Derivatives HW 4

Please hand-write your solutions and submit a PDF file named
`LastName_FirstName_HW4.pdf`

Due Date: October 15th, 3:30pm

1 FRA

1.1

The term structure of interest rates is upward sloping. Put the following in order of magnitude:

1. The **7-year zero rate**
2. The **yield** on a **7-year coupon-bearing bond**
3. The **forward rate** for the period between **6.5 and 7.0 years** in the future

What about when the term structure of interest rates is downward sloping? Provide a detailed explanation.

1.2

Suppose that risk-free zero interest rates with continuous compounding are as follows:

Maturity (months)	**Rate** (% per annum)
3	2.8
6	3.1
9	3.5
12	3.7
15	3.9
18	4.1

1. Calculate forward interest rates for the second, third, fourth, fifth, and sixth quarters.
2. Assuming that SOFR rates are as in the table, what is the value of an FRA where the holder will pay SOFR and receive 4.9% (quarterly compounded) for a three-month period starting in one year on a principal of $1,000,000?

2 Interest Rate Swaps

2.1

A dealer receives fixed and pays 3-month SOFR on an interest rate swap with **10 months** remaining and **quarterly** payments on a notional of $150 million. The fixed rate is **2.25**% per annum (quarterly paid). Continuously compounded zero rates are:

$$R_{1\mathrm{m}} = 1.20\%, \quad R_{4\mathrm{m}} = 1.50\%, \quad R_{7\mathrm{m}} = 1.70\%, \quad R_{10\mathrm{m}} = 1.90\%.$$

The realized continuously compounded short rate for the past two months is **0.90**%. Value the swap to the *fixed-rate receiver* by expressing it as a strip of forward rate agreements (FRAs).

2.2

Two companies are considering borrowing $20 million for five years. They have received the following quotes:

Table 1: *
Quoted Borrowing Rates (per annum)

	Fixed rate	**Floating rate**
Company A	5.0%	SOFR + 0.1%
Company B	6.4%	SOFR + 0.6%

Company A prefers to borrow at a *floating* rate; Company B prefers to borrow at a *fixed* rate. Design a fixed-for-floating interest rate swap (no intermediary spread) that splits the 0.9% total comparative advantage *equally* between Companies A and B.

1. State who should borrow fixed and who should borrow floating in the cash market before entering the swap, and explain why.
2. Design the swap so that the comparative-advantage benefit is split 50/50. (Hint: each firm should end up 0.45% per annum better off than its best direct quote..Why?)
3. Report the final terms of the swap and each firm's resulting effective borrowing rate.

2.3 Plain Vanilla Swap

Given below is a table with spot rates given. Assume you are to pay the floating and receive the fixed interest rate. The swap is 4 years, the notional is $10million, and the payments are made annually. Note that the implied forward rate

for year 1 is equal to the 1-year spot rate. Compute the Implied Forward Rates, Discount Factors, etc. that are necessary to compute the swap rate. After doing so, ensure that the sum of the discounted expected cash flows are approximately = zero.

Year	Receive Fixed Swap Rate	Annual Spot Interest Rate	Pay Floating– Implied Forward Rate
1		5.3866%	
2		5.4047%	
3		5.4136%	
4		5.4225%	

Year	Discount Factor	Disc. Implied FwdRate	Expected CF	Disc. Expected CF
1				
2				
3				
4				
sum				
swap rate =				

Formula Sheet

Implied Forward Rate

$$f_{t_1,t_2} = \left(\frac{(1+r_{t_2})^{t_2}}{(1+r_{t_1})^{t_1}} \right)^{\frac{1}{t_2-t_1}} - 1 \tag{1}$$

Discount Factor

$$DF_t = (1+r_t)^{-t} \tag{2}$$

Discounted Implied Forward Rate

$$\text{Disc. IFR}_{t_i,t_{i+1}} = DF_t \times IFR_{t_i,t_{i+1}} \tag{3}$$

Fixed Swap Rate

$$\text{Fixed Swap Rate} = \frac{\sum \text{Disc. IFR}_t}{\sum DF_t} \tag{4}$$

Expected Net Cash Flow

$$\mathbb{E}[\text{Net Cash Flow}] = N \times (\text{Fixed Swap Rate} - IFR_t) \tag{5}$$

Discounted Expected Net Cash Flow

$$\text{Disc. } \mathbb{E}[\text{Net CF}] = DF_t \times \mathbb{E}[\text{Net CF}] \tag{6}$$

Summing the discounted expected net cash flows should be approximately zero:

$$\sum_t \text{Disc. } \mathbb{E}[\text{Net CF}] \approx \emptyset$$