WARP Shoe Optimization Model

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1.0 Abstract

This report represents an optimization model to maximize the profit of the WARP Shoe company in February 2006. The model identifies the optimal amount of shoes to be produced as the decision variable. The model we created implements constraints such as budget, raw material availability, warehouse capacity, average production time, and demand. Using AMPL and Gurobi solver an optimal production plan was created, maximizing profit while adhering to the provided constraints. The results highlight the optimal profit given various conditions such as changing the budget for raw materials and decreasing machine time available.

2.0 Introduction

The WARP Shoe Company is one of the oldest shoe companies in Canada. At the beginning of 2006, one of the major market competitors of WARP went bankrupt. Through the company's market analysis, it was predicted that the demand for the month of February would be doubled. Thus the company is looking to find the most profitable production plan to meet this predicted demand. We analyzed all of the company's data and created a linear program using AMPL and a Gurobi solver, allowing us to find the best way to maximize profit.

3.0 Methodology:

3.1 Objective Function

To provide the most profitable production plan for the company, we translated the problem into a linear program. We set our decision variable to x, representing each shoe to produce (Answer to question 2). We then formulated our objective function to maximize profit. In this equation, we added the shoe cost of all shoes produced and subtracted the machine operating cost, the average duration of each shoe produced on each machine, warehouse operating cost, raw materials cost required for each shoe, and finally the $10 penalty cost for not meeting demand. This data was provided to us by the WARP Shoe Database. The predicted demand for February 2006, depicted as *d[s]* was calculated using the historical demand of February from 1997 – 2003. By creating an SQL query, we averaged the demand of all 7 years and multiplied by 2 [Appendix A] (Answer to question 1). The final objective function is provided below.

$$\text{Maximize Profit:}$$

$$\sum_{s\in S}\text{Shoe\_Cost}[s]\cdot x[s]-\sum_{m\in M}\text{Machine\_OpCost}[m]\cdot(\text{Machine\_OpTime})-\sum_{s\in S,m\in M}x[s]\cdot\text{Avg\_Duration}[s,m]\cdot\frac{25}{60}$$
$$-\sum_{w\in WH}\text{Warehouse\_OpCost}[w]-\sum_{s\in S,rm\in R}\text{RM\_Cost}[rm]\cdot x[s]\cdot\text{Amt\_RM\_Shoe}[s,rm]-\sum_{s\in S}10\cdot(d[s]-x[s]).$$

3.2 Constraints

The objective function was met with 5 constraints (Answer to question 2). These constraints help to set a limit on how many shoes are produced while ensuring profit is maximized and demand is met. The constraints include a limited budget, amount of raw materials, limited warehouse capacity, and a limit on the amount of time available to use machines.

The first constraint is related to demand. This constraint ensures that the production does not exceed the demand. The constraint is modeled below:

$$0 \;\leq\; x[s] \;\leq\; d[s]$$

The second constraint is related to the budget. The budget provided was $10,000,000 and the total cost of the raw materials needs to be equal to the provided budget in order to produce as many shoes as possible. The constraint is modeled below:

$$\sum_{s \in S, rm \in R} \text{RM\_Cost}[rm] \cdot x[s] \cdot \text{Amt\_RM\_Shoe}[s, rm] = \text{Budget}$$

The third constraint is related to the amount of time available to produce each type of shoe on the provided machines. Since the machines only operate 12 hours per day and there are 72 available machines, there is a total machine operation time of 20,160 minutes for the month. This constraint ensures that production time does not exceed the total machine operation time. The constraint is modeled below:

$$\sum_{s \in S, m \in M} \text{Avg\_Duration}[s, m] \cdot x[s] \leq (\text{Machine\_OpTime} \cdot \text{Num\_Machines})$$

The fourth constraint is related to warehouse capacity. All of the warehouses have a certain maximum capacity of shoes they can hold. This constraint ensures that the amount of shoes produced does not exceed the warehouse capacity. The constraint is modeled below:

$$\sum_{s \in S} x[s] \leq \text{Warehouse\_Capacity}[w], \quad \forall w \in WH$$

The last constraint is related to the amount of raw material available. Each shoe uses different quantities and different types of raw material, and there is a limit to how much of each raw material is available. This constraint ensures that the total amount of raw material used to produce all of the shoes does not exceed the amount of each raw material available. The constraint is modeled below:

$$\sum_{s \in S} \text{Amt\_RM\_Shoe}[s, rm] \cdot x[s] \leq \text{RM\_Amount}[rm], \quad \forall rm \in R$$

4.0 Results

4.1 Obtained results

After having set our decision variable, objective function, and constraints, we used Gurobi solver to solve our linear program. We first created a .dat file that reads the company's data from the WARPShoe database [Appendix B]. We then created a .mod file where we formulated the linear program and a .run file that solves it and prints the final profit and decision variables for each shoe into a .out file [Appendix B]. Our obtained optimal profit is $15,869,430.74. As for the obtained decision variables, it was found optimal to not produce certain types of shoes, for which the production was equal to 0. The constraint related to demand was binding, which means that the solution to our optimization problem exactly satisfies this constraint. The constraints related to warehouse capacity, available time, available raw material, and budget constraints were not binding (Answer to question 4). Since our original method for solving this program was using an LP, no relaxation was used. (Answer to question 3)

4.2 Modeling Different Scenarios

The following scenarios were asked to be modeled:

1. Assume that some additional warehouse space is available at the price of $10/box of shoes. Is it economical to buy it? What is the optimal amount of space to buy in this situation?

No, it's not economical to buy the additional warehouse space since all the shadow prices of the warehouses are 0, which means the warehouse capacity is not a limiting factor in the current solution. So increasing the capacity of the warehouses will not improve the obtained profit.

2. Imagine that machines were available for only 8 hours per day. How would your solution change? Which constraints are binding now? Does the new solution seem realistic to you?

The maximum profit is $16,088,502.74. The only binding constraint is the demand constraint. Although there would be a decrease in machine operating cost, the company would be producing less shoes, thus having a higher profit seems unrealistic.

3. If in addition there was a $7,000,000 budget available to buy raw materials, what would you do? Change your formulation and solve it again.

By changing our budget constraint to be less than $7,000,000, the maximum profit decreased to $11,012,136.47. [Appendix C]

5.0 Conclusion

The optimization model successfully identified an optimal production plan that achieved a maximum profit of $15869430.74. While adhering to the provided constraints, the model showed how the maximum profit changes given certain research bottlenecks such as raw material budget and machine time available. For future implementations, further research conducted on possible constraints and added costs like shipping procedures, could be included to provide a more thorough optimization plan.

Appendix A.

Figure 1. SQL Query in Access Database

Query1

```
SELECT Product_Num,
round(SUM(Demand)*2/7) AS TotalDemand FROM Product_Demand
WHERE Month=2
GROUP BY Product_Num;
```

Figure 2.

Query1

Product_N	TotalDemand
SH001	475
SH002	449
SH003	463
SH004	421
SH005	466
SH006	427
SH007	440
SH008	434
SH009	440
SH010	395

Appendix B.

Figure 3. Reading Access database tables into WARPShoe.dat file

welcome.txt | WARPShoe.mod | WARPShoe.dat | *WARPShoe.run

```
#Lea_Akzam_akzamlea_1009879669
#Anjali_Jammulamadaka_jammula1_1009931870

#The BOM table gives us the quantity of the raw material needed for each shoe
table BOM_table IN "ODBC" "./WARP2011W.mdb" "BOM":
[Product_Num, RM_Num], Amt_RM_Shoe ~ Quantity;
read table BOM_table;

#The Machine Assign table tells us how long each machine takes to produce shoe i
table MAssign_table IN "ODBC" "./WARP2011W.mdb" "Machine_Assign":
[Product_Num, Machine_Num], Avg_Duration ~ Avg_Duration;
read table MAssign_table;

#The Machine Master table tells us the operation cost per minute of each machin
table MMaster_table IN "ODBC" "./WARP2011W.mdb" "Machine_Master":
M <- [Machine_Num], Machine_OpCost ~ OpCost_per_min;
read table MMaster_table;

#Product Demand table tells us the historical demand of shoe i for month 2 (February) from 1997-2003
#which we will use to calculate the demand for February of 2006
table PDemand_table IN "ODBC" "./WARP2011W.mdb" "SQL=SELECT Product_Num, round(SUM(Demand)*2/7) AS TotalDemand FROM Product_Demand WHERE Month=2 GROUP BY Product_Num":
[Product_Num], d ~ TotalDemand;
read table PDemand_table;

#Product Master
table PMaster_table IN "ODBC" "./WARP2011W.mdb" "Product_Master":
S <- [Product_Num], Shoe_Cost ~ Sales_Price;
read table PMaster_table;

#RM Master
table RMMaster_table IN "ODBC" "./WARP2011W.mdb" "RM_Master":
R <- [RM_Num], RM_Cost ~ Cost, RM_Amount ~ S_Quantity;
read table RMMaster_table;

#Warehouse
table Warehouse_table IN "ODBC" "./WARP2011W.mdb" "Warehouse_Master":
WH <- [Warehouse_Num], Warehouse_Capacity ~ Capacity, Warehouse_OpCost ~ Op_Cost; # do we need the last one?
read table Warehouse_table;
```

Figure 4. Formulating LP in WARPShoe.mod file

welcome.txt | WARPShoe.mod | WARPShoe.dat | WARPShoe.run

```
#sets
set S; #set of shoes
set M; #set of machines
set WH; #set of warehouses
set R; #set of raw materials


#parameters
param Shoe_Cost{S}; #cost per shoe for set S
param Labour_Cost := 25; #fixed labour cost
param RM_Amount{R}; #amount of each raw material in set RM that is available
param RM_Cost{R}; #cost per raw material in set RM
param Avg_Duration{S, M} default 0; #Average duration to produce shoe in set S on machine in set M
param Amt_RM_Shoe{S,R} default 0; #Quantity of raw materials in set RM needed for shoe in set S
param Machine_OpCost{M}; #Machine operation cost
param Machine_OpTime := 12*28*60; #total machine operating time in the month of Feb in minutes
param Warehouse_OpCost{WH};#Operation cost of warehouse in set W
param Warehouse_Capacity{WH}; #Capacity of warehouse in set W
param Budget := 10000000; #Budget
param d{S};
param Num_Machines:= 72; # number of available machines (from Machine_Master Table)
param Penalty_Cost := 10; #penalty cost

#decision variables
var x{S} >= 0; # number of shoes produced

#write the objective func
maximize profit: sum{s in S} Shoe_Cost[s]*x[s]
- sum{ m in M} Machine_OpCost[m]*(Machine_OpTime)
- sum{s in S, m in M} x[s]*Avg_Duration[s, m] * (25/60)
- sum{w in WH} Warehouse_OpCost[w]
- sum{s in S, rm in R} RM_Cost[rm]*x[s]*Amt_RM_Shoe[s,rm]
#- sum{s in S, m in M} Labour_Cost*Avg_Duration[s, m]*x[s]
- sum{s in S} 10 * (d[s] - x[s]);


#write the constraints

#demand constraint
subject to Demand_Co{s in S}: 0 <= x[s] <= d[s];
#budget constraint
subject to Budget_Co: sum{s in S, rm in R} RM_Cost[rm]*x[s]*Amt_RM_Shoe[s, rm] = Budget;
#time constraint
subject to Time_Co: sum{s in S, m in M} Avg_Duration[s,m]*x[s] <= (Machine_OpTime*Num_Machines);
#warehouse capacity constraint
subject to Warehouse_Co{w in WH} S: sum{s in S} x[s] <= Warehouse_Capacity[w];
#raw material constraint
subject to RM_Quantity_Available{rm in R}: sum{s in S} (Amt_RM_Shoe[s, rm]*x[s]) <= RM_Amount[rm];
```

Appendix C.

Figure 5. Sample Results for Question 6

WARPShoe6.out - Notepad

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```
The maximum profit is: 16088502.74.
The optimal x-values (shoes produced) are:
:          x        x.rc        :=
SH001      0      -17.0792
SH002    449        0
SH003    463        0
SH004      0       -1.41986
SH005    466        0
SH006    427        0
SH007    440        0
SH008    434        0
SH009    440        0
SH010    395        0
```

Figure 6. Sample Results for Question 7

WARPShoe7.out - Notepad

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```
The maximum profit is: 11012136.47.
The optimal x-values (shoes produced) are:
:          x         x.rc         :=
SH001      0      -23.3233
SH002      0       -7.27898
SH003    463        0
SH004      0      -12.3014
SH005      0       -0.00186413
SH006    427        0
SH007      0       -6.52346
SH008    434        0
SH009    440        0
SH010    395        0
```