DEEN DAYAL UPADHYAYA COLLEGE

UNIVERSITY OF DELHI

(SESSION: 2023-2024)

PRACTICAL FILE

COMPLEX ANALYSIS

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SEMESTER: *6th*

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CONTENTS

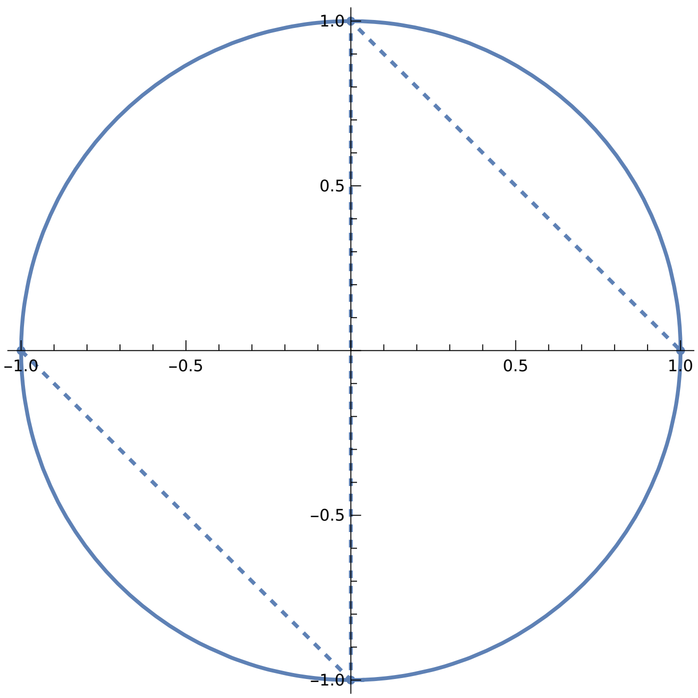
PRACTICAL NO. 1

Objective: Make a geometric plot to show that the nth roots of unity are equally spaced points that lie on the unit circle C1(0) = {z : |z| = 1} and form the vertices of a regular polygon with n sides, for n = 4, 5, 6, 7, 8.

In[83]:=
```
geocomplexroot[n_] := Module[{}, R1 = ComplexExpand[Solve[z^n == 1]];
  C1 = ContourPlot[x^2 + y^2 == 1, {x, -1, 1}, {y, -1, 1}, Axes → True, Frame → False];
  P1 = ListPlot[{{Re[z], Im[z]} /. R1}, PlotStyle → Thick];
  L1 = ListLinePlot[{{Re[z], Im[z]} /. R1}, PlotStyle → Dashed]; Show[C1, P1, L1]]
```

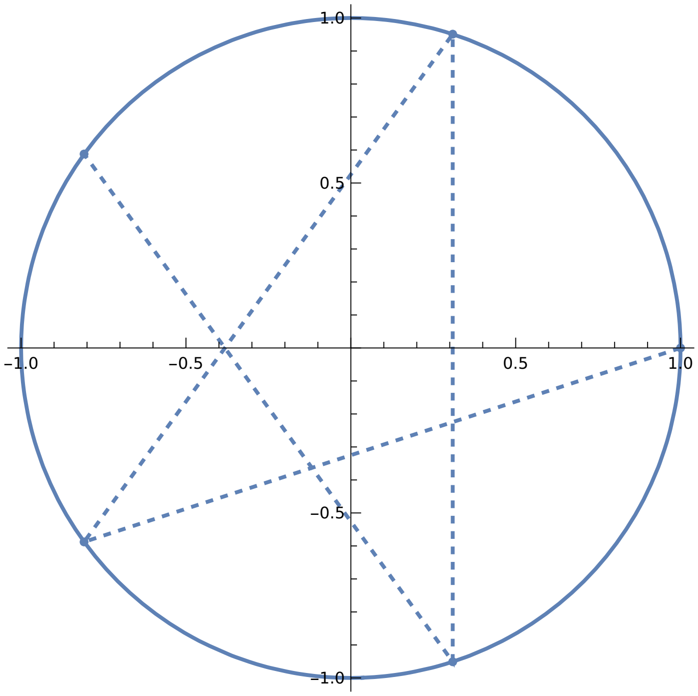
In[84]:=
```
geocomplexroot[4]
```

Out[84]=



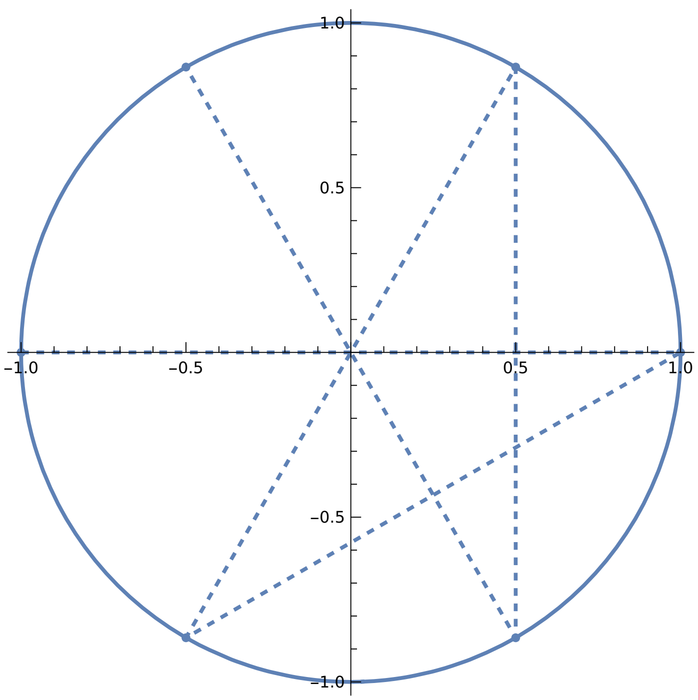
In[85]:= `geocomplexroot[5]`

Out[85]=



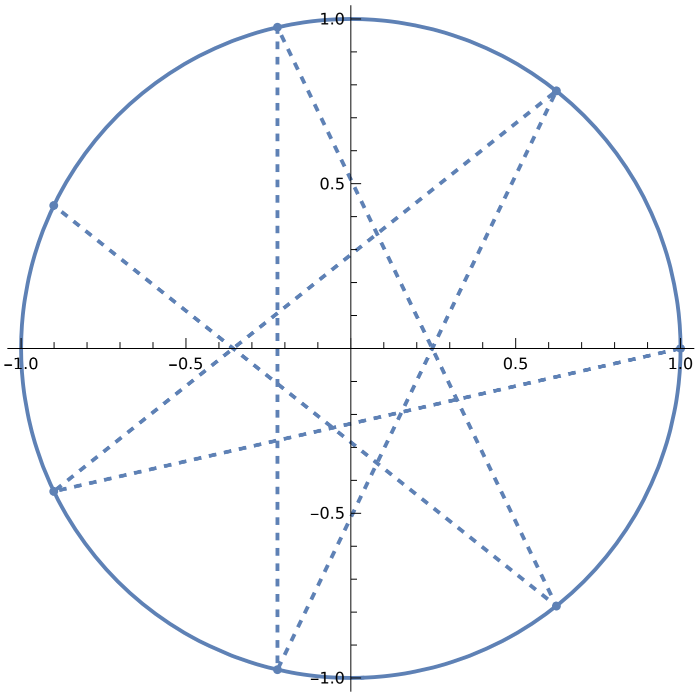
In[86]:= `geocomplexroot[6]`

Out[86]=



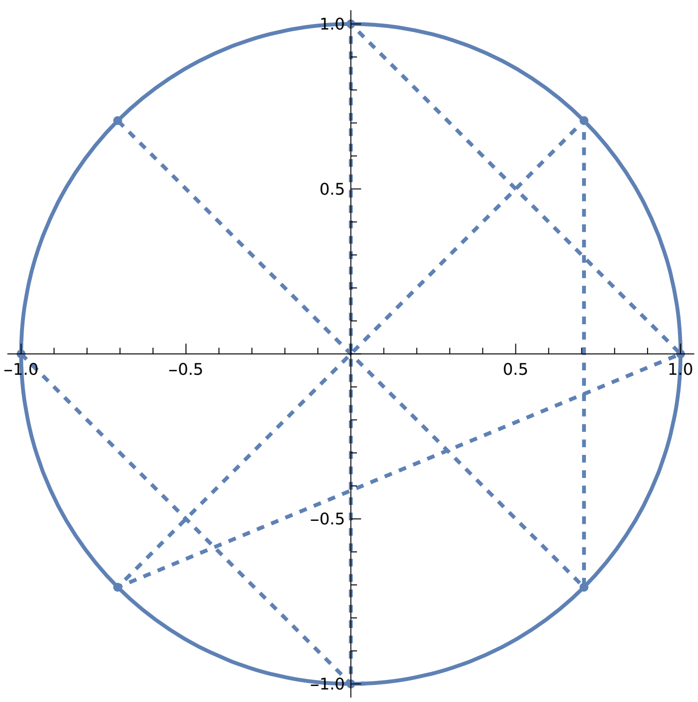
In[87]:= `geocomplexroot[7]`

Out[87]=



In[88]:= `geocomplexroot[8]`

Out[88]=



PRACTICAL NO. 2

Objective: Find all the solutions of the equation z ^3= 8i and represent these geometrically.

In[108]:=

```
R2 = ComplexExpand[Solve[z^3 == 8 * I]]
```

Out[108]=

$\{\{z \to -2 i\}, \{z \to i + \sqrt{3}\}, \{z \to i - \sqrt{3}\}\}$

In[113]:=

```
C2 = ContourPlot[x^2 + y^2 == 4, {x, -2, 2}, {y, -2, 2}];
```

In[114]:=

```
P2 = ListPlot[{{Re[z], Im[z]} /. R2},
   PlotRange → {{-2, 2}, {-2, 2}}, AspectRatio → Automatic, PlotStyle → Black];
```

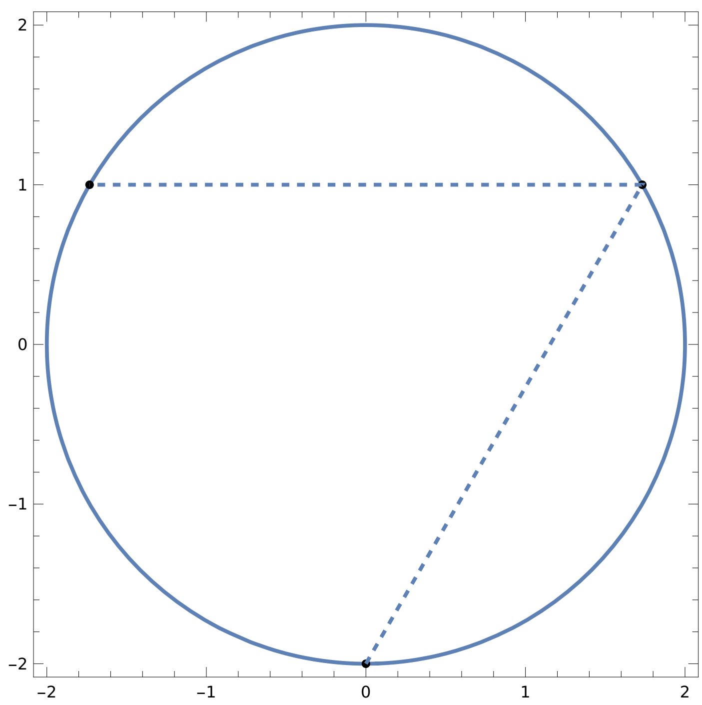
In[115]:=

```
L2 = ListLinePlot[{ {Re[z], Im[z]} /. R2},
   PlotRange → {{-2, 2}, {-2, 2}}, AspectRatio → Automatic, PlotStyle → Dashed];
```

In[116]:=

```
Show[C2, P2, L2]
```

Out[116]=

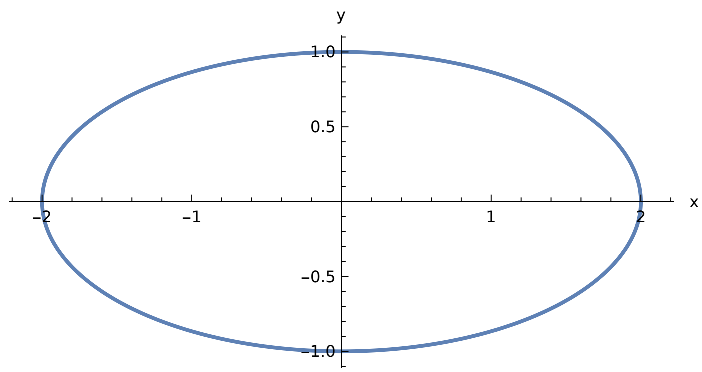


PRACTICAL NO. 3

Objective: Write parametric equations and make a parametric plot for an ellipse centered at the origin with horizontal major axis of 4 units and vertical minor axis of 2 units.Show the effect of rotation of this ellipse by an angle of $\pi/6$ radians and shifting of the centre from (0,0) to (2,1), by making a parametric plot.

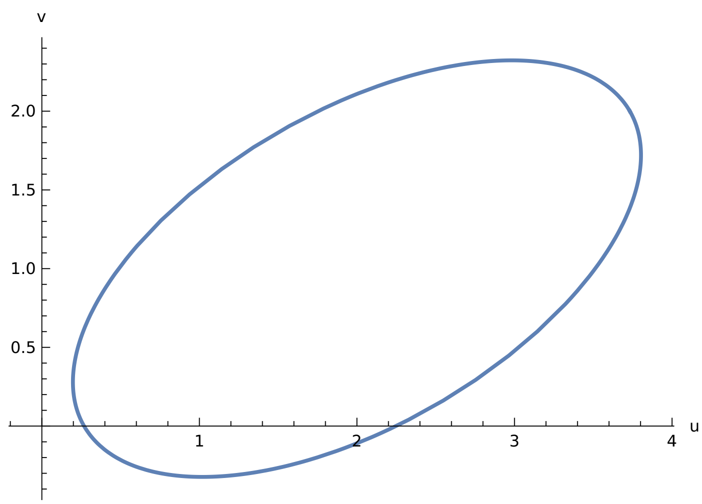
In[2]:=
```
s[t_] := 2 * Cos[t] + I * Sin[t];
r[t_] := s[t] * Exp[I * Pi / 6] + (2 + I);
ParametricPlot[{Re[s[t]], Im[s[t]]}, {t, 0, 2 * Pi}, AxesLabel → {"x", "y"}]
```

Out[4]=



In[5]:=
```
ParametricPlot[{Re[r[t]], Im[r[t]]}, {t, 0, 2 * Pi}, AxesLabel → {"u", "v"}]
```

Out[5]=



PRACTICAL NO. 4

Objective: Show that the image of the open disk $D1(-1-i) = \{z : |z+1+i| < 1\}$ under the linear transformation $w = f(z) = (3-4i)z + 6 + 2i$ is the open disk: $D5\ (-1+3i) = \{w: |w+1-3i| < 5\}$.

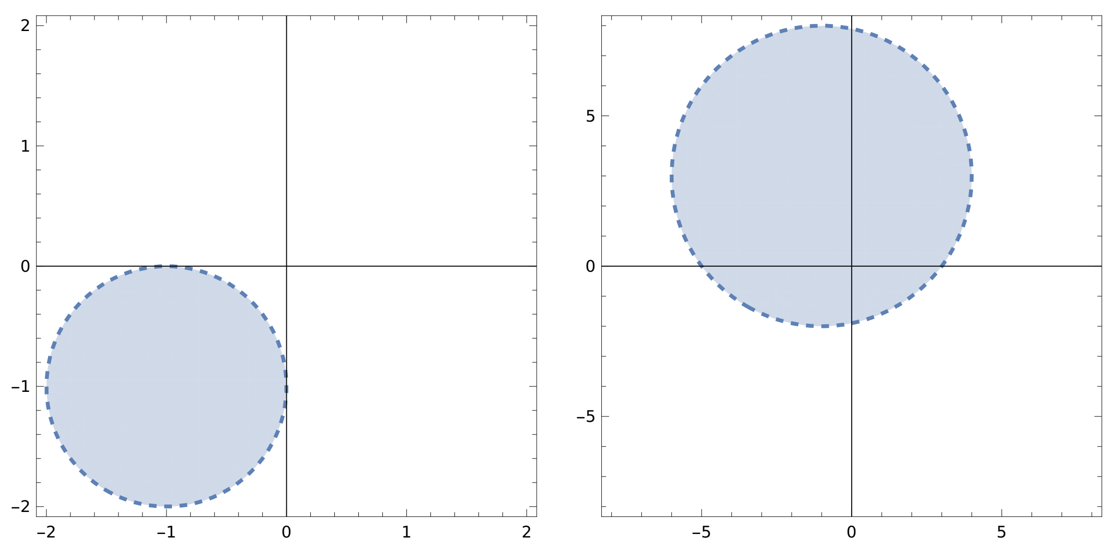
In[9]:=
```
z = x + I * y;
Solve[w1 == (3 - 4 * I) z1 + 6 + 2 * I, z1]
```

Out[10]=

$$\left\{\left\{z1 \to \frac{1}{25}\left(-10 + 3\,w1 + 2\,i\left(-15 + 2\,w1\right)\right)\right\}\right\}$$

In[17]:=
```
A1 = RegionPlot[Abs[z + 1 + I] < 1,
   {x, -2, 2}, {y, -2, 2}, BoundaryStyle → Dashed, Axes → True];
A2 = RegionPlot[Abs[((z - 6 - 2 * I) / (3 - 4 * I)) + 1 + I] < 1,
   {x, -8, 8}, {y, -8, 8}, BoundaryStyle → Dashed, Axes → True];
GraphicsRow[{A1, A2}]
```

Out[19]=



PRACTICAL NO. 5

Objective: Show that the image of the right half plane Re z = x > 1 under the linear transformation w = (−1 + i)z − 2 + 3i is the half plane v > u + 7, where u = Re(w), etc. Plot the map.

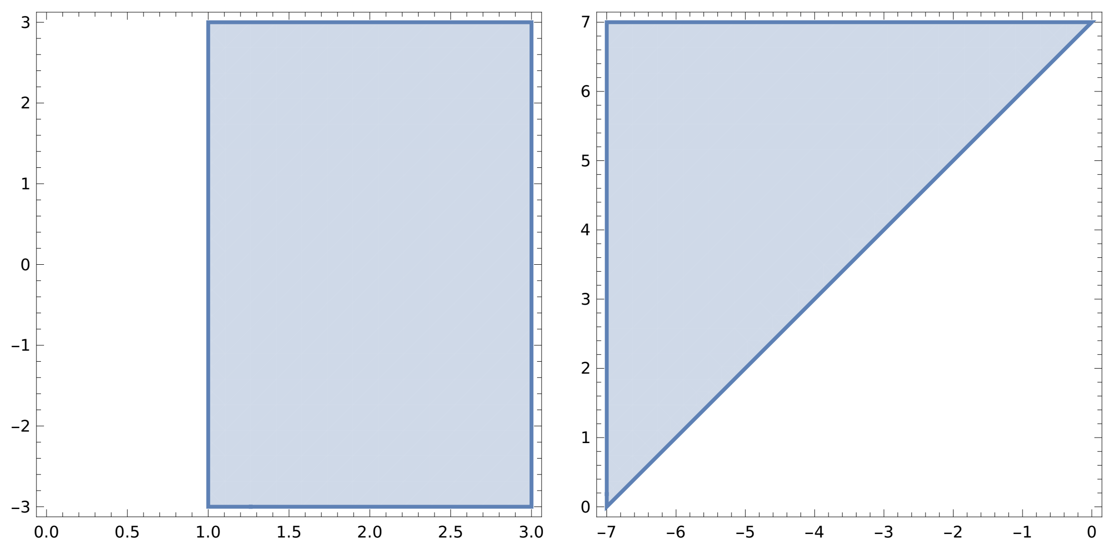
In[21]:=
```
z = x + I * y;
Solve[w1 == (-1 + I) z1 - 2 + 3 * I, z1]
```

Out[22]=

$$\left\{\left\{z1 \to \frac{1}{2}\left(-5 + i\,(1 - w1) - w1\right)\right\}\right\}$$

In[58]:=
```
A1 = RegionPlot[Re[z] > 1, {x, 0, 3}, {y, -3, 3}];
A2 = RegionPlot[Re[(z + 2 - 3 * I) / (-1 + I)] > 1, {x, -7, 0}, {y, 0, 7}];
GraphicsRow[{A1, A2}]
```

Out[60]=



PRACTICAL NO. 6

Objective: Show that the image of the right half plane A = {z : Re z ≥ 1/2} under the mapping w = f(z) = 1/z is the closed disk D(1) = {w : |w – 1| ≤ 1} in the w- plane.

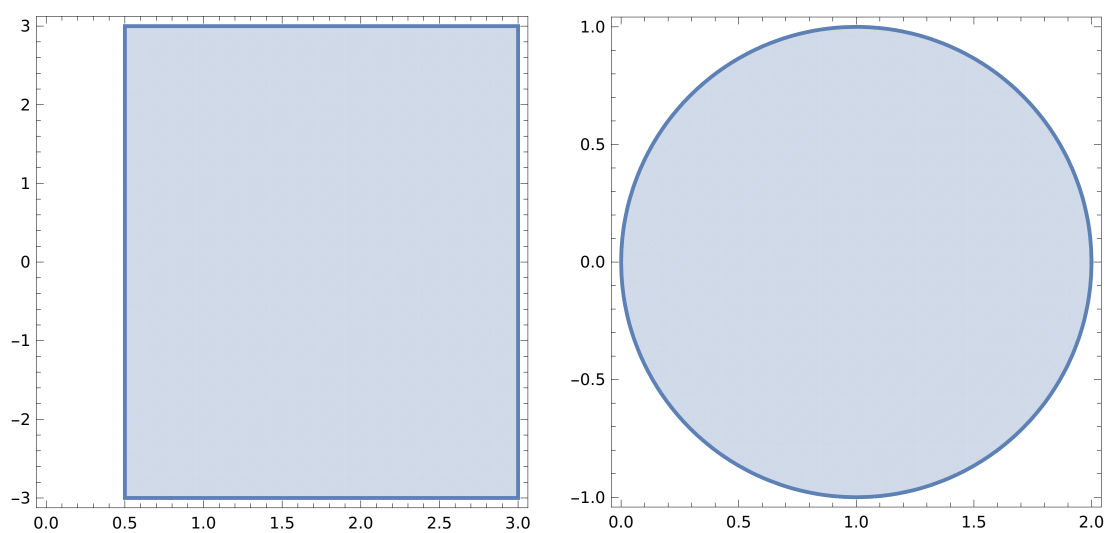
In[26]:=
```
z = x + I * y;
Solve[w1 == 1 / z1, z1]
```

Out[27]=

$$\left\{\left\{z1 \to \frac{1}{w1}\right\}\right\}$$

In[46]:=
```
A1 = RegionPlot[Re[z] ≥ 1 / 2, {x, 0, 3}, {y, -3, 3}];
A2 = RegionPlot[Re[1 / z] ≥ 1 / 2, {x, 0, 2}, {y, -1, 1}];
GraphicsRow[{A1, A2}]
```

Out[48]=



PRACTICAL NO. 7

Objective: Make a plot of the vertical lines x = a, for a = –1,-1/2,1/2 , 1 and the horizontal lines y = b, for b = –1,– 1/2,1/2,1. Find the plot of this grid under the mapping w = f(z) =1/z .

In[68]:=
```
w = 1 / z;
X = ComplexExpand[Re[w]]
Y = ComplexExpand[Im[w]]
```

Out[69]=

$$\frac{x}{x^2 + y^2}$$

Out[70]=

$$-\frac{y}{x^2 + y^2}$$

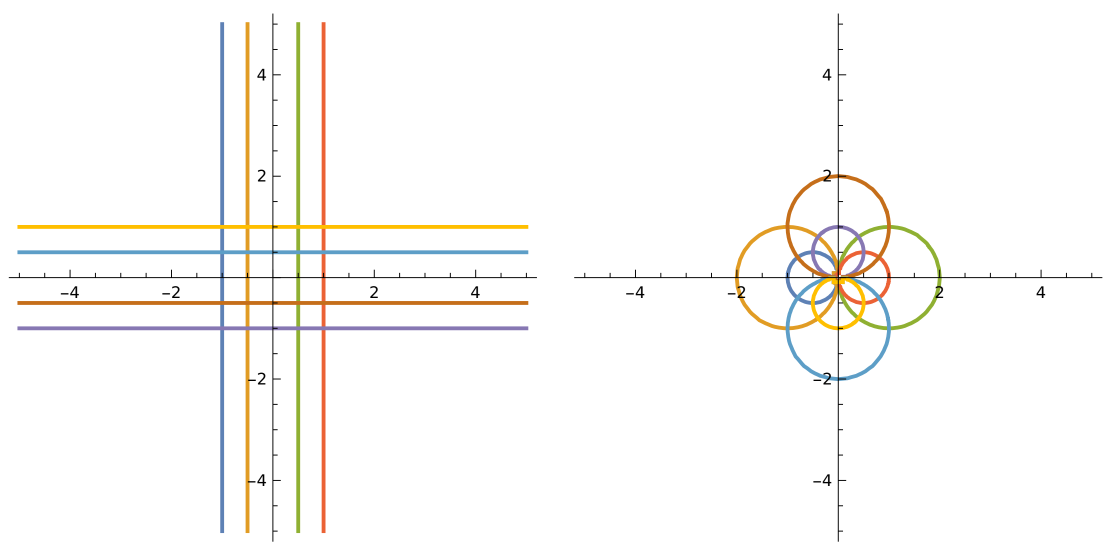
In[71]:=
```
Solve[u == X && v == Y, {x, y}]
```

Out[71]=

$$\left\{\left\{x \to \frac{u}{u^2 + v^2}, y \to -\frac{v}{u^2 + v^2}\right\}\right\}$$

In[81]:=
```
A1 = ContourPlot[{x == -1, x == -1 / 2, x == 1 / 2, x == 1, y == -1, y == -1 / 2,
     y == 1 / 2, y == 1}, {x, -5, 5}, {y, -5, 5}, Axes → True, Frame → False];
A2 = ContourPlot[{u / (u ^ 2 + v ^ 2) == -1, u / (u ^ 2 + v ^ 2) == -1 / 2,
     u / (u ^ 2 + v ^ 2) == 1 / 2, u / (u ^ 2 + v ^ 2) == 1, -v / (u ^ 2 + v ^ 2) == -1,
     -v / (u ^ 2 + v ^ 2) == -1 / 2, -v / (u ^ 2 + v ^ 2) == 1 / 2, -v / (u ^ 2 + v ^ 2) == 1},
   {u, -5, 5}, {v, -5, 5}, Axes → True, Frame → False];
GraphicsRow[{A1, A2}]
```

Out[83]=

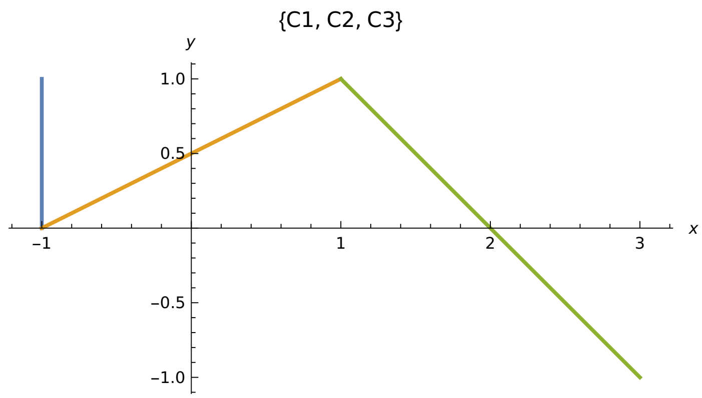


PRACTICAL NO. 8

Objective: Find a parametrization of the polygonal path C = C1 + C2 + C3 from −1 + i to 3 − i, where C1 is the line from: −1 + i to −1, C2 is the line from: −1 to 1 + i and C3 is the line from 1 + i to 3 − i. Make a plot of this path.

In[84]:=
```
C1 = {-1, 1 - t};
C2 = {-1 + 2 * t, t};
C3 = {1 + 2 * t, 1 - 2 * t};
ParametricPlot[{C1, C2, C3}, {t, 0, 1},
 PlotLabel → {"C1", "C2", "C3"}, AxesLabel → {x, y}]
```

Out[87]=

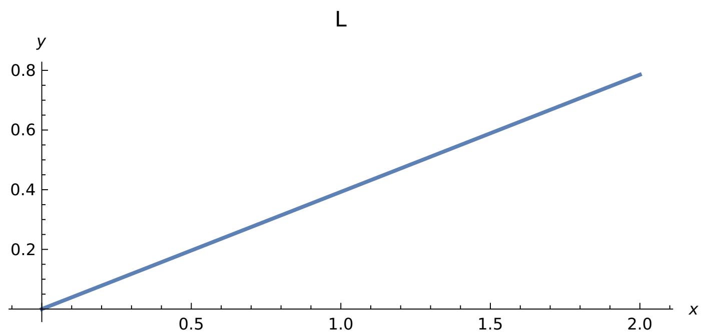


PRACTICAL NO. 9

Objective: Plot the line segment ‘L’ joining the point A = 0 to B = 2 + π/4 i and give an exact calculation of ∫ e^z dz over contour L.

In[94]:=
```
ParametricPlot[{2 * t, (Pi / 4) * t}, {t, 0, 1}, AxesLabel → {x, y}, PlotLabel → "L"]
```

Out[94]=



In[95]:=
```
z = x + I * y;
f[z_] := Exp[z];
g[t_] := (2 + ((Pi * I) / 4)) * t;
Integrate[(f[g[t]] * g'[t]), {t, 0, 1}]
```

Out[98]=

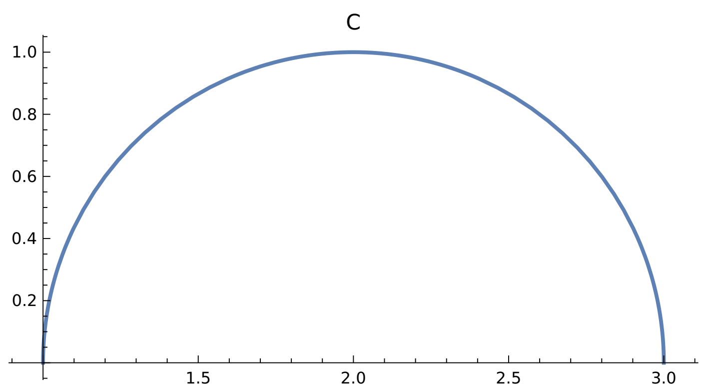
$-1 + (-1)^{1/4} e^2$

PRACTICAL NO. 10

Objective: Plot the semicircle ‘C’ with radius 1 centered at z = 2 and evaluate the contour integral ∫c 1/(z-2) dz .

In[99]:=
```
ParametricPlot[{2 + Cos[t], Sin[t]}, {t, 0, Pi}, PlotLabel → "C"]
```

Out[99]=



In[100]:=
```
z = x + I * y;
f[z_] := 1 / (z - 2);
g[t_] := 2 + Exp[I * t];
Integrate[f[g[t]] * g '[t], {t, 0, Pi}]
```

Out[103]=

$i\pi$

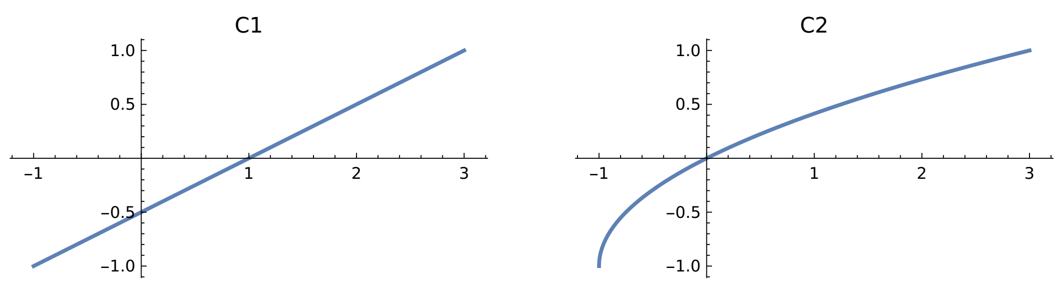
PRACTICAL NO. 11

Objective: Show that $\int_{c1} z\,dz = \int_{c2} z\,dz = 4 + 2i$ where C1 is the line segment from $-1 - i$ to $3 + i$ and C2 is the portion of the parabola $x = y^2 + 2y$ joining $-1 - i$ to $3 + i$. Make plots of two contours C1 and C2 joining $-1 - i$ to $3 + i$.

In[104]:=

```
C1 = ParametricPlot[{2 * t + 1, t}, {t, -1, 1}, PlotLabel → "C1"];
C2 = ParametricPlot[{t ^ 2 + 2 * t, t}, {t, -1, 1}, PlotLabel → "C2"];
GraphicsRow[{C1, C2}]
```

Out[106]=



In[107]:=

```
f[z_] := z;
g[t_] := (2 * t + 1) + I * t;
h[t_] := (t ^ 2 + 2 * t) + I * t;
Integrate[f[g[t]] * g '[t], {t, -1, 1}]
Integrate[f[h[t]] * h '[t], {t, -1, 1}]
```

Out[110]=

$4 + 2 i$

Out[111]=

$4 + 2 i$

PRACTICAL NO. 12

Objective: Use ML inequality to show that |∫c (1/z^2+1) dz | ≤ 1/2(5)^(1/2) , where C is the straight line segment from 2 to 2 + i. While solving, represent the distance from the point z to the points i and – i, respectively, i.e. |z – i| and |z + i| on the complex plane ℂ.

In[113]:=

```
f[z_] := 1 / (z ^ 2 + 1);
g[t_] := 2 + I * t;
c = Integrate[f[g[t]] * g '[t], {t, 0, 1}];
I1 = ComplexExpand[c]
```

Out[116]=

$$\frac{3\pi}{8} - \text{ArcTan}[2] + i\left(-\frac{\text{Log}[2]}{2} + \frac{\text{Log}[8]}{4}\right)$$

In[117]:=

```
N[Abs[I1]]
```

Out[117]=

```
0.187249
```

In[118]:=

```
N[1 / (2 * Sqrt[5])]
```

Out[118]=

```
0.223607
```

In[119]:=

```
N[Abs[I1]] ≤ N[1 / 2 * Sqrt[5]]
```

Out[119]=

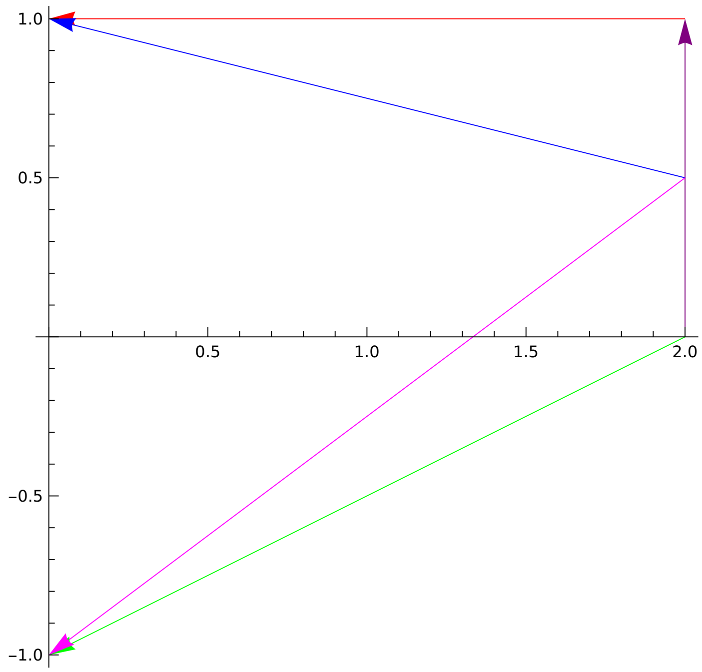
```
True
```

For L =1 and M = 0.5 the equation |∫c (1/z^2+1) dz | ≤ 1/2*(5)^(1/2) | is also satisfied by ML Inequality, which is also verified above.

In[120]:=

```
Show[Graphics[
  {Purple, Arrow[{{2, 0}, {2, 1}}], Red, Arrow[{{2, 1}, {0, 1}}], Green, Arrow[{{2, 0}, {0, -1}}],
   Blue, Arrow[{{2, 1 / 2}, {0, 1}}], Magenta, Arrow[{{2, 1 / 2}, {0, -1}}]}], Axes → True]
```

Out[120]=



PRACTICAL NO. 13

Objective: Show that ∫c dz/(2*z^(1/2)) , where z^(1/2) is the principal branch of the square root function and C is the line segment joining 4 to 8 + 6i. Also plot the path of integration.

In[121]:=

```
f[z_] := 1 / (2 * z ^ (1 / 2));
g[t_] := 4 + (4 + I * 6) * t;
Integrate[f[g[t]] * g '[t], {t, 0, 1}]
```

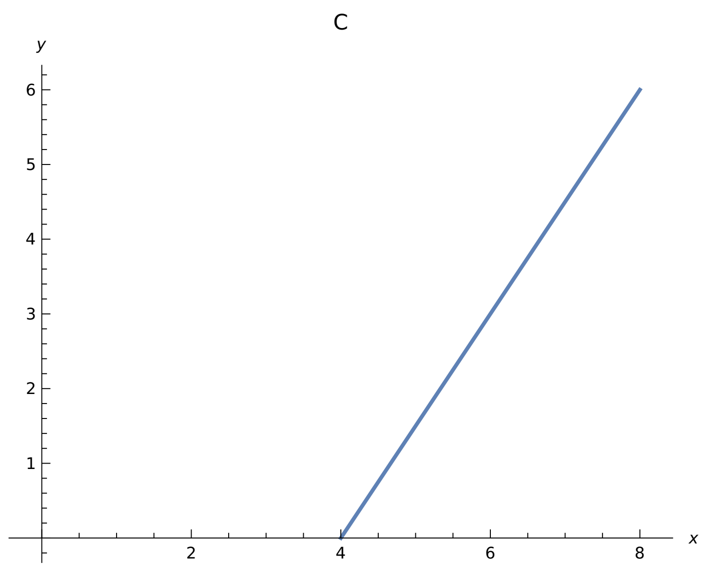
Out[123]=

$1 + i$

In[124]:=

```
ParametricPlot[{4 + 4 * t, 6 * t}, {t, 0, 1},
 AxesOrigin → {0, 0}, AxesLabel → {x, y}, PlotLabel → "C"]
```

Out[124]=



PRACTICAL NO. 14

Objective: Find and plot three different Laurent series representations for the function f(z) =3/(2+z-z^2), involving powers of z.

In[2]:=
```
f[z_] := 3 / (2 + z - z ^ 2);
Solve[Denominator[f[z]] == 0]
```

Out[3]= {{z → -1}, {z → 2}}

In[8]:=
```
n1 = Normal[Series[f[z], {z, -1, 5}]]
```

Out[8]= $\frac{1}{3}+\frac{1}{1+z}+\frac{1+z}{9}+\frac{1}{27}(1+z)^2+\frac{1}{81}(1+z)^3+\frac{1}{243}(1+z)^4+\frac{1}{729}(1+z)^5$

In[9]:=
```
n2 = Normal[Series[f[z], {z, 2, 5}]]
```

Out[9]= $\frac{1}{3}+\frac{2-z}{9}-\frac{1}{-2+z}+\frac{1}{27}(-2+z)^2-\frac{1}{81}(-2+z)^3+\frac{1}{243}(-2+z)^4-\frac{1}{729}(-2+z)^5$

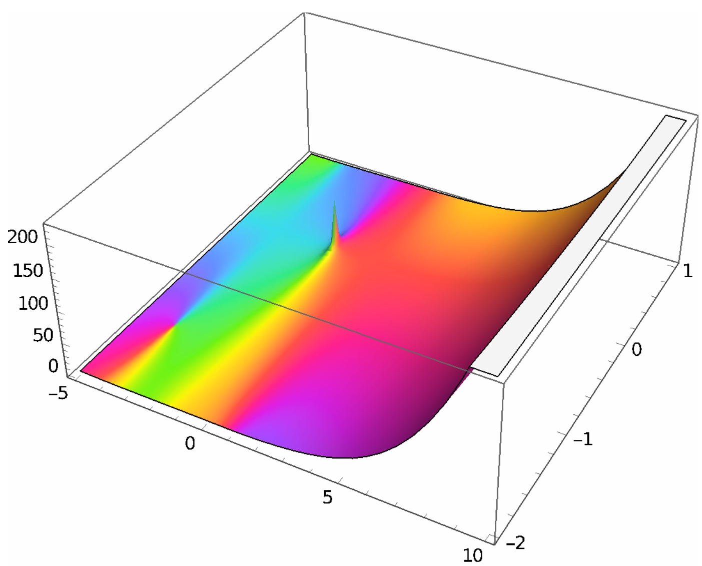
In[10]:=
```
n3 = Normal[Series[f[z], {z, 0, 5}]]
```

Out[10]=

$$\frac{3}{2}-\frac{3z}{4}+\frac{9z^2}{8}-\frac{15z^3}{16}+\frac{33z^4}{32}-\frac{63z^5}{64}$$

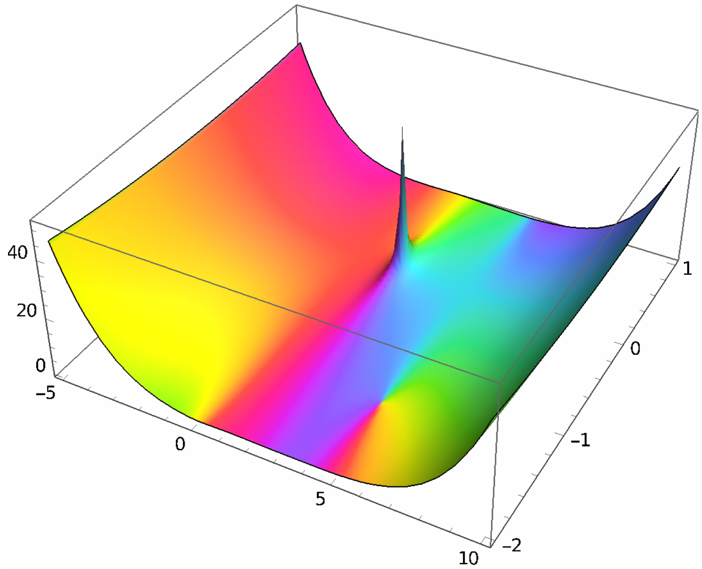
In[11]:=
```
ComplexPlot3D[n1, {z, -5 + I, 10 - 2 * I}]
```

Out[11]=



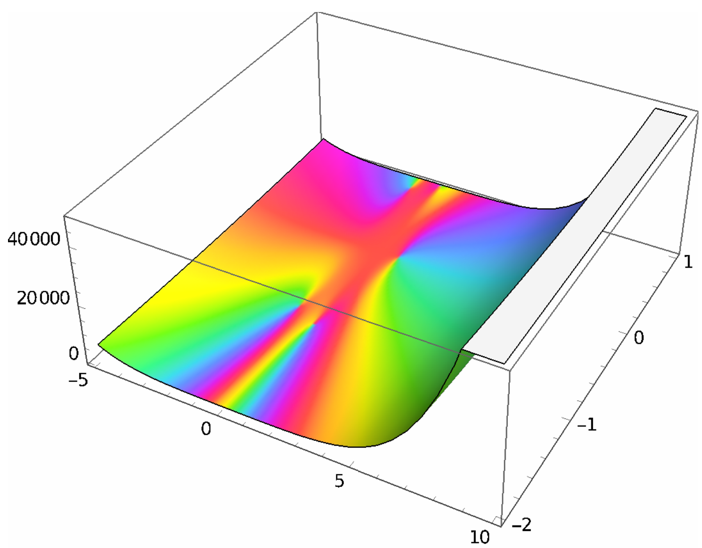
In[12]:= `ComplexPlot3D[n2, {z, -5 + I, 10 - 2 * I}]`

Out[12]=



In[13]:= `ComplexPlot3D[n3, {z, -5 + I, 10 - 2 * I}]`

Out[13]=



PRACTICAL NO. 15

Objective: Locate the poles of f(z) = 1/(5z^4+26z^2+5) and specify their order.

In[33]:=
```
f[z_] := 1 / (5 * z ^ 4 + 26 * z ^ 2 + 5);
```

In[34]:=
```
FunctionPoles[f[z], z]
```

Out[34]=

$$\left\{\left\{-\frac{i}{\sqrt{5}}, 1\right\}, \left\{\frac{i}{\sqrt{5}}, 1\right\}, \left\{-i\sqrt{5}, 1\right\}, \left\{i\sqrt{5}, 1\right\}\right\}$$

- **Hence,Poles are -i, i, -i 5, i 5 and their order are 1 for all poles.**

PRACTICAL NO. 16

Objective: Locate the zeros and poles of g(z) =(π cot(π z))/z^2 and determine their order. Also justify that Res(g,0) = -π^2/3.

In[62]:=
```
g[z_] := Pi * Cot[Pi * z] / z ^ 2;
```

In[56]:=
```
FunctionPoles[g[z], z]
```

Out[56]=

$$\left\{\{0, 3\}, \left\{2\, c_1 \text{ if } c_1 \in \mathbb{Z}, 1\right\}, \left\{\frac{\pi + 2\ \pi\, c_1}{\pi} \text{ if } c_1 \in \mathbb{Z}, 1\right\}\right\}$$

In[57]:=
```
Residue[g[z], {z, 0}]
```

Out[57]=

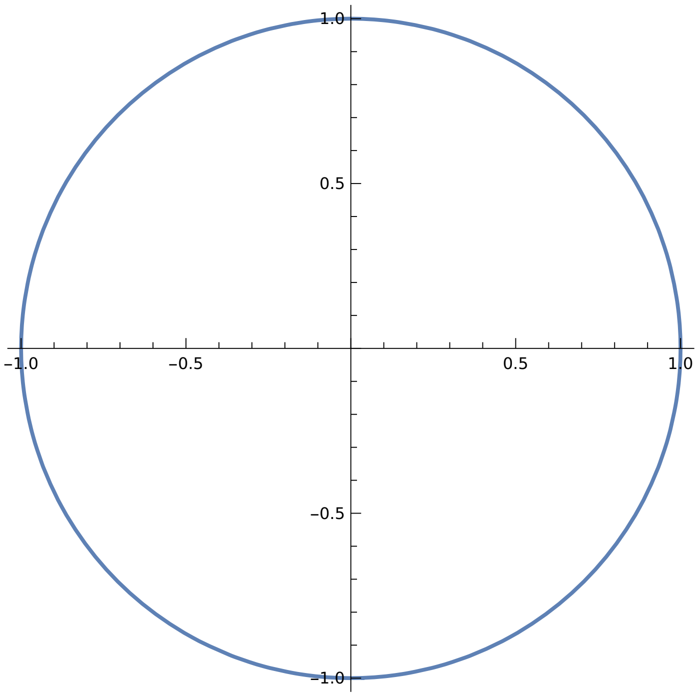
$$-\frac{\pi^2}{3}$$

PRACTICAL NO. 17

Objective: Evaluate ∫C1(0) exp (2/z) dz, where C1(0) denotes the circle {z : |z| = 1} with positive orientation. Similarly evaluate ∫C1(0) 1/(z^4+z^3-2z^2) dz.

In[57]:=
```
c1 = ContourPlot[x^2 + y^2 == 1, {x, -1, 1}, {y, -1, 1}, Axes → True, Frame → False]
```

Out[57]=



In[2]:=
```
g[z_] := Exp[2 / z];
```

In[11]:=
```
Residue[g[z], {z, 0}]
```

Out[11]=

2

In[16]:=
```
I1 = 2 * Pi * I * Residue[g[z], {z, 0}]
```

Out[16]=

$4 i \pi$

Clearly, the integrand is analytic inside and on the contour C1 except at point $z = 0$. Thus by residue theorem, we have **residue of g[z] at z=0 is 2.**

Hence, the integral is, ∫c1+(0) exp (2/z) dz = 4 i π.

In[40]:=
```
f[z_] := 1 / (z^4 + z^3 - 2 * z^2);
```

In[44]:=
```
Solve[Denominator[f[z]] == 0, z]
```

Out[44]=

{{z → -2}, {z → 0}, {z → 0}, {z → 1}}

In[46]:=
```
Residue[f[z], {z, 0}]
```
Out[46]=

$$-\frac{1}{4}$$

In[48]:=
```
Residue[f[z], {z, 1}]
```
Out[48]=

$$\frac{1}{3}$$

In[50]:=
```
I2 = 2 * Pi * I * (Residue[f[z], {z, 0}] + Residue[f[z], {z, 1}])
```
Out[50]=

$$\frac{i \pi}{6}$$

Clearly, the integrand is analytic inside and on the contour C1 except at point z = 0 and z = 1. Thus by residue theorem, we have **residue of f[z] at z = 0 is -1/4 and at z = 1 is 1/3.**
Hence, the integral is, ∫c1+(0) 1/(z^4+z^3-2z^2) dz.= I2= $i\pi$/6 .