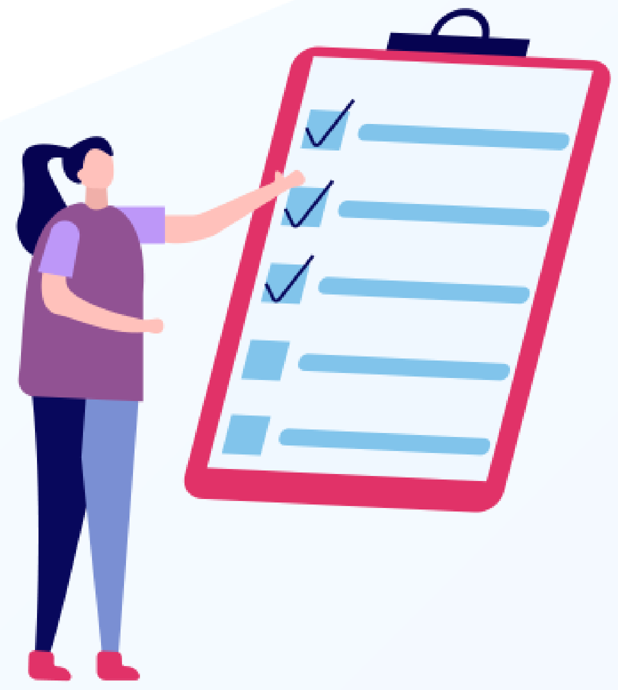
Statistics Essentials for Data Science

Probability

Learning Objectives

By the end of this lesson, you will be able to:

- Understand the concepts of probability
- Comprehend probability and its importance
- Explain the key terms in probability and conditional probability
- Identify the independent and dependent events
- Explain the addition and multiplication theorems of probability
- Understand the Bayes' theorem



Business Scenario

ABC manufactures light bulbs and contemplates adding a new plant in a different state. This expansion holds the potential for increased product output and higher profits.

However, this potential growth also brings risks. The new plant might fail to generate sufficient profits to cover expansion and operational costs. Does this expansion constitute a wise business move? What is the likelihood that it will succeed?

Introduction to Probability and Its Importance

Discussion: Probability

Duration: 15 minutes

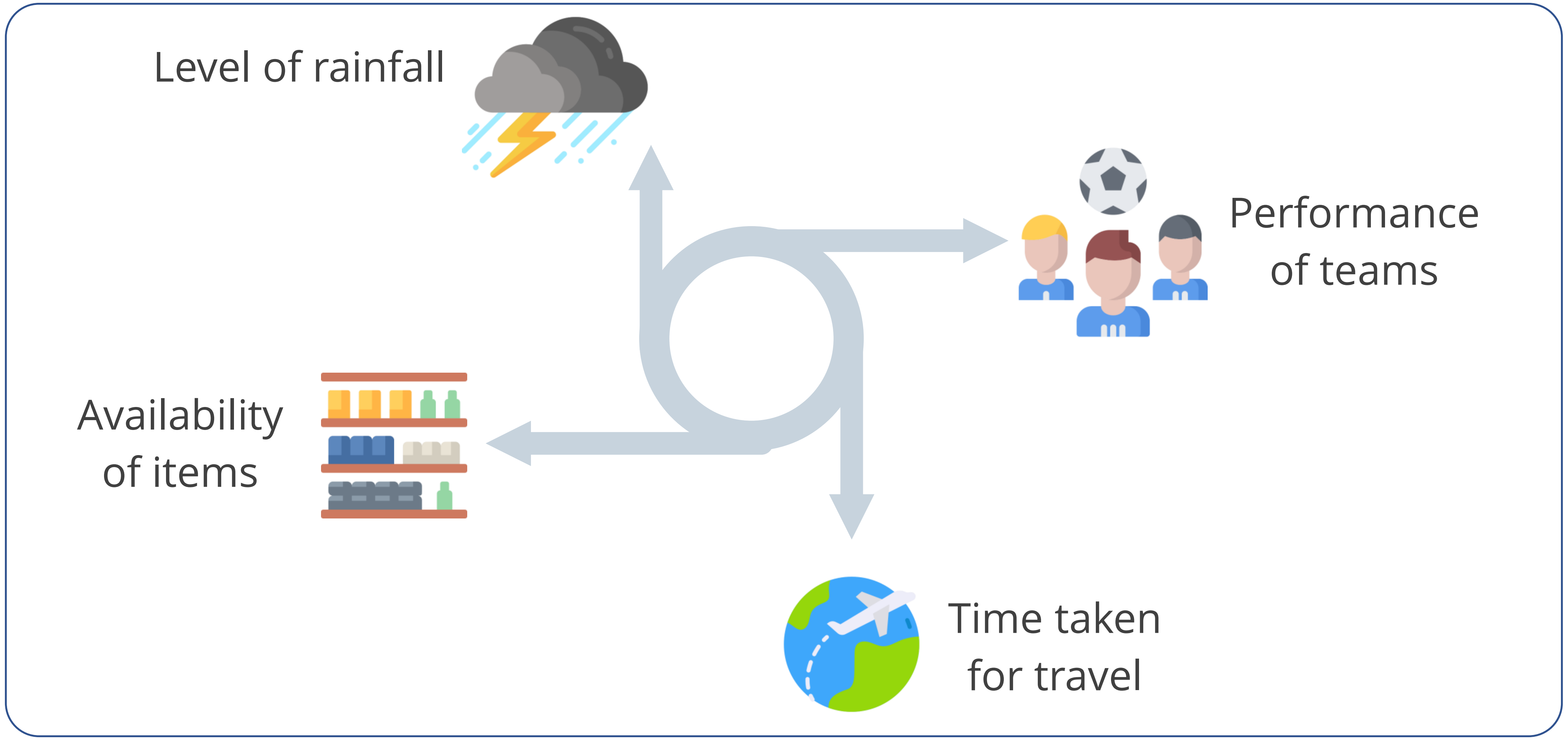
What does probability mean?

- How does probability play a role in decision-making?
- What are the key terms of probability?

Introduction to Probability

A probability is the chance or likelihood of an event occurring.

Examples:



Introduction to Probability

Probability is defined as the ratio of favorable outcomes to the total number of possible outcomes.

Mathematically, it is expressed as:

$$Probability = \frac{Favorable\ outcomes}{Total\ number\ of\ possible\ outcomes}$$

Example: Two coins are tossed simultaneously; what is the probability of getting two heads?
Favorable outcomes: {HH}
Total number of possible outcomes: Ω = {HT, TH, TT, HH}

Introduction to Probability

A probability space is a mathematical construct used to model random experiments or processes.

It consists of three components:

Sample space (Ω): The sample space, denoted by Ω, is the set of all possible outcomes of a random experiment.

Events (A): An event is a subset of the sample space Ω. It represents a particular outcome or a combination of outcomes that we are interested in.

Probability function (P): The probability function, denoted by P, assigns a numerical value to each event in the sample space.

Importance of Probability

Probability is a powerful tool used to address uncertainty in planning and decision-making.

It helps us assess and forecast the outcome of a plan, be prepared, and act as the situation and factors change.

Importance of Probability: Example

Example 1: Consider the case of a supplier who needs to decide on the optimal quantity of items to maintain in inventory to avoid running out of stock.

Importance of Probability: Example

The supplier's goal is to serve customers effectively so that they do not switch to another supplier due to unavailability.

Past data indicates that the average number of units sold is 80, but the sale of these units is influenced by variable factors.

Importance of Probability: Example

The supplier should stock more than 80 units.

Probability can help to statistically determine the appropriate number of units.
This approach incorporates a certain degree of uncertainty.

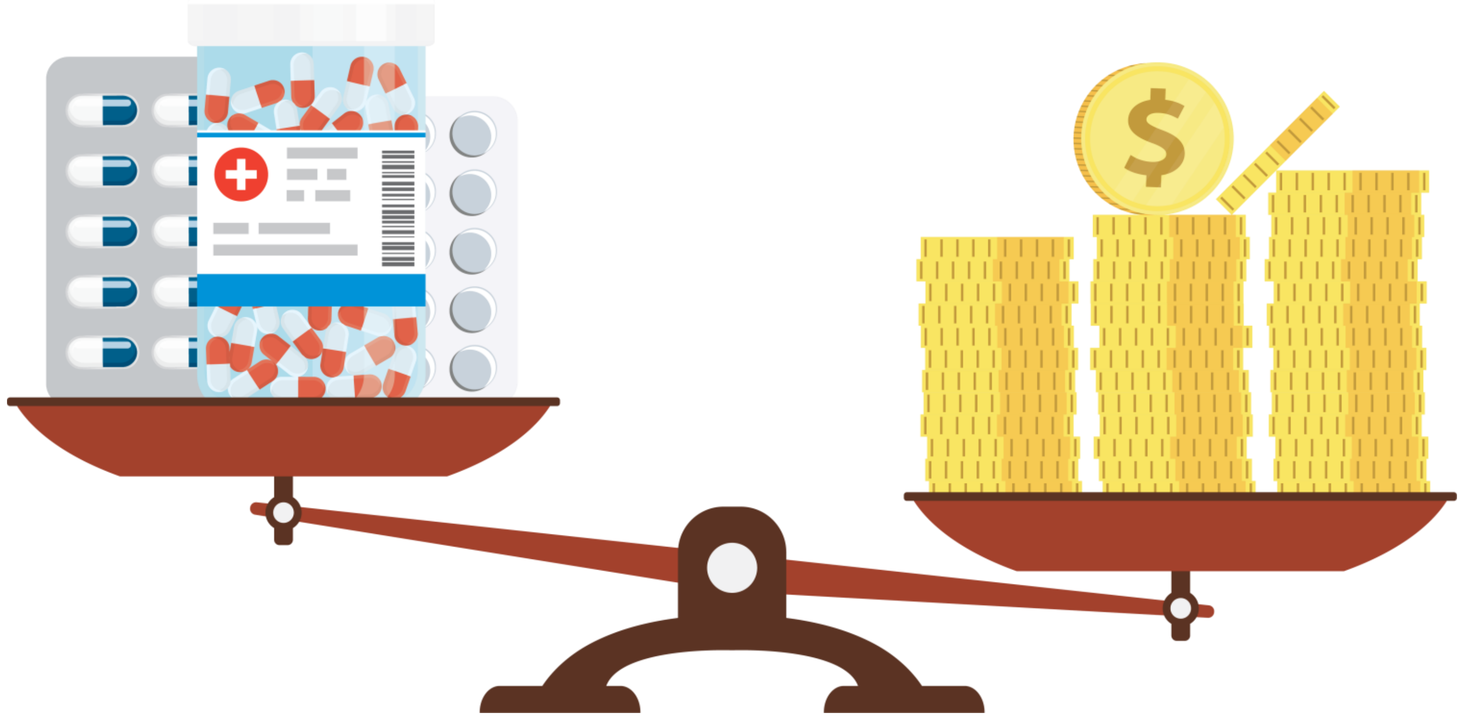
Importance of Probability: Example

Example 2: Pricing insurance policies

Insurance policies are priced after incorporating costs and the prospects of insurance claims.

Importance of Probability: Example

An extremely high price will deter customers from purchasing the insurance policy.



Low prices, on the other hand, will impact the financial performance of the firm.

Importance of Probability: Example

Probability can offer useful insights that help set the price when both variables are considered and assessed.

Uses of Probability

Weather Forecasting: Probability helps to predict the weather by utilizing historical data and probability techniques.

Uses of Probability

Health Insurance: Insurance companies use probability to determine the chances of a person's death by studying their medical history, family history, and specific habits like smoking and drinking.

Uses of Probability

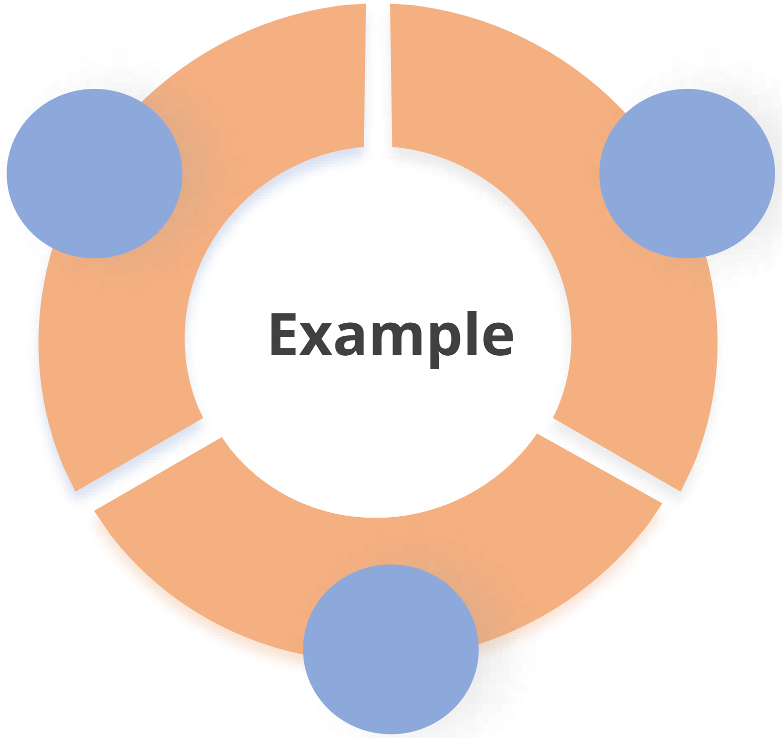
Elections: Probability helps to predict which political party will rise to power by closely studying the results of exit polls.

Key Terms in Probability

Key Terms in Probability

Occurrences are referred to as events, and probabilities are assigned to occurrences.

An even number will appear when a dice is rolled.



A card drawn at random from a deck of 52 cards is a spade.

A resident in the neighborhood owns a Lexus.

Often, two or more events are considered simultaneously in a study or an investigation.

Key Terms in Probability

The following are a few more key terms in probability:

Experiment:

It is a planned activity done under controlled settings.

Outcome:

It is the result of an experiment.

Sample space:

The list of all possible outcomes is called sample space.

Probability function:

It is a function that explains the probability of an outcome.

Relationship Between Events

In a study, the relationship between two or more events is considered.

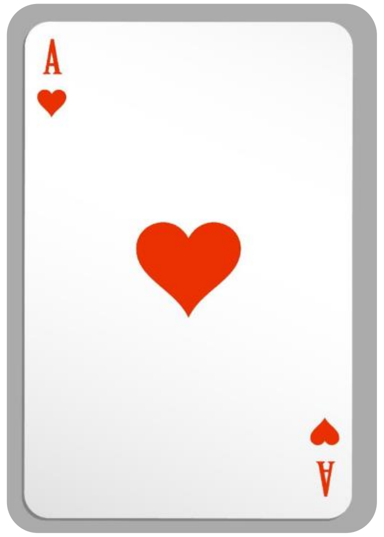
The two important types of relationships are:

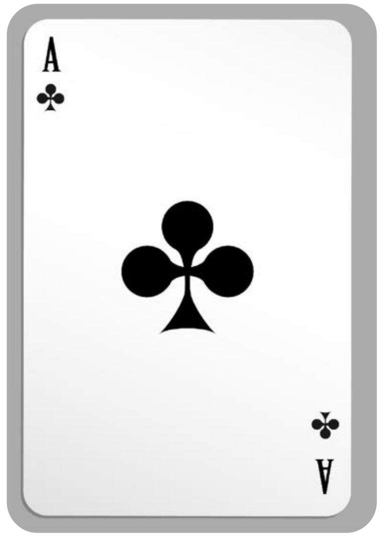
- Mutually exclusive
- Mutually exhaustive or mutually inclusive

Mutually Exclusive

Events are said to be mutually exclusive if they cannot occur simultaneously.

Example:





The event that a card drawn at random is a heart

Mutually exclusive

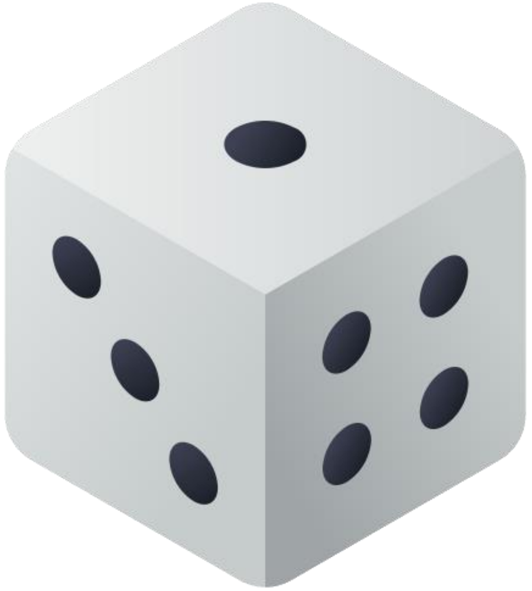
The event that a card drawn at random is a club

Mutually Exhaustive

Events are mutually exhaustive if at least one of the events will occur.

Example:

Rolling a six-sided dice

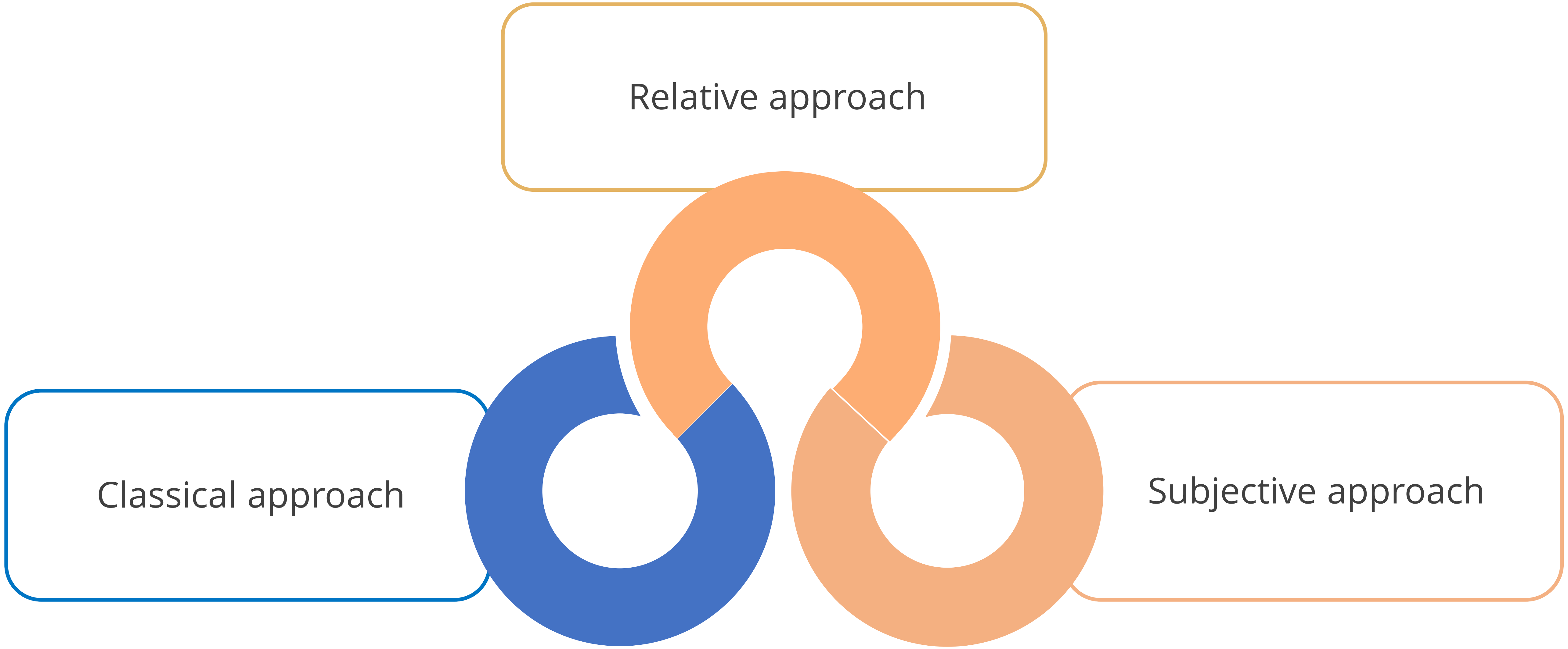


Event A: Getting an odd number

Event B: Getting an even number

Approach of Probability

The various approaches of probability are:



Classical Approach

The probability of an event A, P(A), having n mutually exclusive and equally likely outcomes with k implying outcomes is k/n.

Probability of drawing a club from a deck of 52 cards

13/52 = 1/4

Relative Approach

In the relative approach, probabilities are determined by considering the ratio of favorable outcomes to the total number of possible outcomes.

Past data may be used instead of experimental trials.

Relative Approach

Example of relative approach:

Probability of a product being defective
Number of manufactured pieces: 1000
Number of defective pieces: 5
Probability: 0.005

Subjective Approach

Besides past experiences, the probability of events is estimated using the subjective judgment of experts.

Probability of a customer purchasing different brands

Subjective assessment of experts

Experts’ perception of consumer or market research

Discussion

Duration: 15 minutes

What does probability mean?

- How does probability play a role in decision-making?

Answer: Probability plays a crucial role in decision-making by providing a framework for assessing and quantifying uncertainty, such as risk assessment, decision analysis, forecasting, and optimization.

- What are the key terms of probability?

Answer: Occurrences are referred to as events, and probabilities are assigned to occurrences.

Conditional Probability

Discussion: Conditional Probability

Duration: 15 minutes

You are working in a weather forecast company, and you are supposed to determine the weather based on the data you have.

To do so, determine the following:

- Conditional probability
- Dependent and independent events

Conditional Probability

Conditional probability refers to the probability of an event A occurring given that another event B has occurred.

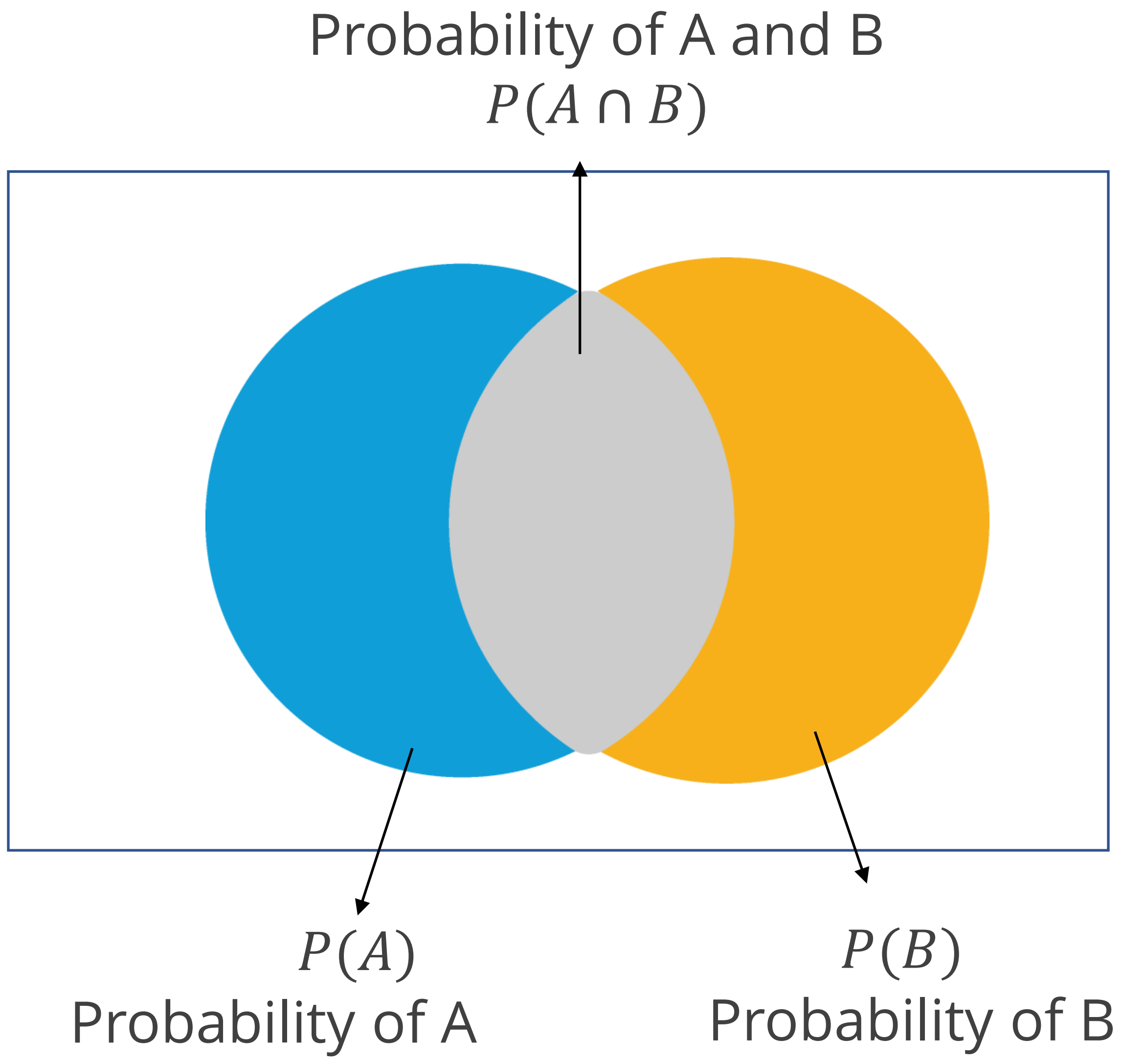
It is denoted by $P(A|B)$, which is read as the probability of A given B.

Example:

For instance, when the rainfall in a particular year is low, the probability of a normal yield of crops will also be lower.

Conditional Probability

Conditional probability and its formula explained with the help of Venn diagram:



Conditional probability formula:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Probability that A occurs given that B has occurred

Calculating Conditional Probability

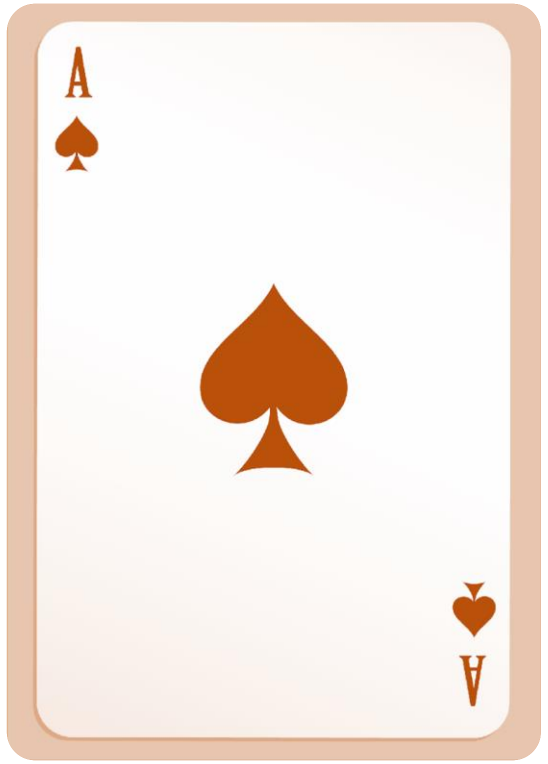
Example 1: Assuming that all cards are equally likely to be selected, draw one card at random from a deck of 52 playing cards

Event A: Drawing a red card

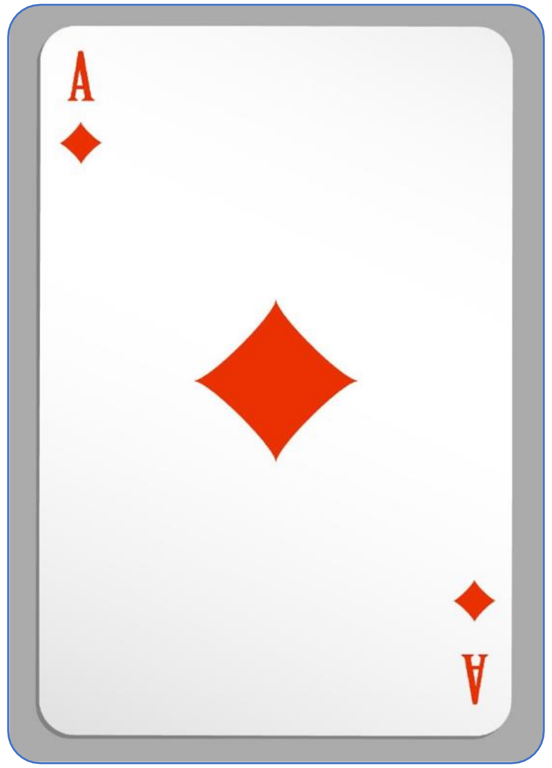
Calculating Conditional Probability

There are 26 red cards (13 hearts and 13 diamonds) and 4 aces (one ace from each suit) in the deck.

Event B: Drawing an ace



Only two aces are red cards

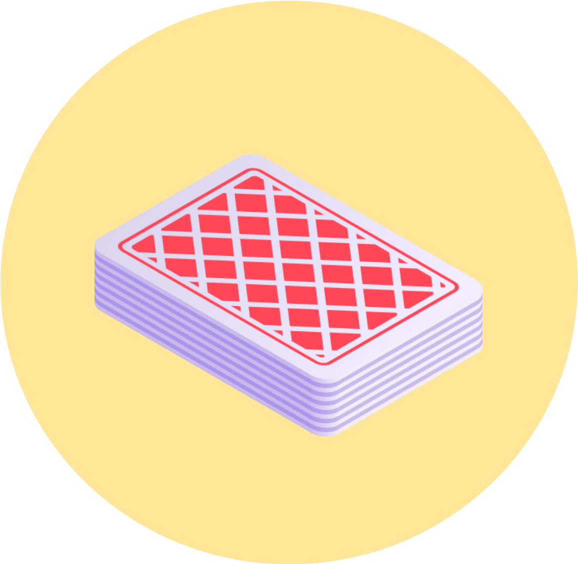


Now, the conditional probability of drawing an ace **(Event B)** given that a red card **(Event A)** has already been drawn is 2/26.

Calculating Conditional Probability

Example 2: A and B denote that a card drawn from a deck is a spade and that the card is black.

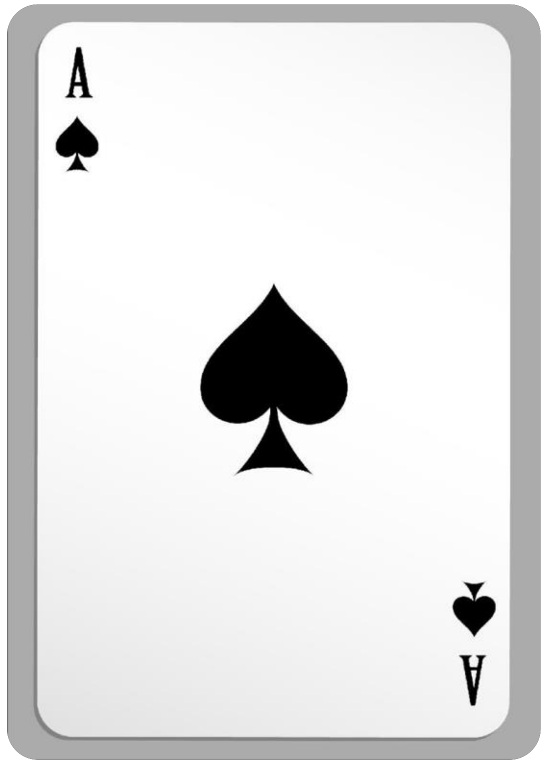
The probability of A denotes that the card drawn from a deck is a spade.
$P(A) = 13/52$



The probability of B denotes that the card drawn from a deck is black.
$P(B) = 26/52$

Calculating Conditional Probability

Example 2: Calculate the probability of a card being a spade given that the card is black.



The formula of conditional probability:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cap B) = \frac{13}{52}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{13}{52} \div \frac{26}{52}$$

$$= \frac{1}{2}$$

Uses of Conditional Probability

Example 1: Results of a market research survey on a new product launch

Disappointing results
Imply a low probability of the product being a hit in the market

Favorable results
Imply a high probability of the product being a hit in the market

Discussion

Duration: 15 minutes

You are working for a weather forecast company, and you are supposed to determine the weather based on the data you have. To do so, determine the following:

- Conditional probability

Answer: The likelihood of an event depends on the occurrence of related events. The possibility of an event is determined by incorporating relevant information.

- Dependent and independent events

Answer: Two events are independent if the occurrence or non-occurrence of one event does not impact the other.

Independent and Dependent Events

Independent and Dependent Events

The properties of the interaction of two or more events are classified under two types :

Independent events: Two events are considered independent if the occurrence of one event does not affect the probability of the other event occurring.

Dependent events: Two events are considered dependent if the occurrence of one event affects the probability of the other event occurring.

Independent Events

Example: Consider an example of rolling a fair dice

Event C: The number appearing is odd → $P(C) = \frac{3}{6} = \frac{1}{2}$

Event D: The number appearing is a multiple of 3 → $P(D) = \frac{2}{6} = \frac{1}{3}$

$C\ and\ D$: The number appearing is odd and a multiple of 3 → $P(C \cap D) = \frac{1}{6}$

Independent Events

By using the conditional probability formula, the influence of the occurrence of D on the occurrence of C can be validated:

$$P(C|D) = \frac{P(C \cap D)}{P(D)} = \frac{\frac{1}{6}}{\frac{1}{3}}$$

$$P(C|D) = \frac{1}{2}$$

The result shows that the occurrence of event D has not affected the probability of the occurrence of event C. Thus, P(C|D) = P(C).

Dependent Event

Example: Draw two cards uniformly at random from a deck of 52 cards, without replacement

Event A: Drawing a king on the first draw

$$P(A) = \frac{4}{52}$$

Dependent Event

After the first draw, there are 51 cards left in the deck, including three kings. One king has already been drawn.

Event B: Drawing a king on the second draw given that one king has already been drawn

$$P(B/A) = \frac{3}{51}$$

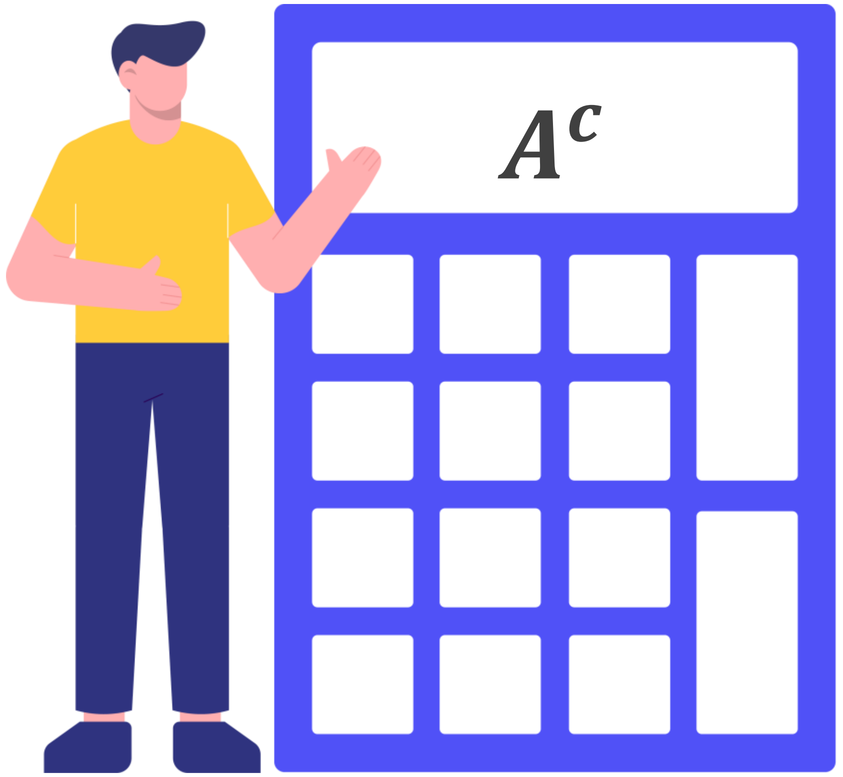
Dependent Event

Since the events are dependent, probability of the second event depends on the outcome of the first draw.

$$P(A \cap B) = P(A) * P(B|A) = \frac{4}{52} * \frac{3}{51}$$

Complement of an Event

The complement of an event A (A^c (consists of all outcomes in the sample space that are not part of event A.



P(A^c) = 1 – P(A)

Addition Theorem of Probability

Addition Theorem of Probability

The addition theorem states that if A_1 ,A_2 ...,A_j ...up to A_n are n mutually exclusive events:

Then, the probability that at least one event of these will occur is the sum of their probabilities, as shown below:

$$P(A_1 \cup A_2 \cup A_3 \cup \ldots..\cup A_n) = P(A_1) + P(A_2) + P(A_3) + \cdots + P(A_n)$$

Addition Theorem of Probability

When A and B are two events that are not necessarily mutually exclusive, then:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$A \cup B$ → All elements that belong to either A or B

$A \cap B$ → All elements that belong to both A and B

Addition Theorem: Example

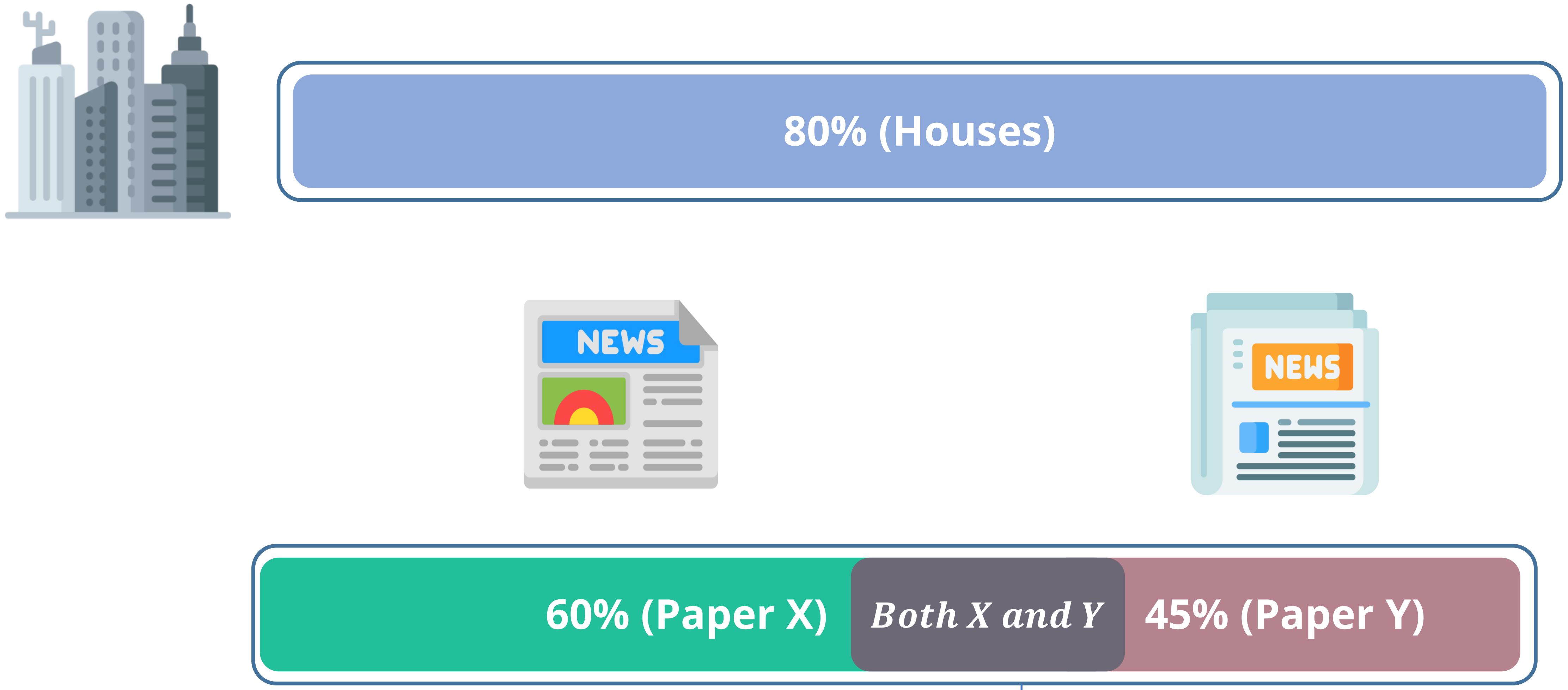
A newspaper vendor delivers two newspapers, X and Y, to houses in a big city.

Newspaper X

Newspaper Y

Addition Theorem: Example

80% of the houses are the vendor's customers.



Determine the percentage of houses that buy both X and Y

Addition Theorem: Example

Event A: Houses that receive paper X

Event B: Houses that receive paper Y

Given:

$$P(A) = 0.6$$

$$P(B) = 0.45$$

$$P(A \cup B) = 0.8$$

Addition Theorem: Example

When A and B are two events that are not necessarily mutually exclusive, then:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Substituting the values:

$$0.8 = 0.6 + 0.45 - P(A \cap B)$$

$$P(A \cap B) = 0.25$$

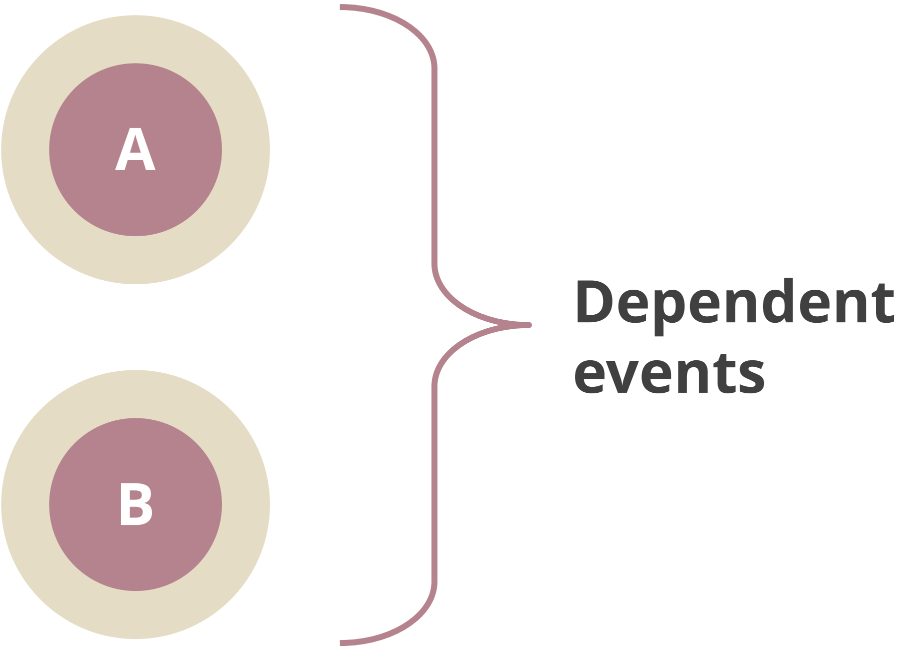
The result shows that 25% of the houses receive both newspapers.

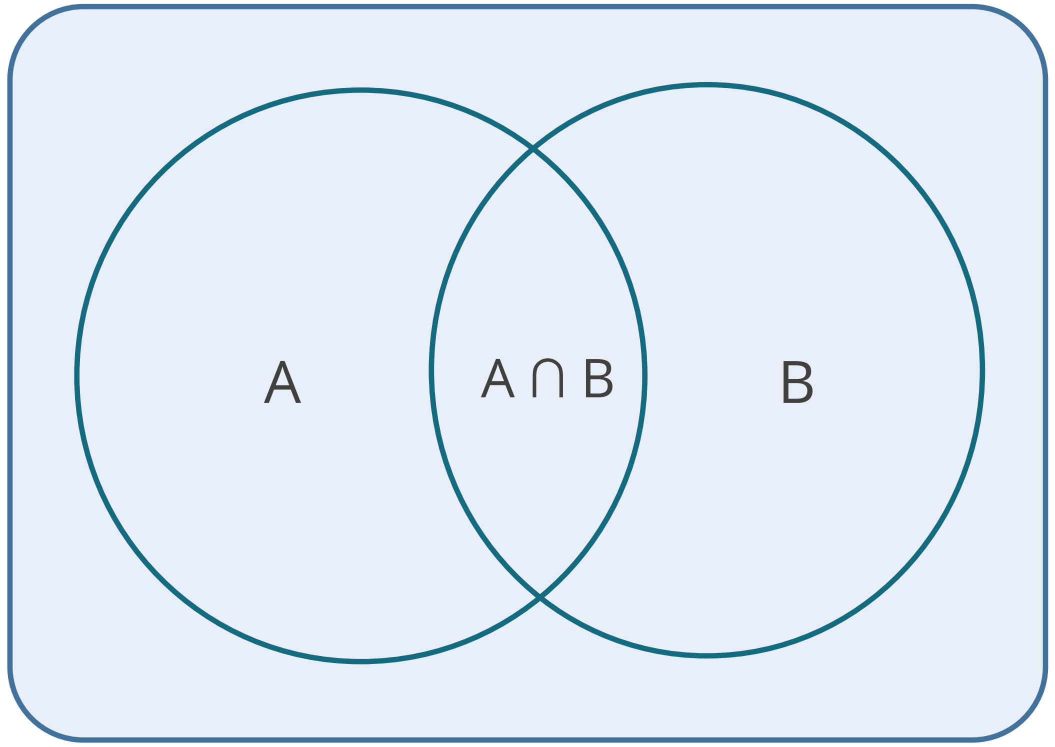
Multiplication Theorem of Probability

Multiplication Theorem of Probability

It states that for two dependent events, A and B, the probability of A and B is equal to the probability of (A/B) multiplied by the probability of B.

$$P(A \cap B) = P(A|B) * P(B)$$

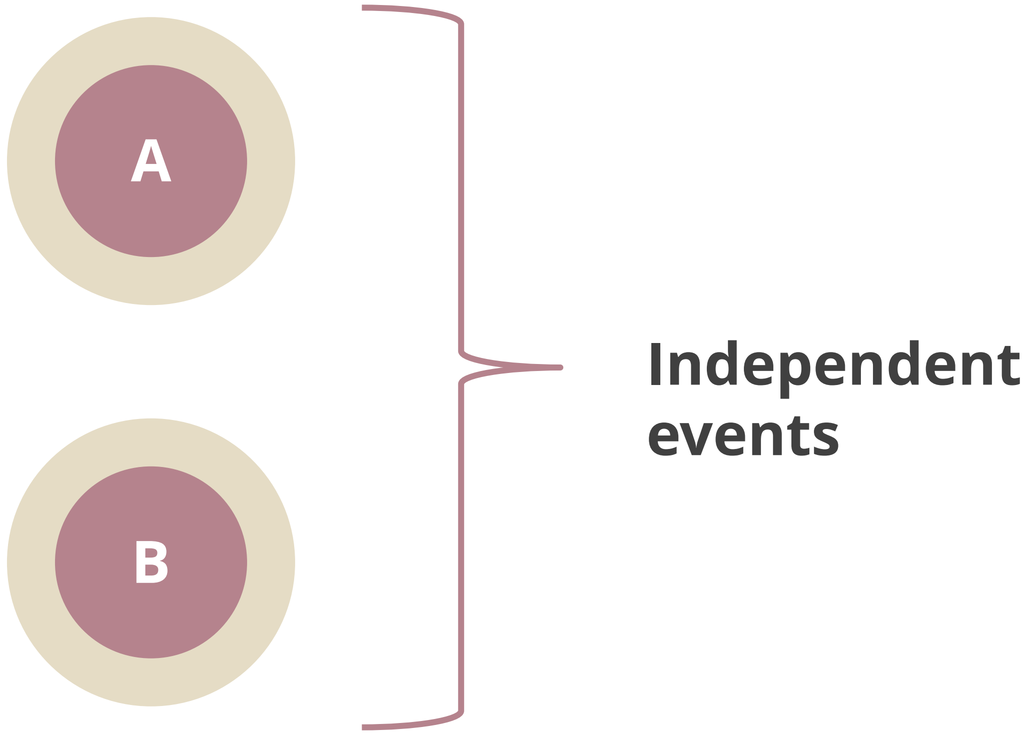


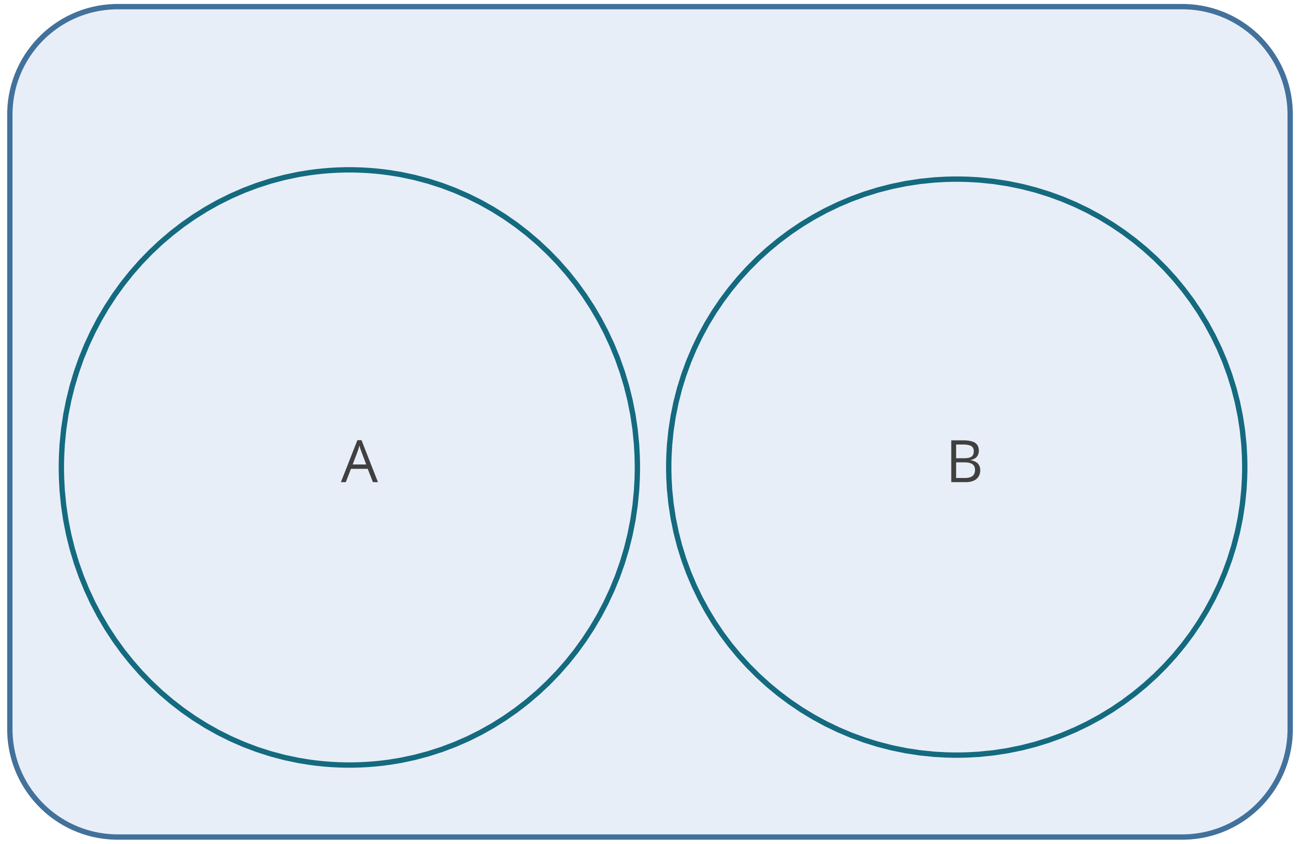


Multiplication Theorem of Probability

When A and B are independent events, the probability of simultaneous occurrence of both A and B is the product of the two probabilities.

$$P(A \cap B) = P(A) * P(B)$$





This is true since $P(A|B) = P(A)$ for independent events.

Multiplication Theorem of Probability

Generally, for n independent events, the probability that all the events occur is the product of their probabilities.

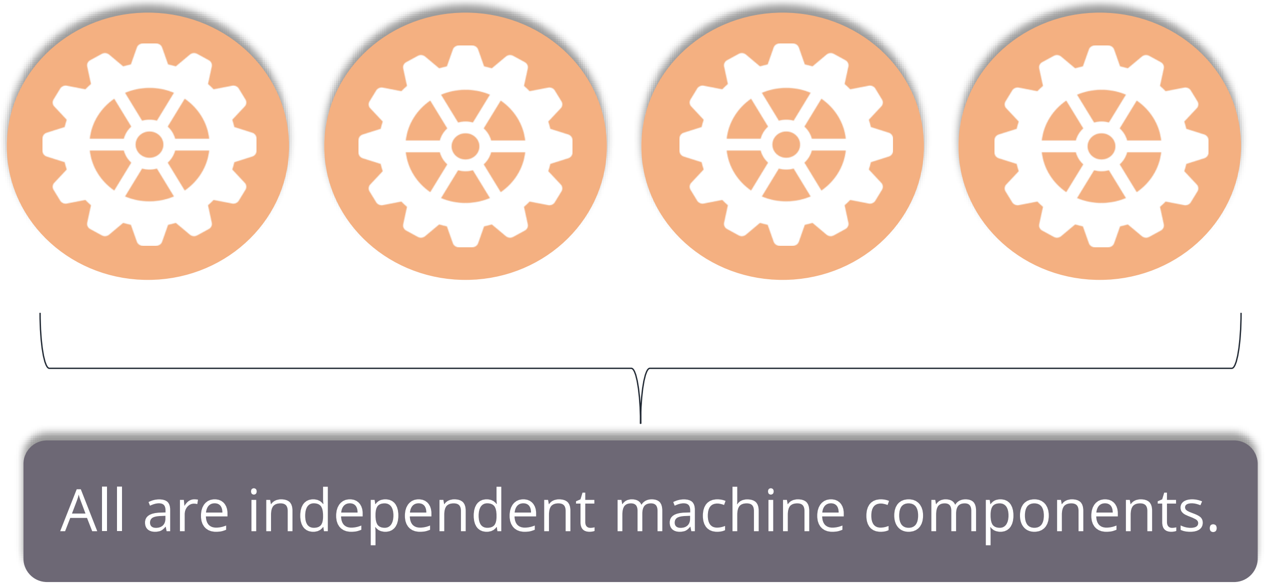
A_1, A_2,… A_j …up to A_n → **n independent events**

$$P(A_1 \cap A_2 \cap A_3 \cap \ldots\ldots \cap A_n) = P(A_1) * P(A_2) * P(A_3) * \cdots * P(A_n)$$

Multiplication Theorem

Example: Components of an electronic system

An electronic system consists of four components of identical functionality.



The system becomes dysfunctional only when all its components are dysfunctional.
The functionality of each component is independent of the other.

Multiplication Theorem: Example

The probability that a component is functional during the warranty period is independent of the others and is 0.9.

Probability that a component is functional = 0.9

What is the probability that the system will be functional during the warranty period?

Multiplication Theorem: Example

A_j denotes the event that component j is functional.

$P(A_j) = 0.9$

j = 1 to 4

$P(A_j^c) = 1 - 0.9 = 0.1$

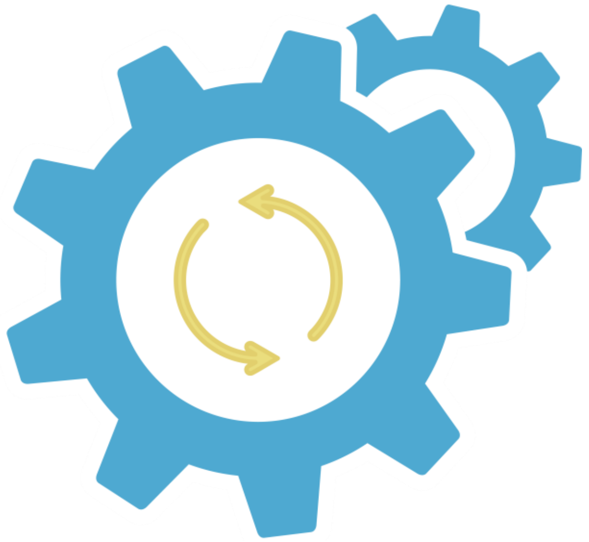
$P(A_j^c)$ =Probability that a component j is dysfunctional.

Multiplication Theorem: Example

Probability of the system being dysfunctional

Probability that all components fail = $P(A_1^c) * P(A_2^c) * P(A_3^c) * P(A_4^c)$

$$= 0.1 * 0.1 * 0.1 * 0.1 = 0.0001$$



The probability of the system being functional = 1- 0.0001 = 0.9999

Bayes' Theorem

Bayes' Theorem with an Example

A company is ready to launch a new product and wants to determine the probability of its success in the market.

Bayes' Theorem with an Example

From the past data, the company knows the probabilities that a product will be:

Success

Disaster

Neither

Bayes' Theorem with an Example

The company also knows the probability of a favorable outcome from a market research survey in each of the three cases.

Success

Disaster

Neither

Bayes' Theorem with an Example

The company would now like to know the revised probabilities of each outcome when the market research survey has projected a favorable outcome.

Revised probability of favorable outcomes

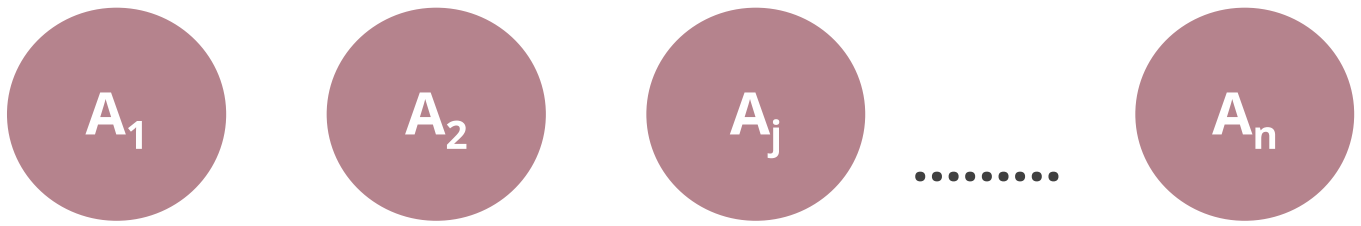
Success

Disaster

Neither

Bayes' Theorem

Consider A_1, A_2,... A_j ...up to A_n are mutually exclusive events with known probabilities



Consider B is another event, and the probability of $P(B/A_j)$ for (j= 1 to n) is also known

Bayes' Theorem

Bayes' theorem states that the probability of A_k and B is the ratio of the simultaneous probability of A_k and B to the probability of B.

$$P(A_k|B) = \frac{P(B|A_k) * P(A_k)}{\sum_{j=1}^{n} P\left(\frac{B}{A_j}\right) * P(A_j)} = \frac{P(B \cap A_k)}{\sum_{j=1}^{n} B \cap A_j}$$

k = 1 to n

The denominator is $P(B)$

Bayes' Theorem

The probability of A_1 ,probability of A_2 ,...,probability of A_j until the probability of A_n is called **prior probabilities.**

$P(A_1)$ $P(A_2)$ .... $P(A_j)$ .... $P(A_n)$

The probability of $P(A_1|B)$, probability of $P(A_j|B)$ until the probability of $P(A_n|B)$ is called **posterior probabilities**.

$P(A_1|B)$ $P(A_2|B)$ .... $P(A_j|B)$ .... $P(A_n|B)$

Bayes' Theorem with an Example

Example: Success of a new product in the market

Consider three events A_1 ,A_2 ,and A_3 with probabilities 0.6, 0.1, and 0.3.

Suppose B is another event and the probability of $(B|A_1)$,$(B|A_2)$,and $(B|A_3)$ are 0.75, 0.1, and 0.4

$P(A_1)$	**0.6**
$P(A_2)$	**0.1**
$P(A_3)$	**0.3**

$P(B\|A_1)$	**0.75**
$P(B\|A_2)$	**0.1**
$P(B\|A_3)$	**0.4**

Bayes' Theorem with an Example

Apply the formula and get the following values:

$$\sum_{j=1}^{n} P(B|A_j) * P(A_j)$$

$$= 0.75 * 0.6 + 0.1 * 0.1 + 0.4 * 0.3$$

$$= 0.45 + 0.01 + 0.12$$

$$= 0.58$$

Bayes' Theorem with an Example

The value of revised probabilities of each outcome is given as:

$$P(A_1/B) = \frac{0.45}{0.58} = 0.78$$

$$P(A_2/B) = \frac{0.01}{0.58} = 0.02$$

$$P(A_3/B) = \frac{0.12}{0.58} = 0.21$$

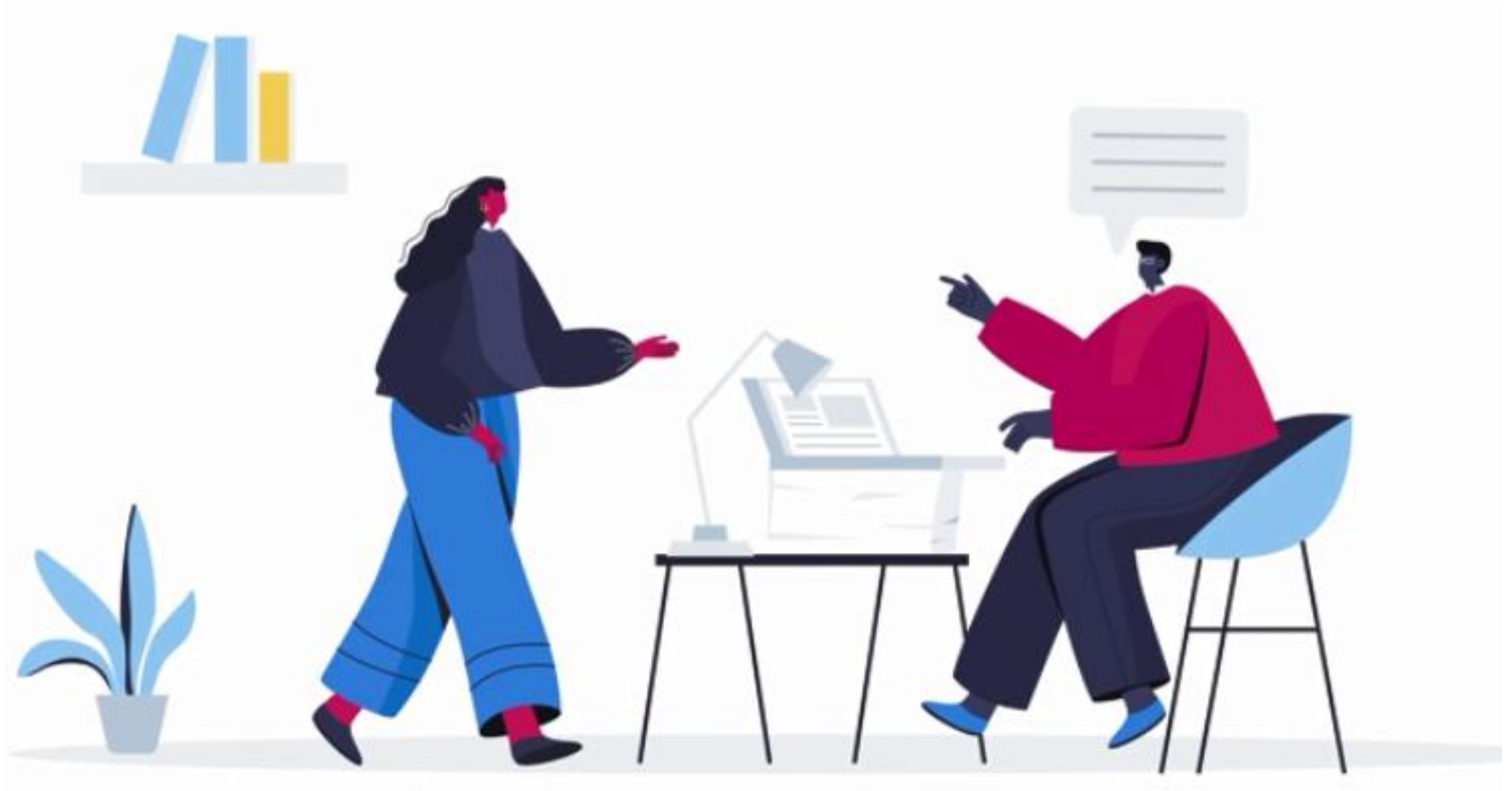
Bayes' Theorem with an Example

A_1 ,A_2 ,and A_3 denote the events of the product being a success, a disaster, or neither of them. B denotes the event of the market survey which predicted a favorable result.

	Success	Disaster	Neither
	A_1	A_2	A_3
Prior probabilities	0.6	0.1	0.3
Posterior probabilities	0.78	0.02	0.21

Bayes' Theorem with an Example

The results show that the market research survey improves outcomes and serves as a guide for decision-making.



Key Takeaways

- The key concepts in probability include the classical approach, the relative frequency approach, and the subjective approach.
- The likelihood of the occurrence of an event may be revised based on the occurrence or nonoccurrence of related events. This is known as conditional probability.
- Independent and dependent events are the forms of interaction of two or more events.

Knowledge Check

Knowledge Check 1

In which approach is the probability of an event A, P(A), in n mutually exclusive and equally likely outcomes with k implying the outcome k/n?

A. Classic approach

B. Relative approach

C. Relative frequency approach

D. Subjective approach

Knowledge Check

1

In which approach is the probability of an event A, P(A), in n mutually exclusive and equally likely outcomes with k implying the outcome k/n?

A. Classic approach

B. Relative approach

C. Relative frequency approach

D. Subjective approach

The correct answer is **A**

In the classic approach, the probability of an event A, P(A), in n mutually exclusive and equally likely outcomes with k implying the outcome is k/n.

Knowledge Check 2

_______________ events cannot occur simultaneously.

A. Mutually exclusive

B. Mutually exhaustive

C. Both A and B

D. None of the above

Knowledge Check

2

________________ events cannot occur simultaneously.

A. Mutually exclusive

B. Mutually exhaustive

C. Both A and B

D. None of the above

The correct answer is **A**

Mutually exclusive events cannot occur simultaneously.

Knowledge Check 3

Which of the following theorems states that for two dependent events, A and B, the $P(A \cap B)$ is equal to the $P(A/B)$ multiplied by the $P(B)$?

A. Additional theorem

B. Multiplication theorem of probability

C. Bayes' theorem

D. None of the above

Knowledge Check

3

Which of the following theorems states that for two dependent events, A and B, the $P(A \cap B)$ is equal to the $P(A/B)$ multiplied by the $P(B)$?

A. Additional theorem

B. Multiplication theorem of probability

C. Bayes' theorem

D. None of the above

The correct answer is **B**

The multiplication theorem of probability states that for two events, A and B, the probability of A and B is equal to the probability of A|B multiplied by the probability of B.

Thank You