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Probability $(0 \le p \le 1)$

Basic terms:

i) Random experiment
: The experiment in which result is unpredictable is called random experiment

ii) Trial
: Performing the random experiment is called trial.

iii) Event
: It is the result / outcome of random experiment

iv) Sample space (S)
: It is the set of all possible outcomes of an experiment

Example: When a single coin is tossed.
$S = \{H, T\}$
When a dice is rolled.
$S = \{1, 2, 3, 4, 5, 6\}$

v) Exhaustive cases
: It is the total number of possible outcomes of an experiment.

Example:
$S = \{HH, HT, TH, TT\}$.
No. of outcomes $(n) = 4$.

vi) Mutually exclusive events

: Two or more events are mutually exclusive if they can't happen simultaneously.

vii) Dependent and Independent events

: Two or more events are said to be independent if probability of any event doesn't affect the probability of other event.

Two or more events are said to be dependent if probability of any event affect the probability of other event.

Example: selection procedure with replacement independent.

Selection procedure without replacement dependent event.

Classical or mathematical or prior definition of probability

: Let m be the favourable number of cases of any event and n be the total number of possible outcomes of an experiment then probability of event is given by:-

$$P(E) = \frac{m}{n}$$

Remarks

: ~~Probability of E~~ $P(E) + P(\bar{E}) = 1$.

Note: Coin Toss 2^n, $n=1$, 2; $n=2$, $2^2=4$; $n=3$, $2^3=8$
Dice 6^n, $n=1$, $6^1=6$; 6^2, $n=2$, 36

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Suppose a coin is tossed three times then find the probability of getting:

a) All head
b) One head and two tail
c) Atleast one head.
d) Atmost one head.

a) All head.
Here

Total no. of outcomes $(n) = 2^3 = 8$

sample space $= \{H, T\}\{H, T\}$
$= \{HH, HT, TH, TT\} \times \{H, T\}$
$= \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

Now

Probability (all head) $= \frac{m}{n} = \frac{1}{8}$

b) One head and two tail
Here.

Probability (one head & 2 tail) $= \frac{3}{8}$

c) Atleast one head.
Here

Probobility (atleast one head) $= \frac{7}{8}$

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d) Atmost one head.
Here,

Probability (atmost one head) $= \frac{4}{8} = \frac{1}{2}$

Combination
: It is the total number of possible outcomes in selection procedure.

* A box contain 5 red chips, 7 black chips and 4 white chips. 3 chips are selected at random at a time. Then, find the probability of getting:-

a) All red chips
b) One red and 2 black.
c) One red and 2 other chips.
d) Same colour chips.
e) Different colour chips.

Soln

Total no. of chips $(n) = {}^{16}C_3 = 560$.

a) Here,

For 3 red chips (E)

$(m) = {}^{5}C_3 = 10$

Now,

Probability of all red chips $P(E) = \frac{m}{n} = \frac{10}{560}$

$= \frac{1}{56}$

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b) Here

For one red and 2 black P(E)

$m = {}^5C_1 * {}^7C_2$

$= 105$

Now

$P(E) = \frac{105}{560} = \frac{3}{16}$

c) Here

for one red and 2 other chips. P(E).

$m = {}^5C_1 \times {}^{11}C_2$

$= 275$

Now,

$P(E) = \frac{m}{n} = \frac{275}{560} = \frac{55}{112}$

d) Here

For different colour (E) = 1R & 1W & 1B.

$m = {}^5C_1 \times {}^7C_1 \times {}^4C_1$

$= 140$

Now,

$P(E) = \frac{m}{n} = \frac{140}{560} = \frac{1}{4}$

e) Here,

for same colour chips (E) = 3R or 3W or 3B.

$m = {}^5C_3 + {}^7C_3 + {}^4C_3$

$= 49.$

$P(E) = \frac{m}{n} = \frac{49}{560} = \frac{7}{80}$

* In a certain community, there are five IT officer, 8 teachers and 9 doctors. A commitee of 3 person is to be formed. Then find the probability that comittee will consist of

a) all IT officer
b) one IT officer and 2 other
c) One of each ~~person~~ profession
d) same of each profession
e) Atleast one IT officer

Here

Total no. of profession $(n) = {}^{22}C_3$
$= 1540$

a) Here

For all IT officer, (E)

$m = {}^{5}C_3$
$= 10$

Now,

$P(E) = \frac{m}{n} = \frac{10}{1540} = \frac{1}{154}$

b) Here

For one IT officer and 2 other (E)

$m = {}^{5}C_1 * {}^{17}C_2$
$= 680$

Now,

$$P(E) = \frac{m}{n} = \frac{680}{1540} =$$

c) One of each profession.

Here,

For one of each profession (E)

$$m = {}^5C_1 \times {}^8C_1 \times {}^9C_1$$
$$= 360$$

Now,

$$P(E) = \frac{m}{n} = \frac{360}{1540} = \frac{18}{77}$$

d) Here

for same of each profession.

$$m = {}^5C_1 + {}^8C_1 + {}^9C_1$$
$$= 22$$

Then,

$$P(E) = \frac{22}{1540} = \frac{1}{70}$$

e) Here

(5) IT officer	(17) Other
1	2
2	1
3	0

Then,

$$P(E) = ({}^5C_1 \times {}^{17}C_2) + ({}^5C_2 \times {}^{17}C_1) + ({}^5C_3 \times {}^{17}C$$
$$= 680 + 170 + 10 = 860$$

$\therefore P(E) = \frac{m}{n} = \frac{860}{1540} = \frac{43}{77}$

OR, $P(E) = 1 - \frac{^{17}C_3}{1540}$. 1 - (Not IT officer).

<u>IV</u> 2080 Question

Three groups of children 2 boys and 2 girls, 3 boys and 1 girl, 1 boy and 3 girl respectively. One child is selected at random from each group. Find the probability of selecting one boy & two girls.

Soln:

I	II	III
2B	3B	1B
2G	1G	3G.

Then

	I	II	III
A	B	G	G
		OR	
B	G	B	G
		OR	
C	G	G	B

Then

$$P(A) = \frac{^2C_1}{^4C_1} \times \frac{^1C_1}{^4C_1} \times \frac{^3C_1}{^4C_1}$$

$$= \frac{3}{32}.$$

Again,

$$P(B) = \frac{^2C_1}{^4C_1} \times \frac{^3C_1}{^4C_1} \times \frac{^3C_1}{^4C_1}$$

$$= \frac{9}{32}$$

Again,

$$P(C) = \frac{^2C_1}{^4C_1} \times \frac{^1C_1}{^4C_1} \times \frac{^1C_1}{^4C_1}$$

$$= \frac{1}{32}$$

At last,

$$P(E) = P(A) + P(B) + P(C)$$

$$= \frac{3}{32} + \frac{9}{32} + \frac{1}{32}$$

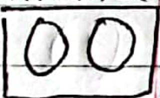
$$= \frac{13}{32}$$

Theorem: Of probability

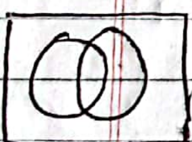
① Addition theorem

: A and B be two events, then probability of occurance of atleast one event ie. either A or B is given by:-

mutually enclusive



$P(A \cup B) = P(A) + P(B)$ (Mutually enclusive)



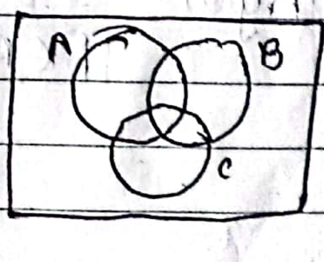
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (Mutually not enclusive)

For three events A, B, C

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C).$$

$P(A \cup B) = 1 - P(\bar{A}) \times P(\bar{B})$

$P(A \cup B \cup C) = 1 - P(\bar{A}) \times P(\bar{B}) \times P(\bar{C})$



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Remarks:

Multiplication theorem
: Let A and B two independent events then probability of occurance of both A and B.

$$P(A \cap B) = P(A) \times P(B).$$
$$P(A \cap B \cap C) = P(A) \times P(B) \times P(C).$$

Remarks:
: Let A and B be two events then,

$$P(A \text{ only}) = P(A \cap \bar{B})$$
$$= P(A) \times P(\bar{B})$$

OR

$$P(A \text{ only}) = P(A) - P(A \cap B)$$

$$P(B \text{ only}) = P(\bar{A} \cap B) = P(\bar{A}) \times P(B).$$

Probability of exactly one = $P_o(A) + P_o(B)$

* Probability that Mr. A will attend the seminar is 0.5. Probability of Mr. B is 0.6. Also, Both can attend the seminar. Then find the probability that:-

a) Both will attend the seminar
b) Either A or B will attend the seminar
c) Neither of them attend the seminar.
d) Only A will attend the seminar.
e) Only B will attend the seminar.

Soln

$P(A) = 0.5$ $\qquad$ $P(\bar{A}) = 0.5$

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$P(B) = 0\cdot6$ $P(\overline{B}) = 0\cdot4$

(1)
ans: Here,

$$= P(A) \times P(B)$$
$$P(A \cap B) = 0\cdot5 \times 0\cdot6$$
$$= 0\cdot3.$$

(11)
ans: Here

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 0\cdot5 + 0\cdot6 - 0\cdot3$$
$$= 0\cdot8$$

(111) Here,

$$= 1 - P(A \cup B)$$
$$P(\overline{A \cup B}) = 1 - 0\cdot8$$
$$= 0\cdot2$$

(IV) Here

$$P(A \text{ only}) = P(A) - P(A \cap B)$$
$$= 0\cdot5 - 0\cdot3$$
$$= 0\cdot2$$

(V) Here

$$P(B \text{ only}) = P(B) - P(A \cap B)$$
$$= 0\cdot6 - 0\cdot3$$
$$= 0\cdot3$$

Independent = x

Lastai imp

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* A problem on statistics is given to three student A, B and C whose chances of solving it are 1/3, 2/3 and 1/4 respectively. If they try the problem independently then what is the probability that:-

a) Problem will be solved.

Sol^n

$$P(A \cup B \cup C) = 1-(P(\bar{A}) * P(\bar{B}) * P(\bar{C}))$$
$$= 1-\left(\left(1-\frac{1}{3}\right) \times \left(1-\frac{2}{3}\right) \times \left(1-\frac{1}{4}\right)\right)$$
$$= \frac{5}{6}$$

b) Problem will n't be solved.

Sol^n

$$P(\overline{A \cup B \cup C}) = 1 - P(A \cup B \cup C)$$
$$= 1 - \frac{5}{6}$$
$$= \frac{1}{6}$$

c) Only A will solve the problem.

Sol^n

$$P(A \text{ only}) = P(A \cap \bar{B} \cap \bar{C})$$
$$= P(A) \times P(\bar{B}) \times P(\bar{C})$$
$$= \frac{1}{3} \times \frac{1}{3} \times \frac{3}{4}$$
$$= \frac{1}{12}$$

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d) B and C only.
Soln

$$P(B \& C \text{ only}) = P(\bar{A} \cap B \cap C)$$
$$= P(\bar{A}) \times P(B) \times P(C)$$
$$= \frac{2}{3} \times \frac{2}{3} \times \frac{1}{4}$$
$$= \frac{1}{9}$$

* The odds that Mr. A will speak truth is 6:8 and the odds that Mr. B will speak truth is 5:7 then find the probability that they will contradict to each other or at what (%)

Soln they will contradict to each other.

$$P(A) = \frac{6}{6+8} = \frac{6}{14}$$ (Favour = 6/14. Against = 8/14.

$$P(B) = \frac{5}{5+7} = \frac{5}{12}$$

Then

$$P(A \cap B) = P(A) \times P(B)$$
$$= \frac{\cancel{6}}{14} \times \frac{5}{\cancel{12}}$$
$$= \frac{5}{28}$$

Now

$$P(A \text{ only}) = P(A) - P(A \cap B)$$
$$= \frac{\cancel{6}}{14} - \frac{5}{28}$$
$$= \frac{1}{4}$$

$P(B \text{ only}) = P(B) - P(A \cap B)$

$= \frac{5}{12} - \frac{5}{28}$

$= \frac{5}{21}$

Now,

$P(A \text{ only} \ \& \ B \text{ only}) = \frac{1}{4} + \frac{5}{21}$

$= \frac{41}{84}$

Conditional Probability

: It is the probability of any event given that another event has already occurred.

The conditional probability of A given that B has already occured.

$P(A/B)$ (In this B has already occurred)

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

* In a classroom, 60% of students were passed in statistics and 90% were passed in OOP, and 50% in both subjects. A student is selected at random, then find the probability that a) student will pass either in stat or OOP

b) Suppose the selected student is passed in stat, Then find the probability that student also pass in OOP.

c) Find the probability that student will pass in stat given that student passed in OOP.

ans Here

Given that:

A → Pass in stat

B → Pass in OOP.

$P(A) = 60\% = 0.6$

$P(B) = 90\% = 0.9.$

Then $P(A \cap B) = 50\% = 0.5$

a)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 0.6 + 0.9 - 0.5$$
$$= \cancel{0.8}\ 1.0$$

b) Here

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$
$$= \frac{0.5}{0.6}$$
$$= \cancel{\frac{7}{8}}\ \frac{5}{6}$$

c) Here

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.5}{0.9} = \frac{5}{9}$$

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② The following table represents employment status of 200 graduates from TU.

	Gender (A)	Employment status (B) Employed (B_1)	Unemployed (B_2)	Total
(A_1)	M	110 ($A_1 \cap B_1$)	20 ($A_1 \cap B_2$)	130 (A_1)
(A_2)	F	60 ($A_2 \cap B_1$)	10 ($A_2 \cap B_2$)	70 (A_2)
	Total	170 (B_1)	30 (B_2)	200

A graduated student is selected random then find the probability that the selected graduate is:-

a) Male
b) female and currently employed.
c) Currently employed
d) Male or currently employed.
e) If the selected graduate is male then what is the probability that he is currently employed.
f) Suppose the selected graduate is unemployed then find the probability that graduate is female.

a) ans Soln.

$$P(A_1) = \frac{m}{n} = \frac{130}{200} = \frac{13}{20}$$

c) Soln

$$P(B_1) = \frac{m}{n} = \frac{170}{200} = \frac{17}{20}$$

c) Sol^n

$$P(A_2 \cap B_1) = \frac{60}{200} = \frac{3}{10}$$

d,

ans Sol^n

$$P(A_1 \cup B_1) = P(A_1) + P(B_1) - P(A_1 \cap B_1)$$

$$= \frac{13}{20} + \frac{17}{20} - \frac{110}{200}$$

$$= \frac{19}{20}$$

e)

ans Sol^n

$$P\left(\frac{B_1}{A_1}\right) = \frac{P(A_1 \cap B_1)}{P(A_1)}$$

$$= \frac{\frac{110}{200}}{\frac{13}{20}}$$

$$= \frac{11}{13}$$

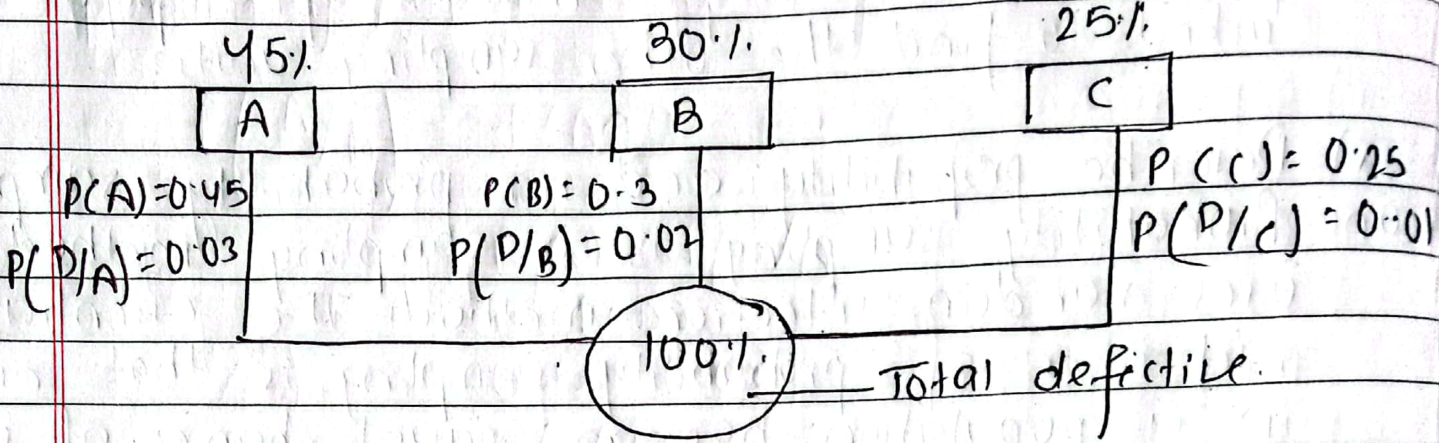
f)

ans Sol^n

$$P\left(\frac{A_2}{B_2}\right) = \frac{P(A_2 \cap B_2)}{P(B_2)}$$

$$= \frac{\frac{10}{200}}{30/200} = \frac{1}{3}$$

Baye's Theorem.



$$P(D) = P(A \cap B) + P(B \cap D) + P(C \cap D)$$
$$= P(A) \times P\left(D/A\right) + P(B) \times P\left(D/B\right) + P(C) \times P\left(D/C\right)$$
$$= 0.022$$

If the selected article is defective then probability that it was from machine A, B and C is given by

$$P\left(A/D\right) = \frac{P(A \cap D)}{P(D)}$$
$$= \frac{P(A) \times P\left(D/A\right)}{0.022}$$
$$= 0.61 \quad (61\%)$$

$$P\left(B/D\right) = \frac{P(B \cap D)}{P(D)}$$
$$= \frac{P(B) \times P\left(D/B\right)}{0.022}$$
$$= 0.27 \quad (27\%)$$

$$P(C/D) = \frac{P(C \cap D)}{P(D)}$$

$$= \frac{P(C) \times P(D/C)}{0.022}$$

$$= 0.11\%$$ (11%)

① In a certain company, machine A, B and C respectively produces 3,000, 2,000 and 1,000 articles. From past record it was found that probability of defective for each machine is 3%, 2% and 1%. An article is selected from total and found to be defective then what is the probability that it was produced by machine A.

Soln

Given that:-

$P(D/A) = 3\% = 0.03$ $P(D/B) = 0.02$ $P(D/C) = 0.0$

$P(A) = \frac{3}{6} = 0.5$ $P(B) = \frac{2}{6} =$ $P(C) = \frac{1}{6}$

Let,

D denotes defective article.

$P(D) = P(A \cap B) + P(B \cap D) + P(C \cap D)$

$= P(A) \times P(D/A) + P(B) \times P(D/B) + P(C) \times P(D/C$

$= \frac{3}{6} \times 0.03 + \frac{2}{6} \times 0.02 + \frac{1}{6} \times 0.01$

$= 0.023$

Then,

If the selected article is defective then probability that it was from A:-

$$P\left(A/D\right) = \frac{P(A\cap D)}{P(D)}$$

$$= \frac{P(A) \times P\left(D/A\right)}{P(D)}$$

$$= \frac{0.5 \times 0.03}{0.023}$$

$$= 0.65$$

(11) In a classroom, 20 students were considered as very intelligent, 40 were average and 10 were below average. The probability that a very intelligent student fails in examination with probability is 0.003 and probability for average and below average is 0.04 and 0.4 respectively. If a student is fail in examination then what is the probability that he is below average.

Solⁿ

Given that:-

$P(A) = \frac{20}{70} = \frac{2}{7}$ $\quad P(B) = \frac{40}{70} = \frac{4}{7}$

$P(C) = \frac{10}{70} = \frac{1}{7}$

$P(F/A) = 0.003$ $\quad P(F/B) = 0.04$ $\quad P(F/C) = 0.4$

Then

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$P(F) = P(A \cap F) + P(B \cap F) + P(C \cap F)$
$= P(A) \times P(F/A) + P(B) * P(F/B) + P(C) \times P(F/C)$

$= \frac{2}{7} \times 0.003 + \frac{4}{7} \times 0.04 + \frac{1}{7} \times 0.4$

$= 0.08$

Now,
If the selected student is fail then probability that it was below average.

$P(C/F) = \frac{P(C \cap F)}{P(F)}$

$= \frac{P(C) \times (P(F/C))}{P(F)}$

$= \frac{1/7 \times 0.4}{0.08}$

$= 0.71$,,

(iii) A survey reported that 95% of highly successful product receive good reviews, 60% of moderate successful product receive good review and 10% of poor product receive good review. In addition to this 40% of the product have been highly successful, 35% of have been moderate successful and 25% have been poor ~~successful~~. product. ~~receive~~ ~~good~~ ~~review~~.

a) What is the probability that a product attain good review

b) If a new product attained good reviews then what is the probability that it will be highly successful product.

ans = Sol^n

Given that:-

$P(H) = 0.4$ $P(M) = 0.35$ $P(P) = 0.25$

$P(G/H) = 0.95$ $P(G/M) = 0.6$ $P(G/P) = 0.1$

Then

a) $P(G) = P(H) \times P(G/H) + P(M) * P(G/M) + P(P) \times P(G/P)$

$= 0.4 \times 0.95 + 0.35 \times 0.6 + 0.25 \times 0.1$

$= 0.615$

b) If the selected product attains good review then probability that it was highly successful product

Sol^n

2080 TU Question.

① What is condition

Soln,

$P(E/A) = \frac{1}{10}$ $P(E/B) = \frac{1}{8}$ $P(E/C) = \frac{1}{5}$

$P(A) = \frac{1}{3}$ $P(B) = \frac{1}{3}$ $P(C) = \frac{1}{3}$

Then,

1) $P(E) = P(A) \times P(E/A) + P(B) \times P(E/B) + P(C) \times P(E/C)$

$= \frac{1}{10} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{8} + \frac{1}{3} \times \frac{1}{5}$

$= 0.14$

II) Here,

If the selected road is escaping then probability that it was from A.

$$P(A/E) = \frac{P(A) \times P(E/A)}{P(E)}$$

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$$= \frac{\frac{1}{3} \times \frac{1}{10}}{0.14}$$

$$= 0.23$$

* The probability that Mr. X, Y, Z becoming principal of Texas college are 3 : 2 : 5. The probability that the scholarship program will be introduced if X, Y, Z becoming principal are 0.7, 0.5 and 0.3 respectively. If the scholarship program has been introduced then what is the probability that X is appointed as a principal.

Sol^n

$P(X) = \frac{3}{10}$ $P(Y) = \frac{2}{10} = \frac{1}{5}$ $P(Z) = \frac{5}{10} = \frac{1}{2}$

$P(S/X) = 0.7$ $P(S/Y) = 0.5$ $P(S/Z) = 0.3$

Then

$$P(S) = P(X) \times P(S/X) + P(Y) \times P(S/Y) + P(Z) \times P(S/Z)$$

$$= \frac{3}{10} \times 0.7 + \frac{1}{5} \times 0.5 + \frac{1}{2} \times 0.3$$

$$= 0.46$$

If the scholarship program has been introduced probability that X is appointed as a principal

$$P(X/S) = \frac{P(X) \times P(S/X)}{P(S)}$$

$$= \frac{\frac{3}{10} \times 0.7}{0.46}$$

$$= 0.45$$

* It was observed that any of the laptop is defected with probability 0.15 If the laptop is defected antivirus software report the virus with probability 0.9. And if laptop is not defected antivirus software still generates false alarm and reports the virus with probability 0.2 Suppose the software reports the virus. Then what is the probability that laptop is defected by virus.

Soln

Here common event is reporting the virus Then,

$P(D) = 0.15$ $\qquad$ $P(N) = 0.85 \ (1-0.15)$

$P(R/D) = 0.9$ $\qquad$ $P(R/N) = 0.2$

Then

$$P(R) = P(D) \times P(R/D) + P(N) \times P(R/N)$$

$= 0.15 \times 0.9 + 0.85 \times 0.2$
$= 0.305$

Probability that it was defected by virus:-

$$P(D/R) = \frac{P(D) \times P(R/D)}{P(R)}$$

$$= \frac{0.15 \times 0.9}{0.305}$$

$$= 0.44$$

Probability distribution

Random variable

: The variable which represents all possible outcomes of an experiment is called random variable.

Types:

1) Discrete random variable
11) Continuous random variable

1) Discrete random variable.

: The random variable which represents the integer number only is called discrete random variable.

11) Continuous random variable.

: The random variable which represents the real number is called continuous random variable.

Suppose, three coins are tossed and let x denotes number of head obtained.

X - head	P(x)
3	1/8
2	3/8
1	3/8
0	1/8

Properties of probability distribution $\Sigma P(x) = 1$

① $\Sigma P(x) = 1$

⑪ $P(x) \geq 0$

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It is the probability distribution when 3 coin are tossed.

Types of probability distribution.
(I) Discrete probability distribution.
(II) Continuous probability distribution.

Imp. 10 marks (I) Discrete probability distribution.

* Binomial distribution
* Poisson probability distribution

(a) Binomial
TU Imp: Conditions for using binomial distribution
i) Trial should be independent.
ii) Number of trial should be finite.
iii) There should be only two outcomes. ie. success (p) and failure (q) such that:-
$p+q=1$.
iv) Probability of success in each trial remains constant.

In binomial distribution probability is estimated by using the relation:-
$$P(X=x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}.$$

(Average) Mean of binomial distribution $= n \cdot p$
Variance of binomial distribution $= npq$.
Mean > Variance.

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* Suppose a coin is tossed 10 times then find the probability of getting:
(i) exactly 3 head.
(ii) no heads.
(iii) atmost two head.
(iv) atleast two head.
What is the average number of head when 10 coins are tossed?
Soln

$p = 0.5$
$q = 0.5$
$n = 10$
$X \rightarrow$ NO. of head.

In biomial distribution,
Probability is estimated by using the relation
$P(X=x) = {}^nC_x p^x q^{n-x}$

a) $P(X=3) = {}^{10}C_3 \, p \, (0.5)^3 (0.5)^7$

$= 0.11$

b) $P(X=0) = {}^{10}C_0 (0.5)^0 (0.5)^{10}$
$= 0.00097$

c) $P(X \leq 2) = P(X=2) + P(X=1) + P(X=0)$
$= {}^{10}C_2 (0.5)^2 (0.5)^8 + {}^{10}C_1 (0.5)^1 (0.5)^9 +$
0.00097
$= 0.054$ //

$p(X > 2) = 1 - p(X \leq 2)$

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d) $P(X \geq 2) = 1 - P(X < 2)$

$= 1 - [P(X=1) + P(X=0)]$

$= 1 - [{}^{10}C_1 (0.5)^1 \times (0.5)^9 + {}^{10}C_0 (0.5)^0 \times (0.5)^{10}]$

$= 0.989$

Then

$\text{Average} = n \cdot p$

$= 10 \times \frac{1}{2}$

$= 5$

* A machine produce 20% defective articles. Then find the probability that out of seven samples there will be:

a) no defective articles
b) exactly 2 defective articles.
c) atmost 2 defective articles.
d) atleast 2 defective article

Sol:

Given that:-

$p = 20\% = 0.2$
$q = 0.8$
$n = 7$

In binomial distribution, Probability is estimated using the relation

$P(X = x) = {}^{n}C_{x} p^{x} q^{n-x}$.

a) Here

$P(X = 0) = {}^{7}C_{0}\, p^{0} q^{7-0}$
$= {}^{7}C_{0} (0.2)^{0} (0.8)^{7}$
$= 0.21$

b) Here

$P(x = 2) = {}^{7}C_{2} (0.2)^{2} (0.8)^{5}$
$= 0.27$

c) Here

$P(X \le 2) = P(x = 2) + P(x = 1) + P(x = 0)$
$= 0.27 + {}^{7}C_{1} (0.2)^{1} (0.8)^{6} + 0.21$
$= 0.83$

d) Here

$$P(X \geq 2) = 1 - P(X < 2)$$
$$= 1 + [P(X=1) + P(X=0)]$$
$$= 1 - [0.36 + 0.2]$$
$$= 0.44$$

A software package consist of 15 programs in which 7 must be packaged. Then find the probability that out of 5 programs.

a) None of them
b) exactly two of them
c) less than 3 of them.

Soln

Given that:-

$$p = \frac{7}{15} = 0.46$$
$$q = 0.54$$
$$n = 5$$

X = No. of programs to be upgraded

In ~~biom~~ binomial distribution

$$(P = X) = {}^{n}C_{x}\, p^{x}\, q^{n-x}.$$

a) Here

$$P(X=0) = {}^{5}C_{0}\,(0.46)^{0}\,(0.54)^{5}$$
$$= 0.04$$

b) Here

$$P(X=2) = {}^{5}C_{2}\,(0.46)^{2}\,(0.54)^{3}$$

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$= 0.33$

c) Here

$P(X<3) = P(X=0) + P(X=1) + P(X=2)$

$= 0.04 + {}^5C_1 (0.46)^1 (0.54)^4 + 0.33$

$= 0.04 + 0.195 + 0.33$

$= 0.565$

2080 T.U. Q:8) 11) The probability of a novice archer hitting the target with any shot is 0.3. Given that the archer shoots six arrows find the probability that the target is hit at least once.

Soln

$p = 0.3$

$q = 0.7$

$n = 6$

X = No. of hit

Then

$P(x \geq 1) = 1 - P(X<1)$

$= 1 - P(X=0)$

$= 1 - [{}^6C_0 (0.3)^0 (0.7)^6]$

$= 0.88$

In binomial distibution, an experiment succeed twice as often as it fails. Then find the probability that in next six trial there will be atleast 4 success.

$p = 2q$

$p = 2(1-q)$

or, $p = 2 - pq$

or, $2q + q = 2$

or, $3q = 2$

$\therefore q = \frac{2}{3}$

$p = 2q$ $\quad$ $q = 1 - p$

or, $p = 2(1-p)$ $\quad$ $= 1 - \frac{2}{3}$

or, $p = 2 - 2p$

or, $3p = 2$ $\quad$ $= \frac{1}{3}$

$\therefore p = \frac{2}{3}$

$n = 6.$

$X \Rightarrow$ No. of success.

Then,

$P(X \geq 4) = P(X=4) + P(X=5) + P(X=6)$

$= {}^6C_4 \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^2 + {}^6C_5 \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right)^1$

$+ {}^6C_6 \left(\frac{2}{3}\right)^6 \left(\frac{1}{3}\right)^0$

$= 0.68$

A large chain retailer purchase a certain kind of electronic device from a manufacture. The manufacturer indicates that the defective rate of the device is 15%. The inspector randomly picks 10 items from a shipment

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What is the probability that there will be atleast one defective item among these 10?

Soln

$p = 0.15$

$q = 0.85$

$n = 10$

Then,

$P(X \geq 1) = 1 - P(X < 1)$

$= 1 - P(X = 0)$

$= 1 - [{}^{10}C_0 (0.15)^0 (0.85)^{10}]$

$= 1 - 0.1986$

$= 0.803$

Imp. Sure.

Fittig of Binomial distribution

* Fittigng

: It is the estimation of expected frequency for any experiment.

In biomial distribution, expected frequency is calculated by using the formula,

$f(x) = N \times P(X = x)$

IV

(1) Suppose 5 coins are tossed ~~200~~ 250 times and following result of head were obtained.

$X \to$ no. of head	f
0	20
1	40
2	60
3	80
4	30
5	20

Fit the binomial distribution and find expected frequency.

Sol^n

$p = 0.5$

$q = 0.5$

$n = 5$

$N = 250$

X = No of head.

In binomial distribution expected frequency is estimated by using the relation

$f(x) = N \times P(X = x)$

$= N \times {}^nC_x p^x q^{n-x}$.

When $x = 0$

$f(0) = 250 \times {}^5C_0 (0.5)^0 (0.5)^5$

$= 7.81 \approx 8$

Again

When $x = 1$.

$f(1) = 250 \times {}^5C_1 (0.5)^1 (0.5)^4$

$= 39$.

Again

When $x = 2$

$f(2) = 250 \times {}^5C_2 (0.5)^2 (0.5)^3$

$= 78.12 \approx 78$

Again

When $x = 3$

$f(3) = 250 \times {}^5C_3 (0.5)^3 (0.5)^2$

$= 78.125 \approx 78$

Again

When $x=4$.

$$f(4) = 250 \times {}^5C_4\,(0.5)^4 (0.5)^1$$
$$= 39.0$$

Again

When $x=5$

$$f(5) = 250 \times {}^5C_5\,(0.5)^5 (0.5)^0$$
$$= 7.81 \approx 8$$

$X=x$	f_o	f_e
0	20	8
1	40	39
2	60	78
3	80	78
4	30	39
5	20	8
	250	250

Exmp.

From the following data set, fit the binomial distribution and find expected frequency.

$X=x$	f	fx
6	30	180
5	70	350
4	110	440
3	200	600
2	100	200
1	30	30
0	20	0
	$N=560$	$\Sigma fx = 1800$

Then,

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$$\bar{x} = \frac{\Sigma f x}{N}$$
$$= \frac{1800}{560}$$
$$= 3.21$$

Then,
$$\bar{x} = np$$
or, $\frac{3.21}{6} = p$
$$\therefore p = 0.53$$

Now
$$q = 1 - 0.53$$
$$= 0.47$$

Now,
In binomial distribution, expected frequency is estimated by using the relation.
$$f(x) = N \times P(X = x)$$
$$= N \times {}^{n}C_{x}\, p^{x} q^{n-x}$$

When $x = 6$.
$$f(6) = \cancel{N \times}\ 560 \times {}^{6}C_{6}\, (0.53)^{6} (0.47)^{0}$$
$$= 12.4 \approx 12$$

When $x = 5$
$$f(5) = 560 \times {}^{6}C_{5}\, (0.53)^{5} (0.47)^{1}$$
$$= 66$$

When $x = 4$
$$f(4) = 560 \times {}^{6}C_{4}\, (0.53)^{4} (0.47)^{2}$$
$$= 146.4 \approx 146$$

When $x = 3$
$$f(3) = 560 \times {}^{6}C_{3}\, (0.53)^{3} (0.47)^{3}$$

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$= 173.1 \approx 173$

When $n=2$,

$f(2) = 560 \times {}^6C_2 (0.53)^2 (0.47)^4$

$= 115.1 \approx 116$

When $n=1$

$f(1) = 560 \times {}^6C_1 (0.53)^1 (0.47)^5$

$= 40.84 \approx 41$

When $n=0$

$f(0) = 560 \times {}^6C_0 (0.53)^0 (0.47)^6$

$= 6.03 \approx 6$

$X = n$
6
5
4
3
2
1
0

Imp sur.

Poisson distribution (per unit time then use this method)

Condition for using Poisson distribution

i) Trial should be independent.

ii) No. of trial should be sufficiently large ie. $n \to \infty$

iii) Probability of success should be sufficiently small ie. $p \to 0$.

iv) Average i.e. (λ) = np should be finite.

In Poisson distribution probability is estimated by using the relation

$$P(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

Mean = Variance = λ

① In Kathmandu city average number of minor accident per hour is 4 then find the probability that during next hour there will be:-

a) no accident
b) exactly 3 accident
c) atmost 2 accident
d) atleast 2 accident

Sol^n

Given that:-

$\lambda = 4$.

$X \rightarrow$ No of accident

In Poisson distribution

$$P(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

a) Here

$$P(X=0) = \frac{e^{-4}(4)^{0}}{0!}$$

$= 0.018$

Poisson ma greater than aayo bhane
less than ma garne.

b) Here

$$P(X=3) = \frac{e^{-4}(4)^3}{3!}$$
$$= 0.19$$

c) Here

$$P(X \le 2) = P(X=2) + P(X=1) + P(X=0)$$
$$= \frac{e^{-4}(4)^2}{2!} + \frac{e^{-4}(4)^1}{1!} + 0.018$$
$$= 0.23$$

d) Here

$$P(X \ge 2) = 1 - \{P(X=1) + P(X=0)\}$$
$$= 1 - 0.09$$
$$= 0.90$$

* ~~An office~~ In a certain departmental store on an average 3 customer arrive per minute then find the probability that in ~~an~~ every minute there will be:-

a) no customer
b) 2 or 3 customer
c) atmost 1 customer
d) atleast 2 customer in 2 minutes.

ans Soln

Given that,
$\lambda = 3$
$X \rightarrow$ No. of customer

In Poisson distribution

$$P(X=x) = \frac{e^{-\lambda} \lambda^{x}}{x!}$$

a) Here,

$$P(X=0) = \frac{e^{-3} \, 3^{0}}{0!}$$

$$= 0.049$$

b) Here,

$$P(X=2 \text{ or } X=3) = P(X=2) + P(X=3)$$

$$= \frac{e^{-3} \, 3^{2}}{2!} + \frac{e^{-3} \, 3^{3}}{3!}$$

$$= 0.22 + 0.224$$

$$= 0.45$$

c) Here,

$$P(X \le 1) = P(X=1) + P(X=0)$$

$$= \frac{e^{-3} \, 3^{1}}{1!} + \frac{e^{-3} \, 3^{0}}{0!}$$

$$= 0.198$$

d) Here,

1 min $\longrightarrow$ 3

2 min $\longrightarrow$ $3 \times 2 = 6$

Then

$$P(X \ge 2) = \cancel{P(ex} \; 1 - P(X<2)$$

$$= 1 - [P(X=0) + P(X=1)]$$

$$= 1 - \left[\frac{e^{-6} \, 6^{0}}{0!} + \frac{e^{-6} \, 6^{1}}{1!} \right]$$

$= 0.98$

* In Kathmandu city, average number of network error per day is 4. Then find the probability that in any given day

a) no network error
b) atleast two error
c) atmost one error.

Soln

Given that:-

$\lambda = 4$

$X \longrightarrow$ No. of network error

In Poisson distribution,

$$P(X=x) = \frac{e^{-\lambda} \lambda^{x}}{x!}$$

a) Here,

$$P(X=0) = \frac{e^{-4} \, 4^{0}}{0!}$$

$= 0.01$

b) Here,

$$P(X \geq 2) = 1 - [P(X < 2)]$$

$= 1 - [P(X=1) + P(X=0)]$

$$= 1 - \left[\frac{e^{-4}\, 4^{1}}{1!} + \frac{e^{-4} \times 4^{0}}{0!}\right]$$

$=$

$= 0.91$

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c) Here,

$$P(X \leq 1) = P(X=1) + P(X=0)$$

$$= \frac{e^{-4} \times 4^1}{1!} + 0.01$$

$$= 0.083$$

Poisson approximation to binomial distribution.

* If $p \leq 5\%$ i.e $p \to 0$
$n > 20$ i.e $n \to \infty$

Then we use Poisson distribution instead of binomial distribution.

It seems to be binomial but we use poisson as $p = 4\% \leq 5\%$

① A machine produce 4% defective article. Then find the probability that in a sample of 125 articles there will be:-

a) exactly two defective article.
b) atleast two defective article.

Soln

Given that:-

$p = 4\% = 0.04$

$n = 125$

Then $X \to$ No. of defective articles.

$$\lambda = np$$

$$= 125 \times 0.04$$

$$= 5$$

In poisson distribution,

$$P(X=x) = \frac{e^{-\lambda} \lambda^{x}}{x!}$$

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a) Here,

$$P(X=2) = \frac{e^{-5} 5^2}{2!}$$
$$= 0.084$$

b) Here

$$P(X \geq 2) = 1 - P[X=1 + P(X=0)]$$
$$= 1 - \left[\frac{e^{-5} 5^1}{1!} + \frac{e^{-5} 5^0}{0!}\right]$$
$$= 0.95$$

Imp

Fitting of Poisson distribution.
In Poisson distribution expected frequency is estimated by using the relation.

$$f(x) = N \times P(X=x)$$
$$= N \times \frac{e^{-\lambda} \lambda^x}{x!}$$

* Under what conditions Binomial distribution tends to Poisson distribution, The number of telephone calls received during the month of May is summarized in the following table.
Fit the Poisson distribution.

x	f_0	fx
0	8	0
1	12	12
2	18	36
3	13	39
4	9	36
~~$\Sigma x = 10$~~	N = 60	$\Sigma fx = 123$

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Then

$$x = \frac{\Sigma fx}{N}$$

$$= \frac{123}{60}$$

$$= \cancel{12.3}\ 2.05$$

i.e. $\lambda = \cancel{12.3}\ 2.05$

Now,

$$f(x) = N \times P(X=x)$$

$$= N \times \frac{e^{-\lambda}\lambda^x}{x!}$$

Then when $x=0$:

$$f(0) = \frac{e^{-2.05}\ 2.05^0}{0!} \times 60$$

$$= 8$$

When $x=1$:

$$f(1) = 60 \times \frac{e^{-2.05} \times 2.05^1}{1!}$$

$$= \cancel{10.09}\ 15.83 = 16.$$

When $x=2$.

$$f(2) = 60 \times \frac{e^{-2.05} \times 2.05^2}{2!}$$

$$= 16$$

When $n = 3$.

$$f(3) = 60 \times \frac{e^{-2.05} \times (2.05)^3}{3!}$$

$$= 12$$

When $n = 4$

$$f(4) = 60 \times \frac{e^{-2.05} \times (2.05)^4}{4!}$$

$$= 8$$

$X = n$	f_o	f_e
		8
		16
		16
		12
		8

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(Range or Interval)

TU 100% fixed

Continuous Probability ~~distillation~~ distribution.

① Normal ~~distillation~~ distribution.

$X \rightarrow$ continuous random variable.

$\mu \rightarrow$ mean

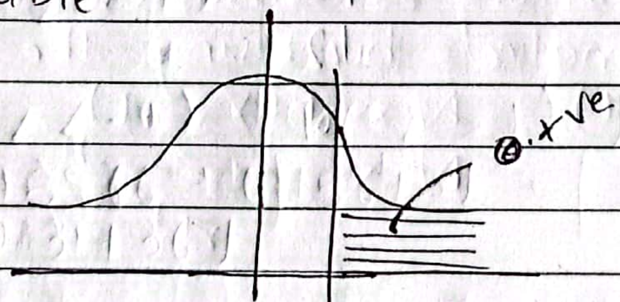
$\sigma \rightarrow$ Standard deviation.

$\left(\frac{X - \mu}{\sigma} \right) = Z$ (Standard Normal variable)

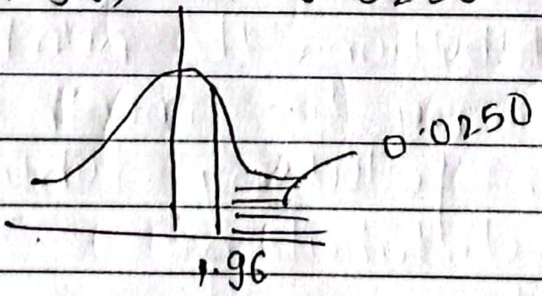
Total probability = 1.

Z^+ or $Z^- = 0.5$

+ve table

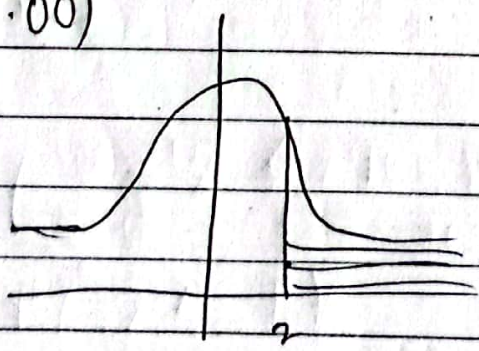


(i) $P(Z > 1.96) = 0.0250$

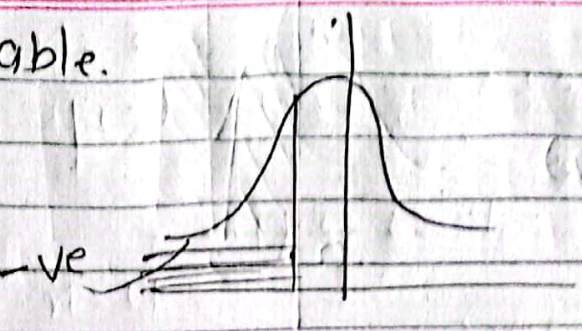


(ii) $P(Z > 2.00)$

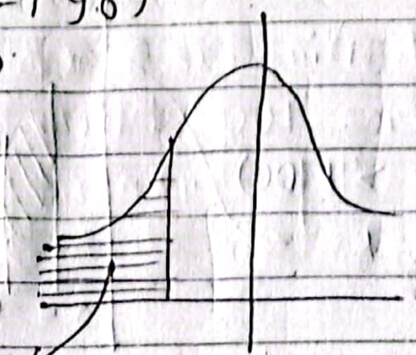
$= 0.0228$



-ve table.



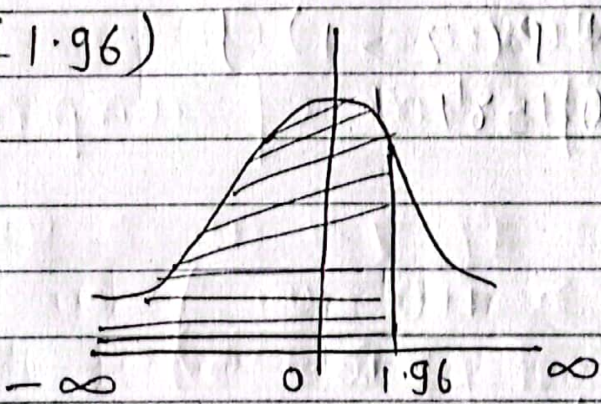
① $P(Z < -1.96)$
$= 0.2500$



0.2500

⑩ Find the following probability:

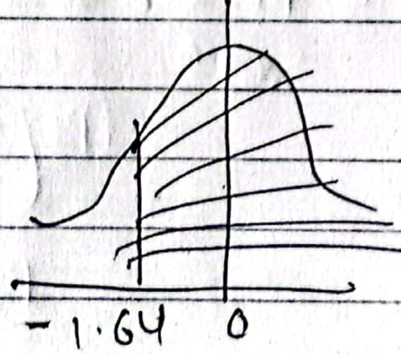
a) $P(Z < 1.96)$



$= 1 - P(Z > 1.96)$
$= 1 - 0.0250$
$= 0.975$

b) $P(Z > -1.64)$
Soln

$= 1 - P(Z < -1.64)$
$= 1 - 0.05050 \quad = 0.949$



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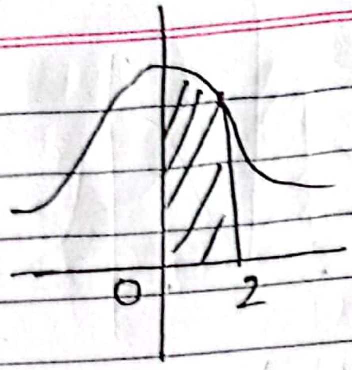
c) $P(0 < z < 2)$.



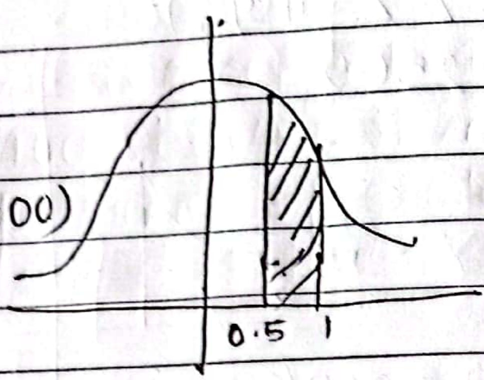
Here

$= 0.5 - P(z > 2)$

$= 0.5 - 0.0228$

$= 0.4772$.

d) $P(0.5 < z < 1.00)$.



Here

$= P(z > 0.5) - P(z > 1.00)$

$= 0.3085 - 0.1587$

$= 0.1498$

e) $P(-1 < z < 1)$

Here

$= 1 - [P(z < -1) + P(z > 1)]$

$= 1 - [0.15866 + 0.1587]$

$= 1 - [0.31736]$

$= 0.6826$

In normal distribution with mean 20 and standard deviation 5. Find the following probability.

a) $P(X > 22)$.

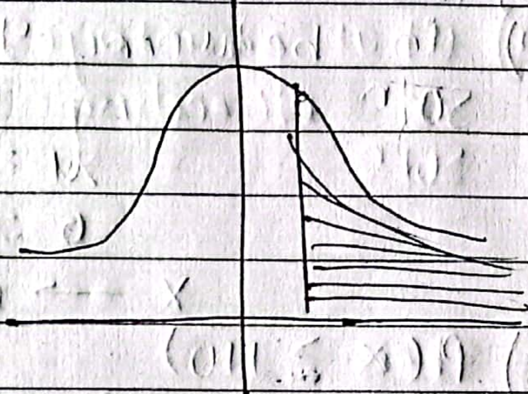
Soln

$= P\left(\frac{X-\mu}{\sigma} > \frac{22-\mu}{\sigma}\right)$

$= P\left(Z > \frac{22-20}{5}\right)$

$= P(Z > 0.4)$

$= 0.3446$.



b) $P(X > 15)$

Soln

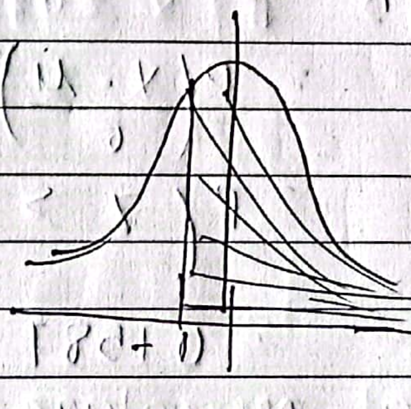
$= P\left(\frac{X-\mu}{\sigma} > \frac{15-20}{5}\right)$

$= P(Z > -1)$

$= 1 - P(Z < -1)$

$= 1 - 0.1587$

$= 0.8413$

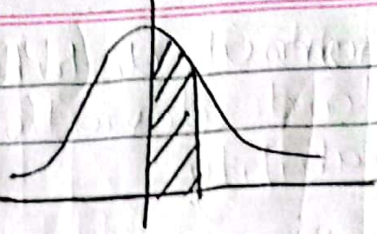


c) $P(20 < X < 30)$

Soln

$= P\left(\frac{20-20}{5} < \frac{X-\mu}{\sigma} < \frac{30-20}{5}\right)$

$\Rightarrow P(0 < Z < 2)$



$= 0.5 - P(Z > 2)$
$= 0.5 - 0.0228$
$= 0.4772$

The lifetime of electronic components is normally distributed with mean 100 hour and $\sigma = 10$ then find the probability that life-time of component is
a) more than 110 hour.
b) less than 90 hour
c) in between 90 to 100 hour.

Soln

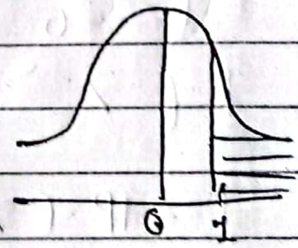
$\mu = 100$
$\sigma = 10$
$X \longrightarrow$ life time of components

a) $P(X \geq 110)$
Soln

$$= P\left(\frac{X-\mu}{\sigma} > \frac{110-100}{10}\right)$$

$$= P(Z > 1)$$

$$= 0.1587$$



b) $P(X < 90)$
Soln

$$= P\left(\frac{X-\mu}{\sigma} < \frac{90-100}{10}\right)$$

$$= P(Z < -1)$$

% = probability × 100

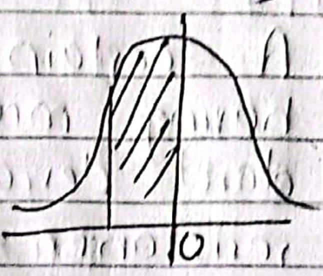
$= 0.1587$

c) ~~in~~ $P(90 < X < 100)$

Soln

$$= P\left(\frac{90-100}{10} < \frac{X-\mu}{\sigma} < \frac{100-100}{10}\right)$$

$$= P(-1 < Z < 0)$$



~~$= 0.15866 + P(Z$~~

~~$= 0.15866 - 0.5$~~

~~$= -0.34134$~~

$$= 0.5 - P(Z < +1)$$
$$= 0.5 - 0.15866$$
$$= 0.34134.$$

In a photographic process, the developing time of prints as a random variable having normal distribution with mean of 18.25 seconds with standard deviation 0.34 second. Find the probability that at least 17.64 seconds to develop one of the prints.

Soln

$\mu = 18.25$

$\sigma = 0.34$

$X \longrightarrow$ Time to print.

Then

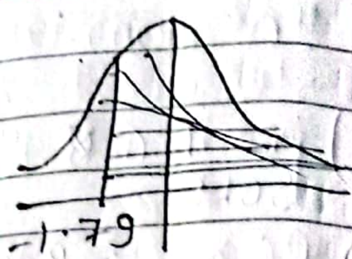
$$P(X \geq 17.64) = P\left(\frac{X-\mu}{\sigma} \geq \frac{17.64 - 18.25}{0.34}\right)$$

$= P(Z \geq -1.794)$

$= 1 - P(Z < -1.8)$

$= 1 - 0.3593$

$= 0.6407$



8) A certain machine makes electrical resistors having mean resistance of 40 ohms and standard deviation of 2 ohms. Assuming that the resistance follows a normal distribution.

1) What percentage of resistors will have a resistance exceeding 43 ohms?

Sol^n

$\mu = 40$

$\sigma = 2.$

Then

$$P(X > 43) = P\left(Z \frac{X-\mu}{\sigma} > \frac{43-40}{2}\right)$$

$= P(Z > 1.5)$

$= 0.0668 \times 100\%$

$= 6.68\%$

II) What percentage of resistors will have a resistance between 30 ohms to 45 ohms?

Sol^n

$$P(30 < X < 45) = P\left(\frac{30-40}{2} < \frac{X-\mu}{\sigma} < \frac{45-40}{2}\right)$$

$= P(-5 < Z < 2.5)$

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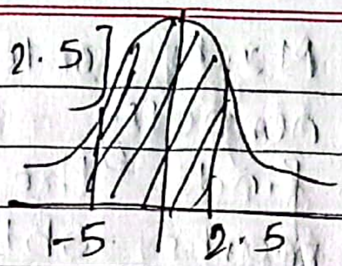
$= 1 - [P(Z < -5) + P(Z > 2.5)]$



$= 1 - [0.0000 + 0.0062]$

$= 0.9938 \times 100\%$

$= 99.38\%$

From a batch of 10,000 the lifetime of laptop batteries has a normal distribution with a mean of 40 months and a standard deviation of 8 months. What is the probability that a laptop selected at random will have life time.

1) Find the number of batteries having the lifetime less than 35.

ans Sol^n =

$= P(X < 35)$

$= P\left(\frac{X-\mu}{\sigma} < \frac{35-40}{8}\right)$

$= P(Z < -0.625)$

$= 0.26763 \times 10000$

$= 2676.3$

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* Marks obtained by the student is distributed normally with mean 60 and S.D 10 then find the probability that:-
1) marks obtained by student is more than 50
ii) % student getting the marks atmost 70
iii) Find the lowest marks of top 15% student
iv) Find the highest marks of lowest 10% student.

Soln

1) $P(X > 50) = P\left(\frac{X-\mu}{\sigma} > \frac{50-60}{10}\right)$

$= P(Z > -1)$

$= 1 - P(Z < -1)$

$= 1 - 0.15866$

$= 0.84134\%$

ii) Here

$P(X \leq 70) = P\left(\frac{X-\mu}{\sigma} < \frac{70-60}{10}\right)$

$= P(Z < 1)$

$= 1 - P(Z > 1)$

$= 1 - 0.1587$

$= 0.8413$

iii) Here,

$\cancel{P(X > 85) = P\left(\frac{X-\mu}{\sigma} > \frac{85-60}{10}\right)}$

$\cancel{= P(Z > 2.5)}$

$\cancel{= 0.0062}$.

c) Here

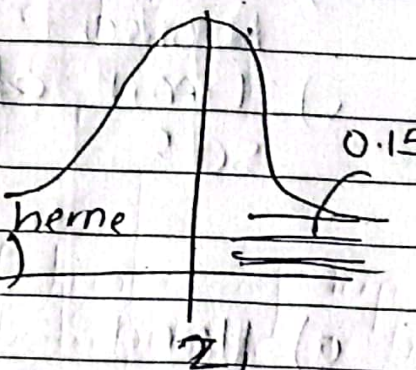
top 15%

x

Then

Probability = 15%

= 0.15



Then 0.15 katana herne

$z_1 = 1.03$ (from table)

Now

$$\frac{x - \mu}{\sigma} = 1.03$$

or, $$\frac{x - 60}{10} = 1.03$$

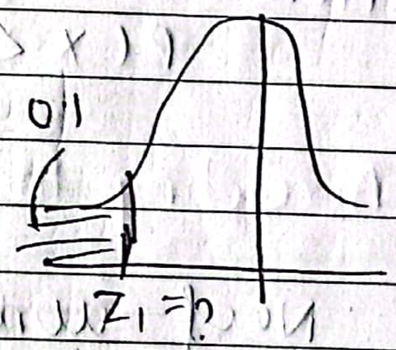
or, $\therefore x = 70.3$

d) Here 10% x

~~600~~

Then

$z_1 = -1.28$



Then

$$\frac{x - 60}{10} = -1.28$$

$\therefore x = 47.2$

Mean & S.D of 10,000 workers are found to be 750 and 50 respectively. Assuming the distribution of income is normal then find:-

) Probability than income of worker is more than 700.

ii) No of worker having income less than 700
iii) ~~Highest~~ Lowest income of top 20% worker.
iv) find the highest income of lowest 25% worker.
v) Limit of income of middle 40% worker.
Soln

$\mu = 750, \quad \sigma = 50$

$X \longrightarrow$ Income of worker

a) Here

$$P(X > 700) = P\left(\frac{X-\mu}{\sigma} > \frac{700-750}{50}\right)$$
$$= P(Z > -1)$$
$$= 1 - P(Z < -1)$$
$$= 1 - 0.15866$$
$$= 0.84134$$

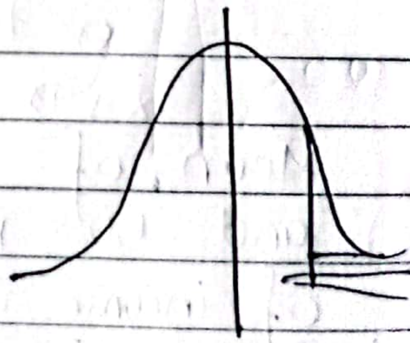
b) Here

$$P(X < 700) = P\left(\frac{X-\mu}{\sigma} < \frac{700-750}{50}\right)$$
$$= P(Z < -1)$$
$$= 0.1587$$

$\therefore$ No of worker $= 0.1587 \times 10{,}000 = 1587$

c)

0.2



Here,

Probability $= 20\%$
$= 0.2$

Then,

Q.2)

$z_1 = 0.84$ (from table).

Now

$$\frac{x-\mu}{\sigma} = z_1$$

or, $$\frac{x-750}{50} = 0.84$$

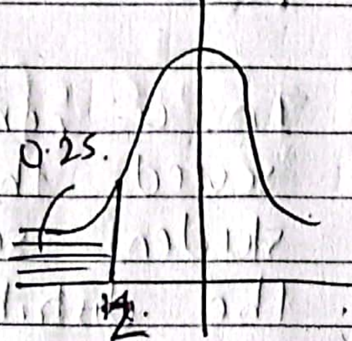
$\therefore x = 792$

iv) Here

25%

z_1

Probability $= \frac{25}{100} = 0.25$



Then

$z_1 = -0.67$

Now

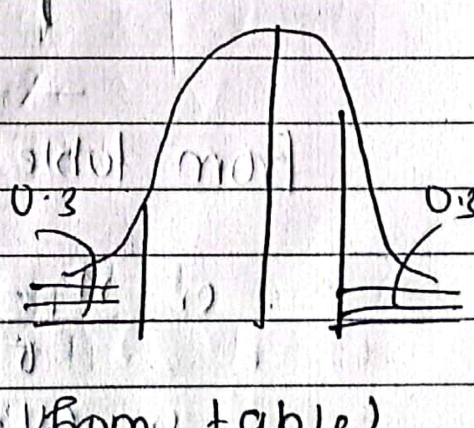
$$\frac{x-750}{50} = -0.67$$

$\therefore x = \boxed{716.5}$

v) Here

30% middle 40% 30%

x_1 (lower) x_2 (upper)



Then

$z_1 = -0.52$ (left 30%) from table

Again

$z_2 = 0.52$ (right 30% from table)

Now

$$\frac{x_1 - \mu_1}{\sigma} = -0.52$$

$$\frac{x_1 - 750}{50} = -0.52$$

$\therefore x_1 = 724$

Again,

$$\frac{x_2 - 750}{50} = 0.52$$

or, $x_2 = 776$

$\therefore$ Limit = 724 – 776 .

* In a certain examination, 8% of students scored the marks above 64 and 31% of student scored the marks below 45. Assuming the distribution of marks is normal then find mean and S.D of distribution.

Here

mean (μ) = ?

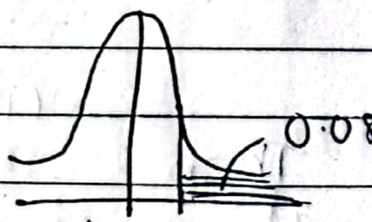
S.D (σ) = ?

$x \rightarrow$ Marks of student.

According to question,

8% = 0.08

64



From table

$z_1 = 1.40$

or, $\frac{x_1 - \mu}{\sigma} = 1.40$

or, $\frac{64 - \mu}{\sigma} = 1.40$

or, $64 = 1.40\sigma + \mu$

Variance = σ

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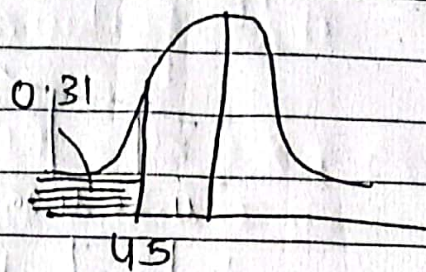
Aggain, from question,

$31\% = 0.31$

45



Then,

$Z_2 = -0.49$

Now,

$\frac{45-\mu}{\sigma} = -0.49$

or, $45 = -0.49\sigma + \mu$.

or, $45 = -0.49\sigma + (64 - 1.40\sigma)$

or, $1.89\sigma = 19$

$\therefore \sigma = 10.05$

Again,

$64 = 1.40 \times 10.05 + \mu$

or, $64 = 14.07 + \mu$

$\therefore \mu = 49.93$

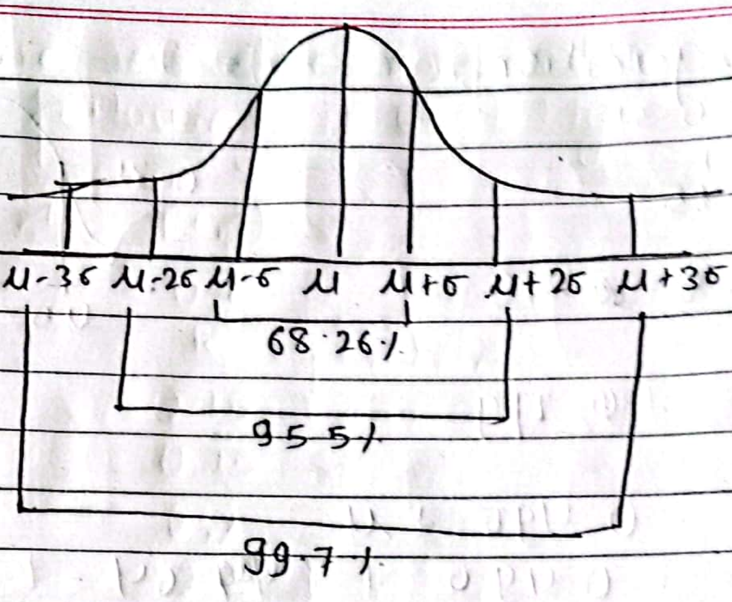
Imp

Characteristics or features or properties of normal distribution

i) It is continuous probability distribution.
ii) It has two parameters i.e. μ and σ^2.
iii) Total probability under the curve is 1.
iv) The curve of normal distribution is bell shape.
v) In normal distribution, mean = median = mode.
vi) Area property: $\mu \pm \sigma = 68.26\%$.

$\mu \pm 2\sigma = 95.5\%$.

$\mu \pm 3\sigma = 99.7\%$



* curve of normal distribution is asymptotic.

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Random variable & Mathematical ~~exception~~ expectation.

* **Probability mass function (p.m.f) (Point probability)**
: Let X be the discrete random variable then probability $(X = x)$ or $p(x)$ is called p.m.f if :-

1) $\sum p(x) = 1$
11) $p(x) \geq 0$

* **Probability density function (p.d.f) (Interval probability)**
: Let X be the continuous random variable then $f(X = x)$ or $f(x)$ is called p.d.f if:

1) $\int f(x)\,dx = 1$
11) $f(x) \geq 0$

Also, $p(a < X < b) = \int_a^b f(x)\,dx$

Cummulative Probability distribution function
: Let ~~cum~~ X be the random variable either discrete or continuous then cumulative distribution ~~frequene~~ function is denoted by $F(x)$ and is given by $F(x) = P(X \leq x)$.

$= \sum^{x} p(x)$ if X is discrete

$= \int_{-\infty}^{x} f(x)\,dx$ if X is continuous

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① Suppose a discrete random variable X have the following probability distribution :-

$X = x$	$P(x)$
0	0.2
1	K
2	0.15
3	0.15
4	0.2
5	2K

a) Determine the value of K.
Here

Since we know that:-

$$\Sigma P(x) = 1$$

or, $0.2 + K + 0.15 + 0.15 + 0.2 + 2K = 1$

or, $0.7 + 3K = 1$

or, $3K = 0.3$

$\therefore K = 0.1$

b) Find the probability : $P(X \geq 3)$.
Here

$P(X = 3) + P(X = 4) + P(X = 5)$

$= 0.15 + 0.2 + 2 \times 0.1$

$= 0.55$

c) $F(2) = P(X \leq 2)$.
Here

$= P(X = 2) + P(X = 1) + P(X = 0)$

$= 0.15 + 0.2 + 0.1$

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$= 0.45$

② Suppose a continuous random variable X has the following pdf

$$f(x) = \begin{cases} kx^2 & 0 \le x \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

a) Determine the value of k.

Here

$$\int_0^1 f(x)dx = 1$$

or, $k \int_0^1 x^2 dx = 1$

or, $k \left[\frac{x^3}{3} \right]_0^1 = 1$

or, $\frac{k}{3}(1^3 - 0^3) = 1$

or, $\frac{k}{3} = 1$

$\therefore k = 3$

$\therefore$ Hence, $f(x) = 3x^2$

b) Find the probability that $P(0.25 < X < 0.5)$

Here,

$$= \int_{0.25}^{0.5} f(x) dx$$

$$= \int_{0.25}^{0.5} 3x^2 dx$$

$$= 3 \left[\frac{x^3}{3} \right]_{0.25}^{0.5}$$

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$= 0.5^3 - 0.25^3$
$= 0.1093$

c) $F(0.3)$ or $F(X=0.3)$
Here

$$= \int_0^{0.3} f(x)\,dx$$

$$= \int_0^{0.3} 3x^2\,dx$$

$$= 3\left[\frac{x^3}{3}\right]_0^{0.3}$$

$$= [0.3^3 - 0^3]$$

$$= 0.027$$

Imp.

Mathematical expectation
: It is the mean of any random variable in terms of probability.
Let X_1, X_2 ~~and~~ X_n be any possible outcomes of an experiment and $P_1, P_2, \ldots, P_n$ be corresponding probability then, expected value is estimated as $E(X) = \Sigma x \cdot P(x)$ if x is discrete.
$E(X^2) = \Sigma x^2 \cdot P(x)$ if X is discrete

If X is continuous then

$$E(x) = \int x f(x)\,dx$$

$$E(x^2) = \int x^2 f(x)\,dx$$

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Also, Variance of X, $V(x) = E(x^2) - [E(x)]^2$

① Suppose a discrete random variable has the following probability distribution :-

$X = x$	$P(x)$	$x \cdot P(x)$	$x^2 \cdot P(x)$
5	0.2	1	5
10	0.3	3	30
15	0.2	3	45
20	0.1	2	40
25	0.15	3.75	93.75
30	0.05	1.5	45
		$\Sigma x \cdot p(x) = 14.25$	$\Sigma x^2 \cdot P(x) = 258.75$

a) Find expected value of X and variance of X. ie $E(X)$ & $V(x)$

b) Also $E(4X+3)$ & $V(4X+3)$.

Here

$$\cancel{E}\ \text{i.e.}\ E(x) = \Sigma x \cdot P(x)$$
$$= 14.25$$

$$V(x) = \Sigma x^2 \cdot P(x) - (E(x))^2$$
$$= 258.75 - (14.25)^2$$
$$= 55.68$$

b) Note:

Properties of mean and variance

(i) $E(5) = 5$ ie. $E(c) = c$. (constant)

(ii) $E(4x) = 4 \times E(X)$

(iii) $V(5) = 0$ ie. $V(c) = 0$

(iv) $V(4x) = 4^2 \times V(X)$ ie. $V(cx) = c^2 \times V(x)$

b) $E(4X+3)$

Here

$$E(4X+3) = 4\,E(X) + E(3)$$
$$= 4 \times 14.25 + 3$$
$$= 60$$

$$V(4X+3) = 4^2 \times V(x) + 0\,V(3)$$
$$= 16 \times 55.68$$
$$= 890.88$$

2079

Q: 2). A coin is tossed two times and if X denotes the number of heads obtained, find (i) $E(x)$ (ii) $E(X^2)$ (iii) $V(x)$ (iv) $E(2X^2+3X-5)$

Here

$S = \{HH, HT, TH, TT\}$

$X =$ No. of head.

$X-x$	$P(x)$	$x \cdot P(x)$	$x^2 P(X)$
0	1/4	0	0
1	1/2	1/2	1/2
2	1/4	1/2	1
		$\Sigma x \cdot P(x) = 1$	$\Sigma x^2 P(x) = 3/2$

Then

$$E(X) = \Sigma X \cdot P(x)$$
$$= 1$$

Again

$$V(x) = \Sigma x^2 P(x) - (\Sigma x \cdot P(x))^2$$
$$= 1.5 - 1$$
$$= 0.5$$

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$E(X^2) = \sum x^2 P(x)$
$= \frac{3}{2}$

$E(2X^2 + 3X - 5) = 2 \times E(X^2) + 3E(X) - E(5$
$= 2 \times 1 - 5 + 3 \times 1 - 5$
$= 1$

Suppose a discrete random variable X has the following probability distribution:-

$X = x$	$P(x)$	$x \cdot P(x)$	$x^2 P(x)$
0	0.05	0	0
1	0.1	0.1	0.1
2	0.15	0.3	0.6
3	0.3	0.9	2.7
4	0.25	1	4
5	0.15	0.75	3.75
		$\sum x \cdot P(x) = 3.05$	$\sum x^2 P(x) = 11.15$

a) find $P(0 \le x \le 2)$
b) $E(X)$ & $V(X)$
c) $E(2X-2)$ & $V(2X+100)$.
Here,

a) Here,
$P(0 \le X \le 2) = P(X=0) + P(X=1) + P(X=2)$
$= 0.05 + 0.1 + 0.15$
$= 0.3$

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b) $E(X) = \sum x \cdot P(x)$
$= 3.05$

Again

$E(X^2) = \sum x^2 \cdot P(X)$
$= 11.15$

Now

$V(X) = \sum x^2 P(X) - (\sum x P(x))^2$
$= 11.15 - (3.05)^2$
$= 1.8475$

c) Here

$E(2X-2) = 2E(X) - E(2)$
$= 2 \times 3.05 - 2$
$= 4.1$

Then

$V(2X-2) = 2V(x) - V(2)$
$= 2 \times 1.8475 - 0$
$= 3.695$

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Suppose a continuous random variable X has the following pdf:

$$f(x) = \begin{cases} 2x & 0 \le x \le 1 \\ 0 & \end{cases}$$

a) Verify the given function is pdf.

Here

Given function is pdf.

$$\int f(x)\,dx = 1$$

or $\int_0^1 2x\,dx = 1$

or $2\left[\frac{x^2}{2}\right]_0^1 = 1$

or $1^2 - 0^2 = 1$

$\therefore 1 = 1.$

Hence the given function is pdf.

b) Find expected value of X & Variance of X

E(x) & V(x).

Here

$$E(x) = \int x \cdot f(x)\,dx$$

$$= \int_0^1 x \cdot 2x\,dx$$

$$= 2\left[\frac{x^3}{3}\right]_0^1$$

$$= \frac{2}{3}\left[1^3 - 0^3\right]$$

$$= \frac{2}{3}$$

$$E(X^2) = \int_0^1 x^2 f(x)\,dx$$

$$= \int_0^1 x^2 \cdot 2x\,dx$$

$$= 2\left[\frac{x^4}{4}\right]_0^1$$

$$= \frac{2}{4}\left[1^4 - 0^4\right]$$

$$= \frac{1}{2}$$

Then

$$V(X) = E(X^2) - (E(X))^2$$

$$= \frac{1}{2} - \left(\frac{2}{3}\right)^2$$

$$= \frac{1}{18}$$

c) Find $E(4X + 2)$ and $V(4x + 2)$.

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* Suppose a continuous random variable x has the following pdf.

$$f(x) = \begin{cases} c(x - x^2) & 0 \le X \le 1 \\ 0 \end{cases}$$

Here Determine the value of c.

$$\int_0^1 f(x)\,dx = 1$$

or, $\int_0^1 c(x - x^2)\,dx = 1$

or. $c\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = 1$

or, $c\left[\frac{1}{2} - \frac{1}{3}\right] = 1$

or, $c\left[\frac{1}{6}\right] = 1$

$\therefore c = 6$

b) $P(0 < X < 0.2)$

Here $\int_0^{0.2} 6(x - x^2)$

$= 6\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^{0.2}$

$= 6\left[\frac{0.2^2}{2} - \frac{0.2^3}{3}\right]$

$= 0.104$

(c) $E(X)$ & $V(x)$.

Here

$$E(X) = \int_0^1 x \cdot f(x)\,dx$$

$$= \int_0^1 x \cdot 6(x - x^2)\,dx$$

$$= 6\int_0^1 x^2 - x^3\,dx$$

$$= 6\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1$$

$$= 6\left[\frac{1}{3} - \frac{1}{4}\right]$$

$$= 0.5$$

Again

$$E(x^2) = \int_0^1 x^2 f(x)\,dx$$

$$= \int_0^1 x^2 \cdot 6(x - x^2)\,dx$$

$$= 6\left[\frac{x^4}{4} - \frac{x^5}{5}\right]_0^1$$

$$= 6\left[\frac{1}{4} - \frac{1}{5}\right]$$

$$= 0.3$$

Then

$$V(x) = E(X^2) - (E(X))^2$$

$$= 0.3 - (0.5)^2$$

$$= 0.05$$

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d) $E(4X+2)$ & $V(2X+2)$.
Sol^n

$$E(4X+2) = 4E(X) + E(2)$$
$$= 4 \times 0.5 + 2$$
$$= 4$$

Again,

$$V(2X+2) = 2V(X) + V(2)$$
$$= 2 \times 0.05 + 0$$
$$= 0.1$$

Suppose a continuous random variable X represents length of telephone calls with following pdf :-

$$f(x) = \begin{cases} \frac{1}{5} e^{-x/5}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

a) Then, find the probability that length of a telephone call is :-
i) less than 5 minutes.
ii) More than 10 minutes.
iii) In between 5 to 10 minutes.

Here,

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i) $P(X < 5)$
Here

$$= \int_0^5 f(x)\,dx.$$

$$= \int_0^5 \frac{1}{5} e^{-x/5} dx$$

$$= \frac{1}{5} \int_0^5 e^{-x/5} dx.$$

$$= \frac{1}{5} \cdot \left[\frac{e^{-x/5}}{-1/5} \right]_0^5$$

$$= -1 \left[e^{-5/5} - e^{-0} \right]$$

$$= e^{-0} - e^{-1}$$

$$= 1 - \frac{1}{e}$$

$$= 0.632.$$

ii) $P(X > 10)$
Here

$$= \int_{10}^{\infty} f(x)\,dx$$

$$= \frac{1}{5} \int_{10}^{\infty} e^{-x/5} dx$$

$$= \frac{1}{5} \left[\frac{e^{-x/5}}{-1/5} \right]_{10}^{\infty}$$

$$= -1 \left[e^{-\infty} - e^{-2} \right]$$

$$=$$

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The number of hardware failure with their corresponding probability.

X = No of failure	P(X)	$x \cdot p(n)$	$x^2 \cdot p(n)$
0	0.1	0	0
1	0.2	0.2	0.2
2	0.3	0.6	1.2
3	0.15	0.45	1.35
4	0.1	0.4	1.6
5	0.05	0.25	1.25
		$\Sigma x \cdot P(n) = 1.9$	$\Sigma x^2 \cdot P(x) = 5.6$

Determine mean and variance number of failure. E(X) and V(X).

Here

$$E(X) = \Sigma X \cdot P(X)$$
$$= 1.9$$

Again,

$$E(X^2) = \Sigma X^2 \cdot P(X)$$
$$= 5.6$$

Then

$$V(X) = E(X^2) - (E(X))^2$$
$$= 5.6 - (1.9)^2$$
$$= 1.99$$

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Theory inviber sur.

Joint Probability distribution

X \ Y	0	1	Total
0	0.2	0.4	0.6
1	0.15	0.25	0.4
	0.35	0.65	1.

Joint Probability

: Simultaneous probability of two variable is called joint probability.

Example:-

$P(X=0, Y=0) = 0.2$

$P(X=0, Y=1) = 0.4$

$P(X=1, Y=0) = 0.15$

$P(X=1, Y=1) = 0.25$

sure 100%

Marginal probability

: The probability of single variable is called marginal probability.

Example:

$P(X=0) = P(X=0, Y=0) + P(X=0, Y=1)$

$= 0.2 + 0.4$

$= 0.6$

$P(X=1) = P(X=1, Y=0) + P(X=1, Y=1)$

$= 0.15 + 0.25$

$= 0.4$

$P(Y=0) = 0.35$

$P(Y=1) = 0.65$

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① Suppose two discrete random variable x and y have the following joint probability distribution.

X\Y	1	2	3	
5	0.1	0.1	0.05	0.25
10	0.2	0.1	0.05	0.35
15	0.1	0.2	0.1	0.4.
	0.4	0.4	0.2	

Find the following probability.

a) Marginal probability of X and Y.

Here

Marginal probability of X.

X : 5 10 15

P(X) : 0.25 0.35 0.4

Marginal probability of Y.

Y: 1 2 3

P(Y): 0.4 0.4 0.2

b) $P(X=5), P(Y=1)$

Here

$P(X=5\ Y=1) = 0.1$

c) $P(X=5\ Y\le 2)$

Here

$= P(X=5\ Y=1) + P(X=5\ Y=2) = 0.1 + 0.1$

$= 0.2.$

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d) $P(X=5/Y=1)$
Here

$$P(X=5 / Y=1) = \frac{P(X=5 \quad Y=1)}{P(Y=1)}$$

$$= \frac{0.1}{0.4}$$

$$= 0.25$$

e) $P(X=10 / Y<2)$
Here,

$$P(X=10 / Y\leq 2) = \frac{P(X=20, \; Y=1)}{P(Y=1)} + \frac{P(X=10, Y=2)}{P(Y=2)}$$

$$= \frac{0.2}{0.4} + \frac{0.1}{0.4}$$

$$= 0.75$$

f) $E(X)$ & $V(X)$
Here

$$E(X) = \sum x \cdot p(x)$$
$$= 5 \times 0.25 + 10 \times 0.35$$
$$+ 15 \times 0.4$$
$$= 10.75$$

X	P(x)
5	0.25
10	0.35
15	0.4

$$E(X^2) = \sum x^2 \cdot P(x)$$
$$= 5^2 \times 0.25 + 10^2 \times 0.35 + 15^2 \times 0.4$$
$$= 131.25$$

$$V(x) = E(X^2) - (E(X))^2$$
$$= 131.25 - (10.75)^2$$

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$= 15.68$

g) $E(Y)$ & $V(Y)$

Here

$E(Y) = \Sigma Y.P(Y)$
$= 1 \times 0.4 + 2 \times 0.4 + 3 \times 0.2$
$= 1.8$

Y	P(Y)
1	0.4
2	0.4
3	0.2

$E(Y^2) = \Sigma Y^2 P(Y)$
$= 1^2 \times 0.4 + 2^2 \times 0.4 + 3^2 \times 0.2$
$= 3.8$

$V(Y) = E(Y^2) - (E(Y))^2$
$= 3.8 - (1.8)^2$
$= 0.56$

COV (X, Y)

: Simultaneous ~~probability~~ variance of two variable is called covariance.

$COV(X, Y) = E(XY) - E(X).E(Y)$

h) COV (X, Y)

Here

$E(XY) = \Sigma\Sigma XY\, P(X,Y)$
$= 5 \times 1 \times 0.1 + 5 \times 2 \times 0.1 + 5 \times 3 \times 0.05 + 10 \times 1 \times 0.2 + 10 \times 2 \times 0.1 + 10 \times 3 \times 0.05 + 15 \times 1 \times 0.1 + 15 \times 2 \times 0.2 + 15 \times 3 \times 0.1$

$= 19.75$

Then,

$$\text{cov}(X,Y) = E(XY) - E(X).E(Y)$$
$$= 19.75 - 10.75 \times 1.8$$
$$= 0.4$$

3)

Soln

X/y	20	30	40	50
4	0.01	0.03	0.05	0.02
5	0.03	0.1	0.08	0.04
6	0.02	0.08	0.12	0.11
7	0.02	0.04	0.07	0.18

a) Fi

Here,

Marginal probability of X.

X	4	5	6	7

c) Are X and Y independent.

ans Here,

$$P(X, Y) = P(X) \times P(Y)$$

$$P(X=4, Y=20) = P(4) \times P(20)$$

or, $0.01 = 0.01 + 0.03 + 0.05 + 0.02 \times$
$0.01 + 0.03 + 0.02 + 0.02$

or, $0.01 = 0.11 \times 0.08$

$\therefore 0.01 \neq 0.0088$.

$\therefore$ X and Y aren't independent.

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Suppose two continuous random variable x and y have the following joint pdf.

$$f(x,y) = \begin{cases} kxy & 0 < x < 1 \\ & 0 < y < 1 \\ 0 & , \text{otherwise} \end{cases}$$

a) Determine the value of k,
Here,

Since we know that,

$$\iint f(x,y)\, dy\, dx = 1.$$

$$\text{or, } \int_0^1 \int_0^1 kxy = 1$$

$$\text{or, } k \int_0^1 x \left[\frac{y^2}{2}\right]_0^1 = 1$$

$$\text{or, } k \int_0^1 \frac{x}{2} = 1$$

$$\text{or, } \frac{k}{2} \left[\frac{x^2}{2}\right]_0^1 = 1$$

$$\text{or, } \frac{k}{2} \times \frac{1}{2} = 1$$

$$\text{or, } \frac{k}{4} = 1$$

$$\therefore k = 4$$

imp sure.

b) Marginal pdf of X & Y.
Here

$$f(x) = \int_y f(x,y)\, dy$$

$$\text{or, } f(x) = \int_0^1 4xy\, dy$$

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$$= 4x \left[\frac{y^2}{2}\right]_0^1$$

$$= \frac{4x}{2}\left[1^2 - 0^2\right]$$

$$= 2x.$$

Illy,

$$f(y) = \int_x f(x,y)\,dx.$$

$$= \int_0^1 4xy\,dx$$

$$= 4y\left[\frac{x^2}{2}\right]_0^1$$

$$= \frac{4y}{2}\left[1^2 - 0^2\right]$$

$$= 2y.$$

99.99%

c) Conditional pdf of x and y.

Here,

Conditional pdf of X for given Y:-

$$f(x/y) = \frac{f(x,y)}{f(y)}$$

$$= \frac{4xy}{2y}$$

$$= 2x.$$

Conditional pdf of Y for given X,

$$f(Y/X) = \frac{f(x,y)}{f(x)}$$

$$= \frac{4xy}{2x}$$

$= 2y.$

d) Are X and Y independent.

ans: Here

$$f(x,y) = f(x) \times f(y)$$

or, $4xy = 2x \times 2y$

$\therefore 4xy = 4xy.$

$\therefore$ X and Y are independent.

2075 5) If two random variable have the joint probability density function.

$$f(x,y) = \begin{cases} k(2x+3y), & \text{for } 0 \le x \le 1,\ 0 \le y \le 1. \\ 0, & \text{otherwise.} \end{cases}$$

1) Find constant k.

Here

Since, we know that

$$\int_0^1 \int_0^1 k(2x+3y)\,dy\,dx = 1.$$

or, $k \int_0^1 \left(2x + \left[\frac{3y^2}{2}\right]_0^1\right) dx = 1$

or, $\frac{k}{1}\left[\frac{3}{2} + \frac{2x^2}{2}\right]_0^1 = 1$

or, $\frac{k}{1}\left[\frac{3}{2} + 1\right] = 1$

or, $\frac{5k}{2} = 1$ $\therefore k = \frac{2}{5}.$

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ii) Marginal pdf of X & Y.
Here

$$f(x) = \int_y \frac{2}{5}(2x+3y)\,dy$$

$$= \int_0^1 \frac{2}{5}(2x+3y)\,dy$$

$$= \frac{2}{5}\int_0 \left(2x + \frac{3y^2}{2}\right)_0^1 dy$$

$$\text{or } \frac{2}{5}\left(\frac{4x+3}{2}\right)$$

$$= \frac{4x+3}{5}$$

$$f(y) = \int_x \frac{2}{5}(2x+3y)\,dx.$$

$$= \frac{2}{5}\int_0^1 (2x+3y)\,dx$$

$$= \frac{2}{5}\left[\frac{2x^2}{2} + 3y\right]_0^1$$

$$= \frac{2}{5}\left(\left[\frac{x^2}{1}\right]_0^1 + 3y\right.$$

$$= \frac{2}{5}\left[1+3y\right)$$

$$= \frac{2+6y}{5}$$

iii) Conditional pdf of X.
Here

$$f(x/y) = \frac{f(x,y)}{f(y)}$$

$$= \frac{\frac{4x+6y}{\cancel{5}}}{\frac{2+6y}{\cancel{5}}}$$

$$= \frac{2(2x+3y)}{2(1+6y)}$$

$$= \frac{2x+3y}{1+6y}.$$

$$f(y/x) = \frac{f(x,y)}{f(x)}$$

$$= \frac{\frac{4x+6y}{\cancel{5}}}{\frac{4x+3}{\cancel{5}}}$$

$$= \frac{4x+6y}{4x+3}$$

iv) X and Y are independent.

Here

$$f(x,y) = f(x).\ f(y)$$

or $4x+6y \neq \frac{4x+3}{5} \cdot \frac{2+6y}{5}$

$\therefore$ X are Y are not independent.

2075

$$f(x,y) = \begin{cases} e^{-(x+y)} & x \geq 0,\ y \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

(i) Verify the given function is joint pdf.

Here,

Given function is joint pdf is:-

$$\int_x \int_y f(x,y)\,dy\,dx = 1$$

$$\text{or, } \int_0^{\infty}\int_0^{\infty} e^{-(x+y)}\,dy\,dx = 1$$

$$\text{or, } \int_0^{\infty} e^{-x}\left[\frac{e^{-y}}{-1}\right]_0^{\infty} dx = 1$$

$$\text{or, } -1\int_0^{\infty} e^{-x}\,dx\left[e^{-\infty} - e^{0}\right] = 1$$

$$\text{or, } (-1)\left[0-1\right]\left[\frac{e^{-\infty} - e^{0}}{-1}\right] = 1$$

$$\text{or, } (-1)(-1)(-1)(0-1) = 1$$

$$\text{or, } (-1)(-1) = 1$$

$$\therefore 1 = 1 \quad \text{proved.}$$

$\therefore$ Hence the given function is joint pdf.

(ii) Marginal pdf of X & Y.

Here,

$$f(x) = \int_0^{\infty} e^{-(x+y)}\,dy$$

$$= e^{-x}\int_0^{\infty} e^{-y}\,dy$$

$$= e^{-x}\left[\frac{e^{-y}}{-1}\right]_0^{\infty}$$

$= e^{-x}(-1)\ [e^{-\infty} + e^{0}]$
$= e^{-x}\ (-1)\ (0-1)$
$= e^{-x}$

Again,

$$f(y) = \int_0^\infty e^{-(x+y)}\,dx$$
$$= \int_0^\infty e^{-x} \cdot e^{-y}\,dx.$$
$$= e^{-y}\left[\frac{e^{-x}}{-1}\right]_0^\infty$$
$$= e^{-y}\,(-1)\,[e^{-\infty} - e^{-0}]$$
$$= e^{-y}\,(-1)\,(-1)$$
$$= e^{-y}.$$

~~Then~~

(iii) ~~Probability~~ ~~distribution~~ of conditional pdf of X and Y.

Here,

$$f(X/Y) = \frac{f(X,Y)}{f(Y)}$$
$$= \frac{e^{-(x+y)}}{e^{-y}}$$
$$= \frac{e^{-x}\,e^{-y}}{e^{-y}}$$
$$= e^{-x}$$

d) Obtain cummulation distribution function.
Here

$$F(x,y) = P_x(X \le x,\ Y \le y)$$

$$= \int_0^x \int_0^y f(x,y)\, dy\, dx$$

$$= \int_0^x \int_0^y e^{-(x+y)}\, dy\, dx$$

$$= \int_0^x \int_0^y e^{-x} \cdot e^{-y}\, dy\, dx$$

$$= \int_0^x e^{-x} \left[\frac{e^{-y}}{(-1)} \right]_0^y dx$$

$$= \int_0^x e^{-x} \left[\frac{e^{-y} - e^{-0}}{(-1)} \right] dx$$

$$= (-1)(e^{-y} - 1) \int_0^x (e^{-x})\, dx$$

$$= (-1)(e^{-y} - 1) \left[\frac{e^{-x}}{-1} \right]_0^x$$

$$= (-1)(-1)(e^{-y} - 1)(e^{-x} - 1)$$

$$= (e^{-y} - 1)(e^{-x} - 1)$$

$$= (1 - e^{-x})(1 - e^{-y}).$$

e) Suppose two continuous random variable X and Y have the following joint pdf.

$$f(x,y) = \begin{cases} 4xy & 0 < x < 1 \\ & 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

Here,

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a) Marginal of X and Y.

Here,

$$f(X) = \int_0^1 4xy\,dy$$

$$= 4x\left[\frac{y^2}{2}\right]_0^1$$

$$= \frac{4x}{2}$$

$$= 2x.$$

$$f(Y) = \int_0^1 4xy\,dx.$$

$$= 4y\left[\frac{x^2}{2}\right]_0^1$$

$$= \frac{4y}{2}$$

$$= 2y$$

b) Find the probability that:- $P(X \le 0.2, Y \le 0.2)$.

Here,

$$P(X, Y) = \int_0^{0.2}\int_0^{0.2} 4xy\,dy\,dx.$$

$$= \int_0^{0.2} 4x\left[\frac{y^2}{2}\right]_0^{0.2} dx$$

$$= \int_0^{0.2} \frac{4x}{2}\left[0.2^2 - 0^2\right] dx$$

$$= 2\int_0^{0.2} x\,(0.04)\,dx$$

$$= 2 \times 0.04 \left[\frac{x^2}{2}\right]_0^{0.2}$$

$= 0.04 \times 0.04$
$= 0.0016.$

c) Obtain cumulative distribution function of X & Y
Here

$$F(x,y) = P(X \le x, Y \le y)$$
$$= \int_0^x \int_0^y f(x,y)\, dy\, dx.$$
$$= \int_0^x \int_0^y 4xy\, dy\, dx$$
$$= \int_0^x 4x \left[\frac{y^2}{2}\right]_0^y dx$$
$$= \int_0^x 2x \left[y^2 - 0\right] dx$$
$$= 2y^2 \left[\frac{x^2}{2}\right]_0^x$$
$$= y^2 \left[x^2 - 0^2\right]$$
$$= x^2 y^2.$$

d) $P(X \le 0.2 / Y \le 0.2)$

Here

$$= \frac{P(X \le 0.2, Y \le 0.2)}{P(Y \le 0.2)}$$
$$= \frac{0.0016}{0.04}$$
$$= 0.04$$

Then

$$P(Y \le 0.2) = \int_0^{0.2} 2y\,dy$$

$$= 2\left[\frac{y^2}{2}\right]_0^{0.2}$$

$$= (0.2)^2 - 0^2$$

$$= 0.04$$

e) $E(X)$ & $V(X)$.

Here

$$E(X) = \int x f(x)\,dx$$

$$= \int_0^1 x \cdot 2x\,dx$$

$$= \int_0^1 2x^3\,dx$$

$$= 2\left[\frac{x^3}{3}\right]_0^1$$

$$= \frac{2}{3}$$

Again,

$$E(X^2) = \int_0^1 x^2 f(x)\,dx$$

$$= \int_0^1 x^2\, 2x\,dx$$

$$= 2\left[\frac{x^4}{4}\right]_0^1$$

$$= \frac{\cancel{2} \times 1}{\cancel{4}\,2}$$

$$= \frac{1}{2}$$

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$$V(X) = E(X^2) - (E(X))^2$$
$$= \frac{1}{2} - \left(\frac{2}{3}\right)^2$$
$$= \frac{1}{18}$$

f) $E(\cancel{X}Y)$ & $V(Y)$

Here

$$E(Y) = \int_0^1 y \cdot 2y \, dy$$
$$= 2\left[\frac{y^3}{3}\right]_0^1$$
$$= \frac{2}{3}.$$

g) $\mathrm{cov}(X, Y)$

Here

$$\mathrm{cov}(X, Y) = E(XY) - E(X) \cdot E(Y)$$

Then

$$E(XY) = \int_0^1 \int_0^1 xy \, f(x, y) \, dy \, dx$$
$$= \int_0^1 \int_0^1 xy \, (4x^2y^2) \, dy \, dx$$
$$= 4\int_0^1 \int_0^1 x^3 y^3 \, dy \, dx$$
$$= 4\int_0^1 x^3 \left[\frac{y^4}{4}\right]_0^1 dx$$
$$= 4\int_0^1 x^3 \left[\frac{1}{4}\right] dx$$

$\sigma_x = \sqrt{var(x)}$ $\sigma_y = \sqrt{v(y)}$.

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$= \frac{4}{3} \left[\frac{x^3}{3} \right]_0^1$

$= \frac{4}{3} \times \frac{1}{3}$

$= \frac{4}{9}$

Then

$Cov(x, y) = \frac{4}{9} - \frac{2}{3} \times \frac{2}{3}$

$= \frac{4}{9} - \frac{4}{9}$

$= 0$

Notes:-

$$r = \frac{n\Sigma XY - \Sigma X \Sigma Y}{\sqrt{n\Sigma x^2 - (\Sigma x)^2} \cdot \sqrt{n\Sigma y^2 - (\Sigma y)^2}}$$

corelation coefficient,

$$r = \frac{Cov(x, y)}{\sigma_x \cdot \sigma_y}$$

h) Here,

$$r = \frac{Cov(x, y)}{\sigma_x \cdot \sigma_y}$$

$= \frac{0}{\quad}$

$= 0$

$\therefore$ There is no relation with corelation coefficient

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A Negative exponential distribution.

Continuous probability distribution
(i) Normal distribution
(ii) Exponential distribution
(iii) Gamma distribution.

Exponential distribution

* A continuous random variable x is said to follow exponential distribution with parameter (λ) if its pdf is given by:-

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0. \end{cases}$$

Here,

Mean $E(x) = 1/\lambda$

Variance $V(x) = 1/\lambda^2$

T.U.

(1) The operating time of certain device follows exponential distribution with parameter $\lambda = \frac{1}{50}$ then find the probability that the operating time of device is more than 100 hours in between 50 to 100 hours. Find mean and variance of lifetime of device.

Here,

$$\lambda = \frac{1}{50}$$

$X \rightarrow$ Operating time

$$f(x) = \lambda e^{-\lambda x}$$

Then,

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a) $P(X > 100) = 1 - P(X \le 100)$ (in calculator)

Here,

$$P(X > 100) = 1 - P(X \le 100)$$
$$= 1 - \int_0^{100} f(x)\,dx$$
$$= 1 - \lambda \int_0^{100} e^{-\lambda x}\,dx$$
$$= 1 - \frac{1}{50}\left[\frac{e^{-x/50}}{\left(-\frac{1}{50}\right)}\right]_0^{100}$$
$$= 1 + \left[e^{-100/50} - e^{-0}\right]$$
$$= \cancel{1} + e^{-2} - \cancel{1}$$
$$= e^{-2}$$
$$= 0.12$$

b) $P(50 < X < 100)$

Here,

$$P(50 < X < 100) = \int_{50}^{100} f(x)\,dx$$
$$= \lambda \int_{50}^{100} e^{-\lambda x}\,dx$$
$$= \frac{1}{50}\left[\frac{e^{-x/50}}{\left(-\frac{1}{50}\right)}\right]_{50}^{100}$$
$$= -\left(e^{-100/50} - e^{-50/50}\right)$$
$$= -e^{-2} + e^{-1}$$
$$= e^{-1} - e^{-2}$$
$$= 0.232$$

$S.D = \sqrt{V(X)}$

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c) $E(X)$

Here,

$$E(X) = \frac{1}{\lambda}$$

$$= \frac{1}{\frac{1}{50}}$$

$$= 50$$

d) $V(X)$

Here

$$V(X) = \frac{1}{\lambda^2}$$

$$= \frac{1}{\left(\frac{1}{50}\right)^2}$$

$$= 2500$$

Gamma distribution

* A continuous random variable x is said to follow gamma distribution with parameter alpha and beta if its pdf is given by:-

$$f(x) = \begin{cases} \frac{1}{\beta^\alpha \gamma\alpha} x^{\alpha-1} e^{-x/\beta}, & x \geq 0 \\ \text{otherwise}, 0 \end{cases}$$

$$E(X) = \alpha \cdot \beta$$

$$V(X) = \alpha \beta^2.$$

$\gamma n = (n-1)!$

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(11) In a certain city, daily consumption of electric power follows gamma distribution with parameter $\alpha = 3$ and $\beta = 2$. If the power plan of this city has daily capacity of 12 millions kwatt. Then, what is the probability that power supply will be insufficient in any given day.

Here

Given that,

$\alpha = 3$,

$\beta = 2$.

$X \rightarrow$ Electric power consumption

$$f(x) = \frac{1}{\beta^{\alpha}\gamma\alpha} x^{\alpha-1} e^{-x/\beta}$$

Then,

$$f(x) = \frac{1}{2^3 \gamma 3} x^2 e^{-x/2}$$

$$= \frac{1}{2^3 (3-1)!} x^2 . e^{-x/2}$$

$$= \frac{1}{16} x^2 e^{-x/2}.$$

Now,

a) $P(X > 12)$

$$= 1 - P(X < 12)$$

$$= 1 - \int_0^{12} f(x)dx$$

$$= 1 - \frac{1}{16}\int_0^{12} x^2 e^{-x/2} dx$$

$$= 0.06$$