Logical Syntax, Semantics & Rules of Inference

Comprehensive reference: symbols, quantifiers, precedence, WFFs, truth tables, semantics (tautology, contradiction, entailment, etc.), and key rules of inference.

1. Basic Logical Symbols

¬P	Negation
P ∧ Q	Conjunction
P ∨ Q	Disjunction (inclusive)
P → Q	Implication
P ↔ Q	Biconditional

2. Quantifiers

∀x P(x)	Universal quantifier: for all x, P(x)
∃x P(x)	Existential quantifier: there exists x such that P(x)
∃!x P(x)	Unique existence quantifier: exactly one x with P(x)

3. Order of Precedence

1	Parentheses (...)
2	Negation ¬
3	Quantifiers ∀, ∃, ∃!
4	Conjunction ∧
5	Disjunction ∨
6	Implication →
7	Biconditional ↔

4. Well-Formed Formulas (WFFs)

Terms: variables, constants, or functions of terms.
Atomic formulas: predicate applied to terms.
Formulas: built from atomic formulas, connectives, quantifiers.
Free variable: not bound by a quantifier.
Bound variable: inside scope of quantifier.
Sentence: formula with no free variables.

5. Truth Tables

P	Q	¬P	P ∧ Q	P ∨ Q	P → Q	P ↔ Q
T	T	F	T	T	T	T
T	F	F	F	T	F	F

F	T	T	F	T	T	F
F	F	T	F	F	T	T

6. Semantic Notions

Tautology: true under every assignment.
Contradiction: false under every assignment.
Contingent: true sometimes, false sometimes.
Satisfiable: true under some assignment.
Unsatisfiable: false under all assignments.
Logical equivalence: $\phi \equiv \psi$ if $\phi \leftrightarrow \psi$ is a tautology.
Entailment: Γ ■ ϕ if every assignment making Γ true also makes ϕ true.
Valid argument: premises ■ conclusion.

7. Rules of Inference

Modus Ponens	From P, $P \rightarrow Q$, infer Q
Modus Tollens	From $\neg Q$, $P \rightarrow Q$, infer $\neg P$
Hypothetical Syllogism	From $P \rightarrow Q$, $Q \rightarrow R$, infer $P \rightarrow R$
Disjunctive Syllogism	From $P \vee Q$, $\neg P$, infer Q
Addition	From P, infer $P \vee Q$
Simplification	From $P \wedge Q$, infer P
Conjunction	From P, Q, infer $P \wedge Q$
Resolution	From $P \vee Q$, $\neg P \vee R$, infer $Q \vee R$

8. Rules with Quantifiers

Universal Instantiation (UI)	From $\forall x\ \phi(x)$, infer $\phi(c)$ for any constant c
Universal Generalization (UG)	From $\phi(x)$ (arbitrary x), infer $\forall x\ \phi(x)$
Existential Instantiation (EI)	From $\exists x\ \phi(x)$, infer $\phi(c)$ for some fresh constant c
Existential Generalization (EG)	From $\phi(c)$, infer $\exists x\ \phi(x)$