Calculus

In this article, I will discuss basic calculus concepts and use graphs to visualize them.
Topics covered in this part will be as follows:

1. Slope, Derivative
2. Derivative of a curve (cosine wave)
3. Maximum and Minimum of a curve, Inflection point

Slope

Measure of the steepness of a line. It can be incline, decline, or flat.
It is typically calculated using the formula:

$$\text{Slope} = \frac{rise}{run} \; or \; \frac{up}{across} \; or \; \frac{\Delta y}{\Delta x}$$

This formula captures how much the line goes up or down (rise) for a horizontal distance (run).

Let's break it down:
Incline (Positive Slope): When a line slants upwards from left to right, it has a positive slope. This means that as you move along the line to the right (run), it goes up (rise). The greater the rise for a given run, the steeper the line and the greater the slope ***(slope > 0)***.

Decline (Negative Slope): A line that slants downwards from left to right has a negative slope. Here, as you move to the right (run), the line goes down (negative rise). A steeper downward slope will have a larger negative value, indicating a sharper decline ***(slope < 0).***

Flat (Zero Slope): A horizontal line has a zero slope. This means there is no vertical change as you move along the line; the rise is zero. No matter how far you run horizontally, if there's no up or down movement, the slope is 0 ***(slope = 0).***

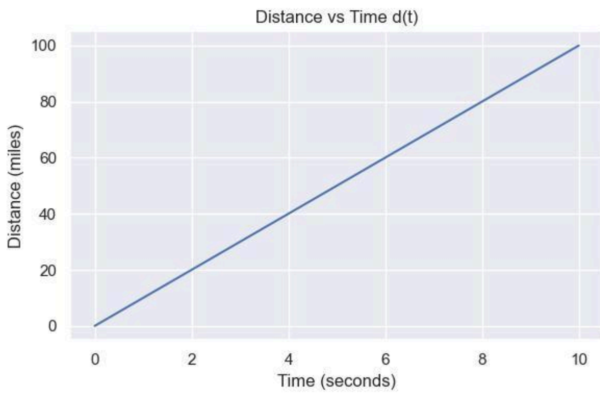
Example:

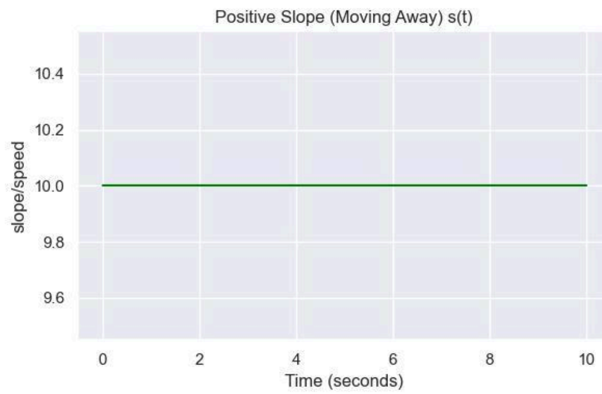
Speed is the slope of distance to time, i.e. speed = $\frac{rise}{run}$ or $\frac{distance}{time}$

Acceleration is the slope of speed to time, i.e. acceleration = $\frac{up}{across}$ or $\frac{speed}{time}$

Visualization of positive, negative, and zero slopes of a curve.

1. Positive Slope (Inclination) (Slope > 0)



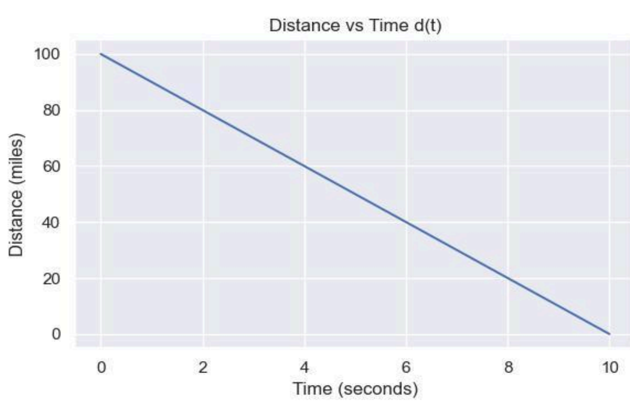


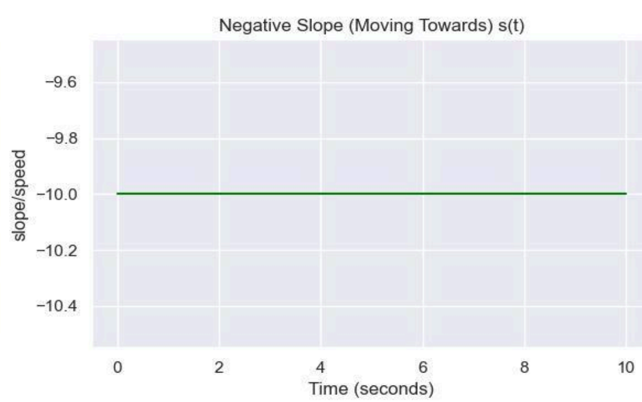
Graph 1 visualizes the function_1 d(t), which provides the distance covered at every time step. Graph 2 visualizes the rate of change of function_1 d(t), which is known as the function_2 slope/speed. Here, as time increases, distance also increases with a constant value of 10 at every time step; hence, it has a slope of 10.

Example:
At 4 seconds
d(t) = 40 miles

$$s(t)\ (\text{slope/speed}) = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{40-0}{4-0} = 10$$

2. Negative slope (Declination) (Slope < 0)





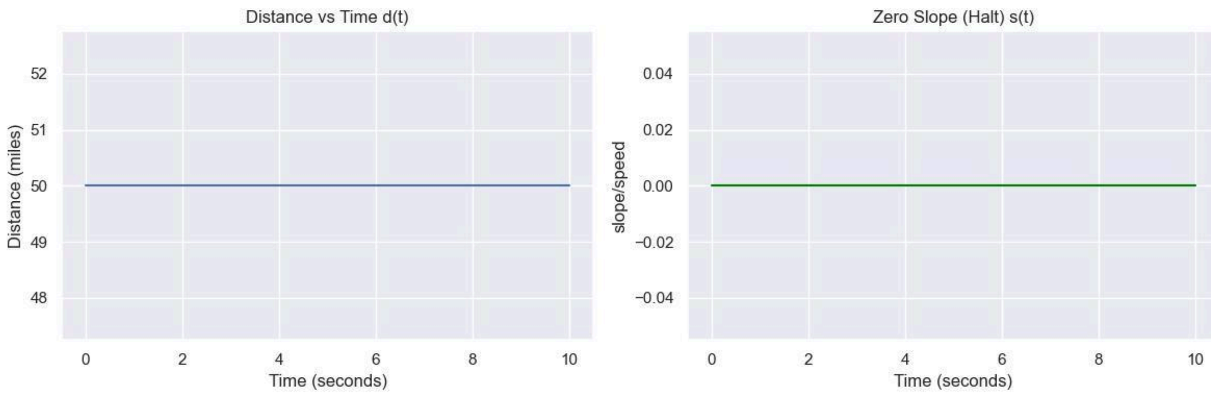
Here, as time increases, the distance covered decreases, with a constant value of 10; hence, it has a slope of -10.

Example:

At 4 seconds
d(t) = 60 miles (It is negative as the distance covered is decreasing)

s(t) (slope/speed) = $\frac{rise}{run}$ = $\frac{\Delta y}{\Delta x}$ = $\frac{60-100}{4-0}$ = -10

3. Constant Slope (Flat) (Slope = 0)



Here, as time increases, the distance covered remains constant, indicating no change in slope; hence, it has a slope of 0.

Example:
At 4 seconds
d(t) = 50 miles (It is negative as the distance covered is decreasing)

s(t) (slope/speed) = $\frac{rise}{run}$ = $\frac{\Delta y}{\Delta x}$ $=$ $\frac{50-50}{4-0}$ = 0

Derivative

The derivative of a function at a given point represents the rate of change of the function at that point. *It gives the slope of the tangent line to the graph of the function at that point.* The slope, on the other hand, refers to the steepness of a line and is a measure of how much the line rises or falls for a given horizontal change.

Derivative is used to visualize how a function changes at each point.

Relationship between Slope and Derivative

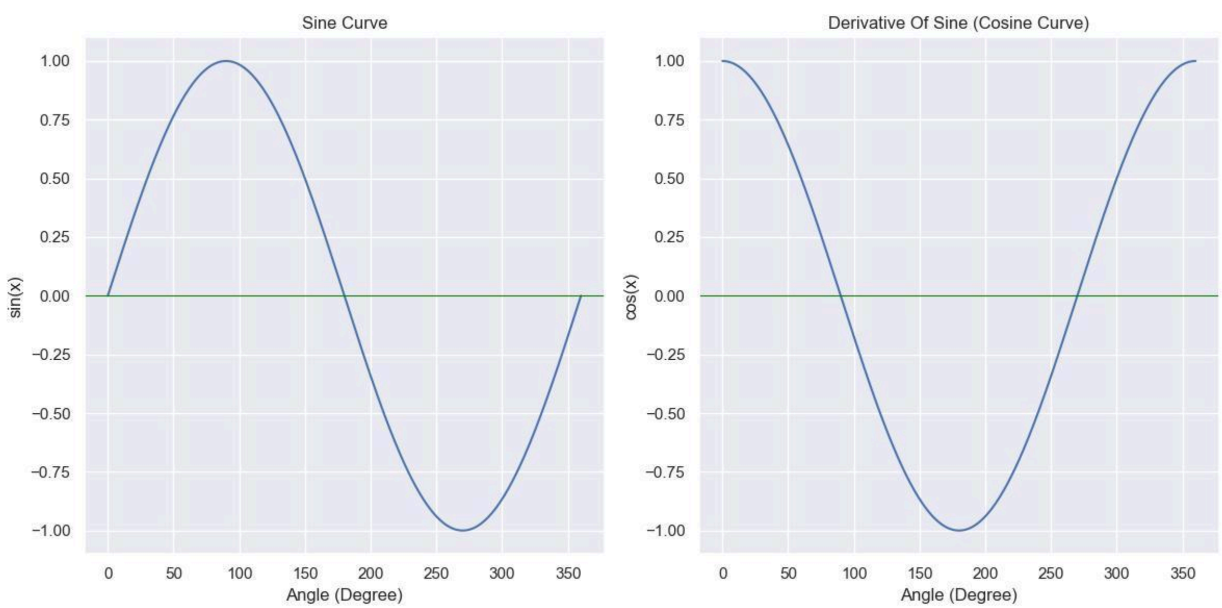
1. Constant Functions: For a linear function (a straight line), the derivative is constant and equal to the slope of the line.

2. Non-Linear Functions: For curves (non-linear functions), the derivative varies at different points. The derivative at any point gives the slope of the tangent line to the curve at that point.

In the upcoming part of the article, we will be discussing the derivative of a function as now we will be looking at instantaneous change and non-linear functions.

Derivative of a non-linear function (Sine Wave)

Derivative of a sine wave is represented by the curve that illustrates the slope of the tangent line to sine wave at every point. Cosine wave is able to represent the slope of sine wave at all instants and hence it is its derivative.



Let’s view at certain angles

1. Angle = 0
   The tangent line at that point in sine curve has a slope of 1, as provided in the cosine curve
2. Angle = $\pi/2$
   The tangent line at that point in sine curve has a slope of 0, as provided in the cosine curve

Starting at 0 degrees angle, the sine curve inclination keeps on decreasing until it reaches 90 degrees angle where it remains constant, which can be visualized by the declining curve from 0 to 90 degrees in cosine curve and finally reaching zero.

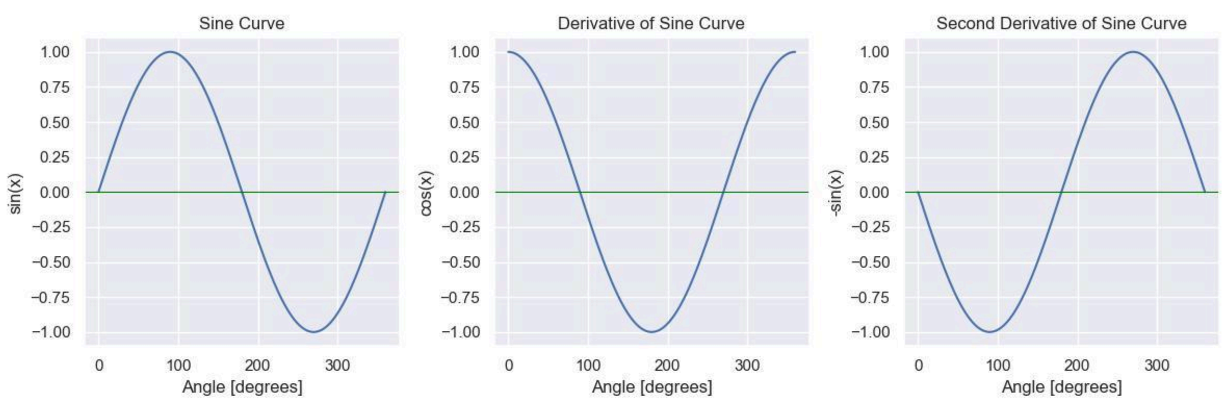
Similarly, the inclination, declination and flatness at every point of sine curve is represented by cosine curve which is it's derivative.

Maximum and Minimum of a curve, Inflection point

At maximum point of a curve:
- First Derivative = 0
- Second Derivative < 0
- The curve is bending downwards
- Concave Curve

At maximum point of a curve:
- First Derivative = 0
- Second Derivative > 0
- The curve is bending upwards
- Convex Curve

Inflection point:
- Second derivative = 0
- Curve changes from concave to convex or vice versa



In the above graph,
Maximum point is at 90 degrees angle
- First derivative at that point = 0
- Second derivative < 0

Minimum point is at 270 degrees angle

- First derivative at that point = 0
- Second derivative > 0

Inflection point is at 180 degrees angle

- Second derivative = 0

Although calculus is huge, these are few fundamentals and their clarity is an essential part in data science. This is the first part of the series expalining the fundamentals, it the next part, I will be illustrating calculus concepts are being implemented in data science.

References:

Gilbert Strang’s Calculus Lectures
3Blue1Brown: Calculus
Quora