Problem set 7

Due on November 3, 2020, at 11:15am

Exercise 1

Consider an option on a stock that has an initial price $S_0 = 100$. The expiration is $T = 1$ (year), but the option can be exercised at four possible dates: at $t_1 = 0.25$, $t_2 = 0.5$, $t_3 = 0.75$, $t_4 = T = 1$. The payout is similar to the payout of a call option, but depends on the average of the stock price over the previous exercise dates: the payout is

$$(S(t_1) - K)^+ \qquad \text{if exercised at } t_1 \tag{1}$$

$$\left(\frac{S(t_1) + S(t_2)}{2} - K\right)^+ \qquad \text{if exercised at } t_2 \tag{2}$$

$$\left(\frac{S(t_1) + S(t_2) + S(t_3)}{3} - K\right)^+ \qquad \text{if exercised at } t_3 \tag{3}$$

$$\left(\frac{S(t_1) + S(t_2) + S(t_3) + S(t_4)}{4} - K\right)^+ \qquad \text{if exercised at } t_4 \tag{4}$$

Use $K = 98$, $r = 0$, $q = 0.02$.

Use the Least-Square Monte Carlo method. Define the process A such that $A(t_1) = S(t_1)$, $A(t_2) = \frac{S(t_1)+S(t_2)}{2}$, $A(t_3) = \frac{S(t_1)+S(t_2)+S(t_3)}{3}$, $A(t_4) = \frac{S(t_1)+S(t_2)+S(t_3)+S(t_4)}{3}$ At each exercise date, regress the continuation value on S, S^2, S^3, A, A^2, A^3 and a constant.

- (80 points) Use a constant volatility equal to 23%. For this part, you can build your paths by simulating the stock price only at the 4 exercise dates.
- (20 points) Use a local volatility model, with the local vols obtained in your previous assignment. For this part, you have to build paths by simulating the stock price at denser intervals. For example, you can use monthly intervals (i.e. simulate the stock price at t =1 month, 2 months, ...)
- Quote your final result for the option price and its standard error.