Problem set 11

Due on December 15, 2020, at 11:15am

Exercise 1

At the initial time $t = 0$ the term structure is flat at 5%. Consider a receiver Bermudan swaption with the following characteristics:

- the first exercise date is $T_s = 1$ (1 year). The last exercise date is $T_x = 2$.
- The underlying swap has quarterly payments: $\tau = 0.25$. The final payment is at $T_N = 4$.

Use the set of dates $[T_0, T_1, ...T_{16}]$, with $T_0 = 0$, $T_i = i * 0.25$ and the corresponding forward rates $F_k,\ k = 1, ..., 16$ (F_1 is the spot Libor rate). Each forward rate F_k has constant volatility σ_k, with

$$
\begin{aligned}
\sigma_k &= 0.2 \qquad for\ k = 2\ to\ 7 \\
\sigma_k &= 0.22 \qquad for\ k = 8\ to\ 11 \\
\sigma_k &= 0.24 \qquad for\ k = 12\ to\ 16
\end{aligned}
$$

Every dW_k driving the corresponding forward rate F_k is a linear combination of two independent Brownian motions $dW^{(1)}$ and $dW^{(2)}$, with $dW^{(1)}dW^{(2)} = 0$. For every $k > 1$

$$dW_k = cos(\theta_k)dW^{(1)} + sin(\theta_k)dW^{(2)} \tag{1}$$

and

$$\theta_k = \frac{\pi}{2}\frac{k-2}{14} \tag{2}$$

so that we have

$$
\begin{aligned}
dW_2 &= dW^{(1)} &(3)\\
dW_{16} &= dW^{(2)} &(4)\\
dW_{j-1}dW_j &= cos(\pi/28) = 0.9937 &(5)
\end{aligned}
$$

Price the Bermudan swaption using the Andersen algorithm.