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% Take Home exam 4 - Anita Mezzetti

clc
clear all
close all

% parameters
sigma = 0.2;
r = 0.015;
K = 1;
T = 1;
S_min = 0;

% S_max is such that the truncation error is less than 10^-6:
max_tol = 1e-6;
S_max = K * exp(0.5 * sigma^2 * T - sigma * sqrt(T) * norminv(max_tol/
K));

%2b
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%--------------------- i) THETA METHOD: ---------------------------

theta = 0.5;    % => Crank-Nicholson
Nx = 60;        % number of intervals in x
Nt = 100;       % number of time steps

% initialisation functions for bs_timestepping:
forcing = @(S,t) zeros(length(S),length(t)); % right-hand-side of the
 equation
bc_right = @(t) zeros(size(t)); % boundary condition on the right
 border
init_cond = @(S) max(K-S,0); % initial condition of the problem

[U_theta, FD_grid, time_steps] = bs_timestepping...
    (sigma, r, forcing, bc_right, init_cond, S_max, Nx, T, Nt, theta);
% U_theta is the option price (solution of the pde)


%----------- ii) approximate rhs of the PDE solved by vega
 -------------

% approximation of U_SS
vega_rhs = find_rhs(U_theta, sigma, FD_grid, S_max);

% vega boundary and initial conditions
vega_bc_right = @(t) zeros(size(t));
vega_initcond = @(S) zeros(size(S));

% finite-difference solver using theta-method
[vega_approx, FD_grid, ] = vega_timestepping(sigma, r, vega_rhs,
 vega_bc_right,...
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    vega_initcond, S_max, Nx, T, Nt, theta);


%------------------- iii) exact vega and plot -----------------------

% exact solution:
vega_exact = blsvega(FD_grid(2:end)',K,r,T,sigma);

% plot 2b iii
figure
plot(FD_grid(2:end),vega_approx(2:end,end),'b', 'LineWidth', 1.3)
hold on
plot(FD_grid(2:end),vega_exact,'r--', 'LineWidth', 1.3)
title('Vega in the Black-Scholes-Merton model')
xlabel('S')
ylabel('Vega')
legend('Numerical solution','Exact solution')

%2c
 ------------------------------------------------------------------


i = [0:5];
Nx = 10 * 2.^i;
Nt = 0.6 * Nx;
h = S_max./Nx;

error = zeros(length(i),1); % initialization error vector

for n = 1:length(i) % loop for getting finer grids
    x = Nx(n); % Nx
    t = Nt(n); % Nt

    % option prices using theta-method:
    [U_theta, FD_grid] = bs_timestepping(...
        sigma, r, forcing, bc_right, init_cond, S_max, x, T, t,
 theta);

    % approximation of U_SS
    vega_rhs = find_rhs( U_theta, sigma, FD_grid, S_max );

    % finite-difference solver using theta-method
    [vega_approx,FD_grid,time_steps] = vega_timestepping(sigma, r,
 vega_rhs,...
        vega_bc_right, vega_initcond, S_max, x, T, t, theta);

    % exact solution
    vega_exact = blsvega(FD_grid(2:end)', K, r, T, sigma);

    % error
    error(n) = max(abs(vega_approx(2:end,end) - vega_exact));
end

% plot 2c
X = linspace(h(1),h(end));
```

```
P1 = polyfit(h,error,1);
Y1 = polyval(P1,X);
P2 = polyfit(h,error,2);
Y2 = polyval(P2,X);

figure
plot(h, error', 'r*')
hold on
plot(X, Y1, 'g')
plot(X, Y2, 'b')
title('Error of the finite difference solution')
xlabel('h')
ylabel('Error')
legend('Error','Interpolating line (degree 1)',...
    'Interpolating line (degree 2)','Location', 'northwest')

% order of convergence:
log_error = log(error);


%2d
 -------------------------------------------------------------------
ds = [0.0005 0.001 0.01 0.02 0.03 0.05];
%ds = [1e-5 1e-6 1e-7];

%------------ i) vega with the central limit diff scheme:
 ----------------
%------------ ii) approximation error and plot:
 --------------------------

error2 = zeros (length(Nx), length(ds)); % error matrix initialisation

for n = 1:length(Nx)

    x = Nx(n); % Nx
    t = Nt(n); % Nt

    for i = 1:length(ds)

        % option prices using theta-method:
        [U_plus, ] = bs_timestepping(sigma+ds(i), r, forcing, ...
            bc_right, init_cond, S_max, x, T, t, theta);

        [U_minus,FD_grid] = bs_timestepping(sigma-ds(i), r,
 forcing,...
            bc_right, init_cond, S_max, x, T, t, theta);

        % approximated vega using the central limit difference scheme:
        vega_sigma = (U_plus(:,end) - U_minus(:,end)) / (2 * ds(i));

        % exact vega
        vega_exact = blsvega(FD_grid(2:end)',K,r,T,sigma);

        % error
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        error2(n,i) = max(abs(vega_sigma(2:end) - vega_exact));
    end
end

% plot

% replot the results of c:
figure
plot(h, error, 'o', 'DisplayName', 'Finite Difference (point c)')

% new plot
for i = 1:length(ds)
    hold on
    tt = sprintf('Central Limit diff scheme ds= %f ', ds(i));
    plot(h, error2(:,i), '*', 'DisplayName', tt);
end
title('Error using the central finite difference scheme')
xlabel('h')
ylabel('Error')
legend show
hold off




%
 functions:-----------------------------------------------------------

function [rhs] = find_rhs ( U, sigma, FD_grid, S_max )

[Nx,Nt] = size(U);
h = S_max/(Nx - 1);      % space step
rhs = zeros(Nx, Nt);     % solution initialisation

for x = 2 : Nx-1
    approx_second_der = ( U(x+1,:) - 2*U(x,:) + U(x-1,:)) / h^2;
    rhs(x,:) = sigma * FD_grid(x)^2 * approx_second_der;
end

end

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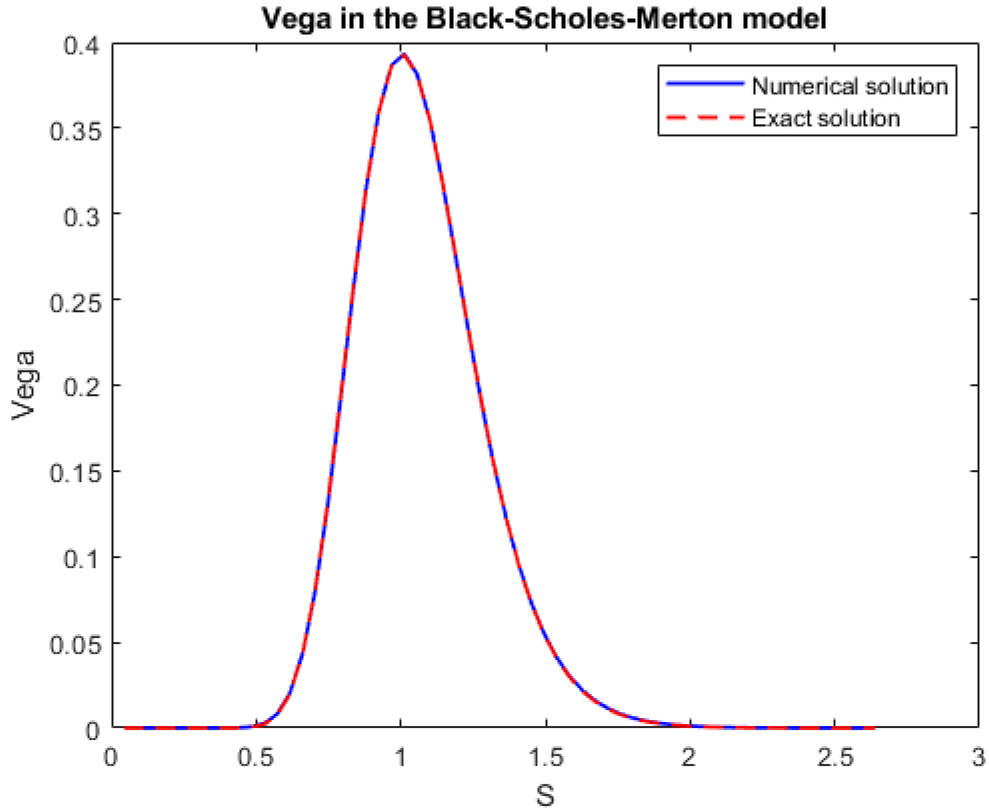
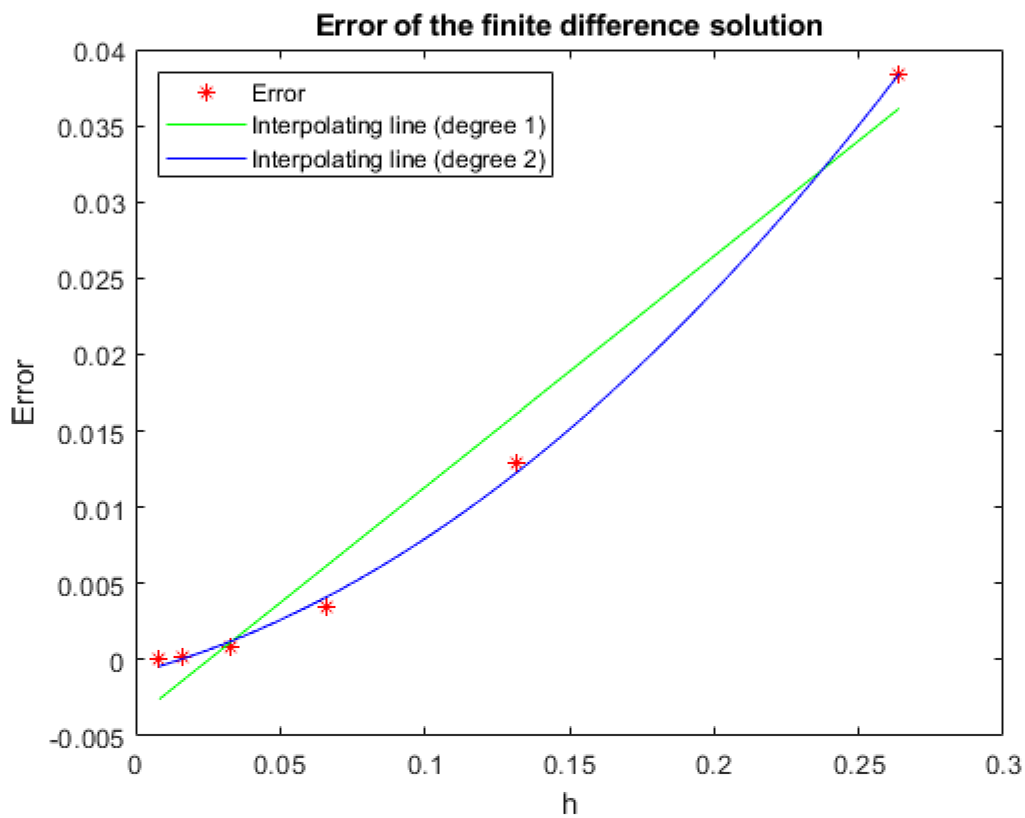
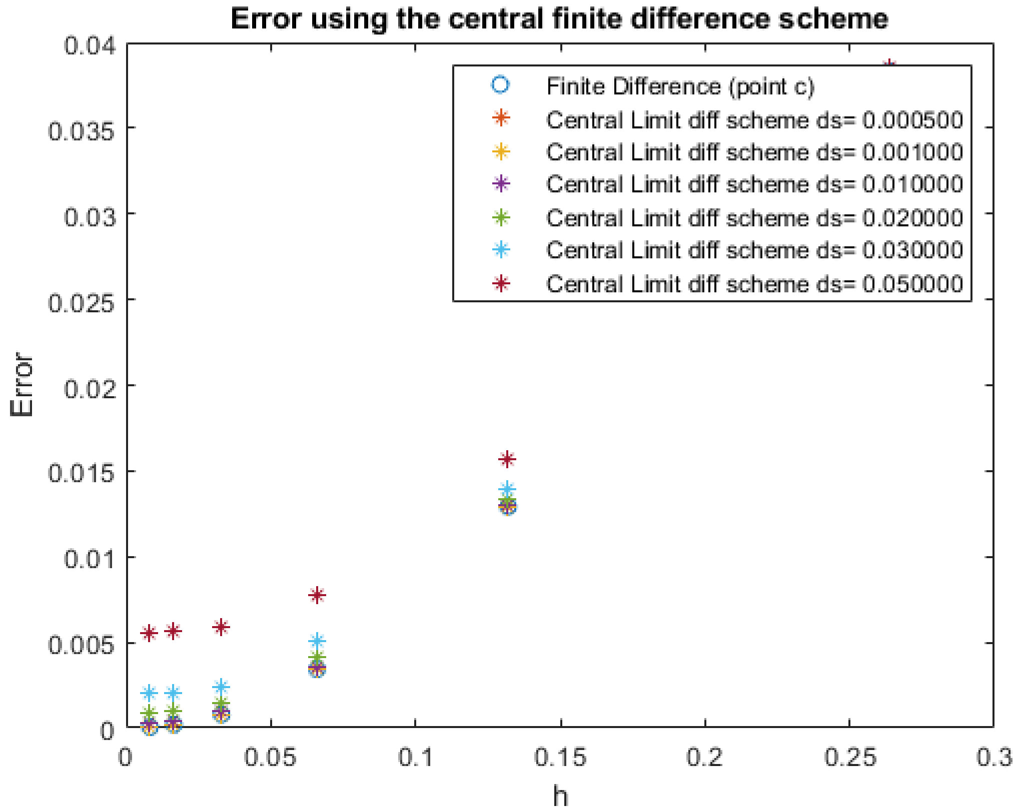

Vega in the Black-Scholes-Merton model
Numerical solution
Exact solution
Vega
S
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
0.5
1
1.5
2
2.5
3


Error of the finite difference solution
Error
Interpolating line (degree 1)
Interpolating line (degree 2)
Error
h
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
-0.005
0
0.05
0.1
0.15
0.2
0.25
0.3



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