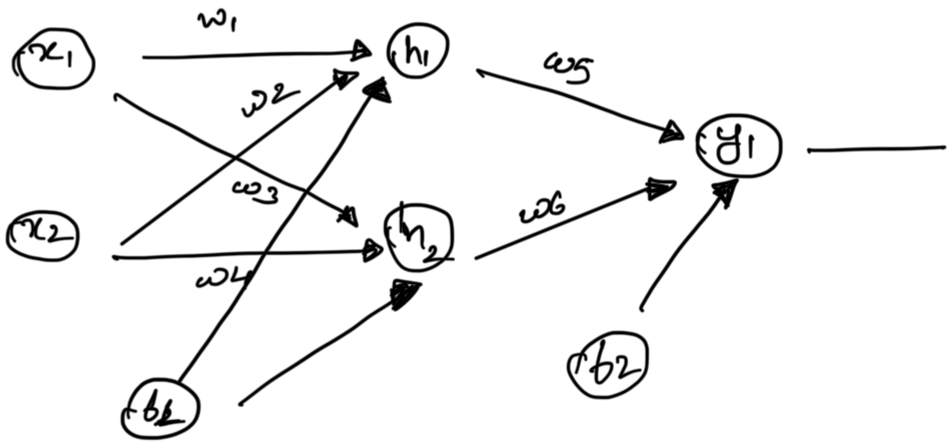
HW2   Anjali H ojha

Step-0



$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} = \begin{bmatrix} .1 \\ .2 \\ .1 \\ .2 \end{bmatrix} \qquad \begin{bmatrix} w_5 \\ w_6 \end{bmatrix} = \begin{bmatrix} .4 \\ .5 \end{bmatrix}$$

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 1 \\ .9 \end{bmatrix} \qquad \begin{bmatrix} y_1 \end{bmatrix} = \begin{bmatrix} .2 \end{bmatrix}$$

Activation function = sigmoid $= \dfrac{1}{1+e^{-2}}$

$\boxed{\text{Forword Pass}}$

$$h_1 = w_1x_1 + w_2x + b_1 = .1\times 1 + .2 \times 2 + 1 = 1.5$$

$$op(h_1) = \frac{1}{1+e^{-h_1}} = \frac{1}{1+e^{-1.5}} = .81$$

$$h_2 = w_2 x + w_4 x_2 + b_1 = .1\times 1 + .2\times 2 + 1 = 1.5$$

$$op(h_1) = \frac{1}{1+e^{-h_2}} = \frac{1}{1+e^{-1.5}} = .81$$

$y_1 = w_5 \text{ outof}(h_1) + w_6 \text{ outof}(h_2) + b_2$

$= .4 \times .81 + .5 \times .81 + .9$
$= 1.629$

$\text{out}(y_1) = \frac{1}{1+e^{-y_1}} = \frac{1}{1+e^{-1.629}} = .8360$

$\text{error} = \frac{1}{n} \sum_{i=1}^{n} (\hat{y_i} - y_i)^2$

$= [(1 - .8360)^2]$

$= (.164)^2$

$= .026$

Backword Pass –

$\text{error at } w_5 = \frac{\partial \text{ error}}{\partial w_5}$ | Expanding using chain rule

$\frac{\partial \text{error}}{\partial w_5} = \frac{\partial \text{ errore}}{\partial \text{ out } y_1} * \frac{\partial \text{ out } y_1}{\partial y_1} * \frac{\partial y_1}{\partial w_5}$

$\frac{\partial \text{ error}}{\partial \text{out } y_1} = \frac{\partial}{\partial \text{out } y_1} [[(\hat{y_1} - \text{out } y_1)^2]]$

$= 2(\hat{y_1} - \text{out } y_1) * -1$

$= -2(1 - .8360)^2$

$= -0.052$

$\frac{\partial \text{ output } y_1}{\partial y_1} = \text{out } y_1 \times (1 - \text{out } y_1)$

$= .8360 \times (1 - .8360)$

$$= -.8360 \times .164$$
$$= .137$$

$$\frac{\partial y_1}{\partial w_5} = \frac{\partial}{\partial w_5}\left[w_5 \cdot out\,h_1 + w_6\, out\,h_2 + b_2 \right]$$

$$= out\,h_1 = .81$$

substituting value in original equation:

$$\frac{\partial error}{\partial w_5} = (-0.052) \times (-.137) \times .81$$
$$= +0.005$$

updating value of w5

$$w_5 = w_5 - \eta \frac{\partial error}{\partial w_5} \qquad \eta = .5$$
$$= .4 + .5 \times [0.005]$$
$$= .4025$$

Simillarly updating values

$$\frac{\partial error}{\partial w_6} = \frac{\partial error}{\partial out\,y_1} \times \frac{\partial out\,y_1}{\partial y_1} \times \frac{\partial y_1}{\partial w_6}$$

$$= [-0.052] \times [-.137] \times [out\,h_2]$$
$$= .052 \times .137 \times .81$$
$$= 0.005$$

$$w_6 = w_6 + \eta \frac{\partial error}{\partial w_6} = .5 + .5 \times [0.005]$$
$$= .5025$$

updating b_2

$$\frac{\partial error}{\partial b_2} = \frac{\partial error}{\partial out\,y_1} \times \frac{\partial out\,y_1}{\partial y_1} \times \frac{\partial y_1}{\partial b_2}$$
$$= [-.052] \times [-.137] \times [1]$$
$$= .0071$$

$$b_2 = b_2 + \eta \frac{\partial error}{\partial b_2} = .9 + .5 \times [.0071]$$
$$= .9035$$

For $w_1 = \frac{\partial error}{} = \frac{\partial error}{} \times \frac{\partial out\,h_1}{} \times \partial h_1$

$$\partial w_1 \qquad \overline{\partial out\, h_1} \qquad \overline{\partial h_1} \qquad \overline{\partial w_1}$$

$$\frac{\partial error}{\partial out\, h_1} = \left[\frac{\partial error\ y_1}{\partial\ out\ h_1}\right]$$

$$\frac{\partial error\, y_1}{\partial out\ h_1} = \frac{\partial error\ y_1}{\partial y_1} \times \frac{\partial y_1}{\partial out\ h_1}$$

$$\frac{\partial error}{\partial y_1} = \frac{\partial error\ y_1}{\partial\ out\ y_1} \times \frac{\partial out\ y_1}{\partial y_1}$$

<u>Putting All values together</u>

$$\frac{\partial error}{\partial w_1} = \frac{\partial error}{\partial out\ y_1} \times \frac{\partial\ out\, y_1}{\partial y_1} \times \frac{\partial y_1}{\partial out\, h_1} \times \frac{\partial\ out\, h_1}{\partial h_1} \times \frac{\partial h_1}{\partial w_1}$$

$$= [-'052] \times [-1.37] \times [w5] \times [out\ h_1(1 - out\ h_1)] \times [x_1]$$

$$= [\cdot 052 \times \cdot 137 \times \cdot 4 \times \cdot 01 \times (1 - \cdot 01) \times 1]$$

$$= -0\cdot 00043$$

$$w_1 = w_1 + \eta \frac{\partial error}{\partial w_1}$$

$$= \cdot 1 + \cdot 5\,[-0.00043]$$

$$= \cdot 099$$

Simillary we will be updating other parameters

$$\frac{\partial error}{\partial w_2} = \frac{\partial error}{\partial out\, y_1} \times \frac{\partial out\, y_1}{\partial y_1} \times \frac{\partial y_1}{\partial out\, h_1} \times \frac{\partial out\, h_1}{\partial h_1} \times \frac{\partial h_1}{\partial w_2}$$

$$= [-\cdot 052] \times [-\cdot 137] \times [\cdot 4][\cdot 01 \times \cdot 19] \times [x_2]$$

$$= [-0.00043] \times [2]$$

$$= -0.00086$$

$$\omega_2 = \omega_2 + \eta \frac{\partial \text{error}}{\partial \omega_2}$$
$$= \cdot 2 + \cdot 5 \left[-0.00086 \right]$$
$$= \cdot 19957$$

$$\frac{\partial \text{error}}{\partial \omega_3} = \frac{\partial \text{error}}{\partial \text{out } y_1} \times \frac{\partial \text{out } y_1}{\partial y_1} * \frac{\partial y_1}{\partial \text{out } h_2} \times \frac{\partial \text{out } h_2}{\partial h_2} \times \frac{\partial h_2}{\partial \omega_3}$$

$$= \left[-0.052 \right] \times \left[-\cdot 137 \right] \times \left[\omega_6 \right] \times \left[\text{out}h_2 \left(1 - \text{out } h_2 \right) \right] \times \left[x_1 \right]$$
$$= 0.052 \times \cdot 137 * \cdot 5 \times \cdot 81 \times \left(-\cdot 19 \right) \times \cdot 1$$
$$= -0.000054$$

$$\omega_3 = \omega_3 + \eta \frac{\partial \text{error}}{\partial \omega_3}$$
$$= \cdot 1 + \cdot 5 \times \left(-0.000054 \right)$$
$$= \cdot 09997$$

Similarly $\boxed{\omega_4 = \cdot 1995}$

$$\frac{\partial \text{error}}{\partial b_1} = \frac{\partial \text{error}}{\partial \text{out } y_1} \times \frac{\partial \text{out } y_1}{\partial y_1} \times \frac{\partial y_1}{\partial b_1}$$

$$\frac{\partial \text{error}}{\partial b_1} = \frac{\partial \text{error}}{\partial y_1} \times \frac{\partial \text{out } y_1}{\partial y_1} \left[\frac{\partial y_1}{\partial \text{out } h_1} \times \frac{\partial \text{out } h_1}{\partial h_1} \times \frac{\partial h_1}{\partial b_1} \right.$$
$$\left. + \left[\frac{\partial y_1}{\partial \text{out } h_2} \times \frac{\partial \text{out } h_2}{\partial h_2} \times \frac{\partial h_2}{\partial b_1} \right] \right.$$

Final b_1

$$b_1 = b_1 + \eta \frac{\partial \text{error}}{\partial b_1}$$
$$= 1 + \cdot 5 \left[-0.0114 \right]$$

$\boxed{\text{Updated values}}$

$w_1 = .099$

$w_2 = .19957$

$w_3 = .0997$

$w_4 = .1995$

$b_1 = .99943$

$w_5 = .4025$

$w_6 = .5025$

$b_2 = .9035$