Logical Syntax, Semantics & Rules of Inference

Reference covering syntax, semantics, and formal rules of inference (with inference rule notation).

1. Propositional Rules of Inference

Modus Ponens	$P \to Q,\ P \blacksquare Q$
Modus Tollens	$P \to Q,\ \neg Q \blacksquare \neg P$
Hypothetical Syllogism	$P \to Q,\ Q \to R \blacksquare P \to R$
Disjunctive Syllogism	$P \vee Q,\ \neg P \blacksquare Q$
Addition	$P \blacksquare P \vee Q$
Simplification	$P \wedge Q \blacksquare P$
Conjunction	$P,\ Q \blacksquare P \wedge Q$
Resolution	$P \vee Q,\ \neg P \vee R \blacksquare Q \vee R$

2. Quantifier Rules of Inference

Universal Instantiation (UI)	$\forall x\ P(x) \blacksquare P(c)$
Universal Generalization (UG)	$P(x) \blacksquare \forall x\ P(x)$
Existential Instantiation (EI)	$\exists x\ P(x) \blacksquare P(c)$ [c new constant]
Existential Generalization (EG)	$P(c) \blacksquare \exists x\ P(x)$

3. Inference Rule Notation

Rules of inference are often written with a horizontal line:

Example (Modus Ponens):

$$\frac{P \to Q,\ P}{Q}$$

The statements above the line are premises; the statement below is the conclusion.