

# Lecture 6

## Chapter-02

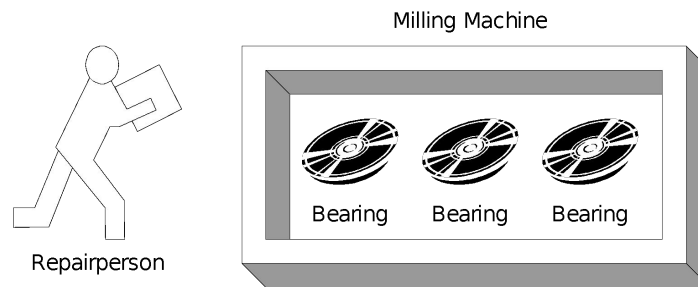
Discrete-Event System Simulation

-Jerry Banks

# Other Examples of Simulation

# Other Examples of Simulation (1)

- Example 2.5 A Reliability Problem



- Downtime for the mill is estimated at \$5 per minute.
- The direct on-site cost of the repairperson is \$15 per hour.
- It takes 20 minutes to change one bearing, 30 minutes to change two bearings, and 40 minutes to change three bearings.
- The bearings cost \$16 each.
- A proposal has been made to replace all three bearings whenever a bearing fails.

# Other Examples of Simulation (2)

- Example 2.5 (Cont.)

**Table 2.22** Bearing-Life Distribution

<i>Bearing Life (Hours)</i>	<i>Probability</i>	<i>Cumulative Probability</i>	<i>Random-Digit Assignment</i>
1000	0.10	0.10	01–10
1100	0.13	0.23	11–23
1200	0.25	0.48	24–48
1300	0.13	0.61	49–61
1400	0.09	0.70	62–70
1500	0.12	0.82	71–82
1600	0.02	0.84	83–84
1700	0.06	0.90	85–90
1800	0.05	0.95	91–95
1900	0.05	1.00	96–00

**Table 2.23** Delay-Time Distribution

<i>Delay Time (Minutes)</i>	<i>Probability</i>	<i>Cumulative Probability</i>	<i>Random-Digit Assignment</i>
5	0.6	0.6	1–6
10	0.3	0.9	7–9
15	0.1	1.0	0

- The delay time of the repairperson's arriving at the milling machine is also a random variable, with the distribution given in Table 2.23.
- The cumulative distribution function of the life of each bearing is identical, as shown in Table 2.22.

**Table 2.24** Bearing Replacement Using Current Method

Bearing 1						Bearing 2						Bearing 3					
Accumulated						Accumulated						Accumulated					
Life		Life		Delay		Life		Life		Delay		Life		Life		Delay	
RD <sup>a</sup>	(Hours)	(Hours)	RD	(Minutes)		RD	(Hours)	(Hours)	RD	(Minutes)		RD	(Hours)	(Hours)	RD	(Minutes)	
1	67	1,400	1,400	2	5	70	1,500	1,500	0	15		76	1,500	1,500	0	15	
2	08	1,000	2,400	3	5	43	1,200	2,700	7	10		65	1,400	2,900	2	5	
3	49	1,300	3,700	1	5	86	1,700	4,400	3	5		61	1,400	4,300	7	10	
4	84	1,600	5,300	7	10	93	1,800	6,200	1	5		96	1,900	6,200	1	5	
5	44	1,200	6,500	8	10	81	1,600	7,800	2	5		65	1,400	7,600	3	5	
6	30	1,200	7,700	1	5	44	1,200	9,000	8	10		56	1,300	8,900	3	5	
7	10	1,000	8,700	2	5	19	1,100	10,100	1	5		11	1,100	10,000	6	5	
8	63	1,400	10,100	8	10	51	1,300	11,400	1	5		86	1,700	11,700	3	5	
9	02	1,000	11,100	3	5	45	1,300	12,700	7	10		57	1,300	13,000	1	5	
10	02	1,000	12,100	8	10	12	1,100	13,800	8	5		49	1,300	14,300	4	5	
11	77	1,500	13,600	7	10	48	1,300	15,100	0	15		36	1,200	15,500	8	10	
12	59	1,300	14,900	5	5	09	1,000	16,100	8	10		44	1,200	16,700	2	5	
13	23	1,100	16,000	5	5	44	1,200	17,300	1	5		94	1,800	18,500	1	5	
14	53	1,300	17,300	9	10	46	1,200	18,500	2	5		78	1,500	20,000	7	10	
15	85	1,700	19,000	6	5	40	1,200	19,700	8	10							
16	75	1,500	20,500	4	5	52	1,300	21,000	5	5							
				110						125						95	

<sup>a</sup>RD, random digits.

## Other Examples of Simulation (3)

- Example 2.5 (Cont.)
  - Table 2.24 represents a simulation of 20,000 hours of operation under the current method of operation.
  - Note that there are instances where more than one bearing fails at the same time.
  - This is unlikely to occur in practice and is due to using a rather coarse grid of 100 hours.
  - It will be assumed in this example that the times are never exactly the same, and thus no more than one bearing is changed at any breakdown. Sixteen bearing changes were made for bearings 1 and 2, but only 14 bearing changes were required for bearing 3.

# Other Examples of Simulation (4)

- Example 2.5 (Cont.)

- The cost of the current system is estimated as follows:
  - Cost of bearings =  $46 \text{ bearings} \times \$16/\text{bearing} = \$736$
  - Cost of delay time =  $(110 + 125 + 95) \text{ minutes} \times \$5/\text{minute} = \$1650$
  - Cost of downtime during repair =  
$$46 \text{ bearings} \times 20 \text{ minutes/bearing} \times \$5/\text{minute} = \$4600$$
  - Cost of repairpersons =  
$$46 \text{ bearings} \times 20 \text{ minutes/bearing} \times \$15/60 \text{ minutes} = \$230$$
  - Total cost =  $\$736 + \$1650 + \$4600 + \$230 = \$7216$
- Table 2.25 is a simulation using the proposed method. Notice that bearing life is taken from Table 2.24, so that for as many bearings as were used in the current method, the bearing life is identical for both methods.



# Other Examples of Simulation (5)

- Example 2.5 (Cont.)
  - Since the proposed method uses more bearings than the current method, the second simulation uses new random digits for generating the additional lifetimes.
  - The random digits that lead to the lives of the additional bearings are shown above the slashed line beginning with the 15<sup>th</sup> replacement of bearing 3.
  - The total cost of the new policy :
    - Cost of bearings = 54 bearings  $\times$  \$16/bearing = \$864
    - Cost of delay time = 125 minutes  $\times$  \$5/minute = \$625
    - Cost of downtime during repairs = 18 sets  $\times$  40 minutes/set  $\times$  \$5/minute = \$3600
    - Cost of repairpersons = 18 sets  $\times$  40 minutes/set  $\times$  \$15/60 minutes = \$180
    - Total cost = \$864 + \$625 + \$3600 + \$180 = \$5269
  - The new policy generates a savings of \$1947 over a 20,000-hour simulation. If the machine runs continuously, the simulated time is about 2 1/4 years. Thus, the savings are about \$865 per year.



# Other Examples of Simulation (12)

- Example 2.6 Lead-Time Demand

- Lead-time demand may occur in an inventory system.
- The lead time is the time from placement of an order until the order is received.
- In a realistic situation, lead time is a random variable.
- During the lead time, demands also occur at random. Lead-time demand is thus a random variable defined as the sum of the demands over the lead time, or

$$\sum_{i=0}^T D_i$$

where  $i$  is the time period of the lead time,  $i = 0, 1, 2, \dots$ ,  $D_i$  is the demand during the  $i^{th}$  time period; and  $T$  is the lead time.

- The distribution of lead-time demand is determined by simulating many cycles of lead time and building a histogram based on the results.

Optional

# Other Examples of Simulation (13)

- Example 2.6 (Cont.)

- The daily demand is given by the following probability distribution:

Daily Demand (Rolls)	3	4	5	6
Probability	0.20	0.35	0.30	0.15

- The lead time is a random variable given by the following distribution:

Lead Time (Days)	1	2	3
Probability	0.36	0.42	0.22

**Table 2.27** Random-Digit Assignment for Demand

<i>Daily Demand</i>	<i>Probability</i>	<i>Cumulative Probability</i>	<i>Random-Digit Assignment</i>
3	0.20	0.20	01–20
4	0.35	0.55	21–55
5	0.30	0.85	56–85
6	0.15	1.00	86–00

**Table 2.28** Random-Digit Assignment for Lead Time

<i>Lead Time (Days)</i>	<i>Probability</i>	<i>Cumulative Probability</i>	<i>Random-Digit Assignment</i>
1	0.36	0.36	01–36
2	0.42	0.78	37–78
3	0.22	1.00	79–00

# Other Examples of Simulation (14)

- Example 2.6 (Cont.)

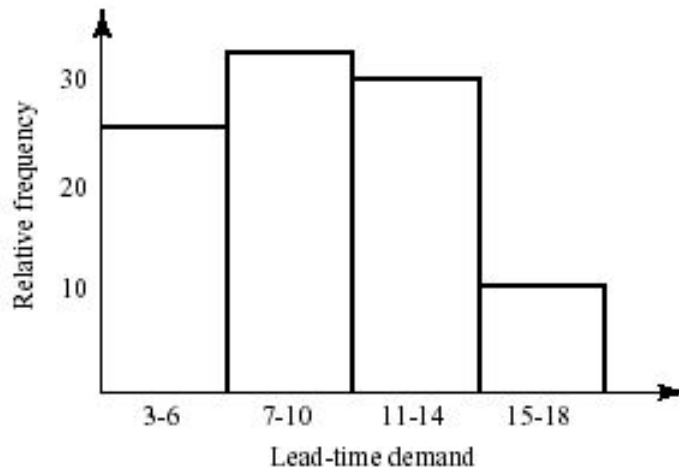
**Table 2.29** Simulation Table for Lead-Time Demand

<i>Cycle</i>	<i>Random Digits for Lead Time</i>	<i>Lead Time (Days)</i>	<i>Random Digits for Demand</i>	<i>Demand</i>	<i>Lead-Time Demand</i>
1	57	2	87	6	
			34	4	10
2	33	1	82	5	5
3	93	3	28	4	
			19	3	
			63	5	12
4	55	2	91	6	
			26	4	10
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.

- The incomplete simulation table is shown in Table 2.29.
- The random digits for the first cycle were 57. This generates a lead time of 2 days.
- Thus, two pairs of random digits must be generated for the daily demand.

# Other Examples of Simulation (15)

- Example 2.6 (Cont.)



**Figure 2.9** Histogram for lead-time demand.

- The histogram might appear as shown in Figure 2.9.
- This example illustrates how simulation can be used to study an unknown distribution by generating a random sample from the distribution.