Basic Discrete structure

Set

A set is a well-defined collection of distinct object that may be numbers, letters, alphabets, etc. The collection of object is written within the braces. i.e $\{ \}$, the set of first four numbers, $\{1, 2, 3, 4\}$.

Set-Builder Notations

These notations are used to characterize all the elements of sets by stating its properties.

For example :- set of all odd positive numbers.

$$O = \{x \mid x \text{ is all odd positive numbers}\}$$

$$O = \{x \in Z^+ \mid x \text{ is all odd}\}$$

$$O = \{x \in Z^+ \mid x \text{ is even}\}$$

Equal set

Two sets are said to be equal set if the elements of set A and set B are same. Eg:- $A = \{1, 3, 5\}$, $B = \{1, 3, 5\}$

Equivalent set

Two sets are said to be equivalent set if the number of set A and set B are equal.

Venn-diagram

Venn-diagram is the graphical representation of the sets.

Null set or empty set $\{ \}$ or $\{\phi\}$

A set is said to be null set if its cardinality is zero. It is denoted by $\{ \}$ or $\{\phi\}$.

Singleton set

A set is said to be singleton set if it has only one

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Subset

A set A is said to be subset of another set iff every element of set is also an element of second set. Eg:- $A = \{1, 2\}$, $B = \{1, 2, 3, 4, 5\}$. i.e $A \subseteq B$.

$$\forall x \{x \in A \rightarrow x \in B\}$$

For every set S

$$\phi \subset S$$

$$S \subseteq S.$$

Proper subset

If A is proper subset of B then $A < B$ denoted by $A \subset B$.

Power set.

A set of all possible subset of a set is called power set. It is denoted ~~P(S)~~ by P(S).

(Q) What is the power set of $S = \{2\}$.

2) Soln:-

Given,

$S = \{2\}$

$n = 1$.

We know,

$P(S) = 2^n = 2^1 = 2.$

$\therefore S = \{\{\phi\}, \{2\}\}$

(Q) What is the power set of $S = \{2, 3\}$.

=) Soln:-

Given,

$S = \{2, 3\}$

$n = 2$

We know,

$P(S) = 2^n = 2^2 = 4$

$\therefore S = \{\{\phi\}, \{2\}, \{3\}, \{2,3\}\}$

Cartesian product

$A = \{1,2\}$, $B = \{a,b,c\}$

$A \times B = \{1,2\} \times \{a,b,c\}$

$= \{(1,a), (1,b), (1,c), (2,a), (2,b), (2,c)\}$.

Set Operation.

① Union $(\cup)$

$A \cup B = \{x \mid x \in A \vee x \in B\}$

$A = \{1,3,5\}$, $B = \{1,2,3\}$

$A \cup B = \{1,3,5\} \cup \{1,2,3\}$

$= \{1,2,3,5\}$.

(ii) Intersection $(\cap)$

$A \cap B = \{x \mid x \in A \wedge x \in B\}$

$A \cap B = \{1,3,5\} \cap \{1,2,3\}$

$= \{1,3\}$.

(iii) Difference $(-)$

$A - B = \{x \mid x \in A \wedge x \notin B\}$ or, $\{x \mid x \in A \wedge x \in \neg B\}$

$A - B = \{1,3,5\} - \{1,2,3\}$

$= \{5\}$.

$B - A = \{1,2,3\} - \{1,3,5\}$

$= \{2\}$.

(iv) Complement.

~~$\bar{A} = U - A$~~ $\bar{A} = \{x \mid x \in U \wedge x \in \neg A\}$

or,

$= \{x \mid x \in \neg A\}$

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$U = \{1, 2, 3, 4, 5\}$

$A = \{1, 3, 5\}$

$U - A = \{1, 2, 3, 4, 5\} - \{1, 3, 5\}$

$\quad\quad = \{2, 4\}$

Symmetrical ~~ed~~ difference.

$(A \Delta B \mid A \oplus B) = (A - B) \cup (B - A) \;/\; (A \cap B) - (A \cap B).$

Let U = Letters of the alphabet.

A = vowel letters

B = letters in 'Discrete math'

Now,

Calculate (i) $A \cup B$ (ii) $A \cap B$ (iii) $A - B$ (iv) $B - A$ (v) $\overline{A \cup B}$ (vi) $\overline{A \cap B}$

(vii) Is $A \subseteq B$?

(i) $A \cup B = \{a, e, i, o, u\} \cup \{d, i, s, c, r, e, t, m, a, \cancel{t}, h\}$

$\quad = \{a, c, d, e, i, h, m, o, r, s, t, u\}$

(ii) $A \cap B = \{a, e, i, o, u\} \cap \{d, i, s, c, r, e, t, m, a, h\}$

$\quad = \{a, e, i\}$

(iii) $A - B = \{a, e, i, o, u\} - \{d, i, s, c, r, e, t, m, a, h\}$

$\quad = \{o, u\}$

(iv) $B - A = \{d, i, s, c, r, e, t, m, a, h\} - \{a, e, i, o, u\}$

$\quad = \{d, s, c, r, t, m, h\}$

(v) $\overline{A \cup B} = U - \{a, c, d, e, i, h, m, o, r, s, t, u\}$

$\quad = \{b, f, g, j, k, l, n, p, q, v, w, x, y, z\}.$

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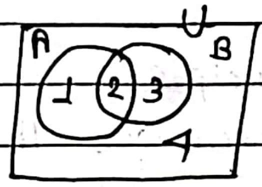
(VI) $\overline{A \cap B} = U - (A \cap B) = U - \{a, e, i\}$
$= \{b, c, d, f, g, h, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z\}$.

(VII) Is $A \subset B$?
, $A \not\subset B$. A is not the subset of B.

Set Identities

Identity law :- $A \cup \phi = A$ | $A \cap U = A$
Domination law :- $A \cup U = U$ | $A \cap \phi = \phi$
Idempotent law :- $A \cup A = A$ | $A \cap A = A$ [unchanged]
Complementation law :- $\overline{\overline{A}} = A$
Commutative law :- $A \cup B = B \cup A$ | $A \cap B = B \cap A$
Associative law :- $A \cup (B \cup C) = (A \cup B) \cup C$ | $A \cap (B \cap C) = (A \cap B) \cap C$.
Distributive law :- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ | $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
De-Morgan's law :- $\overline{A \cup B} = \bar{A} \cap \bar{B}$ | $\overline{A \cap B} = \bar{A} \cup \bar{B}$.
Absorption law :- $A \cup (A \cap B) = A$ | $A \cap (A \cup B) = A$.



$A \cap B = \{1, 2\} \cap \{2, 3\} = \{2\}$
$A \cup (A \cap B) = \{1, 2\} \cup \{2\}$
$= \{1, 2\}$
$A \cup B = \{1, 2\} \cup \{3, 4\} = \{1, 2, 3, 4\}$
$A \cap (A \cup B) = \{1, 2\} \cap \{1, 2, 3, 4\} = \{1, 2\}$
Complement law :- $A \cup \bar{A} = U$ | $A \cap \bar{A} = \phi$

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Prove that :- $\overline{A \cup B} = \bar{A} \cap \bar{B}$ by using set builder notation.

=) Soln :-

$L.H.S = \overline{A \cup B}$

$= \{x \mid x \in (\overline{A \cup B})\}$

$= \{x \mid x \notin (A \cup B)\}$

$= \{x \mid x \notin A \text{ and } x \notin B\}$

$= \{x \mid x \in \bar{A} \cap x \in \bar{B}\}$

$= \{x \mid x \in (\bar{A} \cap \bar{B})\}$

$= \bar{A} \cap \bar{B}$

$\therefore \overline{A \cup B} = \bar{A} \cap \bar{B}$

proved

Truth Table :-

A	B	A	B	$A \cup B$	$\overline{A \cup B}$	$\bar{A}$	$\bar{B}$	$\bar{A} \cap \bar{B}$
0	0	T	T	T	F	F	F	F
0	1	T	F	T	F	F	T	F
1		F	T	T	F	T	F	F
		F	F	F	T	T	T	T

$\therefore \overline{A \cup B} = \bar{A} \cap \bar{B}$

proved

Inclusion - Exclusion Principle

It is a counting technique that computes the number of elements that satisfy at least one of the several properties while guaranteeing that elements satisfying more than one property are not counted twice.

Basic idea :- Summing the number of elements that satisfies at least one of two categories } Inclusion

Subtracting the overlap which prevents double counting.

i.e $|A \cup B| = |A| + |B| - |A \cap B|$

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For finite set A, B and C,

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

Basic principle are used in graph colouring problem.

Eg:- There are 35 students at college who have taken a course in Calculus, 21 who have taken a course in DS, and 18 who have taken both Calculus and DS. How many students have taken the course in either calculus or DS?

⇒ Soln:-

Let C ~~k~~ and D be the number of students who have taken a course in Calculus and DS respectively.

~~$n(U)$ $n(C)$ $= n(C \cup D) = 35$~~

~~$n(C) = 21$~~

$n(C) = 35$

$n(D) = 21$

$n(C \cap D) = 18$

$n(C \cup D) = ?$

We know,

$$n(C \cup D) = n(C) + n(D) - n(C \cap D)$$
$$= 35 + 21 - 18$$
$$= 38.$$

∴ 38 students have taken the course in either calculus or DS.

(2) In the survey of 80 people it is found that 60 likes egg and 30 likes fish. Find the percentage of people who likes both egg and fish.

⇒ Soln:

let E and F be the number of people who like egg and fish respectively.

~~$n(E$~~ $n(E \cup F) = 80$

$n(E) = 60$

$n(F) = 30$

$n(E \cap F) = ?$

Now,

$n(E \cup F) = n(E) + n(F) - n(E \cap F)$

or, $80 = 60 + 30 - n(E \cap F)$

$\therefore n(E \cap F) = 10$

At last,

Percentage of ~~students~~ people who like both egg and fish

$= \frac{10}{80} \times 100\%$

$= 12.5\%$

Computer Representation of set :-

Also called bit string representation.

For universal set $U = \{x_1, x_2, \ldots, x_n\}$ of length n are represented by bit string (0, 1), and all the subsets of U are represented by either 1 (if element exist in that set) or by 0.

For eg:- Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Set A = all odd integer.

$A = \{1, 3, 5, 7, 9\}$

Set B = all even integer.

$B = \{2, 4, 6, 8, 10\}$

Now,

$U = \{1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\}$

$A = \{1\ 0\ 1\ 0\ 1\ 0\ 1\ 0\ 1\ 0\}$

$B = \{0\ 1\ 0\ 1\ 0\ 1\ 0\ 1\ 0\ 1\}$

Again,

$A \cup B = \{1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\}$

$A \cap B = \{0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\}$

Let set $A = \{1,2,3,4,5\}$ and $B = \{1,3,5,7,9\}$.
$U = \{1,2,3,4,5,6,7,8,9,10\}$. Represent the given sets by using bits string and find union and intersection of sets using bit string representation.

=) Soln:

$U = \{1,2,3,4,5,6,7,8,9,10\}$

$U = \{1\,1\,1\,1\,1\;1\,1\,1\,1\,1\}$

$A = \{1\,1\,1\,1\,1\;1\,0\,0\,0\,0\,0\}$

$B = \{1\,0\,1\,0\,1\,0\;1\,0\,1\,0\,1\,0\}$

Now,

$A \cup B = \{1\,1\,1\,1\,1\,1\,0\,1\,0\,1\,0\}$

$A \cap B = \{1\,0\,1\,0\,1\,0\,0\,0\,0\,0\}$

Again,

$\bar{A} = \{0\,0\,0\,0\,0\,1\,1\,1\,1\,1\}$

$\bar{B} = \{0\,1\,0\,1\,0\,1\,0\,1\,0\,1\}$

$\bar{A} \cap \bar{B} = \{0\,0\,0\,0\,0\,1\,0\,1\,0\,1\}$

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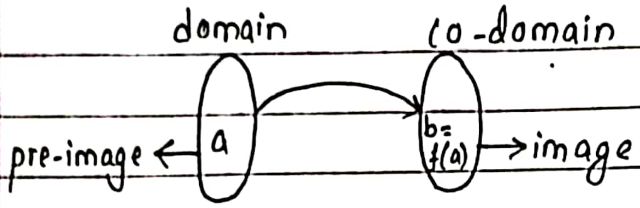
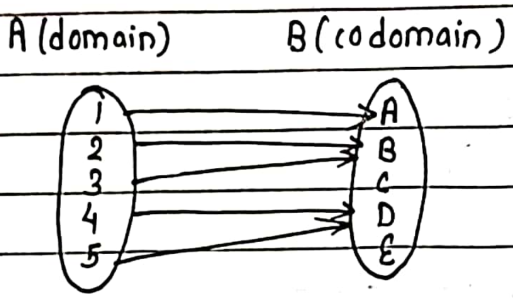
Function

A function f from A to B denoted by $f: A \Rightarrow B$ assigned each elements of A to ~~exactly~~ at least one element of B.
It is also called mapping or Transformation.





$f: A \to B$.

$f(1) = A$

$f(4) = D$

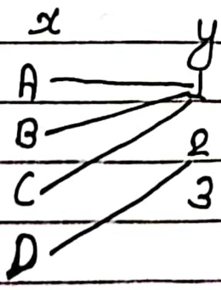
Image :- A is the image of 1.
Pre-image :- 1 is the preimage of A.

$f: A \to B$

images = $\{A, B, D\}$

Range = $\{A, B, D\}$.

Answer the following. $f: x \to y$



$\therefore$ Domain :- x

Codomain = ~~X~~ y

Range = $\{1, 2\}$

Preimage of 2 : $\{D\}$

Preimage of 1 : $\{A, B, C\}$

$f(D) = 2$.

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Types of function

(I) Injective function (one to one) :-

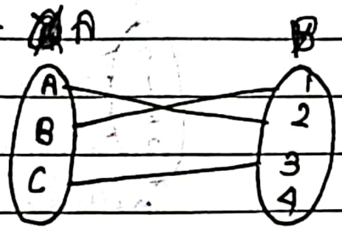
Each value in the range corresponds to exactly one element in the domain.

OR

In $f: A \to B$, B has at most one pre-image.

$$\forall a \forall b \, (a \neq b) \to f(a) \neq f(b).$$

$$\forall a \forall b \, (f(a) = f(b)) \to a = b.$$

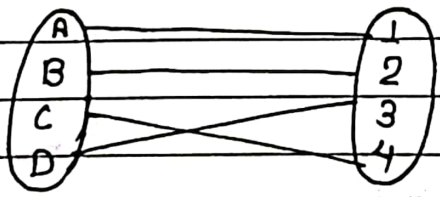
eg:- 

(II) Surjective function (Onto function)

Every element in the codomain maps to at least one element in the domain.

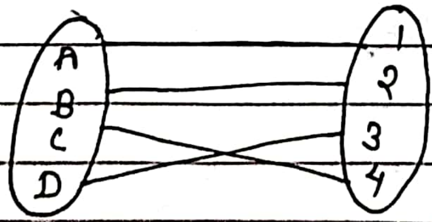
$$\forall y \exists x \, (f(x) = y).$$

Domain   Co-domain



Bijective

A function is said to be bijective if it is both one-to-one and onto.



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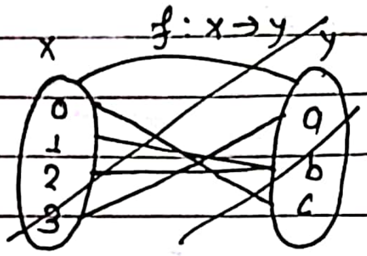
(Q) Let f be a function from $X = \{0, 1, 2, 3\}$ to $Y = \{a, b, c\}$ defined by $f(0) = c$, $f(1) = b$, $f(2) = b$ and $f(3) = c$. Is this $f: x \to y$ is either injective or surjective?

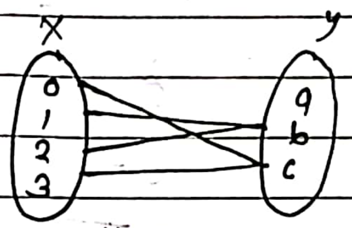
=) Soln:

Given,

$X = \{0, 1, 2, 3\}$

$Y = \{a, b, c\}$





neither nor surjective.

From above, the function is ^injective^ ~~since every element of x has at least one image.~~

(Q) If $f(x) = 5x + 2$ for $\forall x \in R$ then find out the function is surjective or not?

=) Soln:-

Let $f(x) = 5x + 2$

Let $f(x) = y = 5x + 2$

or, $x = \frac{y-2}{5} \in R$

$f(x) = f\left(\frac{y-2}{5}\right) + 2$

$= 5\left(\frac{y-2}{5}\right) + 2$

$= y - 2 + 2$

$= y$

Since $f(x) = y$, the function is surjective.

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Q) From the given $f(x) = x^2$ from the set of integer to set of integers is either one-to-one or onto?

⇒ Soln:-

Given,

$f(x) = x^2$ where $f: Z \rightarrow Z$

Let $f(x_1) = f(x_2)$

or, $x_1^2 = x_2^2$

$\therefore x_1 \neq x_2$

$\therefore f(x) = x^2$ is not one-to-one.

Now,

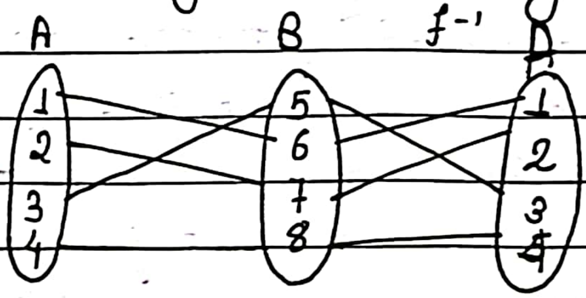
Let $y = x^2$

or, $x = \sqrt{y} \notin Z$

$\therefore f(x) = x^2$ is not onto.

Inverse function

Let f be a bijective function from set A to B. The inverse of f, denoted by f^{-1}, is the function from B to A defined as $f^{-1}(y) = x$ iff $f(x) = y$.



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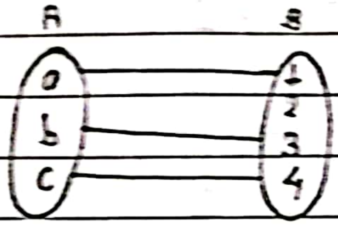
(8) If f is the function from $\{a,b,c\}$ to $\{1,2,3,4\}$ such that $f(a)=1$, $f(b)=3$ and $f(c)=4$, is f invertible? If so, what is its inverse?

$\Rightarrow$ Soln:-

let $A=\{a,b,c\}$
$B=\{1,2,3,4\}$



Since, $f: A \to B$ is one-to-one but not onto, its inverse i.e f^{-1} does not exist.

(9) Let $f: Z \to Z$ such that $f(x) = x+3$, is f invertible? If so what is the inverse of f

$\Rightarrow$ Soln:-

Given,

$f: Z \to Z$ such that $f(x) = x+3$.

For one-to-one,

Let $f(x_1) = f(x_2)$

or, $x_1 + 3 = x_2 + 3$

$\therefore x_1 = x_2$

$\therefore f(x)$ is one-to-one.

For onto,

let $y = x+3$

or, $x = y-3$

Now,

$f(x) = f(y-3) = y-3+3$
$= y$

Since, $f(x) = y$, $f(x)$ is onto.

Hence, $f^{-1}(x)$ exists.

For one-to-one,

We need to show,

$\forall x_1 \forall x_2 (x_1 \neq x_2) \to f(x_1) \neq f(x_2)$

Let $x_1 = 1$, $x_2 = 3$.

$f(1) = 1+3 = 4$

$f(3) = 3+3 = 6$

Since, $x_1 \neq x_2 \to f(x_1) \neq f(x_2)$, $f(x)$ is one-to-one.

For onto,

$\forall y \exists x \, (f(x) = y)$.

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~~No~~ For inverse,

~~Let~~ let $f(x) = y$

or, $f^{-1}(y) = x$ . . . (i)

Now,

let $y = x+3$

or, $x = y-3$ . . . (ii)

~~$f(x) = f(y-3)$~~

~~$= y-3+3$~~

From (i) and (ii),

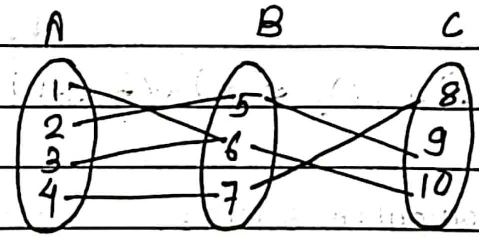
$f^{-1}(y) = y-3$

Interchanging the dummy variables x and y we get,

$f^{-1}(x) = x-3$

Composite function

Let $f: A \to B$ and $g: B \to C$ then composite of g with f, $g \circ f$ is the function from A to C defined by $g \circ f(x) = g(f(x))$



(Q) If $f(x) = x+3$ and $g(x) = x^2-2$ find $fog(1)$, $fog(x)$, $gof(1)$, $gof(x)$.

=) Soln:

Given,

$f(x) = x+3$

$g(x) = x^2-2$

Now,

$fog(x) = fg(x) = f(x^2-2)$

$= x^2-2+3 = x^2+1$

$fog(1) = 1^2+1 = 2$

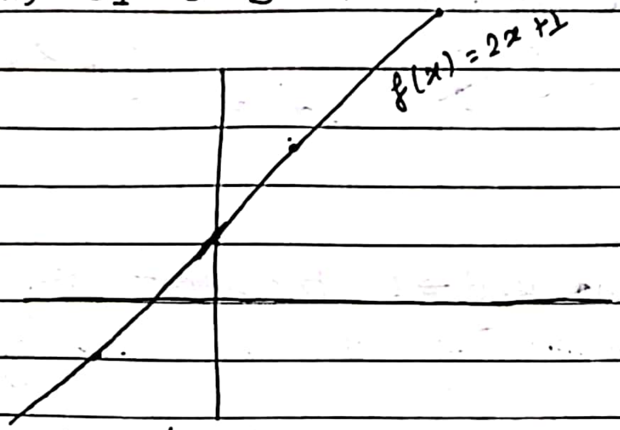
$gof(x) = g f(x) = g(x+3)$
$= (x+3)^2 - 2$
$= x^2 + 6x + 9 - 2$
$= x^2 + 6x + 7$

$gof(1) = 1^2 + 6 \times 1 + 7 = 14$

Graph of a function.

For $f(x) = 2x + 1$ from R to R.

x	0	1	2	3	-1	-2
$f(x)$	1	3	5	7	-1	-3



$G = \{(0,1), (1,3), (2,5), (-1,-1)\}$.

Function for computer science.

The ceiling and floor function.

$\lceil x \rceil \longrightarrow$ ceiling ~~[2.3] = 3~~

$\lfloor x \rfloor \longrightarrow$ floor ~~[2.3] = 2~~

Example :-

$\lceil 2.3 \rceil = 3$ $\qquad$ $\lfloor 2.3 \rfloor = 2$

$\lceil 2.7 \rceil = 3$ $\qquad$ $\lfloor 2.7 \rfloor = 2$

$\lceil -3.7 \rceil = -3$

$\lfloor -3.7 \rfloor = -4$

(2) let $f(x) = \left\lfloor \frac{x^2}{2} \right\rfloor$. Find $F(s)$, if $s = \{0,1,2,3\}$

=) Soln:-

Given,
$F(x) = \left\lfloor \frac{x^2}{2} \right\rfloor$, $F(s) = \left\lfloor \frac{s^2}{2} \right\rfloor$

For $s = 0$
$F(0) = \left\lfloor \frac{0^2}{2} \right\rfloor = 0$

For $s = 1$
$F(1) = \left\lfloor \frac{1^2}{2} \right\rfloor = \lfloor 0.5 \rfloor = 0$

For $s = 2$
$F(2) = \left\lfloor \frac{2^2}{2} \right\rfloor = 4$ $\cancel{2}$

For $s = 3$
$F(3) = \left\lfloor \frac{3^2}{2} \right\rfloor = \lfloor 4.5 \rfloor = 4$

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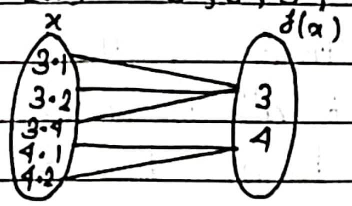
Explain why the floor function $f(x) = \lfloor x \rfloor$ is surjective but not injective where $f(x): R \to R$.

$\Rightarrow$ Let $x = 3.1, 3.2, 3.4, 4.1, 4.2$

$f(x) = \lfloor x \rfloor = 3, 3, 3, 4, 4.$

~~Fa~~



From ~~the~~ mapping we can say that $f(x) = \lfloor x \rfloor$ is surjective since $f(x) = y$. but not injective since every elements in domain do not have unique image in co-domain.

Boolean Function

If $f: x^n \to x$ describes the way which boolean output is derived from boolean input is called Boolean function.

i.e $\left\{ \begin{array}{l} f(x_1, x_2, \ldots \ldots x_n) = X \\ \quad \text{where } x = (0, 1) \\ \quad f(0,1)^n = \{0, 1\} \end{array} \right\}$

For eg:- $f(A, B) = \bar{A} \cdot \bar{B}$.

$\therefore f(1, 0) = \bar{1} \cdot \bar{0}$

$= 0 \cdot 1$

$= 0$

Exponential function

If b is any number such ~~ba~~ that $b > 0$ & $b \neq 1$ then an exponential function is $f(x) = b^x$ where b is base and x is any real number.

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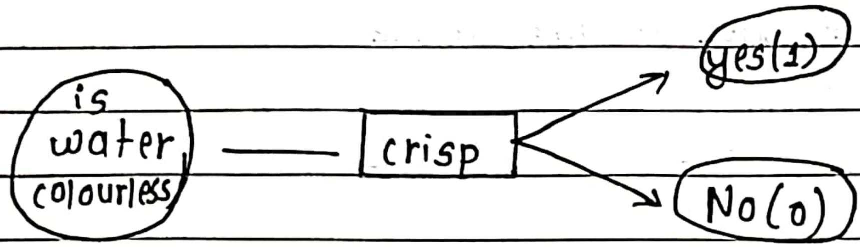
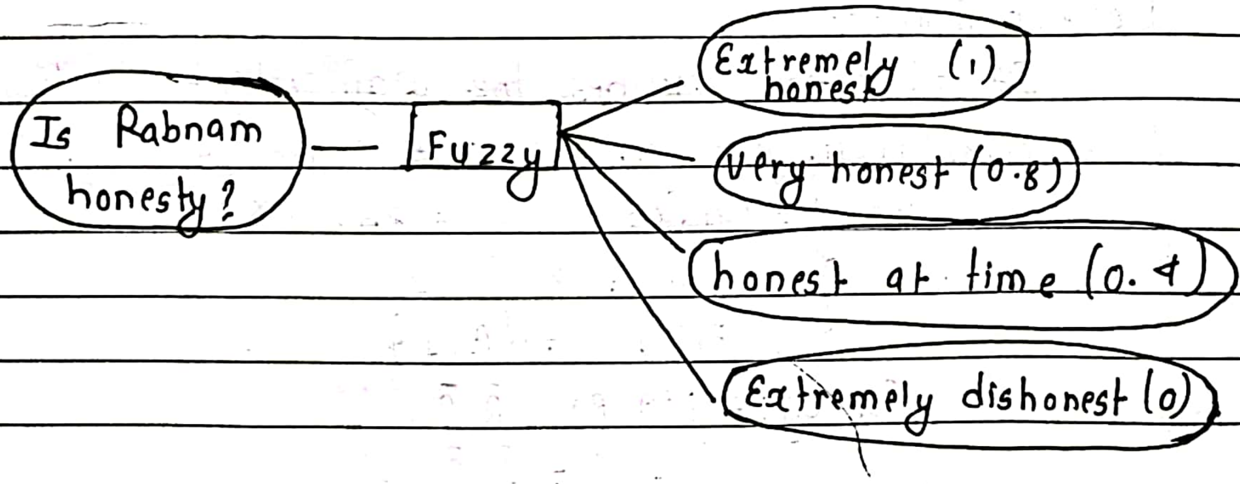
Fuzzy set

→ The word fuzzy means "vagueness (ambiguity)".
→ It occurs when the boundary of piece of information is not clear.
→ In 1965 "Lofti Zadeh" introduced the extension of classical set where classical set allows membership of element in the set ~~of~~ in binary terms but fuzzy set is valued between 0 to 1.





If X is an universe of discourse and x is a particular element of X, where fuzzy set $\tilde{A}$ is defined on x is

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X\}$$

$\mu_{\tilde{A}}$ = membership function.

Eg:- let $X = \{a_1, a_2, a_3\}$ and $\tilde{A}$ is the collection of noisy students then,

$$\tilde{A} = \{(a_1, 0.4), (a_2, 1), (a_3, 0)\}$$

Similarly,

$\tilde{B}$ represents smart students then,

$\tilde{B} = \{(a_1, 0.6), (a_2, 0.4), (a_3, 1)\}$.

Representation of fuzzy set :-

$\tilde{A} = \{(x_1, \mu_{\tilde{A}}(x_1)), \ldots\ldots, \ldots\}$

$\tilde{B} = \left\{\frac{\mu_{\tilde{B}}(y_1)}{y_1}, \ldots\ldots\ldots\right\}$

Fuzzy set operation :-

(a) Union => max of μ_A & μ_B.

(b) Intersection => min of two sets.

(c) Complement => $1 - \mu_A$

Eg :- $\tilde{A} = \left\{\frac{0.3}{x_1}, \frac{0.5}{x_2}, \frac{1}{x_3}\right\}$

$\tilde{B} = \left\{\frac{0.8}{x_1}, \frac{0.2}{x_2}, \frac{1}{x_3}\right\}$

Now, calculate :-

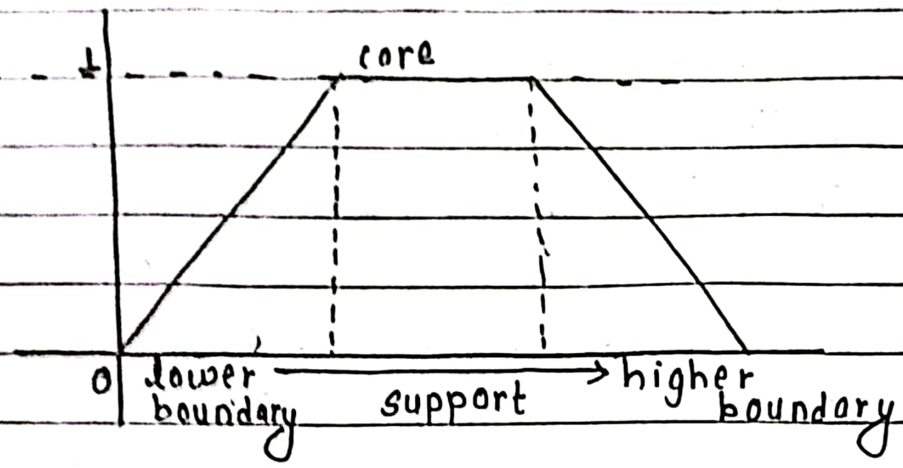
$(\tilde{A} \cup \tilde{B}) = \left\{\frac{0.8}{x_1}, \frac{0.5}{x_2}, \frac{1}{x_3}\right\}$

$(\tilde{A} \cap \tilde{B}) = \left\{\frac{0.3}{x_1}, \frac{0.2}{x_2}, \frac{1}{x_3}\right\}$

$\overline{(\tilde{A})}^{\circ} = \left\{\frac{0.7}{x_1}, \frac{0.5}{x_2}, \frac{1}{x_3}\right\}$

$\overline{(\tilde{B})} = \left\{\frac{0.2}{x_1}, \frac{0.8}{x_2}, \frac{1}{x_3}\right\}$

Features of membership function



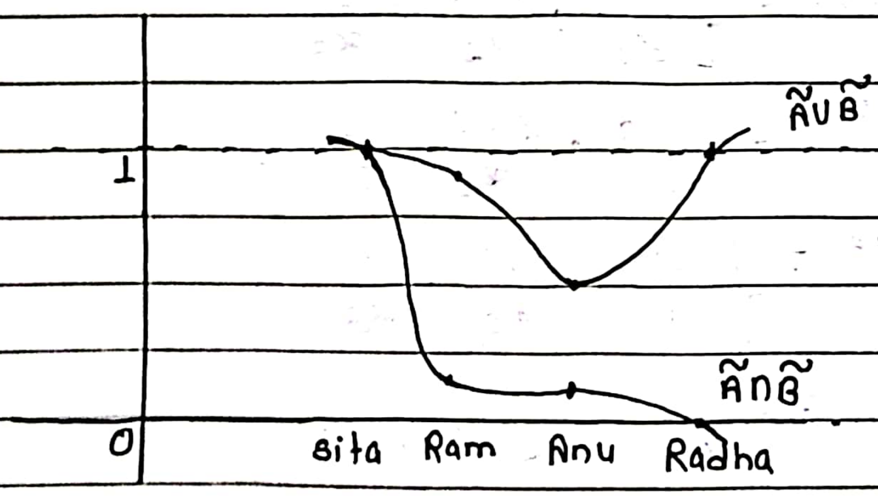
Let $\tilde{A} = \left\{ \left(\frac{1.0}{Sita}, \frac{0.9}{Ram}, \frac{0.2}{Anu}, \frac{0}{Radha} \right) \right\}$

$\tilde{B} = \left\{ \frac{1.0}{Sita}, \frac{0.2}{Ram}, \frac{0.5}{Anu}, \frac{1}{Radha} \right\}$

Now,

$\tilde{A} \cup \tilde{B} = \left\{ \frac{1.0}{Sita}, \frac{0.9}{Ram}, \frac{0.5}{Anu}, \frac{1}{Radha} \right\}$

$\tilde{A} \cap \tilde{B} = \left\{ \frac{1.0}{Sita}, \frac{0.2}{Ram}, \frac{0.2}{Anu}, \frac{0}{Radha} \right\}$



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Sequence and summation

A sequence is a output of function whose input is either the set of positive integers (Z^+) or set of natural number arranged in order pair.

Eg:- $a_0, a_1, a_2, \ldots \ldots a_n, a_{n+1}, \ldots \ldots$

let $a_0 = 0$, $a_1 = 1$, $a_2 = 4 \ldots$ then the sequence will be $0, 1, 4, 9, 16, 25 \ldots \ldots$

Q1) Let $a_n = n^2 \; \forall n$. Now find the sequence for $a_n = n^2 \; \forall n$.

⇒ Soln:-

Let $a_0 = 0^2 = 0$

$a_1 = 1^2 = 1$

$a_2 = 2^2 = 4$

$a_3 = 3^2 = 9$

$\vdots$

(2) Let $a_n = 27^n \; \forall n$ s.t $5 < n < 123$. Find the sequence of a_n.

⇒ Soln:-

Given,

$a_n = 27^n \quad \forall n$

$5 < n < 123$

Now,

$a_6 = 27^6$

$a_7 = 27^7$

$a_8 = 27^8$

$a_9 = 27^9$

$\vdots$

$a_{122} = 27^{122}$

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Progression

If the term of sequence follow particular pattern or rule, then such a sequence is called progression. There are three types of progression:-

(i) A.P (ii) G.P (iii) H.P

Basically progression is used to findout the n^{th} position.

Arithmetic progression (A.P):-

A sequence is called A.P if

$$d = t_{n+1} - t_n$$

OR,

$$a + (1a+d) + (a+2d) + \ldots + (a+(n-1)d)$$

where d is the difference between two terms.

For eg:- let sequence is,

5, 6, 7, 8

Then $a = 5$

$d = 6 - 5 = 1$

& $t_n = a + (n-1)d$

$n = 4$

$t_4 = 5 + (4-1) \times 1$

$= 5 + 3$

$= 8$

$t_3 = 5 + (3-1) \times 1$

$= 7$

(Q) Let $t_n = 7 - 3n$ $\forall n$ with initial term $= 7$ & $d = -3$ then find out $\{t_0, t_1, \ldots t_n\}$.

⇒ Soln:

Given,

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$t_n = 7 - 3n$

$a = 7$

$d = -3$

Now,

$t_n = a + (n-1)d$

$t_0 = 7 + (0-1) \times (-3)$

$= 7 + 3 = 10$

$t_1 = 7 + (1-1) \times (-3)$

$= 7$

$t_2 = 7 + (2-1) \times (-3)$

$= 4$

$\vdots$

$t_n = 7 + (n-1) \times (-3)$

$= 7 - 3n + 3$

$= 10 - 3n$

Geometric Progression

If sequence is in following form then it is called G.P.

$a, ar, ar^2, ar^3, \ldots\ldots ar^{n-1}$

where $r = \dfrac{a_n}{a_{n-1}}$, $a_n = ar^{n-1}$

Eg:- $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$

Find the 10^{th} term of 2, 4, 8, 16.

⇒ Soln:-

$a = 2$

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Summation

Summation is the operation of adding a sequence of numbers. which can be achieved by using summation operation Σ.

The sum of the term sequence.

$a_n = \{a_m, a_{m+1}, \ldots a_n\}$ can be written as

$$\sum_{i=m}^{n} a_i \quad \text{OR} \quad \sum_{m \le i \le n} a_i$$ where i = index of summation
m = lower limit
n = higher limit.

Eg:- Sum of the first seven terms of (n^2) where $n = 1, 2, 3 \ldots 7$

i.e $\sum_{i=1}^{7} a_i = \sum_{i=1}^{7} i^2 = 1 + 4 + \ldots + 49 = 140$

Express the sum of the first 100 terms of sequence $\{a_n\}$ where $a_n = \frac{1}{n}$ and $n = 1, 2, 3 \ldots$

$\therefore a_n = \frac{1}{n}$, $n = 100$

$\sum_{i=1}^{100} 1 + \frac{1}{2} + \frac{1}{3} + \ldots \frac{1}{100}$.

What is the value of $\sum_{i=5}^{9} i^2$?

=) Soln:

$$\sum_{i=5}^{9} i^2 = 5^2 + 6^2 + 7^2 + 8^2 + 9^2$$
$$= 255.$$

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What is the value of $\sum_{i=7}^{10} (-1)^i$?

$\Rightarrow$ Soln:-

$$\sum_{i=7}^{10} (-1)^i = (-1)^7 + (-1)^8 + (-1)^9 + (-1)^{10}$$

$$= -1 + 1 + (-1) + 1$$

$$= 0$$

Double summation

It is used to calculate the sum/complexity of nested loops and 2-dimensional array.

For eg:- $\sum_{i=1}^{n} \sum_{j=1}^{m} f_{ij} = \sum_{i=1}^{n} \left(\sum_{j=1}^{m} f_{ij} \right)$

What is the value of $\sum_{i=1}^{3} \sum_{j=2}^{4} f_{ij}$ where $f_{ij} = (i+j)$

$\Rightarrow$ Soln:-

Given,

$$\sum_{i=1}^{3} \sum_{j=2}^{4} f_{ij}$$

$$f_{ij} = (i+j)$$

Now,

$$\sum_{i=1}^{3} \sum_{j=2}^{4} f_{ij} = \sum_{i=1}^{3} \sum_{j=2}^{4} (i+j)$$

$$= \sum_{i=1}^{3} (i+2)+(i+3)+(i+4)$$

$$= (1+2)+(1+3)+(1+4)+(2+2)+(2+3)+(2+4)+(3+2)+(3+3)+(3+4)$$

$$\cancel{= 20+14} = 45$$

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(9) $\sum_{i=1}^{4} \sum_{j=1}^{2} f_{ij}$ where $f_{ij} = (2i-j)$ and represent this sum in pseudocode.

=) Soln:

Given,

$$\sum_{i=1}^{4} \sum_{j=1}^{2} f_{ij}$$

$$f_{ij} = (2i-j)$$

Now,

$$\sum_{i=1}^{4} \sum_{j=1}^{2} f_{ij} = \sum_{i=1}^{4} \sum_{j=1}^{2} (2i-j)$$

$$= \sum_{i=1}^{4} (2i-1) + (2i-2)$$

$$= (2\times1-1) + (2\times1-2) + (2\times2-1) + (2\times2-2) + (3\times2-1) + (3\times2-2) + (4\times4-1) + (2\times4-2)$$

$$= 28$$

Pseudocode:
```
sum = 0;
for (i = 1 to 4)
{
    for (j = 1 to 2)
    {  sum += 2i - j;
    }
}  return s;
```

(2) What is the value of $\sum_{s \in (0,2,4)} s$?

=) Soln:

$$\sum_{s \in (0,2,4)} s = 0+2+4 = 6.$$

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Some formulas of summation.

$$\sum_{k=0}^{n} ar^k \, (r \neq 0) \Rightarrow \frac{ar^{n+1} - a}{r-1}$$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

$$\sum_{k=0}^{\infty} k \, , \, |x| < 1 = \frac{1}{1-x}$$

(Q) $\sum_{k=50}^{100} k^2$.

$\Rightarrow$ Soln. :-

$$\cancel{\sum_{k=50}^{100} k^2 = \frac{n(n+1)(2n+1)}{6}} \qquad \sum_{k=1}^{100} k^2 - \sum_{k=1}^{50} k^2$$

$$= \frac{100(100+1)(2\times100+1)}{6} - \frac{50(50+1)(2\times50+1)}{6}$$

$= 338350 - 42925$

$= 295425$

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