Assignment - 1

Anita Dash

***Abstract*—This document contains the solution to Exercise 3.36 (a) of Oppenheim.**

Problem 1. If the input $x[n]$ to an LTI System is $x[n] = u[n]$, The output is

$$y[n] = \left(\frac{1}{2}\right)^{n-1} u[n+1] \tag{1}$$

Find $H(z)$, the z-transform of the system impulse response, and plot its pole-zero diagram.
Solution: We know that,

$$H(z) = \frac{Y(z)}{X(z)} \tag{2}$$

$$X(z) = \frac{1}{1-z^{-1}} \quad |z| > 1 \tag{3}$$

$$\sum_{n=-\infty}^{\infty} y(n)z^{-n} = \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^{n-1} u(n+1)z^{-n} \tag{4}$$

$$= z \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^{n-2} u(n)z^{-n} \tag{5}$$

$$= z \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n-2} z^{-n} \tag{6}$$

$$= z \frac{\frac{1}{4}}{1-\frac{1}{2}z^{-1}} \tag{7}$$

Therefore,

$$Y(z) = \frac{1}{4(z^{-1})(1-\frac{1}{2}z^{-1})} \quad |z| > \frac{1}{2} \tag{8}$$

Thus,

$$H(z) = \frac{1}{4(z^{-1})(1-\frac{1}{2}z^{-1})(1-z^{-1})} \quad |z| > 1 \tag{9}$$

The Python code for plotting the pole-zero diagram can be found in the link below. The code yields the graph shown in figure 1

wget https://github.com/anitadash/EE3900/blob/main/Assignment_1/Assgn_1.py

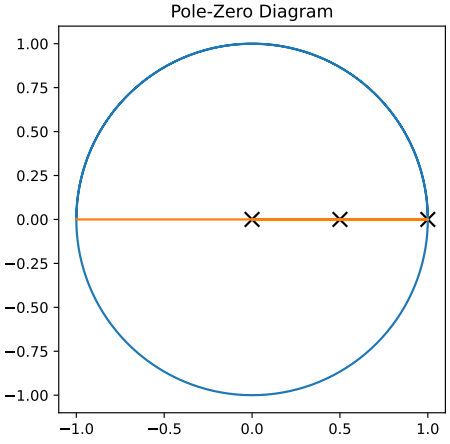

Fig. 1