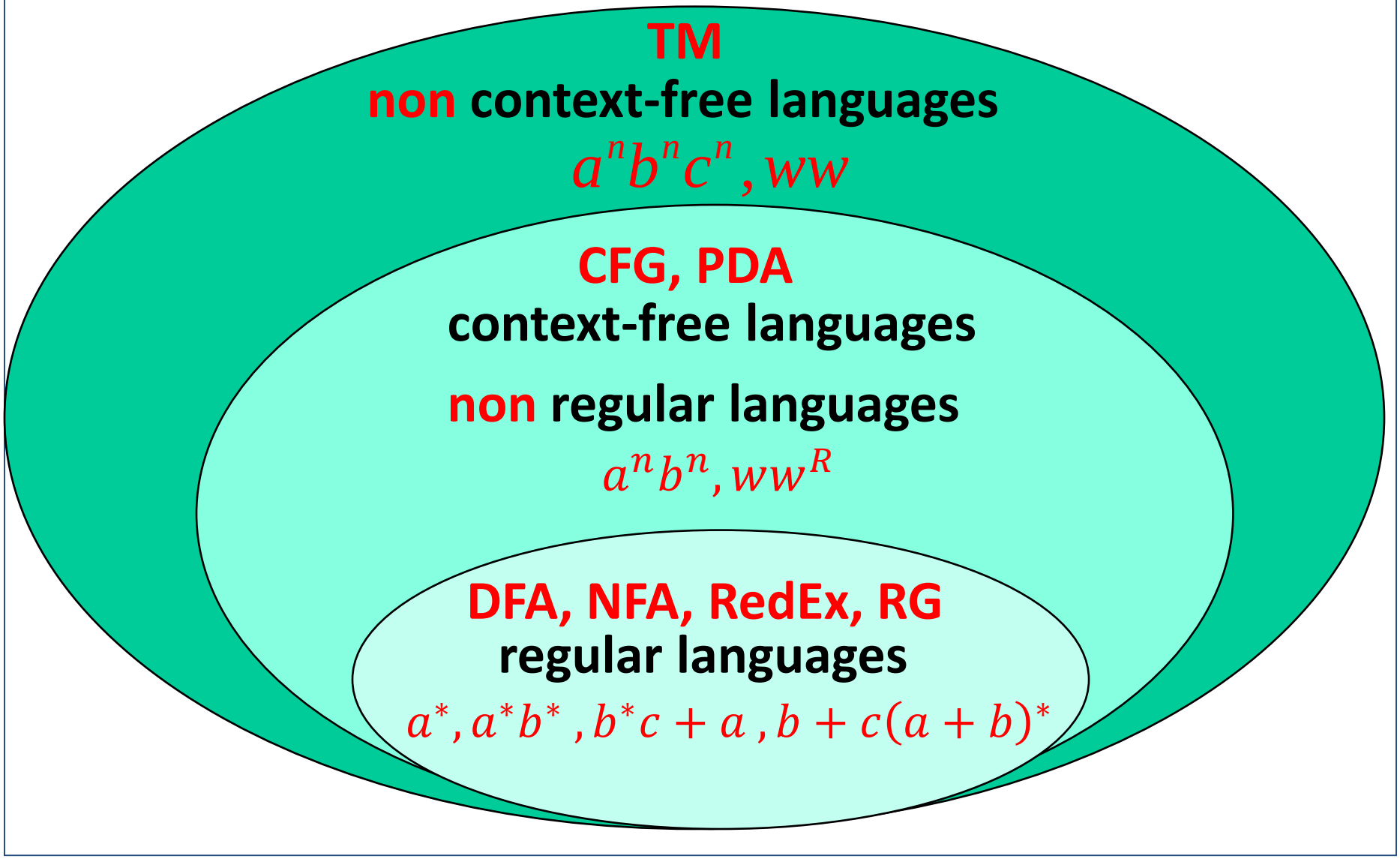
Context-Free grammars (CFG) Examples

The Language Hierarchy



The Language Hierarchy

FA = Limited memory

PDA = FA + Stack

TM = FA + Tape

Examples: CFG

Construct CFG machines that accept the following languages, where $n, m, k, l \geq 0$:

1. $L = \{a^n b^n\}$
2. $L = \{a^n b^n c^m\}$
3. $L = \{a^n b^m c^m\}$
4. $L = \{a^n b^m c^n\}$
5. $L = \{a^n b^m \mid n \leq m\}$
6. $L = \{a^n b^m \mid n < m\}$
7. $L = \{a^n b^m \mid n > m\}$
8. $L = \{a^n b^m \mid n \geq m\}$

Examples: CFG

Construct CFG machines that accept the following languages, where $n, m, k, l \geq 0$:

9. $L = \{a^n b^m c^k \mid n \leq m\}$
10. $L = \{a^n b^m c^k \mid n \leq k\}$
11. $L = \{a^n b^m \mid n \neq m\}\ (meaning, n < m\ or\ n > m)$
12. $L = \{a^n b^m \mid m = n + 2\}$
13. $L = \{a^n b^m \mid m \geq n + 2\}$
14. $L = \{a^n b^m \mid n \neq m - 1\}\ (meaning, m \leq n\ or\ m > n + 2)$

Examples: CFG

Construct CFG machines that accept the following languages, where $n, m, k, l \geq 0$:

1. $L = \{a^n b^n\}$

$$S \rightarrow aSb \mid \lambda$$

Examples: CFG

Construct CFG machines that accept the following languages, where $n, m, k, l \geq 0$:

2. $L = \{a^n b^n c^m\}$

$$S \rightarrow S_1 C \mid \lambda$$

$$S_1 \rightarrow a S_1 b \mid \lambda$$

$$C \rightarrow cC \mid \lambda$$

Examples: CFG

Construct CFG machines that accept the following languages, where $n, m, k, l \geq 0$:

3. $L = \{a^n b^m c^m\}$

$$S \rightarrow AS_1 \mid \lambda$$

$$S_1 \rightarrow bS_1c \mid \lambda$$

$$A \rightarrow aA \mid \lambda$$

Examples: CFG

Construct CFG machines that accept the following languages, where $n, m, k, l \geq 0$:

4. $L = \{a^n b^m c^n\}$

$$S \rightarrow aSc \mid B$$

$$B \rightarrow bB \mid \lambda$$

Examples: CFG

Construct CFG machines that accept the following languages, where $n, m, k, l \geq 0$:

5. $L = \{a^n b^m \mid n \leq m\}$

$$S \rightarrow aSb \mid S_1$$

$$S_1 \rightarrow S_1 b \mid \lambda$$

Examples: CFG

Construct CFG machines that accept the following languages, where $n, m, k, l \geq 0$:

6. $L = \{a^n b^m \mid n < m\}$

$$S \rightarrow aSb \mid S_1 b$$

$$S_1 \rightarrow S_1 b \mid \lambda$$

Examples: CFG

Construct CFG machines that accept the following languages, where $n, m, k, l \geq 0$:

7. $L = \{a^n b^m \mid n > m\}$

$$S \rightarrow aSb \mid aS_1$$

$$S_1 \rightarrow aS_1 \mid \lambda$$

Examples: CFG

Construct CFG machines that accept the following languages, where $n, m, k, l \geq 0$:

8. $L = \{a^n b^m \mid n \geq m\}$

$$S \rightarrow aSb \mid S_1$$

$$S_1 \rightarrow aS_1 \mid \lambda$$

Examples: CFG

Construct CFG machines that accept the following languages, where $n, m, k, l \geq 0$:

9. $L = \{a^n b^m c^k \mid n \leq m\}$

$$S \rightarrow S_1 C$$
$$S_1 \rightarrow aS_1 b \mid S_2$$
$$S_2 \rightarrow S_2 b \mid \lambda$$
$$C \rightarrow cC \mid \lambda$$

$$S \rightarrow aSb \mid S_1$$
$$S_1 \rightarrow S_1 b \mid S_2$$
$$S_2 \rightarrow cS_2 \mid \lambda$$

Examples: CFG

Construct CFG machines that accept the following languages, where $n, m, k, l \geq 0$:

10. $L = \{a^n b^m c^k \mid n \leq k\}$

$$S \rightarrow aSc \mid S_1$$

$$S_1 \rightarrow S_1 c \mid S_2$$

$$S_2 \rightarrow bS_2 \mid \lambda$$

Examples: CFG

Construct CFG machines that accept the following languages, where $n, m, k, l \geq 0$:

11. $L = \{a^n b^m \mid n \neq m\}\ (meaning, n < m\ or\ n > m)$

$$S \rightarrow \text{aSb} \mid \text{aS}_1 \mid \text{S}_2\text{b}$$

$$S_1 \rightarrow aS_1 \mid \lambda$$

$$S_2 \rightarrow S_2 b \mid \lambda$$

Examples: CFG

Construct CFG machines that accept the following languages, where $n, m, k, l \geq 0$:

12. $L = \{a^n b^m \mid m = n + 2\}$

$$S \rightarrow aSb \mid S_1 bb$$

$$S_1 \rightarrow \lambda$$

Examples: CFG

Construct CFG machines that accept the following languages, where $n, m, k, l \geq 0$:

13. $L = \{a^n b^m \mid m \geq n + 2\}$

$$S \rightarrow aSb \mid S_1 bb$$

$$S_1 \rightarrow S_1 b \mid \lambda$$

Examples: CFG

Construct CFG machines that accept the following languages, where $n, m, k, l \geq 0$:

14. $L = \{a^n b^m \mid n \neq m - 1\}\ (meaning, m \leq n\ or\ m > n + 2)$

$$S \rightarrow aSb \mid S_1 \mid S_2 bb$$

$$S_1 \rightarrow aS_1 \mid \lambda$$

$$S_2 \rightarrow S_2 b \mid \lambda$$

$n \neq m - 1$

$6 \neq 7 - 1$

$m \neq n + 1$

Hence

$m \leq n$

or

$m > n + 2$