Math Refresher

Coordinate Geometry

Learning Objectives

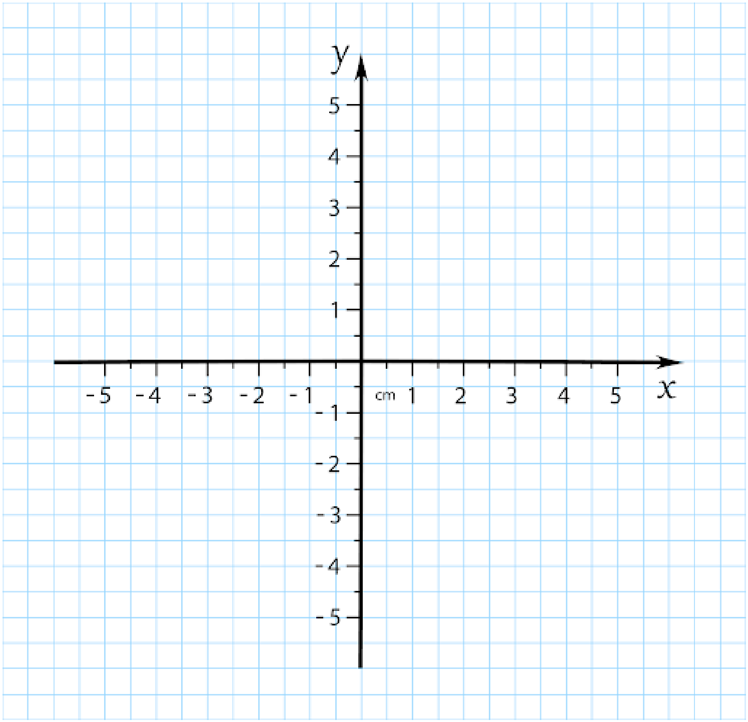
By the end of this lesson, you will be able to:

- Define the coordinate plane and the coordinates of a point in coordinate geometry
- Understand the formulas for distance and slope used in coordinate geometry
- List the various coordinate geometry formulas and their components
- Explain the fundamentals of coordinate geometry and its uses

Introduction to Coordinate Geometry

Coordinate Geometry

Coordinate geometry involves studying geometric figures by plotting them on coordinate axes.

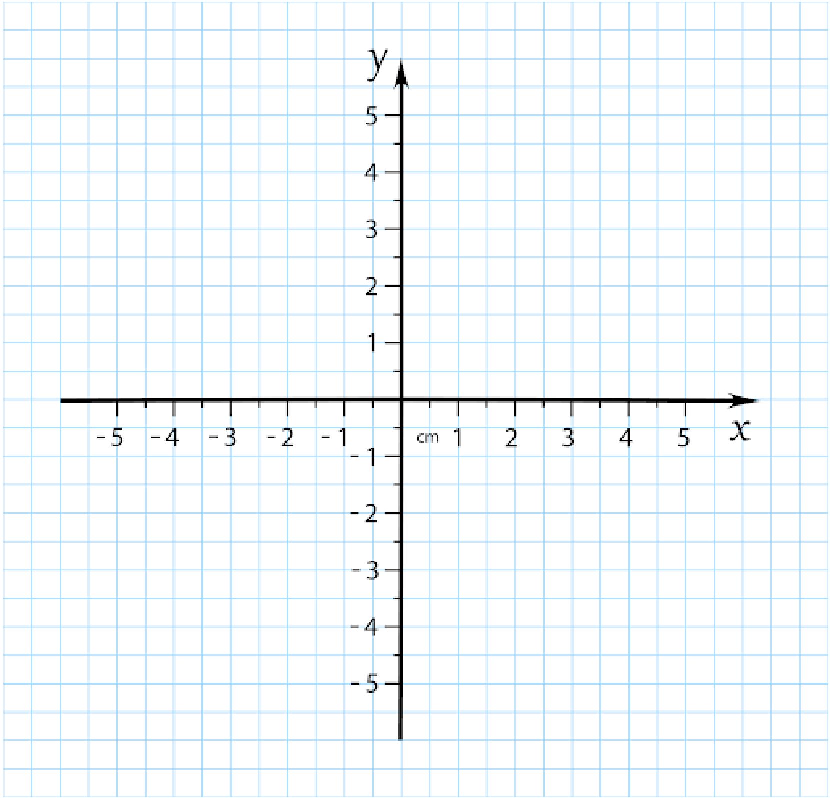


It is a branch of mathematics that enables the representation of geometric figures on a two-dimensional plane and the understanding of their properties.

Straight lines, curves, circles, ellipses, hyperbolas, and polygons can be accurately drawn and represented to scale on the coordinate axes.

Coordinate Geometry

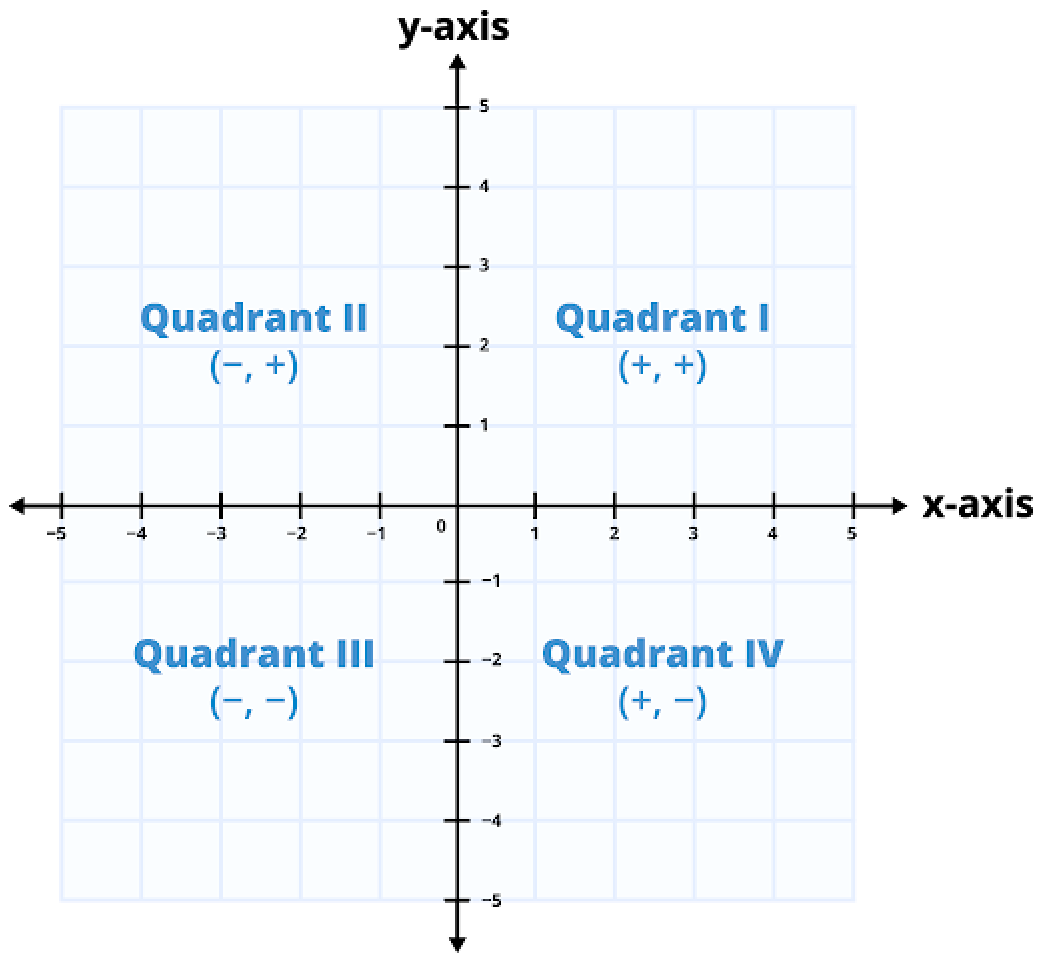
Coordinate geometry also facilitates algebraic computations and the analysis of geometric figure attributes using the coordinate system.



The starting points of the coordinate system are the zero degree of Greenwich Longitude and the zero degree of Equator Latitude.

Coordinate Plane

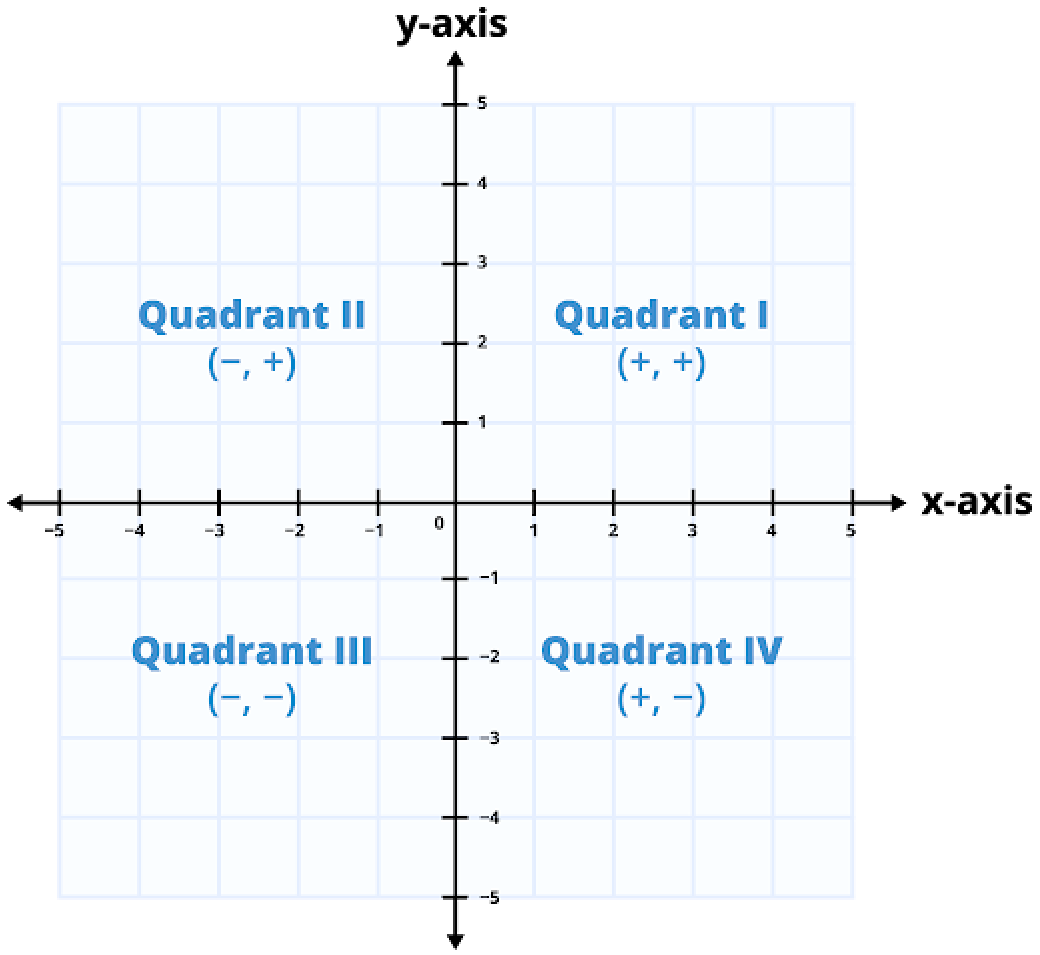
A Cartesian plane divides the plane space into two dimensions, making it convenient to locate points.



The two axes of the coordinate plane are the horizontal x-axis and the vertical y-axis.

Coordinate Plane

Other features of the coordinate plane are as follows:

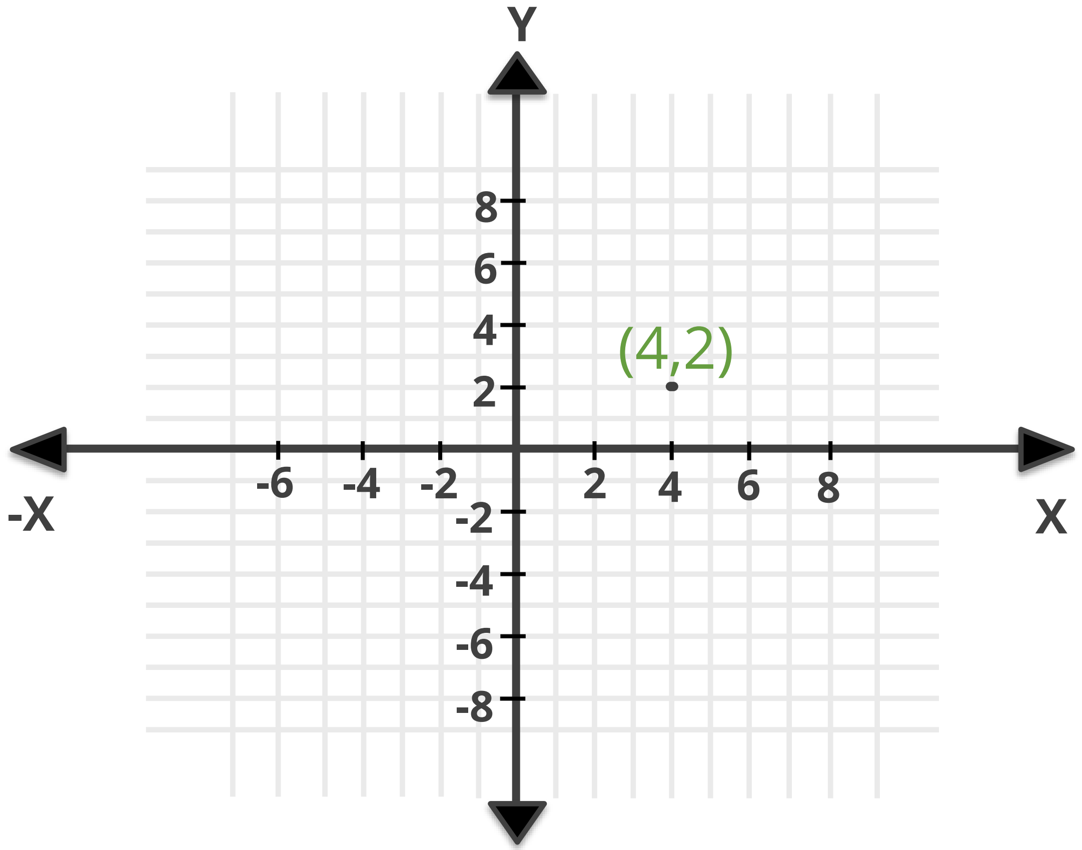


The two coordinate axes divide the plane into four quadrants, and the point at which they intersect is called the origin (0, 0).

In the coordinate plane, points are represented by coordinate pairs (x, y), with x denoting the position along the x-axis and y denoting the position along the y-axis.

Coordinate Plane: Key Terms

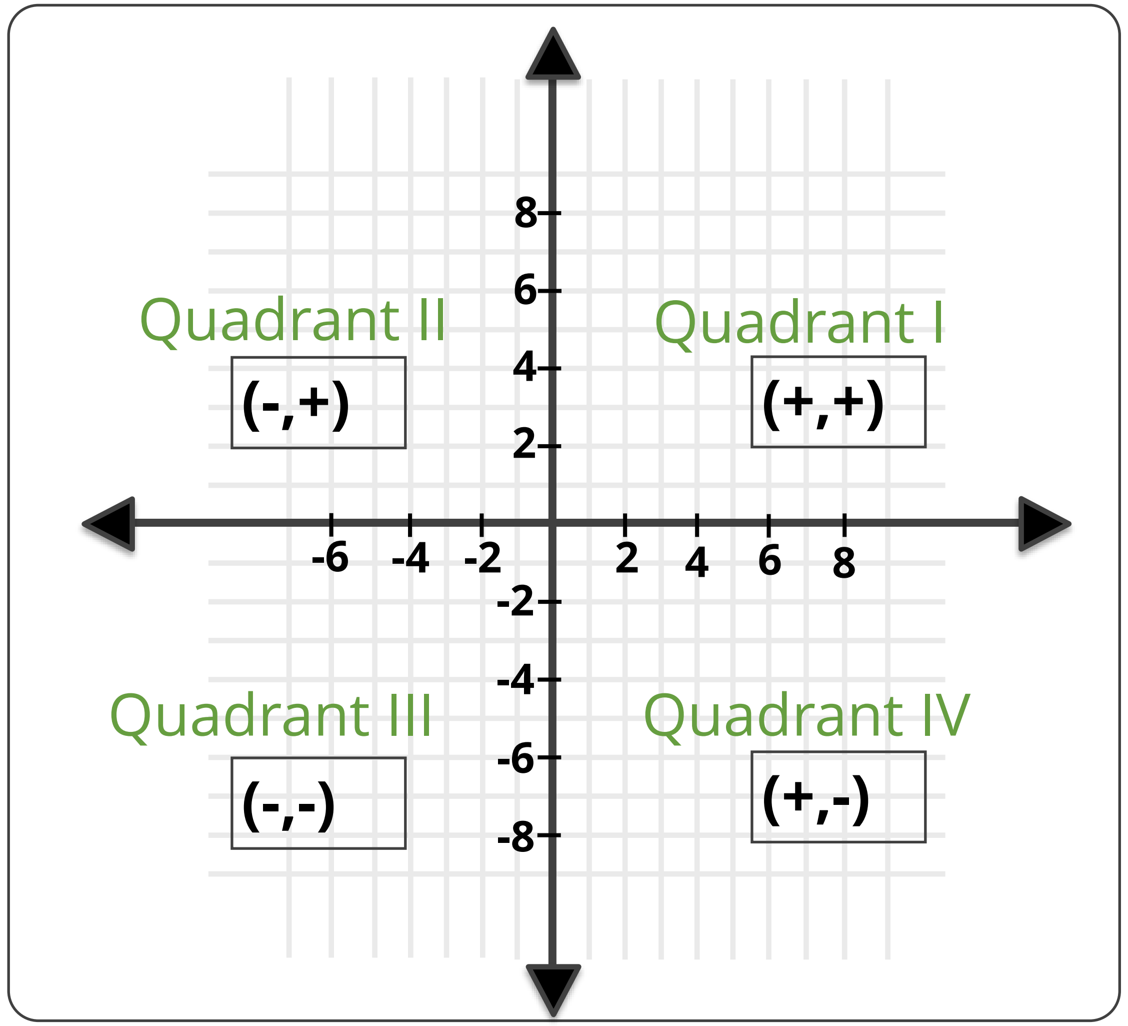
Here are some key terms associated with a coordinate plane:



- **Distance**: Using the distance formula, derived from the Pythagorean theorem, the distance between two points is calculated on a graph.
- **Slope**: The slope of a line, denoted m, represents its steepness, determined by the ratio of vertical to horizontal change.
- **Graphing**: The coordinate plane visually depicts functions, equations, and shapes by plotting points and connecting them with lines or curves.

Coordinate Plane: Properties

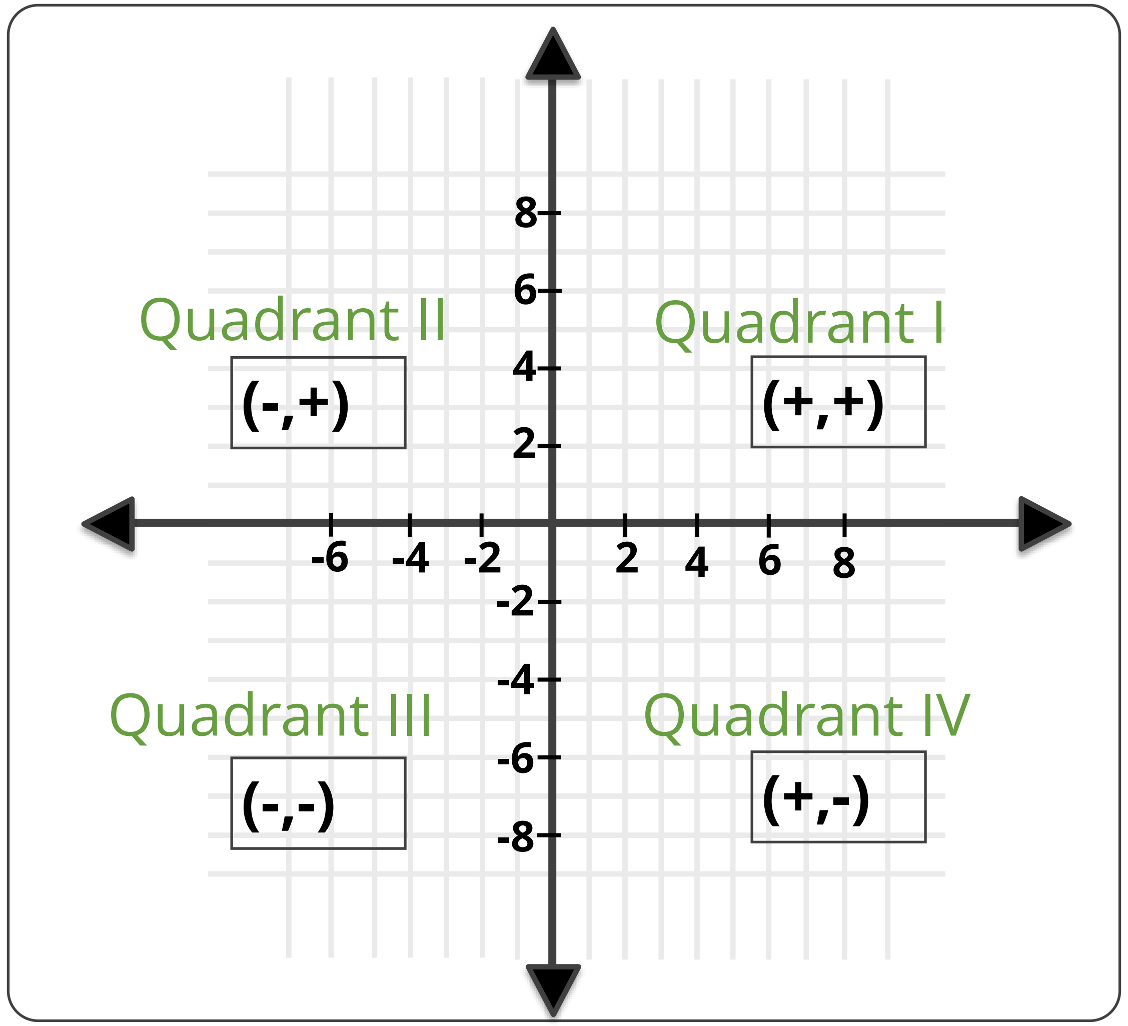
The properties of points in the four quadrants of the coordinate plane are as follows:



- The origin O with coordinates (0, 0) represents the point of intersection between the x-axis and the y-axis.
- The x-axis to the right of the origin O is the positive x-axis, while the x-axis to the left of the origin O is the negative x-axis.

Coordinate Plane: Properties

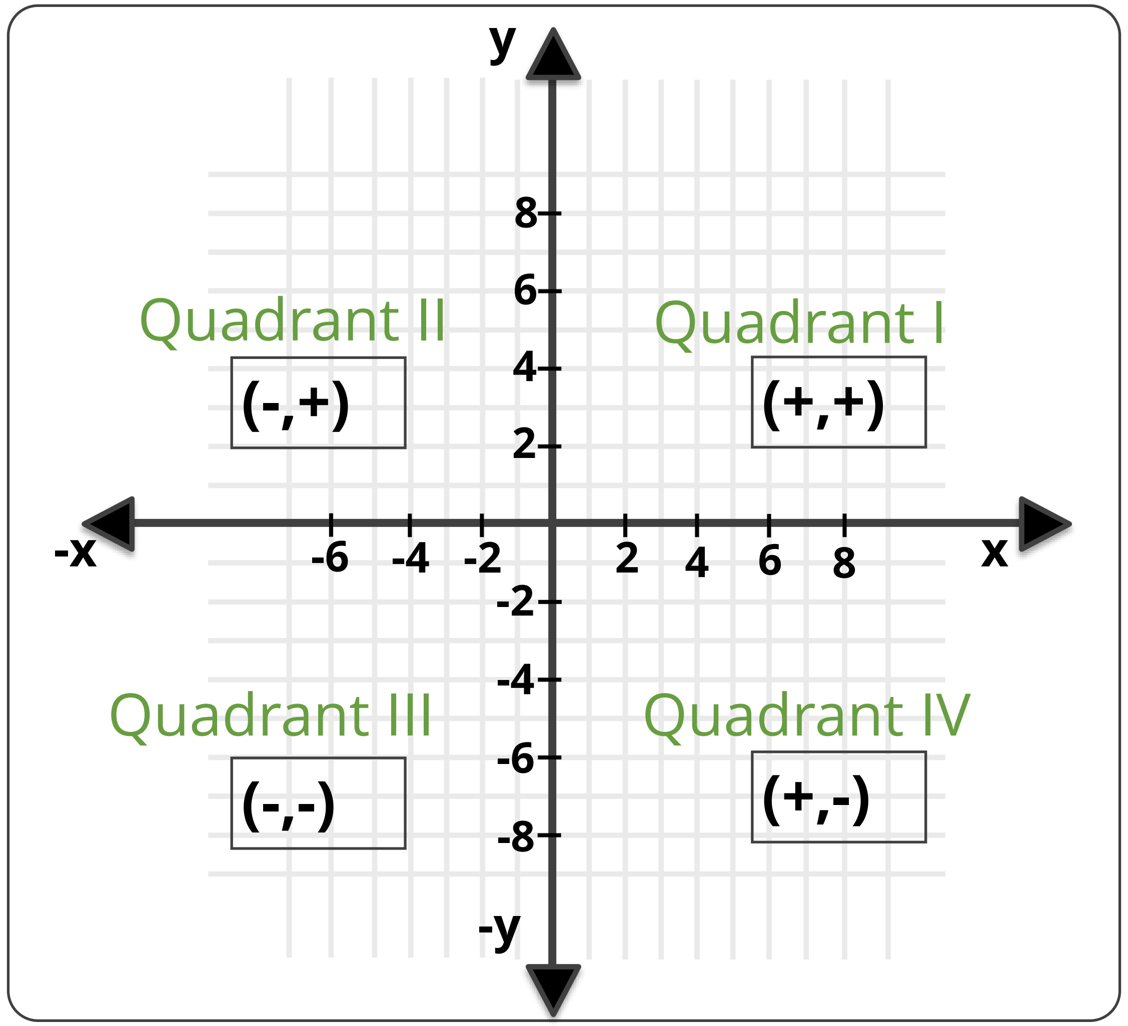
The properties of points in the four quadrants of the coordinate plane are as follows:



- The y-axis above the origin O is the positive y-axis, while the y-axis below the origin O is the negative y-axis.
- A point in the first quadrant (x, y) has positive values for both x and y; it is plotted with regards to the positive x-axis and positive y-axis.

Coordinate Plane: Properties

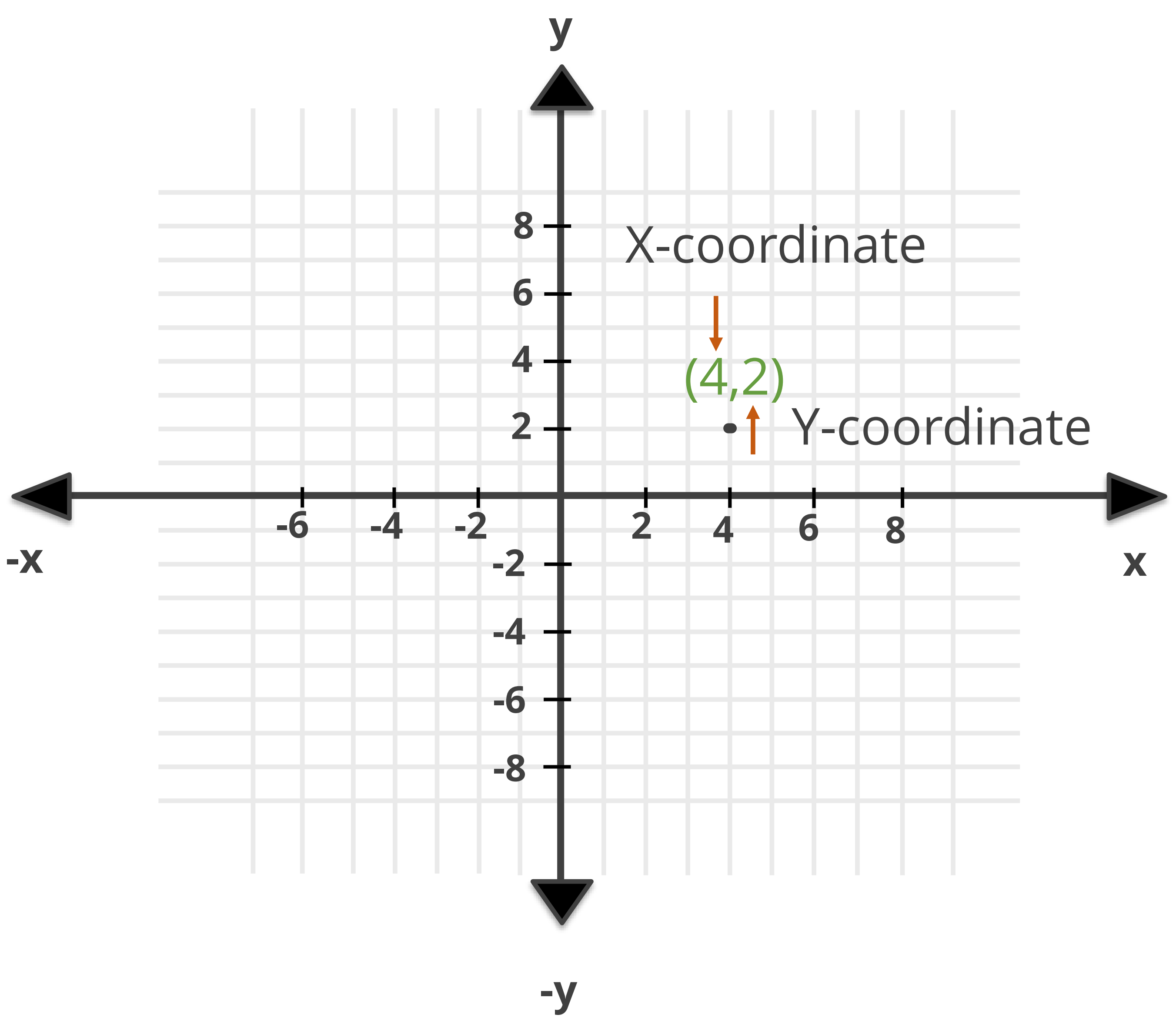
The properties of points in the four quadrants of the coordinate plane are as follows:



- In the second quadrant, a point is represented as (-x, y) and is plotted with reference to the negative x-axis and positive y-axis.
- In the third quadrant, a point is represented as (-x, -y) and is plotted with reference to the negative x-axis and negative y-axis.
- In the fourth quadrant, a point is represented as (x, -y) and is plotted with reference to the positive x-axis and negative y-axis.

Coordinates of a Point

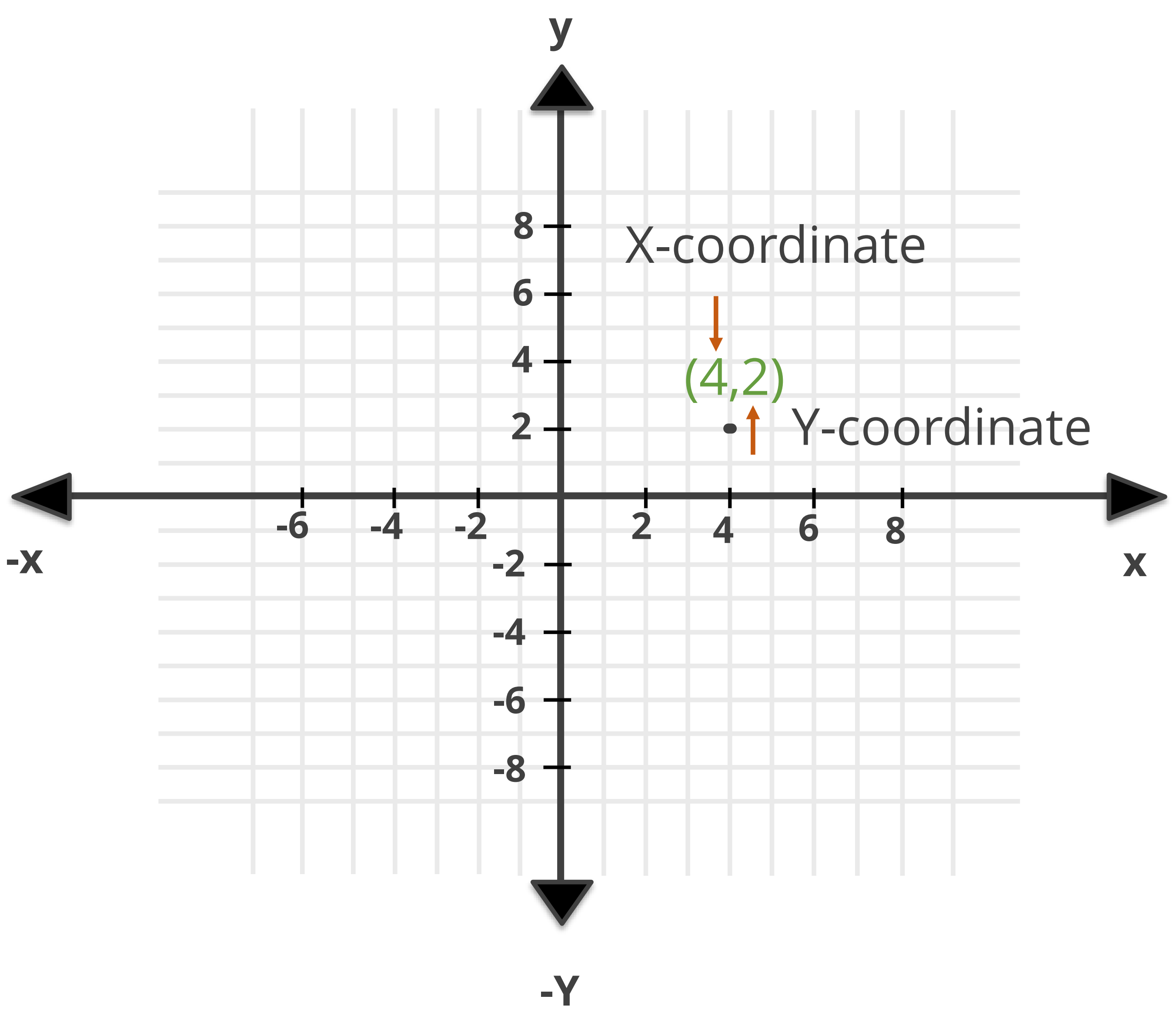
A coordinate is an address that helps locate a point in space.



- In a two-dimensional space, the coordinates of a point are denoted as (x, y).
- **Abscissa** refers to the x value in a point (x, y) and represents the distance of the point along the x-axis from the origin.

Coordinates of a Point

A coordinate is an address that helps locate a point in space.

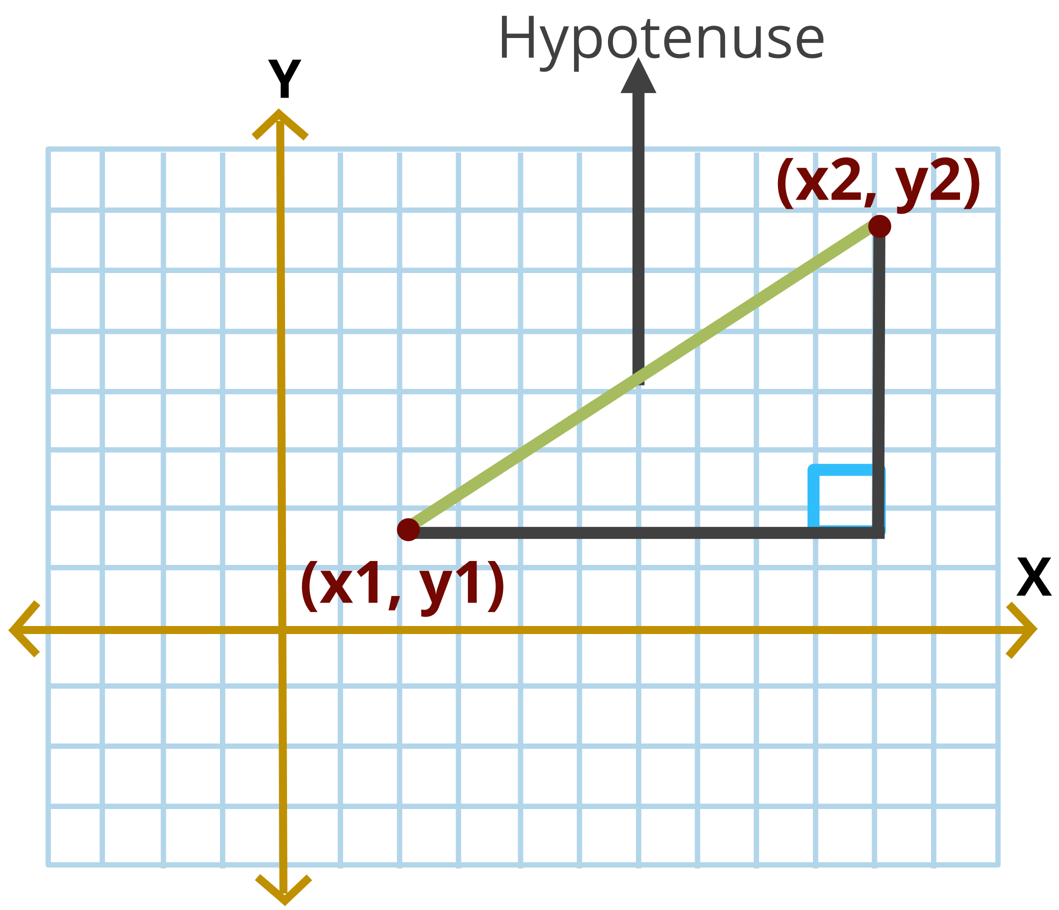


- **Ordinate** refers to the y value in a point (x, y) and represents the perpendicular distance of the point from the x-axis, parallel to the y-axis.
- The coordinates of a point are essential for performing various operations such as calculating distance, finding the midpoint, determining the slope of a line, and deriving the equation of a line.

Coordinate Geometry Formulas

Euclidean Distance Formula

The distance formula calculates the distance between two points. It represents the length of the line segment that connects them.



Euclidean Distance Formula

Let's consider two points, A and B, with coordinates (x_1, y_1) and (x_2, y_2), respectively.

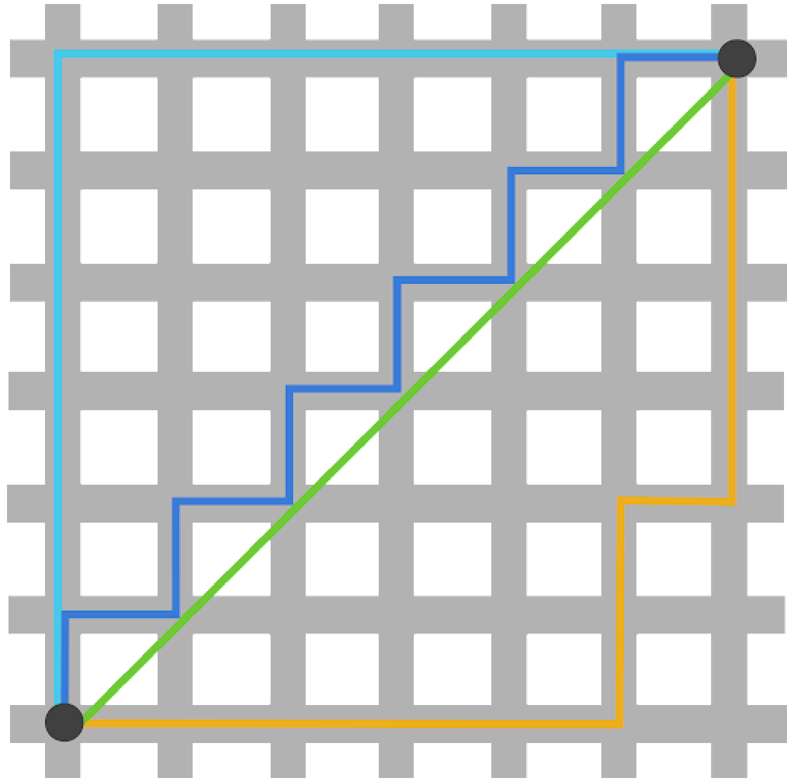
The distance between these two points is calculated as follows:

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Where: $(x_2 - x_1)$ = Change in x
$(y_2 - y_1)$ = Change in y

Manhattan Distance Formula

In some cases, the distance is calculated using the Manhattan distance, also known as the taxicab distance.



This distance is determined by summing the absolute differences in the x-coordinates and y-coordinates between the two points.

Manhattan Distance Formula

Consider two points, A and B, with coordinates (x_1,y_1) and (x_2,y_2), respectively.

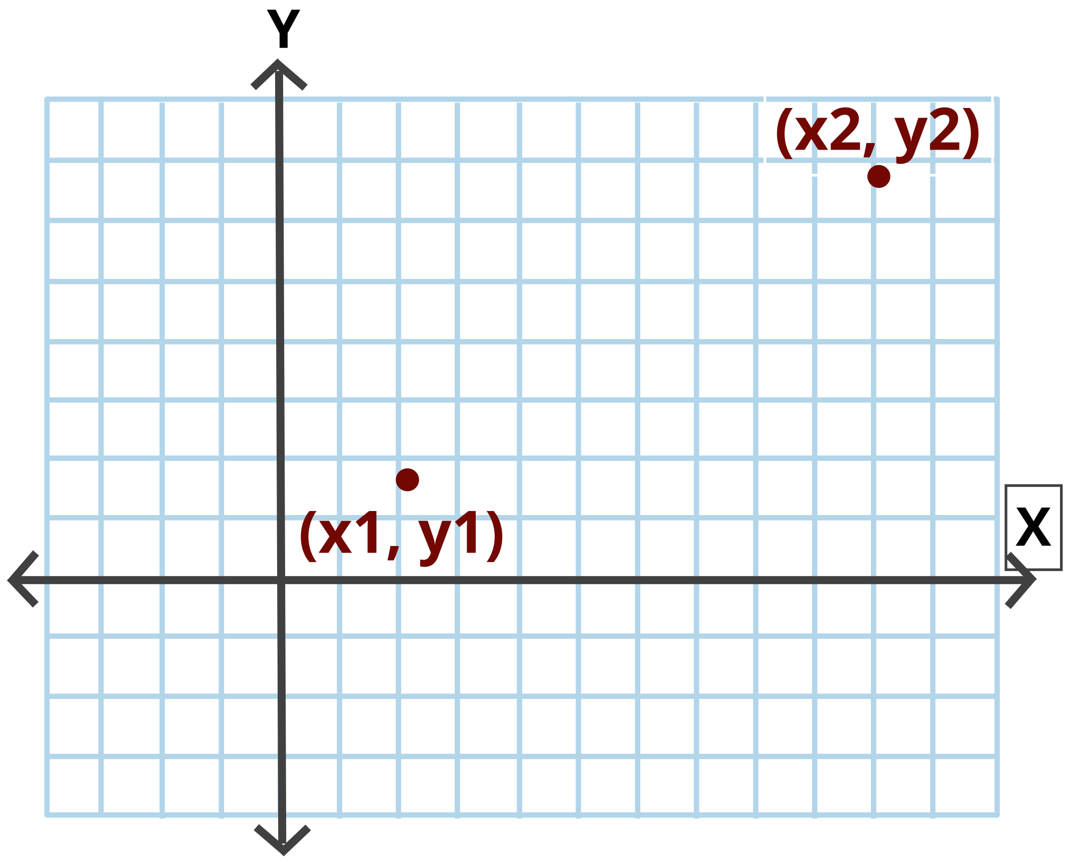
The distance between these two points is calculated as follows:

$$d(A, B) = |x_2 - x_1| + |y_2 - y_1|$$

Where: $(x_2 - x_1)$ = Change in x
$(y_2 - y_1)$ = Change in y

Minkowski Distance Formula

The Minkowski distance is a generalized distance formula that combines the Euclidean distance and Manhattan distance.



It is defined as the pth root of the sum of the absolute differences raised to the power of p.

Minkowski Distance Formula

Consider two points, A and B, with coordinates (x_1, y_1) and (x_2, y_2), respectively.

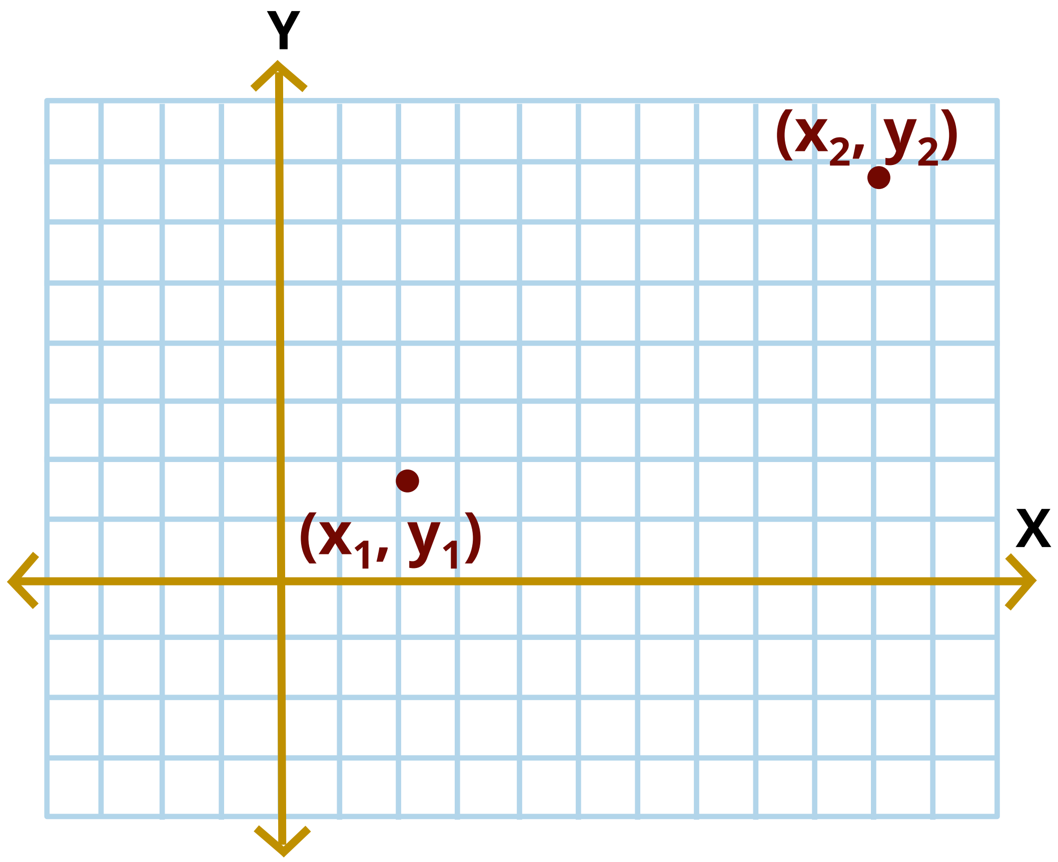
The distance between these two points is calculated as follows:

$$d(A, B) = ((|x_2 - x_1|)^p + (|y_2 - y_1|)^p)^{1/p}$$

Where: $(x_2 - x_1)$ = Change in x
$(y_2 - y_1)$ = Change in y

Midpoint Formula

The midpoint formula is used to find the coordinates of the midpoint between two points in a coordinate plane.



Midpoint Formula

Consider two points, A and B, with coordinates (x_1, y_1) and (x_2, y_2), respectively.

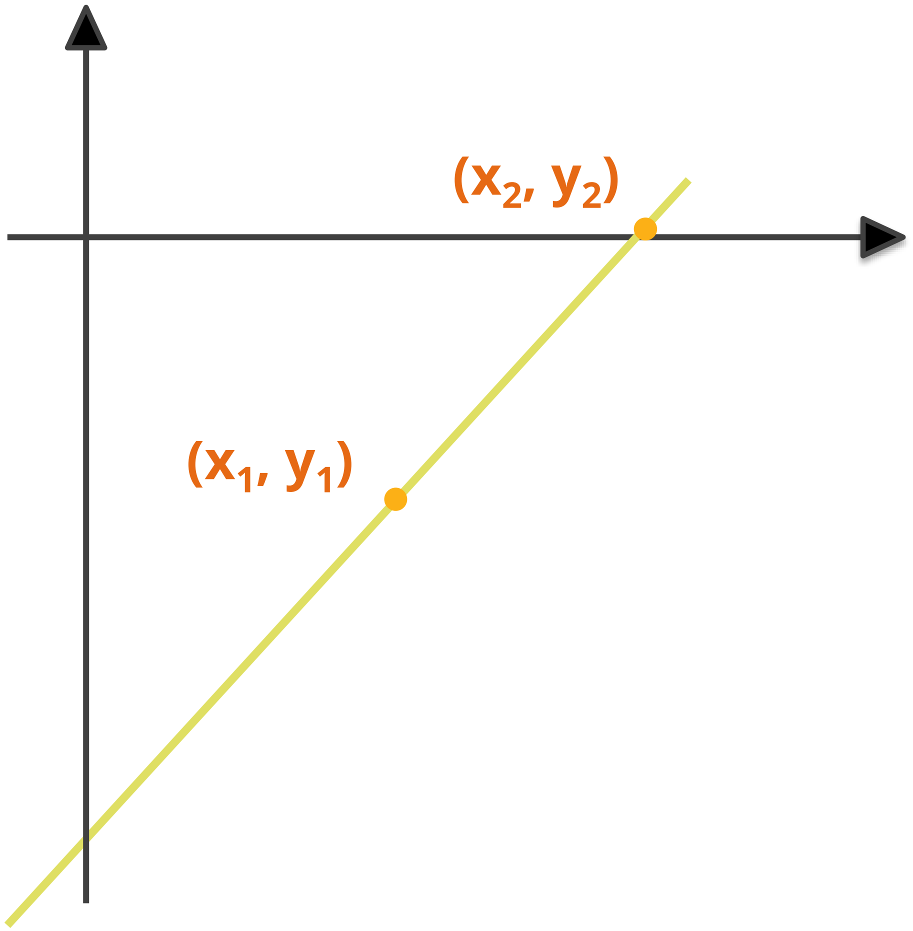
The coordinates of the midpoint, denoted as M(x, y), can be calculated using the following formula:

$$M(x, y) = (\frac{(x_1+x_2)}{2}, \frac{(y_1+y_2)}{2})$$

Coordinate Geometry: Line

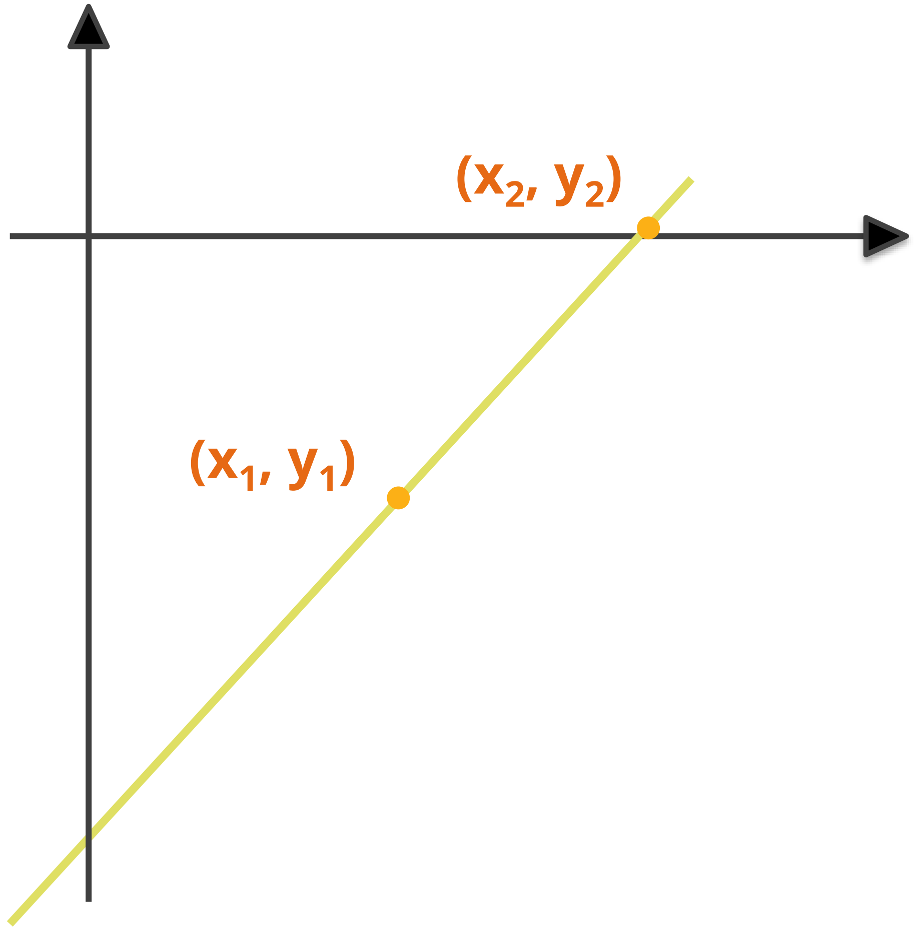
Line

In coordinate geometry, a line is a straight path that extends infinitely in both directions.



Line

The equation of a line relates the x and y coordinates of points on the line.



The y-intercept of a line is the point at which it intersects the y-axis.

The slope of a line represents its steepness or inclination and is defined as the ratio of the vertical change (rise) to the horizontal change (run) between two points on the line.

Equation of a Line

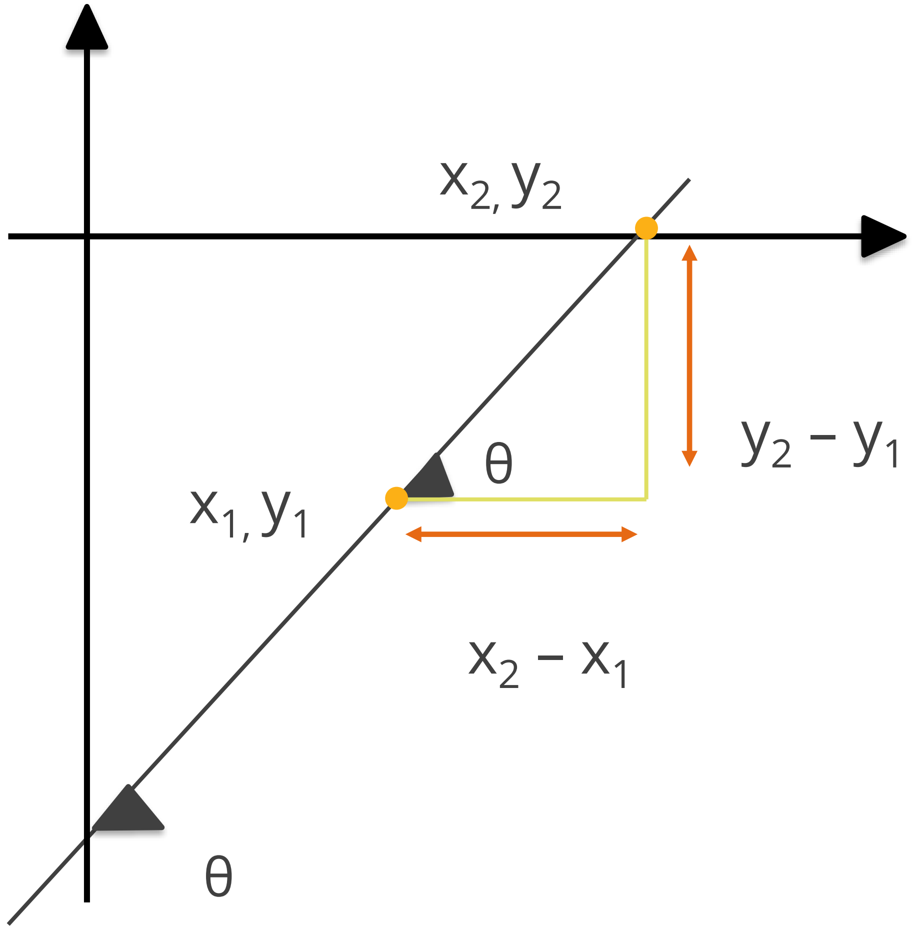
The table below provides the formulae for different forms of a line:

Formula Name	Formula
General Formula	$Ax + By = C$
Slope Intercept Form	$Y = mx + b$
Point Slope Form	$(y - y_1) = m(x - x_1)$

Here, m is the slope and b is the y-intercept.

Slope Formula

The slope of a line, often referred to as its gradient, is a numerical representation of the line's steepness and direction.

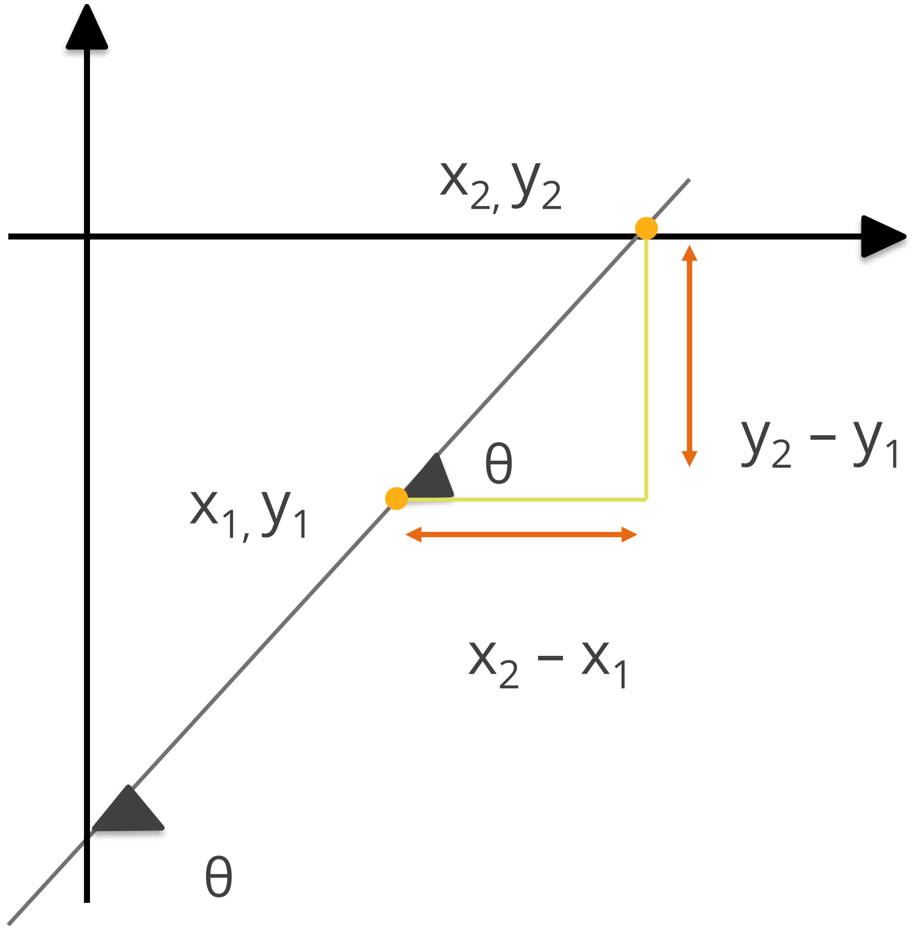


- It is calculated as the difference between the change in the x and y coordinates.
- The slope can be determined by selecting any two consecutive points on the line or using the angle that the line forms with the positive x-axis.

The slope of the vertical line at an angle θ with the x-axis vertical is, **m = tan(θ).**

Slope Formula

For a line segment formed by two points, the slope at an angle θ can be calculated using the following formula:

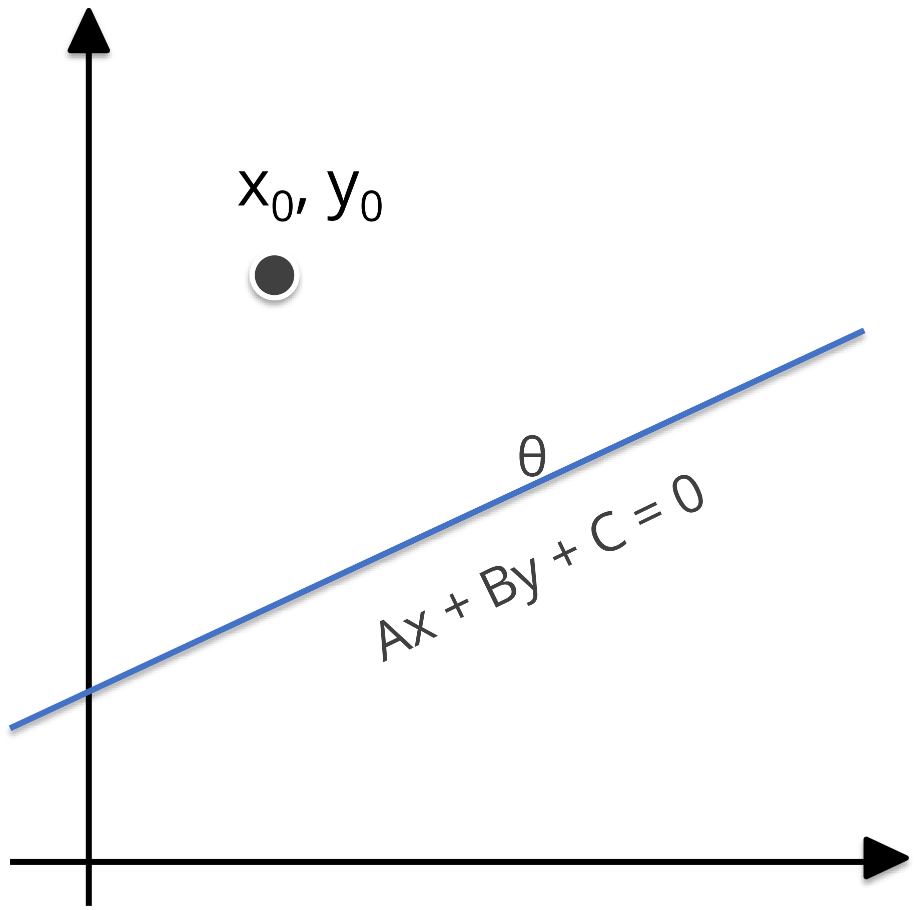


$$\tan(\theta) = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan(\theta)$$

Distance of a Point from Line

Consider a point $P(x_0, y_0)$ and a line represented by the equation $Ax + By + C = 0$.



The distance between the point and the line can be calculated using the following formula:

$$\frac{|Ax_0 + By_0 + C|}{\sqrt{a^2 + b^2}}$$

Applications of Measuring the Distance

Geometric analysis

To determine the spatial relationship and proximity between a point and a line

Line fitting

To evaluate the accuracy of regression models by calculating the distance between data points and the fitted line

Error estimation

To measure accuracy or deviation in numerical approximations or computational algorithms

Optimization problems

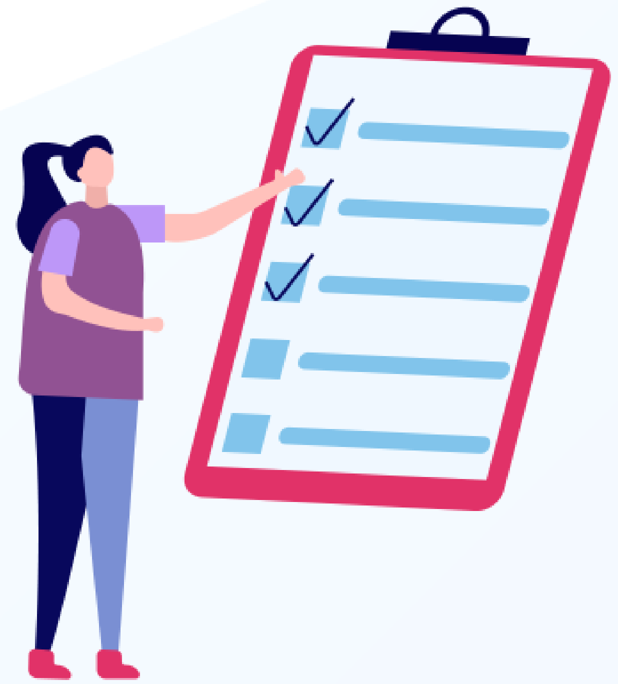
To minimize or maximize criteria by calculating distances to constraint lines

Key Takeaways

- Coordinate geometry is a branch of mathematics that examines geometric figures using a coordinate system and aids in algebraic computations.
- The distance formula is used to determine the distance between two points on a coordinate plane.
- The coordinate plane divides the plane space into two dimensions via the x and y axes, which further divide the plane into four quadrants. The point of intersection of these axes is known as the origin (0,0).

Key Takeaways

- The slope of a line provides a numerical representation of both its steepness and direction.
- The midpoint formula is employed to determine a point that lies exactly halfway between two points on a coordinate plane.



Knowledge Check

Knowledge Check 1

The distance of the point P(2, 3) from the x-axis is

A. 2

B. 3

C. 1

D. 5

Knowledge Check

1

The distance of the point P(2, 3) from the x-axis is

A. 2

B. 3

C. 1

D. 5

The correct answer is **B**

The distance of a point from the x-axis is equal to its ordinate, which is 3.

Knowledge Check

2

The distance of the point (α, β) from the origin is

A. $\propto + \beta$

B. $\propto^2 + \beta^2$

C. $|\propto| + |\beta|$

D. $\sqrt{\propto^2 + \beta^2}$

Knowledge Check 2

The distance of the point (α, β) from the origin is

A. $\propto + \beta$

B. $\propto^2 + \beta^2$

C. $| \propto | + |\beta|$

D. $\sqrt{\propto^2 + \beta^2}$

The correct answer is **D**

The distance of the point (α, β) from the origin can be calculated using the formula $\sqrt{(\propto - 0)^2 + (\beta - 0)^2}$

= $\sqrt{(\propto)^2 + (\beta)^2}$

Knowledge Check

3

What is the distance between the point P(4, -2) and the line 2x + 3y - 6 = 0?

A. $\frac{2}{\sqrt{13}}$

B. $\frac{3}{\sqrt{13}}$

C. $\frac{4}{\sqrt{13}}$

D. $\frac{6}{\sqrt{13}}$

Knowledge Check

3

What is the distance between the point P(4, -2) and the line 2x + 3y - 6 = 0?

A. $\frac{2}{\sqrt{13}}$

B. $\frac{3}{\sqrt{13}}$

C. $\frac{4}{\sqrt{13}}$

D. $\frac{6}{\sqrt{13}}$

The correct answer is **C**

The distance can be calculated using the formula: $|Ax_1 + By_1 + C| / \sqrt{(A^2 + B^2)}$. In this case, A = 2, B = 3, and C = -6. Distance = $|2(4) + 3(-2) - 6| / \sqrt{(2^2 + 3^2)} = |8 - 6 - 6| / \sqrt{(4 + 9)} = |-4| / \sqrt{13} = 4 / \sqrt{13}$.

Knowledge Check

4

Which of the following points has a slope of 2 with respect to the origin (0, 0)?

A. (2, 6)

B. (-3, 6)

C. (-1, -2)

D. (0, 3)

Knowledge Check
4

Which of the following points has a slope of 2 with respect to the origin (0, 0)?

A. (2, 6)

B. (-3, 6)

C. (-1, -2)

D. (0, 3)

The correct answer is **C**

The slope is calculated using the formula (y-0)/(x-0) = (-2 - 0) / (-1 - 0) = -2 / -1 = 2. Hence, the answer is C.

Knowledge Check

5

What is the midpoint between the points A (2, 5) and B (8, -1)?

A. (5, 2)

B. (6, 3)

C. (5, -3)

D. (4, 1)

Knowledge Check

5

What is the midpoint between the points A (2, 5) and B (8, -1)?

A. (5, 2)

B. (6, 3)

C. (5, -3)

D. (4, 1)

The correct answer is **A**

Midpoint is calculated using the formula $[(x_1+x_2)/2, (y_1+y_2)/2]$ = ((2 + 8) / 2, (5 + (-1)) / 2) = (10 / 2, 4 / 2) = (5, 2)