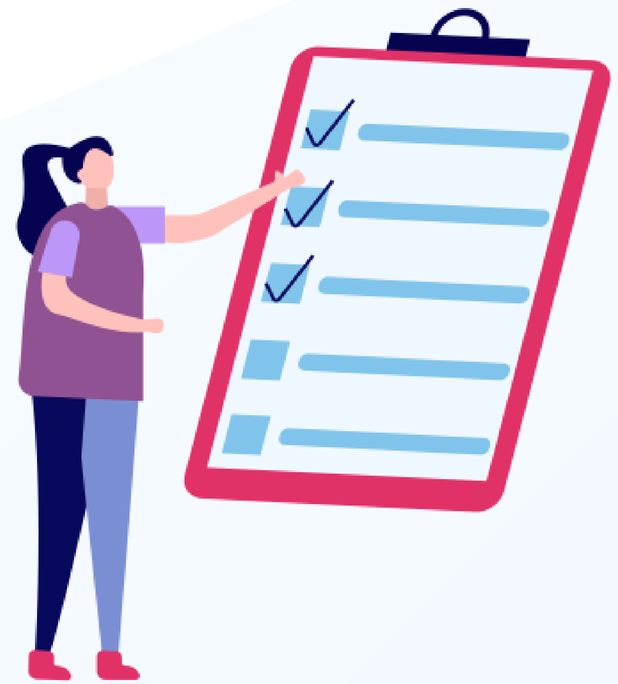
Math Refresher

Linear Algebra

Learning Objectives

By the end of this lesson, you will be able to:

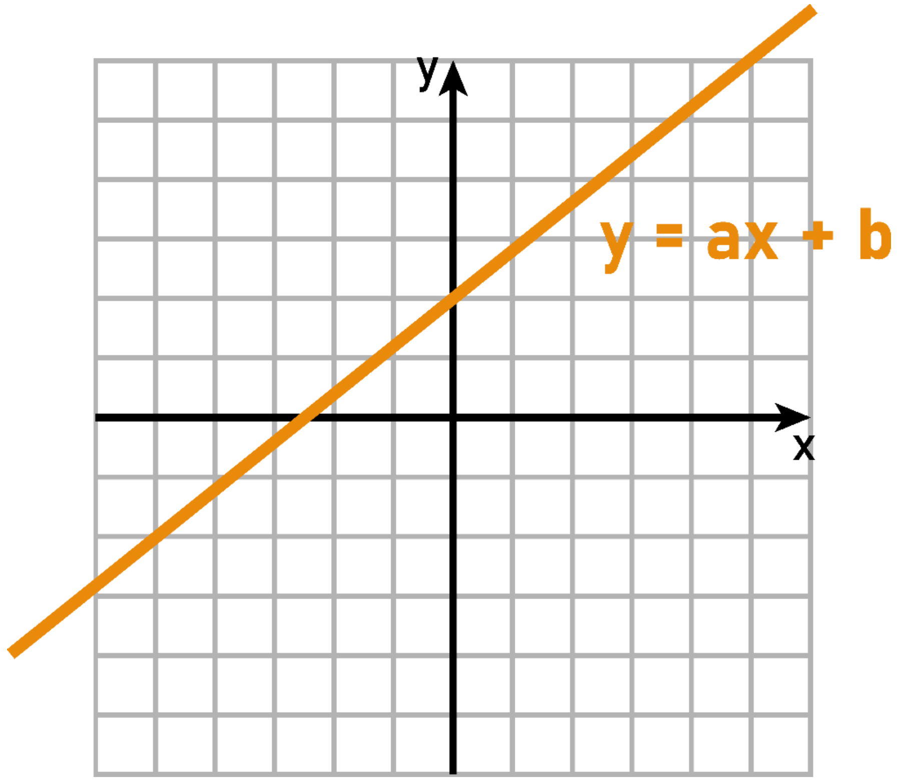
- Explain the concepts of linear algebra
- Apply linear algebraic concepts to solve linear equations
- Understand vectors and matrices
- Implement matrix operations such as addition, subtraction, and multiplication



Introduction to Linear Algebra

Linear Algebra

Linear algebra studies the manipulation and combination of linear equations.

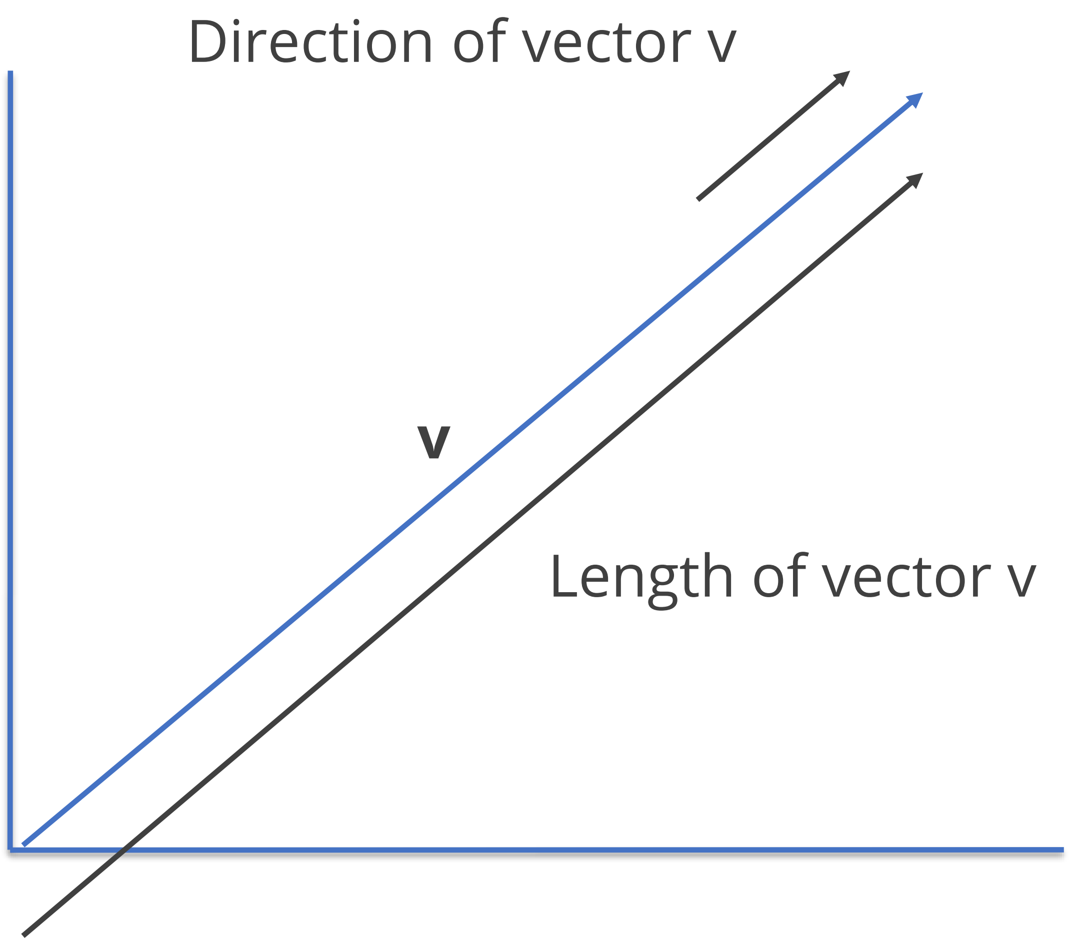


- Understanding vector spaces, lines, planes, and mappings are necessary for performing linear transformations.
- This field addresses linear functions, vectors, and matrices.
- It investigates the characteristics and behaviors of sets transformed using linear operations.

Introduction to Vectors

Vector

Objects with both magnitude and direction are called vectors.

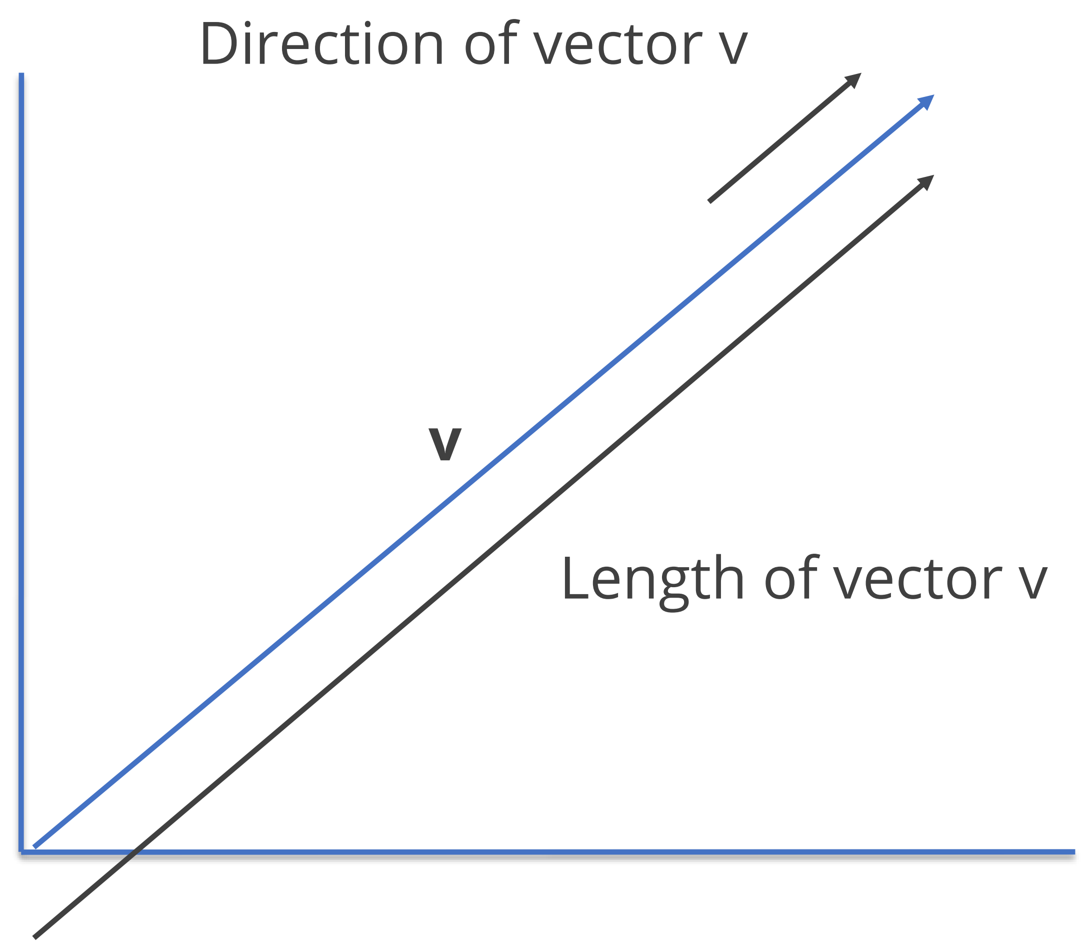


They have wide applications in various fields, including mathematics, physics, computer science, and engineering.

The magnitude of a vector dictates its size.

Vector

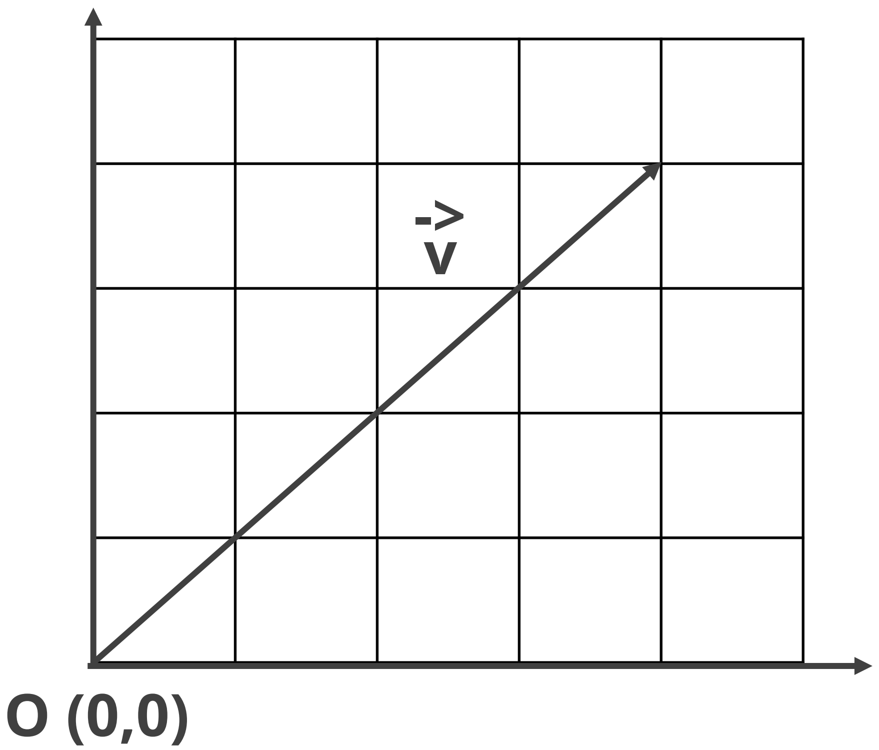
Objects with both magnitude and direction are called vectors.



Vectors can also be represented algebraically as ordered sets of numbers or coordinates.

Vector

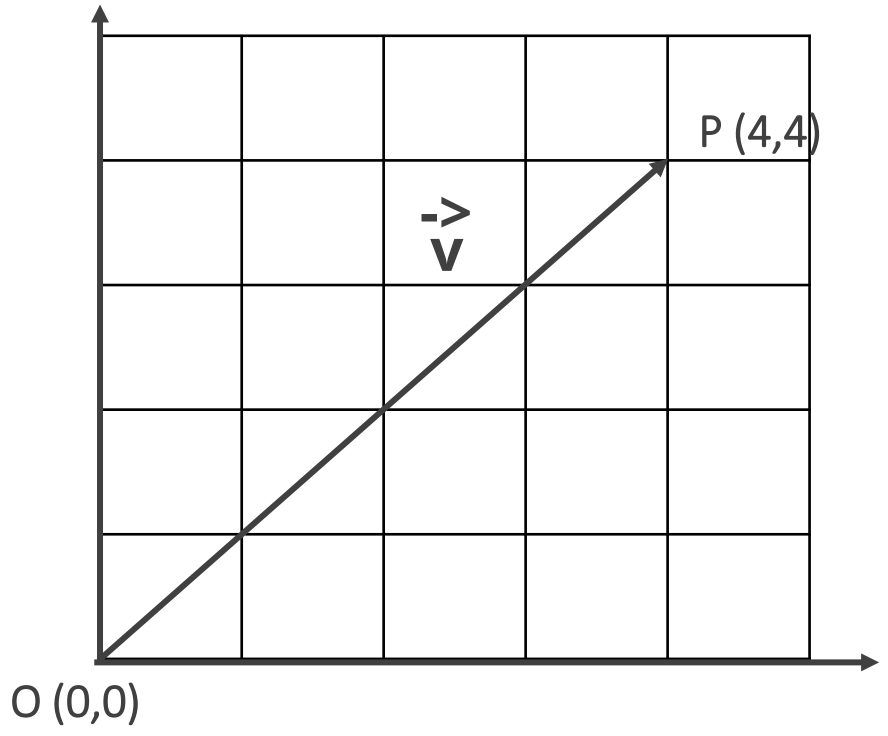
The vector's length, denoted by ||v||, originates at the origin (0,0).



It is also known by various names such as Euclidean vector, geometric vector, spatial vector, or simply a vector.

Vector Representation

Depending on the context and mathematical notation, vectors can be represented in numerous ways. Common representations of vectors include:



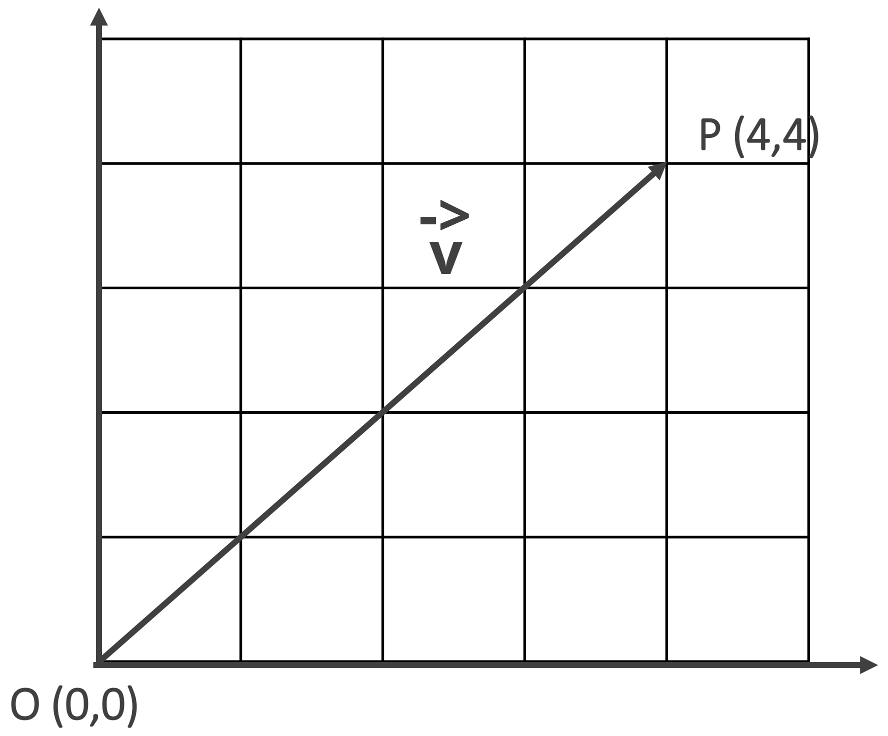
Geometric representation

$$\vec{V} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$$

Algebraic representation

Vector Representation

Geometric representation:



- The length of the arrow corresponds to the magnitude of the vector
- The direction of the arrow represents the direction of the vector.

For example, in a two-dimensional space, a vector v can be represented as an arrow from the origin (0,0) to the point (x, y).

Vector Representation

Algebraic representation:

$$\vec{V} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$$

A vector v can be represented as a column matrix or a tuple:

v = [x, y] or v = (x, y)

Here, x represents the horizontal component (x-coordinate) of the vector.

y represents the vertical component (y-coordinate) of the vector.

Types and Properties of Vectors

Types of Vectors

Vectors are of the following types:

Vector Name	Description
Row vector	It is represented horizontally; typically denoted as [a, b, c, ...]. It consists of a single row of elements.
Column vector	It is represented vertically; typically denoted as [a; b; c; ...]. It consists of a single column of elements.
Zero vector	All its components are zero. Adding the zero vector to any vector does not change the vector.
Unit vector	It has a magnitude (or length) of 1. It represents a direction or orientation without scaling or stretching.

Types of Vectors

Vectors are of the following types:

Vector Name	Description
Position vector	In geometry, a position vector specifies the location of a point relative to a reference point or origin. It represents the displacement from the origin to the point in a vector form.
Displacement vector	It represents the change or difference between two position vectors, describing the movement from one point to another.
Normal vector	In geometry and linear algebra, a normal vector is a perpendicular vector to a surface or object, commonly used to determine orientation, angles, or distances.
Direction vector	It specifies the direction of a line or geometric object without specifying its starting point or length, describing the orientation, slope of a line, or direction of motion.

Types of Vectors

Vectors are of the following types:

Vector Name	Description
Eigenvector	In linear algebra, an eigenvector of a square matrix is a non-zero vector that, when multiplied by the matrix, results in a scalar multiple of itself. Eigenvectors play a crucial role in the study of eigenvectors and eigenvalues.
Orthogonal vector	If xTy = 0, vectors x and y are orthogonal, meaning they are at a 90-degree angle from each other.
orthonormal vector	It is an orthogonal vector with a unit norm.
Basis vector	It forms a set of linearly independent vectors that span a vector space.

Properties of Vectors

Vectors have the following properties:

Commutativity of vector addition
For vectors u and v,
$u + v = v + u$

Associativity of vector addition
For vectors u, v, and w,
$(u + v) + w = u + (v + w)$

Zero vector
For any vector u, $u+0=u$

Additive inverse
For any vector u, $u+(-u)=0$

Properties of Vectors

Vectors have the following properties:

Scalar multiplication associativity
For vectors u, (c * d) * u = c * (d * u)

Scalar multiplication distributivity
For vectors u and v and scalars c and d,
(c + d) * u = c * u + d * u, and
c * (u + v) = c * u + c * v

Scalar multiplication identity
For any vector u, 1 * u = u

Magnitude of scalar multiples
|c * u| = |c| * |u|

Properties of Vectors

Vectors have the following properties:

Orthogonality

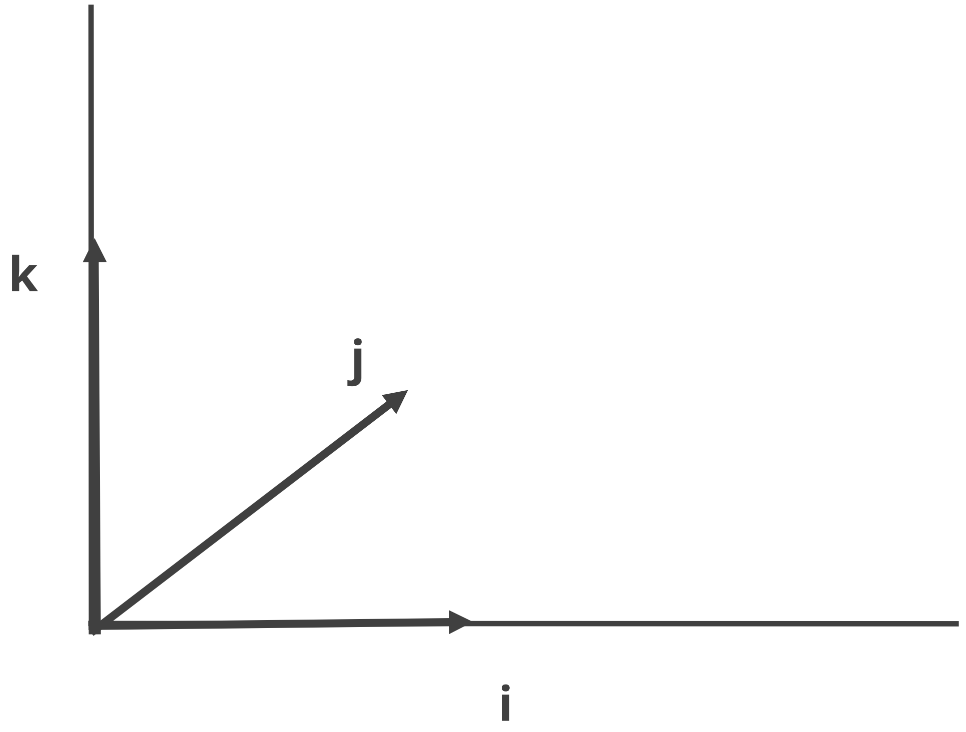
For vectors u and v, $u \cdot v = 0$.

Triangle inequality

For vectors u and v, $|u + v| \leq |u| + |v|$.

Properties of Vectors

In the mentioned example, vectors i and k are orthogonal.



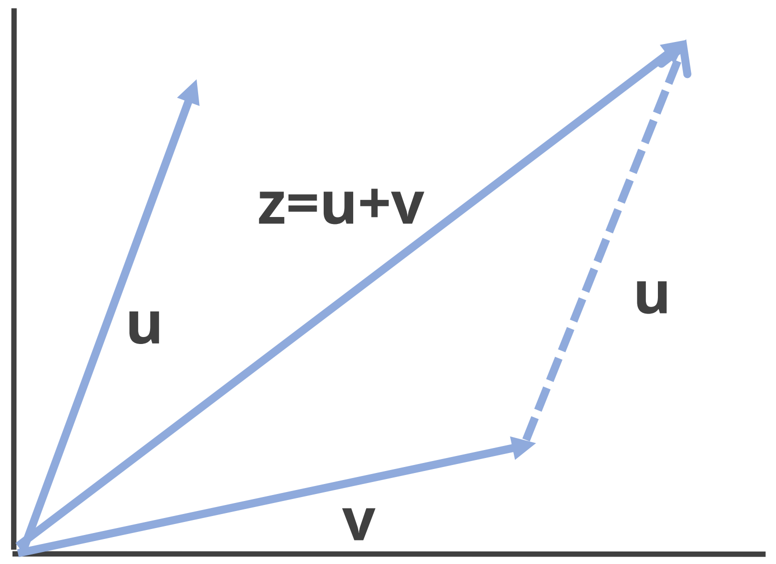
- If considered to have a unit norm, they can be classified as orthonormal.
- Vectors k and j are not orthogonal.
- Similarly, vectors j and i are not orthogonal.

Vector Operations

Vector Operations: Addition

Vector addition refers to the operation of combining two or more vectors to yield a vector sum.

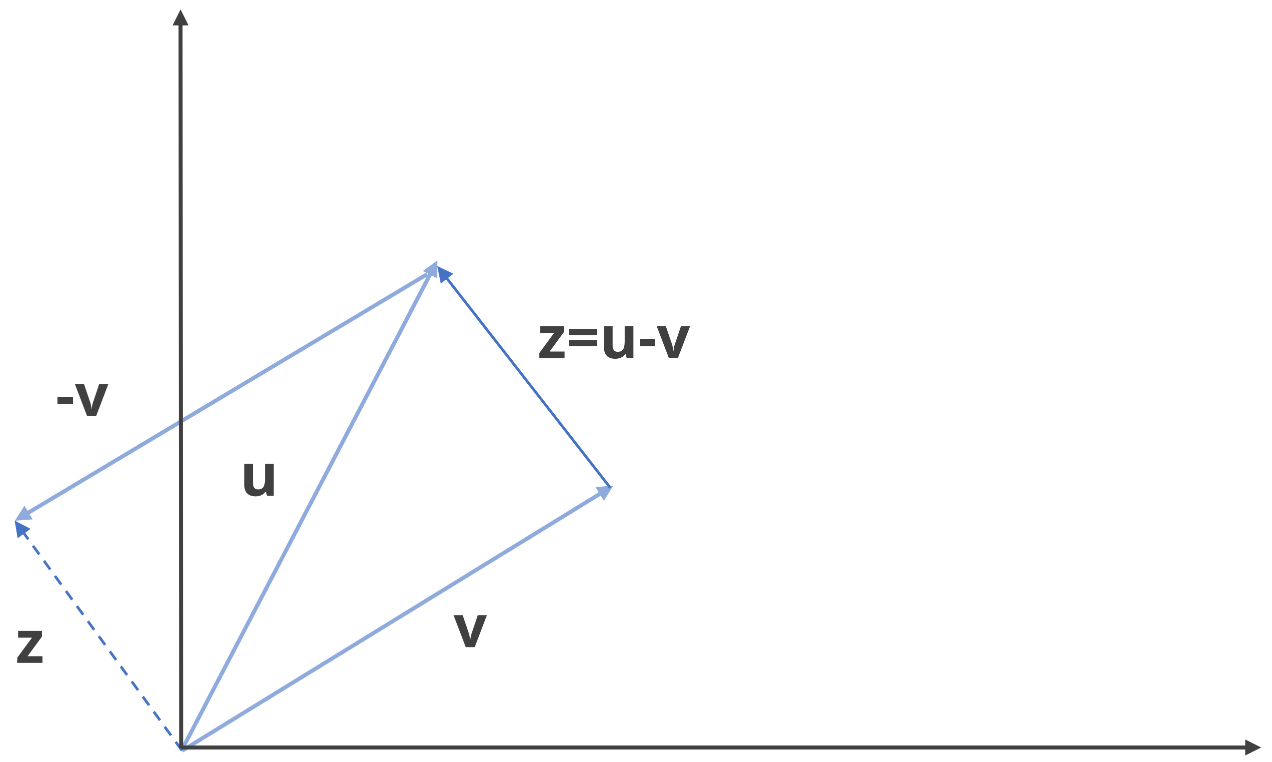
The vector sum for the two vectors **u** and **v** is u + v = z



Vector Operations: Subtraction

Vector subtraction involves the process of deducting two or more vectors to derive a vector difference.

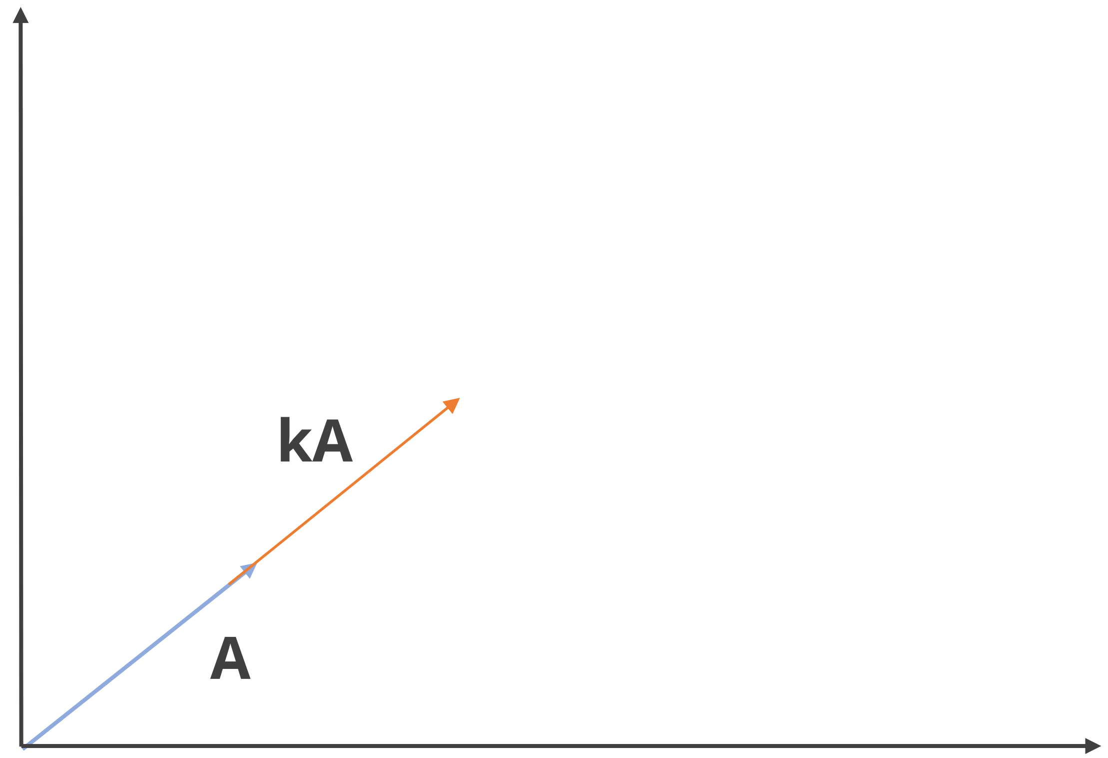
The vector difference for the two vectors **u** and **v** is u - v = z



Vector Operations: Multiplication

Scalar multiplication is the process of multiplying a vector by a scalar value.

For a vector $A = [A_1, A_2, A_3]$ and scalar k,
$kA = [kA_1, kA_2, kA_3]$



Scalar multiplication affects the magnitude and direction of the vector.

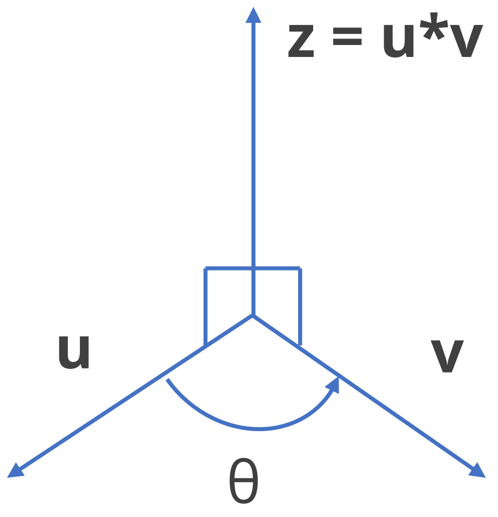
Vector Operations: Multiplication

The multiplication of vectors can be described by cross-products and dot products, as:

u * v = z

$u*v = (x_2y_3 + x_3y_2)\,\hat{\imath} + (x_1y_3 + x_3y_1)\,\hat{\jmath} + (x_1y_2 + x_2y_1)\,k$

$u*v = |x|\,|y|\,\sin\theta$

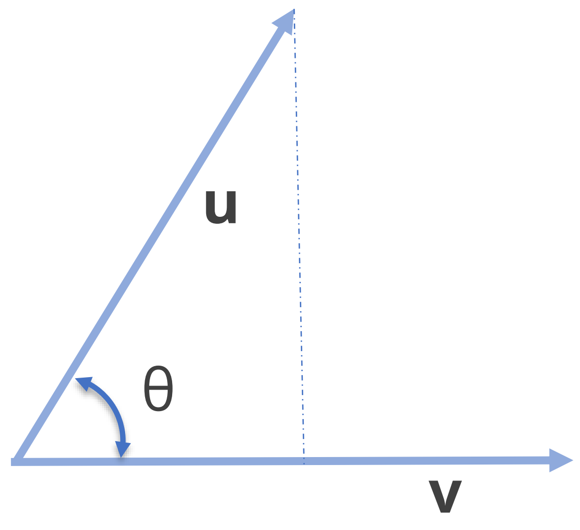


$$\begin{vmatrix} i & j & k \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}$$

u . v = z

$u.v = x_1y_1 + x_2y_2 + x_3y_3$

$u.v = |x|\,|y|\,\cos\theta$



Vector Operations: Dot Product vs. Cross Product

The representations for dot and cross products in vector form are as follows:

Dot product

$$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

$$\vec{a} \bullet \vec{b} = a_1b_1 + a_2b_2$$

Cross product

$$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$$\vec{a} \times \vec{b} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$$

Vector Operations: Norm

In machine learning, the size of a vector is often referred to as its norm, which indicates the distance from the origin to a specific point, represented by the vector's norm.

Consider a vector with an Lp norm where p ≥ 1

$$||X||_p = \left(\sum_i^n |x_i|^p\right)^{\frac{1}{p}}$$

Here:

- X_p symbolizes a vector with xi denoting the vector's individual components.
- $|x_i|^p$ indicates each component of the vector raised to the power of p.
- The entire expression, raised to the power of $(1/p)$, signifies the p-norm or Lp-norm of the vector.

Vector Operations: Norm Features

Here are the norm features for the vector operation:

The L2 norm, also referred to as the Euclidean norm, is the most used norm where p equals 2.

It measures the Euclidean distance between the origin and point x.

Often, the 2 in the L2 norm is dropped, and $||x||_2$ is written as $||x||$.

Vector Operations: Norm Features

The squared L2 norm, denoted as xTx, is often used to measure vector size and is preferred because its derivative depends on x.

$$||X||_2^2 = \sum_i^n |x_i|^2$$

Here,

- X symbolizes a vector with xi representing the vector's components.
- $|x_i|$ signifies the absolute value of x.

Vector Operations: Norm Features

The L1 norm increases whenever an element of x deviates from 0 by ϵ.

The infinite maximum norm can be computed as follows:

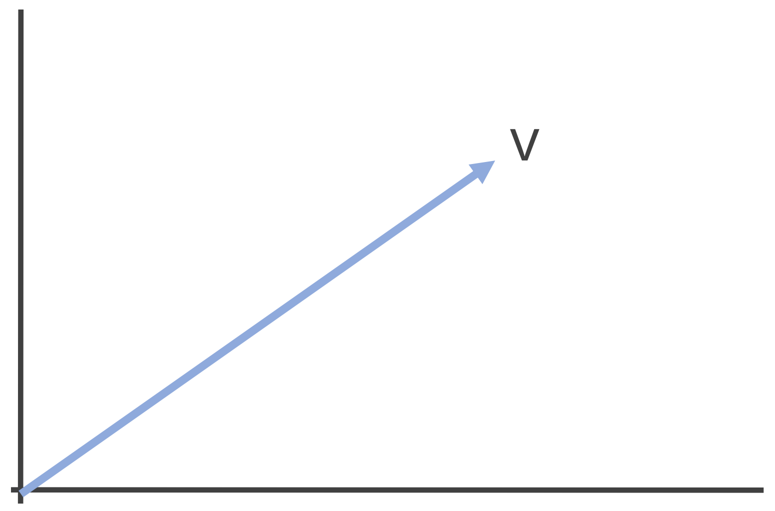
$$P: \|x\|_\infty = max_i |x_i|$$

Where:

- In this context, x represents a vector with xi signifying the vector's individual components.
- $|x_i|$ indicates the absolute value of x_i

Vector: Magnitude and Norm

In vector terminology, **magnitude** and **norm** both denote the size or length of a vector, representing the same concept, and are often used interchangeably.



- The magnitude or norm quantifies a vector's length or size, independent of its direction.
- It is always a non-negative scalar value and it is denoted by ||v|| or |v|.

Vector: Magnitude and Norm

Consider a vector $v = [v_1, v_2, \ldots, v_n]$:

The magnitude can be calculated using the formula:

$$||v|| = \sqrt{{v_1}^2 + {v_2}^2 + \cdots . + vn^2}$$

The magnitude (norm) of v is calculated using the Euclidean norm, also known as the 2-norm.

Here, $v_1, v_2, \ldots, v_n$ represent the components of the vector in n-dimensional space.

Applications of Vectors in data science

Vectors have various applications in Data Science, including:

Feature representation	Similarity and distance metrics	Machine learning algorithms
Vectors effectively represent features in data sets, with each feature corresponding to a dimension of the vector.	Vector operations, such as the dot product and Euclidean distance, serve to gauge the similarity or dissimilarity between data points.	Vectors play a critical role in numerous machine learning algorithms, including linear regression, support vector machines, and neural networks.

Introduction to Matrices

Matrix

A matrix is a rectangular array or table composed of rows and columns filled with numbers, symbols, or expressions, representing a mathematical object or property.

$$A = \begin{pmatrix} 2 & 1 & 3 \\ -1 & 2 & 4 \end{pmatrix} \qquad B = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$$

Matrix Size

The size of the matrix is expressed as **m X n.**

Where:
m is the number of rows
n is the number of columns

A matrix featuring two rows and three columns is known as a two-by-three matrix, a 2 X 3 matrix, or a matrix of dimension 2 X 3.

Notation of Matrix

Below is an illustration of a matrix with m rows and n columns:

$$\begin{array}{cc} & \overbrace{\begin{array}{cccc} 1 & 2 & \cdots\cdots & n \end{array}}^{\text{Columns}} \\ \text{Rows}\left\{\begin{array}{c} 1 \\ 2 \\ 3 \\ \vdots \\ m \end{array}\right. & \begin{bmatrix} a_{11} & a_{12} & \cdots\cdots & a_{1n} \\ a_{21} & a_{22} & \cdots\cdots & a_{2n} \\ a_{31} & a_{32} & \cdots\cdots & a_{3n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots\cdots & a_{mn} \end{bmatrix} = A_{m \times n} \end{array}$$

Forms of Matrix

Different types of matrices:

Square matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Diagonal matrix

$$D = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

Identity matrix

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Forms of Matrix

Different types of matrices:

Upper triangular matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 0 & 0 & 9 \end{bmatrix}$$

Lower triangular matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 5 & 0 \\ 7 & 8 & 9 \end{bmatrix}$$

Symmetric matrix

$$I = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$$

Matrix Operations

Matrix Operations: Addition

Consider the following two matrices:

$$A = \begin{pmatrix} 22 & 32 \\ 11 & 16 \end{pmatrix} \qquad B = \begin{pmatrix} 13 & 8 \\ 13 & 16 \end{pmatrix}$$

$$A + B = \begin{pmatrix} 22 + 13 & 32 + 8 \\ 11 + 13 & 16 + 16 \end{pmatrix}$$

$$A + B = \begin{pmatrix} 35 & 40 \\ 24 & 32 \end{pmatrix}$$

Matrix Operations: Addition Rules

Matrix addition operation follows the following rules:

- Matrices can only be added if they contain the same number of rows and columns.
- Matrix addition follows the commutative property, which can be expressed as A + B equals B + A.

Matrix Operations: Subtraction

Consider the following matrices:

$$A = \begin{pmatrix} 22 & 32 \\ 11 & 16 \end{pmatrix} \qquad B = \begin{pmatrix} 13 & 8 \\ 13 & 16 \end{pmatrix}$$

$$A - B = \begin{pmatrix} 22 - 13 & 32 - 8 \\ 11 - 13 & 16 - 16 \end{pmatrix}$$

$$A - B = \begin{pmatrix} 9 & 24 \\ -2 & 0 \end{pmatrix}$$

Matrix Operations: Subtraction Rules

Matrix subtraction operation follows the following rules:

- Matrices can only be subtracted if they possess the same number of rows and columns.
- Matrix subtraction does not allow the commutative property, which can be illustrated as A - B not being equal to B - A.

Matrix Operations: Subtraction Rules

The subtraction of matrices follows the following properties:

Commutative property: $A - B \neq B - A$

Associative property: $(A - B) - C = A - (B + C)$

Matrix Operations: Multiplication

Consider the following matrices:

$$A = \begin{pmatrix} 22 & 32 \\ 11 & 16 \end{pmatrix} \qquad B = \begin{pmatrix} 13 & 8 \\ 13 & 16 \end{pmatrix}$$

$$A . B = \begin{pmatrix} (22 \times 13) + (32 \times 13) & (22 \times 8) + (32 \times 16) \\ (11 \times 13) + (16 \times 13) & (11 \times 8) + (16 \times 16) \end{pmatrix} = \begin{pmatrix} 702 & 688 \\ 351 & 344 \end{pmatrix}$$

In this scenario, the first and second rows of A are multiplied by the first and second columns of B and then added together.

Matrix Operations: Multiplication Rules

Matrix multiplication operation follows the following rules:

Associativity

For matrices A, B, and C of appropriate dimensions, the following equality holds:

$(AB)C = A(BC)$

Distributivity

For matrices A, B, and C of appropriate dimensions, the following equality holds:

$A(B + C) = AB + AC$

Non-commutativity

$AB = BA$ is true only if it is a diagonal matrix or scalar multiples of the identity matrix.

Identity matrix

Multiplying a matrix by the identity matrix does not change the matrix.

$AI = A$

$IA = A$

Matrix Operations: Multiplication Rules

Matrix multiplication operation follows the following rules:

Zero matrix

Multiplying a matrix by the zero matrix results in the zero matrix.

$A0 = 0$

$0A = 0$

Scalar multiplication

If A is an m x n matrix and k is a scalar, then the following equality holds:

kA = Ak = [k * Aij]

(for each element Aij)

Transposition

If A is an m x n matrix, B is an n x p matrix, and C = AB, then the following equality holds:

$C^t = (AB)^t = B^t A^t$

Inverse matrices

If A is an n x n invertible matrix with inverse A^(-1), then the following equality holds:

$AA^{(-1)} = A^{(-1)}A = I$

Matrix Operations: Transpose

A transpose is a matrix formed by turning all the rows of a given matrix into columns and vice versa. The transpose of matrix A is denoted as A^T.

$$\mathbf{A} = \begin{pmatrix} 22 & 32 \\ 11 & 16 \end{pmatrix} \qquad \mathbf{A^T} = \begin{pmatrix} 22 & 11 \\ 32 & 16 \end{pmatrix}$$

Matrix Operations: Determinant

The determinant is a scalar value that can be computed from the elements of a square matrix and encodes certain properties of the linear transformation described by the matrix

The determinant of a matrix A is denoted as $\det(A)$ or $|A|$.
The determinants for matrices of various sizes are shown below:

a. 1X1 matrix

$A = \begin{bmatrix} a \end{bmatrix}$

$\det(A) = a$

b. 2X2 matrix

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$\det(A) = ad - bc$

c. 3X3 matrix

$$A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$$

$\det(A) = aei + bfg + cdh - ceg - afh - bdi$

Matrix Operations: Determinant

Let’s find the determinant of a 2X2 matrix:

Example

$$A = \begin{pmatrix} 9 & 8 \\ 10 & 11 \end{pmatrix}$$

$\det(A) = 11*9 – 10*8$

$\det(A) = 99 – 80$

$\det(A) = 19$

Matrix Operations: Inverse

If A is a non-singular square matrix, there exists an n x n matrix, identified as A's inverse matrix, which satisfies the following property:

$$AA^{-1} = A^{-1}A = I$$

where I is the Identity matrix

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

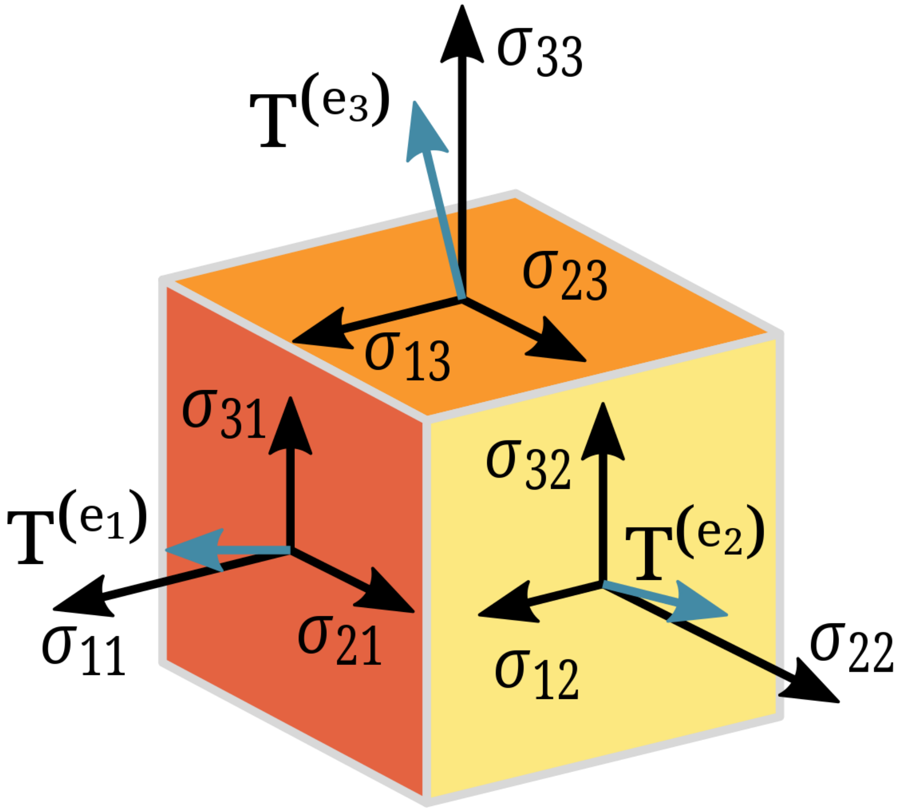
$AB = BA = I^n$

Where:

I^n denotes the nXn identity matrix.

Special Matrix Types: Tensors

Tensors are arrays with more than two axes.



- A tensor can have N dimensions.
- $A_{i,j,k}$ is the value at the coordinates i, j, k.

Source: wikipedia

Matrix: Applications in Data Science

Matrices find wide applications in various fields of Data Science:

- Data representation and manipulation
- Linear transformations
- Linear regression and optimization

$\begin{pmatrix} 1 & 0 \end{pmatrix}$

- Matrix decompositions
- Graph theory and network analysis
- Machine learning algorithms

Linear Equations

Linear Equations

A linear equation is an algebraic equation in which the highest power of the variable(s) is 1. When plotted on a Cartesian plane, it represents a straight line.

A linear equation with n variables has the following form:

$$a_1x_1+a_2x_2+.....+a_nx_n = b$$

Here,

- $x_1+x_2+.....+x_n$ represent the unknown quantities to be found
- $a_1+a_2+.....+a_n$ are the coefficients
- b is the constant term

Linear Equations

A linear equation featuring two variables x, and y, establishes a linear relationship between them.

The value of one variable, often y, depends on the value of the other, usually x.

Here, x serves as the independent variable.

Y is considered the dependent variable, given its dependency on x.

Linear Equations: Example

Here are a few examples of different types of linear equations:

Linear equation with one variable	Linear equation with two variables	Linear equation with three variables
$3x+5=0$	$y+7x=3$	$x + y + z = 0$
$(3/2)x +7 = 0$	$3a+2b = 5$	$a - 3b = c$
$98x = 49$	$6x+9y-12=0$	$3x + 12 y = \frac{1}{2} z$

Identifying Linear and Non-linear Equations

The table below classifies linear and non-linear equations:

Equations	Linear or non-linear
$y = 8x - 9$	Linear
$y = x^2 - 7$	Non-linear, the power of the variable x is 2
$\sqrt{y} + x = 6$	Non-linear, the power of the variable y is 1/2
$y + 3x - 1 = 0$	Linear
$y^2 - x = 9$	Non-linear, the power of the variable y is 2

Forms of Linear Equation

Forms of Linear Equation

Following are the three forms of linear equations:

- Standard form
- Slope intercept form
- Point-slope form

Linear Equation in Standard Form

The formula for one-variable single-line calculations is as follows:

Equation

$$A_x + B = 0$$

Where:

- A and B are real integers
- x is the variable

The formula for two-variable single-line calculations is as follows:

Equation

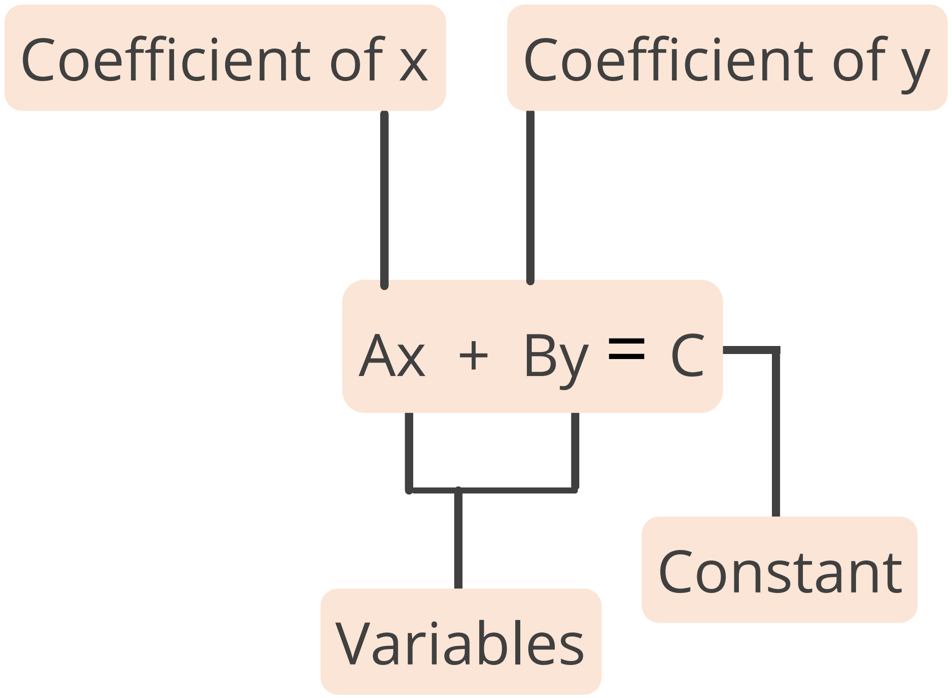
$$A_x + B_y = C$$

Where:

- A, B, and C are real integers
- x and y are the variables

Linear Equation in Standard Form

Linear equations have the following components:



Linear Equation in Slope Intercept Form

The slope of a linear equation can be calculated to understand how a unit change in one variable affects the variation in another variable.

Slope equation

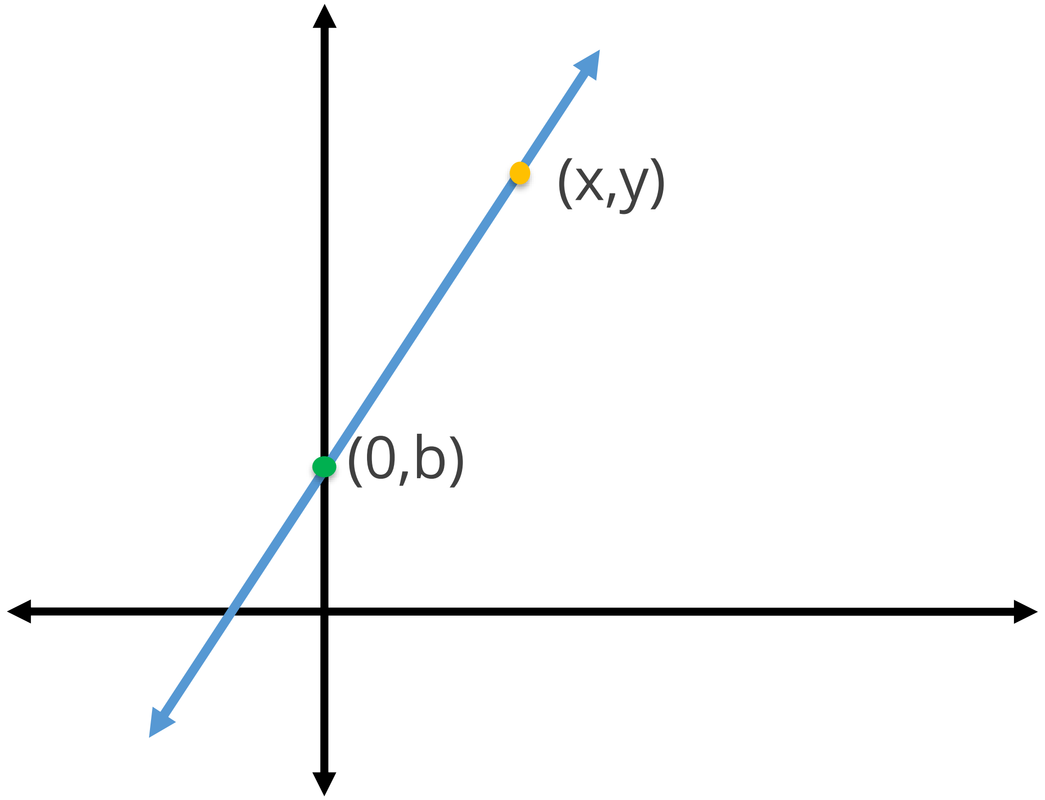
$$y = mx + b$$

Where:

- m is the slope.
- b is the intercept.
- x and y represent the distances of a point on the line from the x-axis and y-axis, respectively.

Linear Equation in Slope Intercept Form

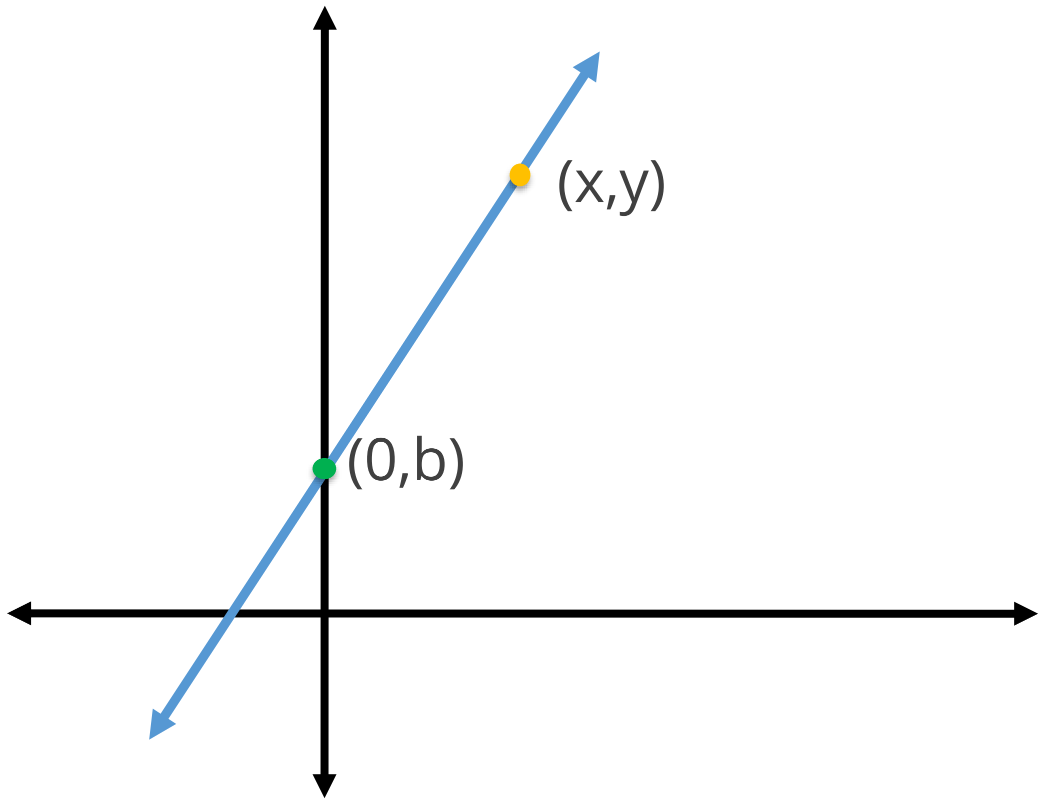
Consider the following graph:



A point (x, y) on the line represents its distance from the x-axis and y-axis, respectively.

The line intercepts the x-axis at the point (0, b).

Linear Equation: Slope



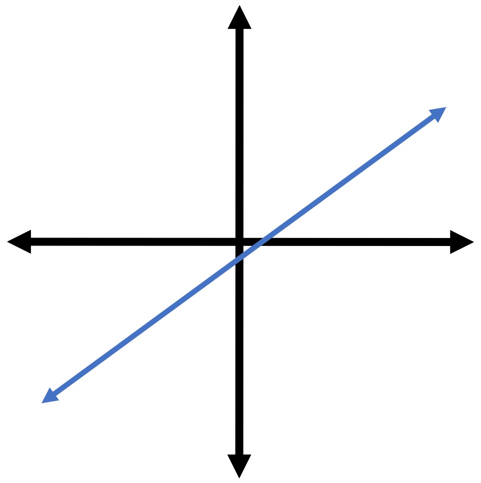
The slope indicates the line's steepness relative to the y-axis.

Viewed from left to right, the slope indicates whether the line ascends or descends.

The slope describes how the independent variable changes in response to variations in the dependent variable.

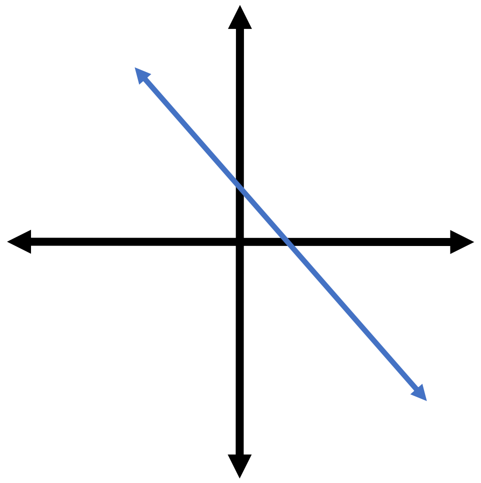
Types of Slope

There are four types of slopes, each representing a different relationship between the variables x and y. These include:



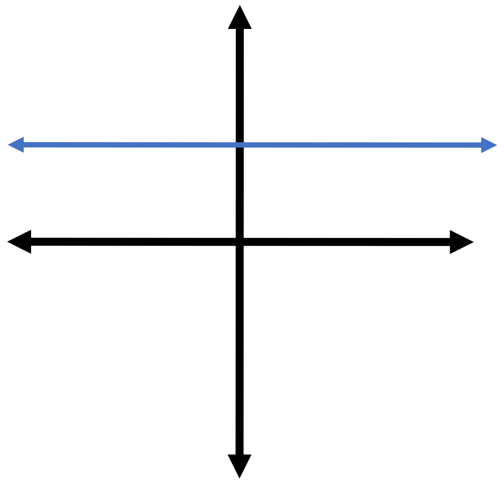
Positive

Slope >= 0



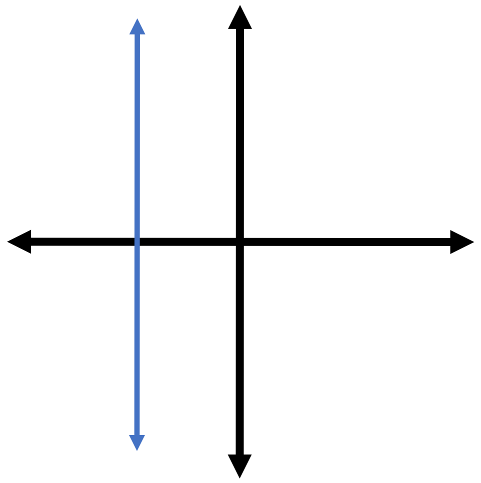
Negative

Slope <= 0



Zero

Slope = 0



Undefined

Slope = ∞

Linear Equation in Point-Slope Form

In point-slope form, a straight line is represented by its slope and a specific point on the line.

Equation:

$$y - y_1 = m(x - x_1)$$

Where

(x_1, y_1) are the coordinates of the point.

Linear Equations Forms: Example

Consider solving the following linear equation:

(2x – 10)/2 = 3(x – 1)

Step 1:
Clear the fraction

x-5 = 3(x-1)

Step 2:
Simplify both sides of the equation

x-5 = 3x-3

x = 3x+2

Step 3:
Solve for x

x-3x = 2

-2x = 2

x = -1

System of Linear Equations

A system of linear equations refers to a finite set of linear equations.

Equation:

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n &= b_2 \\
&\dots\dots\dots\dots \\
a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n &= b_m
\end{aligned}
$$

System of Linear Equations

A system of linear equations has the following features:

A single linear equation within this system may have an infinite number of solutions, one solution, or no solutions at all.

A linear system that offers a solution is labeled as consistent.

Conversely, an inconsistent linear system does not yield any solution.

Solving a Linear Equation

Solving a Linear Equation: Need

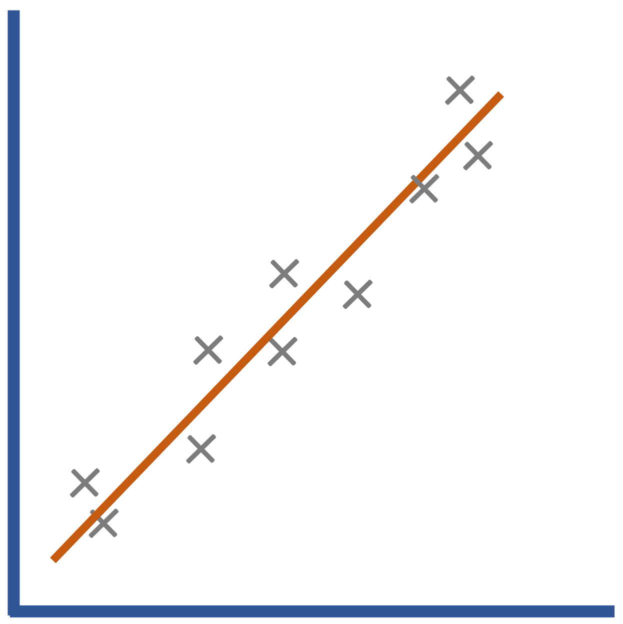
Solving linear equations holds critical importance as it enables the determination of variable values that satisfy the provided equations.

Here are several reasons underscoring the importance of solving linear equations:

- Find solutions
- Understand relationships
- Make predictions
- Optimize solutions
- Validate equations
- Analyze data

Solving a Linear Equation

Several methods can be employed to solve systems of linear equations, including:



1. Graphic method
2. Substitution method
3. Linear combination or elimination method
4. Matrix method

Solving Systems of Linear Equations Using Graphing

Solve the following system of linear equations through graphing:

$y=0.5x+2$; $y=-2x-3$

Both equations are in the slope-intercept form.

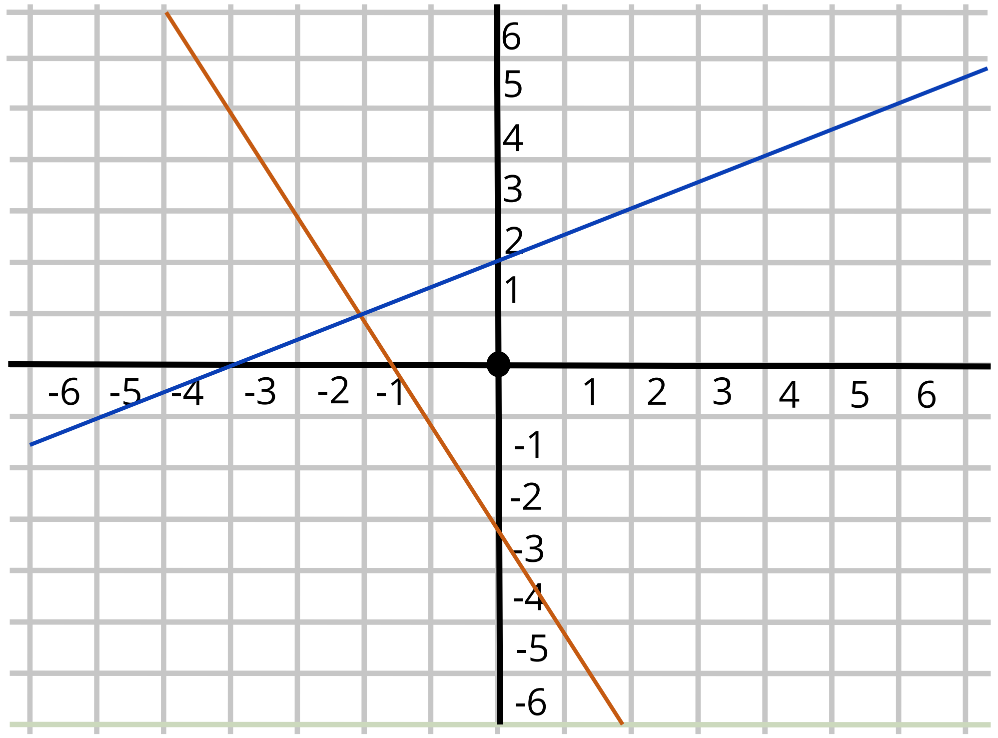
The first line has a slope of 0.5 and a y-intercept of 2.

The second line possesses a slope of -2 and a y-intercept of -3.

Solving Systems of Linear Equations Using Graphing

The graph below illustrates the intersection of two given lines:

Graph for the given lines



The two lines intersect at the point (-2,-1). Therefore, the solution to this system of equations is (-2,-1), where x equals -2 and y equals 1.

Solving Systems of Linear Equations Using Substitution: Steps

Follow these steps to solve systems of linear equations using the substitution method:

01 Rewrite one of the equations in the form "variable =......"

02 Substitute this variable into the other equation

03 Solve the resulting equation

04 If necessary, repeat steps 1 through 3

Solving Systems of Linear Equations Using Substitution

Consider solving the following linear equation: $3x + 2y = 19$; $x + y = 8$

1. Begin with any equation and variable
2. Look at the second equation with the variable y

$$3x+2y = 19 \longrightarrow (1)$$

$$x+y = 8 \longrightarrow (2)$$

Solving Systems of Linear Equations Using Substitution

Follow these steps to solve the linear equations $3x + 2y = 19$ and $x + y = 8$ using the substitution method:

Step 1:
Subtract x from both sides of $x + y = 8$

$$3x + 2y = 19$$
$$y = 8 - x$$

Step 2:
Replace "y" with "8 − x" in the other equation

$$3x + 2(8 - x) = 19$$
$$y = 8 - x$$

Step 3:
Solve using the algebraic methods

$$3x + 16 - 2x = 19$$
$$y = 8 - x$$

Solving Systems of Linear Equations Using Substitution

Step 4:
Solve 3x – 2x

$x + 16 = 19$
$y = 8 - x$

Step 5:
Solve 19 - 16

$x = 3$
$y = 8 - x$

Step 6:
Put x = 3 in the equation y = 8 - x

$x = 3$
$y = 8 - 3$
$y = 5$

Answer:
$x = 3, y = 5$

Solving Systems of Linear Equations Using Elimination: Steps

Steps to solve systems of linear equations using the elimination method:

01 Multiply an equation by a constant (except zero)

02 Add (or subtract) an equation onto another equation

Example:

$$3x + 2y = 19$$

$$x + y = 8$$

Solving Systems of Linear Equations Using Elimination

Follow these steps to solve the linear equations $3x + 2y = 19$ and $x + y = 8$ using the elimination method:

Step 1: Multiply the second equation by 2

$3x+2y=19$
$2x + 2y = 16$

Step 2: Subtract the second equation from the first equation

$x = 3$
$2x + 2y =16$

Step 3: Multiply the second equation by ½

$X =3$
$X + Y = 8$

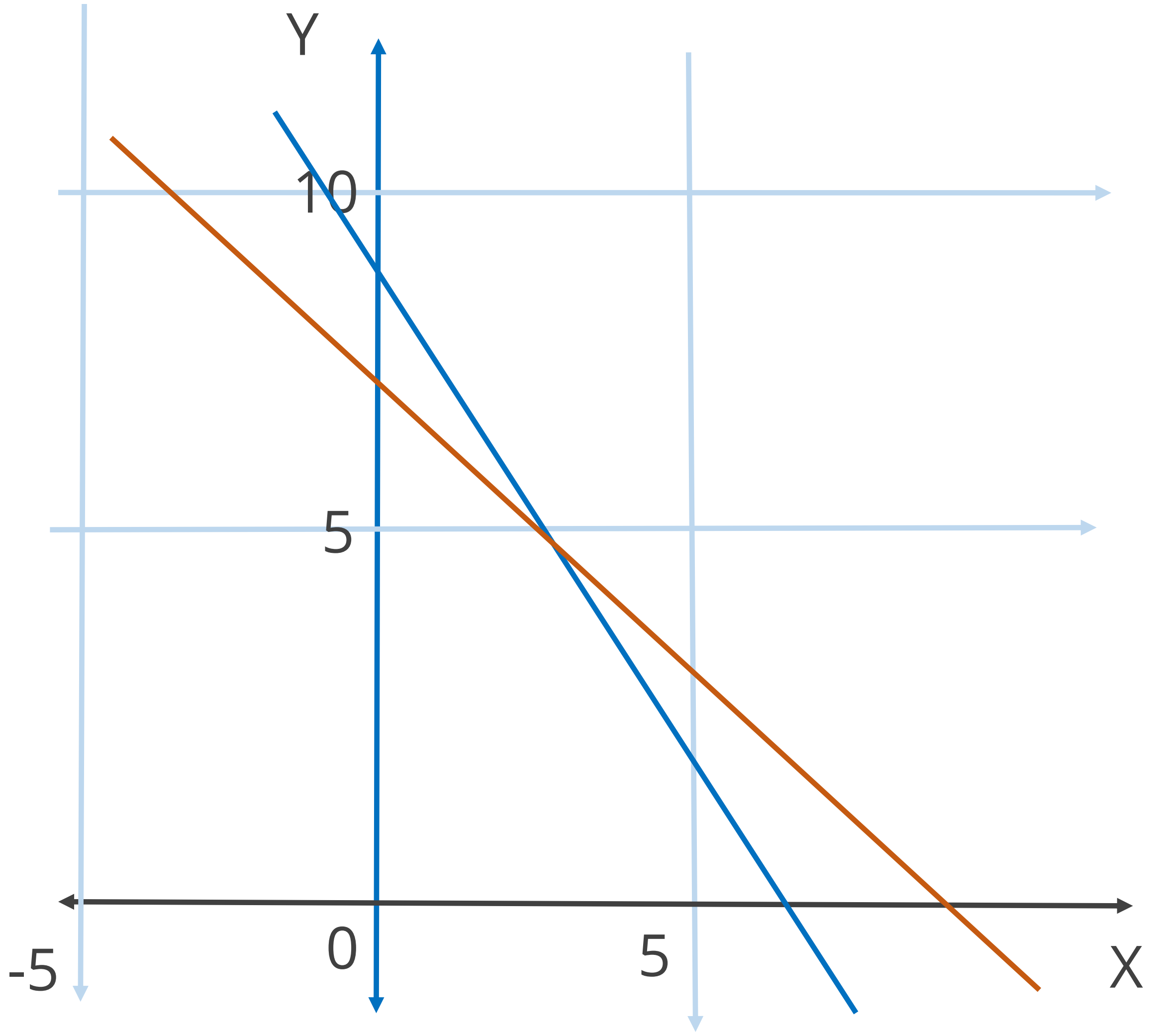
Step 4: Subtract the first equation from the second equation

$y = 5$

Answer: X=3, Y=5

Solving Systems of Linear Equations Using Elimination

The graph below illustrates the intersection of the two equations: $3x + 2y=19$ and $x + y=8$.



The blue line denotes the set of points where $3x + 2y = 19$ holds true.

The red line represents the set of points where $x + y = 8$ is accurate.

The solution to the system is (3, 5), the point where both lines intersect.

Solving Systems of Linear Equations: Applications

The system of linear equations has wide applications, including:

Parameter estimation	In data science, linear equations are often used to model relationships between variables.
Data analysis and visualization	Solving linear equations helps analyze and visualize data.
Optimization problems	Many data science problems involve optimization, where the goal is to maximize or minimize an objective function.
Feature engineering	Solving linear equations helps in feature engineering, which involves transforming or creating new features from existing ones.

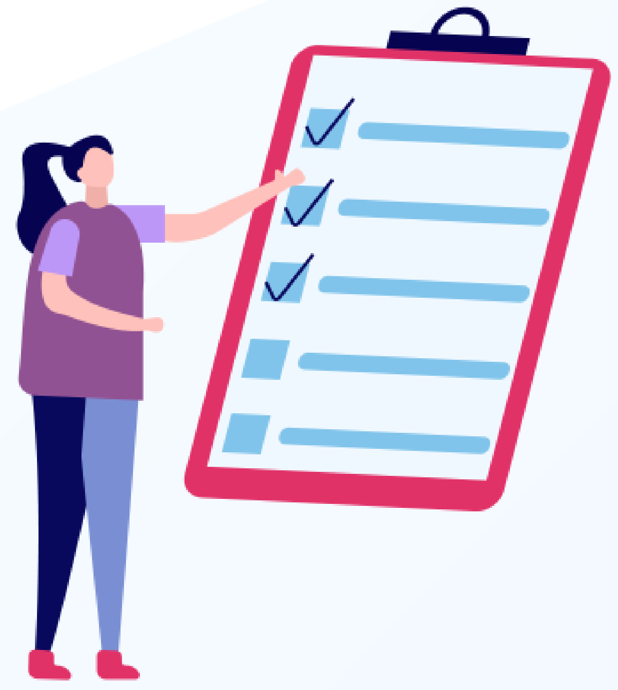
Solving Systems of Linear Equations: Applications

The system of linear equations has wide applications, including:

Dimensionality reduction	Solving linear equations is fundamental to techniques like principal component analysis (PCA).
Model interpretation	Solving linear equations allows for the interpretation of model parameters in linear models.
Signal processing and image analysis	Linear equations and systems of equations are extensively used in signal processing and image analysis.

Key Takeaways

- Linear algebra introduces systematic methods for solving systems of linear equations.
- There are three common forms of linear equations: standard form, slope-intercept form, and point-slope form.
- The graphing method, substitution method, elimination method, or matrix method can be used to solve linear equations.
- A matrix is an $m \times n$ array of scalars drawn from a specified field, with individual values in the matrix referred to as entries.
- Vectors, featuring both magnitude and direction, are essential objects in linear algebra.



Knowledge Check

Knowledge Check 1

What is the necessary condition for a solution to exist for the system Ax = b?

A. a must be invertible

B. b must be linearly dependent on the columns of A

C. b must be linearly independent of the columns of A

D. None of these

Knowledge Check 1

What is the necessary condition for a solution to exist for the system Ax = b?

A. a must be invertible

B. b must be linearly dependent on the columns of A

C. b must be linearly independent of the columns of A

D. None of these

The correct answer is **A**

For the system of linear equations Ax = b to have a solution, it is necessary that the inverse of matrix A exists (i.e., |A| ≠ 0).

Knowledge Check 2

Suppose the cost of 2 balls and 1 bat is 100 units. How can this problem be expressed in linear algebra with the variables x and y?

A. $2x + y = 100$

B. $2x + 2y = 100$

C. $2x + 4y = 200$

D. $x + y = 100$

Knowledge Check 2

Suppose the cost of 2 balls and 1 bat is 100 units. How can this problem be expressed in linear algebra with the variables x and y?

A. $2x + y = 100$

B. $2x + 2y = 100$

C. $2x + 4y = 200$

D. $x + y = 100$

The correct answer is **A**

Assume the price of a bat equates to x units and the price of a ball equates to y units. The values of x and y can vary based on the given scenario, as they are variables. Consequently, the problem translates into the linear algebraic equation: $2x + y = 100$.

Knowledge Check 3

What does a linear equation involving three variables represent?

A. A flat object

B. A line

C. A plane

D. Both A and C

Knowledge Check
3

What does a linear equation involving three variables represent?

A. A flat object

B. A line

C. A plane

D. Both A and C

The correct answer is **C**

A linear equation involving three variables denotes a set of points. The coordinates of these points satisfy the given equation. Essentially, a linear equation with three variables represents a plane.

Knowledge Check

4

Which of the following is NOT a type of matrix?

A. Square matrix

B. Scalar matrix

C. Diagonal matrix

D. Term matrix

Knowledge Check

4

Which of the following is NOT a type of matrix?

A. Square matrix

B. Scalar matrix

C. Diagonal matrix

D. Term matrix

The correct answer is **D**

The term matrix is not a type of matrix in linear algebra.