a^2 - 3b = (2n+1)^2 - 3(2m)$

$= 4n^2 + 4n + 1 - 6m$

$= 2(2n^2 + 2n - 3m) + 1$ $\quad k = 2n^2 + 2n - 3m$

$= 2k + 1 \rightarrow$ odd TRUE

② · let a be even and b odd.

- $a = 2n$ and $b = 2m + 1$
- $a^2 - 3b = (2n)^2 - 3(2m+1)$

$= 4n^2 - 6m - 3$

$= 2(2n^2 - 3m - 2) + 1$ $\quad h = 2n^2 - 3m - 2$

$= 2h + 1 \rightarrow$ odd TRUE