Assignment No. 1

Name - Anjali Kanwasi

Registration Number - 23652008

Signature - Anjali

Course Code - OMC100C

Course Title - Mathematical Foundation of Computer Science

PART - A

Q1. If x is a set and the set contains an integer which is neither positive nor negative then the set x is _____ .

Ans (d) Set is both Non-empty and Finite.

Q2. If a relation R has the property that for every (a, b) in R, (b, a) is also in R, what type of relation is R ?

Ans (b) Symmetric

Q3. What is a partial order relation ?

Ans (c) A relation that is reflexive, antisymmetric and transitive

Q4. In the principle of mathematical induction, find which of the following steps is mandatory ?

Ans (a) induction hypothesis

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Anjali

Q5. According to principle of mathematical induction, if $P(k+1) = m^{k+1} + 5$ is true then ____ must be true.

Ans (b) $P(k) = m^k + 5$

Q6. If $\{1, 3, 5\}$ and $B = \{1, 3, 5, 7\}$ then A is a ____ subset of B

Ans (B) proper

Q7. A ____ is a set S with a relation R on it which is reflexive, antisymmetric and transitive.

Ans (D) Partially ordered set

Q8. If every element in the domain is mapped to a unique element in the codomain, the function is said to be :

Ans (a) One - to - One

Q9. Which function is often used to round down a real number to the nearest integer ?

Ans (9) Floor function

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Anjali

Q10. A function $f: A \to B$ is said to be ____ if for every y in B there exists at least one element x in A such that $f(x) = y$.

Ans (A) surjective

PART - B

Q2. Given $f(x) = 2x$ and $g(x) = x^2$, find the composition $g \circ f$ and evaluate it for $x = 3$.

Ans2.

$f(x) = 2x$

$g(x) = x^2$

$gof(x) = g(f(x)) = g(2x)$

Given $x = 3$,

$g(2 \times 3) = g(6) = 6^2 = 36$

So, $gof(3) = 36$

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Q4. Let $X = \{1, 2, 3, 4, 5\}$ and relation $R = \{(x, y) / x > y\}$. Construct the graph of 'R' and also give its matrix.

Ans4 $R = \{(2,1), (3,1), (3,2), (4,1), (4,2), (4,3), (5,1), (5,2), (5,3), (5,4)\}$

Matrix will be (1 if coordinate is present in above set)

0, 0, 0, 0, 0
1, 0, 0, 0, 0
1, 1, 0, 0, 0
1, 1, 1, 0, 0
1, 1, 1, 1, 0

Q5. Use Mathematical Induction to show that $1^2 + 2^2 + \ldots + 2^n = 2^{n+1} - 1$

Ans5

Step 1: For $n = 0$,

LHS : [1^2] = 1

RHS : 2^(0+1) - 1 = 2^0 = 1

This is true.

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Anjali

Step 2 : For $n = k$, assume true.

$1^2 + 2^2 + \dots + 2^k = 2^{(k+1)} - 1$

Step 3 : Now prove, $n = k + 1$ is true.

$P(k+1) = 1^2 + 2^2 + \dots + 2^k + 2^{(k+1)}$

$= 2^{(k+1)} - 1 + 2^{(k+1)}$

$= 2 \cdot 2^{(k+1)} - 1$

$= 2^{((k+1)+1)} - 1$

Therefore, by mathematical induction, the statement holds for all non-negative integers (n).

$[1^2 + 2^2 + \dots + 2^n = 2^{(n+1)} - 1]$

Q7. Verify by mathematical induction.

Ans7 $1^2 + 2^2 + \dots + n^2 = (n(n+1)(2n+1))/6$

Step 1 : For $n = 1$,

LHS : $[1^2] = 1$

RHS : $(1(1+1)((2\times1)+1))/6 = (1(2)(3))/6$

$= 6/6 = 1$

This is true

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Step 2: For $n=k$, assume true.

$1^2 + 2^2 + \ldots + k^2 = (k(k+1)(2k+1))/6$

Step 3: Now prove, $n=k+1$ is true.

$P(k+1) = 1^2 + 2^2 + \ldots + K^2 (K+1)^2$

$= (K(K+1)(2K+1))/6 + (k+1)^2$

$= ((k(k+1)(2k+1)) + 6(k+1)^2)/6$

$= ((k+1)(k(2k+1) + 6(k+1)))/6$

$= ((k+1)(2k^2 + 7k + 6))/6$

$= ((k+1)(k+2)(2(k+1)+1))/6$

Q8. Use the Euclidean algorithm to compute the greatest common divisor of 46 and 21.

Ans 8 Let $u = 46$ and $v = 21$

Let $r = u \bmod v$

Step 1: $r = 46 \% 21 = 4$

$u = v = 21$

$v = r = 4$ (not equal to 0 goto next step)

Step 2: $r = 21 \% 4 = 1$

$u = v = 4$

$v = r = 1$ (not equal to 0 goto next step)

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Step 3 : $r = 4 \% 1 = 0$

$u = v = 1$

$v = r = 0$ (equal to 0 Stop)

Answer is : 1