___/___/___

Q1 $L(\theta_1, \theta_2 \mid x_1, x_2, \dots, x_n) = \prod_{i=1}^{n} \left(\frac{1}{\sqrt{2\pi\theta_2}} \right)^{-\frac{(x_i-\theta_1)^2}{2\theta_2}}$

$\log L(\theta_1, \theta_2 \mid x_1, x_2, \dots, x_n) = -\frac{n}{2} \log(2\pi\theta_2) - \frac{1}{2\theta_2} \sum_{i=1}^{n} (x_i - \theta_1)^2$

$\Rightarrow$ for θ_1

$\frac{d}{d\theta_1} \log L = \frac{1}{\theta_2} \sum_{i=1}^{n} (x_i - \theta_1) = 0$

$\sum_{i=1}^{n} (x_i - \theta_1) = 0$

$\sum_{i=1}^{n} x_i - n\theta_1 = 0$

$\theta_1 = \frac{1}{n} \sum_{i=1}^{n} x_i$ (MLE of $\theta_1 \rightarrow$ sample mean)

$\Rightarrow$ for θ_2

$\frac{d}{d\theta_2} \log L = \frac{-n}{2\theta_2} + \frac{1}{2\theta_2^2} \sum_{i=1}^{n} (x_i - \theta_1)^2 = 0$

$-n\theta_2 + \sum_{i=1}^{n} (x_i - \theta_1)^2 = 0$

$\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \theta_1)^2$ (MLE of $\theta_2 \rightarrow$ sample variance)

Q2 $PMF = P(X=k) = mk\theta^k (1-\theta)^{m-k}$

$L(\theta \mid x_1, x_2, \dots x_n) = \prod_{i=1}^{n} (m\, x_i)\, \theta^{x_i} (1-\theta)^{m-x_i}$

$\log L(\theta \mid x_1, x_2, \dots, x_n) = \sum_{i=1}^{n} \left[\log(m x_i) + x_i \log\theta + (m - x_i)\log(1-\theta) \right]$

$\frac{d}{d\theta} \log L(\theta \mid x_1, x_2, \dots, x_n) = \sum_{i=1}^{n} \left[\frac{x_i}{\theta} - \frac{m - x_i}{1-\theta} \right]$

$\sum_{i=1}^{n} \left[\frac{x_i}{\theta} - \frac{m - x_i}{1-\theta} \right] = 0$

$\sum_{i=1}^{n} \frac{x_i - \theta m}{\theta(1-\theta)} = 0$

$\sum_{i=1}^{n} (x_i - \theta m) = 0$

$\theta \sum_{i=1}^{n} x_i - n\theta m = 0$

$\theta = \frac{\sum_{i=1}^{n} x_i}{nm}$