Derivatives HW 3

Please hand-write your solutions and submit a PDF file named
`LastName_FirstName_HW3.pdf`

Due Date: October 7th, 3:30pm

1 Interest Rate Futures

1.1 CTD Bond

Suppose that the Treasury bond futures price is 102-15. Which of the following four bonds is cheapest to deliver?

Bond	*Price*	*Conversion factor*
1	124-15	1.2031
2	140-24	1.3643
3	117-31	1.1341
4	138-02	1.2726

1.2 Conversion Factor & Futures Invoice

Consider a deliverable candidate for a 10y treasury note futures contract:

- Futures: September TNote, settlement price: 118-20
- Deliverable Bond: T 4.00%
- Timing assumptions: delivery occurs on a coupon date
- Remaining time to maturity at delivery: 2.5 years
- Conversion Factor convention: discount all cash flows at 6% p.a., semiannual

1. Compute conversion factor
2. Using computed conversion factor, compute the converted futures price per $100,000
3. What is the invoice amount the buyer pays?

1.3 Dirty Price, YTM, Duration, Price change

For a TBond that has 0.1 years to maturity, an annual coupon rate of 3%, and a clean price of 98. Assume continuous compounding. Determine:

1. Dirty Price

2. Yield to Maturity
3. Duration
4. Percentage change in price assuming yields fall by 0.01

1.4 Duration Hedge

On August 1, a portfolio manager has a bond portfolio worth $10 million. The duration of the portfolio in October will be 6.9 years. The December Treasury bond futures price is currently 91-12 and the cheapest-to-deliver bond will have a duration of 8.6 years at maturity.

1. Compute the number of futures contract the PM should long/short to immunize the portfolio against changes in interest rates over the next 2 months
2. Explain why the portfolio manager should long/short the futures contract to have an effective hedge.

1.5 Cheapest-to-Deliver (CTD) for TYZ5 10-Year U.S. Treasury Note Futures

Objective. Using Bloomberg and Python, pull the deliverable bond basket for the December 2025 10-Year U.S. Treasury Note futures contract (`TYZ5 Comdty`) and determine the cheapest-to-deliver (CTD) security. You must *compute the conversion factor (CF) yourself* for each candidate, then use it to evaluate delivery cost.

Data Retrieval (Bloomberg Excel API)

1. Using the Excel Bloomberg add-in, run:

   ```
   =BDS("TYZ5 Comdty","FUT_DELIVERABLE_BONDS")
   ```

   This returns CUSIPs/maturities, coupons, and reference fields for all bonds eligible for delivery into `TYZ5 Comdty`.

2. For each deliverable bond, pull its *clean price* $P_{\text{clean},0}$ and *accrued interest today* AI_0 (e.g., `PX_LAST` and `ACCRUED_INTEREST`).
3. Record the futures price F for `TYZ5 Comdty`.

Assumptions

- **Settlement/Delivery date:** use the *first calendar day* of the delivery month, i.e., 1 Dec 2025. Denote it t_D.
- **Coupon/Day count:** U.S. Treasuries pay semiannual coupons; use ACT/ACT (Treasury) for accrued interest and time fractions.

- **Yield for CF:** Using CBOT convention, conversion factors are defined using a yield of 6% with *semiannual compounding.*
- **Prices:** Distinguish clean vs. dirty consistently. Dirty price at time t is $P_{\text{dirty},t} = P_{\text{clean},t} + \text{AI}_t$.

Task A — Compute the Conversion Factor (you may use Python)

For a deliverable bond with annual coupon rate c (in %), semiannual coupon $C = \frac{c}{2} \times 100$, maturity at t_M, and number of remaining semiannual periods N as of t_D, let the next coupon date after t_D be t_1. Define $k = 1, \ldots, N$ as the semiannual cashflow indices after t_D.

Step A1: Accrual at delivery. Compute accrued interest at the delivery date,

$$\text{AI}_D = \frac{\text{Days from last coupon to } t_D}{\text{Days in coupon period}} \times C.$$

Step A2: Present value at 6% (semiannual). Discount all remaining cashflows from t_D at $y = 3\%$ per half-year:

$$\text{PV}_{6\%} = \sum_{k=1}^{N} \frac{C}{(1+y)^k} + \frac{100}{(1+y)^N}.$$

Step A3: Conversion factor. The conversion factor is the dirty price at a 6% yield, normalized by \$100 par and then adjusted by accrued interest at delivery:

$$\boxed{\text{CF} \;=\; \frac{\text{PV}_{6\%} - \text{AI}_D}{100}}$$

Implementation note: You will need to (i) build the semiannual cashflow schedule from t_D to t_M, (ii) compute ACT/ACT accrual to get AI_D, and (iii) count N correctly when a coupon date falls exactly on t_D.

Task B — Compute Delivery Economics and Identify CTD

1. Compute the **invoice price** if you deliver the bond on t_D:

$$\text{Invoice} \;=\; F \times \text{CF} \;+\; \text{AI}_D.$$

2. Using today's observed bond price, compute the **net basis** (use dirty prices for consistency):

$$\text{Net Basis} \;=\; P_{\text{dirty},0} \;-\; \big(F \times \text{CF} + \text{AI}_D\big).$$

Decision rule: The CTD is typically the bond with the *lowest net basis*, subject to consistent use of dirty prices and the same t_D and conventions across candidates.

Deliverables

1. A table listing, for each deliverable bond: CUSIP/maturity, coupon, your computed CF, $P_{\text{clean},0}$, AI_0, $P_{\text{dirty},0}$, AI_D, invoice price, and net basis.
2. Justification of selected CTD Bond.