Interest Rate Derivatives, Homework #4, SOFR Yield Curve Construction

I will be posting a SOFR swap class as well.

PART I: SOFR Forwards Build Via Convexity Adjustment

Note: This is speced out in the Excel spreadsheet uploaded to Blackboard. All functionalities should be modularized. Ideally every functionality lives in a class. Think of how you would write this code if you wanted to run it every day for 6 months.

1) Make a table of all 3 month SOFR futures going out 4 years. Should be 16 futures. List futures rates (=100-price).

2) Add start date and end date for each futures to the table. You can use the function in DateUtilities.py:

```python
def Convert_CBOT_Month_Code_to_IMM_Date(month_code: str, year: int) -> datetime.date:
```

Using these dates, compute time to start, T=(contract_start – curve_date)/365.25

3) Fix a volatility σ (around 70BPs or so, to be solved for). Use the Convexity Adjustment Formula to convert futures rates to forward rates

$$CA = \frac{\sigma^2 T_1 T_2}{2}$$

$$F^{fut} = F^{fwd} + CA$$

4) Create a table of cumulative discount factors for the IMM dates (SOFR futures start/end dates) out to 4 years. Do this by discounting by the SOFR forward rates using Money Market/LIBOR rate convention.

5) Create a table of discount factors for every date from curve_date out to 4 years using 'log linear' discounting. That is, we take the log of the discount factors in 4) and linearly interpolate dates in between. This is equivalent to fixed forwards between IMM dates (HW exercise!).

6) Exponentiate each log DF to get a SOFR discount factor for every day.

7) From daily discount factors, create a daily SOFR rate (using MM convention).

8) Create an OIS curve by taking the daily SOFR rate and subtracting 6BP.

9) Using the two discount curves, price the 2Y,3Y,4Y SOFR/OIS swaps. SOFR is used for floating rates and OIS is used for discounting. **The two curves will have to be in a single yield curve class in order to interact with the sofr swap**

class! The constructor of a swap requires a yield curve as input and is used in pricing of the swap.

10) Compare price of floating leg with fixed leg with market coupon. This produces 3 errors, one for each of 2Y,3Y,4Y.

11) Solve for volatility until the sum square of errors is minimized.

PART I: SOFR Forwards Build Incorporating FF Build

1) Make a table of FOMC meeting dates and market-implied FFER.

2) Add to the table spreads of SOFR to Fed Funds. These will be solved for, but initially set them all to 6 basis points.

3) Add the spread to get a series of SOFR rates. These are SOFR rates that will prevail between FOMC meetings.

4) Build a series of curves with values for every date going out 1Y+

   - 1 For each date, find the market-implied FFER

   - 2 Then compute the 1-day discount factor using MM convention

- 3 Then compute the cumulative discount factor for every date

- 4 Then find the daily SOFR/FF spread (in step 2)

- 5 Add this spread to the market-implied FFER to get a daily SOFR rate

- 6 Then use this rate to get a daily SOFR discount factor using MM convention

- 7 Then compute the cumulative discount factor for every date

This gives you your new SOFR curve out to 1Y+

5) Price back ZCB yields

For each IMM date (start/end dates for SOFR forwards),

- 1 Compute time to date

- 2 Compute the DF from the original SOFR curve build

- 3 Using time, compute the ZCB yield from original SOFR curve

- 4 Now do the same for the new FF&SOFR integrated curve

- 5 Compute the error between the original ZCB yield and the new

6) We solve for the SOFR/FF spread in 2) using an objective function:

$$OBJ\ FN = \sum_i ERR^2_{ZCB,i} + \frac{1}{10}\sum_i(SOFR/FF_i - SOFR/FF_{i-1})^2$$