Math Refresher

Introduction to Calculus

Learning Objectives

By the end of this lesson, you will be able to:

- Define differential calculus
- Explain the concepts of calculus
- Examine the limits, continuity, and derivatives of a function
- Define integral calculus
- List the differential and integral formulas

Basics of Calculus

What Is Calculus?

Calculus is the branch of mathematics that studies change. It analyzes rates, slopes, areas, volumes, and optimization. Calculus consists of differential calculus, which deals with instantaneous change, and integral calculus, which focuses on accumulation and areas.

Source link: http://wwwmath.mit.edu/~djk/calculus_beginners/chapter01/section02.html

What Is Calculus?

- Calculus is a fundamental area of mathematics that focuses on continuous change.
- It is based on two key concepts: derivatives and integrals.
- Calculus is also referred to as infinitesimal calculus or the calculus of infinitesimals.
- Infinitesimal numbers are quantities that are nearly zero but not exactly zero.
- Traditional calculus primarily revolves around studying continuous changes in functions.

Source link: http://wwwmath.mit.edu/~djk/calculus_beginners/chapter01/section02.html

Derivatives vs. Integrals

Derivatives

- It is a study of the rate at which quantities change.
- They involve concepts like derivatives and differentiability.
- Derivatives represent the instant rate of change of a function at a specific point.
- They are used to analyze motion, calculate slopes, find maximum and minimum values, and study rates of change in various fields.

Integrals

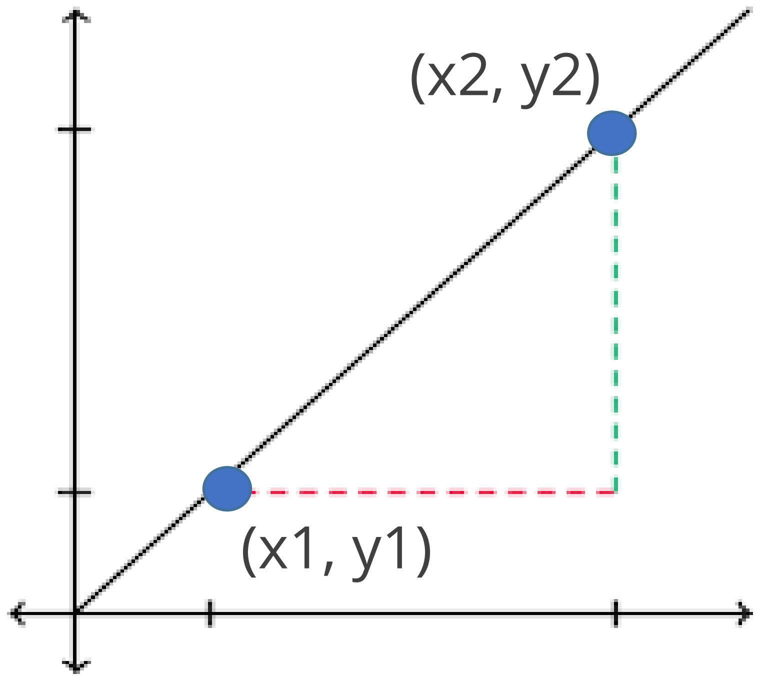
- They deal with finding the accumulation or total value of quantities over an interval.
- Involve concepts like integrals and integration.
- Integrals represent the area under the curve and summation of infinitesimal quantities.
- They are used to calculate areas, volumes, and average values, and solve problems involving accumulation or continuous change.

Differential Calculus

Differential Calculus

Differential calculus is a branch of calculus that focuses on studying the rates at which quantities change.

- Consider a scenario where x and y are real numbers, and y is a function of x, represented as y = f(x).
- If f(x) corresponds to a linear equation, it can be expressed as y = mx + c, where m represents the slope.
- The value of the slope, denoted by m, can be determined using the slope equation.

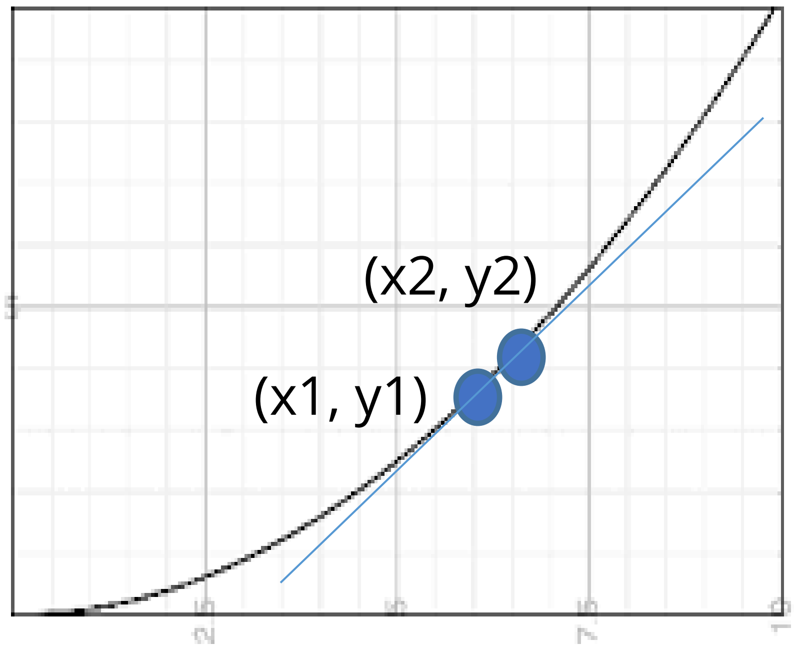


Slope equation

$$m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{\Delta y}{\Delta x}$$

Differential Calculus

It is a branch of calculus that focuses on studying the rates at which quantities change.



- **Δy/Δx**, also represented as **dy/dx**, represents the derivative of y with respect to x. It measures the rate of change of y per unit change in x.
- The slope of the tangent line at a specific point on the curve represents the rate of change at that point.

Limits

Limits are mathematical concepts that explain how a function behaves as the input values approach a particular value.

The limit equation is expressed as follows:

$$\lim_{x \to c} f(x) = A$$

This equation is read as **the limit of f of x as x approaches c equals A**.

The Derivative: Instantaneous Rate of Change

The derivative of a function at a specific point represents the instantaneous rate of change.

It is denoted as $f'(a)$, $df/dx|a$, or $dy/dx|a$.

It provides valuable insights into how the function behaves at a precise location.

Derivatives

Derivatives represent the rate at which a function changes with respect to its independent variable.

The derivative of a function is expressed as follows:

$$f'(x) = \lim_{x \to h} [f(x+h) - f(x)]/h = A$$

Derivatives measure how variables change in response to changes in other variables.

Calculating Derivatives: Fundamental Techniques

There are several rules and techniques available for calculating derivatives, including:

Power Rule:
Differentiate power functions of the form $f(x) = x^n$, where n is a constant.

Quotient Rule:
Differentiate the quotient of two functions.

Trigonometric and Exponential Rules:
Differentiate trigonometric and exponential functions.

Chain Rule:
Differentiate composite functions.

Product Rule:
Differentiate the product of two functions

Continuity

A function f(x) is considered continuous at a specific point x = a, if and only if it satisfies three conditions.

The three conditions of continuity are as follows:

- $f(a)$ is defined.
- $\lim_{x \to a} f(x)$ exists
- $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$ exists

Differential Formulas

Differential Formulas

Differentiation formulas apply to algebraic, trigonometric, inverse trigonometric, and exponential functions.

Differential formula examples

$$\frac{d}{dx} x^n = n x^{n-1}$$

$$\frac{d}{dx} \text{Constant} = 0$$

$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} a^x = a^x \log a$$

$$\frac{d}{dx} \log x = \frac{1}{x}$$

$$\frac{d}{dx} \sin x = \cos x$$

$$\frac{d}{dx} \cos x = -\sin x$$

$$\frac{d}{dx} \tan x = \sec^2 x$$

$$\frac{d}{dx} \cot x = -\operatorname{cosec}^2 x$$

$$\frac{d}{dx} \sec x = \sec x \tan x$$

$$\frac{d}{dx} \operatorname{cosec} x = -\operatorname{cosec} x \cot x$$

Applications of Differentiation

Rate of change	To calculate rate of change in physics, engineering, and economics
Tangent lines and curvature	To analyze motion and determine curve shape in physics and engineering
Optimization	To find maximum or minimum values for profit, cost, and design parameters
Related rates	To solve problems involving changing variables with respect to time

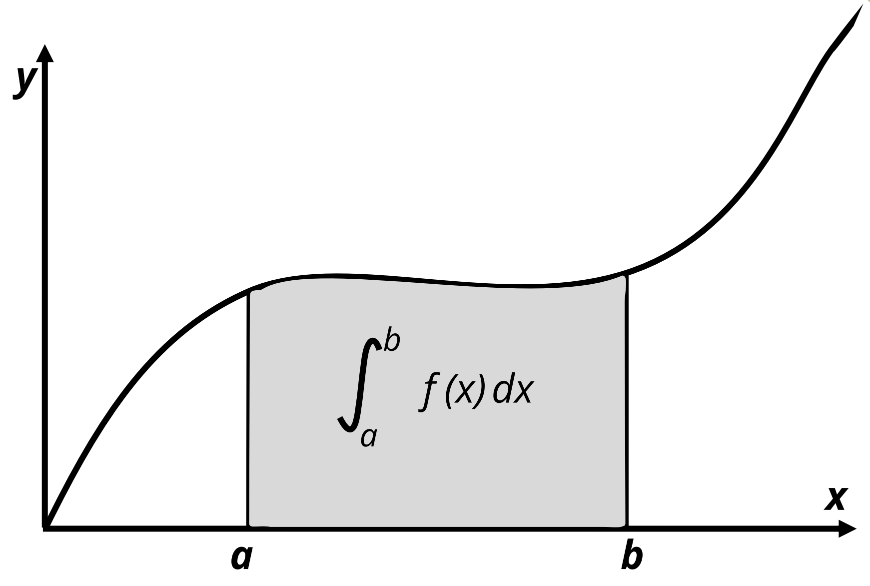
Applications of Differentiation

Growth and decay	To model exponential growth or decay in population dynamics and finance
Curve sketching	To determine critical points, inflection points, and concavity of functions
Linear approximations	To approximate complex functions with simpler linear approximations
Control systems	To model dynamic behavior and develop control strategies in various fields

Integral Calculus

Integral Calculus

Integral calculus assigns numerical values to functions in order to describe concepts such as displacement, area, volume, and other phenomena that result from the combination of infinitesimal data.



- Consider a function **f** that depends on a real variable **x** and an interval **[a, b]** on the real line.
- The definite integral defines the signed area of the region in the xy-plane that is enclosed by the graph of **f**.
- The boundaries of this region are the x-axis, as well as the vertical lines x = a and x = b.

Integral Calculus

The following is an expression for integral calculus:

$$\int_{a}^{b} f(x)\, dx$$

It is read as the integral from a to b of f(x) with respect to x.

Definite Integral

- A definite integral involves specific limits or boundaries for computing the function.
- The upper and lower limits, also known as cutoff points, determine the range of integration for the function.

The following is the equation of a definite integral:

$$\int_a^b f(x)\,dx = F(x)$$

Indefinite Integral

- An indefinite integral does not have specific limits (upper and lower limits).
- The result of the integration is connected to a constant value (C), which accounts for the various possible solutions of the integral.

The following is an equation for the indefinite integral:

$$\int f(x)\,\mathrm{d}x = F(x) + C$$

Integration Formulas

Integration Formulas

Integral formulas can be derived from differentiation formulas and serve as complements to differentiation formulas.

Integration examples

$$\int x^n \, \mathrm{d}x = \frac{x^{n+1}}{n+1} + C$$

$$\int 1 \, \mathrm{d}x = x + c$$

$$\int \mathrm{e}^x \, \mathrm{d}x = \mathrm{e}^x + c$$

$$\int \left(\frac{1}{x}\right) \mathrm{d}x = \log|x| + C$$

$$\int a^x \, \mathrm{d}x = \left(\frac{a^x}{\log a}\right) + C$$

$$\int \cos x \, \mathrm{d}x = \sin x + C$$

$$\int \sin x \, \mathrm{d}x = -\cos x + c$$

$$\int \sec^2 x \, \mathrm{d}x = \tan x + c$$

$$\int \cos ec^2 x \, \mathrm{d}x = -\cot x + c$$

$$\int \sec x \tan x \, \mathrm{d}x = \sec x + c$$

$$\int \operatorname{cosec} x \cot x \, \mathrm{d}x = -\operatorname{cosec} x + c$$

Applications of Integration

Area calculation

Integration is useful for calculating areas under curves, which finds uses in physics, engineering, and geometry.

Accumulation of quantities

Integration sums quantities over intervals, which has applications in physics and economics.

Volume and surface area

Integration determines volumes and surface areas of solids, aiding engineering and manufacturing.

Probability and statistics

Integration calculates probabilities and cumulative distribution functions, important in statistics.

Applications of Integration

Physical systems and dynamics

Integration analyzes physical systems, calculates momentum and energy.

Center of mass and moments

Integration finds center of mass and moments of inertia, relevant in physics and engineering.

Signal processing

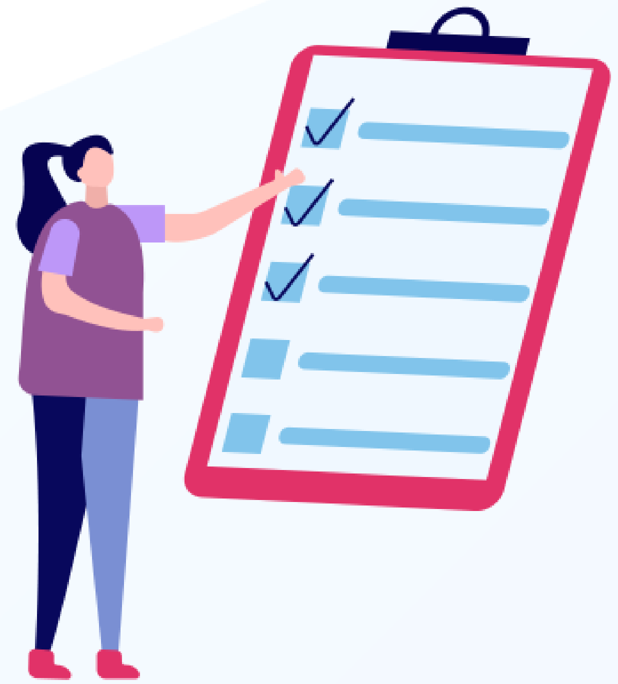
Integration processes signals, removes noise and interference.

Cost analysis and economics

Integration helps with cost, revenue, and profit calculations in economics.

Key Takeaways

- Differential calculus is a branch of calculus that focuses on studying the rates at which quantities change.
- Integral calculus assigns numerical values to functions to describe displacement, area, volume, and other concepts that arise from combining infinitesimal data.
- A definite integral has specific limits or boundaries that determine the computation range for the function.
- Derivatives measure the instantaneous rate of change of one quantity with respect to another.



Knowledge Check

Knowledge Check 1

Let $f(x)\,dx = \frac{x^2-1}{x-1}$, What is $\lim_{x \to 1} f(x)$?

A. 0

B. 1

C. 2

D. None of these

Knowledge Check 1

Let $f(x)\,\mathrm{d}x = \frac{x^2-1}{x-1}$, **What is** $\lim\limits_{x\to 1} f(x)$**?**

A. 0

B. 1

C. 2

D. None of these

The correct answer is **C**

$\boldsymbol{f(x)\,\mathrm{d}x = \frac{x^2-1}{x-1} = \frac{(x-1)(x+1)}{x-1} = x+1}$**, Now as** $\boldsymbol{x \to 1, \lim\limits_{x\to 1}(x+1) = 1+1 = 2}$

Knowledge Check

2

What is meant by the differential?

A. It is a word used a lot on a popular medical television series.

B. It is a method of directly relating how changes in an independent variable affect changes in a dependent variable.

C. It is a gearbox on the back end of your car.

D. None of these

Knowledge Check 2

What is meant by the differential?

A. It is a word used a lot on a popular medical television series.

B. It is a method of directly relating how changes in an independent variable affect changes in a dependent variable.

C. It is a gearbox on the back end of your car.

D. None of these

The correct answer is **B**

The differential is a method of directly relating how changes in an independent variable affect changes in a dependent variable.

Knowledge Check 3

What is the integral of sin(x) with respect to x?

A. $\int \sin x \, \mathrm{d}x = -\cos x + c$

B. $\int \sin x \, \mathrm{d}x = \operatorname{cosec} x + c$

C. $\int \sin x \, \mathrm{d}x = -\sec x + c$

D. $\int \sin x \, \mathrm{d}x = -\cot x + c$

Knowledge Check

3

What is the integral of sin(x) with respect to x?

A. $\int \sin x \, \mathrm{d}x = -\cos x + c$

B. $\int \sin x \, \mathrm{d}x = \operatorname{cosec} x + c$

C. $\int \sin x \, \mathrm{d}x = -\sec x + c$

D. $\int \sin x \, \mathrm{d}x = -\cot x + c$

The correct answer is **A**

The integral of sin(x) with respect to x is indeed -cos(x) + C, where C is the constant of integration.