Problem set 3

Due date: Tuesday, October 6^{th}, 11:15 am.

Exercise 1

In the Merton jump-diffusion model seen in Lecture 2,

$$\frac{dS_t}{S_t} = (r - q + \lambda^Q \gamma)dt = \sigma dW_t - \gamma dN_t \tag{1}$$

with constants $r,\ q,\ \lambda^Q,\ \gamma,\ \sigma$, find a formula to price a contract paying $(S_T)^n$ at time T for an arbitrary power n. Use the same logic as for the call option, done in class.

Exercise 2

A stock subject to the stochastic process (1) jumps at time t. The jump intensity is 0.2 for 1 year. What is the probability that the time to the next jump will be

- Longer than two years
- Shorter than 3 years
- Between 2 and 3 years

(**weight: 10%**)

Exercise 3

Again the stock follows the process (1). Set $q = 0$, $\lambda = 1$ (on average 1 jump per year!), $\gamma = 0.1$, $\sigma = 0.2$, r=0.04, $S_0 = 100$. Write a program to price call options with expiration $T_1 = 0.02$ and $T_2 = 0.08$, with a wide set of strikes (for example ranging from 60 to 150). Use the formula

$$\phi(S, T; S_0) = \left.\frac{\partial^2 \tilde{C}(K, T; S_0)}{\partial K^2}\right|_{K=S} \tag{2}$$

(where $\tilde{C}(K, T : S_0)$ is the *undiscounted* call price for strike K and maturity T given initial stock price S_0) to find the implied probability distribution $\phi(S, T; S_0)$ for the two maturities.

Bonus: extra 10 points. Verify that the probability distribution that you obtain matches the one you obtain using the solution S_T/S_0 of the sde (1).